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MATHEMATICS
GRADE: 10
(24 & 25/09/2021)
Number Patterns
Sessions:

Teacher Section !!!

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Overview of the topic from the CAPS document

NUMBER PATTERN		SERIES AND SEQUENCES
Grade 10	Grade 11	Grade 12
Investigate number patterns leading to those where there is a constant difference between consecutive terms, and the general term is therefore linear.	Investigate number patterns leading to those where there is a constant second difference between consecutive terms, and the general term is therefore quadratic.	1. Number patterns, including arithmetic and geometric sequences and series. 2. Sigma notation. 3. Derivation and application of the formulae for the sum of arithmetic and geometric series $S_n = \frac{n}{2}[2a + (n-1)d;$ $S_n = \frac{n}{2}(a + l);$ $S_n = \frac{a(r^n - 1)}{r-1}; (r \neq 1)$ & $S_\infty = \frac{a}{1-r}; (-1 < r < 1)$ $(r \neq 1)$

Exam Guidelines on Number Patterns

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1. The sequence of first differences of a quadratic number pattern is linear. Therefore, knowledge of linear patterns can be tested in the context of quadratic number patterns.
2. Recursive patterns will not be examined explicitly.
3. Links must be clearly established between patterns done in earlier grades.
4. Questions need not be limited to only quadratic patterns. Questions can be formed by using combinations of quadratic patterns and done in earlier grades.

Lesson !!!

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Exam Paper overview on Number patterns

Weighting of Content Areas			
Description	**Grade 10**	**Grade 11**	**Grade. 12**
PAPER 1 (Grades 12:bookwork: maximum 6 marks)			
Algebra and Equations (and inequalities)	30 ± 3	45 ± 3	25 ± 3
Patterns and Sequences	15 ± 3	25 ± 3	25 ± 3
Finance and Growth	10 ± 3		
Finance, growth and decay		15 ± 3	15 ± 3
Functions and Graphs	30 ± 3	45 ± 3	35 ± 3
Differential Calculus			35 ± 3
Probability	15 ± 3	20 ± 3	15 ± 3
TOTAL	**100**	**150**	**150**

6 Reminders on Number Patterns

1. Arithmetic sequence is done in Grade 12, hence $T_n = a + (n-1)d$ is not used in Grade 10.

$$T_n = bn + c \qquad \textit{instead} \qquad T_n = dn + c$$

2. Consecutive: directly follow one another.

3. Common/constant difference: difference between two consecutive terms in a pattern

$$b = T_2 - T_1 \qquad \textit{instead} \qquad d = T_2 - T_1$$

4. General term T_n: also referred to as the nth term.

   Linear pattern: $T_n = bn + c$

   Quadratic pattern: $T_n = an^2 + bn + c$

6 Reminders on Number Patterns, conti…

5. $T_1;\ T_2; \ldots ; T_{100}$: Terms indicated by T and the number of the term as a subscript.
6. Objectives:

   a) Find the values of the variables.
   b) Use the values to find the general term.

   c) Use the general term to calculate specific term values.

   d) Use specific term values to find the term number.

CAPS Clarification on Number Pattern

Comment:

- Arithmetic sequence is done in Grade 12, hence $T_n = a + (n - 1)d$ is not used in Grade 10.

Examples:

1. Determine the 5^{th} and the n^{th} terms of the number pattern 10; 7; 4; 1; …. There is an algorithmic approach to answering such questions. (R)

2. If the pattern MATHSMATHSMATHS… is continued in this way, what will the 267^{th} letter be? It is not immediately obvious how one should proceed, unless similar questions have been tackled. (P)

Discussion 1. The general term of a linear number pattern.

The n^{th} term of a linear number pattern is given by $T_n = bn + c$.

If we use this formula to calculate the first three terms, we get:

$$T_n = b(1) + c = b + c$$

$$T_n = b(2) + c = 2b + c$$

$$T_n = b(3) + c = 3b + c$$

Let us calculate the difference between consecutive term:

$$\boldsymbol{b+c} \qquad \boldsymbol{2b+c} \qquad \boldsymbol{3b+c}$$

$$b \qquad b$$

Discussion 1. The general term of a linear number pattern, conti…

Note that the first term is $b + c$ and the constant difference is b.

In a linear number pattern:
First term = $b + c$
Constant difference = b

General term: $T_n = bn + c$

Discussion 1.
Examples on Linear pattern

Given the linear pattern: 3 ; 7 ; 11 ; 15

a) Determine the general term (T_n).

$$b = 4$$
$$b + c = 3$$
$$\therefore 4 + c = 3$$
$$c = -1$$
$$\therefore T_n = 4n - 1$$

b) Determine the 20th term.

$$T_n = 4n - 1$$
$$T_{20} = 4(20) - 1$$
$$\therefore T_{20} = 79$$

c) Which term in the number pattern equals 139?

$$T_n = 4n - 1$$
$$139 = 4n - 1$$
$$\therefore n = 35$$

Discussion 1.
Activities on Linear pattern

(a) For each of the following sequences, determine the general rule (nth term) and hence calculate the 100th term.

(1)	$6;9;12;15;$........	(2)	$9;13;17;21;$........	(3)	$3;8;13;18;$........
(4)	$3;7;11;15;$........	(5)	$10;16;22;28;$.....	(6)	$4;11;18;25;$.......
(7)	$5;0;-5;-10;$.......	(8)	$0;-3;-6;$.......	(9)	$-6;-11;-16;$.......
(10)	$5;1;-3;-7;$....	(11)	$-5;-11;-17;$........	(12)	$3\frac{1}{2};4;4\frac{1}{2};$........
(13)	$2\frac{1}{2};4\frac{1}{2};6\frac{1}{2};$.....	(14)	$\frac{1}{4};1;\frac{7}{4};$........	(15)	$0{,}5;0{,}7;0{,}9;$....
(16)	$-13;-7;-1;$.....	(17)	$1;-9;-19;$.......	(18)	$13;12;11;10;$.....

(b) $4;11;18;25;$......... is a given sequence.

(1) Determine the 45th term. (2) Which term of the sequence is 627?

(c) $19;16;13;10;$......... is a given sequence.

(1) Determine the 65th term. (2) Which term of the sequence is -113?

(d) $T_n = 9n - 4$ is the nth term of a linear number pattern (sequence).

(1) Determine the first four terms of the sequence.

(2) Which term is equal to 986?

(e) Consider the number pattern: $4\times7;7\times15;10\times23;13\times31;$......

(1) Determine the nth term.

(2) Determine the 50th term

Discussion 1. Activities on Linear pattern Working Area

Discussion 1. Activities on Linear pattern Solutions

(a)

(1) $T_n = 3n+3$
$\therefore T_{100} = 3(100)+3 = 303$

(2) $T_n = 4n+5$
$\therefore T_{100} = 4(100)+5 = 405$

(3) $T_n = 5n-2$
$\therefore T_{100} = 5(100)-2 = 498$

(4) $T_n = 4n-1$
$\therefore T_{100} = 4(100)-1 = 399$

(5) $T_n = 6n+4$
$\therefore T_{100} = 6(100)+4 = 604$

(6) $T_n = 7n-3$
$\therefore T_{100} = 7(100)-3 = 697$

(7) $T_n = -5n+10$
$\therefore T_{100} = -5(100)+10 = -490$

(8) $T_n = -3n+3$
$\therefore T_{100} = -3(100)+3 = -297$

(9) $T_n = -5n-1$
$\therefore T_{100} = -5(100)-1 = -501$

(10) $T_n = -4n+9$
$\therefore T_{100} = -4(100)+9 = -391$

(11) $T_n = -6n+1$
$\therefore T_{100} = -6(100)+1 = -599$

(12) $T_n = \frac{1}{2}n+3$
$\therefore T_{100} = \frac{1}{2}(100)+3 = 53$

(13) $T_n = 2n+\frac{1}{2}$
$\therefore T_{100} = 2(100)+\frac{1}{2} = 200\frac{1}{2}$

(14) $T_n = \frac{3}{4}n-\frac{1}{2}$
$\therefore T_{100} = \frac{3}{4}(100)-\frac{1}{2} = 74\frac{1}{2}$

(15) $T_n = 0,2n+0,3$
$\therefore T_{100} = 0,2(100)+0,3 = 20,3$

(16) $T_n = 6n-19$
$\therefore T_{100} = 6(100)-19 = 581$

(17) $T_n = -10n+11$
$\therefore T_{100} = -10(100)+11 = -989$

(18) $T_n = -n+14$
$\therefore T_{100} = -(100)+14 = -86$

(b)

(1) $T_n = 7n-3$
$\therefore T_{45} = 7(45)-4 = 311$

(2) $627 = 7n-3$
$\therefore 630 = 7n$
$\therefore n = 90$
$\therefore T_{90} = 627$

(c)

(1) $T_n = -3n+22$
$\therefore T_{65} = -3(65)+22 = -173$

(2) $-113 = -3n+22$
$\therefore 3n = 135$
$\therefore n = 45$
$\therefore T_{45} = -113$

(d)

(1) $T_1 = 9(1)-4 = 5$ $\quad T_2 = 9(2)-4 = 14$
$T_3 = 9(3)-4 = 23$ $\quad T_4 = 9(4)-4 = 32$

(2) $986 = 9n-4$
$\therefore 990 = 9n$
$\therefore n = 110$
$\therefore T_{110} = 996$

(e)

(1) $T_n = (3n+1)(8n-1)$

(2) $\therefore T_{50} = [3(50)+1][8(50)-1] = [151][399] = 60\ 249$

Discussion 2.
Activities on Linear pattern

(a) For each of the following number patterns, determine the general rule and hence the 10th term.

(1)	$2 ; 4 ; 8 ; 16 ; \ldots\ldots$	(2)	$1 ; 3 ; 9 ; 27 ; \ldots\ldots$	(3)	$4 ; 12 ; 36 ; \ldots\ldots$
(4)	$32 ; 16 ; 8 ; 4 ; \ldots\ldots$	(5)	$-2 ; -6 ; -18 ; \ldots\ldots$	(6)	$\frac{1}{2} ; 1 ; 2 ; 4 ; \ldots\ldots$
(7)	$16 ; 4 ; 1 ; \frac{1}{4} \ldots\ldots$	(8)	$\frac{1}{2} ; \frac{1}{4} ; \frac{1}{8} ; \frac{1}{16} ; \ldots\ldots$	(9)	$28 ; 7 ; \frac{7}{4} ; \frac{7}{16} \ldots\ldots$

(b) For each of the following sequences, determine the nth term and hence the 100th term.

(1)	$1 ; 4 ; 9 ; 16 ; \ldots\ldots$	(2)	$2 ; 5 ; 10 ; 17 ; \ldots$	(3)	$4 ; 7 ; 12 ; 19 ; \ldots$
(4)	$5 ; 8 ; 13 ; 20 ; \ldots$	(5)	$0 ; 3 ; 8 ; 15 ; \ldots$	(6)	$-1 ; 2 ; 7 ; 14 ; \ldots$

(c) Determine the general term of the sequence: $\frac{1}{5} ; \frac{3}{8} ; \frac{9}{13} ; \frac{27}{20} ; \ldots\ldots\ldots$

Discussion 2. Activities on Linear pattern Working Area

Discussion 2. Activities on Linear pattern Solutions

(a) (1) $T_n = 2\times 2^{n-1}$ $T_{10} = 2\times 2^9 = 1\,024$

(2) $T_n = 1\times 3^{n-1}$ $T_{10} = 1\times 3^9 = 19\,683$

(3) $T_n = 4\times 3^{n-1}$ $T_{10} = 4\times 3^9 = 78\,732$

(4) $T_n = 32\times (\frac{1}{2})^{n-1}$ $T_{10} = 32\times (\frac{1}{2})^9 = \frac{1}{16}$

(5) $T_n = (-2)\times (3)^{n-1}$ $T_{10} = (-2)\times (3)^9 = -39\,366$

(6) $T_n = (\frac{1}{2})\times (2)^{n-1}$ $T_{10} = (\frac{1}{2})\times (2)^9 = 256$

(7) $T_n = 16\times (\frac{1}{4})^{n-1}$ $T_{10} = 16\times (\frac{1}{4})^9 = \frac{1}{16\,384}$

(8) $T_n = (\frac{1}{2})\times (\frac{1}{2})^{n-1}$ $T_{10} = (\frac{1}{2})\times (\frac{1}{2})^9 = \frac{1}{1\,024}$

(9) $T_n = 28\times (\frac{1}{4})^{n-1}$ $T_{10} = 28\times (\frac{1}{4})^9 = \frac{7}{65\,536}$

(b) (1) $T_n = n^2$ $T_{100} = (100)^2 = 10\,000$

(2) $T_n = n^2 + 1$ $T_{100} = (100)^2 + 1 = 10\,001$

(3) $T_n = n^2 + 3$ $T_{100} = (100)^2 + 3 = 10\,003$

(4) $T_n = n^2 + 4$ $T_{100} = (100)^2 + 4 = 10\,004$

(5) $T_n = n^2 - 1$ $T_{100} = (100)^2 - 1 = 9\,999$

(6) $T_n = n^2 - 2$ $T_{100} = (100)^2 - 2 = 9\,998$

(c) $T_n = \dfrac{1\times 3^{n-1}}{n^2 + 4}$

Discussion 3. Consolidation Activities on Linear pattern

(a) Consider the number pattern: $7 ; 16 ; 25 ; 34 ; \ldots\ldots.$

(1) Determine the nth term and hence the 300th term.

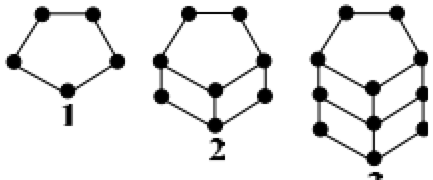
(2) Determine which term of the number pattern equals 448.

(b) Consider the number pattern: $-2 ; -5 ; -8 ; -11 ; \ldots\ldots.$

(1) Determine the nth term and hence the 145th term.

(2) Determine which term of the number pattern equals -389.

(c) Consider the diagram made up of black dots joined by thin black lines.



(1) How many dots are there in figure 4?

(2) How many lines are there in figure 4?

(3) How many dots are there in figure 8?

(4) How many lines are there in figure 8?

(5) Determine the general rule to find the number of dots in the nth figure.

(6) How many dots are there in the 186th figure?

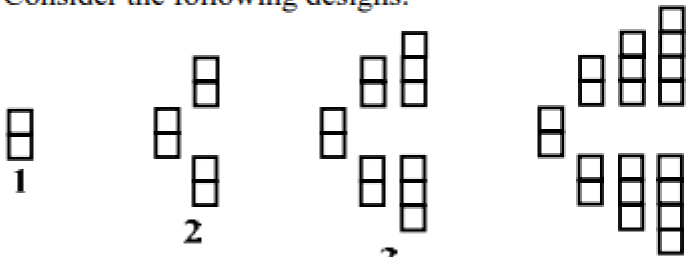
(7) Which figure will contain 272 dots?

(8) Determine the general rule to find the number of lines in the nth figure.

(9) How many lines are there in the 900th figure?

(10) Which figure will contain 650 lines?

(d) Consider the following designs.



(1) Write down the number of squares in design 1, 2, 3, 4, and 5.

(2) Determine the number of squares in design n.

(3) How many squares are there in design 20?

(e) Consider the sequence: $2 ; \frac{5}{4} ; \frac{14}{13} ; 1 ; \frac{22}{23} ; \frac{26}{28} ; \frac{30}{33} ; \ldots\ldots\ldots\ldots$

(1) Determine the nth term.

(2) Calculate the 20th term.

(f) Consider the number pattern: $1 ; 3 ; 1 ; 6 ; 1 ; 9 ; 1 ; 12 ; 1 ; \ldots\ldots.$

Determine the 999th and 1000th terms.

(g) Sipho wrote the name **SWEET** over and over again as follows:

SWEETSWEETSWEETSWEET................................

(1) What is the 23rd letter?

(2) Find the 402nd letter.

(3) The first W is in the second position, the second W is in the seventh position, the third W is in the twelfth position, and so forth. Determine in what position is the 100th W?

Discussion 3. Activities on Linear pattern Working Area

Discussion 3. Activities on Linear pattern Solutions

(a) (1) $T_n = 9n - 2$ $\qquad T_{300} = 9(300) - 2 = 2\ 698$

(2) $\therefore 448 = 9n - 2$

$\therefore 450 = 9n$

$\therefore n = 50$

$\therefore T_{50} = 448$

(b) (1) $T_n = -3n + 1$ $\qquad T_{145} = -3(145) + 1 = -434$

(2) $\therefore -389 = -3n + 1$

$\therefore 3n = 390$

$\therefore n = 130$

$\therefore T_{130} = -389$

(c) (1) Figure 4: 14 $\qquad$ (2) Figure 4: 20

(3) Figure 8: 26 $\qquad$ (4) Figure 8: 40

(5) Figure n: $T_n = 3n + 2$

(6) $T_{186} = 3(186) + 2 = 560$

(7) $272 = 3n + 2$

$\therefore 270 = 3n$

$\therefore n = 90$

Figure 90 will contain 272 dots

(8) Figure n: $T_n = 5n$

(9) $T_{900} = 5(900) = 4\ 500$

(10) $650 = 5n$

$\therefore n = 130$

Figure 130 will contain 650 lines.

(d) (1) Design 1: $2 = T_1 = (1)^2 + (1)$ $\qquad$ Design 2: $6 = T_2 = (2)^2 + (2)$

Design 3: $12 = T_3 = (3)^2 + (3)$ $\qquad$ Design 4: $20 = T_4 = (4)^2 + (4)$

Design 5: $30 = T_5 = (5)^2 + (5)$

(2) Design n: $T_n = n^2 + n$

(3) $T_{20} = (20)^2 + (20) = 420$

(e) (1) $2; \frac{5}{4}; \frac{14}{13}; 1; \frac{22}{23}; \frac{26}{28}; \frac{30}{33}; \ldots\ldots\ldots\ldots$

$= \frac{6}{3}; \frac{10}{8}; \frac{14}{13}; \frac{18}{18}; \frac{22}{23}; \frac{26}{28}; \frac{30}{33}; \ldots\ldots\ldots\ldots$

nth term of number pattern: $T_n = \dfrac{4n+2}{5n-2}$

(2) $T_{20} = \dfrac{4(20)+2}{5(20)-2} = \dfrac{82}{98} = \dfrac{41}{49}$

Discussion 3. Activities on Linear pattern Solutions

(f) $1;3;1;6;1;9;1;12;1;.......$ (original)

All the odd terms are 1. Therefore $T_{999}=1$

The even terms form a linear pattern $3;6;9;12;.......$

The general term is $T_n=3n$

Position of term in original pattern.	Position of term in linear pattern.	Actual term
T_2	T_1	$3(1)=3$
T_4	T_2	$3(2)=6$
T_6	T_3	$3(3)=9$
T_8	T_4	$3(4)=12$
T_{2n}	T_n	$3(n)=3n$
T_{1000}	T_{500}	$3(500)=1500$

Therefore $T_{1000}=1500$

(g) (1) The 23rd letter is E.

(2) All multiples of 5 are the letter T. Therefore the 400th letter is T. This implies that the 402nd letter is W.

(3)

The number of W	The position of W
1	2
2	7
3	12
4	17
5	22
6	27
n	$5n-3$
100	$5(100)-3=497$

Therefore the 100th W is in the 497th position.

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Thank you