Machine Equations

James Brooks - Monday 21 May 2007

Taken from - [Chan 92 p.12]

$$I_d = Y_{\Re}(E''_d - V_d) - Y''_d(E''_q - V_q)$$
$$I_q = Y_{\Re}(E''_q - V_q) + Y''_q(E''_d - V_d)$$
$$pE''_d = \frac{1}{T_{qo''}}\{(x_q - x''_q)I_q - E''_d - aE''_d\}$$
$$pE'_q = \frac{1}{T_{do'}}\{V_f - (x_d - x'_d)I_d - E'_q - aE'_q\}$$
$$pE''_q = \frac{1}{T_{do''}}\{E'_q + aE'_q - (x'_d - x''_d)I_d - E''_q - aE''_q\}$$

Where:

$$Y''_d = -\frac{X''_q}{(R_a)^2 + (X''_d \times X''_q)}$$
$$Y_q = -\frac{X''_d}{(R_a)^2 + (X''_d \times X''_q)}$$
$$Y_{\Re} = \frac{R_a}{(R_a)^2 + (X''_d \times X''_q)}$$

States

I_d Direct axis current
I_q Quadrature axis current
E''_d Direct axis voltage behind transient reactance
E'_q Quadrature axis voltage behind transient reactance
E''_q Quadrature axis voltage behind sub-transient reactance

Variables

V_d Direct access component of terminal voltage
V_q Quadrature axis component of terminal voltage
V_f Field voltage referred to stator terminals

Parameters

T''_{qo} Quadrature axis open-circuit sub-transient time constant
T'_{do} Direct axis open-circuit transient time constant
T''_{do} Direct axis open-circuit sub-transient time constant
X_d Direct axis reactance
X'_d Direct axis transient reactance
X''_d Direct axis sub-transient reactance
X_q Quadrature axis reactance
X''_q Quadrature axis sub-transient reactance
R_a Winding resistance

$\dot{x}_n$ The dot n$^{\text{th}}$ state
x_n The n$^{\text{th}}$ state
u_n The n$^{\text{th}}$ input

$$
\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \end{Bmatrix} = \begin{Bmatrix} 0 & 0 & m_1 & 0 & m_2 \\ 0 & 0 & m_3 & 0 & m_4 \\ 0 & m_5 & m_6 & 0 & 0 \\ m_7 & 0 & 0 & m_8 & 0 \\ m_9 & 0 & 0 & m_{10} & m_{11} \end{Bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{Bmatrix} + \begin{Bmatrix} n_1 & n_2 & 0 \\ n_3 & n_4 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & n_5 \\ 0 & 0 & 0 \end{Bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix}
$$

$$
\begin{aligned}
m_1 &= Y_{\Re} \\
m_2 &= -Y_d'' \\
m_3 &= -Y_q'' \\
m_4 &= Y_{\Re} \\
m_5 &= \frac{X_q - X_q''}{T_{qo}''} \\
m_6 &= \frac{a-1}{T_{qo}''} \\
m_7 &= \frac{X_d - X_d'}{T_{do}'} \\
m_8 &= \frac{a-1}{T_{do}'} \\
m_9 &= \frac{X_d' - X_d''}{T_{do}''} \\
m_{10} &= \frac{a+1}{T_{do}''} \\
m_{11} &= \frac{a-1}{T_{do}''}
\end{aligned}
\qquad
\begin{aligned}
n_1 &= -Y_{\Re} \\
n_2 &= Y_d'' \\
n_3 &= Y_q'' \\
n_4 &= -Y_{\Re} \\
n_5 &= \frac{1}{T_{do}'}
\end{aligned}
$$

Network Interface

$$
(E_{\Re}'' + jE_{\Im}'') - (V_{\Re} + jV_{\Im}) = (R_a + jX'').(I_{\Re} + jI_{\Im})
$$

or if there is a generator transformer:

$$
n \times (E_{\Re}'' + jE_{\Im}'') - (V_{\Re} + jV_{\Im}) = (R_a + jX'' + R_t + jX_t) \times n \times (I_{\Re} + jI_{\Im})
$$

States: $E_d', E_d'', E_q', E_q'', \omega, \delta, V_a$
Inputs: $aE_d', aE_d'', aE_q', aE_q'', T_a, V_d, V_q, E_{fd}$(A.K.A. non-integrable states)
Parameters: $\begin{matrix} x_d, x_d', x_d'', x_q, x_q', x_q'', T_{do}', T_{do}'', T_{qo}', T_{qo}'' \\ M, Y_\Re, Y_\Im, \mathit{Tap}, D_a, T_{lo}, T_{mo}, f_{wm}, f_0 = 50 \end{matrix}$(A.K.A. constants)
States - PSSENG tsmmat.c 762 FormMachineMatrix(),

$$\dot{E}_d' = \frac{1}{T_{qo'}}\{-E_d' - aE_d' + I_q(x_q - x_q')\}$$
$$\dot{E}_d'' = \frac{1}{T_{qo''}}\{E_d' - E_d'' + aE_d' - aE_d'' + I_q(x_q' - x_q'')\}$$
$$\dot{E}_q' = \frac{1}{T_{do'}}\{V_a - E_q' - aE_q' + I_d(x_d - x_d')\}$$
$$\dot{E}_q'' = \frac{1}{T_{do''}}\{E_q' - E_q'' + aE_q' - aE_q'' + I_d(x_d' - x_d'')\}$$
$$\dot{\omega} = T_a \div M$$
$$\dot{\delta} = \omega$$
$$\dot{V}_a = ?$$

Inputs (A.K.A. non-integrable states) - PSSENG tsmgrp.c 722 SolveMachineGroups()

$$aE_d' = \alpha \cdot E_d' \cdot \frac{x_q}{x_d}$$
$$aE_d'' = \alpha \cdot E_d'' \cdot \frac{x_q}{x_d} - (I_q \cdot (x_q - x_q') - E_d' - aE_d') \cdot \frac{T_{qo}''}{T_{qo}'}$$
$$aE_q' = \alpha \cdot E_q'$$
$$aE_q'' = \alpha \cdot E_q'' - (E_{fd} - I_d \cdot (x_d - x_d') - E_q' - aE_q') \cdot \frac{T_{do}''}{T_{do}'}$$
$$T_a = t_m - t_e - T_{lo} - D_a * \omega$$
$$V_d = \frac{\sin(\delta) \cdot V_\Re + \cos(\delta) \cdot V_\Im}{\mathit{Tap}}$$
$$V_q = \frac{\cos(\delta) \cdot V_\Re + \sin(\delta) \cdot V_\Im}{\mathit{Tap}}$$
$$E_{fd} = V_a$$

Outputs:
$I_\Re = \cos(\delta) \cdot I_{njq} + \sin(\delta) \cdot I_{njd}$ $I_{njq} = (Y_{g\Re} \cdot E_q'') + (Y_{g\Im} \cdot E_d'')$
$I_\Im = \sin(\delta) \cdot I_{njq} - \cos(\delta) \cdot I_{njd}$ $I_{njd} = (Y_{g\Re} \cdot E_d'') - (Y_{g\Im} \cdot E_q'')$
Where:

$t_m = \frac{T_{mo}}{1.0 + (\frac{\omega}{2\pi \cdot f_0})}$ $t_e = \frac{(E_q'' \cdot I_q) + (E_d'' \cdot I_d)}{1.0 + \frac{f_{wm}}{2\pi \cdot f_0}}$	$\alpha = \mathit{CalcSat}(\sqrt{(E_q'')^2 + (E_d'')^2}, \mathit{SatTab})$ $I_d = Y_\Re(E_d'' - V_d) - Y_\Im(E_q'' - V_q)$ $I_q = Y_\Re(E_q'' - V_q) + Y_\Im(E_d'' - V_d)$ $(V_\Re, V_\Im)$ Bus Bar Voltages	E_{fd}, V_a Strange or only partly used states. E_{fd} does not connect to a state. V_a cannot change as it doesn't have any elements in it's matrix row.

$$
\begin{gathered}
(I_d + j \cdot I_q) \cdot (R_a + j \cdot X'') = (E''_d + j \cdot E''_q) - (V_d - j \cdot V_q) \\
\ldots \\
I_d = Y_{\Re}(E''_d - V_d) - Y_{\Im}(E''_q - V_q) \\
I_q = Y_{\Re}(E''_q - V_q) + Y_{\Im}(E''_d - V_d) \\
I_{\Re} = \cos(\delta) \cdot I_q + \sin(\delta) \cdot I_d \\
I_{\Im} = \sin(\delta) \cdot I_q - \cos(\delta) \cdot I_d
\end{gathered}
$$