

Q: The probability that a person is not a swimmer is 0.3. The probability that out of 5 persons 4 are swimmers is

Solution:

Parameter	Values	Description
n	5	Number of draws
p	0.3	Probability that person is not a swimmer
q	0.7	Probability that person is a swimmer
μ	3.5	Mean
σ	1.024	Variance
X	0	Swimmer
	1	Not a swimmer
X_i	$\sum_{i=1}^n X$	Bernoulli Random Variable

The gaussian distribution function is defined as:

$$p_Y(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad (1)$$

$$p_Y(4) = \frac{1}{\sqrt{2\pi(1.05)}} e^{-\frac{(4-3.5)^2}{2(1.05)}} \quad (2)$$

$$= \frac{1}{\sqrt{2\pi(1.05)}} e^{-\frac{5}{42}} \quad (3)$$

$$= 0.3456 \quad (4)$$

Probability that out of 5 persons 4 are swimmers using bernoulli distribution is

$$\Pr(Y = 4) = p_Y(4) \quad (5)$$

$$= {}^nC_k p^k (1-p)^{n-k} \quad (6)$$

$$= 0.360 \quad (7)$$

Q function is defined

$$Q(x) = \int_x^\infty f(x) dx \quad (8)$$

then CDF of Y is:

$$\Pr(Y < x) = \int_{-\infty}^x f(x) dx \quad (9)$$

$$= 1 - \int_x^\infty f(x) dx \quad (10)$$

$$= 1 - Q(x) \quad (11)$$

$$\Pr\left(\frac{Y-\mu}{\sigma}\right) \approx \Pr\left(\frac{Y+0.5-\mu}{\sigma} < \frac{Y-\mu}{\sigma} < \frac{Y-0.5-\mu}{\sigma}\right) \quad (12)$$

$$\approx \Pr\left(\frac{Y-\mu}{\sigma} < \frac{Y+0.5-\mu}{\sigma}\right) - \Pr\left(\frac{Y-\mu}{\sigma} < \frac{Y-0.5-\mu}{\sigma}\right) \quad (13)$$

$$\approx Q\left(\frac{Y-0.5-\mu}{\sigma}\right) - Q\left(\frac{Y+0.5-\mu}{\sigma}\right) \quad (14)$$

Hence, probability that out of 5 persons 4 are swimmers using gaussian approximation is

$$\Pr(Y = 4) = \Pr(3.5 < Y < 4.5) \quad (15)$$

$$= \int_0^{0.976} \frac{1}{\sqrt{2\pi}} \times e^{-\frac{x^2}{2}} dx \quad (16)$$

$$(17)$$

$$= 0.335 \quad (18)$$

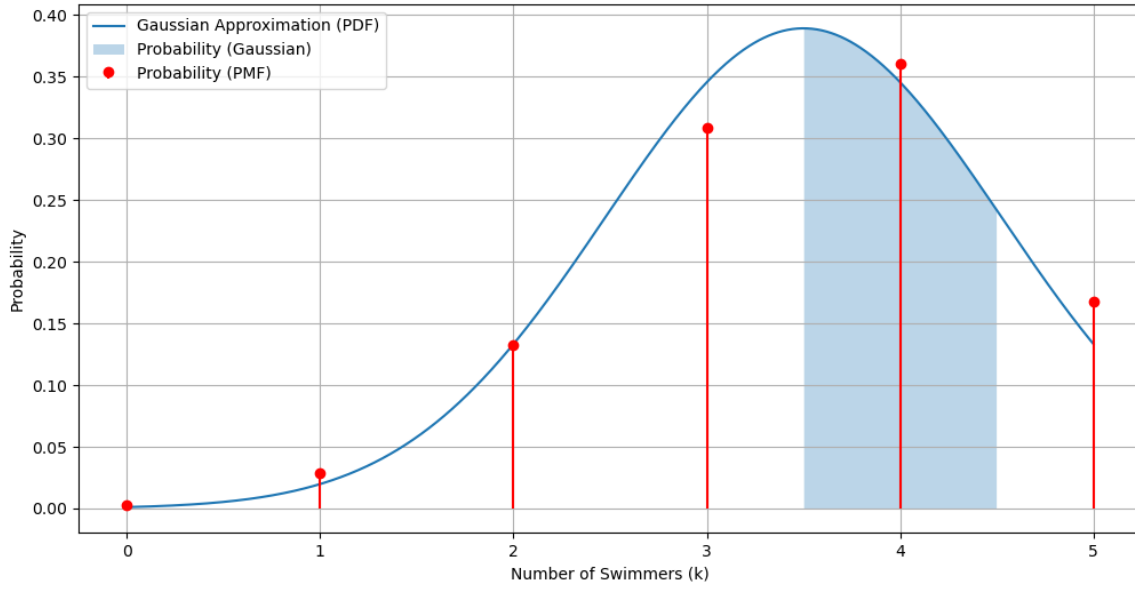


Fig. 0. PDF vs Gaussian