Definition and Classification of Power System Stability

IEEE/CIGRE Joint Task Force on Stability Terms and Definitions

Prabha Kundur (Canada, Convener), *John Paserba (USA, Secretary)*, *Venkat Ajjarapu (USA)*, *Göran Andersson (Switzerland)*, *Anjan Bose (USA)* , *Claudio Canizares (Canada)*, *Nikos Hatziargyriou (Greece)*, *David Hill (Australia)*, *Alex Stankovic (USA)*, *Carson Taylor (USA)*, *Thierry Van Cutsem (Belgium)*, and *Vijay Vittal (USA)*

***Abstract*—The problem of defining and classifying power system stability has been addressed by several previous CIGRE and IEEE Task Force reports. These earlier efforts, however, do not completely reflect current industry needs, experiences and understanding. In particular, the definitions are not precise and the classifications do not encompass all practical instability scenarios.**

This report developed by a Task Force, set up jointly by the CIGRE Study Committee 38 and the IEEE Power System Dynamic Performance Committee, addresses the issue of stability definition and classification in power systems from a fundamental viewpoint and closely examines the practical ramifications. The report aims to define power system stability more precisely, provide a systematic basis for its classification, and discuss linkages to related issues such as power system reliability and security.

***Index Terms*—Frequency stability, Lyapunov stability, oscillatory stability, power system stability, small-signal stability, terms and definitions, transient stability, voltage stability.**

I. Introduction

POWER system stability has been recognized as an important problem for secure system operation since the 1920s [1], [2]. Many major blackouts caused by power system instability have illustrated the importance of this phenomenon [3]. Historically, transient instability has been the dominant stability problem on most systems, and has been the focus of much of the industry's attention concerning system stability. As power systems have evolved through continuing growth in interconnections, use of new technologies and controls, and the increased operation in highly stressed conditions, different forms of system instability have emerged. For example, voltage stability, frequency stability and interarea oscillations have become greater concerns than in the past. This has created a need to review the definition and classification of power system stability. A clear understanding of different types of instability and how they are interrelated is essential for the satisfactory design and operation of power systems. As well, consistent use of terminology is required for developing system design and operating criteria, standard analytical tools, and study procedures.

The problem of defining and classifying power system stability is an old one, and there have been several previous reports on the subject by CIGRE and IEEE Task Forces [4]–[7]. These, however, do not completely reflect current industry needs, experiences, and understanding. In particular, definitions are not precise and the classifications do not encompass all practical instability scenarios.

This report is the result of long deliberations of the Task Force set up jointly by the CIGRE Study Committee 38 and the IEEE Power System Dynamic Performance Committee. Our objectives are to:

- Define power system stability more precisely, inclusive of all forms.
- Provide a systematic basis for classifying power system stability, identifying and defining different categories, and providing a broad picture of the phenomena.
- Discuss linkages to related issues such as power system reliability and security.

Power system stability is similar to the stability of any dynamic system, and has fundamental mathematical underpinnings. Precise definitions of stability can be found in the literature dealing with the rigorous mathematical theory of stability of dynamic systems. Our intent here is to provide a physically motivated definition of power system stability which in broad terms conforms to precise mathematical definitions.

The report is organized as follows. In Section II the definition of Power System Stability is provided. A detailed discussion and elaboration of the definition are presented. The conformance of this definition with the system theoretic definitions is established. Section III provides a detailed classification of power system stability. In Section IV of the report the relationship between the concepts of power system reliability, security, and stability is discussed. A description of how these terms have been defined and used in practice is also provided. Finally, in Section V definitions and concepts of stability from mathematics and control theory are reviewed to provide background information concerning stability of dynamic systems in general and to establish theoretical connections.

The analytical definitions presented in Section V constitute a key aspect of the report. They provide the mathematical underpinnings and bases for the definitions provided in the earlier sections. These details are provided at the end of the report so that interested readers can examine the finer points and assimilate the mathematical rigor.

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II. Definition of Power System Stability

In this section, we provide a formal definition of power system stability. The intent is to provide a physically based definition which, while conforming to definitions from system theory, is easily understood and readily applied by power system engineering practitioners.

A. Proposed Definition

- *Power system stability is the ability of an electric power system, for a given initial operating condition, to regain a state of operating equilibrium after being subjected to a physical disturbance, with most system variables bounded so that practically the entire system remains intact.*

B. Discussion and Elaboration

The definition applies to an interconnected power system as a whole. Often, however, the stability of a particular generator or group of generators is also of interest. A remote generator may lose *stability* (synchronism) without cascading instability of the main system. Similarly, stability of particular loads or load areas may be of interest; motors may lose *stability* (run down and stall) without cascading instability of the main system.

The power system is a highly nonlinear system that operates in a constantly changing environment; loads, generator outputs and key operating parameters change continually. When subjected to a disturbance, the stability of the system depends on the initial operating condition as well as the nature of the disturbance.

Stability of an electric power system is thus a property of the system motion around an equilibrium set, i.e., the initial operating condition. In an equilibrium set, the various opposing forces that exist in the system are equal instantaneously (as in the case of equilibrium points) or over a cycle (as in the case of slow cyclical variations due to continuous small fluctuations in loads or aperiodic attractors).

Power systems are subjected to a wide range of disturbances, small and large. Small disturbances in the form of load changes occur continually; the system must be able to adjust to the changing conditions and operate satisfactorily. It must also be able to survive numerous disturbances of a severe nature, such as a short circuit on a transmission line or loss of a large generator. A large disturbance may lead to structural changes due to the isolation of the faulted elements.

At an equilibrium set, a power system may be stable for a given (large) physical disturbance, and unstable for another. It is impractical and uneconomical to design power systems to be stable for every possible disturbance. The design contingencies are selected on the basis they have a reasonably high probability of occurrence. Hence, large-disturbance stability always refers to a specified disturbance scenario. A stable equilibrium set thus has a finite region of attraction; the larger the region, the more robust the system with respect to large disturbances. The region of attraction changes with the operating condition of the power system.

The response of the power system to a disturbance may involve much of the equipment. For instance, a fault on a critical element followed by its isolation by protective relays will cause variations in power flows, network bus voltages, and machine rotor speeds; the voltage variations will actuate both generator and transmission network voltage regulators; the generator speed variations will actuate prime mover governors; and the voltage and frequency variations will affect the system loads to varying degrees depending on their individual characteristics. Further, devices used to protect individual equipment may respond to variations in system variables and cause tripping of the equipment, thereby weakening the system and possibly leading to system instability.

If following a disturbance the power system is stable, it will reach a new equilibrium state with the system integrity preserved i.e., with practically all generators and loads connected through a single contiguous transmission system. Some generators and loads may be disconnected by the isolation of faulted elements or intentional tripping to preserve the continuity of operation of bulk of the system. Interconnected systems, for certain severe disturbances, may also be intentionally split into two or more "islands" to preserve as much of the generation and load as possible. The actions of automatic controls and possibly human operators will eventually restore the system to normal state. On the other hand, if the system is unstable, it will result in a run-away or run-down situation; for example, a progressive increase in angular separation of generator rotors, or a progressive decrease in bus voltages. An unstable system condition could lead to cascading outages and a shutdown of a major portion of the power system.

Power systems are continually experiencing fluctuations of small magnitudes. However, for assessing stability when subjected to a specified disturbance, it is usually valid to assume that the system is initially in a true steady-state operating condition.

C. Conformance With System—Theoretic Definitions

In Section II-A, we have formulated the definition by considering a given operating condition and the system being subjected to a physical disturbance. Under these conditions we require the system to either regain a new state of operating equilibrium or return to the original operating condition (if no topological changes occurred in the system). These requirements are directly correlated to the system-theoretic definition of asymptotic stability given in Section V-C-I. It should be recognized here that this definition requires the equilibrium to be (a) stable in the sense of Lyapunov, i.e., all initial conditions starting in a small spherical neighborhood of radius δ result in the system trajectory remaining in a cylinder of radius ε for all time $t \geq t_0$, the initial time which corresponds to all of the system state variables being bounded, and (b) at time $t \to \infty$ the system trajectory approaches the equilibrium point which corresponds to the equilibrium point being attractive. As a result, one observes that the analytical definition directly correlates to the expected behavior in a physical system.

III. Classification of Power System Stability

A typical modern power system is a high-order multivariable process whose dynamic response is influenced by a wide array of devices with different characteristics and response rates. Stability is a condition of equilibrium between opposing forces. Depending on the network topology, system operating condition and the form of disturbance, different sets of opposing forces may experience sustained imbalance leading to different forms of instability. In this section, we provide a systematic basis for classification of power system stability.

A. Need for Classification

Power system stability is essentially a single problem; however, the various forms of instabilities that a power system may undergo cannot be properly understood and effectively dealt with by treating it as such. Because of high dimensionality and complexity of stability problems, it helps to make simplifying assumptions to analyze specific types of problems using an appropriate degree of detail of system representation and appropriate analytical techniques. Analysis of stability, including identifying key factors that contribute to instability and devising methods of improving stable operation, is greatly facilitated by classification of stability into appropriate categories [8]. Classification, therefore, is essential for meaningful practical analysis and resolution of power system stability problems. As discussed in Section V-C-I, such classification is entirely justified theoretically by the concept of *partial stability* [9]–[11].

B. Categories of Stability

The classification of power system stability proposed here is based on the following considerations [8]:

- The physical nature of the resulting mode of instability as indicated by the main system variable in which instability can be observed.
- The size of the disturbance considered, which influences the method of calculation and prediction of stability.
- The devices, processes, and the time span that must be taken into consideration in order to assess stability.

Fig. 1 gives the overall picture of the power system stability problem, identifying its categories and subcategories. The following are descriptions of the corresponding forms of stability phenomena.

B.1 Rotor Angle Stability:

Rotor angle stability refers to the ability of synchronous machines of an interconnected power system to remain in synchronism after being subjected to a disturbance. It depends on the ability to maintain/restore equilibrium between electromagnetic torque and mechanical torque of each synchronous machine in the system. Instability that may result occurs in the form of increasing angular swings of some generators leading to their loss of synchronism with other generators.

The rotor angle stability problem involves the study of the electromechanical oscillations inherent in power systems. A fundamental factor in this problem is the manner in which the power outputs of synchronous machines vary as their rotor angles change. Under steady-state conditions, there is equilibrium between the input mechanical torque and the output electromagnetic torque of each generator, and the speed remains constant. If the system is perturbed, this equilibrium is upset, resulting in acceleration or deceleration of the rotors of the machines according to the laws of motion of a rotating body. If one generator temporarily runs faster than another, the angular position of its rotor relative to that of the slower machine will advance. The resulting angular difference transfers part of the load from the slow machine to the fast machine, depending on the power-angle relationship. This tends to reduce the speed difference and hence the angular separation. The power-angle relationship is highly nonlinear. Beyond a certain limit, an increase in angular separation is accompanied by a decrease in power transfer such that the angular separation is increased further. Instability results if the system cannot absorb the kinetic energy corresponding to these rotor speed differences. For any given situation, the stability of the system depends on whether or not the deviations in angular positions of the rotors result in sufficient restoring torques [8]. Loss of synchronism can occur between one machine and the rest of the system, or between groups of machines, with synchronism maintained within each group after separating from each other.

The change in electromagnetic torque of a synchronous machine following a perturbation can be resolved into two components:

- *Synchronizing torque component*, in phase with rotor angle deviation.
- *Damping torque component*, in phase with the speed deviation.

System stability depends on the existence of both components of torque for each of the synchronous machines. Lack of sufficient synchronizing torque results in *aperiodic* or *nonoscillatory instability*, whereas lack of damping torque results in *oscillatory instability*.

For convenience in analysis and for gaining useful insight into the nature of stability problems, it is useful to characterize rotor angle stability in terms of the following two subcategories:

- *Small-disturbance (or small-signal) rotor angle stability* is concerned with the ability of the power system to maintain synchronism under small disturbances. The disturbances are considered to be sufficiently small that linearization of system equations is permissible for purposes of analysis [8], [12], [13].
  - Small-disturbance stability depends on the initial operating state of the system. Instability that may result can be of two forms: i) increase in rotor angle through a nonoscillatory or aperiodic mode due to lack of synchronizing torque, or ii) rotor oscillations of increasing amplitude due to lack of sufficient damping torque.
  - In today's power systems, small-disturbance rotor angle stability problem is usually associated with insufficient damping of oscillations. The aperiodic instability problem has been largely eliminated by use of continuously acting generator voltage regulators; however, this problem can still occur when generators operate with constant excitation when subjected to the actions of excitation limiters (field current limiters).

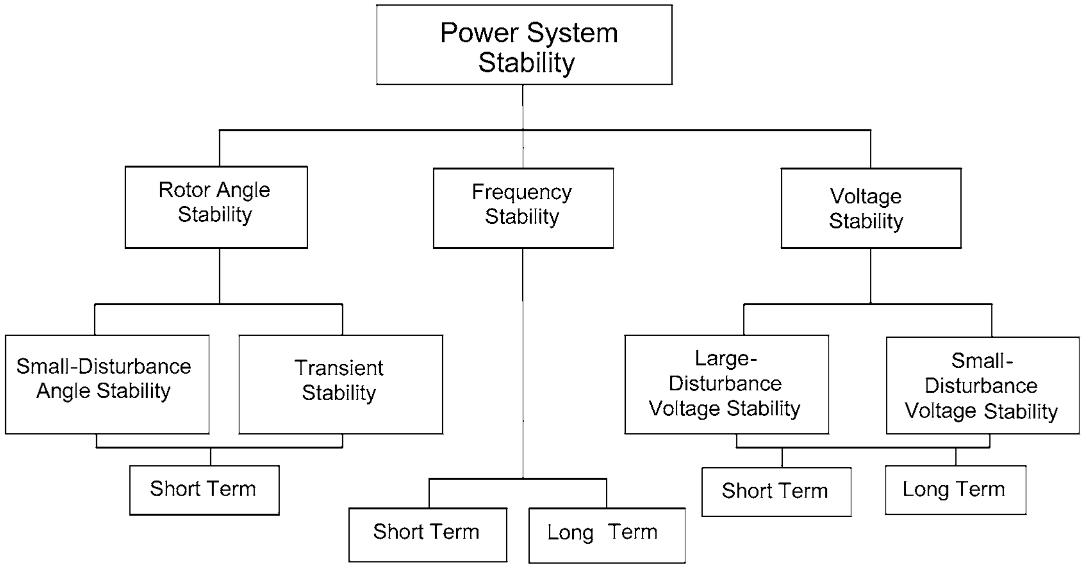

Fig. 1. Classification of power system stability.

- Small-disturbance rotor angle stability problems may be either local or global in nature. Local problems involve a small part of the power system, and are usually associated with rotor angle oscillations of a single power plant against the rest of the power system. Such oscillations are called *local plant mode oscillations*. Stability (damping) of these oscillations depends on the strength of the transmission system as seen by the power plant, generator excitation control systems and plant output [8].

- Global problems are caused by interactions among large groups of generators and have widespread effects. They involve oscillations of a group of generators in one area swinging against a group of generators in another area. Such oscillations are called *interarea mode oscillations*. Their characteristics are very complex and significantly differ from those of local plant mode oscillations. Load characteristics, in particular, have a major effect on the stability of interarea modes [8].

- The time frame of interest in small-disturbance stability studies is on the order of 10 to 20 seconds following a disturbance.

• *Large-disturbance rotor angle stability* or *transient stability*, as it is commonly referred to, is concerned with the ability of the power system to maintain synchronism when subjected to a severe disturbance, such as a short circuit on a transmission line. The resulting system response involves large excursions of generator rotor angles and is influenced by the nonlinear power-angle relationship.

- Transient stability depends on both the initial operating state of the system and the severity of the disturbance. Instability is usually in the form of aperiodic angular separation due to insufficient synchronizing torque, manifesting as *first swing instability*. However, in large power systems, transient instability may not always occur as first swing instability associated with a single mode; it could be a result of superposition of a slow interarea swing mode and a local-plant swing mode causing a large excursion of rotor angle beyond the first swing [8]. It could also be a result of nonlinear effects affecting a single mode causing instability beyond the first swing.

- The time frame of interest in transient stability studies is usually 3 to 5 seconds following the disturbance. It may extend to 10–20 seconds for very large systems with dominant inter-area swings.

As identified in Fig. 1, small-disturbance rotor angle stability as well as transient stability are categorized as *short term* phenomena.

The term *dynamic stability* also appears in the literature as a class of rotor angle stability. However, it has been used to denote different phenomena by different authors. In the North American literature, it has been used mostly to denote small-disturbance stability in the presence of automatic controls (particularly, the generation excitation controls) as distinct from the classical "steady-state stability" with no generator controls [7], [8]. In the European literature, it has been used to denote transient stability. Since much confusion has resulted from the use of the term dynamic stability, we recommend against its usage, as did the previous IEEE and CIGRE Task Forces [6], [7].

B.2 Voltage Stability:

Voltage stability refers to the ability of a power system to maintain steady voltages at all buses in the system after being subjected to a disturbance from a given initial operating condition. It depends on the ability to maintain/restore equilibrium between load demand and load supply from the power system. Instability that may result occurs in the form of a progressive fall or rise of voltages of some buses. A possible outcome of voltage instability is loss of load in an area, or tripping of transmission lines and other elements by their protective systems leading to cascading outages. Loss of synchronism of some generators may result from these outages or from operating conditions that violate field current limit [14].

Progressive drop in bus voltages can also be associated with rotor angle instability. For example, the loss of synchronism of machines as rotor angles between two groups of machines approach 180° causes rapid drop in voltages at intermediate points in the network close to the electrical center [8]. Normally, protective systems operate to separate the two groups of machines and the voltages recover to levels depending on the post-separation conditions. If, however, the system is not so separated, the voltages near the electrical center rapidly oscillate between high and low values as a result of repeated "pole slips" between the two groups of machines. In contrast, the type of sustained fall of voltage that is related to voltage instability involves loads and may occur where rotor angle stability is not an issue.

The term *voltage collapse* is also often used. It is the process by which the sequence of events accompanying voltage instability leads to a blackout or abnormally low voltages in a significant part of the power system [8], [15], and [16]. Stable (steady) operation at low voltage may continue after transformer tap changers reach their boost limit, with intentional and/or unintentional tripping of some load. Remaining load tends to be voltage sensitive, and the connected demand at normal voltage is not met.

The driving force for voltage instability is usually the loads; in response to a disturbance, power consumed by the loads tends to be restored by the action of motor slip adjustment, distribution voltage regulators, tap-changing transformers, and thermostats. Restored loads increase the stress on the high voltage network by increasing the reactive power consumption and causing further voltage reduction. A run-down situation causing voltage instability occurs when load dynamics attempt to restore power consumption beyond the capability of the transmission network and the connected generation [8], [14]–[18].

A major factor contributing to voltage instability is the voltage drop that occurs when active and reactive power flow through inductive reactances of the transmission network; this limits the capability of the transmission network for power transfer and voltage support. The power transfer and voltage support are further limited when some of the generators hit their field or armature current time-overload capability limits. Voltage stability is threatened when a disturbance increases the reactive power demand beyond the sustainable capacity of the available reactive power resources.

While the most common form of voltage instability is the progressive drop of bus voltages, the risk of overvoltage instability also exists and has been experienced at least on one system [19]. It is caused by a capacitive behavior of the network (EHV transmission lines operating below surge impedance loading) as well as by underexcitation limiters preventing generators and/or synchronous compensators from absorbing the excess reactive power. In this case, the instability is associated with the inability of the combined generation and transmission system to operate below some load level. In their attempt to restore this load power, transformer tap changers cause long-term voltage instability.

Voltage stability problems may also be experienced at the terminals of HVDC links used for either long distance or back-to-back applications [20], [21]. They are usually associated with HVDC links connected to weak ac systems and may occur at rectifier or inverter stations, and are associated with the unfavorable reactive power "load" characteristics of the converters. The HVDC link control strategies have a very significant influence on such problems, since the active and reactive power at the ac/dc junction are determined by the controls. If the resulting loading on the ac transmission stresses it beyond its capability, voltage instability occurs. Such a phenomenon is relatively fast with the time frame of interest being in the order of one second or less. Voltage instability may also be associated with converter transformer tap-changer controls, which is a considerably slower phenomenon [21]. Recent developments in HVDC technology (voltage source converters and capacitor commutated converters) have significantly increased the limits for stable operation of HVDC links in weak systems as compared with the limits for line commutated converters.

One form of voltage stability problem that results in uncontrolled overvoltages is the self-excitation of synchronous machines. This can arise if the capacitive load of a synchronous machine is too large. Examples of excessive capacitive loads that can initiate self-excitation are open ended high voltage lines and shunt capacitors and filter banks from HVDC stations [22]. The overvoltages that result when generator load changes to capacitive are characterized by an instantaneous rise at the instant of change followed by a more gradual rise. This latter rise depends on the relation between the capacitive load component and machine reactances together with the excitation system of the synchronous machine. Negative field current capability of the exciter is a feature that has a positive influence on the limits for self-excitation.

As in the case of rotor angle stability, it is useful to classify voltage stability into the following subcategories:

- *Large-disturbance voltage stability* refers to the system's ability to maintain steady voltages following large disturbances such as system faults, loss of generation, or circuit contingencies. This ability is determined by the system and load characteristics, and the interactions of both continuous and discrete controls and protections. Determination of large-disturbance voltage stability requires the examination of the nonlinear response of the power system over a period of time sufficient to capture the performance and interactions of such devices as motors, underload transformer tap changers, and generator field-current limiters. The study period of interest may extend from a few seconds to tens of minutes.
- *Small-disturbance voltage stability* refers to the system's ability to maintain steady voltages when subjected to small perturbations such as incremental changes in system load. This form of stability is influenced by the characteristics of loads, continuous controls, and discrete controls at a given instant of time. This concept is useful in determining, at any instant, how the system voltages will respond to small system changes. With appropriate assumptions, system equations can be linearized for analysis thereby allowing

computation of valuable sensitivity information useful in identifying factors influencing stability. This linearization, however, cannot account for nonlinear effects such as tap changer controls (deadbands, discrete tap steps, and time delays). Therefore, a combination of linear and nonlinear analyzes is used in a complementary manner [23], [24].

As noted above, the time frame of interest for voltage stability problems may vary from a few seconds to tens of minutes. Therefore, voltage stability may be either a short-term or a long-term phenomenon as identified in Figure 1.

- *Short-term voltage stability* involves dynamics of fast acting load components such as induction motors, electronically controlled loads, and HVDC converters. The study period of interest is in the order of several seconds, and analysis requires solution of appropriate system differential equations; this is similar to analysis of rotor angle stability. Dynamic modeling of loads is often essential. In contrast to angle stability, short circuits near loads are important. It is recommended that the term *transient voltage stability* not be used.
- *Long-term voltage stability* involves slower acting equipment such as tap-changing transformers, thermostatically controlled loads, and generator current limiters. The study period of interest may extend to several or many minutes, and long-term simulations are required for analysis of system dynamic performance [17], [23], [25]. Stability is usually determined by the resulting outage of equipment, rather than the severity of the initial disturbance. Instability is due to the loss of long-term equilibrium (e.g., when loads try to restore their power beyond the capability of the transmission network and connected generation), post-disturbance steady-state operating point being small-disturbance unstable, or a lack of attraction toward the stable post-disturbance equilibrium (e.g., when a remedial action is applied too late) [14], [15]. The disturbance could also be a sustained load buildup (e.g., morning load increase). In many cases, static analysis [23], [24], [26], [27] can be used to estimate stability margins, identify factors influencing stability, and screen a wide range of system conditions and a large number of scenarios. Where timing of control actions is important, this should be complemented by quasi-steady-state time-domain simulations [14], [17].

B.3 Basis for Distinction between Voltage and Rotor Angle Stability:

It is important to recognize that the distinction between rotor angle stability and voltage stability is not based on weak coupling between variations in active power/angle and reactive power/voltage magnitude. In fact, coupling is strong for stressed conditions and both rotor angle stability and voltage stability are affected by pre-disturbance active power as well as reactive power flows. Instead, the distinction is based on the specific set of opposing forces that experience sustained imbalance and the principal system variable in which the consequent instability is apparent.

B.4 Frequency Stability:

Frequency stability refers to the ability of a power system to maintain steady frequency following a severe system upset resulting in a significant imbalance between generation and load. It depends on the ability to maintain/restore equilibrium between system generation and load, with minimum unintentional loss of load. Instability that may result occurs in the form of sustained frequency swings leading to tripping of generating units and/or loads.

Severe system upsets generally result in large excursions of frequency, power flows, voltage, and other system variables, thereby invoking the actions of processes, controls, and protections that are not modeled in conventional transient stability or voltage stability studies. These processes may be very slow, such as boiler dynamics, or only triggered for extreme system conditions, such as volts/Hertz protection tripping generators. In large interconnected power systems, this type of situation is most commonly associated with conditions following splitting of systems into islands. Stability in this case is a question of whether or not each island will reach a state of operating equilibrium with minimal unintentional loss of load. It is determined by the overall response of the island as evidenced by its mean frequency, rather than relative motion of machines. Generally, frequency stability problems are associated with inadequacies in equipment responses, poor coordination of control and protection equipment, or insufficient generation reserve. Examples of such problems are reported in references [28]–[31]. In isolated island systems, frequency stability could be of concern for any disturbance causing a relatively significant loss of load or generation [32].

During frequency excursions, the characteristic times of the processes and devices that are activated will range from fraction of seconds, corresponding to the response of devices such as underfrequency load shedding and generator controls and protections, to several minutes, corresponding to the response of devices such as prime mover energy supply systems and load voltage regulators. Therefore, as identified in Fig. 1, frequency stability may be a *short-term* phenomenon or a *long-term* phenomenon. An example of short-term frequency instability is the formation of an undergenerated island with insufficient underfrequency load shedding such that frequency decays rapidly causing blackout of the island within a few seconds [28]. On the other hand, more complex situations in which frequency instability is caused by steam turbine overspeed controls [29] or boiler/reactor protection and controls are longer-term phenomena with the time frame of interest ranging from tens of seconds to several minutes [30], [31], [33].

During frequency excursions, voltage magnitudes may change significantly, especially for islanding conditions with underfrequency load shedding that unloads the system. Voltage magnitude changes, which may be higher in percentage than frequency changes, affect the load-generation imbalance. High voltage may cause undesirable generator tripping by poorly designed or coordinated loss of excitation relays or volts/Hertz relays. In an overloaded system, low voltage may cause undesirable operation of impedance relays.

B.5 Comments on Classification:

We have classified power system stability for convenience in identifying causes of instability, applying suitable analysis tools, and developing corrective measures. In any given situation, however, any one form of instability may not occur in its pure form. This is particularly true in highly stressed systems and for cascading events; as systems fail one form of instability may ultimately lead to another form. However, distinguishing between different forms is important for understanding the underlying causes of the problem in order to develop appropriate design and operating procedures.

While classification of power system stability is an effective and convenient means to deal with the complexities of the problem, the overall stability of the system should always be kept in mind. Solutions to stability problems of one category should not be at the expense of another. It is essential to look at all aspects of the stability phenomenon, and at each aspect from more than one viewpoint.

IV. Relationship Between Reliability, Security, and Stability

In this section, we discuss the relationship between the concepts of power system reliability, security, and stability. We will also briefly describe how these terms have been defined and used in practice.

A. Conceptual Relationship [34], [35]

Reliability of a power system refers to the probability of its satisfactory operation over the long run. It denotes the ability to supply adequate electric service on a nearly continuous basis, with few interruptions over an extended time period.

Security of a power system refers to the degree of risk in its ability to survive imminent disturbances (contingencies) without interruption of customer service. It relates to robustness of the system to imminent disturbances and, hence, depends on the system operating condition as well as the contingent probability of disturbances.

Stability of a power system, as discussed in Section II, refers to the continuance of intact operation following a disturbance. It depends on the operating condition and the nature of the physical disturbance.

The following are the essential differences among the three aspects of power system performance:

1) Reliability is the overall objective in power system design and operation. To be reliable, the power system must be secure most of the time. To be secure, the system must be stable but must also be secure against other contingencies that would not be classified as stability problems e.g., damage to equipment such as an explosive failure of a cable, fall of transmission towers due to ice loading or sabotage. As well, a system may be stable following a contingency, yet insecure due to post-fault system conditions resulting in equipment overloads or voltage violations.
2) System security may be further distinguished from stability in terms of the resulting consequences. For example, two systems may both be stable with equal stability margins, but one may be relatively more secure because the consequences of instability are less severe.
3) Security and stability are time-varying attributes which can be judged by studying the performance of the power system under a particular set of conditions. Reliability, on the other hand, is a function of the time-average performance of the power system; it can only be judged by consideration of the system's behavior over an appreciable period of time.

B. NERC Definition of Reliability [36]

NERC (North American Electric Reliability Council) defines power system reliability as follows.

- *Reliability, in a bulk power electric system, is the degree to which the performance of the elements of that system results in power being delivered to consumers within accepted standards and in the amount desired. The degree of reliability may be measured by the frequency, duration, and magnitude of adverse effects on consumer service.*

Reliability can be addressed by considering two basic functional aspects of the power systems:

Adequacy—the ability of the power system to supply the aggregate electric power and energy requirements of the customer at all times, taking into account scheduled and unscheduled outages of system components.

Security—the ability of the power system to withstand sudden disturbances such as electric short circuits or nonanticipated loss of system components.

The above definitions also appear in several IEEE and CIGRE Working Group/Task Force documents [37], [38].

Other alternative forms of definition of power system security have been proposed in the literature. For example, in reference [39], security is defined in terms of satisfying a set of inequality constraints over a subset of the possible disturbances called the "next contingency set."

C. Analysis of Power System Security

The analysis of security relates to the determination of the robustness of the power system relative to imminent disturbances. There are two important components of security analysis. For a power system subjected to changes (small or large), it is important that when the changes are completed, the system settles to new operating conditions such that no physical constraints are violated. This implies that, in addition to the next operating conditions being acceptable, the system must survive the transition to these conditions.

The above characterization of system security clearly highlights two aspects of its analysis:

- *Static security analysis*—This involves steady-state analysis of post-disturbance system conditions to verify that no equipment ratings and voltage constraints are violated.
- *Dynamic security analysis*—This involves examining different categories of system stability described in Section III.

Stability analysis is thus an integral component of system security and reliability assessment.

The general industry practice for security assessment has been to use a deterministic approach. The power system is designed and operated to withstand a set of contingencies referred to as "normal contingencies" selected on the basis that they have a significant likelihood of occurrence. In practice, they are usually defined as the loss of any single element in a power system either spontaneously or preceded by a single-, double-, or three-phase fault. This is usually referred to as the $N-1$ criterion because it examines the behavior of an N-component grid following the loss of any one of its major components. In addition, loss of load or cascading outages may not be allowed for multiple-related outages such as loss of a double-circuit line. Consideration may be given to extreme contingencies that exceed in severity the normal design contingencies. Emergency controls, such as generation tripping, load shedding, and controlled islanding, may be used to cope with such events and prevent widespread blackouts.

The deterministic approach has served the industry reasonably well in the past—it has resulted in high security levels and the study effort is minimized. Its main limitation, however, is that it treats all security-limiting scenarios as having the same risk. It also does not give adequate consideration as to how likely or unlikely various contingencies are.

In today's utility environment, with a diversity of participants with different business interests, the deterministic approach may not be acceptable. There is a need to account for the probabilistic nature of system conditions and events, and to quantify and manage risk. The trend will be to expand the use of risk-based security assessment. In this approach, the probability of the system becoming unstable and its consequences are examined, and the degree of exposure to system failure is estimated. This approach is computationally intensive but is possible with today's computing and analysis tools.

V. System-Theoretic Foundations of Power System Stability

A. Preliminaries

In this section, we address fundamental issues related to definitions of power system stability from a system-theoretic viewpoint. We assume that the model of a power system is given in the form of explicit first-order differential equations (i.e., a state-space description). While this is quite common in the theory of dynamical systems, it may not always be entirely natural for physical systems such as power systems. First-principle models that are typically used to describe power systems are seldom in this form, and transformations required to bring them to explicit first-order form may, in general, introduce spurious solutions [40].

More importantly, there often exist algebraic (implicit) equations that constrain various quantities, and a set of differential-algebraic equations (DAE) is often used in simulations of power system transients. The algebraic part often arises from a singular perturbation-type reasoning that uses time separation between subsets of variables to postulate that the fast variables have already reached their steady state on the time horizon of interest [41], [42].

Proving the existence of solutions of DAE is a very challenging problem in general. While local results can be derived from the implicit function theorem that specifies rank conditions for the Jacobian of the algebraic part, the non-local results are much harder to obtain. One general approach to non-local study of stability of DAE systems that is based on differential geometry is presented in [43] (for sufficient conditions see [44]). The surfaces on which rank conditions for the Jacobian of the algebraic part do not hold are commonly denoted as *impasse* surfaces, and in the analysis of models of power systems, it is typically assumed that equilibrium sets of interest in stability analysis are disjoint from such surfaces [41], [45].

An often useful approximation of the fast dynamics is based on the concept of dynamic (time-varying) phasors [46] and dynamic symmetrical components [47]. It is also typically assumed that distributed nature of some elements of a power system (e.g., transmission lines) can be approximated with lumped parameter models without a major loss of model fidelity. This is mostly dictated by the fundamental intractability of models that include partial differential equations, and by satisfactory behavior of lumped parameter models (when evaluated on the level of single element—e.g., the use of multiple "π" section models for a long transmission line). Some qualitative aspects of fault propagation in spatially extended power systems can, however, be studied effectively with distributed models [48].

Power systems are also an example of constrained dynamical systems, as their state trajectories are restricted to a particular subset in the state space (phase space in the language of dynamical systems) denoted as the feasible (or technically viable, or permitted) operating region [45]. The trajectories that exit this desired region may either lead to structural changes (e.g., breaker tripping in a power system), or lead to unsafe operation. This type of consideration will introduce restricted stability regions in power system stability analysis.

Several additional issues are raised by the fact that the power system interacts with its (typically unmodeled) environment, making the power system model nonautonomous (or time-varying). Examples include load variations and network topology changes due to switching in substations. Additional interactions with the environment include disturbances whose physical description may include outages of system elements, while a mathematical description may involve variations in the system order, or the number of variables of interest.

Finally, a power system is a controlled (or forced) system with numerous feedback loops, and it is necessary to include the effects of control inputs (including their saturation), especially on longer time horizons.

The outlined modeling problems are typically addressed in a power system analysis framework in the following way:

1) The problem of defining stability for general nonautonomous systems is very challenging even in the theoretical realm [40], and one possible approach is to say that a system to which the environment delivers square-integrable signals as inputs over a time interval is stable if variables of interest (such as outputs) are also square integrable. In a more general setup, we can consider signals truncated in time, and denote the system

as *well-posed* if it maps square integrable truncated signals into signals with the same property. In a power system setting, one typically assumes that the variables at the interface with the environment are known (or predictable)—e.g., that mechanical inputs to all generators are constant, or that they vary according to the known response of turbine regulators.

2) The disturbances of interest will fall into two broad categories—event-type (typically described as outages of specific pieces of equipment) and norm-type (described by their size e.g., in terms of various norms of signals); we will return to this issue shortly. We also observe that in cases when event-type (e.g., switching) disturbances occur repeatedly, a proper analysis framework is that of hybrid systems (for a recent review, see [49]); event-type disturbances may also be initiated by human operators. Our focus is on time horizons of the order of seconds to minutes; on a longer time scale, the effects of market structures may become prominent. In that case, the relevant notion of stability needs re-examination; some leads about systems with distributed decision making may be found in [50].
3) Given our emphasis on stability analysis, we will assume that the actions of all controllers are fully predictable in terms of known system quantities (states), or as functions of time; the dual problem of designing stabilizing controls for nonlinear systems is very challenging, see for example [8], [51].

A typical power system stability study consists of the following steps:

1) Make modeling assumptions and formulate a mathematical model appropriate for the time-scales and phenomena under study;
2) Select an appropriate stability definition;
3) Analyze and/or simulate to determine stability, typically using a scenario of events;
4) Review results in light of assumptions, compare with the engineering experience ("reality"), and repeat if necessary.

Before considering specifics about power system stability, we need to assess the required computational effort. In the case of linear system models, the stability question is decidable, and can be answered efficiently in polynomial time [52]. In the case of nonlinear systems, the available algorithms are inherently inefficient. For example, a related problem of whether all trajectories of a system with a single scalar nonlinearity converge to the origin turns out to be very computationally intensive (i.e., NP-hard), and it is unclear if it is decidable at all [52]. Given the large size of power systems and the need to consider event-type perturbations that will inevitably lead to nonlinear models, it is clear that the task of determining stability of a power system will be a challenging one. It turns out, however, that our main tools in reducing the computational complexity will be our ability (and willingness) to utilize approximations, and the particular nature of event-type disturbances that we are analyzing.

We also want to point out that a possible shift in emphasis regarding various phenomena in power systems (e.g., hybrid aspects) would necessarily entail a reassessment of notions of stability. For a recent review on notions of stability in various types of systems (including infinite dimensional ones), see [53].

B. A Scenario for Stability Analysis

We consider the system

$$\dot{x} = f(t, x)$$

where x is the state vector (a function of time, but we omit explicitly writing the time argument t), $\dot{x}$ is its derivative, f is sufficiently differentiable and its domain includes the origin. The system described above is said to be *autonomous* if $f(t,x)$ is independent of t and is said to be *nonautonomous* otherwise.

A typical scenario for power system stability analysis involves three distinct steps.

1) The system is initially operating in a pre-disturbance equilibrium set X_n (e.g., an equilibrium point or perhaps even a benign limit cycle in the state space); in that set, various driving terms (forces) affecting system variables are balanced (either instantaneously, or over a time interval). We use the notion of an equilibrium set to denote equilibrium points, limit sets and more complicated structures like aperiodic attractors (which may be possible in realistic models of power systems). However, in the vast majority of cases of practical interest today, the equilibrium points are the sets of interest.

   In general, an equilibrium set, or an *attractor*, is a set of trajectories in the *phase space* to which all neighboring trajectories converge. Attractors therefore describe the long-term behavior of a dynamical system. A *Point attractor, or an equilibrium point*, is an attractor consisting of a single point in the phase space. A *Limit cycle attractor*, on the other hand, corresponds to closed curves in phase space; limit cycles imply periodic behavior. *A chaotic (or aperiodic, or strange) attractor* corresponds to a equilibrium set where system trajectories never converge to a point or a closed curve, but remain within the same region of phase space. Unlike limit cycles, strange attractors are non-periodic, and trajectories in such systems are very sensitive to the initial conditions.
2) Next, a disturbance acts on the system. An event-type (or incident-type) disturbance is characterized by a specific fault scenario (e.g., short circuit somewhere in the transmission network followed by a line disconnection including the duration of the event—"fault clearing time"), while norm-type (described by their size in terms of various norms of signals—e.g., load variations) disturbances are described by their size (norm, or signal intensity). A problem of some analytical interest is determining the maximum permissible duration of the fault (the so called "critical clearing time") for which the subsequent system response remains stable. This portion of stability analysis requires the knowledge of actions of protective relaying.
3) After an event-type disturbance, the system dynamics is studied with respect to a known post-disturbance equilibrium set X_p (which may be distinct from X_n). The system initial condition belongs to a (known) starting set χ_p, and we want to characterize the system motion with

respect to X_p i.e., if the system trajectory will remain inside the technically viable set Ω_p (which includes X_p). In the case of norm-type disturbances, very often we have $X_p = X_n$. If the system response turns out to be stable (a precise definition will follow shortly), it is said that X_p (and sometimes X_n as well) are stable. A detected instability (during which system motion crosses the boundary of the technically viable set $\partial\Omega_p$—e.g., causing line tripping or a partial load shedding) may lead to a new stability study for a new (reduced) system with new starting and viable sets, and possibly with different modeling assumptions (or several such studies, if a system gets partitioned into several disconnected parts).

The stability analysis of power systems is in general nonlocal, as various equilibrium sets may get involved. In the case of event-type disturbances, the perturbations of interest are specified deterministically (the same may apply to X_n as well), and it is assumed that the analyst has determined all X_p that are relevant for a given X_n and the disturbance. In the case of norm-type perturbations, the uncertainty structure is different—the perturbation is characterized by size (in the case of the so-called small-disturbance or small-signal analysis this is done implicitly, so that linearized analysis remains valid), and the same equilibrium set typically characterizes the system before and after the disturbance. Note, however, that norm-type perturbations could in principle be used in large-signal analyses as well.

We propose next a formulation of power system stability that will allow us to explore salient features of general stability concepts from system theory.

- *"An equilibrium set of a power system is stable if, when the initial state is in the given starting set, the system motion converges to the equilibrium set, and operating constraints are satisfied for all relevant variables along the entire trajectory."*

The operating constraints are of inequality (and equality) type, and pertain to individual variables and their collections. For example, system connectedness is a collective feature, as it implies that there exist paths (in graph-theoretic terms) from any given bus to all other buses in the network. Note also that some of the operating constraints (e.g., voltage levels) are inherently soft i.e., the power system analyst may be interested in stability characterization with and without these constraints. Note that we assume that our model is accurate within Ω_p in the sense that there are no further system changes (e.g., relay-initiated line tripping) until the trajectory crosses the boundary $\partial\Omega_p$.

C. Stability Definitions From System Theory

In this section, we provide detailed analytical definitions of several types of stability including Lyapunov stability, input-output stability, stability of linear systems, and partial stability. Of these various types, the Lyapunov stability definitions related to stability and asymptotic stability are the ones most applicable to power system nonlinear behavior under large disturbances. The definition of stability related to linear systems finds wide use in small-signal stability analysis of power systems. The concept of partial stability is useful in the classification of power system stability into different categories.

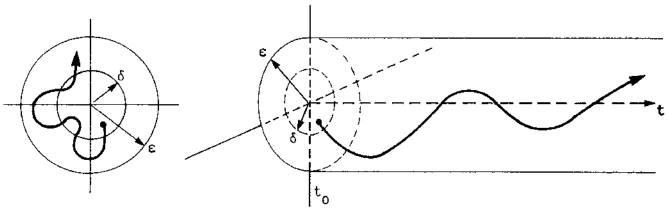

Fig. 2. Illustration of the definition of stability [57].

C.1 Lyapunov Stability:

The definitions collected here mostly follow the presentation in [54]; we present definitions for cases that are typical for power system models (e.g., assuming differentiability of functions involved), and not necessarily in the most general context. We will concentrate on the study of stability of equilibrium points; a study of more intricate equilibrium sets, like periodic orbits, can often be reduced to the study of equilibrium points of an associated system whose states are deviations of the states of the original system from the periodic orbit; another possibility is to study periodic orbits via sampled states and Poincare maps [55].

We again consider the nonautonomous system:

$$\dot{x} = f(t, x) \tag{1}$$

where x is the state vector, $\dot{x}$ is its derivative, f is sufficiently differentiable and its domain includes the origin. Note that the forcing (control input) term is not included i.e., we do not write $f(t, x, u)$. In stability analysis in the Lyapunov framework that is not a limitation, since all control inputs are assumed to be known functions of time t and/or known functions of states x. For technical reasons, we will assume that the origin is an equilibrium point (meaning that $f(t, 0) = 0, \forall\, t \geq 0$). An equilibrium at the origin could be a translation of a nonzero equilibrium after a suitable coordinate transformation [54, p. 132]

The equilibrium point $x = 0$ of (1) is:

- *stable* if, for each $\varepsilon > 0$, there is $\delta = \delta(\varepsilon, t_0) > 0$ such that:

$$\|x(t_0)\| < \delta \Rightarrow \|x(t)\| < \varepsilon, \quad \forall\, t \geq t_0 \geq 0 \tag{2}$$

Note that in (2) any norm can be used due to topological equivalence of all norms. In Figure 2, we depict the behavior of trajectories in the vicinity of a stable equilibrium for the case $x \in R^2$ (a two-dimensional system in the space of real variables). By choosing the initial conditions in a sufficiently small spherical neighborhood of radius δ, we can force the trajectory of the system for all time $t \geq t_0$ to lie entirely in a given cylinder of radius ε.

- *uniformly stable* if, for each $\varepsilon > 0$, there is $\delta = \delta(\varepsilon) > 0$, independent of t_0, such that (2) is satisfied;
- *unstable* if not stable;
- *asymptotically stable* if it is stable and in addition there is $\eta(t_0) > 0$ such that:

$$\|x(t_0)\| < \eta(t_0) \Rightarrow x(t) \to 0 \text{ as } t \to \infty$$

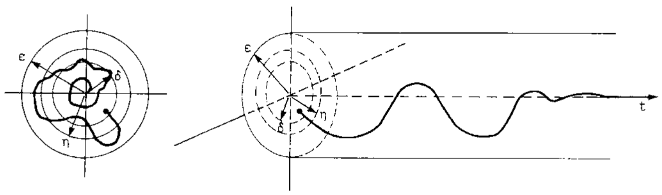

Fig. 3. Illustration of the definition of asymptotic stability [57].

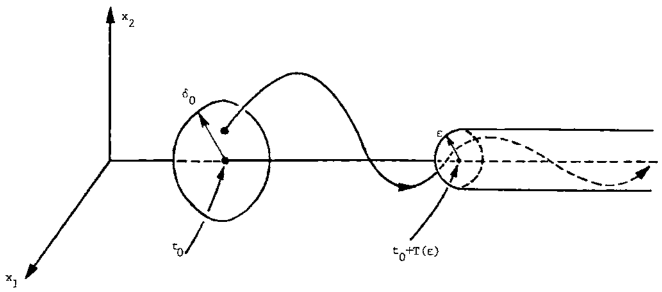

Fig. 4. Illustration of the definition of uniform asymptotic stability [58].

It is important to note that the definition of asymptotic stability combines the aspect of stability as well as attractivity of the equilibrium. This is a stricter requirement of the system behavior to eventually return to the equilibrium point. This concept is pictorially presented in Figure 3.

- *uniformly asymptotically stable* if it is uniformly stable and there is $\delta_0 > 0$, independent of t_0, such that for all $\|x(t_0)\| < \delta_0$, $x(t) \to 0$ as $t \to \infty$, uniformly in t_0 and $x(t_0)$; that is, for each $\varepsilon > 0$, there is $T = T(\varepsilon, \delta_0) > 0$ such that

$$\|x(t_0)\| < \delta_0 \Rightarrow \|x(t)\| < \varepsilon, \quad \forall\, t \geq t_0 + T(\varepsilon, \delta_0).$$

In Figure 4, we depict the property of uniform asymptotic stability pictorially. By choosing the initial operating points in a sufficiently small spherical neighborhood at $t = t_0$, we can force the trajectory of the solution to lie inside a given cylinder for all $t > t_0 + T(\varepsilon, \delta_0)$.

- *globally uniformly asymptotically stable* if it is uniformly stable, and, for each pair of positive numbers ε and δ_0, there is $T = T(\varepsilon, \delta_0) > 0$ such that:

$$\|x(t_0)\| < \delta_0 \Rightarrow \|x(t)\| < \varepsilon, \quad \forall\, t \geq t_0 + T(\varepsilon, \delta_0).$$

- *exponentially stable* if there are $\delta > 0, \varepsilon > 0, \alpha > 0$ such that:

$$\|x(t_0)\| < \delta \Rightarrow \|x(t)\| \leq \varepsilon \|x(t_0)\| e^{-\alpha(t-t_0)}, \quad t \geq t_0.$$

In Figure 5 the behavior of a solution in the vicinity of an exponentially stable equilibrium point is shown.

- *globally exponentially stable* if the exponential stability condition is satisfied for any initial state.

These definitions form the foundation of the Lyapunov approach to system stability, and can be most naturally checked for a specific system via so called Lyapunov functions. Qualitatively speaking, i.e., disregarding subtleties due to the nonautonomous characteristics of the system, we are to construct a smooth positive definite "energy" function whose time derivative (along trajectories of (1)) is negative definite. While unfortunately there is no systematic method to generate such functions (some leads for the case of simple power systems are given in [56], [57]), the so called converse Lyapunov theorems establish the existence of such functions if the system is stable in a certain sense.

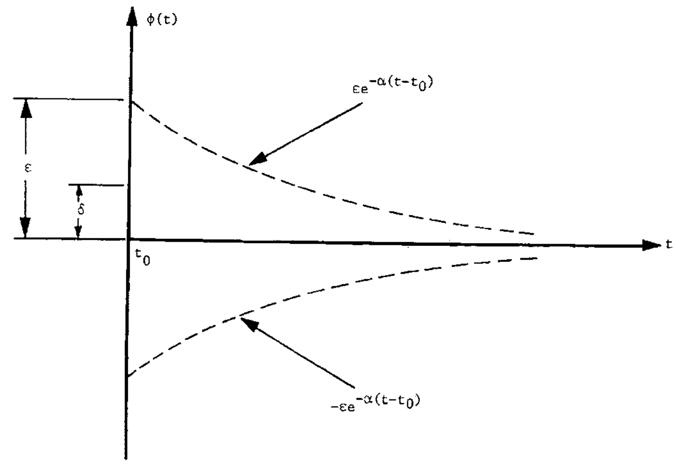

Fig. 5. Illustration of the definition of exponential stability [58].

In power systems we are interested in, the region of attraction $R(X_p)$ of a given equilibrium set X_p, namely the set of points in the state space with the property that all trajectories initiated at the points will converge to the equilibrium set X_p. If the equilibrium set is a point that is asymptotically stable, then it can be shown that the region of attraction has nice analytical properties (it is an open and connected set, and its boundary is formed by system trajectories). In the case of large scale power systems, we are naturally interested in the effects of approximations and idealizations that are necessary because of system size. Even if the nominal system has a stable equilibrium at the origin, this may not be the case for the actual perturbed system, which is not entirely known to the analyst. We cannot necessarily expect that the solution of the perturbed system will approach the origin, but could if the solution is *ultimately bounded* i.e., $\|x(t)\|$ is bounded by a fixed constant, given that the initial condition is in a ball of a fixed radius, and for sufficiently large time t. Characterization of stability in this case requires knowledge of the size of the perturbation term, and of a Lyapunov function for the nominal (non-perturbed) system. A related notion of *practical stability* is motivated by the idea that a system may be "considered stable if the deviations of motions from the equilibrium remain within certain bounds determined by the physical situation, in case the initial values and the perturbation are bounded by suitable constants" [59]. One does not require a more narrow interpretation that the deviation from the origin of $\|x(t)\|$ can be made arbitrarily small by a suitable choice of the constants, as is the case with *total stability*. Roughly speaking, for practical stability, we allow that the system will move away from the origin even for small perturbations, and we cannot make that motion arbitrarily small by reducing the model perturbation term.

Another concept of interest in power systems is that of *partial stability* [9]–[11], introduced already by Lyapunov himself. The basic idea is to relax the condition for stability from one that requires stable behavior from all variables (because of the properties of the norm used in (2) and elsewhere) to one that requires such behavior from only some of the variables. This formulation is natural in some engineered systems [9], and leads to substantial simplifications in others (e.g., in some adaptive systems). It has been used in the context of power system stability as well [60].

A power system is often modeled as an interconnection of lower-order subsystems, and we may be interested in a hierarchical (two-level) approach to stability determination [61]. At the first step, we analyze the stability of each subsystem separately (i.e., while ignoring the interconnections). In the second step, we combine the results of the first step with information about the interconnections to analyze the stability of the overall system. In a Lyapunov framework, this results in the study of *composite* Lyapunov functions. An important qualitative result is that if the isolated subsystems are sufficiently stable, compared to the strength of the interconnections, then the overall system is uniformly asymptotically stable at the origin.

C.2 Input/Output Stability:

This approach considers the system description of the form:

$$y = H(u) \tag{3}$$

where H is an operator (nonlinear in general) that specifies the q-dimensional output vector y in terms of the m-dimensional input u. The input u belongs to a normed linear space of vector signals $\mathcal{L}_e^m$—e.g., extended bounded or square integrable signals, meaning that all truncations u_τ of such signals (set to zero for $t > \tau$) are bounded or square integrable (this allows inclusion of "growing" signals like ramps etc. that are of interest in stability analysis).

- Definition: A continuous function $\alpha : [0, \alpha) \to [0, \infty)$ is said to belong to class $\mathbf{K}$ if it is strictly increasing and $\alpha(0) = 0$.
- Definition: A continuous function $\beta : [0, a) \times [0, \infty) \to [0, \infty)$ is said to belong to class $\mathbf{KL}$ if, for each fixed s, the mapping $\beta(r, s)$ belongs to class $\mathbf{K}$ with respect to r and, for each fixed r, the mapping $\beta(r, s)$ is decreasing with respect to s and $\beta(r, s) \to 0$ as $s \to \infty$.
- A mapping H: $\mathcal{L}_e^m \to \mathcal{L}_e^q$ is L *stable* if there exists a class-$\mathbf{K}$ function $\alpha(\cdot)$ defined on $[0, \infty)$, and a non-negative constant β such that:

$$\|(Hu)_\tau\|_{\mathcal{L}} \le \alpha(\|u_\tau\|_{\mathcal{L}}) + \beta \tag{4}$$

for all $u \in \mathcal{L}_e^m$ and $\tau \in [0, \infty)$.

It is *finite-gain L stable* if there exist non-negative constants γ and β such that:

$$\|(Hu)_\tau\|_{\mathcal{L}} \le \gamma(\|u_\tau\|_{\mathcal{L}}) + \beta \tag{5}$$

for all $u \in \mathcal{L}_e^m$ and $\tau \in [0, \infty)$.

Note that if $\mathcal{L}$ is the space of uniformly bounded signals, then this definition yields the familiar notion of bounded-input, bounded-output stability. The above definitions exclude systems for which inequalities (4) and (5) are defined only for a subset of the input space; this is allowed in the notion of *small-signal $\mathcal{L}$ stability*, where the norm of the input signals is constrained.

Let us consider a nonautonomous system with input:

$$\dot{x} = f(t, x, u). \tag{6}$$

Note the shift in analytical framework, as in input/output stability inputs are not assumed to be known functions of time, but assumed to be in a known class typically described by a norm.

A system (6) is said to be *locally input-to-state stable* if there exists a class-$\mathbf{KL}$ function $\beta(\cdot, \cdot)$, a class-$\mathbf{K}$ function $\alpha(\cdot)$, and positive constants k_1 and k_2 such that for any initial state $x(t_0)$ with $\|x(t_0)\| < k_1$ and any input $u(t)$ with $\sup_{t \ge t_0} \|u(t)\| < k_2$, the solution $x(t)$ exists and satisfies:

$$\|x(t)\| \le \beta(\|x(t_0)\|, t - t_0) + \alpha\left(\sup_{t_0 \le \tau \le t} \|u(\tau)\|\right) \tag{7}$$

for all $t \ge t_0 \ge 0$.

It is said to be *input-to-state stable* if the local input-to-state property holds for the entire input and output spaces, and inequality (7) is satisfied for any initial state $x(t_0)$ and any bounded input $u(t)$. This property is typically established by Lyapunov-type arguments [54].

Next, we consider the system (6) with the output y determined from:

$$y = h(t, x, u) \tag{8}$$

where h is again assumed smooth.

A system (6) is said to be *locally input-to-output stable* if there exists a class-$\mathbf{KL}$ function β, a class-$\mathbf{K}$ function α, and positive constants k_1 and k_2 such that for any initial state $x(t_0)$ with $\|x(t_0)\| < k_1$ and any input $u(t)$ with $\sup_{t \ge t_0} \|u(t)\| < k_2$, the solution $x(t)$ exists and the output $y(t)$ satisfies:

$$\|y(t)\| \le \beta(\|x(t_0)\|, t - t_0) + \alpha\left(\sup_{t_0 \le \tau \le t} \|u(\tau)\|\right) \tag{9}$$

for all $t \ge t_0 \ge 0$.

It is said to be *input-to-output stable* if the local input-to-state property holds for entire input and output spaces, and inequality (9) is satisfied for any initial state $x(t_0)$ and any bounded input $u(t)$.

The first term describes the (decreasing) effects of the initial condition, while the function α in the second term bounds the "amplification" of the input through the system. In the case of square-integrable signals, the maximal amplification from a given input to a given output is denoted as the $\mathcal{L}_2$ gain of the system. This gain can be easily calculated, in general, only for linear systems, where it equals the maximal singular value (the supremum of the two-norm of the transfer function evaluated along the imaginary axis, or the H_∞ norm) of the transfer function. One of main goals of control design is then to minimize this gain, if the input represents a disturbance. There exist a number of theorems relating Lyapunov and input-to-output stability, and some of the main tools for establishing input-to-output stability come from the Lyapunov approach. Note, however, that input-to-output stability describes global properties of a system, so in its standard form, it is not suitable for study of individual equilibrium sets. Input-to-output stability results are thus sometimes used in the stability analysis to establish Lyapunov stability results in a global sense [62]. For a more sophisticated use of the input-to-output stability concept, in which the input-output properties are indexed by the operating equilibrium, see [63].

C.3 Stability of Linear Systems:

The direct ways to establish stability in terms of the preceding definitions are constructive; the long experience with Lyapunov stability offers guidelines for generating candidate Lyapunov functions for various classes of systems, but no general systematic procedures. For the case of power systems, Lyapunov functions are known to exist for simplified models with special features [56], [57], but again not for many realistic models. Similarly, there are no general constructive methods to establish input-to-output stability using (9) for nonlinear systems.

One approach of utmost importance in power engineering practice is then to try to relate stability of a nonlinear system to the properties of a linearized model at a certain operating point. While such results are necessarily local, they are still of great practical interest, especially if the operating point is judiciously selected. This is the method of choice for analytical (as contrasted with simulation-based) software packages used in the power industry today. The precise technical conditions required from the linearization procedure are given, for example, in [62, p. 209-211]. The essence of the approach is that if the linearized system is uniformly asymptotically stable (in the nonautonomous case, where it is equivalent to exponential stability), or if all eigenvalues have negative real parts (in the autonomous case), then the original nonlinear system is also locally stable in the suitable sense. The autonomous system case when some eigenvalues have zero real parts, and others have negative real parts, is covered by the center manifold theory; see [54] for an introduction.

In this subsection, we consider a system of the form:

$$\dot{x} = \mathbf{A}(t)x(t) \tag{10}$$

which is the linearization of (1) around the equilibrium at the origin. General stability conditions for the nonautonomous case are given in terms of the state transition matrix $\mathbf{\Phi}(t, t_0)$:

$$x(t) = \mathbf{\Phi}(t, t_0)x(t_0). \tag{11}$$

While such conditions [62, p. 193-196] are of little computational value, as it is impossible to derive an analytical expression for $\mathbf{\Phi}(t, t_0)$ in the general case, they are of significant conceptual value.

In the case of autonomous systems (i.e., $\mathbf{A}(t) = \mathbf{A}$): The origin of (10) is *(globally) asymptotically (exponentially) stable* if and only if all eigenvalues of $\mathbf{A}$ have negative real parts. The origin is stable if and only if all eigenvalues of $\mathbf{A}$ have nonpositive real parts, and in addition, every eigenvalue of $\mathbf{A}$ having a zero real part is a simple zero of the minimal characteristic polynomial of $\mathbf{A}$.

In the autonomous case, an alternative to calculating eigenvalues of $\mathbf{A}$ is to solve a linear *Lyapunov Matrix Equation* for a positive definite matrix solution; if such solution exists, it corresponds to a quadratic Lyapunov function that establishes stability of the system.

D. Stability Definitions and Power Systems

D.1 Complementarity of Different Approaches:

While Lyapunov and input/output approaches to defining system stability have different flavors, they serve complementary roles in stability analysis of power systems. Intuitively speaking, "input/output stability protects against noise disturbances, whereas Lyapunov stability protects against a single impulse-like disturbance" [55, p. 103].

The ability to select specific equilibrium sets for analysis is a major advantage of the Lyapunov approach; it also connects naturally with studies of bifurcations [64] that have been of great interest in power systems, mostly related to the topic of voltage collapse. Note, however, that standard definitions like (2) are not directly applicable, as both the starting set χ_p and the technically viable set Ω_p are difficult to characterize with the norm-type bounds used in (2). An attempt to use such bounds would produce results that are too conservative for practical use. For outage of a single element (e.g., transmission line), and assuming a known post-equilibrium set X_p and an autonomous system model, the starting set is a point; for a finite list of different outages of this type, the starting set will be a collection of distinct points. The requirement that such χ_p be "covered" by a norm-type bound is not very suitable, as it would likely include many other disturbances to which the system may not be stable, and which are not of interest to the power system analyst. Note also that partial stability may be very suitable for some system models [60]. In a very straightforward example, we are typically not interested in some states, like generator angles, but only in their differences. The concept of partial stability is hence of fundamental importance in voltage and angle stability studies. In such studies, we focus on a subset of variables describing the power system, and we assume that the disregarded variables will not influence the outcome of the analysis in a significant way. In practice, we tend to use simpler reduced models where the ignored variables do not appear, but conceptually we effectively use partial stability. The other key difficulty in analysis stems from the fact that the construction of Lyapunov functions for detailed power system models, particularly accounting for load models, is still an open question, as we commented earlier. Because of these two reasons, the stability of power systems to large disturbances is typically explored in simulations. Advances in this direction come from improved computer technology and from efficient power system models and algorithms; for a recent review of key issues in power system simulations that are related to stability analysis see [65]. In the case of power system models for which there exist energy functions, it is possible to approximate the viable set Ω_p using the so-called BCU method and related ideas; a detailed exposition with geometric and topological emphasis is presented in [65].

The input/output framework is a natural choice for analysis of some persistent disturbances acting on power systems (e.g., load variations). Note, however, that conditions like (9) are difficult to establish in a non-local (large signal) setting, and simulations are again the main option available today.

The two approaches of stability coalesce in the case of linear system models. The use of such models is typically justified with the assumed small size of the signals involved. A range of powerful analysis tools (like participation factors [8]) has been developed or adapted to power system models. For noise-type disturbances, an interesting nonstochastic approach to the worst-case analysis is offered by the set-based description of noise detailed in [66]. Small signal analyses are a part of standard practice of power system operation today.

D.2 An Illustration of a Typical Analysis Scenario:

In terms of the notation introduced here, a scenario leading toward a blackout is as follows: Following a disturbance, X_p^1 turns out to be unstable, and the system trajectory passes through $\partial\Omega_p^1$. After actions of relays and line tripping, the system splits into k_2 (mutually disconnected) components. The post-fault equilibria in each component are $X_p^{2,l}, l = 1, \ldots, k_2$, and some of them again turn out to be unstable, as their boundaries $\partial\Omega_p^{2,l}$ are crossed by corresponding (sub)system trajectories. Note that up to this point $k_2 + 1$, stability analyses have been performed. Then the stability assessment process repeats on the third step and so on. In this framework under a "power system," we understand a set of elements that is supplying a given *set of loads*, and if it becomes disconnected (in graph-theoretic sense) at any point, we have to consider as many newly created power systems as there are connected components.

There exists a point of difference between theorists and practitioners that we want to comment upon here: Stability theorists tend to see a new system after the initial event (e.g., a line switching), while practitioners tend to keep referring back to the original (pre-disturbance) system. This is because stability limits are specified in terms of pre-disturbance system conditions. While this is typically not a major obstacle, it points out toward the need for a more comprehensive treatment of stability theory for power systems as discussed in this section.

VI. Summary

This report has addressed the issue of stability definition and classification in power systems from a fundamental viewpoint and has examined the practical ramifications of stability phenomena in significant detail. A precise definition of power system stability that is inclusive of all forms is provided. A salient feature of the report is a systematic classification of power system stability, and the identification of different categories of stability behavior. Linkages between power system reliability, security, and stability are also established and discussed. The report also includes a rigorous treatment of definitions and concepts of stability from mathematics and control theory. This material is provided as background information and to establish theoretical connections.

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