SUBJECT : MATHEMATICS
GRADE : 10

TOPIC: FINANCIAL MATHEMATICS
DATE : 06 & 08 AUGUST 2022

Putting things into perspective

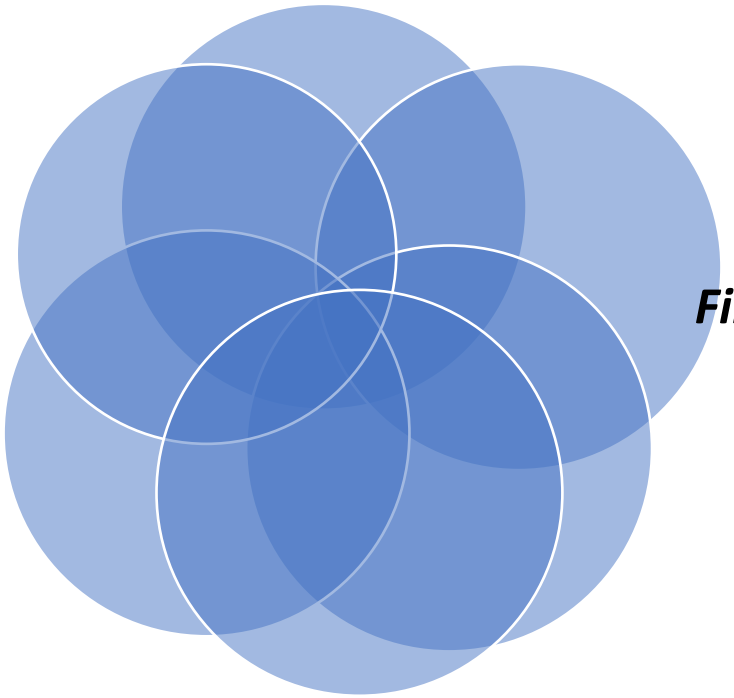
Focus :

FINANCIAL MATHEMATICS

Saturday: 04 OCTOBER 2021

Lesson Topic :

Finance and Growth (Face-to-Face)



Monday: 06 AUGUST 2022

Lesson Topic :

Finance and Growth (Face to Face)

Weighting of Financial Mathematics According to CAPS

Weighting of Content Areas			
Description	**Grade 10**	**Grade 11**	**Grade. 12**
PAPER 1 (Grades 12:bookwork: maximum 6 marks)			
Algebra and Equations (and inequalities)	30 ± 3	45 ± 3	25 ± 3
Patterns and Sequences	15 ± 3	25 ± 3	25 ± 3
Finance and Growth	10 ± 3		
Finance, growth and decay		15 ± 3	15 ± 3
Functions and Graphs	30 ± 3	45 ± 3	35 ± 3
Differential Calculus			35 ± 3
Probability	15 ± 3	20 ± 3	15 ± 3
TOTAL	**100**	**150**	**150**

SECTION 1

Grade 9 ***Finance and Growth (recovery)***

TERMINOLOGY

- **LOAN:** A sum of money that is borrowed, usually from a bank or other financial institution.
- **INVESTMENT:** Money that is saved by depositing it into a bank.
- **PRINCIPAL (P):** The initial value of a loan investment. Some books use P_v meaning "Present Value"
- **AMOUNT (A):** The value of the principal together with the interest earned over a period. This amount may also be referred to as the **maturity value** of the investment. Some books use F_v meaning "**Future Value**".
- **INTEREST (I)**: The amount of money earned on an investment. Interest on a loan charged by a bank is referred to a **finance charges**.

TERMINOLOGY CONTINUED

- **RATE OF INTEREST (i):** The rate at which interest is calculated. The rate is given a s a percent per annum. (p.a.)
- An interest rate of 6% p.a. is given as i = 0,06
- **TIME (n):** The period over which the money is invested or a loan is taken. The time is usually given in years, half-years, quarters, months or days.
- **LUMP SUM INVESTMENT:** A once off investment is usually called a lump sum investment.
- **ANNUITY**: Payments made at regular intervals or drawn at regular intervals.

What is Simple Interest?

- Simple interest is a fixed percentage of the amount invested or borrowed and is calculated on the original amount.

- When **simple interest** is applied to an investment, the value of the investment **increases** by an agreed **fixed percentage** at specific **regular time intervals**.

- Simple interest entails adding a constant amount to the principal amount at regular intervals.

FORMULAE FOR GROWTH AND DECAY (DECAY -11 &12 ONLY)

GROWTH

DECAY

SIMPLE INTEREST

$$A = P(1 + ni)$$

Simple interest / growth (Hire Purchase)

$$A = P(1 - ni)$$

Simple depreciation / straight line depreciation

COMPOUND INTEREST

$$A = P(1 + i)^n$$

Compound interest / growth. Also used for inflation.

$$A = P(1 - i)^n$$

Reducing balance depreciation

SIMPLE INTEREST Explanation

Assume that :

P is the amount borrowed or invested, called the **principal.**

r is the **interest rate** or growth rate.

n is the number of investment (growth) periods.

Simple Growth formula :

$$A = P + \frac{P \times n \times r}{100} = P\left(1 + \frac{n \times r}{100}\right) = P(1 + ni)$$

where $i = \frac{r}{100}$.

A is the **accrued** or final **amount.**

Interest earned :

$$I = P \bullet i \bullet n$$

IMPORTANT NOTES

In grade 10 you will only be asked to calculate **A, P, I and n**. But in compound interest you won't be asked to calculate **n**

> **Note**:
> Calculating the value of n in the compound interest formula is not part of the Grade 10 syllabus since this requires the use of logarithms which are studied in Grade 12.

TRIAL AND ERROR METHOD CAN BE USED FOR GRADE 11 BUT IT NOT EXAMINABLE

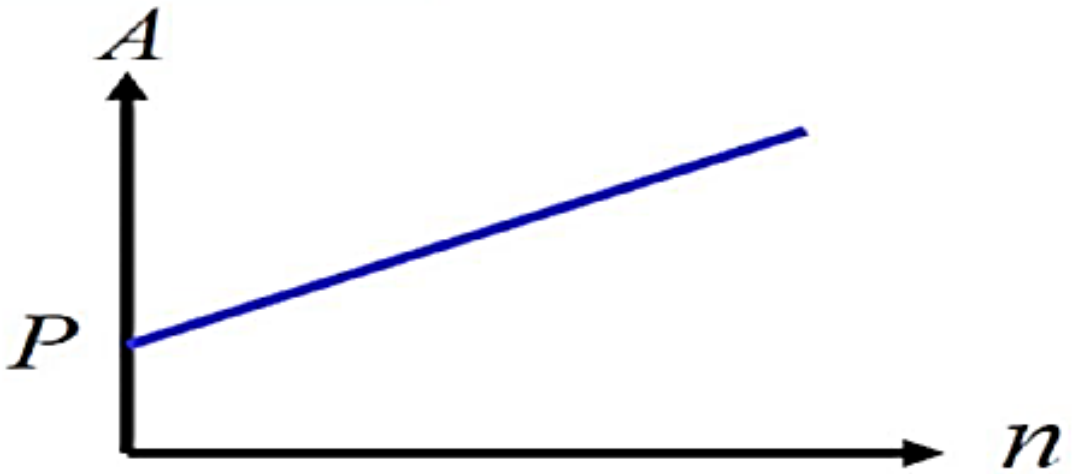
Explanation Continued

$$A = P(1 + ni)$$

$$\therefore A = P + (Pi)n$$

Linear relationship with:

1. A a function of n
2. $c = P$ and $m = Pi;\ m > 0$



Calculate Simple Interest Growth

EXAMPLE 1

Example 1: An amount of R1 050 is invested at a simple interest rate of $4\frac{3}{4}\%$ p.a. for a period of 9 months. Calculate:

(a) The simple interest (growth) earned on the investment;

(b) Final value of the investment (accrued value).

Discussion

Example 1- Solutions

EXAMPLE 1 SOLUTION

(a) Amount of growth (Interest earned):

$$I = P \times i \times n = 1\,050 \times 0,0475 \times 0,75 = R37,41$$

(b) Final value of investment:

$$A = P + I = R1\,050 + R37,41 = R1\,087,41$$

Alternatively, calculate directly by means of the formula $A = P(1+ni)$.

Know that:

$$P = 1\,050$$

$$r = 4\frac{3}{4}\% = 4,75\%$$

$$n = 9 \text{ months}$$

$$= \frac{9}{12} \text{ years}$$

$$= 0,75 \text{ years}$$

$$i = \frac{r}{100}$$

$$= \frac{4,75}{100}$$

$$= 0,0475$$

Discussion Area

ACTIVITY 1.1

1. A student borrows R15 000 for 3 years at simple interest to pay for his studies. If he has to pay back R20 625 at the end of the three years, what rate of (simple) interest per annum is he paying?

2. If I invest R15 000 in a fixed deposit account at 10% p.a. simple interest, how long will it take before I will have R30 000?

3.How much money should you invest in a fixed deposit account paying 9% p.a. simple interest if you would like to receive R8 000 at the end of 5 years?

Discussion

ACT 1.1 - SOLUTION

Discussion Area

1. A student borrows R15 000 for 3 years at simple interest to pay for his studies. If he has to pay back R20 625 at the end of the three years, what rate of (simple) interest per annum is he paying?

Possible Method:

- Use formula $A = P(1 + i \cdot n)$
- Substitute in given values
- Make i subject of formula

Given:

$P = 15\ 000$

$n = 3$

$A = 20\ 625$

$i = ?$

$$20\ 625 = 15\ 000(1 + i \times 3)$$

$$\therefore\ 3i + 1 = \frac{20\ 625}{15\ 000}$$

$$\therefore\ i = \frac{1}{3}\left(\frac{20\ 625}{15\ 000} - 1\right) = 0{,}125 = 12{,}5\%$$

ACT1.1 - SOLUTION

Discussion Area

2. If I invest R15 000 in a fixed deposit account at 10% p.a. simple interest, how long will it take before I will have R30 000?

$$n = \frac{30\ 000 - 15\ 000}{15\ 000 \times 0,1}$$

$$= 10 \text{ years}$$

$$A = P(1 + i \times n)$$

$$\Rightarrow A = P + P \times i \times n$$

$$\Rightarrow P \times i \times n = A - P$$

$$\Rightarrow n = \frac{A - P}{P \times i}$$

Recalculate A by means of $A = P(1 + i \times n)$ to check!

ACT1.1 - SOLUTION

3.How much money should you invest in a fixed deposit account paying 9% p.a. simple interest if you would like to receive R8 000 at the end of 5 years?

Discussion Area

$$A = P(1 + i \times n) \Rightarrow P = \frac{A}{1 + i \times n}$$

$$P = \frac{8\ 000}{1 + 0,09 \times 5} \approx \text{R}5\ 517,24$$

Given:

$A = 8\ 000$

$i = 0,09$

$n = 5$

$P = ?$

What is Compound Interest?

- Simple interest is calculated on the original amount borrowed or invested.
- When calculating **compound interest** the **interest charged or earned in each period is added to the principal.**
- This means that the principal increases (grows).
- **Interest earned** or charged for the next period **is calculated on the increased principal amount.**

COMPOUND INTEREST Explanation

Assume that :

P is the amount borrowed or invested, called the **principal**.

r is the **interest** rate or growth **rate**.

n is the **number** of investment or **growth periods**.

Compound Growth Formula :

$$A = P\left(1 + \frac{r}{100}\right)^n = P(1+i)^n \quad \text{where} \quad i = \frac{r}{100}$$

A is the **accrued** or final **amount**.

Explanation Continued

Discussion Area

$$A = P(1+i)^n$$

Exponential Growth relationship with:

(1) A a function of n

(2) Vertical intercept at P

(3) Base $(1+i)>1$

Compound Interest

EXAMPLE

If R5 000 is invested at 6% p.a. interest compounded annually for 4 years

$$A = P(1 + i)^n$$

$$A = 5\,000\,(1 + 0{,}06)^4$$

$$\therefore A = R6\,312{,}38$$

Activity 1.2

1. Lola invested R1 200 at 10% p.a. compound interest. How much will she receive in 5 years time?
2. Brett has R7 500 which he wants to invest for 5 years. Which savings plan will yield more interest: simple interest at 14% p.a. or compound interest at 12% p.a.?
3. How much was invested for an investment to yield R103 808 at 11% p.a. compound interest for 7 years (round your answer to the nearest rand)
4. R10 000 is deposited into a savings account at a bank. Calculate the compound interest rate (round to the nearest integer) that will produce a balance of R17 490 in the account after 4 years.
5. R1 000 is deposited in a savings account and two years later R2 000 is added to the savings. The interest rate is 11,5% p.a. compounded annually. Calculate the amount of money in the account at the end of 5 years.

Discussion

Activity 1.2 - Solutions

1. Lola invested R1 200 at 10% p.a. compound interest. How much will she receive in 5 years time?

Discussion Area

$$A = P(1 + i)^n$$

$$A = 1200\ (1 + 0{,}10)^5$$

$$\therefore A = R1\ 932{,}61$$

Activity 1.2 Solutions

2. Brett has R7 500 which he wants to invest for 5 years. Which savings plan will yield more interest: simple interest at 14% p.a. or compound interest at 12% p.a.?

Discussion Area

compound interest

$A = P(1 + i)^n$

$A = R7\,500\,(1 + 0{,}12)^5$

$\therefore A = R13\,217,56$

simple interest

$A = P(1 + i\,n)$

$A = R7\,500\,(1 + 0{,}12 \times 5)$

$\therefore A = R12\,000$

$\therefore$ Compound interest yields R1 217,56 more

Activity 1.2 Solutions

3. How much was invested for an investment to yield R103 808 at 11% p.a. compound interest for 7 years (round your answer to the nearest rand)

Discussion Area

$A = P(1 + i)^n$

$R103\ 808 = P\ (1 + 0{,}11)^7$

$\therefore P = \frac{103\ 808}{(1+0{,}11)^7}$

$\therefore P = 49\ 999,\ 99631775 \approx R50\ 000$

Activity 1.2 Solutions

4. R10 000 is deposited into a savings account at a bank. Calculate the compound interest rate (round to nearest integer) that will produce a balance of R17 490 in the account after 4 years.

Discussion Area

$A = P(1 + i)^n$

$R17\ 490 = R10\ 000\ (1 + i)^4$

$\therefore i = \sqrt[4]{\frac{17\ 490}{10\ 000}} - 1$

$\therefore\ i = 0{,}1499989 \times 100 = 15\%$

Activity 1.2 Solutions

5. R1 000 is deposited in a savings account and two years later R2 000 is added to the savings. The interest rate is 11,5% p.a. compounded annually. Calculate the amount of money in the account at the end of 5 years.

$A = P(1 + i)^n$

$A = R1\,000\,(1 + 0{,}115)^5 + 2000\,(1 + 0{,}115)^3$

$\therefore A = R1\,723,35 + R2\,772,39$

$A = R4\,495,74$

Discussion Area

HIRE PURCHASE

- A hire purchase agreement is a short term loan given to a buyer in order to buy items such as furniture and household appliances.
- The buyer signs an agreement with the seller to pay a certain amount per month for the article.
- **Simple interest** is calculated on the full value of the loan over the repayment period. The lending institution also adds on insurance as a cost. If the buyer stops paying his instalments, the seller can repossess the item.

HIRE PURCHASE (IN SIMPLE TERMS)

HIRE PURCHASE AGREEMENT : It is where by a customer buy equipment without cash and sign an agreement with owner of the equipment to pay on monthly bases and pay more money because there will be some interest to be added. In **HIRE PURCHASE** we use **SIMPLE INTEREST** formula to do calculations

- ✓ in hire purchase if there is no deposit paid, **P (principal amount)** will be equal to the cash price(take cash price as it is)…
  Total interest paid = final amount paid – initial amount

- ✓ in hire purchase if there is deposit paid, **P (principal amount)** will be equal to the cash price – deposit (deposit it is there to reduce amount of the loan)…
  Total paid = final amount paid(A) + deposit

EXAMPLE

John wants to purchase a refrigerator and a stove for R11 500. The items are bought on hire purchase involving equal monthly payments over 36 months at 14% p.a. Monthly insurance of R24,75 is added. Calculate what John's monthly payments will be.

PLEASE NOTE

36 months = 3 years

Purchase price = R11 500 + 14% simple interest.

$A = P(1 + i\,n)$

$A = 11\,500\,(1 + (0{,}14)(3))$

$A = R16\,330.$

Monthly payments = R16 330/36+ R24,75 = R478,36

Activity 1.3

1. Sizwe purchases household goods for R6 200. He pays 15% deposit and pays the balance over 36 months at an interest rate of 11,5% p.a. Monthly insurance of R13,60 is added. Calculate the monthly payments.

2. Terence purchased a stove for R4 300. 10% deposit is paid and the balance is paid over 24 months at 13,2% p.a. Total insurance over 24 months is R912,00. Calculate his monthly installment.

3. A newspaper advert read, “Buy a colour laser printer for R290,00 per month. 4 years to pay.” Calculate the cash price assuming that the advertisement is based on a hire purchase agreement with an interest rate of 15% p.a.

Discussion

Activity 1.3 Solutions

1. Sizwe purchases household goods for R6 200. He pays 15% deposit and pays the balance over 36 months at an interest rate of 11,5% p.a. Monthly insurance of R13,60 is added. Calculate the monthly payments.

Loan Amount = Cost – Deposit = R6 200 – (6 200 × 0,15) = R5 270

PLEASE NOTE

36 months = 3 years

Purchase price = R5 270 + 11,5% simple interest.

$$A = P(1 + i\,n)$$

$$A = 5\,270\,(1 + (0{,}115)(3))$$

$$A = R7\,088{,}15.$$

Monthly payments = R7088,15/36+ R13,60 = R210,49

Activity 1.3 Solutions

2. Terence purchased a stove for R4 300. 10% deposit is paid and the balance is paid over 24 months at 13,2% p.a. Total insurance over 24 months is R912,00. Calculate his monthly installment.

Loan Amount = Cost – Deposit = R4 300 – (4 300 × 0,10) = R3 870

PLEASE NOTE

24 months = 2 years

Purchase price = R3 870 + 13,2% simple interest.

$$A = P(1 + i\,n)$$

$$A = 3\,870\,(1 + (0{,}132)(2))$$

$$A = R4\,891{,}68.$$

Monthly payments = R4891,68/24 + R912,00/24

= R203,82 + R3,83 = R207,65

Activity 1,3 Solutions

3. A newspaper advert read, “Buy a colour laser printer for R290,00 per month. 4 years to pay.” Calculate the cash price assuming that the advertisement is based on a hire purchase agreement with an interest rate of 15% p.a.

Total paid = R290,00 × 48 = R13 920

PLEASE NOTE

4 years = 48 months

Purchase price:

$$A = P(1 + i\,n)$$

$$R13\,920 = P\,(1 + (0{,}15)(4))$$

$$P = R13\,920/\,(1+ (0{,}15)(4))$$

$$P = R\,8\,700$$

INFLATION

- Inflation is a continuous increase in the cost of goods and services from one year to the next.

- We use **compound interest** formulae to calculate the expected increase in the cost of goods and services over a period of time.

INFLATION

EXAMPLE

Assume that the current rate of inflation is 9,6% p.a. The price of a litre of milk is R5,50. What will the price of a litre of milk be in 2025 if the rate of inflation remains at 9,6% (the current year is 2006)

$$A = P(1 + i)^n$$

$$A = R5,50\ (1 + 0,096)^{19}$$

$$A = R31,39$$

Activity 1.4

1. A 150g packet of chips was priced at R2,20 in July 2005.

1.1 Calculate the price in July 2025 if the rate of inflation is 12,2% p.a.

1.2 Calculate the price in July 2055 if the rate of inflation remains the same.

2. Assume that the rate of inflation is 7,8%.

2.1 Calculate the expected price of a 2,5 kg packet of sugar in 10 years time if the present price is R12,99

2.2 How much did the same packet of sugar cost 10 years ago?

3. A block of yellow margarine is R8,70. Calculate the price in 6 years time at an inflation rate of 9,2% p.a.

Discussion

Activity 1.4 Solutions

1.1. $A = P(1 + i)^n$

$A = R2,20\ (1 + 0,122)^{20}$

$A= R21,99$

1.2. $A = P(1 + i)^n$

$A = R21,99\ (1 + 0,122)^{30}$

$A= R695,04$

3. $A = P(1 + i)^n$

$A = R8,70\ (1 + 0,092)^{6}$

$A= R14,75$

2.1. $A = P(1 + i)^n$

$A = R12,99\ (1 + 0,078)^{10}$

$A= R27,53$

2.2. $A = P(1 - i)^n$

$A = R12,99\ (1 - 0,078)^{10}$

$A= R5, 77$

Discussion Area

EXCHANGE RATES - Currencies

- Currency is the term used to describe the money system of a country. Different countries have different currencies. E.g. Current exchange rates in 2021

Country	Currency	Symbol	Rand rate
South Africa	Rand	R	R1
United States	U.S. Dollar	$	R14, 73
United Kingdom	British pound	£	R20,27
Botswana	Botswana pula	P	R1,32
Europe	Euro	€	R17,38
Nigeria	Nigerian naira	₦	R0,036
China	Chinese yuan	¥	R2,28
Congo	Congolese franc	Fr	R0,0074
United Arab Emirates	Dirham	د.إ	R4,01

EXCHANGE RATES - Currencies

- When visiting countries abroad, you cannot use your South African Rand notes and coins to pay for goods and services.
- The changing of money from one currency to another is done by a foreign exchange agency called a **Bureau de Change** found at some branches of commercial banks, travel agents and airports.

CALCULATING FOREIGN EXCHANGE

EXAMPLE

Assume that the current rand / dollar exchange rate is R6,23 to the dollar. How much will it cost to buy a book that costs $50,00 (ignore commission and other costs)

$1 = R6,23

Cost of book = 50 × R6,23

= R311,50

Activity 1.5

1. A clothing store earned $45 000 from exports to the United States in November 2002 when the exchange rate was R10,23. In December the exchange rate dropped to R8,73 to the dollar. Find the loss made from November to December.

2. Debbie travelled to London and spent £1 200, then travelled to Paris where she spent €8 000 and then to Tokyo where she spent 40 000 Japanese yen. Use the exchange rates given to calculate the total amount spent in the three cities.

Exchange Rates

United Kingdom (£)	Euro (€)	Japan (yen)
R13,67	R8,81	R0,0707

3. A Zimbabwean sells shoes to a South African wholesaler to the value of R52 000. Calculate how many Zimbabwean dollars this is if the exchange rate is R0,12 to a Zimbabwean Dollar.

Discussion

Activity 1.5 Solutions

Discussion Area

1. November 2002:

   Rand Value = 45000 x 10,23 = R460 350

December 2002:

   Rand Value = 45000 x 8,73 = R392 850
   Loss = R460 350 - R392 850 = R67 500

3. Total spent = 1200 x 13,63 + 8000 x 8,81 + 40 000 x 0,0707
   = R16 356 + R70 480 + R2 828
   = R89 664

3. Value = 52 000 / 0,12 = Zim $433 333, 33

EXAM TYPE QUESTIONS

QUESTION 4

4.1 Thando has R4 500 in his savings account. The bank pays him a compound interest rate of 4,25% p.a. Calculate the amount Thando will receive if he decides to withdraw the money after 30 months. (3)

4.2 The following advertisement appeared with regard to buying a bicycle on a hire-purchase agreement loan:

Purchase price	*R5 999*
Required deposit	*R600*
Loan term	*Only 18 months, at 8% p.a. simple interest*

4.2.1 Calculate the monthly amount that a person has to budget for in order to pay for the bicycle. (6)

4.2.2 How much interest does one have to pay over the full term of the loan? (1)

4.3 The following information is given:

1 ounce = 28,35 g
$1 = R8,79

Calculate the rand value of a 1 kg gold bar, if 1 ounce of gold is worth $978, 34. (4)

[14]

Discussion

SOLUTIONS

QUESTION 4

4.1	$A = P(1+i)^n$ $= 4500\left(1+\frac{4.25}{100}\right)^{2.5}$ $= R\ 4993.47$	✓ $n = 2.5$ ✓ substitution ✓ answer (3)
4.2.1	Loan amount = R5 999 – R600 = R5 399 Total amount owed = 5 399[1+(0,08)(1,5)] = R6 046,88 Monthly instalment $= \frac{6046.88}{18}$ = R335,94	✓ $y = 0$ ✓ 5 399 ✓ $n = 1,5$ ✓ Substitution ✓ R6 046,88 ✓ ÷ 18 ✓ R335,94 (6)
4.2.2	R6 046,88 - R5 399 = R647,88	✓ answer (1)
4.3	1 kg = 1 000 g $\frac{1000}{28,35} = 35,27336861\ldots$ ounces 35,27336861… x 978,34 x 8,79 = R303 337,16	✓ conversion ✓ division ✓ multiplication ✓ answer (4) **[14]**

(NOV. 2019)

QUESTION 4

4.1 In June 2019, the pound to rand exchange rate was £1 = R18,18. Zola, travelled to the United Kingdom to watch some WWE wrestling matches. The total cost needed for the trip was £3 569. Convert this amount into rands. (1)

4.2 Sipho bought a brand-new Ford Ranger in April 2015 on hire purchase at a cost of R379 000. He agreed on paying 15% deposit and took out a loan for the remaining balance at an interest rate of 22,5%.

4.2.1 How much deposit did Sipho pay? (1)

4.2.2 Hence, calculate the initial value of the loan. (1)

4.2.3 Calculate the value of the loan with interest in April 2019. (3)

4.2.4 Calculate the monthly instalments if he paid off the loan after the four-year period. (2)

4.3 A sum of money was invested 6 years ago, earning interest at a rate of 6,7% p.a. compounded annually. The investment is currently worth R 96 714,02. Calculate how much was originally invested 6 years ago. (3)

[11]

Discussion

SOLUTIONS

QUESTION 4/*VRAAG 4*

4.1	Amount/*Bedrag* = 18, 18 x 3569 = R64 884, 42		✓R64 884, 42	(1)
4.2	4.2.1	Loan/*Lening* = 0, 85 x 379 000 = R322 150 **OR/OF** Loan/*Lening* = 379 000 – 0,15 x 379 000 = R322 150	✓0, 85 x 379 000 ✓R322 150 **OR/OF** ✓379 000 – 0,15 x 379 000 ✓R322 150	(2) (2)
	4.2.2	$A = P(1+in)$ $A = 322150(1+0{,}225\times 4)$ $A = R612\ 085$	✓ $A = P(1+in)$ ✓ $A = 322150(1+0{,}225\times 4)$ ✓ $A = R612\ 085$	(3)
	4.2.3	Instalment/*Paaiement* $= \frac{612085}{48} = R12751{,}77$	✓ 48 ✓ $R12751{,}77$	(2)
4.3		$A = P(1+i)^n$ $96714{,}02 = P(1+0{,}067)^6$ $P = \frac{96714{,}02}{(1{,}067)^6}$ $P = R65539{,}47$	✓ $A = P(1+i)^n$ ✓ $96714{,}02 = P(1+0{,}067)^6$ ✓ $P = R65539{,}47$	(3)
				[11]

Concluding Remarks

Following our today lesson, I want you to :

- Read through what the learner **need to understand and master** in your learner material.
- **Complete** the activities
- Attempt as many as possible other similar examples on your own from the **Text-Book and the past exam papers**.

Repeat this procedure until you are **confident**.

- Do not forget: **Practice makes perfect!**

Thank you