SUBJECT : MATHEMATICS

GRADE : 10

LESSON TOPIC: TRIGONOMETRY

DATE : 25 JUNE 2022

Putting things into perspective

Focus :

Trigonometry

Lesson Duration : 3hrs

Saturday (25 June 2022):

Lesson Topic : Trig ratios; Reciprocals ;Substitutions & Equations

Monday (27 June 2022):

Lesson Topic :Right Angle triangle, Angle of elevation and depression; Special Angles

Tuesday (28 June 2022):

Lesson Topic : Special Angles (continuation) ;Cartesian Plane (C.A.S.T)

Wednesday (29 June 2022):

Lesson Topic : Trig functions

Thursday (30 June 2022):

Lesson Topic : Trig functions

Friday (01 July 2022):

Lesson Topic : Trig functions

WHAT IS TRIGONOMETRY?

Pocket Oxford Dictionary

Trigonometry is that branch of mathematics concerned with the relationships between the sides and angles of triangles and with the functions of angles

Longman South African School Dictionary

the part of mathematics concerned with the relationships between the angles and sides of triangles

ORIGINS OF THE WORD

- **GREEK:**

-tri	: three
-gon	: sides
-metron	: measure

APPLICATION OF TRIGONOMETRY

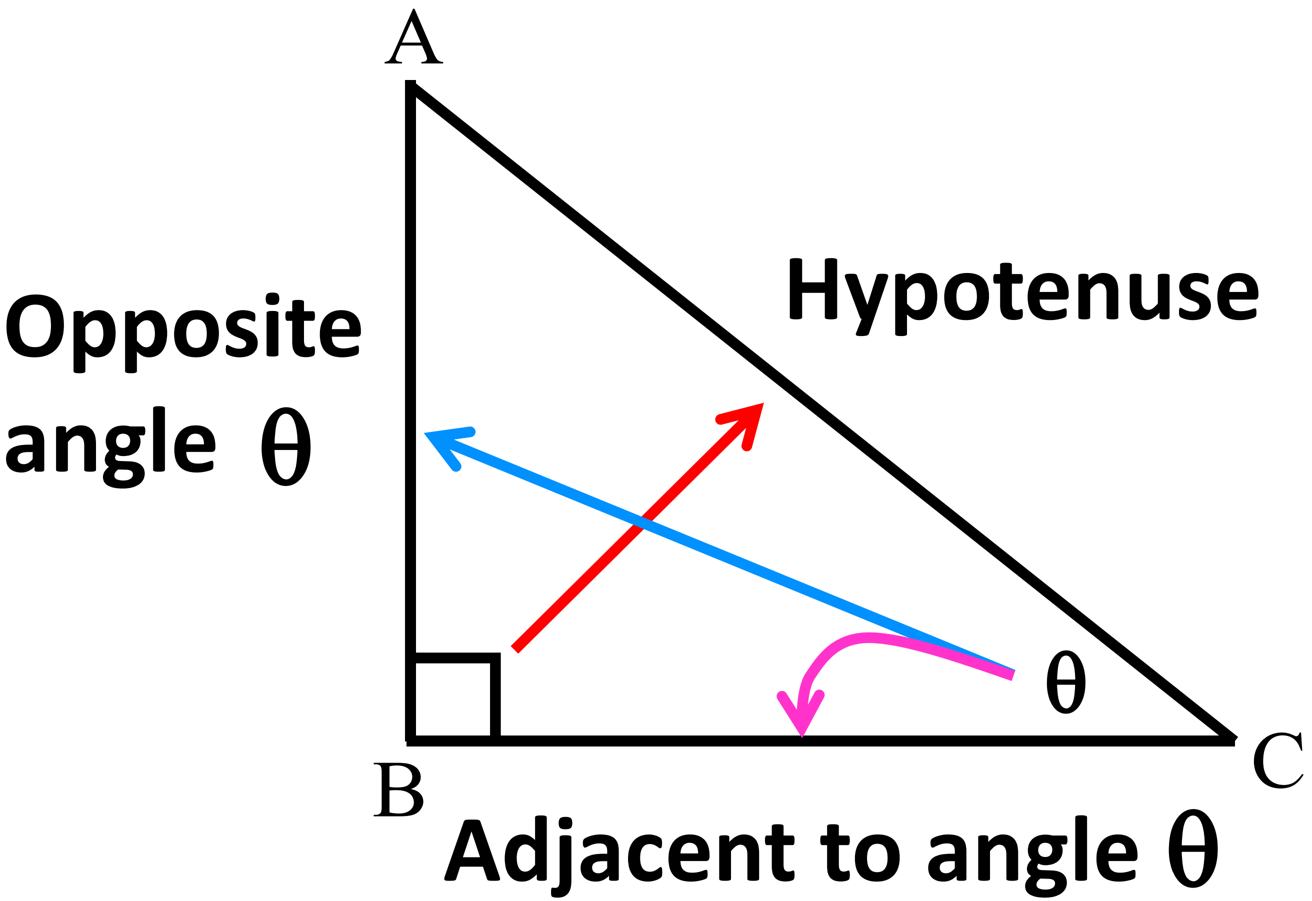
Applications of Trigonometry:

- Navigation
- Architecture
- Engineering
- Astronomy

EXPECTED OUTCOMES

- Definitions if the Trigonometric ratios: $\sin\theta$, $\cos\theta$ and $\tan\theta$ in a right – angled triangle.
- Define the reciprocals of the trigonometric rations: $\operatorname{cosec}\theta$, $\sec\theta$, $\cot\theta$ using right – angled triangles.
- Solve simple trigonometric equations for angles between 0° and 90°
- Derive and use the values of the trigonometric ratios (without using a calculator) for the special angles. i.e. $\theta \epsilon \{0°, 30°, 45°, 60°, 90°\}$.
- Extension of the trigonometric definitions to $0° \leq \theta \leq 360°$.
- Use diagrams to determine the numerical values of ratios for angles of $0°$ and $360°$
- Solve problem in TWO dimensions

Definition: Trigonometry Ratios



Definition: Trigonometry Ratios

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

RECIPROCALS OF THE TRIGONOMETRIC RATIOS

We can define reciprocal funtions for the sine, cosine and tangent ratios.

The **cosecant** function **(cosec $\boldsymbol{\theta}$)** is the reciprocal of $\sin\theta$

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{\text{hyp}}{\text{opp}}$$

The **secant** function **sec $\boldsymbol{\theta}$** is the reciprocal of $\cos\theta$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{\text{hyp}}{\text{adj}}$$

The **cotangent** function **cot $\boldsymbol{\theta}$** is the reciprocal of $\tan\theta$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{\text{adj}}{\text{opp}}$$

Summary

-$sin\theta = \frac{y}{r}$

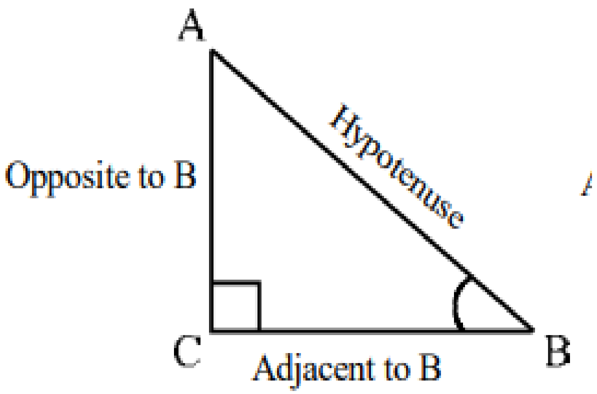
-$cos\theta = \frac{x}{r}$

-$tan\theta = \frac{y}{x}$

-$cosec\theta = \frac{r}{y}$

-$sec\theta = \frac{r}{x}$

-$cot\theta = \frac{x}{y}$



EXAMPLE 1

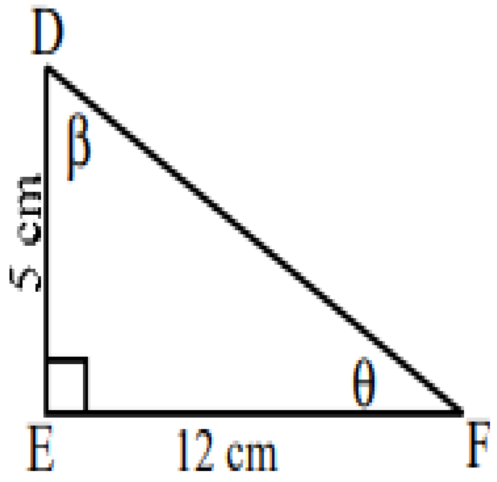
EXAMPLE 1

In ΔDEF, $DE = 5$, $EF = 12$, $\hat{E} = 90°$, $\hat{D} = \beta$ and $\hat{F} = \theta$.

(a) Determine the length of the hypotenuse DF.

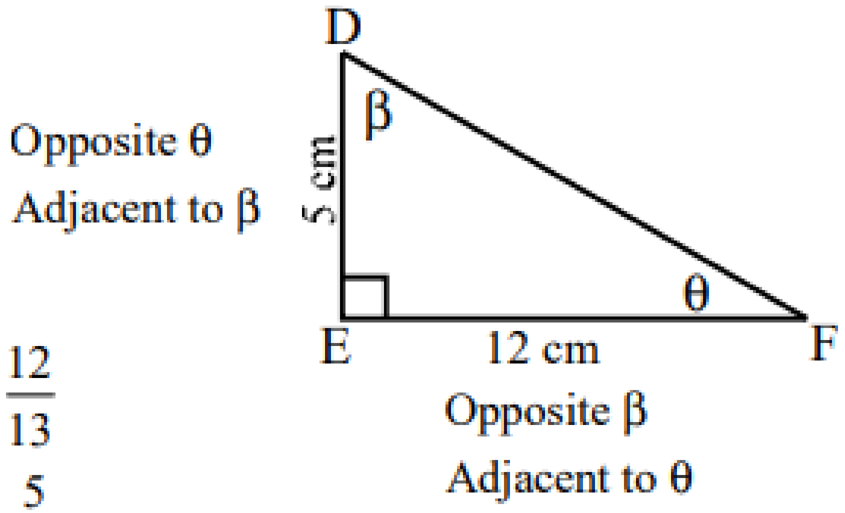
(b) Write the value of $\sin\theta$, $\cos\theta$ and $\tan\theta$.

(c) Write the value of $\sin\beta$, $\cos\beta$ and $\tan\beta$.



Solution:

Solutions



(a) $DF^2 = 5^2 + 12^2$ [Pythagoras]

$\therefore DF^2 = 169$

$\therefore DF = 13 \text{ cm}$

(b) $\sin\theta = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{5}{13}$

$\cos\theta = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{12}{13}$

$\tan\theta = \dfrac{\text{opp}}{\text{adj}} = \dfrac{5}{12}$

(c) $\sin\beta = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{12}{13}$

$\cos\beta = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{5}{13}$

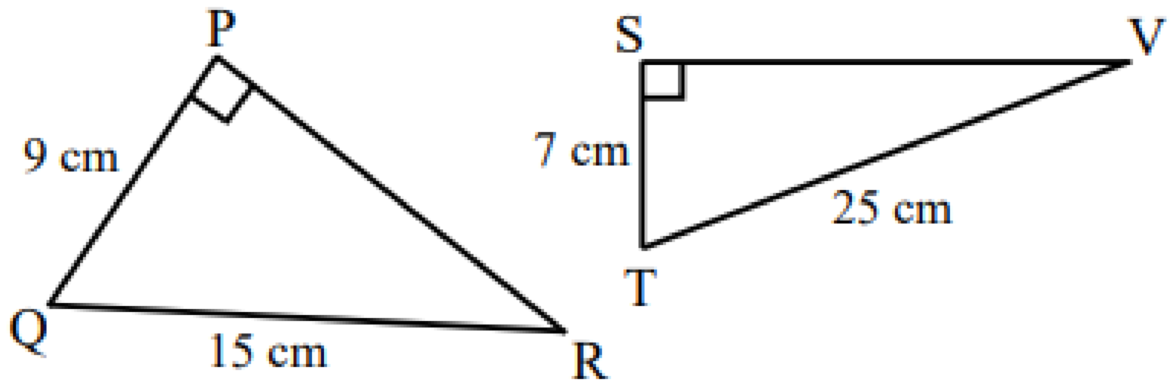
$\tan\beta = \dfrac{\text{opp}}{\text{adj}} = \dfrac{12}{5}$

Example: 2

EXAMPLE 2

Determine: (a) $\tan Q$

(b) $\cos V$



Solutions

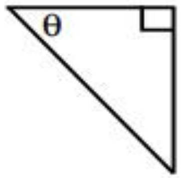
(a)
$$PR^2 = 15^2 - 9^2$$
$$\therefore PR^2 = 144$$
$$\therefore PR = \sqrt{144} = 12\,\text{cm}$$
$$\therefore \tan Q = \frac{\text{opp}}{\text{adj}} = \frac{12}{9} = \frac{4}{3}$$

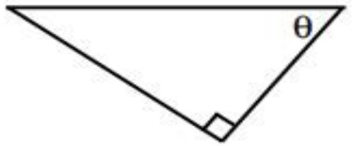
(b)
$$SV^2 = 25^2 - 7^2$$
$$\therefore SV^2 = 576$$
$$\therefore SV = \sqrt{576} = 24\,\text{cm}$$
$$\therefore \cos V = \frac{\text{adj}}{\text{hyp}} = \frac{24}{25}$$

Activity: 1

EXERCISE 1

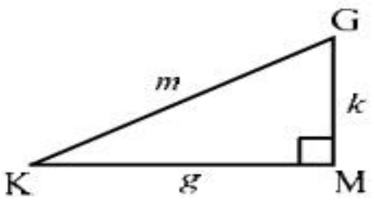
(a) Redraw the triangles below and indicate which sides are opposite, adjacent and hypotenuse with respect to θ.





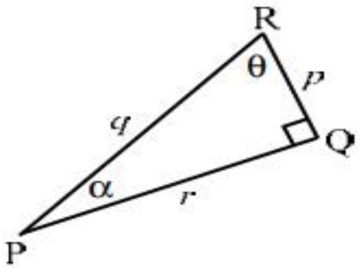
(b) State the following in terms of g, k and m:

(1)	$\sin K$	(2)	$\cos K$
(3)	$\tan K$	(4)	$\sin G$
(5)	$\cos G$	(6)	$\tan G$



(c) State the following in terms of p, q and r:

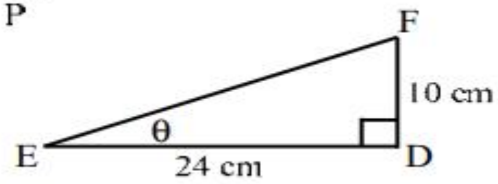
(1)	$\sin \theta$	(2)	$\cos \theta$
(3)	$\tan \theta$	(4)	$\sin \alpha$
(5)	$\cos \alpha$	(6)	$\tan \alpha$



(d) ΔDEF is shown with given dimensions.

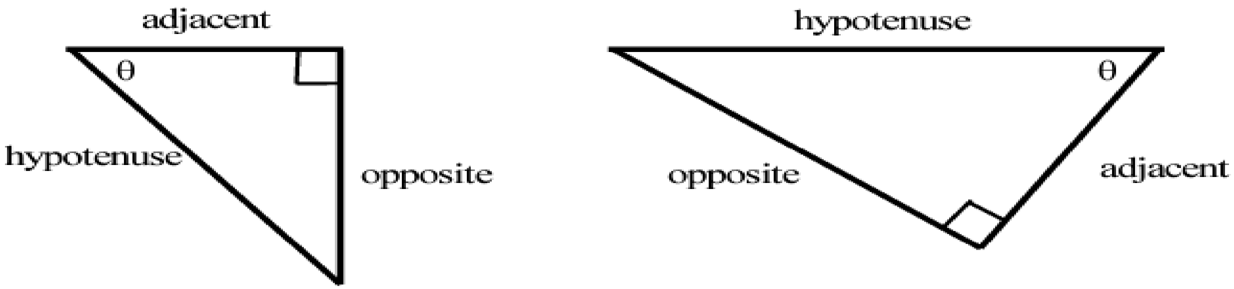
(1) Determine the length of EF.

(2) Write the value of $\sin \theta$, $\cos \theta$ and $\tan \theta$



Solution:

(a)



(b) (1) $\sin K = \dfrac{k}{m}$ (2) $\cos K = \dfrac{g}{m}$ (3) $\tan K = \dfrac{k}{g}$

(4) $\sin G = \dfrac{g}{m}$ (5) $\cos G = \dfrac{k}{m}$ (6) $\tan G = \dfrac{g}{k}$

(c) (1) $\sin\theta = \dfrac{r}{q}$ (2) $\cos\theta = \dfrac{p}{q}$ (3) $\tan\theta = \dfrac{r}{p}$

(4) $\sin\alpha = \dfrac{p}{q}$ (5) $\cos\alpha = \dfrac{r}{q}$ (6) $\tan\alpha = \dfrac{p}{r}$

(d) (1) $EF^2 = 10^2 + 24^2 = 676$

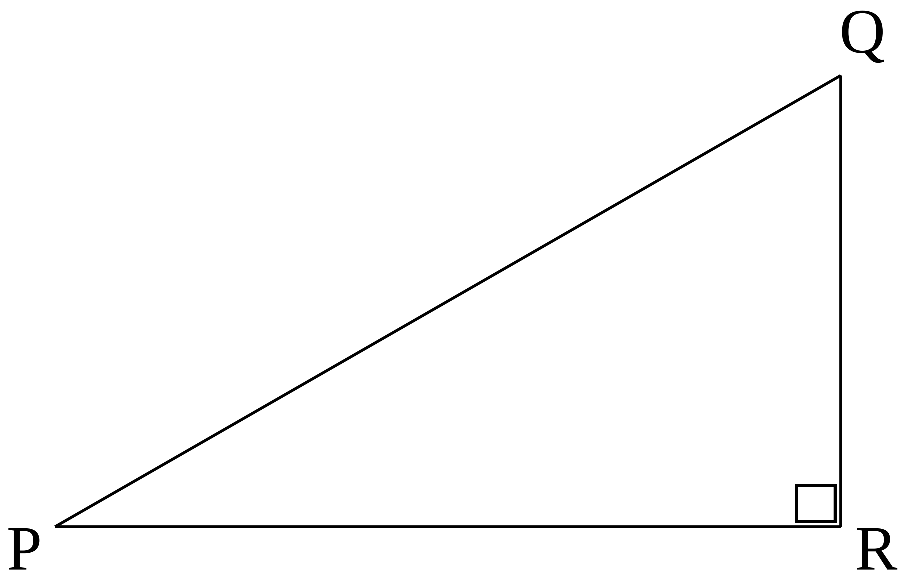
$\therefore\ EF = \sqrt{676} = 26$

(2) $\sin\theta = \dfrac{10}{26} = \dfrac{5}{13}$ $\cos\theta = \dfrac{24}{26} = \dfrac{12}{13}$

$\tan\theta = \dfrac{10}{24} = \dfrac{5}{12}$

#Reciprocals:

Write down the trigonometric ratios for angle P and angle Q.



$\operatorname{cosec} P =$

$\operatorname{cosec} Q =$

$\sec P =$

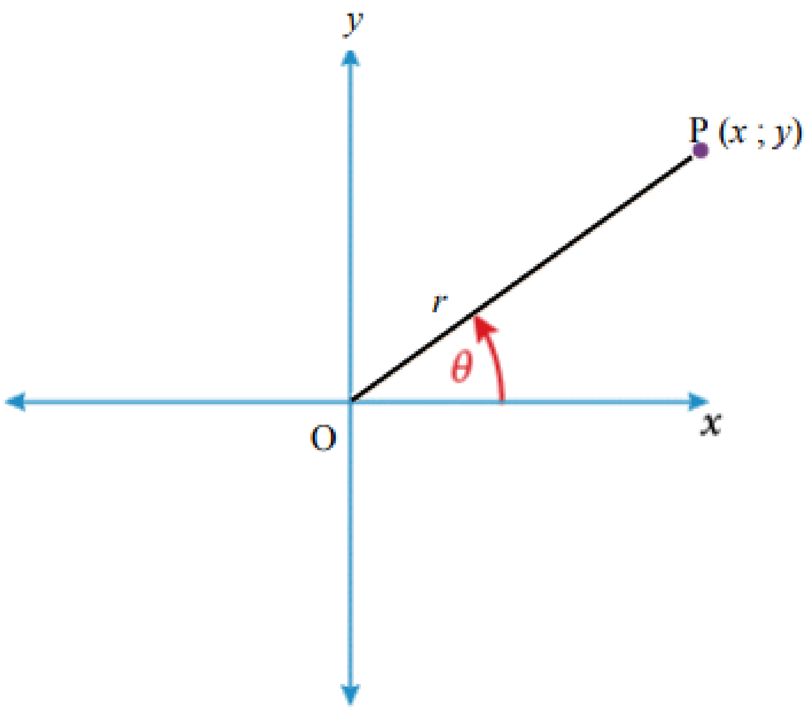
$\sec Q =$

$\cot P =$

$\cot Q =$

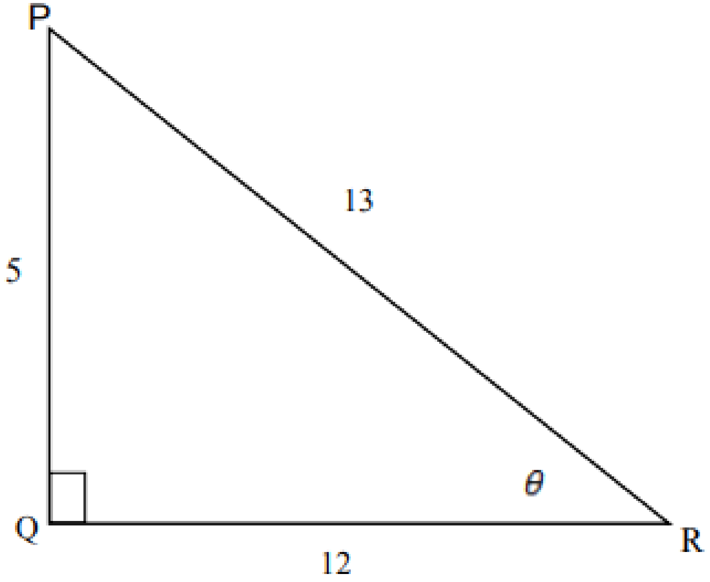
Exam type:

If point $P(x\,;y)$ is a point on the Cartesian plane and OP $= r$ units. Determine $\dfrac{\sin\theta}{\cos\theta}$.



Exam type:

In ΔPQR, $\hat{Q} = 90°$ and $\hat{R} = \theta$. $PQ = 5$ units, $QR = 12$ units and $PR = 13$ units.



Write down the values of:

1.2.1 $\sin\theta$
1.2.2 $\sec\theta$
1.2.3 $\tan\theta$

Solutions:

1.1 $$\frac{\sin\theta}{\cos\theta} = \frac{y}{r} \div \frac{x}{r}$$
$$= \frac{y}{r} \times \frac{r}{x}$$
$$= \frac{y}{x} = \tan\theta$$

1.2

1.2.1	$\sin\ \theta = \frac{PQ}{PR} = \frac{5}{13}$
1.2.2	$\sec\ \theta = \frac{PR}{QR} = \frac{13}{12}$
1.2.3	$\tan\ \theta = \frac{PQ}{QR} = \frac{5}{12}$

Calculating the trigonometric ratios:

The **sine, cosine and tangent ratio** can be calculated with the use of a calculator which must be on the DEG (degree) mode.

Example:

Evaluate the following trigonometric ratios rounded off to two decimal places where necessary.

(a) $\cos 35°$ (b) $\tan 36°$ (c) $\sin 83°$ (d) $\sin 43°$

(e) $\cos 14°$ (f) $\tan 45°$ (g) $5\cos 60°$ (h) $\dfrac{\sin 12°}{12}$

(i) $\dfrac{1}{3}\tan 36°$ (j) $\dfrac{20{,}35}{\sin 38°}$ (k) $\sin(35°+75°)=0{,}94$

(l) $\sin 35°+\sin 75°=1{,}54$

Solutions:

Solutions

(a)	$\cos(35°) = 0,82$	(b)	$\tan(36°) = 0,73$	(c)	$\sin(83°) = 0,99$
(d)	$\sin(43°) = 0,68$	(e)	$\cos(14°) = 0,97$	(f)	$\tan(45°) = 1$
(g)	$5\cos(60°) = 2,5$	(h)	$\dfrac{\sin(12°)}{12} = 0,02$	(i)	$\dfrac{1}{3}\tan(36°) = 0,24$
(j)	$\dfrac{20,35}{\sin(38°)} = 33,05$	(k)	$\sin(35° + 75°) = 0,94$	(l)	$\sin 35° + \sin 75° = 1,54$

Take note: $\sin(35° + 75°) \neq \sin 35° + \sin 75°$ (An important result)

Activity: 2

Calculate with the use of a calculator the following rounded off to two decimal places where appropriate.

(1)	$\cos 23°$	(2)	$\sin 68°$	(3)	$\tan 64°$
(4)	$\cos 34°$	(5)	$2\cos 45°$	(6)	$5\sin 65°$
(7)	$7\tan 58°$	(8)	$-4\sin 30°$	(9)	$\frac{1}{3}\sin 70°$
(10)	$\dfrac{\sin 60°}{20}$	(11)	$\dfrac{\cos 24°}{24}$	(12)	$\tan(54° + 25°)$
(13)	$\tan 54° + \tan 25°$	(14)	$\dfrac{40}{-25\sin 45°}$	(15)	$\dfrac{\sin 60° \cos 70°}{\tan 46° \sin 30°}$

Solutions:

(1) $\cos 23^\circ = 0,92$

(2) $\sin 68^\circ = 0,93$

(3) $\tan 64^\circ = 2,05$

(4) $\cos 34^\circ = 0,83$

(5) $2\cos 45^\circ = 1,41$

(6) $5\sin 65^\circ = 4,53$

(7) $7\tan 58^\circ = 11{,}2$

(8) $-4\sin 30^\circ = -2$

(9) $\frac{1}{3}\sin 70^\circ = 0,31$

(10) $\dfrac{\sin 60^\circ}{20} = 0,04$

(11) $\dfrac{\cos 24^\circ}{24} = 0,04$

(12) $\tan(54^\circ + 25^\circ) = 5,14$

(13) $\tan 54^\circ + \tan 25^\circ = 1,84$

(14) $\dfrac{40}{-25\sin 45^\circ} = -2,26$

(15) $\dfrac{(\sin 60^\circ \cos 70^\circ)}{(\tan 46^\circ \sin 30^\circ)} = 0,57$

Activity:3

Evaluate each of the following correct to two decimal places:

NB: Degree Mode

$$\operatorname{cosec} 36^\circ$$

$$8\cot 88^\circ \times \frac{1}{\sec 32^\circ}$$

$$\sec^2 52^\circ$$

Substitutions:

Example:

Determine the decimal value of the following if $A = 23,8°$ and $B = 18,1°$
(Round off your answers to one decimal place)

(a) $\sin(A + B)$ (b) $\tan 2B$ (c) $\cos^2(2A - 10°)$

Solutions

(a)
$$\sin(23,8° + 18,1°)$$
$$= \sin(41,9°)$$
$$= 0,667... \approx 0,7$$

(b)
$$\tan(2(18,1°))$$
$$= \tan(36,2)$$
$$= 0,7318... \approx 0,7$$

(c)
$$\cos^2(2(23,8°) - 10°)$$
$$= \cos^2(37,6°)$$
$$= (\cos(37,6°))^2$$
$$= 0,627... \approx 0,6$$

Reciprocals:

Given that $\hat{A} = 38{,}2°$ and $\hat{B} = 146{,}4°$.

Calculate the value of $2\operatorname{cosec}A + \cos 3B$.

Solution:

$$2\operatorname{cosec} 38{,}2° + \cos 3(146{,}4°)$$

$$= 2\left(\frac{1}{\sin 38.2°}\right) + \cos 3(146{,}4°)$$

$$= 3{,}42$$

Activity: 3

Determine the decimal value of the following if $A = 35°$ and $B = 52°$
(Round off your answers to two decimal places)

(1) $\cos(A + B)$ (2) $\cos A + \cos B$ (3) $3\sin 2B$

(4) $3\tan\frac{1}{3}A$ (5) $2\sin^2(2A - B)$ (6) $\sqrt{\cos 3A + \sin B}$

Solution:

(1) $\cos(35° + 52°) = \cos 87° = 0,05$

(2) $\cos 35° + \cos 52° = 1,43$

(3) $3\sin 2(52°) = 3\sin 104° = 2,91$

(4) $3\tan\left[\frac{1}{3}(35°)\right] = 0,62$

(5) $2[\sin(2(35°) - 52°)]^2 = 0,19$

(6) $\sqrt{\cos[3(35°)] + \sin 52°} = 0,73$

DETERMINING THE SIZE OF THE ANGLE GIVEN THE TRIGONOTRIC RATIO: (Equations)

Promaths ONLINE

(a) Determine the size of the acute angle θ in each of the following trigonometric equations. Round your answers off to one decimal place where necessary.

(1) $\cos\theta = 0,5$ (2) $\tan\theta = 4,123$ (3) $\sin\theta = 0,707$

(b) Solve the following equations. Round your answers off to one decimal place where necessary. All angles are acute.

(1) $2\sin\theta = 1,124$ (2) $\sin 2\theta = 0,435$

(3) $\frac{1}{2}\tan 2x = 3$ (4) $1 + 2\cos(x + 10°) = 2,356$

Solutions:

Solutions

(a) (1) $\cos\theta = 0,5$ — press [shift] [cos] then 0,5

$\therefore \theta = \cos^{-1}(0,5)$ — as it appears on the calculator screen

$\therefore \theta = 60°$

(2) $\tan\theta = 4,123$ — press [shift] [tan] then 4,123

$\therefore \theta = \tan^{-1}(4,123)$ — as it appears on the calculator screen

$\therefore \theta = 76,4°$

(3) $\sin\theta = 0,706$ — press [shift] [sin] then 0,706

$\therefore \theta = \sin^{-1}(0,706)$ — as it appears on the calculator screen

$\therefore \theta = 44,9°$

(b) (1) $2\sin\theta = 1,124$ — $\sin\theta$ has been multiplied by 2

$\therefore \sin\theta = 0,562$ — isolate $\sin\theta$ by dividing by 2

$\therefore \theta = \sin^{-1}(0,562)$

$\therefore \theta = 34,2°$

(2) $\sin(2\theta) = 0,435$ — insert the brackets

$\therefore 2\theta = \sin^{-1}(0,435)$

$\therefore 2\theta = 25,78529...°$ — determine θ

$\therefore \theta = 12,9°$ — divide by 2 and then determine θ

(3) $\frac{1}{2}\tan 2x = 3$ — LCD: 2

$\therefore 2\times\frac{1}{2}\tan 2x = 3\times 2$ — isolate $\tan 2x$

$\therefore \tan 2x = 6$

$\therefore 2x = \tan^{-1}(6)$

$\therefore 2x = 80,537...°$

$\therefore x = 40,3°$

(4) $1+2\cos(x+10°) = 2,356$

$\therefore 2\cos(x+10°) = 1,356$

$\therefore \cos(x+10°) = 0,678$

$\therefore x+10° = \cos^{-1}(0,678)$

$\therefore x+10° = 47,3124...°$

$\therefore x = 37,3°$

Exam Type:

Solve for x, correct to ONE decimal place, where $0° \leq x \leq 90°$:

4.2.1 $\tan x = 2{,}01$

4.2.2 $5\cos x + 2 = 4$

4.2.3 $\dfrac{\operatorname{cosec} x}{2} = 3$

Solutions:

$\tan x = 2{,}01$ $x = 63{,}5^\circ$	If the rounding is incorrect: max 1/2 marks
$5\cos x + 2 = 4$ $5\cos x = 2$ $\cos x = \frac{2}{5}$ $x = 66{,}4218...^\circ$ $x = 66{,}4^\circ$	
$\frac{\text{cosec } x}{2} = 3$ $\text{cos ec } x = 6$ $\frac{1}{\sin x} = 6$ $\sin x = \frac{1}{6}$ $x = 9{,}6^\circ$	

Exam type:

In each of the following equations, solve for x where $0^\circ \leq x \leq 90^\circ$. Give your answers correct to TWO decimal places.

5.2.1 $\tan x = 2{,}22$

5.2.2 $\sec(x + 10^\circ) = 5{,}759$

5.2.3 $\dfrac{\sin x}{0{,}2} - 2 = 1{,}24$

Solutions:

$$\tan x = 2{,}22$$
$$x = 65{,}75°$$

$$\sec(x + 10°) = 5{,}759$$
$$\cos(x + 10°) = 0{,}173\ldots \quad \textbf{OR/\textit{OF}} \quad \cos(x + 10°) = \frac{1}{5{,}759}$$
$$x + 10° = 80{,}0°$$
$$x = 70{,}0°$$

$$\frac{\sin x}{0{,}2} - 2 = 1{,}24$$
$$\frac{\sin x}{0{,}2} = 3{,}24$$
$$\sin x = 0{,}648$$
$$x = 40{,}39°$$

Concluding Remarks

Following our today lesson, I want you to do the to:

Read through what the learner **need to understand and master** in your learner material.

Complete the activities

Attempt as many as possible other similar examples on your own from the **Text-Book and the past exam papers**.

Repeat this procedure until you are **confident**.

Do not forget: **Practice makes perfect!**

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ONLINE

Thank you

Putting things into perspective

Focus :

<u>Trigonometry</u>

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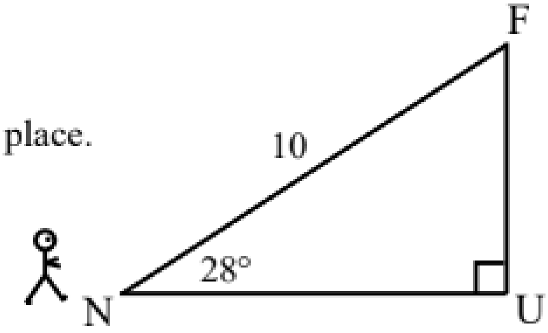
Monday: lesson

FINDING SIDES AND ANGLES USING TRIGONOMETRIC RATIOS

EXAMPLE 9 (Finding the length of a side)

In ΔFUN, $FN = 10$, $\hat{U} = 90°$ and $\hat{N} = 28°$.
Calculate the length of FU, rounded off to one decimal place.



Solution

Let N be your point of reference (given angle).

Side FU is **opposite** 28° and side FN is the **hypotenuse**.
You now need to create an equation involving

the ratio $\frac{\text{opp}}{\text{hyp}}$ and the angle 28°.

$$\frac{\text{opp}}{\text{hyp}} = \sin 28°$$

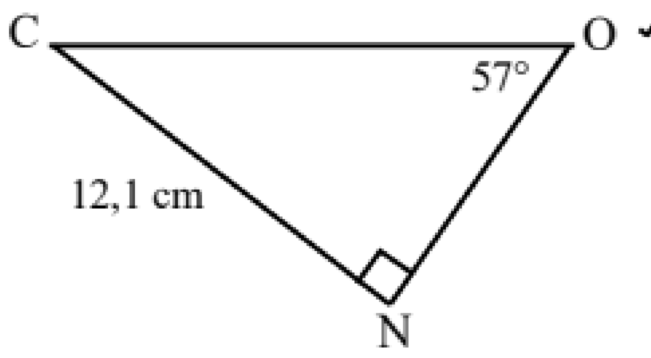
$$\therefore \frac{FU}{10} = \sin 28°$$

$\therefore FU = 10 \sin 28°$ multiply by LCD

$\therefore FU = 4,7$ units

Example:2

Consider the triangle sketched alongside.
Calculate the length of ON correct to one decimal place.



Solution

Let O be your point of reference (given angle).
You want side ON, which is **adjacent** to 57°.
You have side NC, which is **opposite to** 57°.

You now need to create an equation involving the ratio $\frac{\text{opp}}{\text{adj}}$ and the angle 57°:

$$\frac{\text{opp}}{\text{adj}} = \tan 57°$$

$$\therefore \frac{12{,}1}{\text{ON}} = \tan 57°$$

$$\therefore 12{,}1 = \text{ON} \tan 57°$$

$$\therefore \frac{12{,}1}{\tan 57°} = \text{ON}$$

$$\therefore \text{ON} = 7{,}9\,\text{cm}$$

Alternatively: $\hat{C} = 33°$ (int $\angle$s of Δ)

From $\hat{C}$: $\frac{\text{opp}}{\text{adj}} = \tan 33°$

$$\therefore \frac{\text{ON}}{12{,}1} = \tan 33°$$

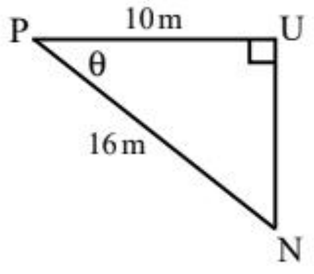
$$\therefore \text{ON} = 12{,}1 \tan 33°$$

$$\therefore \text{ON} = 7{,}9\,\text{cm}$$

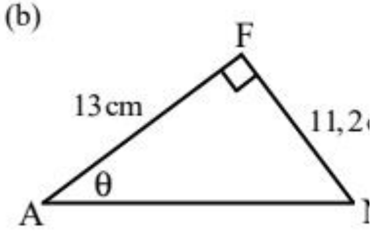
Example:3

Calculate the size of θ correct to one decimal place in each case.

(a)



(b)



Solutions

(a) We have side PU, which is **adjacent** to θ.

Side PN is the **hypotenuse**.

Now form an equation involving the ratio $\frac{\text{adj}}{\text{hyp}}$ and angle θ:

$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$

$$\therefore \cos\theta = \frac{10}{16}$$

$$\therefore \cos\theta = 0{,}625$$

$$\therefore \theta = 51{,}3°$$

(b) We have side AF, which is **adjacent** to θ. Side FN is **opposite** θ.

Now form an equation involving the ratio $\frac{\text{opp}}{\text{adj}}$ and angle θ:

$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$

$$\therefore \tan\theta = \frac{11{,}2}{13}$$

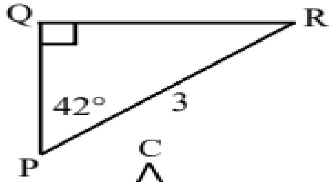
$$\therefore \tan\theta = 0{,}8615384615 \quad \text{(don't round off)}$$

$$\therefore \theta = 40{,}7°$$

Activity:1

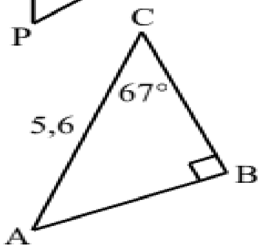
(Round answers off to one decimal place in this exercise)

(a) Calculate the length of PQ in ΔPQR .



(b) (1) Calculate the length of AB.

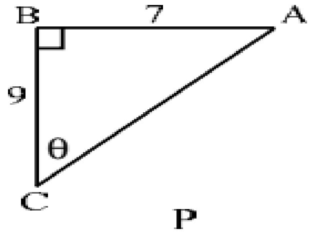
(2) Calculate the length of BC.



(c) Calculate:

(1) the size of θ.

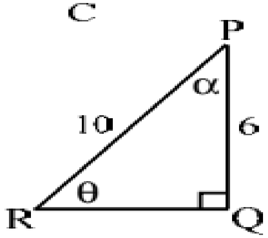
(2) the the length of AC.



(d) Calculate:

(1) the size of α .

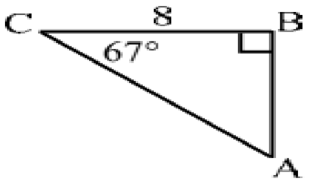
(2) the size of θ.



(e) Calculate:

(1) the length of AC.

(2) the length of AB.



Solutions:

(a) $\frac{PQ}{3} = \cos 42°$

$\therefore PQ = 3\cos 42°$

$\therefore PQ = 2,2$ units

(b) (1) $\frac{AB}{5,6} = \sin 67°$

$\therefore AB = 5,6\sin 67°$

$\therefore AB = 5,2$ units

(2) $\frac{BC}{5,6} = \cos 67°$

$\therefore BC = 5,6\cos 67°$

$\therefore BC = 2,2$ units

(c) (1) $\tan\theta = \frac{7}{9}$

$\therefore \theta = 37,9°$

(2) $\frac{7}{AC} = \sin 37,9°$

$\therefore 7 = AC\sin 37,9°$

$\therefore \frac{7}{\sin 37,9°} = AC$

$\therefore AC = 11,4$ units

(d) (1) $\cos\alpha = \frac{6}{10}$

$\therefore \alpha = 53,1°$

(2) $\theta = 36,9°$

(e) (1) $\frac{8}{AC} = \cos 67°$

$\therefore 8 = AC\cos 67°$

$\therefore \frac{8}{\cos 67°} = AC$

$\therefore AC = 20,5$ units

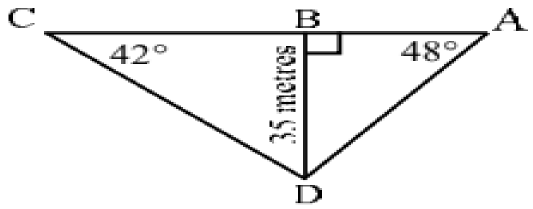
(2) $\frac{AB}{8} = \tan 67°$

$\therefore AB = 8\tan 67°$

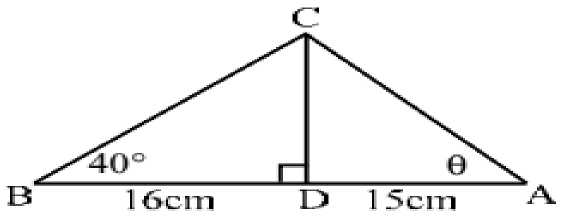
$\therefore AB = 18,8$ units

<u>Activity</u>:2

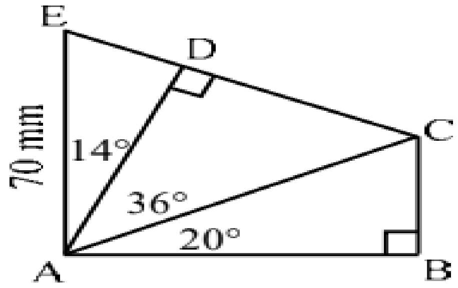
(f) In the diagram, $BD \perp AC$.
Using the information provided, calculate the length of AC.



(g) In ΔABC, $CD \perp AB$, $\hat{A} = \theta$, $\hat{B} = 40°$,
$AD = 15$ cm and $DB = 16$ cm.
Calculate the size of θ.



(h) Using the information provided on the given diagram, calculate the length of BC.

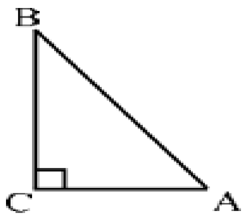


(i) In the given diagram, ΔABC is right-angled at C.
It is given that $AC = 4$ units, $\tan A = \frac{3}{2}$ and $\hat{A} \in (0° ; 90°)$.

(1) Determine the length of BC without solving for $\hat{A}$.

(2) Calculate the size of $\hat{B}$.

(3) Determine the length of AB.



Solutions:

(f) $\frac{35 \text{ metres}}{BC} = \tan 42^\circ$

$\therefore 35 \text{ metres} = BC \tan 42^\circ$

$\therefore \frac{35 \text{ metres}}{\tan 42^\circ} = BC$

$\therefore BC = 38{,}87\ldots \text{ metres}$

$\frac{35 \text{ metres}}{AB} = \tan 48^\circ$

$\therefore 35 \text{ metres} = AB \tan 48^\circ$

$\therefore \frac{35 \text{ metres}}{\tan 48^\circ} = AB$

$\therefore AB = 31{,}51\ldots \text{ metres}$

$AC = AB + BC$

$\therefore AC = 31{,}51\ldots \text{ metres} + 38{,}87\ldots \text{ metres}$

$\therefore AC = 70{,}4 \text{ metres}$

(g) $\frac{CD}{16\text{cm}} = \tan 40^\circ$

$\therefore CD = 16\text{cm} \times \tan 40^\circ$

$\therefore CD = 13{,}4255941\text{cm}$

$\tan\theta = \frac{13{,}4255941\text{cm}}{15\text{cm}}$

$\therefore \tan\theta = 0{,}895039606$

$\therefore \theta = 41{,}8^\circ$

(h) $\frac{AD}{70\text{mm}} = \cos 14^\circ$

$\therefore AD = 70\text{mm} \times \cos 14^\circ$

$\therefore AD = 67{,}92070084\text{mm}$

$\frac{67{,}92070084\text{mm}}{AC} = \cos 36^\circ$

$\therefore 67{,}92070084\text{mm} = AC \cos 36^\circ$

$\therefore \frac{67{,}92070084\text{mm}}{\cos 36^\circ} = AC$

$\therefore AC = 83{,}95460332\text{mm}$

$\frac{BC}{83{,}95460332\text{mm}} = \sin 20^\circ$

$\therefore BC = 83{,}95460332\text{mm} \times \sin 20^\circ$

$\therefore BC = 28{,}7\text{mm}$

(i) (1) $\tan A = \frac{3}{2}$ and $\tan A = \frac{BC}{AC} = \frac{BC}{4}$

$\therefore \frac{3}{2} = \frac{BC}{4}$

$\therefore BC = 6 \text{ units}$

(2) $\tan B = \frac{4}{6}$

$\therefore \hat{B} = 33{,}7^\circ$

(3) $AB^2 = AC^2 + BC^2$

$\therefore AB^2 = (4)^2 + (6)^2$

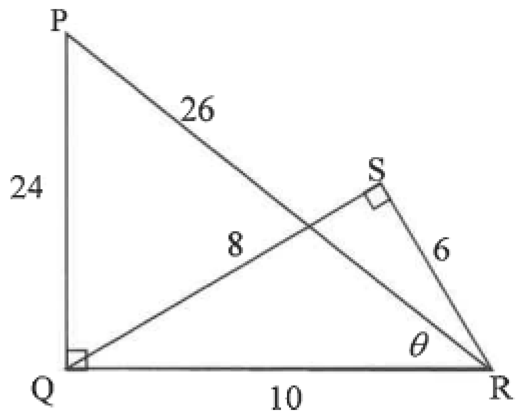
$\therefore AB^2 = 52$

$\therefore AB = \sqrt{52}$

$\therefore AB = 7{,}2 \text{ units}$

Activity:3

ΔPQR and ΔSQR are right-angled triangles as shown in the diagram below.
$PR = 26$, $PQ = 24$, $QS = 8$, $SR = 6$, $QR = 10$ and $P\hat{R}Q = \theta$.



4.1 Refer to the diagram above and, WITHOUT using a calculator, write down the value of:

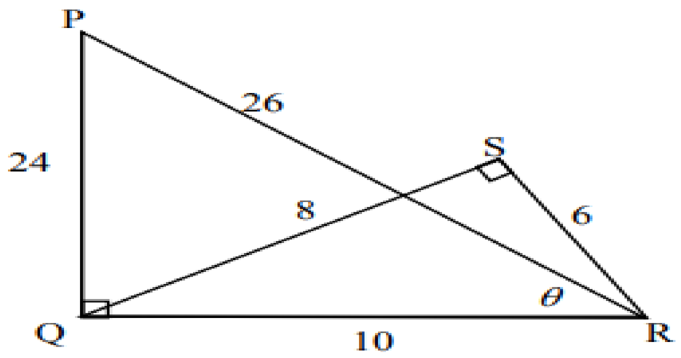
4.1.1 $\tan \hat{P}$

4.1.2 $\sin S\hat{Q}R$

4.1.3 $\cos\theta$

4.1.4 $\sec S\hat{R}Q$

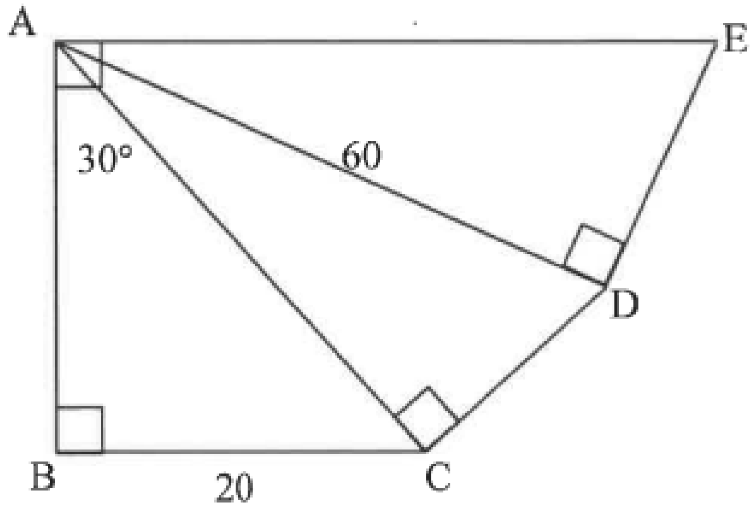
Solution:



4.1.1	$\tan\ \hat{P} = \frac{10}{24} = \frac{5}{12}$	Accept answers as unsimplified fractions.
4.1.2	$\sin\ S\hat{Q}R = \frac{6}{10} = \frac{3}{5}$	
4.1.3	$\cos\ \theta = \frac{10}{26} = \frac{5}{13}$	*Aanvaar antwoorde as nie-vereenvoudigde breuke.*
4.1.4	$\sec\ S\hat{R}Q = \frac{10}{6} = \frac{5}{3}$	
4.2	$\frac{\cot\theta}{\operatorname{cosec}\ Q\hat{R}S}$ $= \frac{10}{24} \div \frac{10}{8}$ $= \frac{1}{3}$	

Exam Type:

In the diagram below, ABC, ACD and ADE are right-angled tirangles.
$B\hat{A}E = 90°$ and $B\hat{A}C = 30°$. $BC = 20$ units and $AD = 60$ units.



Calculate the:

4.1.1 Length of AC

4.1.2 Size of $C\hat{A}D$

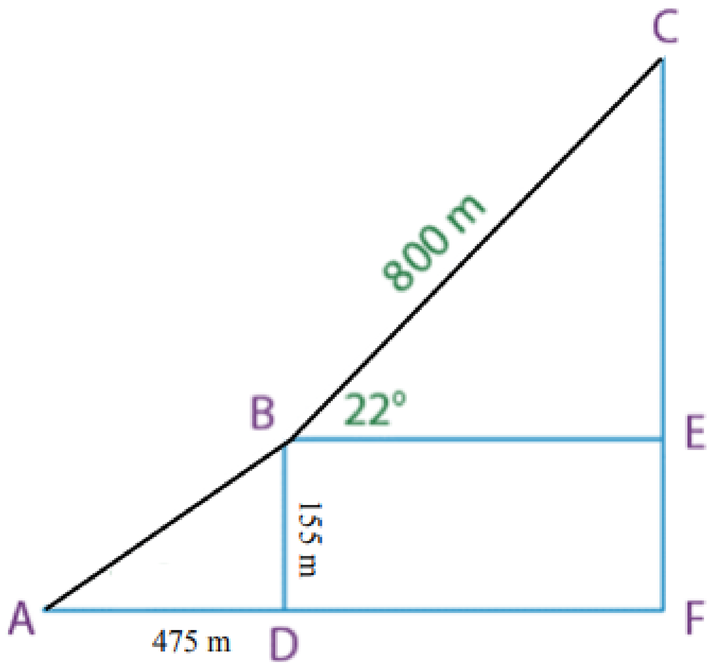
4.1.3 Length of DE

Solution:

4.1.1	$\sin 30^\circ = \dfrac{20}{AC}$ $AC = \dfrac{20}{\sin 30^\circ}$ $AC = 40$ **OR/*OF*** $\text{cosec } 30^\circ = \dfrac{AC}{20}$ $AC = \dfrac{20}{\sin 30^\circ}$ $AC = 40$ **OR/*OF*** $\cos 60^\circ = \dfrac{20}{AC}$ $AC = \dfrac{20}{\cos 60^\circ}$ $AC = 40$ **OR/*OF*** $\sec 60^\circ = \dfrac{AC}{20}$ $AC = \dfrac{20}{\cos 60^\circ}$ $AC = 40$
4.1.2	$\cos C\hat{A}D = \dfrac{AC}{60}$ $\cos C\hat{A}D = \dfrac{40}{60}$ $C\hat{A}D = 48{,}19^\circ$
4.1.3	$D\hat{A}E = 90^\circ - (30^\circ + C\hat{A}D)$ $D\hat{A}E = 90^\circ - (30^\circ + 48{,}19^\circ)$ $= 11{,}81^\circ$ $\tan 11{,}81^\circ = \dfrac{DE}{60}$ $DE = 60 \tan 11{,}81^\circ$ $DE = 12{,}55$

Exam type:

4.1 In the diagram below BDFE is a rectangle with BD = 155 m. AD = 475 m and BC = 800 m. Th
angle of elevation from B to C is 22°.



Calculate:

4.1.1 $\hat{A}$

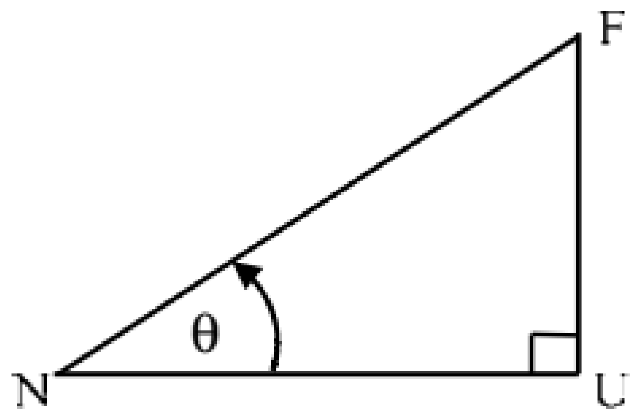
4.1.2 CF

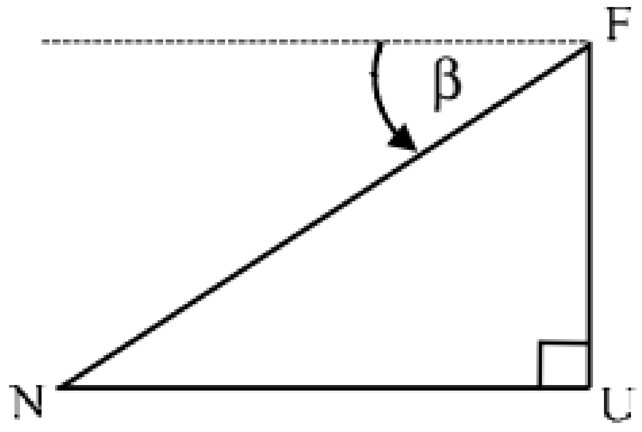
Solution:

4.1.1
In ΔABD $\frac{155}{475} = \tan \hat{A}$ $\hat{A} = 18{,}07°$
4.1.2
In ΔBCE $\frac{CE}{800} = \sin 22°$ $CE = 299{,}69m$ $\therefore CF = 454{,}69m$

Heights and Distance:

ANGLES OF ELEVATION AND DEPRESSION





θ is the **angle of elevation** of F from N.

β is the **angle of depression** of N from F.

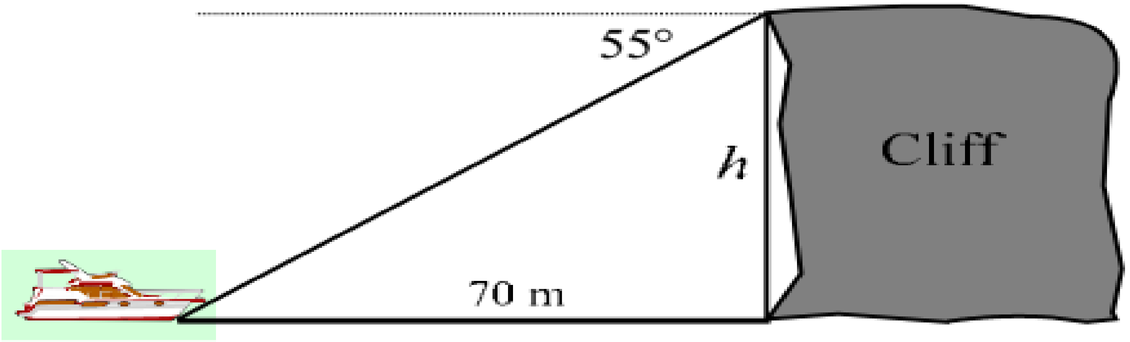
[Note that $\hat{N} = \beta$ since alt $\angle$s are equal]

Example:

The angle of depression of a boat on the ocean from the top of a cliff is $55°$. The boat is 70 metres from the foot of the cliff.

(a) What is the angle of elevation of the top of the cliff from the boat?

(b) Calculate the height of the cliff.

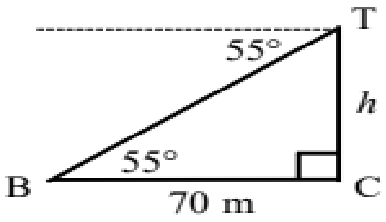


Solutions

(a) The angle of elevation of the top of the cliff from the boat is $55°$, i.e. $\hat{B} = 55°$.

(b) We can calculate the height of the cliff as follows:

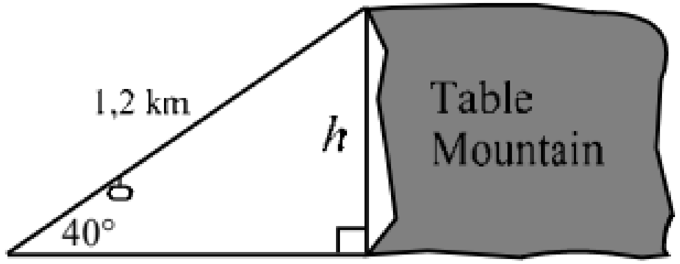
$$\frac{h}{70 \text{ m}} = \tan 55°$$
$$\therefore h = (70 \text{ m}) \tan 55°$$
$$\therefore h = 100 \text{ m}$$



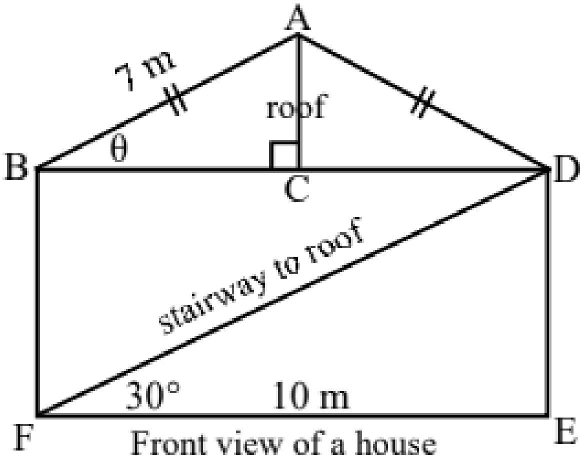
Activity:1

(Round your answers off to one decimal place in this exercise)

(a) The Cape Town cable car takes tourists to the top of Table Mountain. The cable is 1,2 kilometers in length and makes an angle of 40° with the ground. Calculate the height (h) of the mountain.



(b) An architectural design of the front of a house is given below. The length of the house is to be 10 metres. An exterior stairway leading to the roof is to form an angle of elevation of 30° with ground level. The slanted part of the roof must be 7 metres in length.



(1) Calculate the height of the vertical wall (DE).

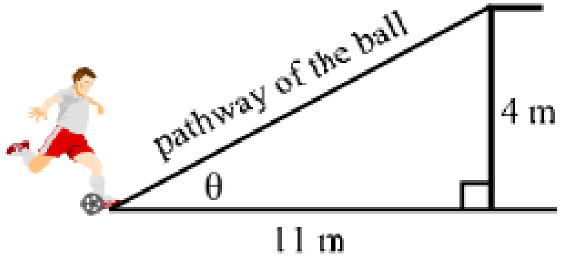
(2) Calculate the size of θ, the angle of elevation of the top of the roof (A) from the ceiling BCD.

(3) Calculate the length of the beam AC.

Solution:

Solution:2

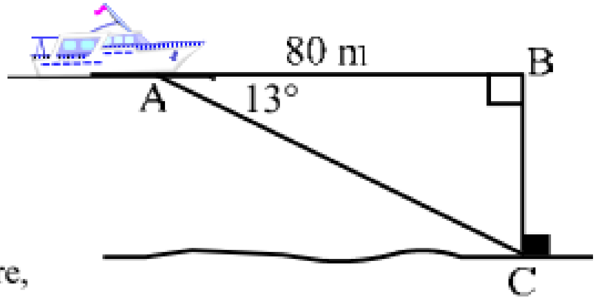
(c) In a soccer World Cup, a player kicked the ball from a distance of 11 metres from the goalposts (4 metres high) in order to score a goal for his team. The shortest distance travelled by the ball is in a straight line. The angle formed by the pathway of the ball and the ground is represented by θ.



(1) Calculate the largest angle θ for which the player will possibly score a goal.

(2) Will the player score a goal if the angle θ is $22°$? Explain.

(d) Treasure hunters in a boat, at point A, detect a treasure chest at the bottom of the ocean (C) at an angle of depression of $13°$ from the boat to the treasure chest. They then sail for 80 metres so that they are directly above the treasure chest at point B. In order to determine the amount of oxygen they will need when diving for the treasure, they must first calculate the depth of the treasure (BC). Calculate the depth of the treasure for the treasure hunters.



Solution:

Concluding Remarks

Following today 's lesson, I want you to do the following

Read through what the learner **needs to understand and master** in your learner material.

Complete the activities

Attempt as many as possible other similar examples on your own from the **Text-Book and the past exam papers**.

Repeat this procedure until you are **confident**.

Do not forget: **Practice makes perfect!**

Thank you

Concluding Remarks

The NEXT lesson will still focus on C.A.S.T and Special Angles , which links with the work we completed today