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MATHEMATICS
Grade 10

LINEAR & QUADRATIC FUNCTIONS
LESSON: 1 &2
09 April 2022

Introduction

In this chapter, you will revise the graphs of linear functions (straight lines). Then you will explore the graphs of quadratic functions (parabolas), hyperbolic functions (hyperbolas) and exponential functions (exponential graphs).

FUNCTIONS AND GRAPHS

A function is a relationship between a set of input values (x-values or the domain) and output values(y-values or the range).

Introduction: Linear Function

LINEAR FUNCTIONS (SKETCHING STRAIGHT LINE GRAPHS)

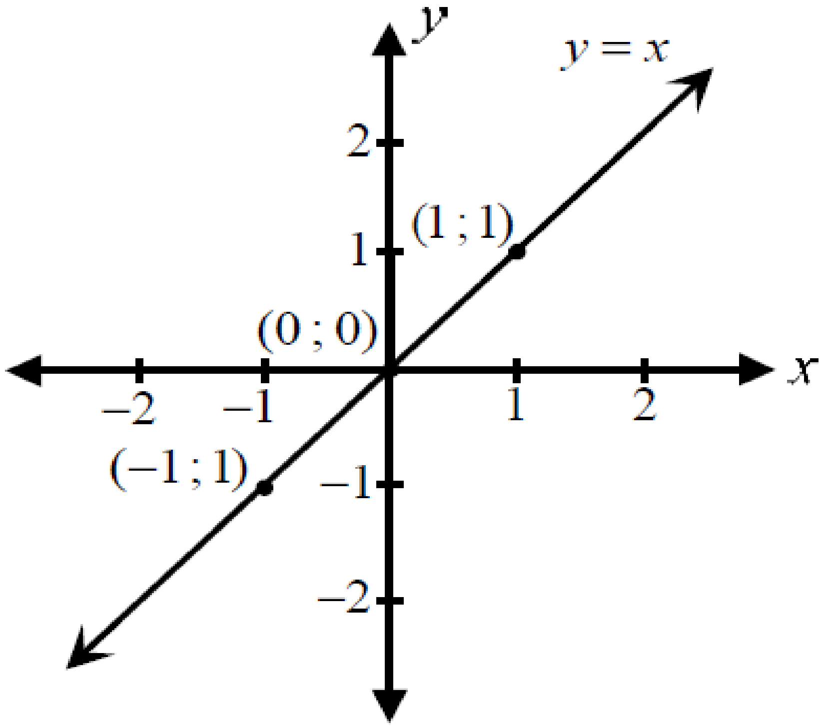
Consider the graph of $y = x$

We can select a few input values (x-values) and hence determine the corresponding output values (y-values). These values will be represented in a table.

x	-1	0	1
y	-1	0	1

The graph of this line is obtained by plotting the points on the Cartesian plane and drawing a solid line through the points.

……Introduction



This graph is referred to as the "mother" graph of straight lines and based on this graph, we can generate different types of straight lines depending on the value of a and q in the general equation of a line, which is $y = ax + q$.

……Introduction

Investigation of the effect of the value of *a*

In Grade 9, you learnt about the gradient and steepness of a straight line. Let's briefly summarise these concepts.

The gradient of a line represents the ratio of the change of the y-values with respect to the x-values. In other words, gradient tells us the direction (or slope) of the line.

We will revise the concept of gradient later on in this chapter. However, the focus now will be on the steepness of a line, which is the way that the line slants upwards or downwards from left to right. We will compare the steepness of different lines to the mother graph and show how the mother graph is transformed by changing the value of a (the coefficient of x).

……Introduction

Let's investigate this by comparing the graphs of the following straight lines:

A: $y = x$ B: $y = \frac{1}{2}x$ C: $y = 2x$ D: $y = 3x$

Lines A, B, C and D pass through the origin since for all of the given graphs, the y-value is 0 if $x = 0$.

To sketch the graphs of these lines, select one x-value and then determine the corresponding y-value. For all four graphs, choose $x = 1$.

A: $y = (1) = 1$ Line A passes through the points $(0\,;0)$ and $(1\,;1)$

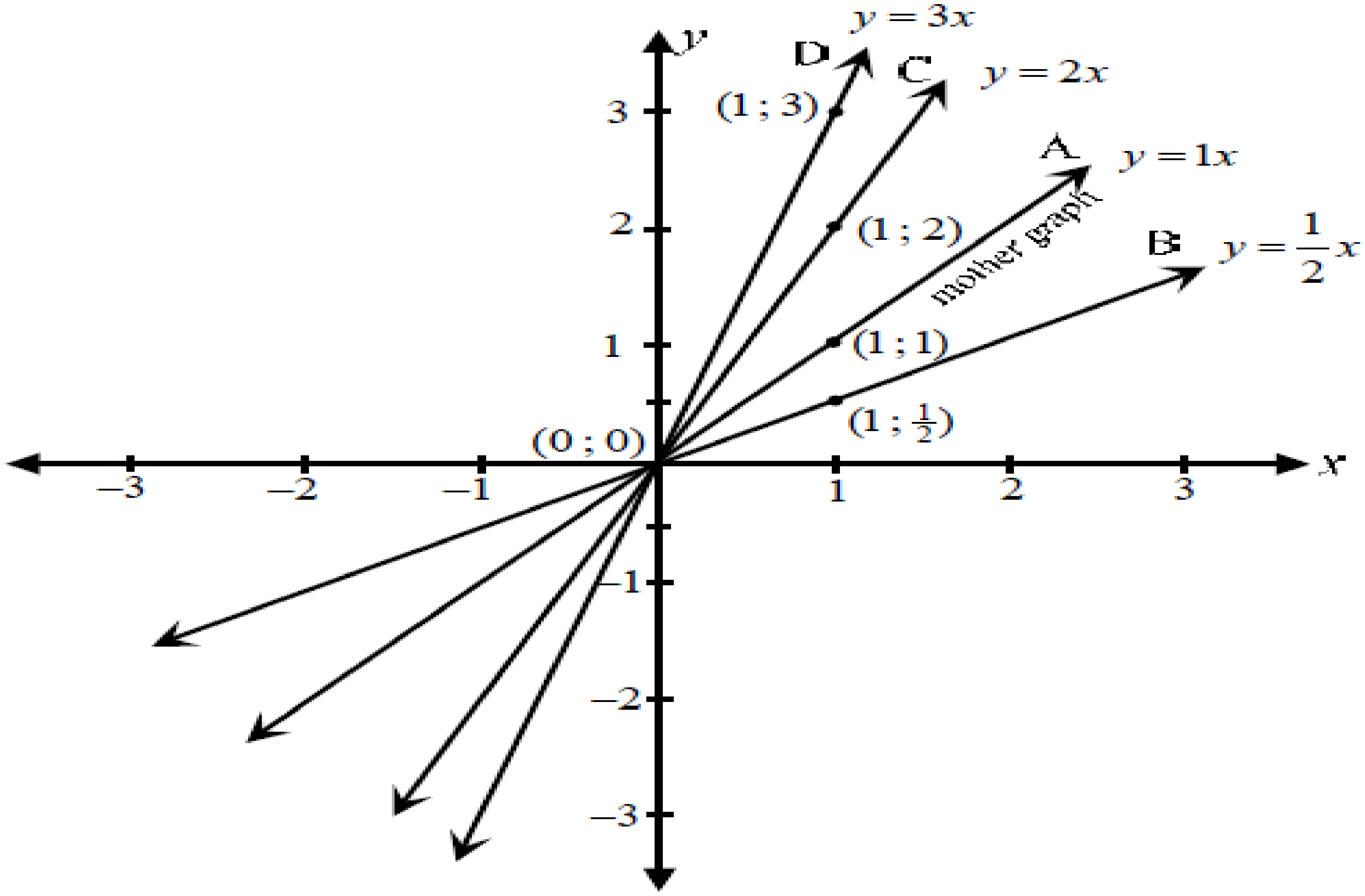
B: $y = \frac{1}{2}(1) = \frac{1}{2}$ Line B passes through the points $(0\,;0)$ and $\left(1\,;\frac{1}{2}\right)$

C: $y = 2(1) = 2$ Line C passes through the points $(0\,;0)$ and $(1\,;2)$

D: $y = 3(1) = 3$ Line D passes through the points $(0\,;0)$ and $(1\,;3)$

……Introduction

We will now plot the points and then draw the lines on the same set of axes.



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Notice that line A (the mother graph) is closer to the y-axis than line B. We say that line A is steeper than line B. The coefficient of x in the equation of line A is greater than the coefficient of x in line B ($1 > \frac{1}{2}$). Line C is steeper line A ($2 > 1$) and line D is steeper than line C ($3 > 2$). Line D is the steepest of all the lines.

......Introduction

Let's now consider what happens if the value of the coefficient of x is **negative**.

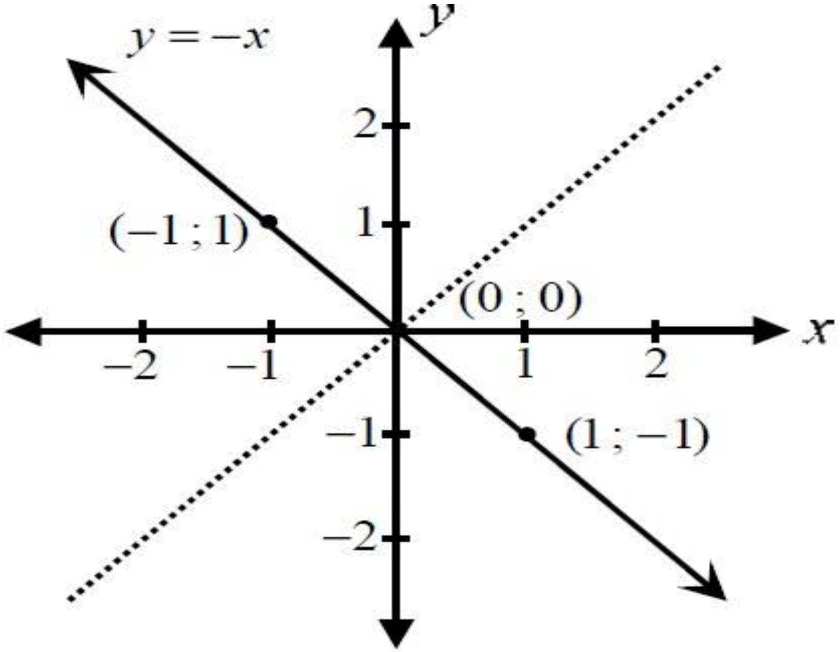
(a) Consider the graph of $y = -x$

We can select a few input values (x-values) and hence determine the corresponding output values (y-values). These values will be represented in a table.

x	-1	0	1
y	1	0	-1

The graph of this line is obtained by plotting the points on the Cartesian plane and drawing a solid line through the points.

……Introduction



If you now compare the mother graph $y = 1x$ to the graph of $y = -1x$, it is interesting to note that the graph of $y = -x$ is the reflection of the mother graph in the x-axis. The negative sign therefore causes a **reflection in the x-axis**.

......Introduction

Conclusion:

The value of a in the equation $y = ax + q$ (ignoring negative signs), determines the steepness of the line (closeness to the y-axis). The larger the value of a, the steeper the line. A negative sign will cause a reflection in the x-axis.

Investigation of the effect of the value of q

(a) Consider the graphs of the following: $y = x + 3$ and $y = x - 2$

For $y = x + 3$:

If $x = 0$ then $y = 0 + 3 = 3$

If $x = 1$ then $y = 1 + 3 = 4$

The line passes through the points $(0\,;\,3)$ and $(1\,;\,4)$

……Introduction

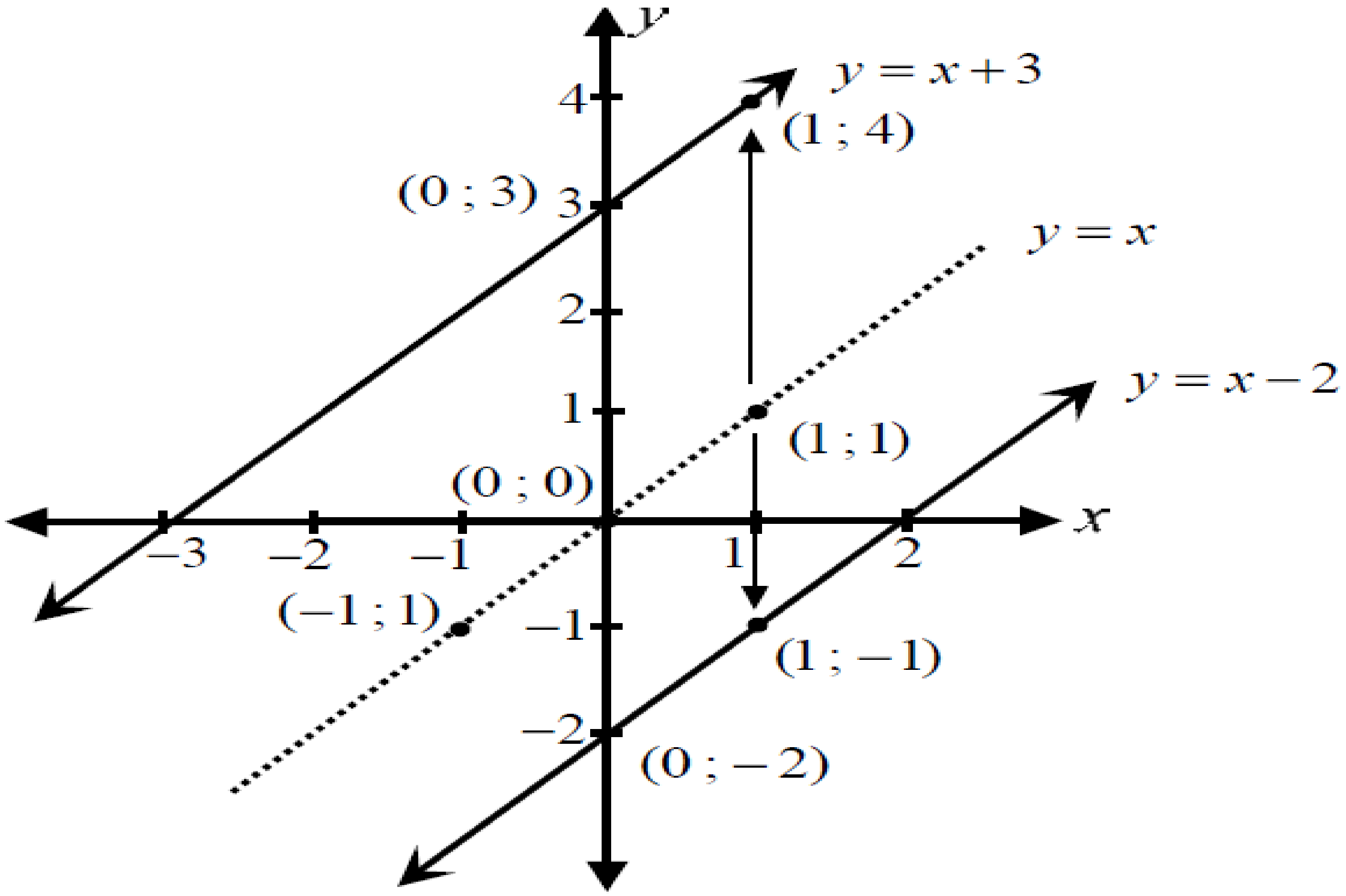
For $y = x - 2$:

If $x = 0$ then $y = 0 - 2 = -2$

If $x = 1$ then $y = 1 - 2 = -1$

The line passes through the points $(0\,;-2)$ and $(1\,;-1)$

……Introduction



......Introduction

It should be clear to you from the graphs that $y = x + 3$ is the mother graph $y = x$ shifted 3 units up and $y = x - 2$ is the mother graph $y = x$ shifted 2 units down.

Also, the y-intercept of $y = x + 3$ is 3 and the y-intercept of $y = x - 2$ is -2.

SUMMARY

Conclusion:

The value of q in the equation $y = ax + q$ determines the shift of the graph of $y = ax$ up or down. It also represents the y-intercept of the graph of $y = ax + q$.

ACTIVITY

Draw a neat sketch graph of $y = -2x + 4$

SOLUTION

First draw the graph of $y = 2x$, reflect this graph in the x-axis to form $y = -2x$ and then shift $y = -2x$ four units up to form $y = -2x + 4$.

The line $y = 2x$ cuts the y-axis at 0.

Now choose $x = 1$

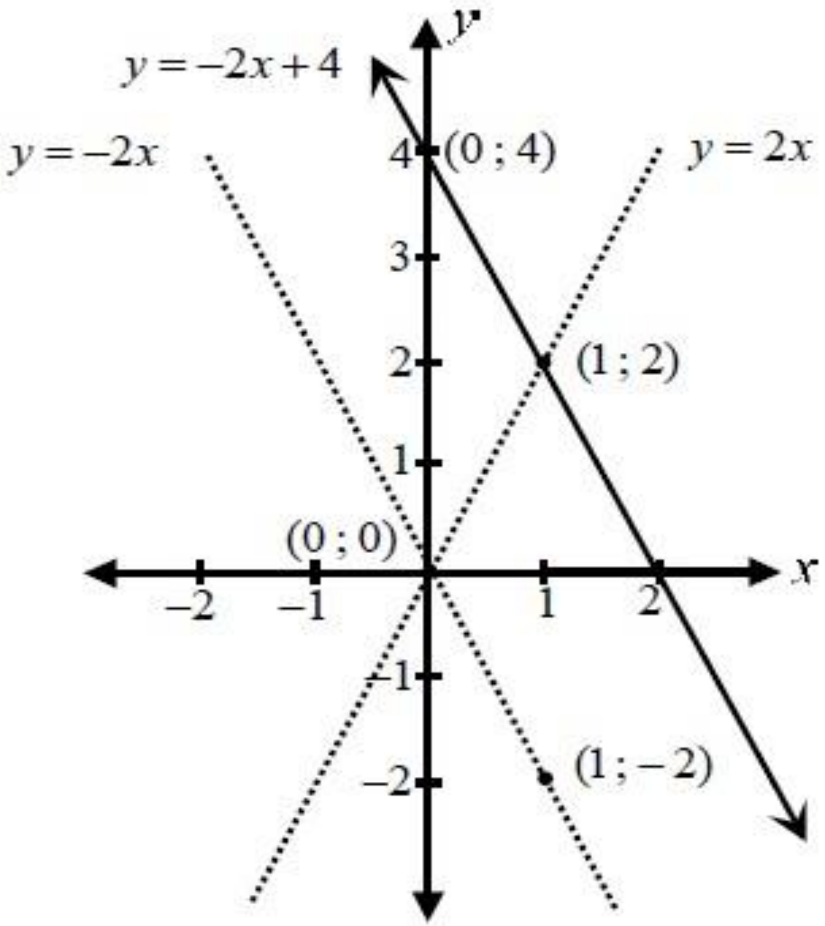
$\therefore y = 2(1) = 2$

Plot the point $(1 ; 2)$ and draw the line $y = 2x$

Now reflect $y = 2x$ in the x-axis to form $y = -2x$. The point $(1 ; 2)$ transforms into the point $(1 ; -2)$.

Then shift $y = -2x$ four units up.

Draw the graph of $y = -2x + 4$.



SOLUTION CONTI..

Alternatively, you can make use of the **dual-intercept method** that you studied in Grade 9. This method involves determining the intercepts with the axes algebraically. Let's use this method for the line $y = -2x + 4$

y-intercept: Let $x = 0$

$$y = -2(0) + 4 = 4$$

x-intercept: Let $y = 0$

$$0 = -2x + 4$$
$$\therefore 2x = 4$$
$$\therefore x = 2$$

The coordinates of the y-intercept are $(0 ; 4)$ and the coordinates of the x-intercept are $(2 ; 0)$.

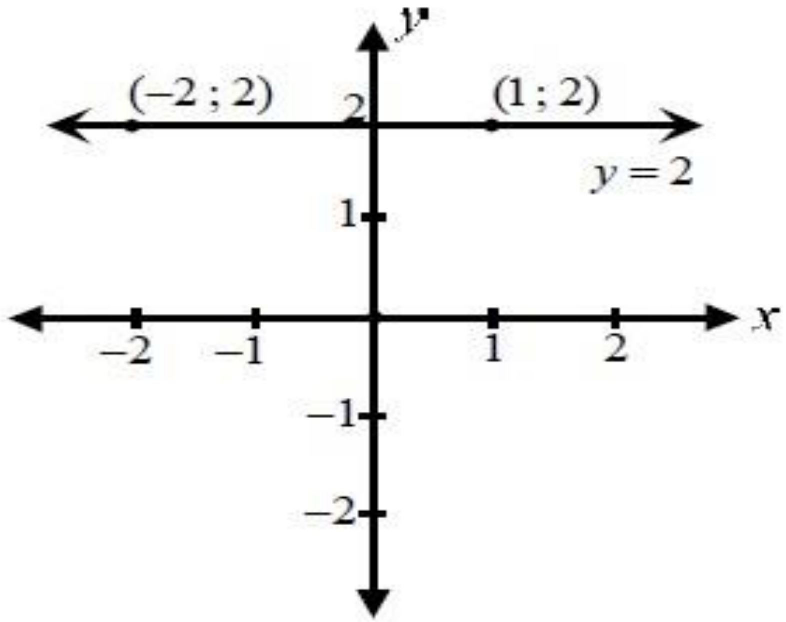
You would now plot these points and draw the straight line.

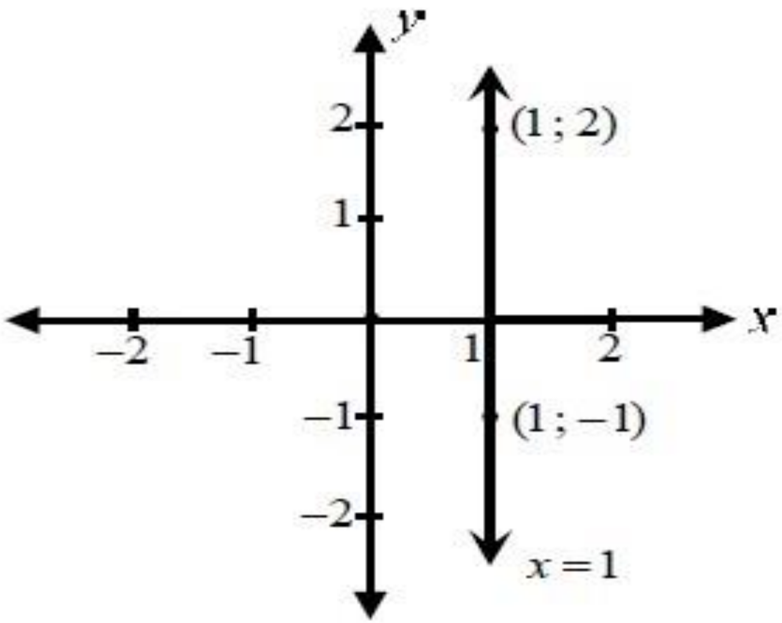
DISCUSSION

Revision of horizontal and vertical lines

In Grade 9 you learnt that horizontal lines have the general equation $y = n$ where n is any real number. Vertical lines have the general equation $x = n$ where n is any real number.

For example, the graphs of the lines $y = 2$ and $x = 1$ are shown below.





For the line $y = 2$, the y-values will be constant but the x-values will vary.
For the line $x = 1$, the x-values will be constant but the y-values will vary.

ACTIVITY

(a) Given: $y = x$ $y = \frac{3}{2}x$ $y = \frac{1}{4}x$

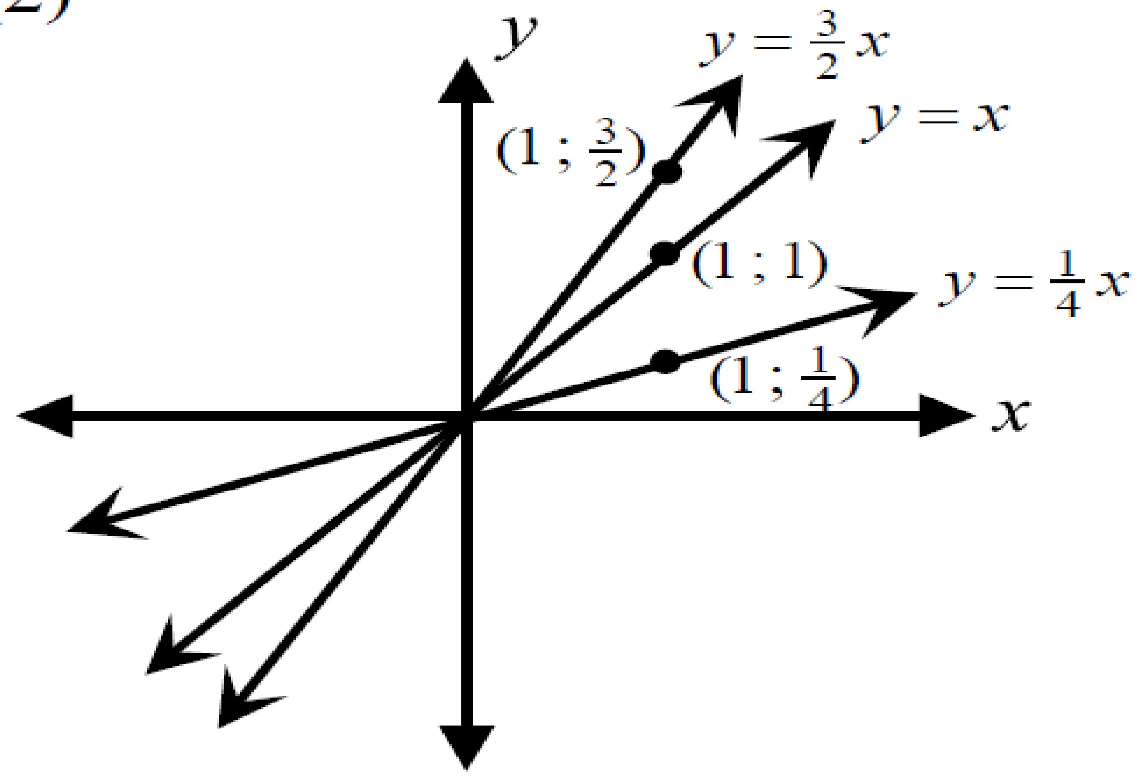
(1) Which of the three lines is the steepest? Explain

(2) Sketch the three graphs on the same set of axes.

SOLUTION

(a) (1) $y = \frac{3}{2}x$ since $\frac{3}{2} > 1 > \frac{1}{4}$

(2)



ACTIVITY

Match the equations on the left to the graphs on the right.

(1) $y = -2x$

(2) $y = \frac{1}{4}x + 2$

(3) $y = -x - 2$

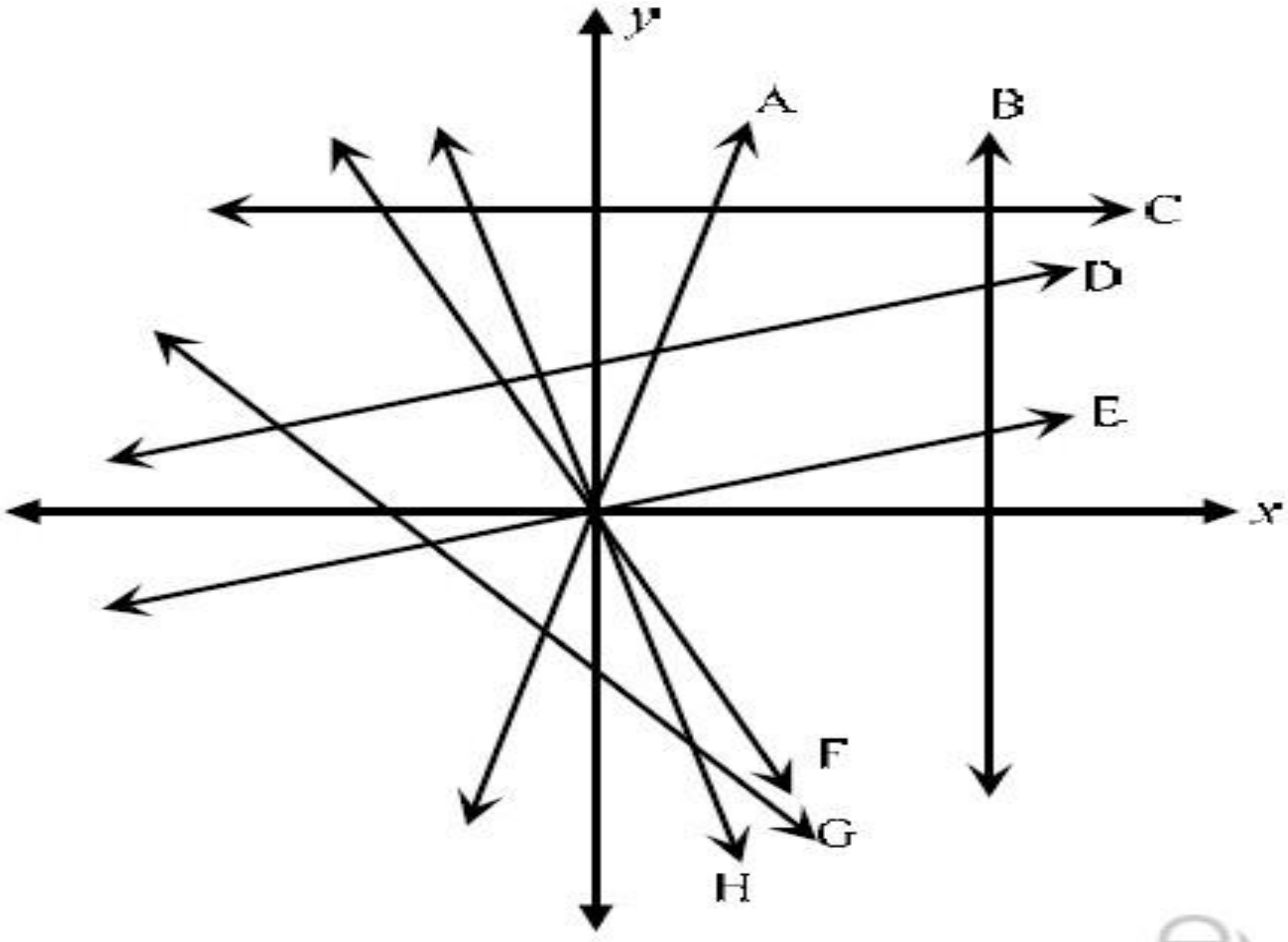
(4) $y = 4x$

(5) $y = \frac{1}{4}x$

(6) $y = -4x$

(7) $x = 4$

(8) $y = 4$



SOLUTION

(1)	F	(2)	D	(3)	G	(4)	A
(5)	E	(6)	H	(7)	B	(8)	C

DISCUSSION

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FUNCTIONAL NOTATION

A function may be represented by means of **functional notation.**

Consider the function $f(x)=3x$

The symbol $f(x)$ is used to represent the value of the output given an input value.

In other words, the y-values corresponding to the x-values are given by $f(x)$, i.e. $y=f(x)$.

For example, if $x=4$, then the corresponding y-value is obtained by substituting $x=4$ into $3x$. For $x=4$, the y-value is $f(4)=3(4)=12$.

The brackets in the symbol $f(4)$ do not mean f multiplied by 4, but rather the y-value when $x=4$. Also, $f(x)=3x$ is read as "f of x is equal to $3x$".

We can also use other letters to name functions. For example, $g(x)$, $h(x)$ and $p(x)$ may be used.

EXAMPLES

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If $f(x)=3x^2-1$, determine the value of:

(a) $f(2)$ (b) $f(-3)$ (c) $f(a)$

(d) $f(3x)$ (e) $3f(x)+1$ (f) x if $f(x)=2$

Solutions

(a)
$$\begin{aligned} f(x) &= 3x^2-1 \\ f(2) &= 3(2)^2-1 \\ &= 3(4)-1 \\ &= 11 \end{aligned}$$

(b)
$$\begin{aligned} f(x) &= 3x^2-1 \\ f(-3) &= 3(-3)^2-1 \\ &= 3(9)-1 \\ &= 26 \end{aligned}$$

EXAMPLES

(c)

$$f(x) = 3x^2 - 1$$
$$f(a) = 3(a)^2 - 1$$
$$= 3a^2 - 1$$

(d)

$$f(x) = 3x^2 - 1$$
$$f(3x) = 3(3x)^2 - 1$$
$$= 3(9x^2) - 1$$
$$= 27x^2 - 1$$

(e)

$$f(x) = 3x^2 - 1$$
$$\therefore 3f(x) = 3(3x^2 - 1)$$
$$\therefore 3f(x) = 9x^2 - 3$$
$$\therefore 3f(x) + 1 = 9x^2 - 3 + 1$$
$$\therefore 3f(x) + 1 = 9x^2 - 2$$

(f)

$$f(x) = 3x^2 - 1$$
$$\therefore 2 = 3x^2 - 1$$
$$\therefore 0 = 3x^2 - 3$$
$$\therefore 0 = x^2 - 1$$
$$\therefore 0 = (x+1)(x-1)$$
$$\therefore x = -1 \quad \text{or} \quad x = 1$$

ACTIVITY

(a) If $f(x) = 2x^2 - x + 1$, determine the value of:

(1) $f(1)$ (2) $f(-1)$ (3) $f(2)$

(4) $f(-2)$ (5) $f\left(\frac{1}{2}\right)$ (6) $f\left(-\frac{1}{2}\right)$

(7) $f(a)$ (8) $f(2x)$ (9) $2f(x)$

SOLUTIONS

(a)

(1)
$$f(x) = 2x^2 - x + 1$$
$$\therefore f(1) = 2(1)^2 - (1) + 1$$
$$\therefore f(1) = 2$$

(2)
$$f(x) = 2x^2 - x + 1$$
$$\therefore f(-1) = 2(-1)^2 - (-1) + 1$$
$$\therefore f(-1) = 4$$

(3)
$$f(x) = 2x^2 - x + 1$$
$$\therefore f(2) = 2(2)^2 - (2) + 1$$
$$\therefore f(2) = 7$$

(4)
$$f(x) = 2x^2 - x + 1$$
$$\therefore f(-2) = 2(-2)^2 - (-2) + 1$$
$$\therefore f(-2) = 11$$

(5)
$$f(x) = 2x^2 - x + 1$$
$$\therefore f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right) + 1$$
$$\therefore f\left(\frac{1}{2}\right) = 2\left(\frac{1}{4}\right) - \frac{1}{2} + 1$$
$$\therefore f\left(\frac{1}{2}\right) = \frac{1}{2} - \frac{1}{2} + 1$$
$$\therefore f\left(\frac{1}{2}\right) = 1$$

(6)
$$f(x) = 2x^2 - x + 1$$
$$\therefore f\left(-\frac{1}{2}\right) = 2\left(-\frac{1}{2}\right)^2 - \left(-\frac{1}{2}\right) + 1$$
$$\therefore f\left(-\frac{1}{2}\right) = 2\left(\frac{1}{4}\right) + \frac{1}{2} + 1$$
$$\therefore f\left(-\frac{1}{2}\right) = \frac{1}{2} + \frac{1}{2} + 1$$
$$\therefore f\left(-\frac{1}{2}\right) = 2$$

(7)
$$f(x) = 2x^2 - x + 1$$
$$\therefore f(a) = 2(a)^2 - (a) + 1$$
$$\therefore f(a) = 2a^2 - a + 1$$

(8)
$$f(x) = 2x^2 - x + 1$$
$$\therefore f(2x) = 2(2x)^2 - (2x) + 1$$
$$\therefore f(2x) = 8x^2 - 2x + 1$$

DISCUSSION

<u>**THE LINEAR FUNCTION**</u>

We will now briefly revise lines of the form $ax + by = c$, the gradient of a line and points of intersection of lines.

EXAMPLES

In the diagram, two lines are drawn:
$3x+2y=6$ and $x-4y=16$
The first line cuts the y-axis at A and the x-axis at B.
The two lines intersect at C. Determine:

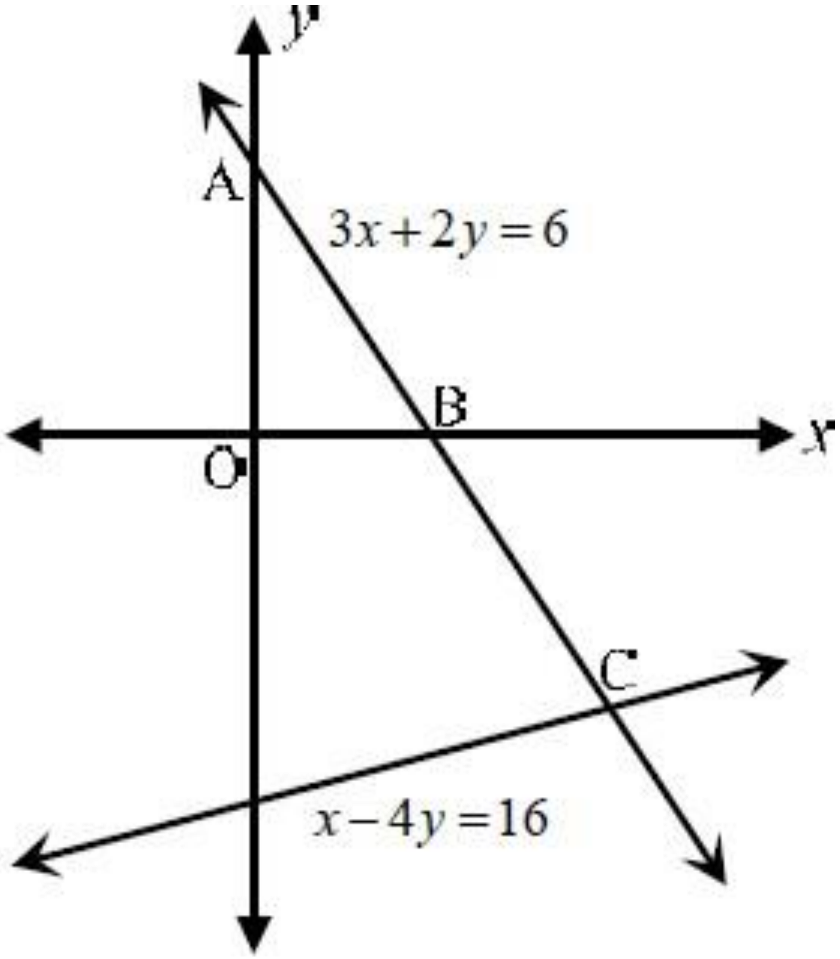
(a) the coordinates of A and B.

(b) the gradient of $3x+2y=6$

(c) the gradient and y-intercept of $x-4y=16$

(d) the coordinates of C

(e) the values of x for which the lines are increasing or decreasing.



EXAMPLE

Solutions

(a) We can use the dual-intercept method:

x-intercept: Let $y=0$

$\therefore 3x+2(0)=6$

$\therefore 3x=6$

$\therefore x=2$

$B(2\,;0)$

y-intercept: Let $x=0$

$\therefore 3(0)+2y=6$

$\therefore 2y=6$

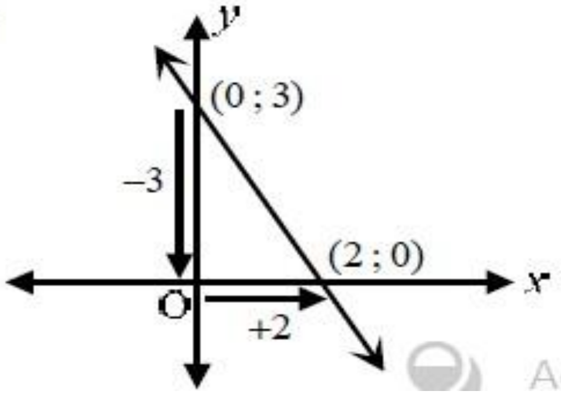
$\therefore y=3$

$A(0\,;3)$

(b) In Grade 9, you learnt that the gradient of a line between any two points on the line is given by the ratio: $\dfrac{\text{change in } y\text{-values}}{\text{change in } x\text{-values}}$

We can determine the gradient by referring to the diagram or by re-writing the equation of the line in the form $y=ax+q$, bearing in mind that the value of a represents the gradient of the line. Between the points $(0\,;3)$ and $(2\,;0)$, the y-values decrease (-3) as the x-values increase $(+2)$.

The gradient is therefore $\dfrac{-3}{2}$



We can also determine the gradient as follows:

$3x+2y=6$

$\therefore 2y=-3x+6$

$\therefore y=\dfrac{-3}{2}x+3$ [The coefficient of x is the gradient]

EXAMPLE

(c) $x-4y=16$

$\therefore -4y=-x+16$

$\therefore y=\dfrac{1}{4}x-4$

The gradient is $\dfrac{1}{4}$ and the y-intercept is -4

(d) In order to determine the coordinates of C, we need to solve simultaneous equations:

$3x+2y=6$	(1)
$x-4y=16$	(2)
$6x+4y=12$	(1)×2
$x-4y=16$	(2)
$\therefore 7x=28$	Add like terms and constants
$\therefore x=4$	
$\therefore 4-4y=16$	Substitute $x=4$ into (2) to get the corresponding y-value
$\therefore -4y=12$	
$\therefore y=-3$	

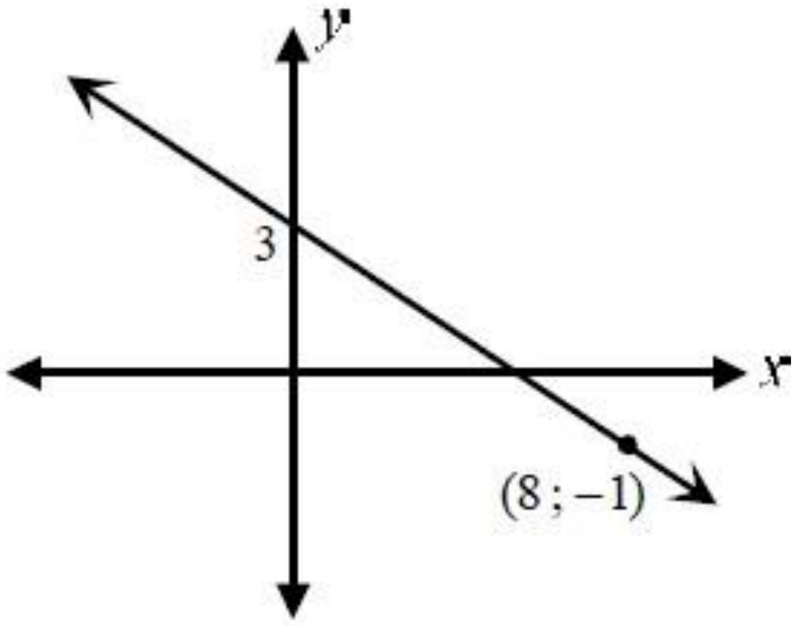
The coordinates of C are $C(4\,;-3)$

(e) $3x+2y=6$ decreases for all real values of x.

$x-4y=16$ increases for all real values of x.

FUNDING THE EQUATION OF LINEAR

(a) Determine the equation of the following line in the form $f(x) = ax + q$.



Solution

The y-intercept is 3.
Therefore $q = 3$.
$\therefore y = ax + 3$
Substitute the point $(8\,;-1)$ to get a:

$$-1 = a(8) + 3$$

$$-1 = 8a + 3$$

$$-8a = 4$$

$$a = -\frac{1}{2}$$

Therefore the equation is $f(x) = -\frac{1}{2}x + 3$

EXAMPLE

(b) Determine the equation of the following line in the form $g(x) = ax + q$.

<u>Solution</u>

Method 1

The y-intercept is 4.
Therefore $q = 4$.
$\therefore y = ax + 4$
Substitute the point $(-2\,;0)$ to get a:

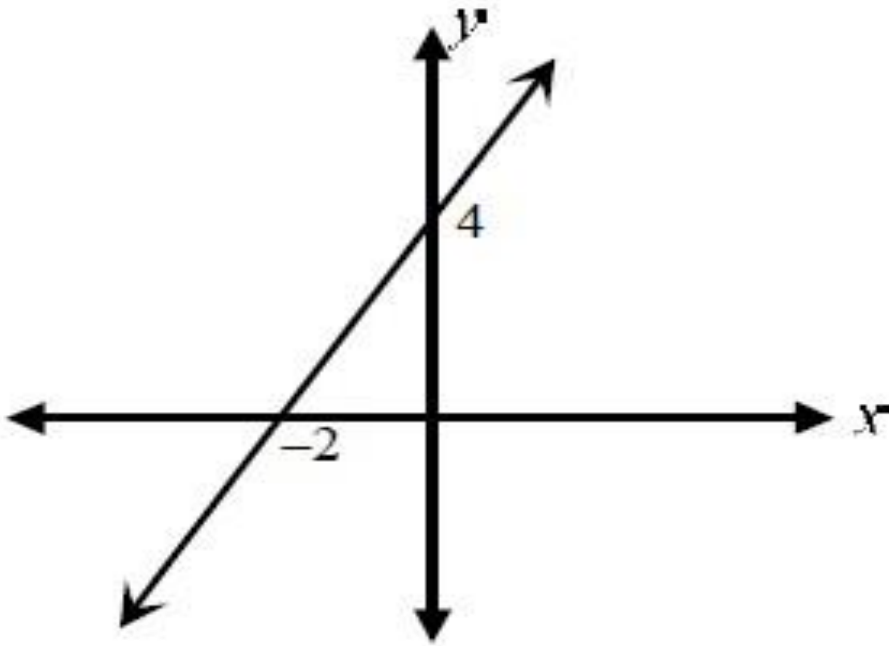
$0 = a(-2) + 4$

$0 = -2a + 4$

$2a = 4$

$a = 2$

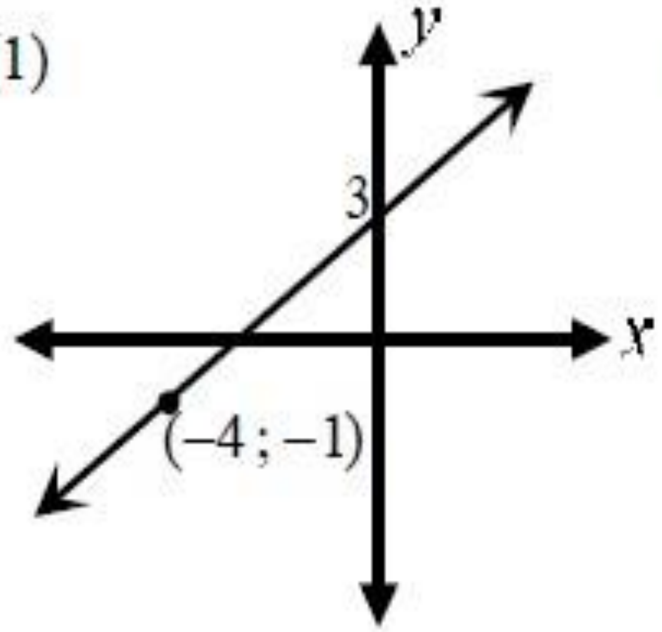
Therefore the equation is $g(x) = 2x + 4$



ACTIVITY

Determine the equations of the following lines in the form $f(x) = ax + q$:

(1)



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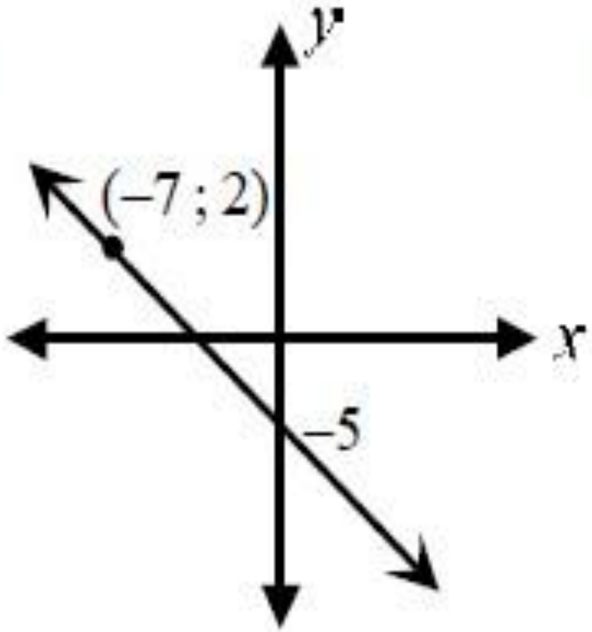
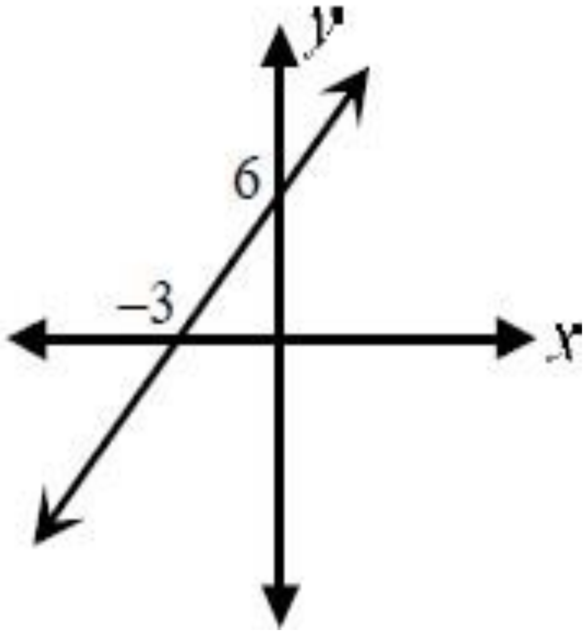


(3)



SOLUTION

(1) The y-intercept is 3.

Therefore $q = 3$.

$\therefore y = ax + 3$

Substitute $(-4\,;-1)$:

$\therefore -1 = a(-4) + 3$

$\therefore -1 = -4a + 3$

$\therefore 4a = 4$

$\therefore a = 1$

$\therefore y = x + 3$

(2) The y-intercept is -5

Therefore $q = -5$.

$\therefore y = ax - 5$

Substitute $(-7\,;2)$:

$\therefore 2 = a(-7) - 5$

$\therefore 2 = -7a - 5$

$\therefore 7a = -7$

$\therefore a = -1$

$\therefore y = -x - 5$

SOLUTION CONTI..

(3) The y-intercept is 6.

Therefore $q = 6$.

$\therefore y = ax + 6$

Substitute $(-3 ; 0)$:

$\therefore 0 = a(-3) + 6$

$\therefore 0 = -3a + 6$

$\therefore 3a = 6$

$\therefore a = 2$

$\therefore y = 2x + 6$

DISCUSSION

QUADRATIC FUNCTIONS (SKETCHING PARABOLAS)

Consider the graph of $y = x^2$

We can select a few input values (x-values) and hence determine the corresponding output values (y-values). These values will be represented in a table.

x	-2	-1	0	1	2
y	4	1	0	1	4

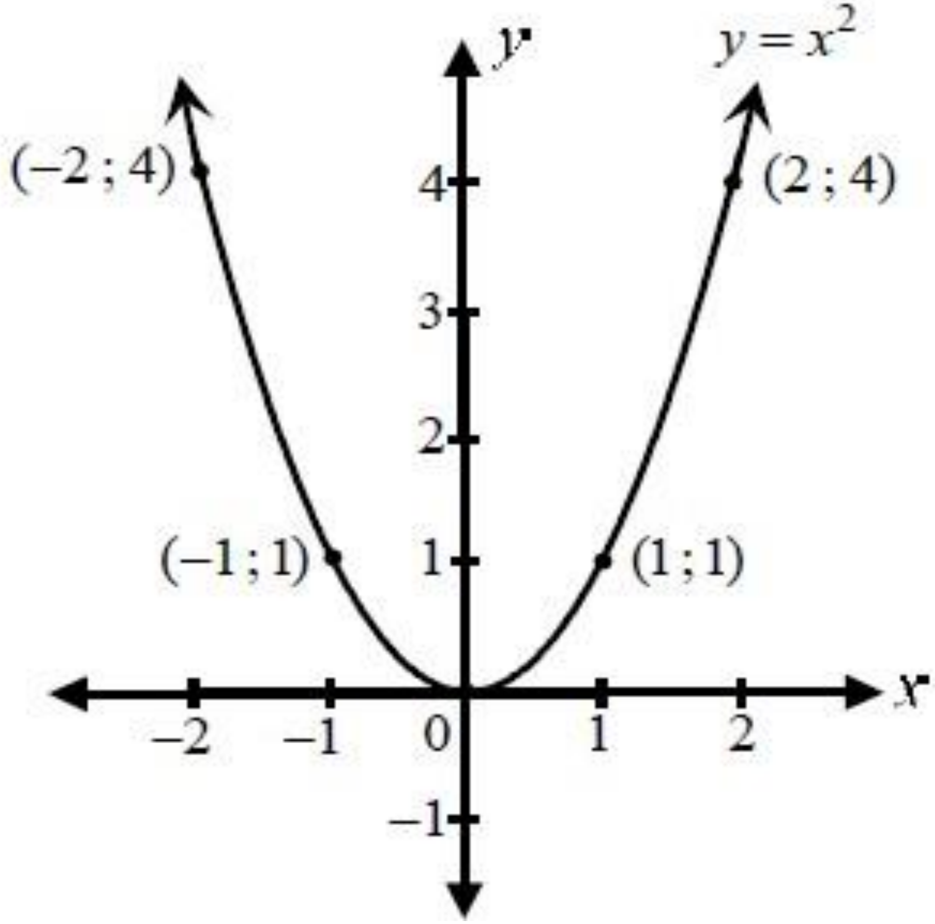
The graph of $y = x^2$ is obtained by plotting the points on the Cartesian plane and drawing a curve through the points.

SUMMARY

The graph of $y = x^2$ is obtained by plotting the points on the Cartesian plane and drawing a curve through the points.

Notice that:

- all output values are positive
- the graph is not linear but rather a curve referred to as the graph of a **parabola**



SUMMARY

This graph is referred to as the "mother" graph of parabolas and based on these graphs, we can generate different types of parabolas depending on the value of a and q in the general equation of a parabola, which is $y = ax^2 + q$.

DISCUSSION

Let's now consider what happens if the value of the coefficient of x is **negative**.

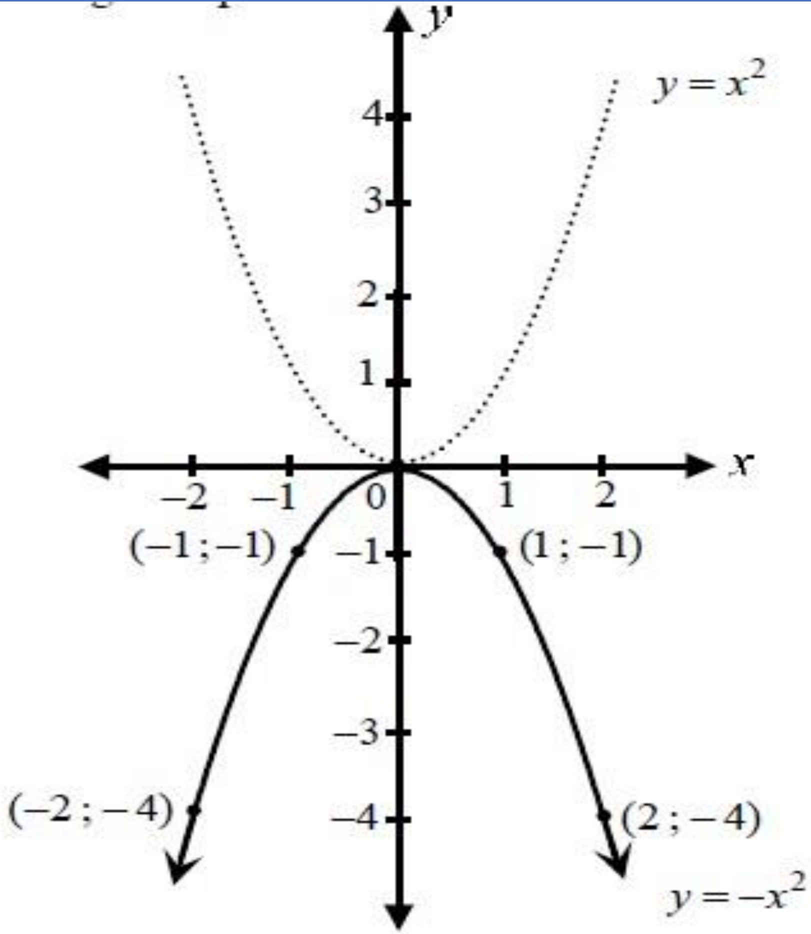
(a) Consider the graph of $y = -x^2$

We can select a few input values (x-values) and hence determine the corresponding output values (y-values). These values will be represented in a table.

x	-2	-1	0	1	2
y	-4	-1	0	-1	-4

The graph of $y = -x^2$ is obtained by plotting the points on the Cartesian plane and drawing a curve through the points.

SAMMARY



If you now compare the mother graph $y = 1x^2$ to the graph of $y = -1x^2$, it is interesting to note that the graph of $y = -x^2$ is the reflection of the mother graph in the x-axis.

The negative sign therefore causes a **reflection in the x-axis**.

Concluding Remarks

Following our today lesson, I want you to do the to:

Read through what the learner **need to understand and master** in your learner material.

Complete the activities

Attempt as many as possible other similar examples on your own from the **Text-Book and the past exam papers**.

Repeat this procedure until you are **confident**.

Do not forget: **Practice makes perfect!**

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Thank you