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MATHEMATICS

GENERAL ALGEBRA & EXPONENTS

GRADE 10

25/26 AUGUST 2022

General Algebra

Guiding activities to clarify the core content

1.1 Expand and simplify:

1.1.1 $(2x-3y)(2x+3y)$ (1)

1.1.2 $(3x+2)^2+(2x-3)^2$ (3)

1.1.3 $(m-2n)^3$ (4)

1.1.4 $(4-3a)(4-3a^2)-(a-2)(a^2+2a+4)$ (4)

WORKING AREA

Solution

1.1.1	$(2x-3y)(2x+3y)$ $=4x^2-9y^2$
1.1.2	$(3x+2)^2+(2x-3)^2$ $=9x^2+12x+4+4x^2-12x+9$ $=13x^2+13$

Solution

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1.1.3

$$(m-2n)^3$$
$$=(m-2n)(m-2n)^2$$
$$=(m-2n)(m^2-4mn+4n^2)$$
$$=m^3-4m^2n+4mn^2-2m^2n+8mn^2-8n^3$$
$$=m^3-6m^2n+12mn^2-8n^3$$

1.1.4

$$(4-3a)(4-3a^2)-(a-2)(a^2+2a+4)$$
$$=16-12a^2-12a+9a^3-(a^3-8)$$
$$=16-12a^2-12a+9a^3-a^3+8$$
$$=8a^3-12a^2-12a+24$$

Guiding activities to clarify the core content

1.2 Factorise fully:

1.2.1 $24x^2 - 54$ (2)

1.2.2 $9x^4 - 9$ (4)

1.2.3 $6a^2 - 5a - 4$ (2)

1.2.4 $a^3 - 4a^2 - 4a + 16$ (3)

1.2.5 $x^3 + 64$ (2)

Working Area

Solution

1.2.1	$24x^2 - 54$ $= 6(4x^2 - 9)$ $= 6(2x+3)(2x-3)$
1.2.2	$9x^4 - 9$ $= 9(x^4 - 1)$ $= 9(x^2+1)(x^2-1)$ $= 9(x^2+1)(x+1)(x-1)$
1.2.3	$6a^2 - 5a - 4$ $= (3a-4)(2a+1)$

Solution

1.2.4	$= a^3 - 4a - 4a^2 + 16$ $= a(a^2 - 4) - 4(a^2 - 4)$ $= (a^2 - 4)(a - 4)$ $= (a + 2)(a - 2)(a - 4)$
1.2.5	$x^3 + 64$ $= (x + 4)(x^2 - 4x + 16)$

Guiding activities to clarify the core content

Simplify:

2.1 $$\frac{x^2-12x+27}{4x^2-12x}$$ (3)

2.2 $$1+\frac{1}{4x^2y}-\frac{(x-2)}{3x^3}$$ (5)

2.3 $$\frac{3}{x^2-3x-4}-\frac{x+1}{4-x}$$ (6)

2.4 $$\left(2x-\frac{4x^2-3}{2x}\right)\left(\frac{3}{2x}\right)^{-2}$$ (5)

Working Area

Solution

2.1

$$\frac{x^2-12x+27}{4x^2-12x}$$

$$=\frac{(x-9)(x-3)}{4x(x-3)}$$

$$=\frac{x-9}{4x}$$

2.2

$$1+\frac{1}{4x^2y}-\frac{(x-2)}{3x^3}$$

$$=\frac{1}{1}\times\frac{12x^3y}{12x^3y}+\frac{1}{4x^2}\times\frac{3x}{3x}-\frac{(x-2)}{3x^3}\times\frac{4y}{4y}$$

$$=\frac{12x^3y}{12x^3y}+\frac{3x}{12x^3y}-\frac{4y(x-2)}{12x^3y}$$

$$=\frac{12x^3y+3x-4y(x-2)}{12x^3y}$$

$$=\frac{12x^3y+3x-4xy+8y}{12x^3y}$$

Solution

2.3

$$\frac{3}{x^2-3x-4}-\frac{x+1}{4-x}$$

$$=\frac{3}{(x-4)(x+1)}+\frac{x+1}{(x-4)}$$

$$=\frac{3+(x+1)(x+1)}{(x-4)(x+1)}$$

$$=\frac{3+x^2+2x+1}{(x-4)(x+1)}$$

$$=\frac{x^2+2x+4}{(x-4)(x+1)}$$

Solution

2.4

$$\left(2x-\frac{4x^2-3}{2x}\right)\left(\frac{3}{2x}\right)^{-2}$$

$$=\left(\frac{4x^2-4x^2+3}{2x}\right)\left(\frac{2x}{3}\right)^{2}$$

$$=\left(\frac{3}{2x}\right)\left(\frac{4x^2}{9}\right)$$

$$=\frac{2x}{3}$$

Guiding activities to clarify the core content

3.1 Solve for x:

3.1.1 $3x^2 = 48$ (3)

3.1.2 $(x-7)(x+3) = 24$ (4)

3.1.3 $\dfrac{3x-4}{4} = \dfrac{4}{3x-4}$ (6)

3.2 Solve for x and represent the solution on a number line:

$\dfrac{x-6}{3} - \dfrac{3x+2}{4} < 0$ (4)

3.3 Solve for x and y simultaneously: $3x + 2y = 12$ and $x + 4y = 14$ (4)

Working Area

Solution

$$3x^2 = 48$$

$$\therefore x^2 = 16$$

$$\therefore x^2 - 16 = 0$$

$$\therefore (x+4)(x-4) = 0$$

$$\therefore x = -4 \quad \text{or} \quad x = 4$$

$$(x-7)(x+3) = 24$$

$$\therefore x^2 - 4x - 21 = 24$$

$$\therefore x^2 - 4x - 45 = 0$$

$$\therefore (x-9)(x+5) = 0$$

$$\therefore x = 9 \quad \text{or} \quad x = -5$$

Solution

$$\frac{3x-4}{4} = \frac{4}{3x-4}$$

$$\text{LCD} = 4(3x-4)$$

$$\therefore (3x-4)(3x-4) = 16$$

$$\therefore 9x^2 - 24x + 16 = 16$$

$$\therefore 9x^2 - 24x = 0$$

$$\therefore x(9x-24) = 0$$

$$\therefore x = 0 \quad \text{or} \quad x = \frac{24}{9}$$

$$\frac{x-6}{3} - \frac{3x+2}{4} < 0$$

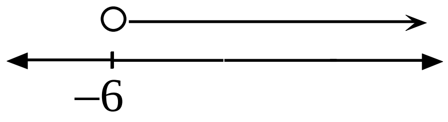
$$\therefore 4(x-6) - 3(3x+2) < 0$$

$$\therefore 4x - 24 - 9x - 6 < 0$$

$$\therefore -5x - 30 < 0$$

$$\therefore -5x < 30$$

$$\therefore x > -6$$



Solution

$$x+4y=14 \qquad A$$
$$3x+2y=12 \qquad B$$
$$x+4y=14 \qquad A$$
$$-6x-4y=-24 \qquad B\times -2$$
$$-5x=-10 \qquad \text{Add}$$
$$\therefore x=2$$
$$\therefore 2+4y=14$$
$$\therefore 4y=12$$
$$\therefore y=3$$

OR

$$x=14-4y$$
$$3(14-4y)+2y=12$$
$$\therefore 42-12y+2y=12$$
$$\therefore -10y=-30$$
$$\therefore y=3$$
$$\therefore x=14-4(3)$$
$$\therefore x=2$$

Exponents

Guiding activities to clarify the core content

4.1 Simplify:

4.1.1 $\dfrac{9^{x-1}.24^{x+1}.8^{-x}}{27^{x}}$ (6)

4.1.2 $\dfrac{2^{x+2}+5.2^{x}+2^{x}}{5.2^{x}}$ (3)

4.2 Solve for x:

4.2.1 $5.125^{x+3}=\dfrac{1}{25}$ (4)

4.2.2 $0,4^{x}=0,064$ (4)

4.2.3 $2x^{\frac{3}{2}}=250$ (3)

Working Area

Solution

4.1.1

$$\frac{9^{x-1}.24^{x+1}.8^{-x}}{27^x}$$

$$=\frac{(3^2)^{x-1}.(2^3.3)^{x+1}.(2^3)^{-x}}{(3^3)^x}$$

$$=\frac{3^{2x-2}.2^{3x+3}.3^{x+1}.2^{-3x}}{3^{3x}}$$

$$=\frac{3^{3x-1}.2^3}{3^{3x}}$$

$$=3^{3x-1-3x}.2^3$$

$$=3^{-1}.2^3$$

$$=\frac{8}{3}$$

4.1.2

$$\frac{2^{x+2}+5.2^x+2^x}{5.2^x}$$

$$=\frac{2^x.2^2+5.2^x+2^x}{5.2^x}$$

$$=\frac{2^x(2^2+5+1)}{5.2^x}=\frac{10}{5}=2$$

Solution

4.2.1

$$5 \, . \, 125^{x+3} = \frac{1}{25}$$

$$\therefore 125^{x+3} = \frac{1}{125}$$

$$\therefore (5^3)^{x+3} = 5^{-3}$$

$$\therefore 5^{3x+9} = 5^{-3}$$

$$\therefore 3x+9 = -3$$

$$\therefore 3x = -12$$

$$\therefore x = -4$$

4.2.2

$$0{,}4^x = 0{,}064$$

$$\therefore \left(\frac{4}{10}\right)^x = \frac{64}{1000}$$

$$\therefore \left(\frac{2}{5}\right)^x = \frac{8}{125}$$

$$\therefore \left(\frac{2}{5}\right)^x = \left(\frac{2}{5}\right)^3$$

$$\therefore x = 3$$

Solution

4.2.3

$$2x^{\frac{3}{2}} = 250$$

$$\therefore x^{\frac{3}{2}} = 125$$

$$\therefore (x^{\frac{3}{2}})^{\frac{2}{3}} = (5^3)^{\frac{2}{3}}$$

$$\therefore x = 25$$

Guiding activities to clarify the core content

4.1 Simplify:

4.1.1 $\dfrac{12^x . 3^{-x}}{2 . 4^x}$ (4)

4.1.2 $\dfrac{4^x + 3 . 2^{2x+1}}{7 . 2^{2x+1}}$ (5)

4.2 Solve for x:

4.2.1 $4 . 25^{x+3} = 4$ (3)

4.2.2 $(0,2)^{x-2} = 0,04$ (4)

4.2.3 $2^{x+1} + 2^{x+2} = 24$ (3)

Working Area

Solution

4.1.1

$$\frac{12^x . 3^{-x}}{2 . 4^x}$$

$$= \frac{(2^2 . 3)^x . 3^{-x}}{2 . (2^2)^x}$$

$$= \frac{2^{2x} . 3^x . 3^{-x}}{2 . 2^{2x}}$$

$$= \frac{2^{2x} . 3^0}{2 . 2^{2x}} = \frac{1}{2}$$

4.1.2

$$\frac{4^x + 3 . 2^{2x+1}}{7 . 2^{2x+1}}$$

$$= \frac{(2^2)^x + 3 . 2^{2x} . 2^1}{7 . 2^{2x} . 2^1}$$

$$= \frac{2^{2x} + 3 . 2^{2x} . 2^1}{7 . 2^{2x} . 2^1}$$

$$= \frac{2^{2x}(1+6)}{14 . 2^{2x}}$$

$$= \frac{7}{14} = \frac{1}{2}$$

Solution

4.2.1

$$4 \cdot 25^{x+3} = 4$$
$$\therefore 25^{x+3} = 1$$
$$\therefore (5^2)^{x+3} = 5^0$$
$$\therefore 5^{2x+6} = 5^0$$
$$\therefore 2x+6 = 0$$
$$\therefore 2x = -6$$
$$\therefore x = -3$$

4.2.2

$$(0,2)^{x-2} = 0,04$$
$$\therefore \left(\frac{2}{10}\right)^{x-2} = \left(\frac{4}{100}\right)$$
$$\therefore \left(\frac{1}{5}\right)^{x-2} = \left(\frac{1}{25}\right)$$
$$\therefore (5^{-1})^{x-2} = 5^{-2}$$
$$\therefore 5^{-x+2} = 5^{-2}$$
$$\therefore -x+2 = -2$$
$$\therefore -x = -4$$
$$\therefore x = 4$$

Solution

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4.2.3

$$2^{x+1}+2^{x+2}=24$$

$$\therefore 2^x.2^1+2^x.2^2=24$$

$$\therefore 2^x(2^1+2^2)=24$$

$$\therefore 2^x(6)=24$$

$$\therefore 2^x=4$$

$$\therefore x=2$$

Guiding activities to clarify the core content

5.1 Given the linear pattern $14\,;9\,;4\,;-1;\ldots$

5.1.1 Determine the nth term of the pattern. (2)

5.1.2 Which term of the pattern is equal to -492? (2)

5.2 Consider the number pattern: $1\,;6\,;1\,;9\,;1\,;12\,;1\,;15\,;1\,;\ldots\ldots$

5.2.1 Determine the 999^{th} term. (1)

5.2.2 Determine the $1\ 000^{\text{th}}$ term. (2)

Working Area

Solution

5.1.1	$14 ; 9 ; 4 ; \ldots$ $T_n = -5n + 19$
5.1.2	$-492 = -5n + 19$ $\therefore 5n = 500$ $\therefore n = 100$ $\therefore T_{100} = -492$
5.2.1	All the odd terms are 1. Therefore $T_{999} = 1$
5.2.2	The even terms form a linear pattern $6 ; 9 ; 12 ; \ldots$ The general term for the even terms in the original pattern is $T_{2n} = 3n + 3$. $T_{1\,000} = T_{2(500)} = 3(500) + 3 = 1\,503$

Concluding Remarks

Following our today lesson, I want you to do the to:

Read through what the learner **need to understand and master** in your learner material.

Complete the activities

Attempt as many as possible other similar examples on your own from the **Text-Book and the past exam papers**.

Repeat this procedure until you are **confident**.

Do not forget: **Practice makes perfect!**

Thank you