MATHEMATICS

GRADE :10

ALGEBRIC EXPRESSIONS

18 MARCH 2023

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Putting Lessons Into Perspective

Saturday : 18 MARCH 2023

ALGEBRIC EXPRESSIONS

Teacher Section!!!

ACCODING TO CAPS DOCUMENT GRADE 10

Topic	Curriculum statement	Clarification
		Where an example is given, the cognitive demand is suggested: knowledge (K), routine procedure (R), complex procedure (C) or problem-solving (P)
Algebraic expressions	1. Understand that real numbers can be rational or irrational. 2. Establish between which two integers a given simple surd lies. 3. Round real numbers to an appropriate degree of accuracy. 4. Multiplication of a binomial by a trinomial. 5. Factorisation to include types taught in grade 9 and: • trinomials • grouping in pairs • sum and difference of two cubes 6. Simplification of algebraic fractions using factorization with denominators of cubes (limited to sum and difference of cubes).	**Examples to illustrate the different cognitive levels involved in factorisation:** 1. Factorise fully: 1.1. $m^2 - 2m + 1$ (revision) Learners must be able to recognise the simplest perfect squares. (R) 1.2. $2x^2 - x - 3$ This type is routine and appears in all texts. (R) 1.3. $\frac{y^2}{2} - \frac{13y}{2} + 18$ Learners are required to work with fractions and identify when an expression has been "fully factorised". (R) 2. Simplify $\frac{1-2x}{4x^2-1} - \frac{x+4}{2x^2-3x+1} + \frac{1}{1-x}$ (C)

Lesson !!!

REVISION OF NUMBER SYSTEM

NUMBER SYSTEM

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Rational numbers

A **rational number** is a number that can be expressed in the form $\frac{a}{b}$ where $b \neq 0$ and where a and b are integers.

Rational numbers include all of the following numbers which can be expressed as common fractions.

(a) **Integers, whole numbers and natural numbers**

For example: $6 = \frac{6}{1}$ $\quad -2 = \frac{-2}{1}$ $\quad 0 = \frac{0}{1}$

(b) **Mixed numbers**

For example: $2\frac{1}{2} = \frac{5}{2}$ where 5 and 2 are integers

(c) **Terminating decimals**

For example: $0{,}125 = \frac{125}{1000} = \frac{1}{8}$

(d) **Recurring decimals**

A recurring decimal has an infinite pattern. For example, $0{,}\dot{1} = 0{,}111111...$ and $0{,}\dot{5}\dot{2} = 0{,}52525252..........$ are examples of recurring decimals.

A recurring decimal like $0{,}111111....$ can be expressed as a fraction in the form $\frac{a}{b}$.

SUMMARY OF REAL NO: SYSTEM

Irrational numbers

Irrational numbers are non-terminating, non-recurring decimals. They cannot be expressed as a ratio between integers. Examples include:

- Square roots of numbers that are not perfect squares. For example: $\sqrt{2}$, $\sqrt{6}$, $\sqrt{8}$
- Cube roots of numbers that are not perfect cubes. For example: $\sqrt[3]{2}$, $\sqrt[3]{5}$, $\sqrt[3]{9}$

Not all calculations will produce real numbers. There are two such calculations:

- **Square roots of negative numbers** do not produce real numbers.
  $\sqrt{-2}$; $\sqrt{-3}$; $\sqrt{-4}$; $\sqrt{-\pi}$; ..
  These numbers exist, but don't have a position on the number line. We call them non-real numbers. This can be extended to any even root of a negative number.
- **Division by zero** does not produce a real number.
  There are no real numbers resulting from dividing by zero. Division by zero is undefined.

1. **Natural numbers:** $\mathbb{N} = \{1; 2; 3; 4; 5; ...\}$

2. **Whole numbers:** $\mathbb{N}_0 = \{0; 1; 2; 3; 4; 5; ...\}$

3. **Integers:** $\mathbb{Z} = \{...; -3; -2; -1; 0; 1; 2; 3; ...\}$

4. **Rational numbers:** $\mathbb{Q} = \left\{\frac{a}{b} : a \in \mathbb{Z} ; b \in \mathbb{Z} ; b \neq 0\right\}$

- Whole numbers and integers
- Proper fractions
- Improper fractions and mixed numbers
- Terminating decimals
- Recurring decimals

5. **Irrational numbers:** $\mathbb{Q}'$

- Non-terminating, non-recurring decimals.
- Square roots of numbers that are not perfect squares, cube roots of numbers that are not perfect cubes etc.
- π

6. **Real numbers:** $\mathbb{R}$

Any number on the number line. All rational and irrational numbers put together.

7. **Calculations that do NOT produce real numbers:**

- Square roots of negative numbers
- Division by zero

ACTIVITIES

SOLUTIONS

State whether the following numbers are **rational**, **irrational** or **neither**:

(a)	$0{,}25$	(b)	-2	(c)	$\pi + 6$	(d)	$\sqrt{10}$
(e)	$\sqrt{-16}$	(f)	$\sqrt{16}$	(g)	0	(h)	$\frac{5}{0}$
(i)	$0{,}543215432154321\ldots$			(j)	$0{,}7931156480518346\ldots$		
(k)	$\sqrt[3]{-27}$	(l)	$\sqrt[3]{-9}$	(m)	$\frac{\pi}{3}$	(n)	$\sqrt{\frac{9}{16}}$

(a) Rational (terminating decimal)
(b) Rational (integer)
(c) Irrational (the sum of an irrational number and a rational number is always irrational)
(d) Irrational (10 is not a perfect square, therefore the square root of 10 is irrational)
(e) Neither (16 is a perfect square but, a square root of a negative is non-real)
(f) Rational (16 is a perfect square, therefore the square root of 16 is rational)
(g) Rational (whole number)
(h) Neither (division by 0 is undefined)
(i) Rational (recurring decimal)
(j) Irrational (non-terminating, non-recurring decimal)
(k) Rational (this equals -3 and -3 is an integer and an integer is rational)
(l) Irrational (real since cube roots of negatives are real, but, since 9 is not a perfect cube, the cube root of -9 is irrational).
(m) Irrational (any fraction of an irrational number is irrational)
(n) Rational (9 and 16 are perfect squares and $\sqrt{\frac{9}{16}} = \frac{3}{4}$ which is a fraction)

SHOW THAT THE RECURRING DECIMALS ARE RATIONAL

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EXAMPLE

(a) $0,\dot{1}$ (b) $1,\dot{7}\dot{5}$

$0,\dot{1}$ can be shown to be rational by expressing it as a common fraction.

Let $x = 0,1111111...$

Now what you need to do is multiply both sides by 10, 100, 1000 and so forth to get the following equations:

$x = 0,111111111...$ (1)

$10x = 1,111111111...$ (2)

$100x = 11,111111111...$ (3)

Look for two equations where the **decimals after the comma are the same** and then subtract the equations.

Two equations where the decimals are the same after the comma are (2) and (1). Subtract (1) from (2) as follows:

$10x - x = 1,111111111... - 0,111111111...$

$\therefore 9x = 1,000000000...$

$\therefore 9x = 1$

$\therefore x = \frac{1}{9}$ which is a rational number.

You could have also used equation (2) and (3) or (1) and (3).

(b) $x = 1,75757575...$ (1)

$10x = 17,57575757...$ (2)

$100x = 175,75757575...$ (3)

Carefully consider the decimals of each line. It should be clear that the decimals of (2) and (1) are not the same. Two equations where the **decimals are the same after the comma are** (3) and (1). Subtract (1) from (3).

$\therefore 99x = 174,00000.........$

$\therefore x = \frac{174}{99} = \frac{58}{33}$ which is a rational number.

It may be necessary in other cases to continue multiplying by 10 until you have established that the decimals to the right of the comma are equal.

SHOW THAT THE RECURRING DECIMALS ARE RATIONAL

SOLUTIONS

ACTIVITY

(1) $0,\dot{4}$ (2) $0,\dot{2}\dot{1}$ (3) $0,1\dot{4}$ (4) $19,4\dot{5}$

SOLUTION

(1) Let $x = 0,4444444.....$

$\therefore 10x = 4,444444.....$

$\therefore 9x = 4$

$\therefore x = \frac{4}{9}$

(2) Let $x = 0,21212121.....$

$\therefore 100x = 21,212121......$

$\therefore 99x = 21$

$\therefore x = \frac{21}{99} = \frac{7}{33}$

(3) Let $x = 0,1444444......$

$\therefore 100x = 14,4444.......$

$\therefore 10x = 1,4444........$

$\therefore 90x = 13$

$\therefore x = \frac{13}{90}$

(4) Let $x = 19,455555.....$

$\therefore 100x = 1945,5555...$

$\therefore 10x = 194,5555....$

$\therefore 90x = 1751$

$\therefore x = \frac{1751}{90}$

BETWEEN WHICH TWO CONSECUTIVE INTEGERS DO IRRATIONAL NUMBERS LIES?

MIXED ACTIVITIES

(a) $\sqrt{12}$ (b) $-\sqrt{12}$ (c) $\sqrt[3]{20}$ (d) -2π

Solutions

(a) $9 < 12 < 16$

$\therefore \sqrt{9} < \sqrt{12} < \sqrt{16}$

$\therefore 3 < \sqrt{12} < 4$

$\therefore \sqrt{12}$ lies between 3 and 4.

(b) $9 < 12 < 16$

$\therefore 3 < \sqrt{12} < 4$

$\therefore -3 > -\sqrt{12} > -4$

$\therefore -\sqrt{12}$ lies between -4 and -3.

(c) $8 < 20 < 27$

$\therefore \sqrt[3]{8} < \sqrt[3]{20} < \sqrt[3]{27}$

$\therefore 2 < \sqrt[3]{20} < 3$

$\therefore \sqrt[3]{20}$ lies between 2 and 3.

(d) $-2\pi = -6{,}2833185307...$

$-7 < -6{,}283318... < -6$

$\therefore -7 < -2\pi < -6$

$\therefore -2\pi$ lies between -7 and -6

SOLUTIONS

(a) From the list of numbers: -3; $\frac{3}{4}$; $\sqrt{2}$; $\sqrt{9}$; 0; 2; $\sqrt{-4}$, write down all the

(1) natural numbers (2) whole numbers (3) integers

(4) rational numbers (5) irrational numbers (6) real numbers

(b) State whether each of the following numbers are rational, irrational or neither.

(1) $\sqrt[3]{33}$ (2) $\frac{\pi}{2}$ (3) 4,01345 (4) $\sqrt{\frac{1}{121}}$

(5) $\sqrt{-2}$ (6) $-\sqrt{2}$ (7) $\frac{\sqrt{2}}{\sqrt{2}}$ (8) $\sqrt[3]{27+1}$

SOLUTIONS

ACTIVITY

(a)

(1) $2; \sqrt{9}$ (2) $0; 2; \sqrt{9}$ (3) $-3; 0; 2; \sqrt{9}$

(4) $-3; \frac{3}{4}; 0; 2; \sqrt{9}$ (5) $\sqrt{2}$ (6) $-3; \frac{3}{4}; \sqrt{2}; \sqrt{9}; 0; 2$

(b)

(1) Irrational (2) Irrational (3) Rational (4) Rational

(5) Neither (6) Irrational (7) Rational (8) Irrational

(c)

(1) Mixed number (2) Mixed number (3) Natural number

(4) Integer (5) Terminating decimal (6) Terminating decimal

SOLUTIONS

(a) Without using a calculator, determine between which two integers the following irrational numbers lie. Then verify your answers by using a calculator.

(1) $\sqrt{50}$ (2) $\sqrt{29}$ (3) $\sqrt[3]{45}$ (4) $-\sqrt{54}$

(5) $\sqrt[5]{30}$ (6) π

ROUNDING OFF NUMBERS TO A CERTAIN DECIMALS

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(a)

(1) $\sqrt{49} < \sqrt{50} < \sqrt{64}$

$\therefore 7 < \sqrt{50} < 8$

(2) $\sqrt{25} < \sqrt{29} < \sqrt{36}$

$\therefore 5 < \sqrt{29} < 6$

(3) $\sqrt[3]{27} < \sqrt[3]{45} < \sqrt[3]{64}$

$\therefore 3 < \sqrt[3]{45} < 4$

(4) $-\sqrt{64} < -\sqrt{54} < -\sqrt{49}$

$\therefore -8 < -\sqrt{54} < -7$

(5) $\sqrt[5]{1} < \sqrt[5]{30} < \sqrt[5]{32}$

$\therefore 1 < \sqrt[5]{30} < 2$

(6) $3 < \pi < 4$

ROUNDING OFF NUMBERS TO A CERTAIN DECIMALS

The rules for rounding off numbers to certain decimal places are as follows:

- Count to the number of decimal places ***after the comma*** that you want to round off to.
- Look at the digit to the right of this decimal place.
  - If it is lower than 5, drop it and all the digits to the right of it.
  - If it is 5 or more than 5, then add one digit to the digit immediately to the left of it and drop it and all the digits to the right of it.
  - If necessary, keep or add zeros as place holders.

EXAMPLE 5

MULTIPLICATION OF ALGEBRIC EXPRESSIONS

EXAMPLE 5

Round off the following numbers to the number of decimal places indicated:

(a) 4,31437 (2 decimal places):

$4{,}31\underline{4}37$ and the answer: 4,31

(b) 1,77777 (3 decimal places)

$1{,}777\underline{7}7$ and the answer: 1,778

(c) 365,1534 (1 decimal place)

$365{,}1\underline{5}34$ and the answer: 365,2

(d) 594,2 (2 decimal places)

$594{,}20\underline{0}$ and the answer: 594,20

(e) 12,07963 (3 decimal places)

$12{,}079\underline{6}3$ and the answer: 12,080

(f) 9,998 (1 decimal place)

$9{,}99\underline{8}$ and the anwer: 10,0

MULTIPLICATION OF ALGEBRIC EXPRESSIONS

The distributive law (revision)

$$a(b+c) = ab + ac \qquad a(b+c+d) = ab + ac + ad$$

The variable "a" is distributed to and multiplied with all the other terms in the brackets.

The product of two binomials (revision)

The **FOIL** method can be used to multiply two binomials. Here you must first multiply the first terms in each bracket. Then you multiply the outer terms, then the inner terms and finally the last terms.

F = Firsts
O = Outers
I = Inners
L = Lasts

OUTERS

FIRSTS

$$(a+b)(c+d) = ac + ad + bc + bd$$

INNERS

LASTS

The product of a binomial and a trinomial

The method revised above can be extended to the product of a binomial and a trinomial.

$$(x+y)(a+b+c)$$
$$=(x+y)a+(x+y)b+(x+y)c$$
$$=ax+ay+bx+by+cx+cy$$

or

$$(x+y)(a+b+c)$$
$$=ax+bx+cx+ay+by+cy$$

MULTIPLICATION OF ALGEBRIC EXPRESSIONS

EXAMPLE 8

The examples below illustrate the methods of multiplying binomials and trinomials.

(a) $(x+3)(x+2)$
$=x^2+2x+3x+6$
$=x^2+5x+6$

(b) $(3y-1)(2y+4)$
$=6y^2+12y-2y-4$
$=6y^2+10y-4$

(c) $(2x-4y)(x-3y)$
$=2x^2-6xy-4xy+12y^2$
$=2x^2-10xy+12y^2$

(d) $(2ab+3)(ab-5)$
$=2a^2b^2-10ab+3ab-15$
$=2a^2b^2-7ab-15$

(e) $(2x^3+7y)(x^3-2y)$
$=2x^6-4x^3y+7x^3y-14y^2$
$=2x^6+3x^3y-14y^2$

(f) $(x-y)(x^2-3xy+2y^2)$
$=x^3-3x^2y+2xy^2-x^2y+3xy^2+2y^3$
$=x^3-4x^2y+5xy^2+2y^3$

EXAMPLES

ACTIVITIES

SOLUTIONS

(a) Expand and simplify:

(1) $3x(x+3)$ (2) $-3a(3a^3-6a^2+a)$ (3) $(x+5)(x+2)$

(4) $(x-5)(x-2)$ (5) $(x+5)(x-2)$ (4) $(x-5)(x+2)$

(7) $(3x-1)(2x+3)$ (8) $(7m-2n)(3m+4n)$

(9) $(2x^4-3y^2)(3x^4+2y^2)$ (10) $(4x^4+3y^5)(2x^4-4y^3)$

(b) Expand and simplify:

(1) $(x+1)(x^2+2x+3)$ (2) $(x-1)(x^2-2x+3)$

(3) $(2x+4)(x^2-3x+1)$ (4) $(2x-4)(x^2-3x+1)$

(5) $(3x-y)(2x^2+4xy-y^2)$ (6) $(a+2b)(4a^2-3ab+b^2)$

(7) $(3x-2y)(9x^2+6xy+4y^2)$ (8) $(3x+2y)(9x^2-6xy+4y^2)$

(c) Expand and simplify:

(1) $2x(3x-4y)-(7x^2-2xy)$ (2) $(5y+1)^2-(3y+4)(2-3y)$

(3) $(2x+y)^2-(3x-2y)^2+(x-4y)(x+4y)$

(4) $x^6+(x^3-3y)(x^3+3y)$ (5) $(3a+b)(3a-b)(2a+5b)$

(a)

(1) $3x^2+9x$

(2) $-9a^4+18a^3-3a^2$

(3) $x^2+7x+10$

(4) $x^2-7x+10$

(5) $x^2+3x-10$

(6) $x^2-3x-10$

(7) $6x^2+7x-3$

(8) $21m^2+22m-8n^2$

(9) $6x^8-5x^4y^2-6y^4$

(10) $8x^8-16x^4y^3+6x^4y^5-12y^8$

(b)

(1) $(x+1)(x^2+2x+3)$
$=x^3+2x^2+3x+x^2+2x+3$
$=x^3+3x^2+5x+3$

(2) $(x-1)(x^2-2x+3)$
$=x^3-2x^2+3x-x^2+2x-3$
$=x^3-3x^2+5x-3$

(3) $(2x+4)(x^2-3x+1)$
$=2x^3-6x^2+2x+4x^2-12x+4$
$=2x^3-2x^2-10x+4$

(4) $(2x-4)(x^2-3x+1)$
$=2x^3-6x^2+2x-4x^2+12x-4$
$=2x^3-10x^2+14x-4$

(5) $(3x-y)(2x^2+4xy-y^2)$
$=6x^3+12x^2y-3xy^2-2x^2y-4xy^2+y^3$
$=6x^3+10x^2y-7xy^2+y^3$

(6) $(a+2b)(4a^2-3ab+b^2)$
$=4a^3-3a^2b+ab^2+8a^2b-6ab^2+2b^3$
$=4a^3+5a^2b-5ab^2+2b^3$

(7) $(3x-2y)(9x^2+6xy+4y^2)$
$=27x^3+18x^2y+12xy^2-18x^2y-12xy^2-8y^3$
$=27x^3-8y^3$

(8) $(3x+2y)(9x^2-6xy+4y^2)$
$=27x^3-18x^2y+12xy^2+18x^2y-12xy^2+8y^3$
$=27x^3+8y^3$

SOLUTIONS

SPECIAL PRODUCTS

(c) (1) $2x(3x-4y)-(7x^2-2xy)$
$=6x^2-8xy-7x^2+2xy$
$=-x^2-6xy$

(2) $(5y+1)^2-(3y+4)(2-3y)$
$=25y^2+10y+1-(6y-9y^2+8-12y)$
$=25y^2+10y+1-(-9y^2-6y+8)$
$=25y^2+10y+1+9y^2+6y-8$
$=34y^2+16y-7$

(3) $(2x+y)^2-(3x-2y)^2+(x-4y)(x+4y)$
$=4x^2+4xy+y^2-(9x^2-12xy+4y^2)+x^2-16y^2$
$=4x^2+4xy+y^2-9x^2+12xy-4y^2+x^2-16y^2$
$=-4x^2+16xy-19y^2$

(4) $x^6+(x^3-3y)(x^3+3y)$
$=x^6+x^6-9y^2$
$=2x^6-9y^2$

(5) $(3a+b)(3a-b)(2a+5b)$
$=(9a^2-b^2)(2a+5b)$
$=18a^3+45a^2b-2ab^2-5b^3$

SPECIAL PRODUCTS

We will now consider the following special products:

- Products which lead to the difference of two squares
- Squaring a binomial
- Cubing a binomial

Products which lead to the difference of two squares (revision)

Consider the product $(x+y)(x-y)$. The product can be simplified as follows:

$$(x+y)(x-y)$$
$$=x^2-xy+xy-y^2$$
$$=x^2-y^2$$

In other words, the pattern is as follows:

$(\text{first term} + \text{last term})(\text{first term} - \text{last term}) = (\text{first term})^2 - (\text{last term})^2$ or

$(\text{first term} - \text{last term})(\text{first term} + \text{last term}) = (\text{first term})^2 - (\text{last term})^2$

Expand and simplify the following:

(a) $(3x+2y)(3x-2y)$

$=9x^2-4y^2$

(b) $(a-5b)(a+5b)$

$=a^2-25b^2$

(c) $(4x^4-3y^3)(4x^4+3y^3)$

$=16x^8-9y^6$

(d) $(a-b+d)(a-b-d)$

$=[(a-b)+d][(a-b)-d]$

$=(a-b)^2-d^2$

$=(a-b)(a-b)-d^2$

$=a^2-2ab+b^2-d^2$

Alternatively we can substitute $(a-b)=k$

$\therefore [(a-b)+d][(a-b)-d]$

$=(k+d)(k-d)$

$=k^2-d^2$

$=(a-b)^2-d^2$

$=a^2-2ab+b^2-d^2$

The alternative method makes it easier to recognise that the product leads to the difference of two squares.

EXAMPLES

SPECIAL PRODUCTS

ACTIVITY

EXAMPLES

Squaring a binomial (revision)

Consider the squares of the following binomials:

$$\begin{aligned}&(a+b)^2\\&=(a+b)(a+b)\\&=a^2+ab+ba+b^2\\&=a^2+2ab+b^2\end{aligned}$$

$$\begin{aligned}&(a-b)^2\\&=(a-b)(a-b)\\&=a^2-ab-ba+b^2\\&=a^2-2ab+b^2\end{aligned}$$

Therefore:

$(a+b)^2=a^2+2ab+b^2$ and $(a-b)^2=a^2-2ab+b^2$

SOLUTIONS

(a) Expand and simplify:

(1) $(x+7)(x-7)$ (2) $(x-3)(x+3)$ (3) $(2x-1)(2x+1)$

(4) $(9x+4)(9x-4)$ (5) $(3x-2y)(3x+2y)$ (6) $(4a^3b+3)(4a^3b-3)$

(7) $(-3a+5b)^2$ (8) $3(x-3y)^2$ (9) $[2(m-4n)]^2$

(10) $(x^3-3y^6)^2$ (11) $(2a+3b)^3$ (12) $(2a-3b)^3$

(a) (1) x^2-49 (2) x^2-9 (3) $4x^2-1$ (4) $81x^2-16$

(5) $9x^2-4y^2$ (6) $16a^6b^2-9$ (7) $36-9x^8y^2$

(7) $9a^2-30ab+25b^2$ (8) $3(x^2-6xy+9y^2)=3x^2-18xy+27y^2$

(9) $4(m^2-8mn+16n^2)=4m^2-32mn+64n^2$

(10) $x^6-6x^3y^6+9y^{12}$

(11) $(2a+3b)(2a+3b)^2$

$$=(2a+3b)(4a^2+12ab+9b^2)$$

$$=8a^3+24a^2b+18ab^2+12a^2b+36ab^2+27b^3$$

$$=8a^3+36a^2b+54ab^2+27b^3$$

(12) $(2a-3b)(2a-3b)^2$

$$=(2a-3b)(4a^2-12ab+9b^2)$$

$$=8a^3-24a^2b+18ab^2-12a^2b+36ab^2-27b^3$$

$$=8a^3-36a^2b+54ab^2-27b^3$$

Thank you

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