

Q: An item is manufactured by three machines A, B and C. Out of the total number of items manufactured during a specified period, 50% are manufactured on A, 30% on B and 20% on C, 2% of the items produced on A and 2% of items produced on B are defective, and 3% of these products produced on C are defective. All the items are stored at one godown. One item is drawn at random and is found to be defective. What is the probability that it was manufactured on machine A?

Solution:

Parameter	Values	Description
X	0	not defective
	1	defective
Y	1	manufactured on A
	2	manufactured on B
	3	manufactured on C

TABLE 0

TABLE 1

Given,

$$\Pr(Y = 1) = \frac{50}{100} \quad (1)$$

$$= 0.5 \quad (2)$$

$$\Pr(Y = 2) = \frac{30}{100} \quad (3)$$

$$= 0.3 \quad (4)$$

$$\Pr(Y = 3) = \frac{20}{100} \quad (5)$$

$$= 0.2 \quad (6)$$

$$\Pr(X = 1|Y = 1) = \frac{2}{100} \quad (7)$$

$$= 0.02 \quad (8)$$

$$\Pr(X = 1|Y = 2) = \frac{2}{100} \quad (9)$$

$$= 0.02 \quad (10)$$

$$\Pr(X = 1|Y = 3) = \frac{3}{100} \quad (11)$$

$$= 0.03 \quad (12)$$

We need to find $\Pr(Y = 1|X = 1)$,

$$\Pr(Y = 1|X = 1) = \frac{\Pr(Y = 1) \cdot \Pr(X = 1|Y = 1)}{\Pr(Y = 1) \cdot \Pr(X = 1|Y = 1) + \Pr(Y = 2) \cdot \Pr(X = 1|Y = 2) + \Pr(Y = 3) \cdot \Pr(X = 1|Y = 3)} \quad (13)$$

$$= \frac{0.5 \cdot 0.02}{0.5 \cdot 0.02 + 0.3 \cdot 0.02 + 0.2 \cdot 0.03} \quad (14)$$

$$= \frac{5}{11} \quad (15)$$

$$(16)$$