Q: A factory produces bulbs. The probability that any one bulb is defective is $\frac{1}{50}$ and they are packed in boxes of 10. From a single box, find the probability that

1) none of the bulb is defective
2) exactly two bulbs are defective
3) more than 8 bulbs are working properly

Solution: :

Parameter	Values	Description
n	10	Number of boxes
p	$\frac{1}{50}$	Probability of being defective
X	1 if defective 0 if not defective	Bernoulli Random Variable
Y	$\sum_{i=1}^{n} X_i$	Binomial Random Variable

TABLE 3
TABLE 1

$$p_Y(k) = \Pr(Y = k) \tag{1}$$
$$= {}^nC_k p^k (1-p)^{n-k}, (1 \le k \le n) \tag{2}$$

1) none of the bulb is defective
We require $\Pr(Y = 0)$.

$$\Pr(Y = 0) = p_Y(0) \tag{3}$$
$$= {}^nC_k p^k (1-p)^{n-k} \tag{4}$$
$$= \left(\frac{49}{50}\right)^{10} \tag{5}$$

2) exactly two bulbs are defective
We require $\Pr(Y = 2)$.

$$\Pr(Y = 2) = p_Y(2) \tag{6}$$
$$= {}^nC_k p^k (1-p)^{n-k} \tag{7}$$
$$= 45\left(\frac{1}{50}\right)^2\left(\frac{49}{50}\right)^8 \tag{8}$$

3) more than 8 bulbs are working properly

$$\implies \text{Atmost 1 is defective} \tag{9}$$

Since,

$$F_Y(k) = \Pr(Y \le k) \tag{10}$$
$$= \sum_{i=0}^{k} {}^nC_i p^i (1-p)^{n-i} \qquad 0 \le k \le n \tag{11}$$
$$\tag{12}$$

We require $\Pr(Y \leq 1)$.

$$\Pr Y \leq 1 = \sum_{i=0}^{1} {}^{n}C_{i} p^{i} \left(1 - p\right)^{n-i} \tag{13}$$

$$= {}^{10}C_{0} \left(\frac{1}{50}\right)^{0} \left(\frac{49}{50}\right)^{10} + {}^{10}C_{1} \left(\frac{1}{50}\right)^{1} \left(\frac{49}{50}\right)^{9} \tag{14}$$

$$= \frac{59 \cdot 49^{9}}{50^{10}} \tag{15}$$