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MATHEMATICS
GRADE: 10
(23/05/2022)
Analytical Geometry
Sessions: 27 & 28 May

Exam Paper overview on Analytical Geometry Grade 10

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PAPER 2: Grade 11 and 12: theorems and / or trigonometric proofs: maximum 12 marks			
Description	**Grade 10**	**Grade 11**	**Grade 12**
Statistics	15 ± 3	20 ± 3	20 ± 3
Analytical Geometry	15 ± 3	30 ± 3	40 ± 3
Trigonometry	40 ± 3	50 ± 3	50 ± 3
Euclidean Geometry	30 ± 3	50 ± 3	40 ± 3
TOTAL	**100**	**150**	**150**

Note:

- Modelling as a process should be included in all papers, the contextual questions can be set on any topic.
- Questions will not necessarily be compartmentalised in sections, as the table indicates. Various topics can be integrated in the same question.
- **Formula sheet must be provided for the final examinations in Grade 10 and 11**

Teachers must emphasis the last point.

Discussion (1.1) of concepts on Analytical Geometry with Step-by-step solution
Distance between two points

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DISTANCE BETWEEN TWO POINTS

The distance between two points $(x_1; y_1)$ and $(x_2; y_2)$ is given by:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

1. Determine the length of PQ if $P(-1;4)$ and $Q(4;-2)$

Working area

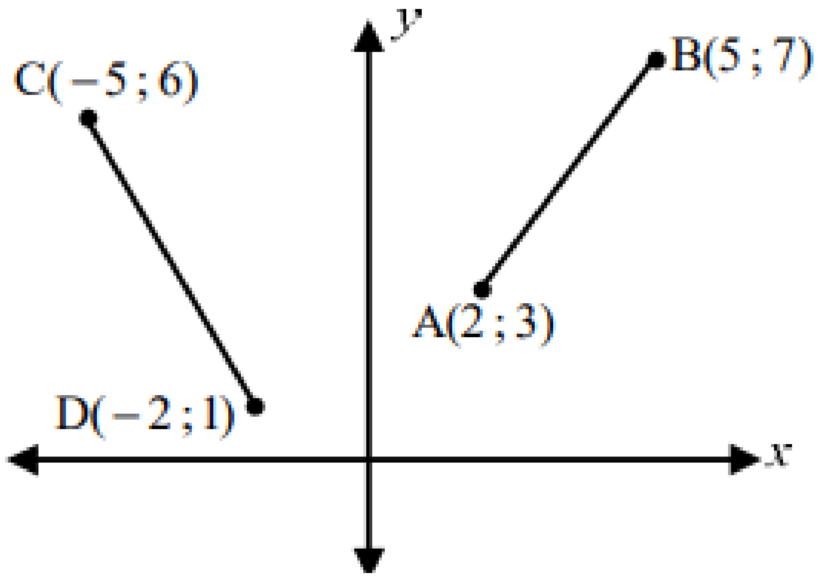
Solution (1.1): Distance between two points

1. Determine the length of PQ if $P(-1;4)$ and $Q(4;-2)$

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-1-4)^2 + (4-(-2))^2} \\ &= \sqrt{61} \\ &= 7{,}81 \end{aligned}$$

Discussion (1.2) of concepts on Analytical Geometry with Step-by-step solution Distance between two points

Calculate the lengths of line segments AB and CD in the given diagram.



Polling Activity

- Is the following state True or False

a) The Distance between the two points can be calculated with the formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)}$

Working area

Solution(1.2): Distance between two points

(a)

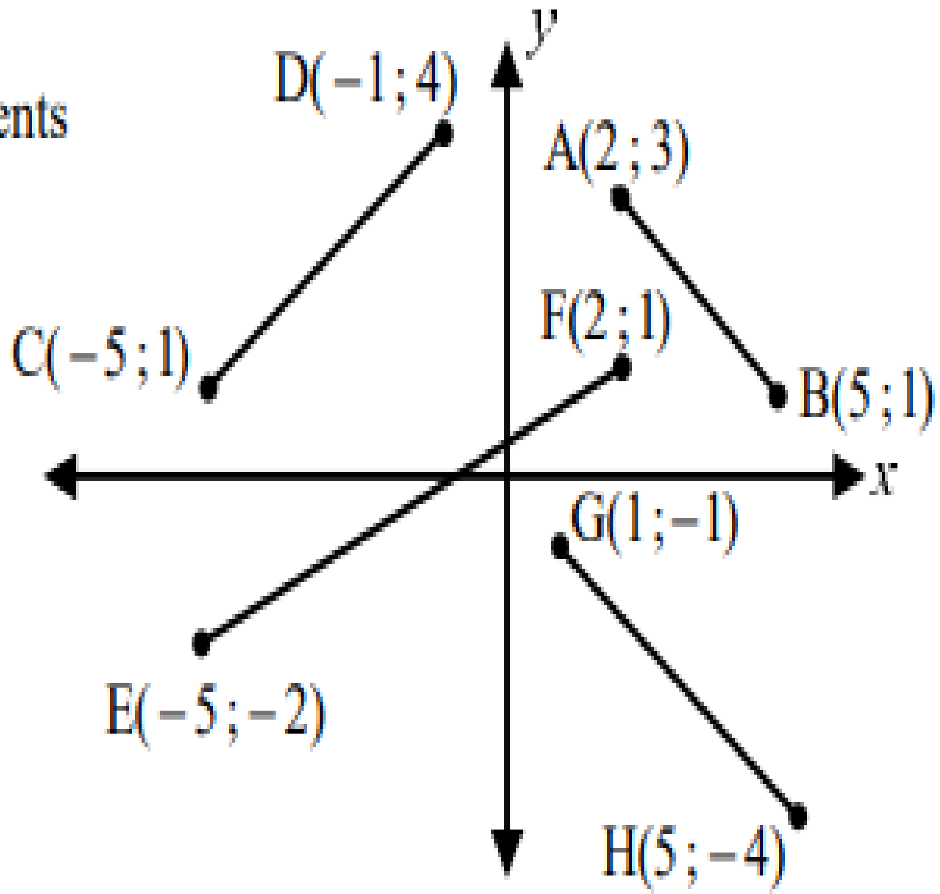
$$AB^2 = (x_B - x_A)^2 + (y_B - y_A)^2$$
$$AB^2 = (5-2)^2 + (7-3)^2$$
$$AB^2 = (3)^2 + (4)^2$$
$$AB^2 = 25$$
$$AB = \sqrt{25} = 5 \text{ units}$$

(b)

$$CD^2 = (x_D - x_C)^2 + (y_D - y_C)^2$$
$$CD^2 = (-2-(-5))^2 + (1-6)^2$$
$$CD^2 = (3)^2 + (-5)^2$$
$$CD^2 = 34$$
$$CD = \sqrt{34} \approx 5{,}83 \text{ units}$$

Discussion (1.3) of concepts on Analytical Geometry with Step-by-step solution Distance between two points

(a) Calculate the lengths of the line segments in the given diagram.



Working area

Solution(1.3): Distance between two points

(a)

$$AB^2 = (5-2)^2 + (1-3)^2$$
$$\therefore AB^2 = 9+4$$
$$\therefore AB^2 = 13$$
$$\therefore AB = \sqrt{13} \text{ units}$$

$$CD^2 = (-1-(-5))^2 + (4-1)^2$$
$$\therefore CD^2 = (-1+5)^2 + (3)^2$$
$$\therefore CD^2 = 25$$
$$\therefore CD = 5 \text{ units}$$

$$EF^2 = (2-(-5))^2 + (1-(-2))^2$$
$$\therefore EF^2 = 49+9$$
$$\therefore EF^2 = 58$$
$$\therefore EF = \sqrt{58} \text{ units}$$

$$GH^2 = (5-1)^2 + (-4-(-1))^2$$
$$\therefore GH^2 = 16 + (-4+1)^2$$
$$\therefore GH^2 = 25$$
$$\therefore GH = 5 \text{ units}$$

Discussion (2.1) of concepts on Analytical Geometry with Step-by-step solution Midpoint of a line segment

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MIDPOINT OF A LINE SEGMENT

The midpoint between $(x_1; y_1)$ and $(x_2; y_2)$ is given by:

$$M(x; y) = \left(\frac{x_2 + x_1}{2}; \frac{y_2 + y_1}{2}\right)$$

1. Determine the midpoint of $P(-1;4)$ and $Q(4;-2)$

Working area

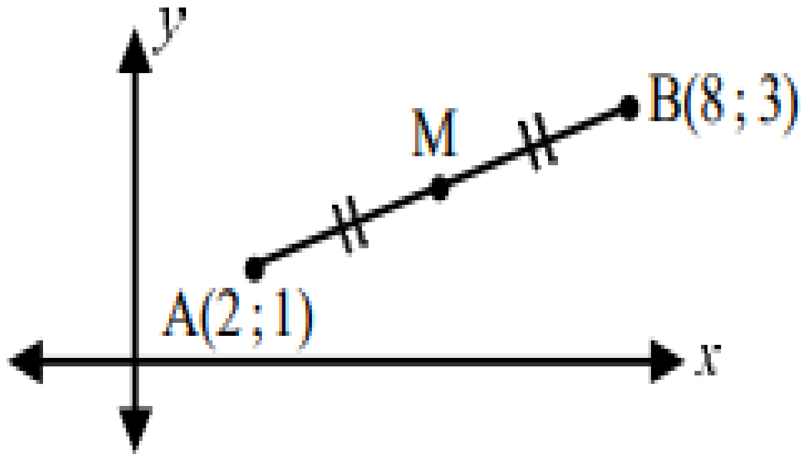
Solution (2.1): Midpoint of a line segment

1. Determine the midpoint of $P(-1;4)$ and $Q(4;-2)$

$$\begin{aligned}\text{Midpnt} &= \left(\frac{x_2+x_1}{2};\frac{y_2+y_1}{2}\right)\\ &= \left(\frac{-1+4}{2};\frac{4-2}{2}\right)\\ &= \left(\frac{3}{2};1\right)\end{aligned}$$

Discussion (2.2) of concepts on Analytical Geometry with Step-by-step solution Midpoint of a line segment

Determine the coordinates of M, if M is the midpoint of line segment AB, where $A(2\,;1)$ and $B(8\,;3)$.



Working area

Solution (2.2): Midpoint of a line segment

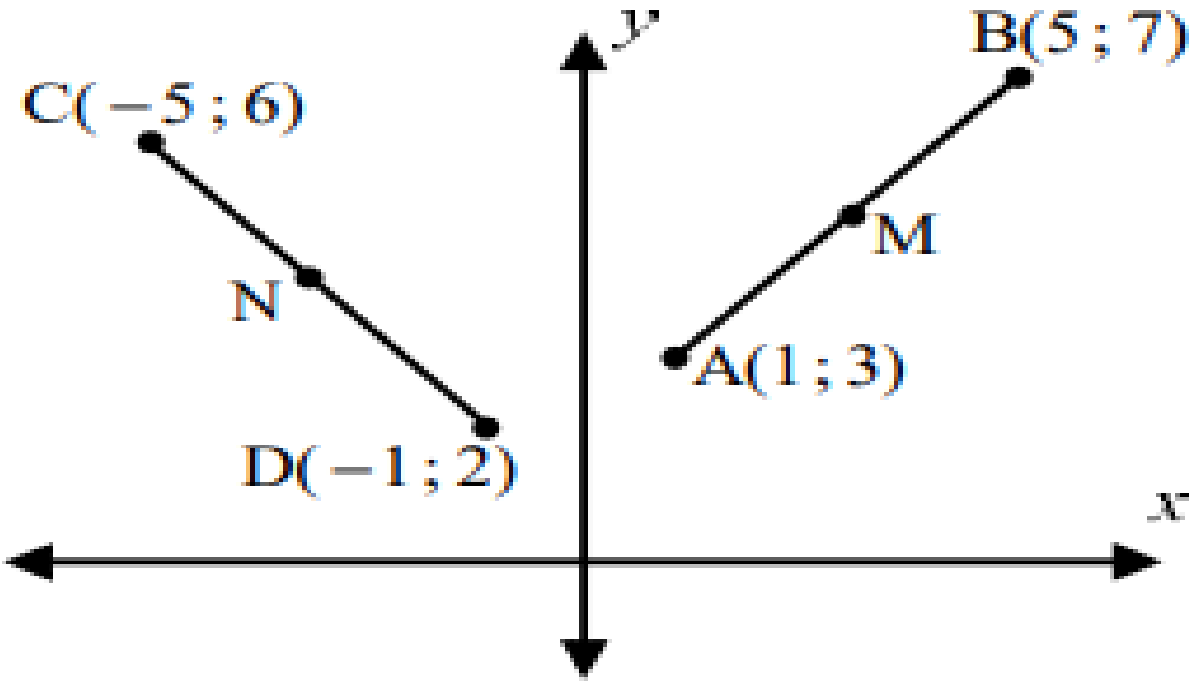
$$\mathrm{M}\left(\frac{x_A + x_B}{2} ; \frac{y_A + y_B}{2}\right)$$

$$= \mathrm{M}\left(\frac{2+8}{2} ; \frac{1+3}{2}\right)$$

$$= \mathrm{M}(5 ; 2)$$

Discussion (2.3) of concepts on Analytical Geometry with Step-by-step solution Midpoint of a line segment

Calculate the midpoints of line segments AB and CD in the given sketch.



Working area

Solution (2.3): Midpoint of a line segment

Midpoint of AB is M:

$$M(x_M\,;y_M)=M\left(\frac{x_A+x_B}{2};\frac{y_A+y_B}{2}\right)$$

$$\therefore M\left(\frac{1+5}{2};\frac{3+7}{2}\right)$$

$$\therefore M(3\,;5)$$

Midpoint of CD is N:

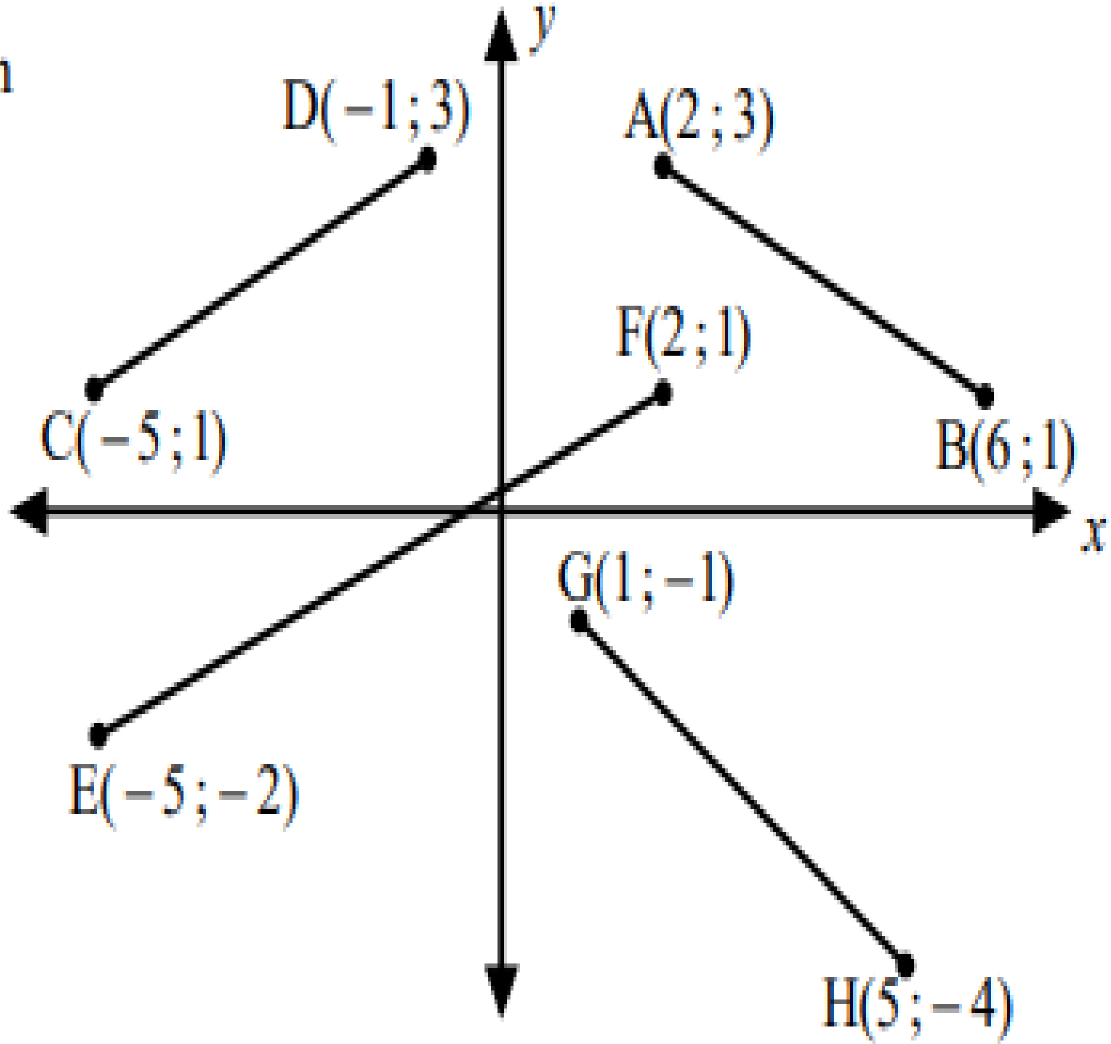
$$N(x_N\,;y_N)=N\left(\frac{x_C+x_D}{2};\frac{y_C+y_D}{2}\right)$$

$$\therefore N\left(\frac{-5+(-1)}{2};\frac{6+2}{2}\right)$$

$$\therefore N(-3\,;4)$$

Discussion (2.4) of concepts on Analytical Geometry with Step-by-step solution Midpoint of a line segment

(a) Determine the midpoints of the given line segments.
Use the midpoint formula.



Working area

Solution (2.4): Midpoint of a line segment

$$\text{Midpoint of AB} = \left(\frac{x_A + x_B}{2} ; \frac{y_A + y_B}{2} \right)$$

$$\therefore \text{Midpoint of AB} = (4 ; 2)$$

$$\text{Midpoint of CD} = \left(\frac{x_C + x_D}{2} ; \frac{y_C + y_D}{2} \right)$$

$$\therefore \text{Midpoint of CD} = \left(\frac{(-5)+(-1)}{2} ; \frac{1+3}{2} \right)$$

$$\therefore \text{Midpoint of CD} = (-3 ; 2)$$

$$\text{Midpoint of EF} = \left(\frac{x_E + x_F}{2} ; \frac{y_E + y_F}{2} \right)$$

$$\therefore \text{Midpoint of EF} = \left(\frac{(-5)+(2)}{2} ; \frac{(-2)+(1)}{2} \right)$$

$$\therefore \text{Midpoint of EF} = \left(\frac{-3}{2} ; \frac{-1}{2} \right) = \left(-1\frac{1}{2} ; -\frac{1}{2} \right)$$

$$\text{Midpoint of GH} = \left(\frac{x_G + x_H}{2} ; \frac{y_G + y_H}{2} \right)$$

$$\therefore \text{Midpoint of GH} = \left(\frac{(1)+(5)}{2} ; \frac{(-1)+(-4)}{2} \right)$$

$$\therefore \text{Midpoint of GH} = \left(3 ; \frac{-5}{2} \right) = \left(3 ; -2\frac{1}{2} \right)$$

Gradient of a Straight Line*!!!*


20%
Keep in
low gear

Discussion (3.1) of concepts on Analytical Geometry with Step-by-step solution Gradient of a line

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GRADIENT OF A LINE

The gradient of a straight line between $(x_1; y_1)$ and $(x_2; y_2)$ is given by:

$$m = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)$$

REMEMBER:

- Parallel (||) lines: $m_1 = m_2$
- Perpendicular (⊥) lines: $m_1 \times m_2 = -1$
- Horizontal (–) lines $[y = c]$: $m = 0$
- Vertical (|) lines $[x = c]$: m is undefined

Given $A(2; 3)$ and $B(-3; 1)$.

1. Determine the gradient of the line AB

Working area

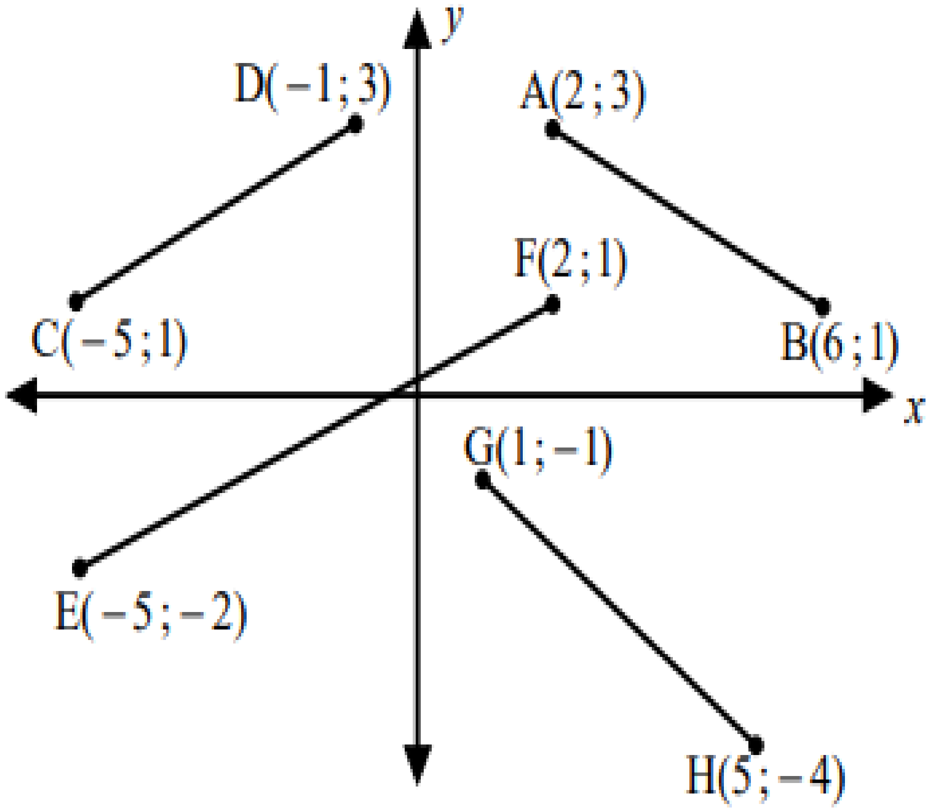
Solution (3.1): Gradient of a line

1. Determine the gradient of the line AB

$$m_{AB} = \left(\frac{y_2 - y_1}{x_2 - x_1}\right) = \left(\frac{3-1}{2+3}\right) = \left(\frac{2}{5}\right)$$

Discussion (3.2) of concepts on Analytical Geometry with Step-by-step solution Gradient of a line

Calculate the gradients of the following lines using the formula for gradient.



Working area

Solution (3.2): Gradient of a line

$$m_{AB} = \frac{y_B - y_A}{x_B - x_A} = \frac{1-3}{6-2} = \frac{-2}{4} = -\frac{1}{2}$$ (slopes down from left to right)

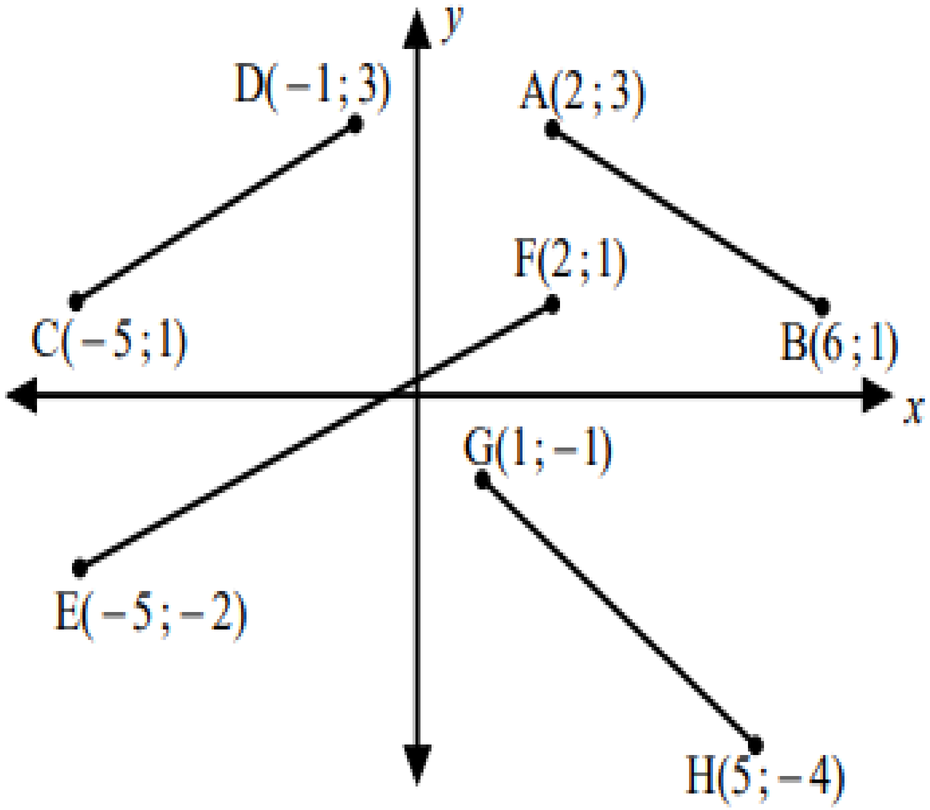
$$m_{CD} = \frac{y_D - y_C}{x_D - x_C} = \frac{3-1}{-1-(-5)} = \frac{2}{4} = \frac{1}{2}$$ (slopes up from left to right)

$$m_{EF} = \frac{y_F - y_E}{x_F - x_E} = \frac{1-(-2)}{2-(-5)} = \frac{3}{7}$$ (slopes up from left to right)

$$m_{GH} = \frac{y_H - y_G}{x_H - x_G} = \frac{-4-(-1)}{5-1} = \frac{-3}{4} = -\frac{3}{4}$$ (slopes down from left to right)

Discuss with learners the –ve & + ve Gradient

Calculate the gradients of the following lines using the formula for gradient.



Can we talk about Increasing & Decreasing Gradient!!!

Discussion (3.3) of concepts on Analytical Geometry with Step-by-step solution Gradient of a line

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(a) Calculate the gradients of the lines joining the following points.

(1) $A(1;3)$ and $B(5;7)$

(2) $A(1;3)$ and $B(-5;-7)$

(3) $A(-1;-3)$ and $B(-5;-7)$

(4) $A(-1;3)$ and $B(5;-7)$

Working area

Solution (3.3): Gradient of a line

(a) (1) $\text{grad}_{AB} = \dfrac{y_B - y_A}{x_B - x_A}$

$\therefore \text{grad}_{AB} = \dfrac{7-3}{5-1}$

$\therefore \text{grad}_{AB} = \dfrac{4}{4}$

$\therefore \text{grad}_{AB} = 1$

(2) $\text{grad}_{AB} = \dfrac{y_B - y_A}{x_B - x_A}$

$\therefore \text{grad}_{AB} = \dfrac{-7-3}{-5-1}$

$\therefore \text{grad}_{AB} = \dfrac{-10}{-6}$

$\therefore \text{grad}_{AB} = \dfrac{5}{3}$

(3) $\text{grad}_{AB} = \dfrac{y_B - y_A}{x_B - x_A}$

$\therefore \text{grad}_{AB} = \dfrac{-7-(-3)}{-5-(-1)}$

$\therefore \text{grad}_{AB} = \dfrac{-4}{-4}$

$\therefore \text{grad}_{AB} = 1$

(4) $\text{grad}_{AB} = \dfrac{y_B - y_A}{x_B - x_A}$

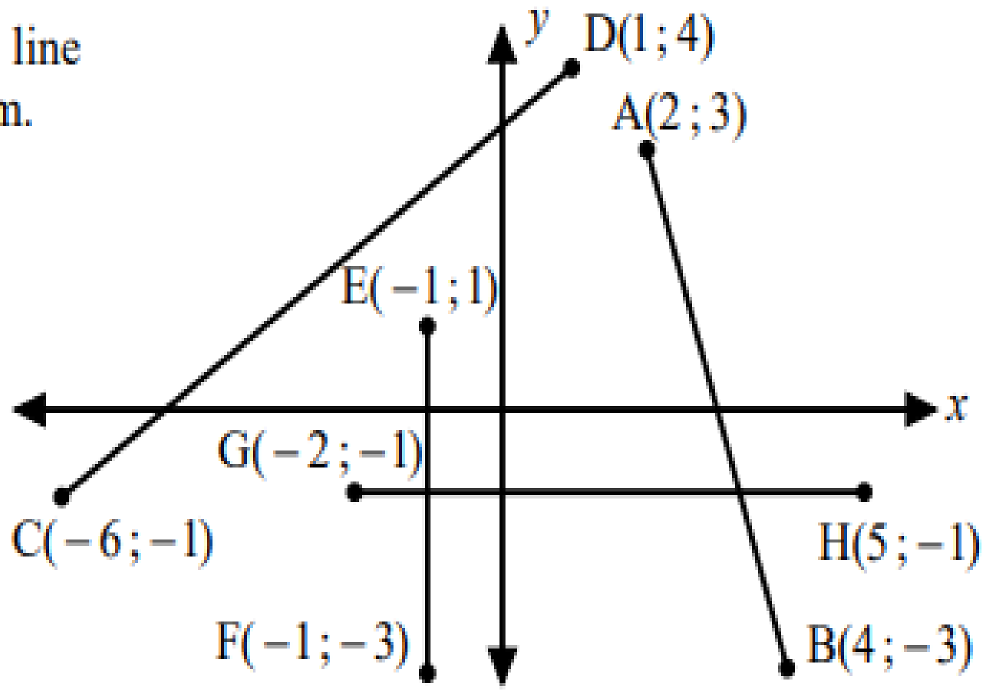
$\therefore \text{grad}_{AB} = \dfrac{-7-3}{5-(-1)}$

$\therefore \text{grad}_{AB} = \dfrac{-10}{6}$

$\therefore \text{grad}_{AB} = -\dfrac{5}{3}$

Discussion (3.4) of concepts on Analytical Geometry with Step-by-step solution Gradient of a line

(b) Calculate the gradients of the line segments in the given diagram.



Working area

Solution (3.4): Gradient of a line

(b)

$$\text{grad}_{AB} = \frac{y_B - y_A}{x_B - x_A}$$

$$\therefore \text{grad}_{AB} = \frac{-3-3}{4-2}$$

$$\therefore \text{grad}_{AB} = -3$$

$$\text{grad}_{CD} = \frac{y_D - y_C}{x_D - x_C}$$

$$\therefore \text{grad}_{CD} = \frac{4-(-1)}{1-(-6)}$$

$$\therefore \text{grad}_{CD} = \frac{5}{7}$$

Gradient of EF is undefined because EF is a vertical line.

Gradient of GH $= 0$ because GH is a horizontal line.

Discussion (3.5) of concepts on Analytical Geometry Application on Gradient of a line

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GRADIENTS OF HORIZONTAL AND VERTICAL LINES

Between any two points on a horizontal line there is no vertical movement (the vertical movement is zero). Thers is only a horizontal movement.

The gradient of a *horizontal line* is always *zero.*

$$\therefore \text{gradient}_{\text{horizontal line}} = \frac{\text{change in } y \text{ values}}{\text{change in } x \text{ values}} = \frac{0}{\text{horizontal movement}} = 0.$$

Between any two points on a vertical line there is no horizontal movement (the horizontal movement is zero). There is only a vertical movement.

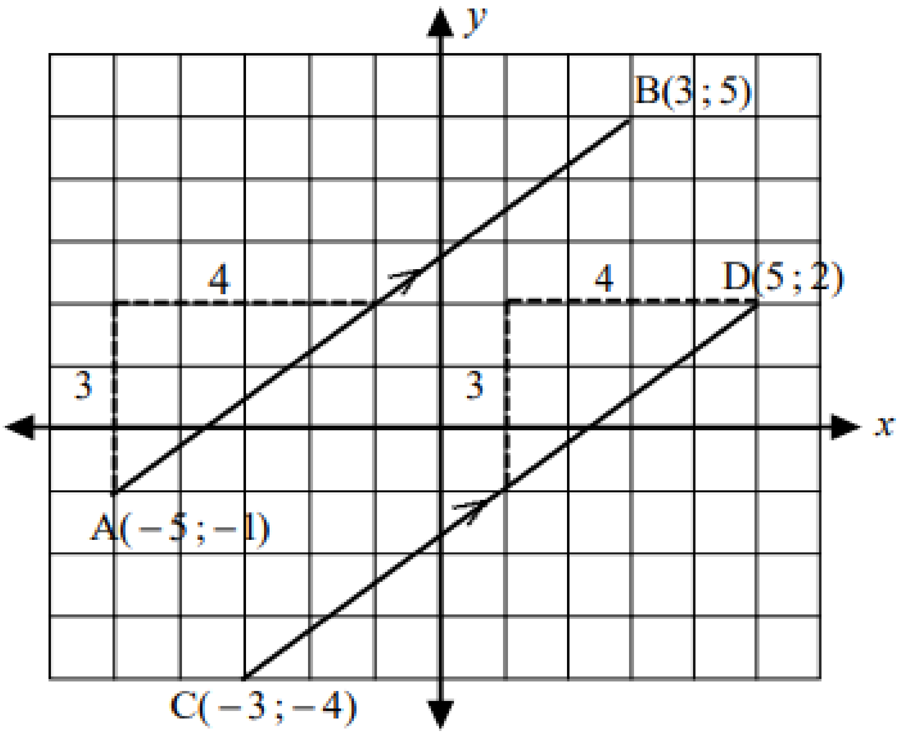
The gradient of a *vertical line* is always *undefined.*

$$\therefore \text{gradient}_{\text{vertical line}} = \frac{\text{change in } y \text{ values}}{\text{change in } x \text{ values}} = \frac{\text{vertical movement}}{0} \text{ which is undefined.}$$

Discussion (3.5) of concepts on Analytical Geometry Application on Gradient of a line. Conti…

Parallel lines

Parallel lines slope in the exactly the same direction and will therefore never intersect. Differently stated: **Lines that are parallel have equal gradients.**



For any pair of parallel lines AB and CD: $m_{AB} = m_{CD}$

Discussion (3.5) of concepts on Analytical Geometry
Application on Gradient of a line. Example

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Given are the points $A(-1;5)$, $B(-2;3)$, $C(9;10)$ and $D(5;2)$. Show that $AB \parallel CD$.

Working area

Solution (3.5): Parallel lines

$$m_{AB} = \frac{y_B - y_A}{x_B - x_A} = \frac{3-5}{-2-(-1)} = \frac{-2}{-2+1} = \frac{-2}{-1} = 2$$

$$m_{CD} = \frac{y_D - y_C}{x_D - x_C} = \frac{2-10}{5-9} = \frac{-8}{-4} = 2$$

$\therefore m_{AB} = m_{CD}$

$\therefore AB \parallel CD$

Discussion (3.6) of concepts on Analytical Geometry
Application on Gradient of a line

COLLINEAR POINTS

Points on the same line, hence, gradients between the points are equal.

When points A, B and C are collinear: $m_{\mathrm{AB}} = m_{\mathrm{AC}} = m_{\mathrm{BC}}$
In other words: $m_{\mathrm{AB}} = m_{\mathrm{AC}}$ and $m_{\mathrm{AB}} = m_{\mathrm{BC}}$ and $m_{\mathrm{AC}} = m_{\mathrm{BC}}$

If $T(5;2)$, $U(7;4)$ and $V(b;-5)$ are collinear, calculate the value of b.

Working area

Solution (3.6): Collinear points

If $T(5;2)$, $U(7;4)$ and $V(b;-5)$ are collinear, calculate the value of b.

Collinear $\therefore m_{TU} = m_{UV}$

$$\frac{2-4}{5-7} = \frac{4+5}{7-b}$$

$$1 = \frac{9}{7-b}$$

$$7-b = 9$$

$$b = -2$$

Discussion (3.7) of concepts on Analytical Geometry
Application on Gradient of a line

Show that the points A, B and C are collinear if the coordinates of the points are:
$A(2;-2)$, $B(1;1)$ and $C(-1;7)$.

Working area

Solution (3.7): Collinear points

We will consider the gradients of AB and BC, but any other pair could have been used.

$$m_{AB} = \frac{y_B - y_A}{x_B - x_A} = \frac{1-(-2)}{1-2} = \frac{3}{-1} = -3 \qquad \text{and} \qquad m_{BC} = \frac{y_C - y_B}{x_C - x_B} = \frac{7-1}{-1-1} = \frac{6}{-2} = -3$$

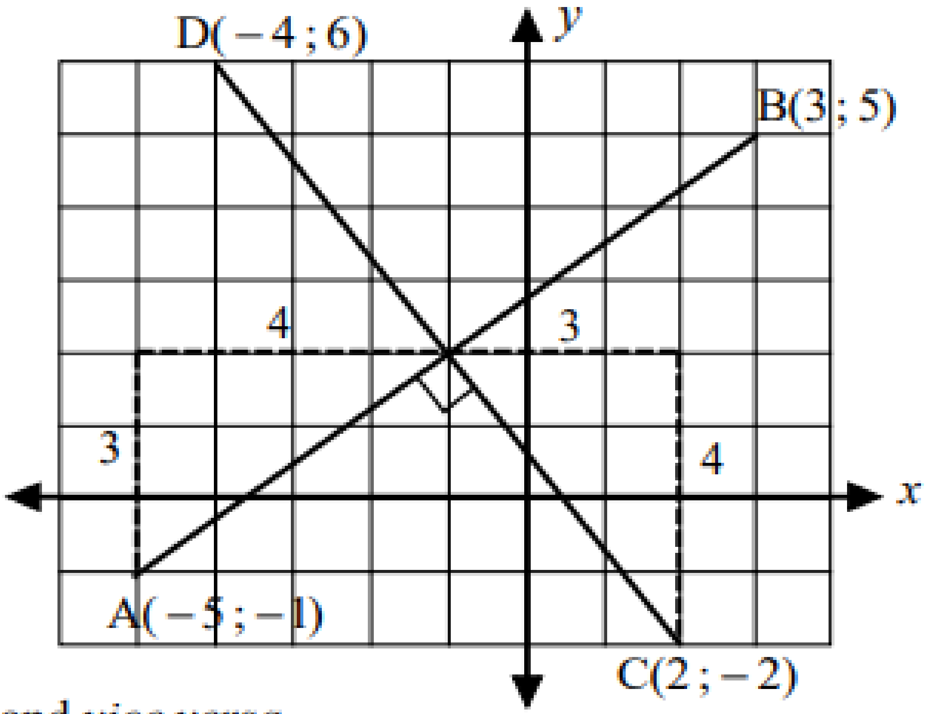
$$\therefore m_{AB} = m_{BC}$$

Therefore A, B and C are collinear.

Discussion (3.8) of concepts on Analytical Geometry with Step-by-step solution based Perpendicular lines

Perpendicular lines

Perpendicular lines intersect at a 90° angle. The gradients of perpendicular lines have a particular property. Consider the diagram on the right.



Firstly:
The gradients of the lines have opposite signs. AB has a positive gradient whereas CD has a negative gradient.
Secondly:
The gradients (ignoring signs) are reciprocals of one another. In other words, the horizontal movement of AB is the vertical movement of CD and *vice versa*.

This can be summarised by the following relationship: The product of the gradients of AB and CD will equal −1 when AB is perpendicular to CD.

For any pair of *perpendicular lines* AB and CD: $m_{AB} \times m_{CD} = -1$

Discussion (3.8) of concepts on Analytical Geometry with Step-by-step solution Application on Gradient of a line

Given are the points $A(3\,;-3)$, $B(6\,;-7)$, $C(-5\,;0)$ and $D(-1\,;3)$.

Show that AB is perpendicular to CD.

Working area

Solution (3.8): Perpendicular lines

$$m_{AB} = \frac{y_B - y_A}{x_B - x_A} = \frac{-7-(-3)}{6-3} = \frac{-7+3}{3} = \frac{-4}{3}$$

$$m_{CD} = \frac{y_D - y_C}{x_D - x_C} = \frac{3-0}{-1-(-5)} = \frac{3}{-1+5} = \frac{3}{4}$$

$$\therefore m_{AB} \times m_{CD} = \frac{-4}{3} \times \frac{3}{4} = -1$$

$$\therefore \text{AB} \perp \text{CD}$$

Thank you