```python
In [1]: import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from scipy.optimize import minimize
from abc import ABC, abstractmethod
import matplotlib as mpl
```

Model fitting and Pavlovian biases. Pavlovian-instrumental interactions

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Download the dataset .csv from Slack. It contains the data of 10 subjects (see column "ID" for the subject identifier), performing a go/no-go task, each for 600 trials. The column "cue" informs you about the presented trial type (see the "cue mapping" variable in our template). The column "pressed" contains the response of the participant (0 is no-go, 1 is go) and "outcome" contains whether a reward was delivered (1), nothing was delivered (0), a punishment was given (-1).

Part 1

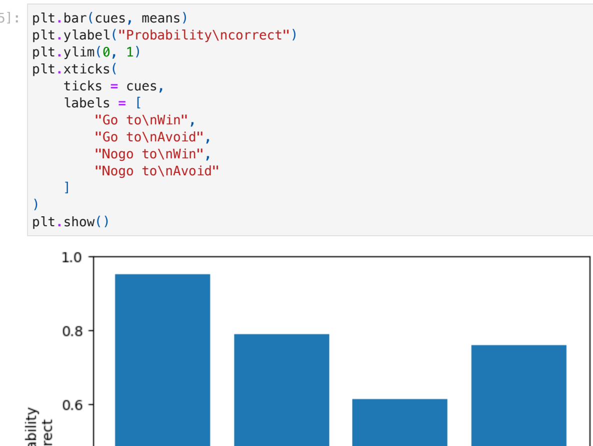
Recreate figure 2E of the paper "Go and no-go learning in reward and punishment: Interactions between affect and effect" with the data you have. Only the bar plots are important here, no need for error bars or significance tests.

```python
In [2]: data = pd.read_csv("gen_data.csv")

# Go+ = Go to win
# Go- = go to avoid losing
# NoGo+ = don't go to win
# NoGo- = don't go to avoid losing
cue_mapping = { 1: "Go+", 2: "Go-", 3: "NoGo+", 4: "NoGo-" }

data["CueFactor"] = [cue_mapping.get(cue) for cue in data.cue]
data["CueFactor"] = data["CueFactor"].astype("category")
```

```python
In [3]: # Determine if each trial is correct or not
data["Correct"] = False
for i, trial in data.iterrows():
    if trial.CueFactor == "Go+" or trial.CueFactor == "Go-":
        data.loc[i, "Correct"] = trial.pressed == 1

    if trial.CueFactor == "NoGo+" or trial.CueFactor == "NoGo-":
        data.loc[i, "Correct"] = trial.pressed != 1

# Compute the average of correct values for each cue
cue_means = data.groupby(["ID", "CueFactor"], observed = False)["Correct"].mean()
```

```python
In [4]: means = []
cues = []
for cue in data.CueFactor.unique():
    cue_data = cue_means.xs(cue, level = "CueFactor")
    means.append(np.mean(cue_data))
    cues.append(cue)
```

```python
In [5]: plt.bar(cues, means)
plt.ylabel("Probability\ncorrect")
plt.ylim(0, 1)
plt.xticks(
    ticks = cues,
    labels = [
        "Go to\nWin",
        "Go to\nAvoid",
        "Nogo to\nWin",
        "Nogo to\nAvoid"
    ]
)
plt.show()
```



Part 2

Program the log likelihood functions of the models 1 to 7 (including) presented in "Disentangling the Roles of Approach, Activation and Valence in Instrumental and Pavlovian Responding" (see Table 2 of that paper for the model numbering and relevant parameters). The paper uses these parameters

- learning rate ϵ
- feedback sensitivity β – the general feedback sensitivity β can be replaced by separate reward and punishment sensitivities ρ (we don't include a sensitivity for omission)
- there can be different learning rates ϵ for reward, feedback omission, and punishment (the paper doesn't make use of omissions, so they use only two learning rates, you will need three)
- there can be a general bias to approach $bias_{app}$, and a general bias to withhold responding $bias_{wth}$

General model

s_t^T - the instrumental stimulus, presented at trial t (one out of four: Go+, Go-, NoGo+, and NoGo-).

a_t - the action (choice) at trial t. The action can be either go (1) or no-go (0).

r_t - the reinforcement obtained, $r_t \in \{-1, 0, 1\}$ where -1 marks a punishment, 0 marks no reinforcement (feedback omission), and -1 marks a reward.

The probability of action a_t in the presence of stimulus s_t^T is a standard probabilistic function:

$$p(a_t|s_t^T) = \frac{exp(\mathcal{W}^T(s_t^T, a_t))}{\sum_{a'} exp(\mathcal{W}^T(s_t^T, a'))}$$

Here, $\mathcal{W}^T$ is the instrumental weight of action a_t:

$$\mathcal{W}^T(s_t^T, a_t) = \mathcal{Q}(s_t^T, a_t) + b(a_t)$$

The variable $b(a_t)$ can take on value $bias_{wth}$ for withhold actions, or $bias_{app}$ for approach actions.

The Q-values are updated according to a Rescorla-Wagner-like rule with a fixed learning rate ϵ. The immediate, intrinsic, value of the reinforcements may have a different meaning for different subjects. To measure this effect, two further parameters are added: ρ_{rew} for the reward sensitivity and ρ_{pun} for the punishment sensitivity. Update equation for the expectations is thus:

$$\mathcal{Q}_{t+1}(s_t^T, a_t) = \mathcal{Q}_t(s_t^T, a_t) + \epsilon(\mathcal{R}_t - \mathcal{Q}_t(s_t^T, a_t))$$

$$\mathcal{R}_t = \begin{cases} \rho_{rew} & \text{if } r_t > 0 \\ \rho_{pun} & \text{if } r_t < 0 \end{cases}$$

```python
In [6]: class Model(ABC):
    def __init__(self, initial):
        # Init model fit bounds
        self.epsilon_bounds = (0.0000001, 0.99999)
        self.beta_bounds = (0.0001, 9.9999)
        self.bias_bounds = (-0.99999, 0.99999)
        self.p_bounds = (0.0001, 0.9999)
        self.initial = initial

    @staticmethod
    def softmax(x):
        exp = np.exp(x)
        return exp / np.sum(exp)

    @staticmethod
    def rescorla_wagner(q_val, epsilon_rew, epsilon_pun, epsilon_omi, reward):
        if reward > 0: return q_val + epsilon_rew * (reward - q_val)
        if reward < 0: return q_val + epsilon_pun * (reward - q_val)
        return q_val + epsilon_omi * (reward - q_val)

    @staticmethod
    def reward(r_t, rho_rew, rho_pun):
        if r_t > 0: return rho_rew
        if r_t < 0: return rho_pun
        return 0

    def log_likelihood(self, cues, actions, rewards, epsilon_rew, epsilon_pun, epsilon_omi, rho_rew
        n_stimuli = len(set(cues))
        n_actions = len(set(actions))

        q_vals = np.zeros((n_stimuli, n_actions))

        log_likelihood = 0

        for t, a_t in enumerate(actions):
            s_t = cues[t] - 1
            r_t = self.reward(rewards[t], rho_rew, rho_pun)

            qs = q_vals[s_t] + [ bias_wth, bias_app ]

            probs = self.softmax(qs)
            log_likelihood += np.log(probs[a_t])

            # Update the Q-values using Rescorla-Wagner
            q_vals[s_t, a_t] = self.rescorla_wagner(
                q_val = q_vals[s_t, a_t],
                epsilon_rew = epsilon_rew,
                epsilon_pun = epsilon_pun,
                epsilon_omi = epsilon_omi,
                reward = r_t
            )

        return log_likelihood

    @abstractmethod
    def loss(self, params, cues, actions, rewards):
        pass

    @abstractmethod
    def minimize_loss(self, cues, actions, rewards, initial = None):
        pass

    def fit(self, data, initial = None):
        fit_result = []

        for subject_id in data.ID.unique():
            subject_data = data[ data.ID == subject_id ]
            subject_data = subject_data.reset_index(drop = True)

            cues = subject_data.cue.tolist()
            actions = subject_data.pressed.tolist()
            rewards = subject_data.outcome.tolist()

            loss, x = self.minimize_loss(cues, actions, rewards, initial)

            x["ID"] = subject_id
            x["loss"] = loss
            fit_result.append(x)

        fit_result = pd.concat(fit_result)
        fit_result.reset_index(drop = True, inplace = True)

        return fit_result
```

Model 1

Model 1 assumes that $-\rho_{pun} = \rho_{rew} = \beta$ and that $bias_{wth} = bias_{app} = 0$.

```python
In [7]: class Model1(Model):
    def __init__(self, initial = None):
        if initial is None:
            initial = [0.5, 5]
        super().__init__(initial = initial)

    def loss(self, params, cues, actions, rewards):
        epsilon, beta = params
        return -self.log_likelihood(
            cues = cues,
            actions = actions,
            rewards = rewards,
            epsilon_rew = epsilon,
            epsilon_pun = epsilon,
            epsilon_omi = epsilon,
            rho_rew = beta,
            rho_pun = -beta,
            bias_wth = 0,
            bias_app = 0
        )

    def minimize_loss(self, cues, actions, rewards, initial = None):
        if initial is None:
            initial = self.initial

        result = minimize(
            fun = self.loss,
            x0 = initial,
            bounds = [self.epsilon_bounds, self.beta_bounds],
            args = (cues, actions, rewards),
            method = "Nelder-Mead"
        )

        fit_params = pd.DataFrame([result.x])
        fit_params.columns = ["epsilon", "beta"]

        return result.fun, fit_params
```

Model 2

Model 2 includes separate reward and punishment sensitivities ρ_{rew} and ρ_{pun} with no action bias

[illegible]

```python
In [10]: class Model4(Model):
    def __init__(self, initial = None):
        if initial is None:
            initial = [0.5, 5, 0, 0]
        super().__init__(initial = initial)

    def loss(self, params, cues, actions, rewards):
        epsilon, beta, bias_app, bias_wth = params
        return -self.log_likelihood(
            cues = cues,
            actions = actions,
            rewards = rewards,
            epsilon_rew = epsilon,
            epsilon_pun = epsilon,
            epsilon_omi = epsilon,
            rho_rew = beta,
            rho_pun = -beta,
            bias_wth = bias_wth,
            bias_app = bias_app
        )

    def minimize_loss(self, cues, actions, rewards, initial = None):
        if initial is None:
            initial = self.initial

        result = minimize(
            fun = self.loss,
            x0 = initial,
            bounds = [
                self.epsilon_bounds,
                self.beta_bounds,
                self.bias_bounds,
                self.bias_bounds,
            ],
            args = (cues, actions, rewards),
            method = "Nelder-Mead"
        )

        fit_params = pd.DataFrame([result.x])
        fit_params.columns = ["epsilon", "beta", "bias_app", "bias_wth"]

        return result.fun, fit_params
```

Model 5

Model 5 assumes a common learning rate ϵ with reward and punishment sensitivities ρ_{rew} and ρ_{pun}, and includes action biases for withdrawal actions $bias_{wth}$ and approach actions $bias_{app}$.

```python
In [11]: class Model5(Model):
    def __init__(self, initial = None):
        if initial is None:
            initial = [0.5, 5, 5, 0, 0]
        super().__init__(initial = initial)

    def loss(self, params, cues, actions, rewards):
        epsilon, rho_rew, rho_pun, bias_app, bias_wth = params
        return -self.log_likelihood(
            cues = cues,
            actions = actions,
            rewards = rewards,
            epsilon_rew = epsilon,
            epsilon_pun = epsilon,
            epsilon_omi = epsilon,
            rho_rew = rho_rew,
            rho_pun = -rho_pun,
            bias_wth = bias_wth,
            bias_app = bias_app
        )

    def minimize_loss(self, cues, actions, rewards, initial = None):
        if initial is None:
            initial = self.initial

        result = minimize(
            fun = self.loss,
            x0 = initial,
            bounds = [
                self.epsilon_bounds,
                self.beta_bounds,
                self.beta_bounds,
                self.bias_bounds,
                self.bias_bounds,
            ],
            args = (cues, actions, rewards),
            method = "Nelder-Mead"
        )

        fit_params = pd.DataFrame([result.x])
        fit_params.columns = ["epsilon", "rho_rew", "rho_pun", "bias_app", "bias_wth"]

        return result.fun, fit_params
```

Model 6

Model 6 allows for separate reward and punishment sensitivities for the approach and withhold actions: ρ^{app}_{rew}, ρ^{wth}_{rew}, ρ^{app}_{pun}, and ρ^{wth}_{pun}.

```python
In [12]: class Model6(Model):
    def __init__(self, initial = None):
        if initial is None:
            initial = [0.5, 5, 5, 5, 5, 0, 0]
        super().__init__(initial = initial)

    @staticmethod
    def reward(a_t, r_t, rho_rew_app, rho_rew_wth, rho_pun_app, rho_pun_wth):
        if a_t == 1 and r_t > 0: return rho_rew_app
        if a_t == 1 and r_t < 0: return -rho_pun_app

        if a_t == 0 and r_t > 0: return rho_rew_wth
        if a_t == 0 and r_t < 0: return -rho_pun_wth

        return 0

    def log_likelihood(self, cues, actions, rewards, epsilon_rew, epsilon_pun, epsilon_omi, rho_rew
        n_stimuli = len(set(cues))
        n_actions = len(set(actions))

        q_vals = np.zeros((n_stimuli, n_actions))

        log_likelihood = 0

        for t, a_t in enumerate(actions):
            s_t = cues[t] - 1
            r_t = self.reward(a_t, rewards[t], rho_rew_app, rho_rew_wth, rho_pun_app, rho_pun_wth)

            qs = q_vals[s_t] + [bias_wth, bias_app]

            probs = self.softmax(qs)
            log_likelihood += np.log(probs[a_t])

            # Update the Q-values using Rescorla-Wagner
            q_vals[s_t, a_t] = self.rescorla_wagner(
                q_val = q_vals[s_t, a_t],
                epsilon_rew = epsilon_rew,
                epsilon_pun = epsilon_pun,
                epsilon_omi = epsilon_omi,
                reward = r_t
            )

        return log_likelihood

    def loss(self, params, cues, actions, rewards):
        epsilon, rho_rew_app, rho_rew_wth, rho_pun_app, rho_pun_wth, bias_app, bias_wth = params
        return -self.log_likelihood(
            cues = cues,
            actions = actions,
            rewards = rewards,
            epsilon_rew = epsilon,
            epsilon_pun = epsilon,
            epsilon_omi = epsilon,
            rho_rew_app = rho_rew_app,
            rho_rew_wth = rho_rew_wth,
            rho_pun_app = rho_pun_app,
            rho_pun_wth = rho_pun_wth,
            bias_wth = bias_wth,
            bias_app = bias_app
        )

    def minimize_loss(self, cues, actions, rewards, initial = None):
        if initial is None:
            initial = self.initial

        result = minimize(
            fun = self.loss,
            x0 = initial,
            bounds = [
                self.epsilon_bounds,
                self.beta_bounds,
                self.beta_bounds,
                self.beta_bounds,
                self.beta_bounds,
                self.bias_bounds,
                self.bias_bounds
            ],
            args = (cues, actions, rewards),
            method = "Nelder-Mead"
        )

        fit_params = pd.DataFrame([result.x])
        fit_params.columns = ["epsilon", "rho_rew_app", "rho_rew_wth", "rho_pun_app", "rho_pun_wth"
        fit_params["rho_pun_app"] = -fit_params["rho_pun_app"]
        fit_params["rho_pun_wth"] = -fit_params["rho_pun_wth"]

        return result.fun, fit_params
```

Model 7

Model 7 includes two learning rates for approach actions ϵ_{app} and withhold actions ϵ_{wth}, as well as the reward and punishment sensitivities ρ_{rew} and ρ_{pun} and the action biases for withdrawal actions $bias_{wth}$ and approach actions $bias_{app}$.

```python
In [13]: class Model7(Model):
    def __init__(self, initial = None):
        if initial is None:
            initial = [0.5, 0.5, 5, 5, 0, 0]
        super().__init__(initial = initial)

    @staticmethod
    def rescorla_wagner(q_val, epsilon_app, epsilon_wth, action, reward):
        if action == 1:
            return q_val + epsilon_app * (reward - q_val)

        return q_val + epsilon_wth * (reward - q_val)

    def log_likelihood(self, cues, actions, rewards, epsilon_app, epsilon_wth, rho_rew, rho_pun, bi
        n_stimuli = len(set(cues))
        n_actions = len(set(actions))

        q_vals = np.zeros((n_stimuli, n_actions))

        log_likelihood = 0

        for t, a_t in enumerate(actions):
            s_t = cues[t] - 1
            r_t = self.reward(rewards[t], rho_rew, rho_pun)

            qs = q_vals[s_t] + [ bias_wth, bias_app ]

            probs = self.softmax(qs)
            log_likelihood += np.log(probs[a_t])

            # Update the Q-values using Rescorla-Wagner
            q_vals[s_t, a_t] = self.rescorla_wagner(
                q_val = q_vals[s_t, a_t],
                epsilon_app = epsilon_app,
                epsilon_wth = epsilon_wth,
                action = a_t,
                reward = r_t
            )

        return log_likelihood

    def loss(self, params, cues, actions, rewards):
        epsilon_app, epsilon_wth, rho_rew, rho_pun, bias_app, bias_wth = params
        return -self.log_likelihood(
            cues = cues,
            actions = actions,
            rewards = rewards,
            epsilon_app = epsilon_app,
            epsilon_wth = epsilon_wth,
            rho_rew = rho_rew,
            rho_pun = -rho_pun,
            bias_wth = bias_wth,
            bias_app = bias_app
        )

    def minimize_loss(self, cues, actions, rewards, initial = None):
        if initial is None:
            initial = self.initial

        result = minimize(
            fun = self.loss,
            x0 = initial,
            bounds = [
                self.epsilon_bounds,
                self.epsilon_bounds,
                self.beta_bounds,
                self.beta_bounds,
                self.bias_bounds,
                self.bias_bounds
            ],
            args = (cues, actions, rewards),
            method = "Nelder-Mead"
        )

        fit_params = pd.DataFrame([result.x])
        fit_params.columns = ["epsilon_app", "epsilon_wth", "rho_rew", "rho_pun", "bias_app", "bias
        fit_params["rho_pun"] = -fit_params["rho_pun"]

        return result.fun, fit_params
```

Part 3

Create an additional model which takes into account Pavlovian biases. Use model 7 as a starting point for this. Add a parameter p to the model. To determine the action values to put into the softmax function for a given cue, take the Q-values, add the general bias to approach or withhold (as in equation 1 of the paper), and add p to the Q-value for approaching if the maximum Q-value for the current cue is positive, or add p to the Q-value for withholding if the maximal Q-value for the current cue is negative. That is, equation (1) of the paper becomes:

$$W(s_t, a_t) = Q_t(s_t, a_t) + b(a_t) + p_t'(a_t)$$

with $p_t'(a_t) = \begin{cases} p & \text{if } max_a Q(s_t, a) > 0 \text{ and } a_t = 1 \\ p & \text{if } max_a Q(s_t, a) < 0 \text{ and } a_t = 0 \\ 0 & \text{else} \end{cases}$

Though note that for this specific model we also overwrite $b(a_t)$ with a different bias.

```python
In [14]: class Model8(Model):
    def __init__(self, initial = None):
        if initial is None:
            initial = [0.5, 0.5, 5, 5, 0, 0, 0.5]
        super().__init__(initial = initial)

    @staticmethod
    def rescorla_wagner(q_val, epsilon_app, epsilon_wth, action, reward):
        if action == 1:
            return q_val + epsilon_app * (reward - q_val)

        return q_val + epsilon_wth * (reward - q_val)

    def log_likelihood(self, cues, actions, rewards, epsilon_app, epsilon_wth, rho_rew, rho_pun, bi
        n_stimuli = len(set(cues))
        n_actions = len(set(actions))

        q_vals = np.zeros((n_stimuli, n_actions))

        log_likelihood = 0

        for t, a_t in enumerate(actions):
            s_t = cues[t] - 1
            r_t = self.reward(rewards[t], rho_rew, rho_pun)

            qs = q_vals[s_t] + [ bias_wth, bias_app ]

            max_q = np.max(q_vals[s_t])
            if max_q < 0: qs[0] += p
            if max_q > 0: qs[1] += p

            probs = self.softmax(qs)
            log_likelihood += np.log(probs[a_t])

            # Update the Q-values using Rescorla-Wagner
            q_vals[s_t, a_t] = self.rescorla_wagner(
                q_val = q_vals[s_t, a_t],
                epsilon_app = epsilon_app,
                epsilon_wth = epsilon_wth,
                action = a_t,
                reward = r_t
            )

        return log_likelihood

    def loss(self, params, cues, actions, rewards):
        epsilon_app, epsilon_wth, rho_rew, rho_pun, bias_app, bias_wth, p = params
        return -self.log_likelihood(
            cues = cues,
            actions = actions,
            rewards = rewards,
            epsilon_app = epsilon_app,
            epsilon_wth = epsilon_wth,
            rho_rew = rho_rew,
            rho_pun = -rho_pun,
            bias_wth = bias_wth,
            bias_app = bias_app,
            p = p
        )

    def minimize_loss(self, cues, actions, rewards, initial = None):
        if initial is None:
            initial = self.initial

        result = minimize(
            fun = self.loss,
            x0 = initial,
            bounds = [
                self.epsilon_bounds,
                self.epsilon_bounds,
                self.beta_bounds,
                self.beta_bounds,
                self.bias_bounds,
                self.bias_bounds,
                self.p_bounds
            ],
            args = (cues, actions, rewards),
            method = "Nelder-Mead"
        )

        fit_params = pd.DataFrame([result.x])
        fit_params.columns = ["epsilon_app", "epsilon_wth", "rho_rew", "rho_pun", "bias_app", "bias
        fit_params["rho_pun"] = -fit_params["rho_pun"]

        return result.fun, fit_params
```

Part 4

Optimize the models, by fitting all the parameters of each model to each individual subject, using the scipy minimize function. Pay attention to initialize the parameters to reasonable values and set sensible bounds for each parameter (since Q-values get turned into probabilities through a softmax, which uses an exponential function, you may have to limit some of the parameters to certain magnitudes, to prevent overflow errors). Given the number of models this can take some minutes, to save time you can e.g. only apply the logarithm at the end, rather than during every iteration of your for-loop.

```python
In [15]: data = pd.read_csv("gen_data.csv")
```

```python
In [16]: models = [
    Model1(),
    Model2(),
    Model3(),
    Model4(),
    Model5(),
    Model6(),
    Model7(),
    Model8()
]

model_fits = []

for i, model in enumerate(models):
    print(f"Fitting model {i + 1}")
    model_fit = model.fit(data)
    print(f"Model {i + 1} loss: {round(model_fit["loss"].sum(), 2)}")
    model_fits.append(model_fit)
```

```
Fitting model 1
Model 1 loss: 2866.06
Fitting model 2
Model 2 loss: 2862.93
Fitting model 3
Model 3 loss: 2792.92
Fitting model 4
Model 4 loss: 2736.41
Fitting model 5
Model 5 loss: 2732.86
Fitting model 6
Model 6 loss: 2655.65
Fitting model 7
Model 7 loss: 2682.4
Fitting model 8
Model 8 loss: 2653.68
```

Part 5

Sum up the optimized log-likelihoods across all subjects for each model. Use this and all other relevant values to compute the BIC score for each model (using e.g. the BIC equation of Wikipedia). What does this tell you about which model describes the data best?

```python
In [17]: def BIC(k, n, l):
    """
    Calculate BIC criteria.
    :param k: number of parameters estimated by the model.
    :param n: sample size.
    :param l: maximized value of the log-likelihood function of the model
    :return: BIC
    """
    return k * np.log(n) - 2 * l
```

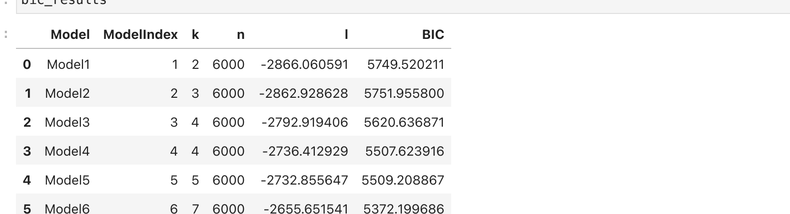
```python
In [18]: bic_results = []

for i, model_fit in enumerate(model_fits):
    k = len(model_fit.columns) - 2
    n = len(data)
    l = - model_fit.loss.sum()
    bic = BIC(k = k, n = n, l = l)

    bic_results.append(
        pd.DataFrame({
            "Model": [f"Model{i + 1}"],
            "ModelIndex": [i + 1],
            "k": [k],
            "n": [n],
            "l": [l],
            "BIC": [bic]
        })
    )

bic_results = pd.concat(bic_results)
bic_results = bic_results.reset_index(drop = True)
```

```python
In [19]: bic_results
```

Out[19]:

	Model	ModelIndex	k	n	l	BIC
0	Model1	1	2	6000	-2866.060591	5749.520211
1	Model2	2	3	6000	-2862.928628	5751.955800
2	Model3	3	4	6000	-2792.919406	5620.636871
3	Model4	4	4	6000	-2736.412929	5507.623916
4	Model5	5	5	6000	-2732.855647	5509.208867
5	Model6	6	7	6000	-2655.651541	5372.199686
6	Model7	7	6	6000	-2682.400884	5416.998856
7	Model8	8	7	6000	-2653.682366	5368.261335

```python
In [20]: fig, axs = plt.subplots(nrows = 1, ncols = 2, figsize = (10, 4))

axs[0].plot(bic_results.ModelIndex, bic_results.BIC, marker = "o", markersize = 4)
axs[0].set_xlabel("Model")
axs[0].set_ylabel("BIC")
axs[0].set_title("BIC for different models")

axs[1].plot(bic_results.ModelIndex, bic_results.l, marker = "o", markersize = 4)
axs[1].set_xlabel("Model")
axs[1].set_ylabel("Log-likelihood")
axs[1].set_title("Log-likelihood for different models")

fig.tight_layout()
plt.show()
```



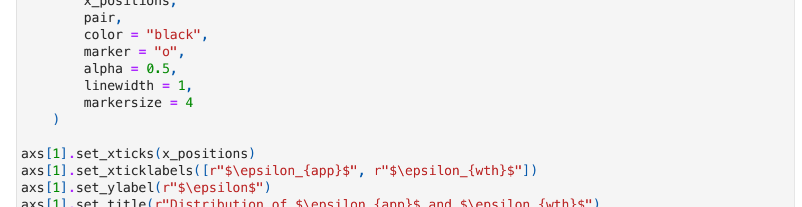
Based on the BIC score (lowest score) and the log-likelihood (highest score), we conclude that the 8th model describes the data best, shortly followed by the 6th model.

Part 6

Compare the fitted ϵ_{app} and ϵ_{wth} for the last model. How do you interpret the difference in their means?

```python
In [21]: last = model_fits[-1]
```

```python
In [22]: fig, axs = plt.subplots(nrows = 1, ncols = 2, figsize = (8, 4), width_ratios = [0.8, 0.2], sharey=T

axs[0].plot(last.ID, last.epsilon_app, label = r"$\epsilon_{app}$", marker = "o", markersize = 4)
axs[0].axhline(last.epsilon_app.mean(), color=mpl.colormaps["tab10"](0), linestyle="--", label = r"
axs[0].plot(last.ID, last.epsilon_wth, label = r"$\epsilon_{wth}$", marker = "o", markersize = 4)
axs[0].axhline(last.epsilon_wth.mean(), color=mpl.colormaps["tab10"](1), linestyle="--", label = r"
axs[0].set_ylabel(r"$\epsilon$")
axs[0].set_xlabel("Subject ID")
axs[0].set_xticks(last.ID)
axs[0].legend()
axs[0].set_title("Learning rates of all subjects")
axs[0].set_ylim(0, 0.5)

## Boxplot

epsilons = np.vstack([last.epsilon_app, last.epsilon_wth]).T

axs[1].boxplot(
    epsilons,
    patch_artist = True,
    widths = 0.5,
    boxprops = dict(facecolor = "white", color = "black"),
    medianprops = dict(color = "gray"),
    showfliers = False
)

x_positions = [1, 2]
for pair in epsilons:
    axs[1].plot(
        x_positions,
        pair,
        color = "black",
        marker = "o",
        alpha = 0.5,
        linewidth = 1,
        markersize = 4
    )

axs[1].set_xticks(x_positions)
axs[1].set_xticklabels([r"$\epsilon_{app}$", r"$\epsilon_{wth}$"])
axs[1].set_ylabel(r"$\epsilon$")
axs[1].set_title(r"Distribution of $\epsilon_{app}$ and $\epsilon_{wth}$")
axs[1].set_ylim(0, 0.5)

fig.tight_layout()
plt.show()
```



The difference in means between the learning rate for approach action ϵ_{app} and withhold action ϵ_{wth} shows that the learning rate for different action type (approach or withhold) differs. Majority of the subjects have a higher learning rate for the approach action rather than withhold action. This could be related to the dopaminergic system and its role in learning - it is easier to learn the association between a reward and action than between a reward and inaction.

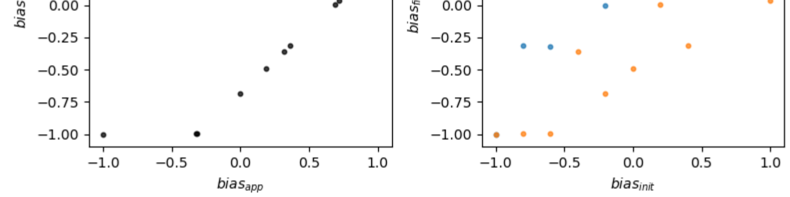
Part 7

Bonus: Fit the first subject 10 times with the last model, using different initial parameters. Create a scatter plot between the fitted $bias_{app}$ and $bias_{wth}$ across the fits. How do you explain this plot?

```python
In [25]: subject_data = data[ data.ID == data.ID[0] ]
subject_data = subject_data.reset_index(drop = True)

model8 = Model8()
# We fit the model 11 times to include the bounds and the middle val (zero)
bias_initials = np.linspace(model8.bias_bounds[0], model8.bias_bounds[1], 11)

fits = []

for i, bias_initial in enumerate(bias_initials):
    print(f"Fitting model with initial bias = {round(bias_initial, 2)}")
    initial = model8.initial

    initial[-2] = bias_initial
    initial[-3] = bias_initial

    model_fit = model8.fit(subject_data, initial = initial)
    model_fit["initial_bias"] = bias_initial

    print(f"Model loss: {round(model_fit["loss"].sum(), 2)}")
    fits.append(model_fit)

fits = pd.concat(fits)
fits = fits.reset_index(drop = True)
```

```
Fitting model with initial bias = -1.0
Model loss: 288.12
Fitting model with initial bias = -0.8
Model loss: 280.41
Fitting model with initial bias = -0.6
Model loss: 280.41
Fitting model with initial bias = -0.4
Model loss: 280.41
Fitting model with initial bias = -0.2
Model loss: 280.41
Fitting model with initial bias = 0.0
Model loss: 280.42
Fitting model with initial bias = 0.2
Model loss: 280.41
Fitting model with initial bias = 0.4
Model loss: 280.41
Fitting model with initial bias = 0.6
Model loss: 280.41
Fitting model with initial bias = 0.8
Model loss: 280.41
Fitting model with initial bias = 1.0
Model loss: 280.41
```

```python
In [26]: fig, axs = plt.subplots(nrows = 1, ncols = 2, figsize = (8, 4))

axs[0].scatter(fits.bias_app, fits.bias_wth, c = "black", alpha = 0.75, s = 10)
axs[0].set_xlabel(r"$bias_{app}$")
axs[0].set_ylabel(r"$bias_{wth}$")
axs[0].set_xlim(-1.1, 1.1)
axs[0].set_ylim(-1.1, 1.1)
axs[0].set_title("Dependency between\napproach and withhold bias")

axs[1].scatter(fits.initial_bias, fits.bias_app, alpha = 0.75, s = 10, label = r"$bias_{app}$")
axs[1].scatter(fits.initial_bias, fits.bias_wth, alpha = 0.75, s = 10, label = r"$bias_{wth}$")
axs[1].set_xlabel(r"$bias_{init}$")
axs[1].set_ylabel(r"$bias_{fitted}$")
axs[1].set_xlim(-1.1, 1.1)
axs[1].set_ylim(-1.1, 1.1)
axs[1].legend()
axs[1].set_title("Dependency between\ninitial and fitted biases")

fig.tight_layout()
plt.show()
```



On the left hand side we show the correlation between approach and withhold bias fitted on model 8 with different initial biases. We observe an almost perfect linear correlation between values larger than -1 and smaller than 1. For the intermediate values, the model fits the withhold and approach bias on a diagonal from ~(-0.3, -1) to ~(1, 0.3).

On the right hand side we plot the initial bias vs the fitted bias. We can observe that the fitted approach bias is always greater or equal to the withhold bias.

In all model fits, we notice that the approach bias is always stronger than the withhold bias, indicating that individuals prefer performing an action rather than withholding responding. The correlation between the two could indicate that the two processes (go/no-go) are related, that is, an individual with a strong tendency to approach also exhibits a strong tendency to withhold when appropriate.