Q: Let $\{-1, -\frac{1}{2}, 1, \frac{5}{2}, 3\}$ be a realization of a random sample of size 5 from a population having $N\left(\frac{1}{2}, \sigma^2\right)$ distribution, where $\sigma > 0$ is an unknown parameter. Let T be an unbiased estimator of σ^2 whose variance attains the Cramer-Rao lower bound. Then, based on the above data, the realized value of T (rounded off to two decimal places) equals

Solution:

Definition 1. *Unbiased Estimator is defined as*

$$E(\hat{\theta}) = \theta$$

where, $E(\hat{\theta})$ represents the expected value of the estimator $\hat{\theta}$ and θ represents the true parameter

Definition 2. *Cramer-rao bound can be defined as*

$$Var(T) \geq \frac{1}{I(\theta)}$$

where, $I(\theta)$ represents a measure of the amount of information in the data about the parameter θ

Definition 3. *Variance of T attains Cramer-Rao lower bound*

$\implies$ *T has attained minimum possible variance and T is an efficient estimator*

Therefore,

$$T = \frac{\sum(X_i - \mu)^2}{n} \tag{1}$$

$$n = 5 \tag{2}$$

$$\mu = \frac{1}{2} \tag{3}$$

X_i	-1	$-\frac{1}{2}$	1	$\frac{5}{2}$	3
$(X_i - \mu)^2$	$\frac{9}{4}$	1	$\frac{1}{4}$	4	$\frac{25}{4}$

TABLE 0
TABLE 1

$$\frac{\sum(X_i - \mu)^2}{n} = 13.75 \tag{4}$$

Hence,

$$T = 2.75 \tag{5}$$