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A transient energy function for power systems including the induction motor model

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A construction method for power system transient energy function is studied in the paper, which is simple and universal, and can unify the forms of some current energy functions. A transient energy function including the induction motor model is derived using the method. The unintegrable term is dealt with to get an approximate energy function. Simulations in a 3-bus system and in the WSCC 4-generator system verify the validity of the proposed energy function. The function can be applied to direct transient stability analysis of multi-machine large power systems and provides a tool for analysis of the interaction between the generator angle stability and the load voltage stability.

power system stability, transient stability, energy function, direct method, induction motor

There are two types of methods for power system transient stability analysis: the time domain approach and the direct method[1,2]. The time domain approach is applicable to detailed power system models, so it is dominant in transient stability analysis of power systems. However, it has the disadvantage of intensive time consumption. The direct method is based on the second method of Lyapunov and LaSalle's invariance principle. Early researches of the direct method focused on developing Lyapunov functions for power systems with different approaches. In the end of 1970s, Athay, Kakimoto et al.[3,4] proposed the concept of transient energy function. They developed a transient energy function that could better reflect the physical characteristics of practical power systems from the viewpoint of system energy, and promoted the development of the direct method. The transient energy function is a possible Lyapunov function but loosens the constraint of being positive definite[5]. Compared with the time domain approach, the direct method based on transient energy functions has the advantage of speed and can provide information regarding the degree of stability. So it is suitable for on-line applications. However, one of the disadvantages of the direct method is that the model used is excessively simple. Construction of

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transient energy functions including more detailed device models is always a big problem in the research of the direct method[2,6,7].

At present, the analysis software of direct method uses the second-order classical model of the generator and constant impedance load model, and the network is reduced. Under the structure preserving model several types of energy functions have also been derived[8−10]. In ref. [8] a structure preserving energy function was constructed. The load model was constant power and the generator model could be classical model or the third-order one-axis model. The obtained energy function was strict Lyapunov function. In ref. [11], a method to derive energy functions based on the energy conservation law was proposed, and energy functions including the third-order one-axis generator model and the classical generator model with the governor were constructed.

For a long time, the study of power system transient stability has been focusing on transient angle stability. Many effective methods to improve transient angle stability, such as the excitation control, have been developed[12]. However, in recent years, voltage stability of the load has become more and more important in power systems. It may bring a new approach for direct voltage stability analysis to include the dynamic load model in energy functions[13]. In ref. [14] a dynamic load model was used and a Lyapunov function of the power system with this model was obtained. However, the load model adopted was not widely admitted or accepted. The most widely used dynamic load model in power systems is the induction motor model. In ref. [15] an energy function with the induction motor model was constructed, but the method was too complicated to apply to large systems. So, there has not been a proper energy function including the induction motor model by now.

This paper further develops the method proposed in ref. [11] and shows that the method can unify the forms of some current energy functions. Compared with other methods of energy function construction, like the first integral approach and the method based on the Popov criterion[6,7], the method is simple, strict and universal. The paper also constructs a transient energy function including the induction motor model that is applicable to large systems using the method, and tests the function in a 3-bus system and the WSCC 4-generator 11-bus system. With the application of the obtained energy function, we can consider both angle stability of the generator and voltage stability of the induction motor load in the direct transient stability analysis synthetically.

1 An energy function construction method

This section first introduces the energy function construction method proposed in ref. [11] and then develops it to construct energy functions including different generator and load models.

Consider a power system with n buses. The structure preserving model is adopted. According to Kirhoff's current law, the current equation of the buses can be denoted as

$$\boldsymbol{Y}_{\mathrm{BUS}}\boldsymbol{V}_{\mathrm{BUS}} - \boldsymbol{I}_{\mathrm{G}} + \boldsymbol{I}_{\mathrm{L}} = 0, \tag{1}$$

where $\boldsymbol{V}_{\mathrm{BUS}}$, $\boldsymbol{I}_{\mathrm{G}}$, and $\boldsymbol{I}_{\mathrm{L}}$ are bus voltages, injection currents of generators and load currents, respectively. They are all n-dimensional complex vectors. $\boldsymbol{Y}_{\mathrm{BUS}}$ is the admittance matrix. The equation is satisfied at any time. So on any point of a system trajectory, there is

$$\left[\left(\boldsymbol{Y}_{\mathrm{BUS}}\boldsymbol{V}_{\mathrm{BUS}} - \boldsymbol{I}_{\mathrm{G}} + \boldsymbol{I}_{\mathrm{L}}\right)^{*}\right]^{\mathrm{T}} \mathrm{d}\boldsymbol{V}_{\mathrm{BUS}} = 0. \tag{2}$$

Taking the imaginary part of the above equation (represented by the function $\mathrm{Im}(\cdot)$) and integrating it along the trajectory, we get

$$\int \mathrm{Im}\left\{\left[\left(\boldsymbol{Y}_{\mathrm{BUS}}\boldsymbol{V}_{\mathrm{BUS}}-\boldsymbol{I}_{\mathrm{G}}+\boldsymbol{I}_{\mathrm{L}}\right)^{*}\right]^{\mathrm{T}}\mathrm{d}\boldsymbol{V}_{\mathrm{BUS}}\right\}=\int \mathrm{Im}\left[\sum_{i=1}^{n}\left(\sum_{j=1}^{n}\boldsymbol{Y}_{ij}^{*}\boldsymbol{V}_{j}^{*}\right)\mathrm{d}\boldsymbol{V}_{i}-\sum_{i\in i_G}\boldsymbol{I}_{Gi}^{*}\mathrm{d}\boldsymbol{V}_{i}+\sum_{i\in i_L}\boldsymbol{I}_{Li}^{*}\mathrm{d}\boldsymbol{V}_{i}\right], \quad (3)$$

where $\boldsymbol{V}_i = V_i e^{\mathrm{j}\theta_i}$ and $\mathrm{d}\boldsymbol{V}_i = e^{\mathrm{j}\theta_i}\mathrm{d}V_i + \mathrm{j}V_i e^{\mathrm{j}\theta_i}\mathrm{d}\theta_i$.

The energy function of the system can be obtained from eq. (3).

1.1 The network term

Let $\boldsymbol{Y}_{ij} = G_{ij} + \mathrm{j}B_{ij}$. When using the structure preserving model we can assume that the network is lossless, namely $G_{ij} = 0$.

$$\begin{aligned}\mathrm{Im}\left[\sum_{i=1}^{n}\left(\sum_{j=1}^{n}\boldsymbol{Y}_{ij}^{*}\boldsymbol{V}_{j}^{*}\right)\mathrm{d}\boldsymbol{V}_{i}\right] &= \mathrm{Im}\left(\sum_{i=1}^{n}\left(\sum_{j=1}^{n}(-\mathrm{j}B_{ij})V_j e^{-\mathrm{j}\theta_j}\right)(e^{\mathrm{j}\theta_i}\mathrm{d}V_i + \mathrm{j}V_i e^{\mathrm{j}\theta_i}\mathrm{d}\theta_i)\right)\\ &= -\sum_{i=1}^{n}\sum_{j=1}^{n}(B_{ij}V_j\cos\theta_{ij}\mathrm{d}V_i - B_{ij}V_iV_j\sin\theta_{ij}\mathrm{d}\theta_i).\end{aligned}$$

Take the terms where $i = j$, and combine the terms where $i \neq j$. Then the following can be derived:

$$\mathrm{Im}\left[\sum_{i=1}^{n}\left(\sum_{j=1}^{n}\boldsymbol{Y}_{ij}^{*}\boldsymbol{V}_{j}^{*}\right)\mathrm{d}\boldsymbol{V}_{i}\right] = -\sum_{i=1}^{n}B_{ii}V_i\mathrm{d}V_i - \sum_{i=1}^{n-1}\sum_{j=i+1}^{n}B_{ij}\mathrm{d}(V_iV_j\cos\theta_{ij}).$$

Integrate it along the trajectory, i.e.,

$$\int \mathrm{Im}\left[\sum_{i=1}^{n}\left(\sum_{j=1}^{n}\boldsymbol{Y}_{ij}^{*}\boldsymbol{V}_{j}^{*}\right)\mathrm{d}\boldsymbol{V}_{i}\right] = \left(-\frac{1}{2}\sum_{i=1}^{n}B_{ii}V_i^2 - \sum_{i=1}^{n-1}\sum_{j=i+1}^{n}B_{ij}V_iV_j\cos\theta_{ij}\right)\Bigg|_{(V_0,\theta_0)}^{(V,\theta)}. \quad (4)$$

Eq. (4) is the magnetic potential energy in refs. [8] and [16].

1.2 The generator term

The classical model of the generator is adopted. The network is expanded to include generator internal buses. The voltage of the internal bus V_i is constant. The state equations are

$$\begin{aligned}\frac{\mathrm{d}\delta_i}{\mathrm{d}t} &= \omega_i,\\ M_i\frac{\mathrm{d}\omega_i}{\mathrm{d}t} &= P_{mi} - P_{ei} - D_i\omega_i,\end{aligned} \quad (5)$$

where δ_i is the rotor angle in rad, ω_i the difference between the rotor angular speed and the synchronous speed in rad/s, M_i the inertia constant, and D_i the damping coefficient. P_{mi} and P_{ei} are the mechanical torque and the electromagnetic torque in p.u., respectively.

$$\begin{aligned}\mathrm{Im}(-\boldsymbol{I}_{Gi}^{*}\mathrm{d}\boldsymbol{V}_i) &= -I_{xi}\mathrm{d}V_{yi} + I_{yi}\mathrm{d}V_{xi} = -I_{xi}\mathrm{d}(V_i\sin\delta_i) + I_{yi}\mathrm{d}(V_i\cos\delta_i)\\ &= -(I_{xi}V_{xi} + I_{yi}V_{yi})\mathrm{d}\delta_i = -P_{ei}\mathrm{d}\delta_i\\ &= -\left(P_{mi} - D_i\omega_i - M_i\frac{\mathrm{d}\omega_i}{\mathrm{d}t}\right)\mathrm{d}\delta_i = -P_{mi}\mathrm{d}\delta_i + D_i\omega_i^2\mathrm{d}t + M_i\omega_i\mathrm{d}\omega_i.\end{aligned}$$

Integrating along the trajectory, we get

$$\int \mathrm{Im}(-\boldsymbol{I}_{Gi}^{*}\mathrm{d}\boldsymbol{V}_i) = \left(\frac{1}{2}M_i\omega_i^2 - P_{mi}\delta_i\right)\Bigg|_{x_0}^{x} + \int D_i\omega_i^2\mathrm{d}t. \tag{6}$$

The first term on the right-hand side of eq. (6) is the traditional energy function of the classical generator model.

1.3 The load term

The constant power static load model is adopted.

$$\mathrm{Im}(\boldsymbol{I}_{Li}^{*}\mathrm{d}\boldsymbol{V}_i) = \mathrm{Im}\left[\frac{P_{Li}+\mathrm{j}Q_{Li}}{V_i e^{\mathrm{j}\theta_i}}(e^{\mathrm{j}\theta_i}\mathrm{d}V_i + \mathrm{j}V_i e^{\mathrm{j}\theta_i}\mathrm{d}\theta_i)\right] = \frac{Q_{Li}}{V_i}\mathrm{d}V_i + P_{Li}\mathrm{d}\theta_i.$$

Integrating along the trajectory, we get

$$\int \mathrm{Im}(\boldsymbol{I}_{Li}^{*}\mathrm{d}\boldsymbol{V}_i) = Q_{Li}\ln V_i\big|_{V_0}^{V} + P_{Li}\theta_i\big|_{\theta_0}^{\theta}. \tag{7}$$

1.4 The transient energy function of power systems

The classical generator model and the constant power load model are adopted. The network is lossless. Let

$$W_1 = \left(-\frac{1}{2}\sum_{i=1}^{n}B_{ii}V_i^2 - \sum_{i=1}^{n-1}\sum_{j=i+1}^{n}B_{ij}V_iV_j\cos\theta_{ij}\right)\Bigg|_{(V_0,\theta_0)}^{(V,\theta)} + \sum_{i\in i_G}\left(\frac{1}{2}M_i\omega_i^2 - P_{mi}\delta_i\right)\Bigg|_{x_0}^{x}$$
$$+\sum_{i\in i_L}(Q_{Li}\ln V_i\big|_{V_0}^{V} + P_{Li}\theta_i\big|_{\theta_0}^{\theta}). \tag{8}$$

Adding eqs. (4), (6), and (7) together and calculating the derivative, we get

$$\frac{\mathrm{d}W_1}{\mathrm{d}t} + \sum_{i\in i_G}D_i\omega_i^2 = \mathrm{Im}\left\{\left[\left(\boldsymbol{Y}_{\mathrm{BUS}}\boldsymbol{V}_{\mathrm{BUS}} - \boldsymbol{I}_{\mathrm{G}} + \boldsymbol{I}_{\mathrm{L}}\right)^{*}\right]^{\mathrm{T}}\mathrm{d}\boldsymbol{V}_{\mathrm{BUS}}\right\} = 0.$$

The above equation shows that along any system trajectory $\frac{\mathrm{d}W_1}{\mathrm{d}t} < 0$. Thus, W_1 is a transient energy function of the system and it has the same form as the energy function in ref. [8].

1.5 The universalness of the method

This energy function construction method does not rely on any specific device models in principle, so it can be used to construct energy functions including different models. It is found that with the application of this method we can get many energy function forms derived by other methods. The followings are the terms corresponding to different device models in energy functions derived using the method. The detailed deriving procedure is not presented for simplicity.

(i) The generator term

The constant E_q' model: $\left(\frac{1}{2}M\omega^2 - P_m\delta + \frac{1}{2}X_qI_q^2 + \frac{1}{2}X_d'I_d^2\right)\Big|_{x_0}^{x}$.

The third-order practical model:

$$\left(\frac{1}{2}M\ \omega^2 + \frac{1}{2}\left(X_d'I_d^2 + X_qI_q^2 + \frac{1}{X_d - X_d'}(E_q')^2\right) - \frac{E_{fd}E_q'}{X_d - X_d'} - P_m\delta\right)\Bigg|_{x_0}^{x}.$$

(ii) The load term

The dynamic load model in ref. [14]: $\left[x_q \ln\frac{V}{\mu} - \frac{Q_s^0}{Q_t^0}x_q + \frac{{x_q}^2}{2Q_t^0} + \frac{Q_t^0}{2}\left(\ln\frac{V}{\mu}\right)^2 + P_d^0\theta\right]\Bigg|_{x_0}^{x}$.

The above energy function corresponding to the third-order model of the generator is the same as that in ref. [8]. Furthermore, when using the dynamic load model in ref. [14], the same energy function is derived but the method in this paper is much simpler.

2 An energy function including the induction motor model

This section uses the above method to derive the energy function including the induction motor model. The induction motor model considering the mechanical and electrical transients is

$$\begin{aligned}
\frac{\mathrm{d}\dot{E}_i'}{\mathrm{d}t} &= -\mathrm{j}s_i\dot{E}_i' - \frac{1}{T_{d0i}'}[\dot{E}_i' - \mathrm{j}(X_i - X_i')\dot{I}_i],\\
\frac{\mathrm{d}s_i}{\mathrm{d}t} &= \frac{1}{M_i}(T_{mi} - T_{ei}).
\end{aligned}$$

Let $\dot{E}_i' = E_i'\angle\delta_i$. The above equation can be transformed to the following third-order model.

$$\begin{aligned}
\frac{\mathrm{d}E_i'}{\mathrm{d}t} &= -\frac{1}{T_{d0i}'}[E_i' - (X_i - X_i')I_{di}],\\
\frac{\mathrm{d}\delta_i}{\mathrm{d}t} &= -s_i + \frac{X_i - X_i'}{T_{d0i}{}'E_i'}I_{qi},\\
\frac{\mathrm{d}s_i}{\mathrm{d}t} &= \frac{1}{M_i}(T_{mi} - T_{ei}),
\end{aligned} \tag{9}$$

where $E_i'\angle\delta_i$ is the internal potential, T_{d0i}' the open-loop time constant of the stator, s_i the slip, and M_i the inertia constant. T_{mi} and T_{ei} are the mechanical torque and the electromagnetic torque, respectively. The other parameters are referred to ref. [16].

Now we derive the term in eq. (3) that corresponds to the induction motor load.

$$\begin{aligned}
\mathrm{Im}(\boldsymbol{I}_{Li}^*\mathrm{d}\boldsymbol{V}_i) &= I_{xi}\mathrm{d}V_{yi} - I_{yi}\mathrm{d}V_{xi} = I_{xi}\mathrm{d}(E_{yi}' + X_i'I_{xi}) - I_{yi}\mathrm{d}(E_{xi}' - X_i'I_{yi})\\
&= I_{xi}\mathrm{d}(E_i'\sin\delta_i) - I_{yi}\mathrm{d}(E_i'\cos\delta_i) + X_i'I_{xi}\mathrm{d}I_{xi} + X'I_{yi}\mathrm{d}I_{yi}\\
&= I_{di}\mathrm{d}E_i' + I_{qi}E_i'\mathrm{d}\delta_i + X_i'I_{xi}\mathrm{d}I_{xi} + X_i'I_{yi}\mathrm{d}I_{yi}.
\end{aligned}$$

We get the expression of I_{di} from the first equation of eq. (9), and consider that the electromagnetic torque $T_{ei} = E_i'I_{qi}$. The above equation can be transformed to

$$\begin{aligned}
\mathrm{Im}(\boldsymbol{I}_{Li}^*\mathrm{d}\boldsymbol{V}_i) &= \left(\frac{E_i'}{X_i - X_i'} + \frac{T_{d0i}'}{X_i - X_i'}\frac{\mathrm{d}E_i'}{\mathrm{d}t}\right)\mathrm{d}E_i' + \left(T_{mi} - M_i\frac{\mathrm{d}s_i}{\mathrm{d}t}\right)\mathrm{d}\delta_i + X_i'I_{xi}\mathrm{d}I_{xi} + X_i'I_{yi}\mathrm{d}I_{yi}\\
&= \frac{E_i'\mathrm{d}E_i'}{X_i - X_i'} + \frac{T_{d0i}'\mathrm{d}t}{X_i - X_i'}\left(\frac{\mathrm{d}E_i'}{\mathrm{d}t}\right)^2 + T_{mi}\mathrm{d}\delta_i + M_i\mathrm{d}s_i\left(s_i - \frac{X_i - X_i'}{T_{d0i}'E_i'}I_{qi}\right) + X_i'I_{xi}\mathrm{d}I_{xi} + X_i'I_{yi}\mathrm{d}I_{yi}.
\end{aligned}$$

Integrating along the trajectory, we get

$$\int \mathrm{Im}(\boldsymbol{I}_{Li}^{*}\mathrm{d}\boldsymbol{V}_i) = \left(\frac{1}{2}M_i s_i^{\,2} + T_{mi}\delta_i + \frac{1}{2(X_i - X_i')}(E_i')^2 + \frac{1}{2}X_i' I_{xi}^{\,2} + \frac{1}{2}X_i' I_{yi}^{\,2}\right)\Bigg|_{x_0}^{x}$$

$$-\int \frac{M_i(X_i - X_i')}{T_{d0i}'}\frac{I_{qi}}{E_i'}\mathrm{d}s_i + \frac{T_{d0i}'}{X_i - X_i'}\int\left(\frac{\mathrm{d}E_i'}{\mathrm{d}t}\right)^2 \mathrm{d}t. \tag{10}$$

We assume that the classical generator model and the induction motor load model are adopted and the network is lossless. Let

$$W_2 = \left(-\frac{1}{2}\sum_{i=1}^{n} B_{ii}V_i^2 - \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} B_{ij}V_iV_j\cos\theta_{ij}\right)\Bigg|_{(V_0,\theta_0)}^{(V,\theta)} + \sum_{i\in i_G}\left(\frac{1}{2}M_i\omega_i^{\,2} - P_{mi}\delta_i\right)\Bigg|_{x_0}^{x}$$

$$+\sum_{i\in i_L}\left(\frac{M_i s_i^2}{2} + T_{mi}\delta_i + \frac{(E_i')^2}{2(X_i - X_i')} + \frac{X_i' I_{xi}^2 + X_i' I_{yi}^2}{2}\right)\Bigg|_{x_0}^{x} - \sum_{i\in i_L}\int \frac{M_i(X_i - X_i')}{T_{d0i}'}\frac{I_{qi}}{E_i'}\mathrm{d}s_i. \tag{11}$$

Using the same processing method in the above section we can get

$$\frac{\mathrm{d}W_2}{\mathrm{d}t} + \sum_{i\in i_G} D_i\omega_i^2 + \sum_{i\in i_L}\frac{T_{d0i}'}{X_i - X_i'}\left(\frac{\mathrm{d}E_i'}{\mathrm{d}t}\right)^2 = 0.$$

It means that along any trajectory of the system, $\frac{\mathrm{d}W_2}{\mathrm{d}t} < 0$. Therefore, W_2 is a transient energy function including the induction motor model. However, there is an unintegrable term, which will be dealt with below.

In the third-order model of the induction motor, the dynamics of the internal potential $E_i'\angle\delta_i$ correspond to the electromagnetic transients of the rotor. They are much faster than those of s_i which correspond to the mechanical transients. If we just consider the mechanical transients, the dynamics of $E_i'\angle\delta_i$ can be neglected, and the corresponding differential equations are transformed to algebraic equations. Then the model of the induction motor is the following first-order model.

$$\begin{aligned} &\frac{\mathrm{d}s_i}{\mathrm{d}t} = \frac{1}{M_i}(T_{mi} - E_i' I_{qi}), \\ &0 = E_i' - (X_i - X_i')I_{di}, \\ &0 = -s_i + \frac{X_i - X_i'}{T_{d0i}'E_i'}I_{qi}. \end{aligned} \tag{12}$$

The equivalent impedance of the induction motor derived from eq. (12) is

$$Z_{eq} = \mathrm{j}X_s + (\mathrm{j}X_m)/\left(\frac{R_r}{s} + \mathrm{j}X_r\right),$$

which is the same as that of the traditional first-order mechanical transient model. When using the model of eq. (12), the corresponding energy term can be simplified to

$$W_M = \left(\frac{M_i s_i^{\,2}}{2} + T_{mi}\delta_i + \frac{(E_i')^2}{2(X_i - X_i')} + \frac{X_i' I_{xi}^2 + X_i' I_{yi}^2}{2} \right)\Bigg|_{x_0}^{x} - \int \frac{M_i (X_i - X_i')}{T_{d0i}'} \frac{I_{qi}}{E_i'} \mathrm{d}s_i$$

$$= \left(T_{mi}\delta_i + \frac{1}{2} X_i I_{di}^2 + \frac{1}{2} X_i' I_{qi}^2 \right)\Bigg|_{x_0}^{x}. \tag{13}$$

Eq. (13) is the approximate energy function corresponding to the first-order induction motor model. We first derive the energy function of the third-order model, then neglect the fast dynamics and transform the third-order model to the first-order model, finally get eq. (13) from the energy function of the third-order model. So the energy function is not strict. However, the function is verified to be effective in engineering by the simulations in the next section.

In summary, the approximate energy function including the classical generator model and the first-order induction motor model is

$$W_3 = \left(-\frac{1}{2}\sum_{i=1}^{n} B_{ii} V_i^2 - \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} B_{ij} V_i V_j \cos\theta_{ij} \right)\Bigg|_{(V_0,\theta_0)}^{(V,\theta)} + \sum_{i\in i_G} \left(\frac{1}{2} M_i \omega_i^2 - P_{mi}\delta_i \right)\Bigg|_{x_0}^{x}$$

$$+ \sum_{i\in i_L} \left(T_{mi}\delta_i + \frac{1}{2} X_i I_{di}^2 + \frac{1}{2} X_i' I_{qi}^2 \right)\Bigg|_{x_0}^{x}. \tag{14}$$

3 Simulation results

3.1 A three-bus system

Consider the 3-bus system shown in Figure 1, where the parameters of the buses and the branches are also shown. The classical model is adopted for the generator at bus 1, with the parameters X_d' =0.30, M =0.0223 $\mathrm{s^2 \cdot rad^{-1}}$, and D =0.02 $\mathrm{s \cdot rad^{-1}}$. The load at bus 2 is an induction motor which is represented by the first-order model in eq. (12). The parameters on its own MVA base are M =0.0064 $\mathrm{s^2 \cdot rad^{-1}}$, R_r=0.02, X_s =0.295, X_r =0.120, X_m =3.5, and s_0 =4.3982 $\mathrm{rad \cdot s^{-1}}$. The system frequency is 50 Hz.

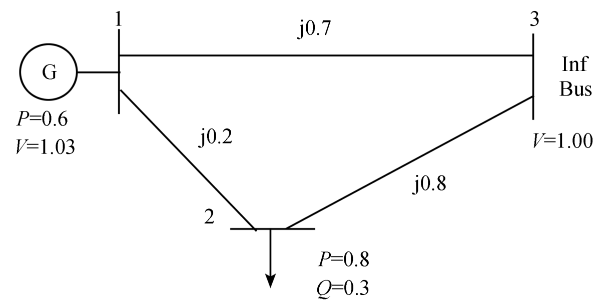

Figure 1 A 3-bus system.

Assume that instantaneous three-phase short-circuit faults occur at the generator bus and the load bus, respectively. The curve of the values of the energy function in eq. (14) is shown in Figure 2. As the continuance of the fault, the energy of the system increases. After the fault is cleared, the energy of the system decreases monotonously. The performance of the energy satisfies the demand of the energy function.

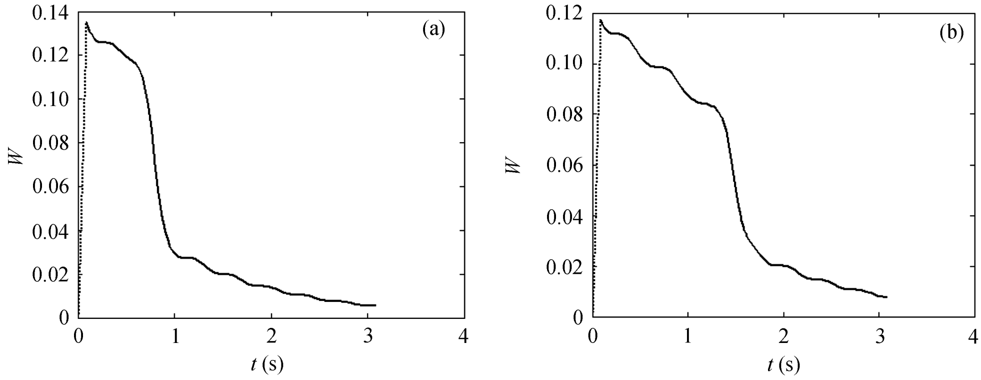

Figure 2 The change of the energy in the 3-bus system when bus faults occur. (a) The generator bus; (b) the load bus.

3.2 WSCC 4-generator 11-bus system

Consider the WSCC 4-generator 11-bus system shown in Figure 3. The bus 4 is set to be the infinite bus. The loads at buses 5, 8, and 10 are all induction motors. The parameters of the system are shown in Tables 1—3. The damping coefficients of the generators are all 0.10 s·rad^{-1}. The impedance parameters of the induction motors in self MVA base are the same as those in the 3-bus system. Assume that instantaneous three-phase short-circuit faults occur at different buses. The change of the energy of the system is shown in Figure 4, which also verifies the validity of the derived energy function.

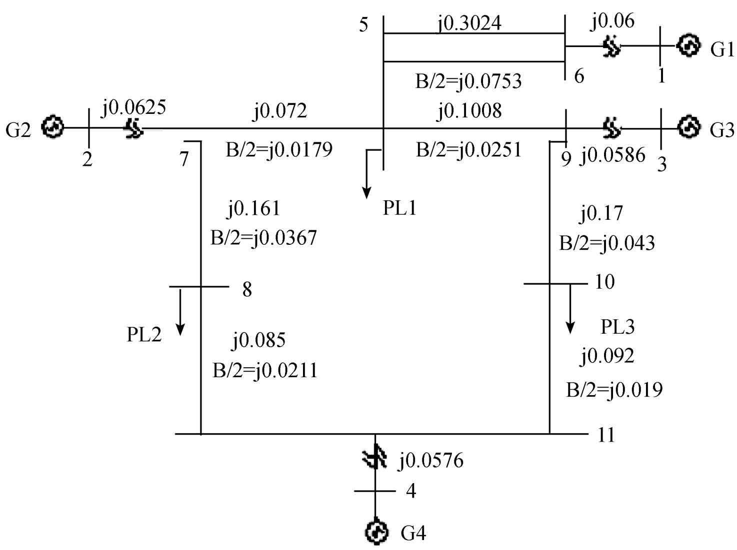

Figure 3 The WSCC 4-generator 11-bus system.

Table 1 The parameters of the generators

No.	G1	G2	G3
X'_d	0.1198	0.1196	0.1813
M (s^2·rad^{-1})	0.0407	0.0407	0.0192

Table 2 The parameters of the induction motors

Bus No.	5	8	10
s_0(rad·s^{-1})	4.3982	4.7124	5.6549
M (s^2·rad^{-1})	0.0048	0.0057	0.0064

Table 3 Load modes

	Bus 1		Bus 2		Bus 3		Bus 4		Bus 5		Bus 8		Bus 10	
	P	V	P	V	P	V	V	θ	P	Q	P	Q	P	Q
Mode	1.4	1.03	1.4	1.04	0.8	1.04	1.03	0	2.0	0.6	1.8	0.4	1.8	0.4

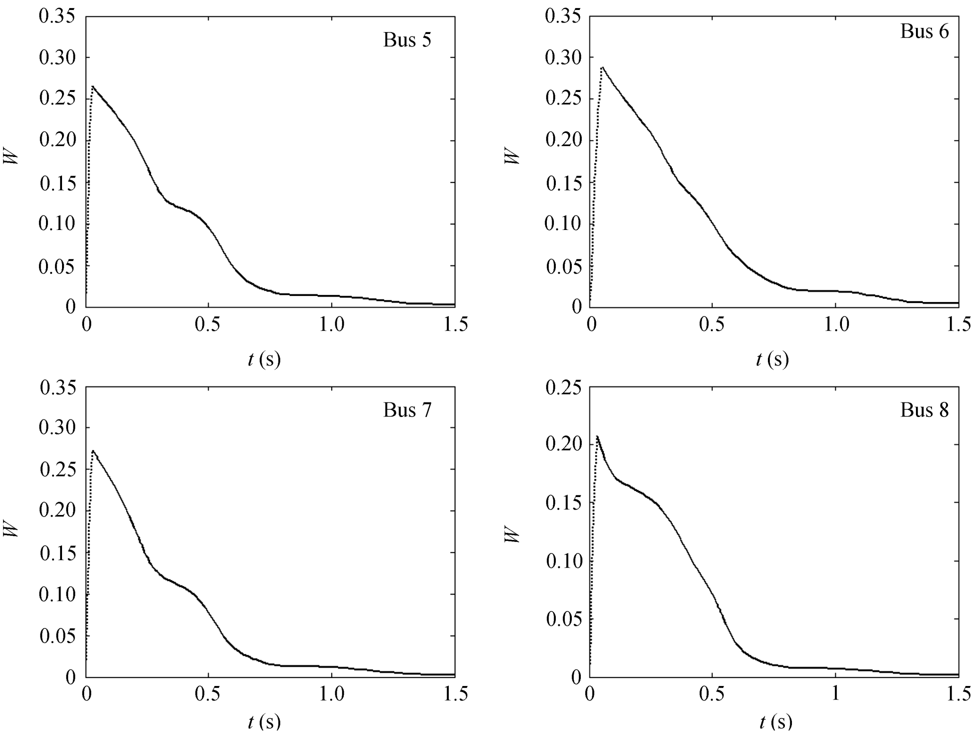

Figure 4 The change of the energy in the WSCC 4-generaot 11-bus system.

4 Conclusion

This paper studies a simple and universal energy function construction method, and constructs a transient energy function for power systems including the induction motor model. This energy function can be applied to multi-machine large power systems. The simulations in a 3-bus system and in the WSCC 4-generator 11-bus system verify the validity of the obtained energy function. The incorporation of the induction motor model into the energy function opens up a number of possibilities for direct voltage stability analysis, and also provides a tool for analysis of the interaction between the generator angle stability and the load voltage stability.

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