Q: The probability that a person is not a swimmer is 0.3. The probability that out of 5 persons 4 are swimmers is

Solution:

Parameter	Values	Description
n	5	Number of draws
p	0.3	Probability that person is not a swimmer
q	0.7	Probability that person is a swimmer
μ	3.5	Mean
σ	1.024	Variance
X	0	Swimmer
	1	Not a swimmer
Y	4	Number of swimmers

$$X \sim \text{Bin}\,(n, p) \tag{1}$$

Probability that out of 5 persons 4 are swimmers using bernoulli distribution is

$$\Pr(Y = 4) = p_Y\,(4) \tag{2}$$
$$= {}^{n}C_k p^k\,(1-p)^{n-k} \tag{3}$$
$$= 0.360 \tag{4}$$

Q function is defined

$$Q\,(x) = \int_x^{\infty} f\,(x)\,dx \tag{5}$$
$$\tag{6}$$

Let Y be the gaussian random variable,

$$Y \sim \mathcal{N}\left(\mu, \sigma^2\right) \tag{7}$$

then CDF of Y is:

$$F_Y\,(x) = \Pr\,(Y < x) \tag{8}$$
$$= \Pr\left(\frac{Y-\mu}{\sigma} < \frac{x-\mu}{\sigma}\right) \tag{9}$$
$$\implies \frac{Y-\mu}{\sigma} \sim N\,(0,1) \tag{10}$$
$$= 1 - \Pr\left(\frac{Y-\mu}{\sigma} > \frac{x-\mu}{\sigma}\right) \tag{11}$$
$$= \begin{cases} 1 - Q\left(\frac{x-\mu}{\sigma}\right) & x \geq \mu \\ Q\left(\frac{\mu-x}{\sigma}\right) & x < \mu \end{cases} \tag{12}$$

Due to continuity correction $\Pr\,(X = x)$ can be approximated using gaussian distribution as

$$p_Y\,(x) \approx \Pr\,(x - 0.5 < Y < x + 0.5) \tag{13}$$
$$\approx F_Y\,(x + 0.5) - F_Y\,(x - 0.5) \tag{14}$$
$$\approx Q\left(\frac{(x-0.5)-\mu}{\sigma}\right) - Q\left(\frac{(x+0.5)-\mu}{\sigma}\right) \tag{15}$$

Hence, probability that out of 5 persons 4 are swimmers using gaussian approximation is

$$\Pr\,(Y = 4) = \Pr\,(3.5 < Y < 4.5) \tag{16}$$
$$= \int_0^{0.976} \frac{1}{\sqrt{2\pi}} \times e^{-\frac{x^2}{2}}\,dx \tag{17}$$
$$= 0.335 \tag{18}$$

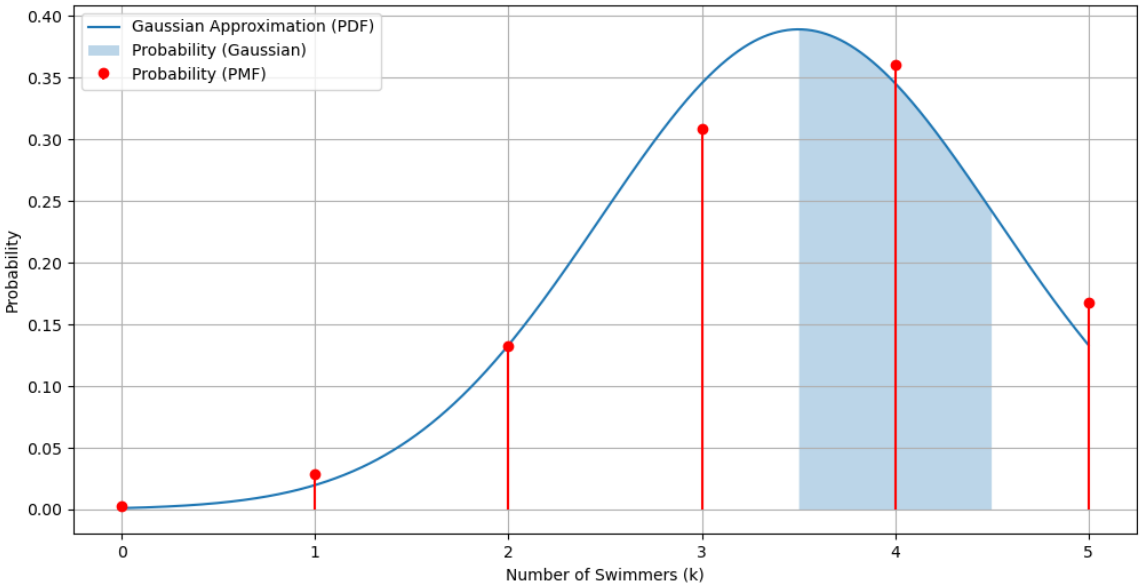

Fig. 0. PDF vs Gaussian