

Q: Let $\{-1, -\frac{1}{2}, 1, \frac{5}{2}, 3\}$ be a realization of a random sample of size 5 from a population having $N(\frac{1}{2}, \sigma^2)$ distribution, where $\sigma > 0$ is an unknown parameter. Let T be an unbiased estimator of σ^2 whose variance attains the Cramer-Rao lower bound. Then, based on the above data, the realized value of T (rounded off to two decimal places) equals

Solution:

Definition 1. *Unbiased Estimator is defined as*

$$E(\hat{\theta}) = \theta$$

where, $E(\hat{\theta})$ represents the expected value of the estimator $\hat{\theta}$ and θ represents the true parameter

Definition 2. *Variance of T attains Cramer-Rao lower bound implies that T has attained minimum possible variance and T is an efficient estimator*

Therefore,

$$T = \frac{\sum(X_i - \mu)^2}{n} \quad (1)$$

$$n = 5 \quad (2)$$

$$\mu = \frac{1}{2} \quad (3)$$

X_i	-1	$-\frac{1}{2}$	1	$\frac{5}{2}$	3
$(X_i - \mu)^2$	$\frac{9}{4}$	1	$\frac{1}{4}$	4	$\frac{25}{4}$

TABLE 0

TABLE 1

$$\frac{\sum(X_i - \mu)^2}{n} = 13.75 \quad (4)$$

Hence,

$$T = 2.75 \quad (5)$$