

Q: The probability that a person is not a swimmer is 0.3. The probability that out of 5 persons 4 are swimmers is

**Solution:** Let  $Y$  be the gaussian random variable,

Parameter	Values	Description
$n$	5	Number of draws
$p$	0.3	Probability that person is not a swimmer
$q$	0.7	Probability that person is a swimmer
$\mu = np$	3.5	Mean
$\sigma^2 = npq$	1.05	Variance

$$Y \sim \mathcal{N}(\mu, \sigma^2) \quad (1)$$

$$\sim \mathcal{N}(3.5, 1.05) \quad (2)$$

Due to continuity correction  $\Pr(X = x)$  can be approximated using gaussian distribution as

$$p_Y(x) \approx \Pr(x - 0.5 < Y < x + 0.5) \quad (3)$$

$$\approx \Pr(Y < x + 0.5) - \Pr(Y < x - 0.5) \quad (4)$$

$$\approx F_Y(x + 0.5) - F_Y(x - 0.5) \quad (5)$$

$$(6)$$

then CDF of  $Y$  is:

$$F_Y(x) = \Pr(Y < x) \quad (7)$$

$$= \Pr\left(\frac{Y - \mu}{\sigma} < \frac{x - \mu}{\sigma}\right) \quad (8)$$

$$\Rightarrow \frac{Y - \mu}{\sigma} \sim N(0, 1) \quad (9)$$

$$= 1 - \Pr\left(\frac{Y - \mu}{\sigma} > \frac{x - \mu}{\sigma}\right) \quad (10)$$

$$= \begin{cases} 1 - Q\left(\frac{x - \mu}{\sigma}\right) & x \geq \mu \\ Q\left(\frac{\mu - x}{\sigma}\right) & x < \mu \end{cases} \quad (11)$$

Hence, probability that out of 5 persons 4 are swimmers using gaussian approximation is

$$\Pr(Y = 4) = \Pr(3.5 < Y < 4.5) \quad (12)$$

$$= 0.335 \quad (13)$$

Probability that out of 5 persons 4 are swimmers using bernoulli distribution is

$$\Pr(Y = 4) = p_Y(4) \quad (14)$$

$$= {}^nC_k p^k (1 - p)^{n-k} \quad (15)$$

$$= 0.360 \quad (16)$$

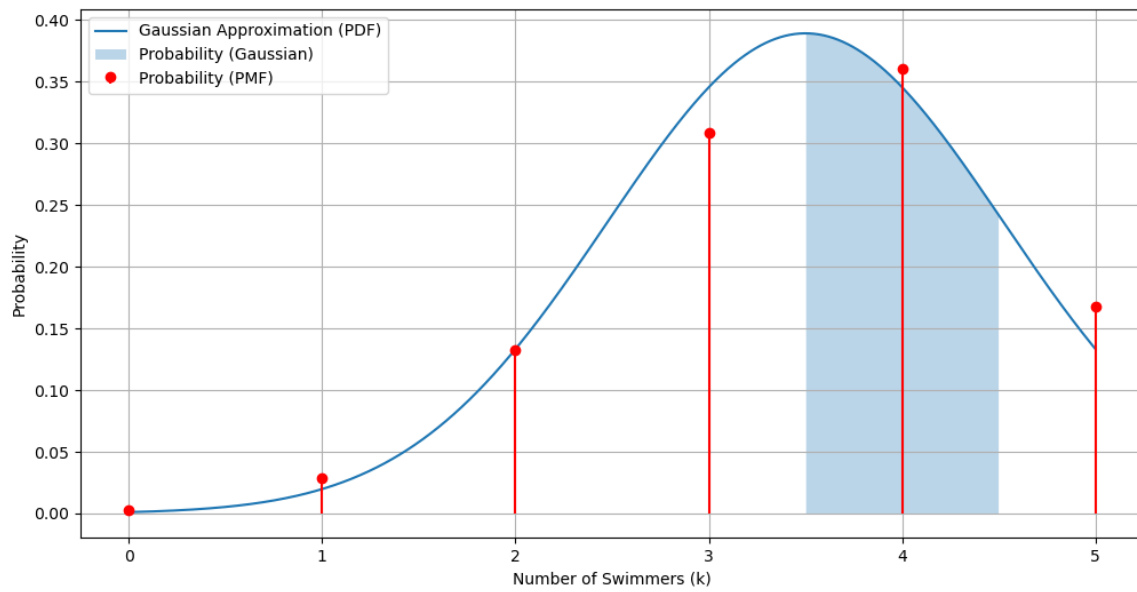


Fig. 0. PDF vs Gaussian