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DIP : Assignment 1
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Q1. Bi-quadratic interpolation

$$v(x,y) = \sum_{i=0}^{2} \sum_{j=0}^{2} a_{ij} x^i y^j$$

$$\Rightarrow v(x,y) = a_{00} + a_{01} y + a_{02} y^2 + a_{10} x + a_{11} xy + a_{12} xy^2 + a_{20} x^2 + a_{21} x^2 y + a_{22} x^2 y^2$$

We need nine equations here.

$$v_1 = a_{00} + a_{01} y_1 + \dots + a_{11} x_1 y_1 + \dots + a_{22} x_1^2 y_1^2$$

$$v_2 = a_{00} + a_{01} y_2 + \dots + a_{11} x_2 y_2 + \dots + a_{22} x_2^2 y_2^2$$

$$\vdots$$

$$v_9 = a_{00} + a_{01} y_9 + \dots + a_{11} x_9 y_9 + \dots + a_{22} x_9^2 y_9^2$$

$$\Rightarrow \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ \vdots \\ v_9 \end{bmatrix} = \begin{bmatrix} 1 & y_1 & y_1^2 & x_1 & x_1 y_1 & x_1 y_1^2 & x_1^2 & x_1^2 y_1 & x_1^2 y_1^2 \\ 1 & y_2 & y_2^2 & x_2 & x_2 y_2 & x_2 y_2^2 & x_2^2 & x_2^2 y_2 & x_2^2 y_2^2 \\ 1 & y_3 & y_3^2 & x_3 & x_3 y_3 & x_3 y_3^2 & x_3^2 & x_3^2 y_3 & x_3^2 y_3^2 \\ & & & & \vdots & & & & \\ 1 & y_9 & y_9^2 & x_9 & x_9 y_9 & x_9 y_9^2 & x_9^2 & x_9^2 y_9 & x_9^2 y_9^2 \end{bmatrix} \begin{bmatrix} a_{00} \\ a_{01} \\ a_{02} \\ \vdots \\ a_{22} \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \\ v_9 \end{bmatrix}}_{V} = \underbrace{\begin{bmatrix} 1 & y_1 & y_1^2 & x_1 & x_1y_1 & x_1y_1^2 & x_1^2 & x_1^2y_1 & x_1^2y_1^2 \\ 1 & y_2 & y_2^2 & x_2 & x_2y_2 & x_2y_2^2 & x_2^2 & x_2^2y_2 & x_2^2y_2^2 \\ 1 & y_3 & y_3^2 & x_3 & x_3y_3 & x_3y_3^2 & x_3^2 & x_3^2y_3 & x_3^2y_3^2 \\ 1 & y_4 & y_4^2 & x_4 & x_4y_4 & x_4y_4^2 & x_4^2 & x_4^2y_4 & x_4^2y_4^2 \\ 1 & y_5 & y_5^2 & x_5 & x_5y_5 & x_5y_5^2 & x_5^2 & x_5^2y_5 & x_5^2y_5^2 \\ 1 & y_6 & y_6^2 & x_6 & x_6y_6 & x_6y_6^2 & x_6^2 & x_6^2y_6 & x_6^2y_6^2 \\ 1 & y_7 & y_7^2 & x_7 & x_7y_7 & x_7y_7^2 & x_7^2 & x_7^2y_7 & x_7^2y_7^2 \\ 1 & y_8 & y_8^2 & x_8 & x_8y_8 & x_8y_8^2 & x_8^2 & x_8^2y_8 & x_8^2y_8^2 \\ 1 & y_9 & y_9^2 & x_9 & x_9y_9 & x_9y_9^2 & x_9^2 & x_9^2y_9 & x_9^2y_9^2 \end{bmatrix}}_{X} \underbrace{\begin{bmatrix} a_{00} \\ a_{01} \\ a_{02} \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{20} \\ a_{21} \\ a_{22} \end{bmatrix}}_{A}$$

$$\Rightarrow \boxed{V = XA}$$

Q2. Given 2x2 image

$$\begin{bmatrix} 5 & 10 \\ 10 & 20 \end{bmatrix}$$

The dimension of the output image is 3×3.
(0,0) in the input matrix will map to (0,0) in the output matrix.
(0,1) in output matrix will map to (0, 2/3) in input matrix.
(1,0) in output " " " " (2/3, 0) in input matrix.

(1,1) in output is mapped to (2/3, 2/3) in input.

Now, four nearest neighbours will be

$$(x_1, y_1) = \left(\text{round}\left(\frac{2}{3}\right), \text{round}\left(\frac{2}{3}\right)\right) = (0,0)$$

$$(x_2, y_2) = \left(\text{round}\left(\frac{1+1.5}{1.5}\right), \text{round}\left(\frac{2}{3}\right)\right) = (1,0)$$

$$(x_3, y_3) = \left(\text{round}\left(\frac{2}{3}\right), \text{round}\left(\frac{5}{3}\right)\right) = (0,1)$$

$$(x_4, y_4) = \left(\text{round}\left(\frac{5}{3}\right), \text{round}\left(\frac{5}{3}\right)\right) = (1,1)$$

The four equations become

(i) $5 = a(0) + b(0) + c(0)(0) + d \Rightarrow d = 5$
(ii) $10 = a(0) + b(1) + c(0)(1) + d \Rightarrow b = 5$
(iii) $10 = a(1) + b(0) + c(1)(0) + d \Rightarrow a = 5$
(iv) $20 = a(1) + b(1) + c(1)(1) + d \Rightarrow c = 5$

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Now, $V(\text{at } (2/3, 2/3))$

$$= a\left(\frac{2}{3}\right) + b\left(\frac{2}{3}\right) + c\left(\frac{2}{3}\right)\left(\frac{2}{3}\right) + d$$

$$= \frac{10}{3} + \frac{10}{3} + \frac{20}{9} + 5$$

$$= \frac{30 + 30 + 20 + 45}{9}$$

$$= \frac{125}{9} \approx 13.89$$

Ans. Input $\leftrightarrow$ Output

Input	Output
(0,0)	(0,0)
(0,2/3)	(0,1)
(2/3,0)	(1,0)

Interpolated pixel value at (1,1) = 13.89