Robust Sensor Fault Detection for Linear Parameter-Varying Systems using Interval Observer

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This paper proposes a new interval observer for continuous-time linear parameter-varying systems with an unmeasurable parameter vector subject to unknown but bounded disturbances. The parameter-varying matrices are assumed to be elementwise bounded. This observer is used to compute a so-called residual interval used for sensor fault detection by checking if zero is contained in the interval. To attenuate the effect of the system's uncertainties on the detectability of faults, additional weighting matrices and different upper and lower observer gains are introduced, providing more degrees of freedom than the classical interval observer strategies. In addition, a L_∞ procedure is proposed to tune the value of the observer gains, this procedure being easy to modify to introduce additional constraints on the estimation algorithm. Simulations are run to show the efficiency of the proposed fault detection strategy.

Keywords: Fault detection, Robust fault detection, Sensor fault, Linear Parameter-Varying system, Interval observer, Continuous-time systems.

1. Introduction

Most real-life systems obey nonlinear dynamics, making the design of control and estimation algorithms a complex task. A common and powerful way to reduce this complexity is to use a linear parameter-varying (LPV) plant (Shamma, 2012). Indeed, due to their partial linearity, methods developed for linear systems can be applied to such plants. In addition, during their evolution, systems can be subject to faults that could provoke serious damages if they go undetected. Therefore, strategies have to be developed to detect such faults. The case of sensor fault detection has been widely studied in the literature (Varga, 2017). Passive fault detection strategies usually rely on the comparison of the system's output with the estimate of the output computed from a fault-free model (Li et al., 2020). However, when the system is subject to unknown but bounded disturbances, classical strategies are limited (Robinson et al., 2020), difficult to implement and could lead to a false positive (Lamouchi et al., 2018).

Set-based estimation algorithms have been introduced to deal with the problem of state estimation for uncertain systems, provided that the uncertainties are bounded. Two classes of algorithms have been developed: set-membership estimation (Combastel, 2005; Li et al., 2020) and interval observers (Mazenc and Bernard, 2011; Raïssi et al., 2011; Wang et al., 2018; Garbouj et al., 2020). In this paper, interval observers are considered to design a passive sensor fault detection strategy. While a classical pointwise observer computes an estimate of the true value of the system's state based on the system's dynamics and output, an interval observer uses two sub-observers to provide bounds for the system's state by also taking into account the uncertainties' bounds. The advantage of such estimation strategies is that they are often more computationally efficient than set-membership algorithms.

Several passive set-based sensor fault detection strategies for LPV systems have been proposed, both based on set-membership estimation (Nejjari et al., 2009; Blesa et al., 2012; Wan et al., 2020) and interval observers (Lamouchi et al., 2018;

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Rotondo et al., 2018, 2019; Ifqir et al., 2020; Garbouj et al., 2020). The classical approach consists in computing a so-called residual interval. If the interval does not contain zero, then the system is subject to a fault. However, due to this strategy, low-magnitude faults could go undetected. Therefore, to ensure that the maximum range of faults is detected, the residual interval has to be tight. Interval observers are based on a change of coordinates meant to ensure that they satisfy the cooperativity property (Mazenc and Bernard, 2011) (i.e. the estimation error state matrix is Metzler and the estimation error dynamics are stable). The performance of the fault detection algorithm (i.e. the tightness of the residual interval) is then heavily influenced by the choice of target coordinates. For this reason, Wang et al. (2018) propose the so-called TNL design strategy (where T, N and L denote the weighting matrices and gain used in this strategy), based on the introduction of additional weighting matrices in the observer design. This approach provides more degrees of freedom in the observer design since a change of coordinates is no longer necessary to ensure cooperativity. In addition, several interval observers for LPV systems have been proposed under the assumption that the vector of scheduling parameters is always available (Wang et al., 2015; Li et al., 2019). In the general case, only the bounds of the parameters (and thus of the parameter-varying matrices) are known (Efimov et al., 2012, 2013). This is why Chebotarev et al. (2015) introduce an interval observer with different gains for the upper and lower sub-observers.

Following Wang et al. (2018), Chebotarev et al. (2015) and Zammali et al. (2021), this study therefore proposes a sensor fault detection strategy for LPV systems based on a robust interval observer. The contributions of this paper are twofold: (i) a novel interval observer structure for continuous-time LPV systems, based on the TNL approach and used to build a residual interval, is introduced; (ii) a new modular L_∞ gain design procedure, ensuring the cooperativity of the estimation error dynamics and the tightness of the residual interval, is proposed.

The remainder of this paper is organized as follows. Section 2 presents general prerequisites and formulates the problem addressed. Section 3 introduces the proposed structure and design procedure for the interval observer. Simulation results are given in Section 4 to assess the efficiency of the proposed fault detection strategy. Finally, Section 5 draws concluding remarks and perspectives.

2. Prerequisites and problem formulation

2.1. *Notations*

The sets of positive integers, real numbers and positive real numbers are denoted respectively by $\mathbb{N}$, $\mathbb{R}$ and $\mathbb{R}_+$. The matrices I_n and $\mathbf{0}$ are respectively the identity matrix of size $n \in \mathbb{N}$ and the matrix filled with zeros of appropriate size. The matrices $A^\top$ and $A^\dagger$, with $A \in \mathbb{R}^{n\times m}$, denote respectively the transpose and the Moore-Penrose inverse of matrix A. The fact that a matrix $A \in \mathbb{R}^{n\times n}$ is positive definite (resp. semidefinite) is denoted by $A \succ 0$ (resp. $A \succeq 0$) and the fact that A is negative semidefinite is denoted by $A \preceq 0$. The matrix $\operatorname{diag}(A_1, \ldots, A_n)$ is the block diagonal matrix with diagonal blocks $A_1, \ldots, A_n$. Given a signal $x : \mathbb{R}_+ \to \mathbb{R}^n$, x_t denotes the value of x at time t (the notation $x(t)$ is alternatively used in the literature). The Euclidean norm of x_t is $\|x_t\|_2^2 = x_t^\top x_t$ and the L_∞ norm of x is $\|x\|_\infty = \sup\{\|x_t\|_2, t \in \mathbb{R}_+\}$. The set of all signals x satisfying $\|x\|_\infty < \infty$ is denoted by $\mathcal{L}_\infty^n$. The Kronecker product of two matrices A and B is denoted by $A \otimes B$. Finally, $\star$ is a placeholder denoting the transpose of a term placed symmetrically in a matrix.

2.2. *Interval analysis*

Let $A, B \in \mathbb{R}^{n\times m}$ be two matrices. Then the inequality $A \leq B$ is understood elementwise. The matrix A can be written as $A = A^+ - A^-$, with $A^+ = \max\{\mathbf{0}, A\}$ (the maximum operator is understood elementwise), such that $A^+, A^- \geq 0$. The above operations are extended without modification to any vector $x \in \mathbb{R}^n$.

Lemma 2.1 (Efimov et al. (2012)). *Let $x, \underline{x}, \overline{x} \in \mathbb{R}^n$ be vectors satisfying $\underline{x} \leq x \leq \overline{x}$.*

(i) Let $A \in \mathbb{R}^{m\times n}$ be a constant matrix. Then:

$$A^+\underline{x} - A^-\overline{x} \leq Ax \leq A^+\overline{x} - A^-\underline{x}.$$

(ii) Let $A, \underline{A}, \overline{A} \in \mathbb{R}^{m\times n}$ be matrices such that $\underline{A} \leq A \leq \overline{A}$. Then:

$$\begin{aligned}&\underline{A}^+\underline{x}^+ - \overline{A}^+\underline{x}^- - \underline{A}^-\overline{x}^+ + \overline{A}^-\overline{x}^- \leq Ax\\ &\leq \overline{A}^+\overline{x}^+ - \underline{A}^+\overline{x}^- - \overline{A}^-\underline{x}^+ + \underline{A}^-\underline{x}^-.\end{aligned}$$

Remark 2.1. If $x \in \mathbb{R}^n$ is a constant vector and $A, \underline{A}, \overline{A} \in \mathbb{R}^{m\times n}$ are matrices such that $\underline{A} \leq A \leq \overline{A}$, then, by (i) of Lemma 2.1:

$$\underline{A}x^+ - \overline{A}x^- \leq Ax \leq \overline{A}x^+ - \underline{A}x^-.$$

In addition to the above results, it is necessary to introduce a particular kind of matrix for the development of interval observers for continuous-time systems.

Definition 2.1 (Chebotarev et al. (2015)). *A matrix $A \in \mathbb{R}^{n\times n}$ is called* Metzler *if all its off-diagonal elements are nonnegative. The matrix A is Metzler if there exists a diagonal matrix $D \in \mathbb{R}_+^{n\times n}$ such that $A + D \geq \mathbf{0}$.*

Finally, the following lemma gives properties on the growth of a particular kind of nonlinear functions.

Lemma 2.2 (Zheng et al. (2016)). *Let $F(z,\rho) = A(\rho)x$ be a function of x and ρ, with $A(\rho)$ a matrix and $\underline{x} \leq x \leq \overline{x}$. If there exist two matrices $\underline{A}$ and $\overline{A}$ and two functions $\underline{F}(\underline{x},\overline{x})$ and $\overline{F}(\underline{x},\overline{x})$ satisfying, for all possible values of ρ and x, $\underline{A} \leq A(\rho) \leq \overline{A}$ and $\underline{F}(\underline{x},\overline{x}) \leq F(x,\rho) \leq \overline{F}(\underline{x},\overline{x})$ then:*

$$\begin{cases} \left\|\overline{F}(\underline{x},\overline{x}) - F(x,\rho)\right\|_2 \leq \bar{l}_F \left\|\overline{x}-x\right\|_2 \\ \qquad + \underline{l}_F \left\|\underline{x}-x\right\|_2 + \overline{m}_F \\ \left\|\underline{F}(\underline{x},\overline{x}) - F(x,\rho)\right\|_2 \leq \bar{l}_F \left\|\overline{x}-x\right\|_2 \\ \qquad + \underline{l}_F \left\|\underline{x}-x\right\|_2 + \underline{m}_F \end{cases}$$

where $\overline{m}_F$ and $\underline{m}_F$ are positive constants depending on the values of A and x and:

$$\begin{cases} \bar{l}_F = \left\|\overline{A}^+\right\|_2 + \left\|\underline{A}^+\right\|_2 \\ \underline{l}_F = \left\|\overline{A}^-\right\|_2 + \left\|\underline{A}^-\right\|_2 \end{cases}$$

2.3. *Problem formulation*

Consider the following continuous-time LPV system:

$$\begin{cases} \dot{x}_t = A(\rho_t)x_t + B(\rho_t)u_t + D(\rho_t)w_t \\ y_t = Cx_t + f_t \end{cases} \tag{1}$$

where $x_t \in \mathbb{R}^{n_x}$, $u_t \in \mathbb{R}^{n_u}$, $y_t \in \mathbb{R}^{n_y}$, $w_t \in \mathbb{R}^{n_w}$, $f_t \in \mathbb{R}^{n_f}$ and $\rho_t \in \mathbb{R}^{n_\rho}$ are respectively the state, input, output, disturbance, fault and unmeasurable parameter vectors and A, B, C and D are matrices of appropriate dimensions such that, for $M \in \{A, B, D\}$:

$$M(\rho_t) = M_0 + \Delta M(\rho_t), \tag{2}$$

Assumption 2.1. *The measurement vector y_t is obtained by several sensors through the matrix C. Only one is potentially affected by a fault f_t.*

Assumption 2.2. *The initial state vector x_0 and the disturbance vector w_t are unknown but bounded and satisfy $\underline{x}_0 \leq x_0 \leq \overline{x}_0$ and $\underline{w}_t \leq w_t \leq \overline{w}_t$, $\forall t \in \mathbb{R}_+$, with $\underline{x}_0, \overline{x}_0 \in \mathbb{R}^{n_x}$ such that $\left\|\underline{x}_0\right\|_2, \left\|\overline{x}_0\right\|_2 \leq \infty$ and $\underline{w}_t, \overline{w}_t \in \mathcal{L}_\infty^{n_w}$.*

The parameter vector ρ_t is not measurable. Then, the value of the matrices ΔM, with $M \in \{A, B, D\}$, is not accessible.

Assumption 2.3. *The matrices ΔM, with $M \in \{A, B, D\}$, are unknown but bounded and satisfy $\underline{\Delta M} \leq \Delta M(\rho_t) \leq \overline{\Delta M}$, $\forall t \in \mathbb{R}_+$.*

The following assumption is necessary to avoid the design of a controller for system (1), which is out of the scope of the present paper.

Assumption 2.4. *The input vector u_t and state vector x_t are such that $u_t \in \mathcal{L}_\infty^{n_u}$ and $x_t \in \mathcal{L}_\infty^{n_x}$. As a direct consequence, when $f_t = 0$, $y_t \in \mathcal{L}_\infty^{n_y}$.*

Classical passive fault detection strategies consist in obtaining an estimate $\widehat{y}_t \in \mathbb{R}^{n_y}$ of the output y_t from an observer built upon a faultless model of the system. This estimate is then used to compute a so-called residual signal $r_t = \widehat{y}_t - y_t$. By the way it is defined, this residual diverges from zero when the system is subject to a fault. However, in the presence of disturbances and uncertainties on the model as in (1), the residual signal might deviate from zero even in a fault-free situation.

To cope with this issue, another passive strategy can be used, consisting in using an interval observer to find guaranteed bounds $\underline{x}_t, \overline{x}_t \in \mathbb{R}^{n_x}$ for the system's state vector such that, in a fault-free situation, $\underline{x}_t \leq x_t \leq \overline{x}_t$. Then, based on Lemma 2.1, it is possible to compute $\underline{y}_t, \overline{y}_t \in \mathbb{R}^{n_y}$ as:

$$\begin{cases} \underline{y}_t = C^+\underline{x}_t - C^-\overline{x}_t \\ \overline{y}_t = C^+\overline{x}_t - C^-\underline{x}_t \end{cases} \tag{3}$$

Therefore, by Lemma 2.1 and the definition of $\underline{x}_t$ and $\overline{x}_t$, $y_t \in [\underline{y}_t, \overline{y}_t]$ if the system is fault-free and $y_t \notin [\underline{y}_t, \overline{y}_t]$ else. An equivalent way to detect a fault in this context is to build the residual framer:

$$\begin{cases} \underline{r}_t = \underline{y}_t - y_t \\ \overline{r}_t = \overline{y}_t - y_t \end{cases} \tag{4}$$

so that the system is fault-free if $\mathbf{0} \in [\underline{r}_t, \overline{r}_t]$ and subject to a fault if $\mathbf{0} \notin [\underline{r}_t, \overline{r}_t]$. While such a strategy limits false alarms due to unknown but bounded disturbances and uncertainties, low-magnitude faults might not be detected if the bounds defined in Assumptions 2.2 and 2.3 are too large.

To perform the passive fault-detection task described above, based on the work of Wang et al. (2018); Chebotarev et al. (2015) and Zammali et al. (2021), the following presents a new residual framer structure for continuous-time linear parameter-varying systems with an unmeasurable parameter vector subject to disturbances. To allow for the detection of low-magnitude faults, this framer should attenuate the effect of the uncertainties on the interval $[\underline{r}_t, \overline{r}_t]$.

3. Main result

3.1. *Framer design*

Let T and N be matrices satisfying:

$$T + NC = I_{n_x}. \tag{5}$$

Assumption 3.1. *The pair (TA_0, C) is detectable.*

Inspired by Li et al. (2019), the proposed framer for the state of system (1) is:

$$\begin{cases} \dot{\underline{\xi}}_t = (TA_0 - \underline{L}C)\,\underline{x}_t + TB_0u_t \\ \qquad\quad + \underline{L}y_t + \underline{\phi}_t + \underline{\chi}_t + \underline{\omega}_t & \text{(6a)} \\ \underline{x}_t = \underline{\xi}_t + Ny_t & \text{(6b)} \\ \dot{\overline{\xi}}_t = \left(TA_0 - \overline{L}C\right)\overline{x}_t + TB_0u_t & \text{(6c)} \\ \qquad\quad + \overline{L}y_t + \overline{\phi}_t + \overline{\chi}_t + \overline{\omega}_t \\ \overline{x}_t = \overline{\xi}_t + Ny_t & \text{(6d)} \end{cases}$$

where:

$$\begin{cases} \underline{\phi}_t = T^+\underline{\delta}_t(A,x) - T^-\overline{\delta}_t(A,x) \\ \overline{\phi}_t = T^+\overline{\delta}_t(A,x) - T^-\underline{\delta}_t(A,x) \end{cases} \tag{7}$$

$$\begin{cases} \underline{\chi}_t = T^+\left(\underline{\Delta B}u_t^+ - \overline{\Delta B}u_t^-\right) \\ \qquad - T^-\left(\overline{\Delta B}u_t^+ - \underline{\Delta B}u_t^-\right) \\ \overline{\chi}_t = T^+\left(\overline{\Delta B}u_t^+ - \underline{\Delta B}u_t^-\right) \\ \qquad - T^-\left(\underline{\Delta B}u_t^+ - \overline{\Delta B}u_t^-\right) \end{cases} \tag{8}$$

$$\begin{cases} \underline{\omega}_t = (TD_0)^+\underline{w}_t - (TD_0)^-\overline{w}_t \\ \qquad + T^+\underline{\delta}_t(D,w) - T^-\overline{\delta}_t(D,w) \\ \overline{\omega}_t = (TD_0)^+\overline{w}_t - (TD_0)^-\underline{w}_t \\ \qquad + T^+\overline{\delta}_t(D,w) - T^-\underline{\delta}_t(D,w) \end{cases} \tag{9}$$

with:

$$\begin{cases} \underline{\delta}_t(M,a) = \underline{\Delta M}^+\underline{a}_t^+ - \overline{\Delta M}^+\underline{a}_t^- \\ \qquad - \underline{\Delta M}^-\overline{a}_t^+ + \overline{\Delta M}^-\overline{a}_t^- \\ \overline{\delta}_t(M,a) = \overline{\Delta M}^+\overline{a}_t^+ - \underline{\Delta M}^+\overline{a}_t^- \\ \qquad - \overline{\Delta M}^-\underline{a}_t^+ + \underline{\Delta M}^-\underline{a}_t^- \end{cases} \tag{10}$$

and $\underline{L}$ and $\overline{L}$ are observer gains such that $TA_0 - \underline{L}C$ and $TA_0 - \overline{L}C$ are Metzler. The matrices T and N are weighting matrices introduced in Wang et al. (2018) to add more degrees of freedom when tuning the values of $\underline{L}$ and $\overline{L}$.

Theorem 3.1. *Let Assumptions 2.2 and 2.3 hold and $TA_0 - \underline{L}C$ and $TA_0 - \overline{L}C$ be Metzler matrices. Then, $\underline{x}_t$ and $\overline{x}_t$ obeying the dynamics* (6) *satisfy, in the fault-free case (i.e. for $f_t = \mathbf{0}$):*

$$\underline{x}_t \le x_t \le \overline{x}_t, \forall t \in \mathbb{R}_+. \tag{11}$$

Proof. In the following, it is assumed that $f_t = \mathbf{0}$, $\forall t \in \mathbb{R}_+$. Let $\underline{e}_t^\xi = \underline{\xi}_t - Tx_t$, $\overline{e}_t^\xi = \overline{\xi}_t - Tx_t$, $\underline{e}_t = \underline{x}_t - x_t$ and $\overline{e}_t = \overline{x}_t - x_t$. Then:

$$\begin{aligned} \dot{\underline{e}}_t^\xi &= (TA_0 - \underline{L}C)\,\underline{x}_t + TB_0u_t + \underline{L}y_t \\ &\quad + \underline{\phi}_t + \underline{\chi}_t + \underline{\omega}_t - TA(\rho_t)x_t \\ &\quad - TB(\rho_t)u_t - TD(\rho_t)w_t \\ &= (TA_0 - \underline{L}C)\,\underline{e}_t + \underline{\Phi}_t + \underline{\mathrm{X}}_t + \underline{\Omega}_t \end{aligned}$$

where $\underline{\Phi}_t = \underline{\phi}_t - T\Delta A(\rho_t)x_t$, $\underline{\mathrm{X}}_t = \underline{\chi}_t - T\Delta B(\rho_t)u_t$ and $\underline{\Omega}_t = \underline{\omega}_t - TD(\rho_t)w_t$. In addition:

$$\begin{aligned} \underline{e}_t &= \underline{\xi}_t + Ny_t - (T + NC)x_t \\ &= \underline{\xi}_t - Tx_t + Nf_t = \underline{e}_t^\xi. \end{aligned}$$

Therefore:

$$\dot{\underline{e}}_t = (TA_0 - \underline{L}C)\,\underline{e}_t + \underline{\Phi}_t + \underline{\mathrm{X}}_t + \underline{\Omega}_t. \tag{12}$$

Following the same procedure:

$$\dot{\overline{e}}_t = \left(TA_0 - \overline{L}C\right)\underline{e}_t + \overline{\Phi}_t + \overline{\mathrm{X}}_t + \overline{\Omega}_t \tag{13}$$

where $\overline{\Phi}_t = \overline{\phi}_t - T\Delta A(\rho_t)x_t$, $\overline{\mathrm{X}}_t = \overline{\chi}_t - T\Delta B(\rho_t)u_t$ and $\overline{\Omega}_t = \overline{\omega}_t - TD(\rho_t)w_t$.

By Assumptions 2.2 and 2.3 and Lemma 2.1, $\underline{\mathrm{X}}_t \le \mathbf{0}$, $\overline{\mathrm{X}}_t \ge \mathbf{0}$, $\underline{\Omega}_t \le \mathbf{0}$ and $\overline{\Omega}_t \ge \mathbf{0}$. To prove that $\underline{x}_t \le x_t \le \overline{x}_t$, it is enough to prove that whenever the j-th component of $\underline{e}_t$ (respectively $\overline{e}_t$) is equal to 0, the j-th component of the derivative $\dot{\underline{e}}_t$ (resp. $\dot{\overline{e}}_t$) is nonpositive (resp. nonnegative). Indeed, in this case, $\underline{e}_t$ (resp. $\overline{e}_t$) can never become positive (resp. negative).

Let a_t^j denote the j-th component of a_t, where a can be any of the vector previously defined. If $\underline{e}_t^j = 0$ and $\overline{e}_t^j \ge 0$, then $\underline{x}_t^j = x_t^j \le \overline{x}_t^j$ and, by Lemma 2.1, $\underline{\Phi}_t^j \le 0$. Moreover:

$$\dot{\underline{e}}_t^j = \sum_{i=1}^{n_x} (TA_0 - \underline{L}C)_{ji}\,\underline{e}_t^i + \underline{\Phi}_t^i + \underline{\mathrm{X}}_t^i + \underline{\Omega}_t^i$$

where $(TA_0 - \underline{L}C)_{ji}$ is the component located at the j-th row and i-th column of $TA_0 - \underline{L}C$. Considering $TA_0 - \underline{L}C$ is Metzler, by Definition 2.1, $(TA_0 - \underline{L}C)_{ji}\,\underline{e}_t^i \le 0$, $\forall i \in \overline{1, n_x}$ such that $i \ne j$. Since $\underline{e}_t^j = 0$, it does not contribute to $\dot{\underline{e}}_t^j$ and $\dot{\underline{e}}_t^j \le 0$. The same reasoning on $\overline{e}_t^j$ leads to $\dot{\overline{e}}_t^j \ge 0$.

With what precedes and the fact that, by Assumption 2.2, $\underline{\Phi}_0 \le \mathbf{0}$, $\overline{\Phi}_0 \ge \mathbf{0}$, $\underline{e}_0 \le \mathbf{0}$ and $\overline{e}_0 \ge \mathbf{0}$, (11) is satisfied. □

The desired residual framer (4) can then be constructed from the proposed state framer (6).

3.2. *Stability and L_∞ performance*

For the framer (6) to be an interval observer, it is necessary that the estimation errors $\underline{e}_t$ and $\overline{e}_t$ are input-to-state stable (ISS). Accordingly, to guarantee input-to-state stability and to attenuate the effect of the uncertainties on the residual framer, as per the design requirements stated in Section 2.3, a L_∞ procedure is proposed to tune the observer gains $\underline{L}$ and $\overline{L}$. To unroll this procedure, the following lemma is recalled.

Lemma 3.1 (Rao and Mitra (1972)). *Given matrices $X \in \mathbb{R}^{n\times m}$, $Y \in \mathbb{R}^{m\times p}$ and $Z \in \mathbb{R}^{n\times p}$ with $\operatorname{rank} Y = p$, the general solution X of the equation $XY = Z$ is:*

$$X = ZY^\dagger + \Xi\left(I_m - YY^\dagger\right)$$

where $\Xi \in \mathbb{R}^{n\times m}$ is an arbitrary matrix.

From this lemma and the relations (5):

$$T = \Theta^\dagger\lambda_1 + \Xi\Psi\lambda_1, N = \Theta^\dagger\lambda_2 + \Xi\Psi\lambda_2, \quad (14)$$

with $\Xi \in \mathbb{R}^{n_x\times(n_x+n_y)}$ a free matrix and:

$$\Theta = \begin{bmatrix} I_{n_x} \\ C \end{bmatrix}, \qquad \Psi = I_{n_x+n_y} - \Theta\Theta^\dagger,$$

$$\lambda_1 = \begin{bmatrix} I_{n_x} \\ \mathbf{0} \end{bmatrix}, \qquad \lambda_2 = \begin{bmatrix} \mathbf{0} \\ I_{n_y} \end{bmatrix}.$$

The following theorem then presents the gain tuning procedure.

Theorem 3.2. *Let all the assumptions of Theorem 3.1 hold. For two given positive scalars α and η, if there exist two positive scalars γ and μ, a positive definite diagonal matrix $P \in \mathbb{R}^{2n_x\times 2n_x}$ and a block diagonal matrix $Y \in \mathbb{R}^{2n_x\times 2n_y}$ such that:*

$$S + \eta P \geq \mathbf{0} \quad (15a)$$

$$\begin{bmatrix} \Lambda_{11} & \star & \star \\ P & -\gamma I_{2n_x} & \star \\ P & \mathbf{0} & -\gamma I_{2n_x} \end{bmatrix} \preceq 0 \quad (15b)$$

$$\begin{bmatrix} P & \star & \star \\ \mathbf{0} & \mu-\gamma & \star \\ \mathcal{C} & \mathbf{0} & \mu I_{2n_y} \end{bmatrix} \succeq 0 \quad (15c)$$

where $\Lambda_{11} = S + S^\top + \alpha P + \gamma Q$, $S = P(I_2 \otimes T)(I_2 \otimes A_0) - Y\Upsilon$, $\Upsilon = I_2 \otimes C$, $Q = 6 \cdot \operatorname{diag}(\underline{l}_\phi^2, \overline{l}_\phi^2)$ and:

$$\mathcal{C} = \begin{bmatrix} C^+ & -C^- \\ -C^- & C^+ \end{bmatrix},$$

then, (6) *is a robust interval observer for system* (1) *in the fault-free case. This interval observer has the performance:*

$$\|R_t\|_2^2 \leq \mu V_0 \mathrm{e}^{-\alpha t} + \mu^2 \|\varepsilon\|_\infty^2 \quad (16)$$

where $R_t^\top = \begin{bmatrix}\underline{r}_t^\top & \overline{r}_t^\top\end{bmatrix}$, $V_0 = E_0^\top P E_0$, $E_t^\top = \begin{bmatrix}\underline{e}_t^\top & \overline{e}_t^\top\end{bmatrix}$ and:

$$\varepsilon_t = \begin{bmatrix} \underline{\mathrm{X}}_t + \underline{\Omega}_t \\ \overline{\mathrm{X}}_t + \overline{\Omega}_t \end{bmatrix}.$$

Proof. In the following, it is assumed that $f_t = \mathbf{0}$. Since $P \succ 0$, all its diagonal terms are strictly positive. Considering $Y = P\operatorname{diag}(\underline{L}, \overline{L})$, the matrices $TA_0 - \underline{L}C$ and $TA_0 - \overline{L}C$ are Metzler if condition (15a) is satisfied, according to Definition 2.1.

The dynamics of E_t are:

$$\dot{E}_t = \Pi E_t + \Phi_t + \varepsilon_t \quad (17)$$

where $\Pi = (I_2 \otimes T)(I_2 \otimes A_0) - L\Upsilon$, $L = \operatorname{diag}(\underline{L}, \overline{L})$ and $\Phi_t^\top = \begin{bmatrix}\underline{\Phi}_t^\top & \overline{\Phi}_t^\top\end{bmatrix}$.

The functions $T\Delta A(\rho_t)x_t$, $\underline{\phi}_t$ and $\overline{\phi}_t$ satisfy the assumptions of Lemma 2.2 so that $\underline{\Phi}_t$ and $\overline{\Phi}_t$ are globally Lipschitz. In addition, by Lemma 2.2:

$$\begin{cases} \underline{\Phi}_t^\top \underline{\Phi}_t \leq \\ \quad \left(\overline{l}_\phi \|\overline{x}_t - x_t\|_2 + \underline{l}_\phi \|\underline{x}_t - x_t\|_2 + \underline{m}_\phi\right)^2 \\ \overline{\Phi}_t^\top \overline{\Phi}_t \leq \\ \quad \left(\overline{l}_\phi \|\overline{x}_t - x_t\|_2 + \underline{l}_\phi \|\underline{x}_t - x_t\|_2 + \overline{m}_\phi\right)^2 \end{cases}$$

where $\|\overline{x}_t - x_t\|_2^2 = \overline{e}_t^\top \overline{e}_t$ and $\|\underline{x}_t - x_t\|_2^2 = \underline{e}_t^\top \underline{e}_t$. Then, by Cauchy-Schwarz inequality:

$$\begin{cases} \underline{\Phi}_t^\top \underline{\Phi}_t \leq 3\left(\overline{l}_\phi^2 \overline{e}_t^\top \overline{e}_t + \underline{l}_\phi^2 \underline{e}_t^\top \underline{e}_t + \underline{m}_\phi^2\right) \\ \overline{\Phi}_t^\top \overline{\Phi}_t \leq 3\left(\overline{l}_\phi^2 \overline{e}_t^\top \overline{e}_t + \underline{l}_\phi^2 \underline{e}_t^\top \underline{e}_t + \overline{m}_\phi^2\right) \end{cases}$$

so that:

$$\Phi_t^\top \Phi_t \leq E_t^\top Q E_t + \beta \quad (18)$$

with $\beta = 3\left(\underline{m}_\phi^2 + \overline{m}_\phi^2\right)$.

Consider now the candidate Lyapunov function $V_t = E_t^\top P E_t$. The time derivative of V is:

$$\begin{aligned} \dot{V}_t &= \dot{E}_t^\top P E_t + E_t^\top P \dot{E}_t \\ &= E_t^\top (S^\top + S) E_t + \Phi_t^\top P E_t + E_t^\top P \Phi_t \\ &\qquad + E_t^\top P \varepsilon_t + \varepsilon_t^\top P E_t \\ &= E_t^\top (S^\top + S + \alpha P) E_t + \Phi_t^\top P E_t \\ &\quad + E_t^\top P \Phi_t + E_t^\top P \varepsilon_t + \varepsilon_t^\top P E_t \\ &\quad - \alpha E_t^\top P E_t + \gamma \Phi_t^\top \Phi_t - \gamma \Phi_t^\top \Phi_t \\ &\qquad + \gamma \varepsilon_t^\top \varepsilon_t - \gamma \varepsilon_t^\top \varepsilon_t \end{aligned}$$

so that, with (18):

$$\dot{V}_t \leq \begin{bmatrix} E_t \\ \Phi_t \\ \varepsilon_t \end{bmatrix}^\top \Lambda \begin{bmatrix} E_t \\ \Phi_t \\ \varepsilon_t \end{bmatrix} - \alpha V_t + \gamma \|\varepsilon_t\|_2^2 + \gamma\beta \quad (19)$$

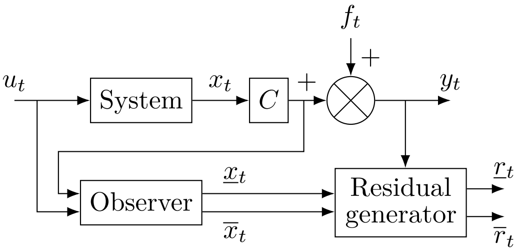

Figure 1. Structure of the fault detection strategy.

where:

$$\Lambda = \begin{bmatrix} \Lambda_{11} & P & P \\ P & -\gamma I_{2n_x} & \mathbf{0} \\ P & \mathbf{0} & -\gamma I_{2n_x} \end{bmatrix}.$$

Since $\gamma\beta$ is a positive constant, (19) is true if the following inequality is satisfied:

$$\dot{V}_t \leq \begin{bmatrix} E_t \\ \Phi_t \\ \varepsilon_t \end{bmatrix}^\top \Lambda \begin{bmatrix} E_t \\ \Phi_t \\ \varepsilon_t \end{bmatrix} - \alpha V_t + \gamma \left\|\varepsilon_t\right\|_2^2. \quad (20)$$

According to Sontag and Wang (1995), the system (17) is ISS if $\Lambda \preceq 0$ since $\left\|\varepsilon_t\right\|_2 < \infty$ by Assumptions 2.2 and 2.4. Then, if condition (15b) is satisfied, the framer (6) is a robust interval observer for (1) in the fault-free case.

In addition, since $\Lambda \preceq 0$, the quadratic term in the right hand side of inequality (20) is negative. Therefore, the following inequality is satisfied:

$$\dot{V}_t \leq -\alpha V_t + \gamma \left\|\varepsilon\right\|_\infty^2$$

or, integrating the differential inequality:

$$\begin{aligned} V_t &\leq V_0 \mathrm{e}^{-\alpha t} + \gamma \left(1 - \mathrm{e}^{-\alpha t}\right) \left\|\varepsilon\right\|_\infty^2 \\ &\leq V_0 \mathrm{e}^{-\alpha t} + \gamma \left\|\varepsilon\right\|_\infty^2 \end{aligned} \quad (21)$$

since $\mathrm{e}^{-\alpha t} \leq 1$. From (3) and (4), $R_t = \mathcal{C} E_t$ (since $f_t = \mathbf{0}$). If:

$$\left\|R_t\right\|_2^2 \leq \mu \left(V_t + (\mu - \gamma) \left\|\varepsilon\right\|_\infty^2\right) \quad (22)$$

then (16) is satisfied since V_t satisfies (21). Moreover, condition (22) is alternatively written:

$$\begin{bmatrix} P - \mathcal{C}^\top \mathcal{C}/\mu & \mathbf{0} \\ \mathbf{0} & \mu - \gamma \end{bmatrix} \succeq 0$$

which is equivalent, by Schur complement (Boyd et al., 1994), to condition (15c). □

From Theorem 3.2, the matrices $\underline{L}$ and $\overline{L}$ are obtained as $\mathrm{diag}(\underline{L}, \overline{L}) = P^{-1} Z$ by minimizing μ subject to the constraints (15a) to (15c).

3.3. *Fault detection*

The interval observer (6) tuned with the procedure described in Theorem 3.2 is used for fault detection. The structure of the fault detector is presented in Figure 1.

The fault-free output signal is injected into the interval observer (6). Then, based on equations (3) and (4), the residual generator provides the residual interval $[\underline{r}_t, \overline{r}_t]$. Therefore, if $\mathbf{0} \notin [\underline{r}_t, \overline{r}_t]$, the system is subject to a fault $f_t \neq \mathbf{0}$ and if $\mathbf{0} \in [\underline{r}_t, \overline{r}_t]$, the system is fault-free or subject to a low-magnitude fault which cannot be detected by the proposed fault detection scheme.

Remark 3.1. The present paper focuses on the design of a TNL interval observer with L_∞ performance for robust sensor fault detection. The sensibility analysis of the proposed interval observer, characterized for example by the minimum detectable fault approach (Meseguer et al., 2010; Pourasghar et al., 2020), is therefore not addressed here and left to subsequent works.

4. Simulation results

To assess the efficiency of the proposed fault detection algorithm, the example of a dampened mass-spring system (Scherer, 2012) is considered. This system obeys the LPV dynamics:

$$\begin{aligned} \dot{x}_t &= \begin{bmatrix} 0 & 1 \\ 2 + \rho_t & -1 \end{bmatrix} x_t + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_t + w_t \\ y_t &= [1 \ 0]\, x_t + f_t \end{aligned}$$

where $x_t^\top = [p_t \ \dot{p}_t]$, with p_t the position of the mass, so that:

$$A_0 = \begin{bmatrix} 0 & 1 \\ -2 & -1 \end{bmatrix}, \qquad \Delta A(\rho_t) = \begin{bmatrix} 0 & 0 \\ \rho_t & 0 \end{bmatrix},$$

$$B_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \qquad \Delta B(\rho_t) = \mathbf{0},$$

$D_0 = I_2$, $\Delta D(\rho_t) = \mathbf{0}$ and $C = [1 \ 0]$.

For this example, it is assumed that $\rho_t = \sin(0.3t)$, u_t is the square wave $u_t = \mathrm{sgn}(\sin(t))$ and $w_t^\top = 0.1\,[\cos(2t) \ \sin(3t)]$. Therefore, $\overline{w}_t = -\underline{w}_t = 0.1$ and:

$$\overline{\Delta A} = -\underline{\Delta A} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}.$$

The initial state vector is $x_0 = \mathbf{0}$ and the bounds for the initial state are $\overline{x}_0 = -\underline{x}_0 = 0.1 \cdot \mathbf{1}_2$.

The value of the matrix Ξ is a design parameter of the proposed interval observer. It can be chosen in different ways, such as to minimize the values of $\underline{l}_\phi$ and $\overline{l}_\phi$. For the sake of simplicity, it is chosen here as:

$$\Xi = \begin{bmatrix} 0.1 & 0 & -0.1 \\ -3 & 0 & 3 \end{bmatrix}$$

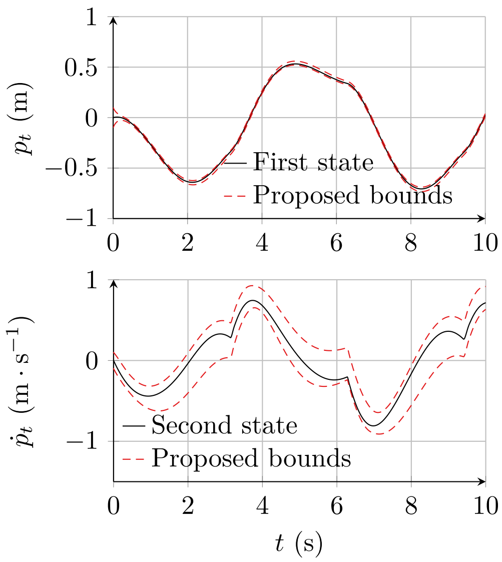

Figure 2. States and guaranteed bounds from the proposed interval observer.

so that:

$$T = \begin{bmatrix} 0.6 & 0 \\ -3 & 1 \end{bmatrix} \qquad N = \begin{bmatrix} 0.4 \\ 3 \end{bmatrix}$$

and $\underline{l}_\phi = \overline{l}_\phi = 1$. Minimizing μ under the constraints (15) with $\alpha = 0.1$ and $\eta = 10$, $\mu = \gamma = 0.3384$ and:

$$\underline{L} = \overline{L} = \begin{bmatrix} 10 \\ -2 \end{bmatrix}.$$

Remark 4.1. Due to the symmetry of the constraints (15), the values of $\underline{L}$ and $\overline{L}$ are equal. However, the L_∞ design procedure proposed in Theorem 3.2 allows for the introduction of additional constraints on the observer gains that could lead to different values.

Finally, the sensor fault signal considered is:

$$f_t = \begin{cases} 0.1 & \text{if } 2 \le t \le 4 \\ 0.05 \cdot (t-7) & \text{if } 7 \le t \le 9 \\ 0 & \text{otherwise} \end{cases}$$

The intervals obtained with the proposed interval observer for the two states are presented in Figure 2 and the residual interval is presented in Figure 3. The states are contained in the computed intervals. Despite the presence of a disturbance signal w_t acting on the system, the fault f_t is detected since, between $t = 2$ s and $t = 4$ s, 0 is not contained in the residual interval $[\underline{r}_t, \overline{r}_t]$.. However, due to this same perturbation signal, the fault f_t appearing between $t = 7$ s and $t = 9$ s is not detected for the first 0.3 s. Indeed, between $t = 7$ s and $t = 7.3$ s, the magnitude of the fault is too low to be detectable. The sensibility analysis mentioned in Remark 3.1 would characterize the minimal level of detectable fault. Moreover, there is no false positive since $0 \in [\underline{r}_t, \overline{r}_t]$ when the system is fault-free.

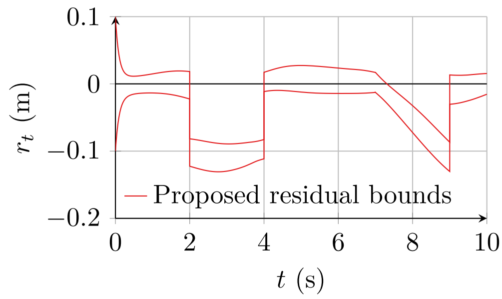

Figure 3. Proposed residual bounds.

5. Conclusion

This paper has presented a new strategy for robust sensor fault detection for continuous-time linear parameter-varying systems with unmeasurable parameter vector subject to unknown but bounded disturbances. An interval observer based on the TNL formalism is used to provide bounds for the residual signal, thus reducing the conservatism of classical interval observer approaches by introducing weighting matrices to provide more degrees of freedom in the observer design. A L_∞ procedure is proposed to tune the gains of the observer, thus attenuating the effect of the unknown disturbance on the estimation process. With this procedure, additional constraints can be easily introduced in the design problem. Simulation results are presented to assess the efficiency of the proposed fault detection algorithm. This method can then be applied to any linear system presenting bounded parametric uncertainties, subject to bounded perturbations and equipped with sensors providing continuous-time measurements. In future work, the sensibility analysis of the proposed observer has to be studied. In addition, the proposed strategy could be adapted to systems subject to actuator or input sensor faults.

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