CBSE MATHS PAPER

KESAVA

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Questions

1 Matrices

1. If A is a square matrix of order 3 with $\left|A\right|$=4,then write the value of $\left|-2A\right|$.

2. A = $\begin{bmatrix} 3 & 9 & 0 \\ 1 & 8 & -2 \\ 7 & 5 & 4 \end{bmatrix}$ and B= $\begin{bmatrix} 4 & 0 & 2 \\ 7 & 1 & 4 \\ 2 & 2 & 6 \end{bmatrix}$, then find the matrix B'A'.

3. Using properties of determinants,prove that $\begin{bmatrix} b+c & a & c \\ b & c+a & b \\ c & c & a+b \end{bmatrix}$=$4abc$.

4. If A = $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix}$,find A^{-1}. Hence, solve the system of equations:
$x+y+z=6$,
$y+3z=11$,
$x-2y+z=0$.

5. Find the inverse of the following matrix, using elementary transformations: A=$\begin{bmatrix} 2 & 3 & 1 \\ 2 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$.

2 Integrations

6. $\int_a^b \frac{logx}{x}dx$.
7. $\int \frac{sin(x-a)}{sin(x+a)}dx$.
8. $\int (logx)^2$.
9. Find the value of x,if tan[sec$^{-1}(\frac{1}{x})$] =sin$(tan^{-1}2)$,$x > 0$.
10. $\int \frac{sin(2x)}{(sin^2x+1)(sin^2x+3)}dx$.
11. Prove that $\int_a^b f(x)dx$=$\int_a^b f(a+b-x)dx$ and hence evaluate.
12. $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\tan x}}$.
13. Find $\int_4^1 (1+x+e^{2x})$ evaluate aslimits as sum.
14. Find the area of the region $(x,y) : 0 \leq y \leq x^2, 0 \leq y \leq x+2, –1 \leq x \leq 3$.

3 Differentiations

15. Find the Integral factor of the differential equation: $x\frac{dy}{dx} - 2y$=$2x^2$.
16. If y=$\sin^{-1} x + \cos^{-1} x$,find $\frac{dy}{dx}$.
17. If $e^y(x+1)$=1, Then shows that $\frac{d^2y}{dx^2}$=$(\frac{dy}{dx})^2$.
18. Find $\frac{dy}{dx}$,if y=$\sin^{-1}[\frac{2^{x+1}}{1+4^x}]$.
19. If y=$(\sec^{-1} x)^2$,$x > 0$,show that $x^2(x^2-1)\frac{d^2y}{dx^2} + (2x^3 - x)\frac{dy}{dx} - 2$= 0.
20. Solve the differential equation: $\frac{dy}{dx}$= $\frac{x+y}{x-y}$.
21. Solve the differential equation: $(1+x^2)dy + 2xydy$=$\cot x dx$.
22. Form the differential equation representing the family of curves y^2= $m(a^2–x^2)$ by eliminating the arbitrary constants ‘m’ and ‘a’.

4 Geometry

23. Find the values of p for which the following lines are perpendicular $\frac{1-x}{3} = \frac{2y-14}{2p} = \frac{z-3}{2}; \frac{1-x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$.

24. Find the direction cosines of the line joining the points P$(4, 3, -5)$ and Q$(-2, 1, -8)$.

25. Find the value of λ for which the following lines are per pendicular to each other: $\frac{x-5}{5\lambda+2} = \frac{2-y}{5} = \frac{1-z}{-1}; \frac{x}{1} = \frac{y+\frac{1}{2}}{2\lambda} = \frac{z-1}{3}$. Hence,find whether the lines insert or not.

26. Show that the height of a cylinder, which is open at the top, ha ving a given surface area and greatest volume, is equal to the radius of its base.

5 Vectors

27. Find a unit vector perpendicular to both vectors $\overrightarrow{a}, \overrightarrow{b}$,where $\mathbf{a}$=$\hat{i} - 7\hat{j} + 7\hat{k}$ and $\mathbf{b}$=$3\hat{i} - 2\hat{j} + 2\hat{k}$.

28. Shows that the vector $\hat{i} - 2\hat{j} + 3\hat{k}$ and $\hat{i} - 3\hat{j} + 5\hat{k}$ are coplanar.

29. Let $\overrightarrow{a}, \overrightarrow{b}$ and $\overrightarrow{c}$ be three vectors such that $\left|\overrightarrow{a}\right| = 1, \left|\overrightarrow{b}\right| = 2$ and $\overrightarrow{|c|} = 3$ .If the projection of $\overrightarrow{b}$ along $\overrightarrow{a}$ is equal to the projection of $\overrightarrow{c}$ along $\overrightarrow{a}$; and $\overrightarrow{b}, \overrightarrow{c}$ are perpendicular to each other, then find $|3\overrightarrow{a} - 2\overrightarrow{b} + 2\overrightarrow{c}|$.

30. Find the vector and Cartesian equations of the plane passing through the points $(2, 5, -3), (-2, -3, 5)$, and $(5, 3, -3)$.Also, find the point of intersection of this plane with the line passing through points $(3, 1, 5)$ and $(-1, -3, -1)$.

31. Find the equation of the plane passing through the intersection of the planes $\overrightarrow{r}.\left(\hat{i} + \hat{j} + \hat{k}\right) = 1$ and $\overrightarrow{r}.2\hat{i} + 3\hat{j} - \hat{k} + 4 = 0$ and parallel to $x - axis$. Hence, find the distance of the plane from $x - axis$.

6 Probability

32. Let X be a random variable which assumes values x1,x2,x3,x4 such that $2P(X = x1) = 3p(X = x2) = p(X = x3) = 5P(X = x4)$. Find the probability distribution of X.

7 Functions

33. If $*$ is defined on the set of all real numbers by$*$:$a * b = \sqrt{a^2 + b^2}$,find the identity element,if it exists in with respect to $*$.

34. Find the intervals in which the function f given by $f(x)$=$4x^3$–$6x^2$–$72x+$ 30 is (a) strictly increasing, (b) strictly decreasing.

35. Show that the relation R on the set Z of integers,given by R=$\{$(a, b):2 *divides* $(a–b)\}$. is an equivalence relation.

36. If f(x)=$\frac{4x+3}{6x-4}$, $x \neq \frac{2}{3}$,show that fof(x)=x for all $x \neq \frac{2}{3}$. Also, find the inverse of f.

37. Mother,father and son line up at random for a family phot. If A and B are two events given by A=Son on one end,B=Father in the middle,find P($\frac{b}{a}$).

38. Find the intervals in which the function f given by $f(x)$ =$4x^3$–$6x^2$–$72x+$ 30 is (a) strictly increasing, (b) strictly decreasing.

8 Optimization

39. A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type A require 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B require 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours and 20 minutes available for cutting and 4 hours available for assembling. The profit is ₹ 50 each for type A and ₹ 60 each for type B

souvenirs. How many souvenirs of each type should the company manufacture in order to maximize profit ? Formulate the above LPP and solve it graphically and also find the maximum profit.