

Q: Let $\{-1, -\frac{1}{2}, 1, \frac{5}{2}, 3\}$ be a realization of a random sample of size 5 from a population having $N(\frac{1}{2}, \sigma^2)$ distribution, where $\sigma > 0$ is an unknown parameter. Let T be an unbiased estimator of σ^2 whose variance attains the Cramer-Rao lower bound. Then, based on the above data, the realized value of T (rounded off to two decimal places) equals

Solution:

Definition 1. *Unbiased Estimator is defined as*

$$E(\hat{\sigma}^2) = \sigma^2 \quad (1)$$

where, $E(\hat{\sigma}^2)$ represents the expected value of the estimator $\hat{\sigma}^2$ and σ^2 represents the true parameter

Definition 2. *The Cramér-Rao bound can be defined as follows:*

$$\text{Var}(\sigma^2) \geq \frac{1}{I(\sigma^2)} \quad (2)$$

where $I(\sigma^2)$ represents a measure of the amount of information in the data about the parameter σ^2 .

Definition 3. *Variance of T attains Cramer-Rao lower bound*

$\Rightarrow T$ has attained minimum possible variance and T is an efficient estimator

Therefore,

$$T = \frac{\sum(X_i - \mu)^2}{n} \quad (3)$$

$$n = 5 \quad (4)$$

$$\mu = \frac{1}{2} \quad (5)$$

| | | | | | |
|-----------------|---------------|----------------|---------------|---------------|----------------|
| X_i | -1 | $-\frac{1}{2}$ | 1 | $\frac{5}{2}$ | 3 |
| $(X_i - \mu)^2$ | $\frac{9}{4}$ | 1 | $\frac{1}{4}$ | 4 | $\frac{25}{4}$ |

TABLE 0

TABLE 1

$$\sum(X_i - \mu)^2 = 13.75 \quad (6)$$

Hence,

$$T = 2.75 \quad (7)$$