

Q: A factory produces bulbs. The probability that any one bulb is defective is $\frac{1}{50}$ and they are packed in boxes of 10. From a single box, find the probability that

- 1) none of the bulb is defective
- 2) exactly two bulbs are defective
- 3) more than 8 bulbs are working properly

Solution: :

Parameter	Values	Description
n	10	Number of boxes
p	$\frac{1}{50}$	Probability of being defective
X	1 if defective 0 if not defective	Bernoulli Random Variable
Y	$\sum_{i=1}^n X_i$	Binomial Random Variable

TABLE 3

TABLE 1

$$p_Y(k) = \Pr(Y = k) \quad (1)$$

$$= {}^nC_k p^k (1 - p)^{n-k}, (1 \leq k \leq n) \quad (2)$$

- 1) none of the bulb is defective

We require $\Pr(Y = 0)$. Since $n = 10$,

$$\Pr(Y = 0) = p_Y(0) \quad (3)$$

$$= {}^nC_k p^k (1 - p)^{n-k} \quad (4)$$

$$= \left(\frac{49}{50}\right)^{10} \quad (5)$$

- 2) exactly two bulbs are defective

We require $\Pr(Y = 2)$. Since $n = 10$,

$$\Pr(Y = 2) = p_Y(2) \quad (6)$$

$$= {}^nC_k p^k (1 - p)^{n-k} \quad (7)$$

$$= 45 \left(\frac{1}{50}\right)^2 \left(\frac{49}{50}\right)^8 \quad (8)$$

- 3) more than 8 bulbs are working properly

$$\Rightarrow \text{Atmost 1 is defective} \quad (9)$$

$$F_Y(k) = \Pr Y \leq k \quad (10)$$

$$= \sum_{i=0}^k {}^nC_i p^i (1 - p)^{n-i} \quad 0 \leq k \leq n \quad (11)$$

$$(12)$$

We require $\Pr(Y \leq 1)$. Since $n = 10$,

$$\Pr Y \leq 1 = \sum_{i=0}^1 {}^nC_i p^i (1-p)^{n-i} \quad (13)$$

$$= {}^{10}C_0 \left(\frac{1}{50}\right)^0 \left(\frac{49}{50}\right)^{10} + {}^{10}C_1 \left(\frac{1}{50}\right)^1 \left(\frac{49}{50}\right)^9 \quad (14)$$

$$= \frac{59 \cdot 49^9}{50^{10}} \quad (15)$$