Q: Let  $\{-1, -\frac{1}{2}, 1, \frac{5}{2}, 3\}$  be a realization of a random sample of size 5 from a population having  $N\left(\frac{1}{2}, \sigma^2\right)$ distribution, where  $\sigma > 0$  is an unknown parameter. Let T be an unbiased estimator of  $\sigma^2$  whose variance attains the Cramer-Rao lower bound. Then, based on the above data, the realized value of T (rounded off to two decimal places) equals

## **Solution:**

**Definition 1.** Unbiased Estimator is defined as

$$E(\hat{\sigma^2}) = \sigma^2 \tag{1}$$

where,  $E(\hat{\sigma^2})$  represents the expected value of the estimator  $\hat{\sigma^2}$  and  $\sigma^2$  represents the true parameter

**Definition 2.** The Cramér-Rao bound can be defined as follows:

$$Var(\sigma^2) \ge \frac{1}{I(\sigma^2)}$$
 (2)

where  $I(\sigma^2)$  represents a measure of the amount of information in the data about the parameter  $\sigma^2$ .

**Definition 3.** Variance of T attains Cramer-Rao lower bound

 $\implies$  T has attained minimum possible variance and T is an efficient estimator

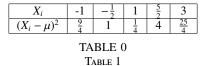
Therefore,

$$T = \frac{\sum (X_i - \mu)^2}{n}$$

$$n = 5$$
(3)

$$n = 5 \tag{4}$$

$$\mu = \frac{1}{2} \tag{5}$$



$$\sum (X_i - \mu)^2 = 13.75 \tag{6}$$

Hence,

$$T = 2.75 \tag{7}$$