

1) Complex Fourier Series

Consider,

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f t} \quad (1)$$

where c_n is the exponential fourier coefficient.

$$c_n = \frac{1}{T} \int_0^T x(t) e^{-j2\pi n f t} dt \quad (2)$$

where T is the time period of function $x(t)$.

2) Trigonometric Fourier Series

We can write:

$$e^{j2\pi n f t} = \cos(2\pi n f t) + j \sin(2\pi n f t) \quad (3)$$

Substituting (3) in (1)

$$x(t) = \sum_{n=-\infty}^{\infty} c_n (\cos(2\pi n f t) + j \sin(2\pi n f t)) \quad (4)$$

$$= a_0 + \sum_{n=1}^{\infty} (a_n \cos(2\pi n f t) + b_n \sin(2\pi n f t)) \quad (5)$$

where a_0, a_n and b_n are trigonometric fourier series.

$$a_0 = c_0 \quad (6)$$

$$= \frac{1}{T} \int_0^T x(t) dt \quad (7)$$

$$a_n = 2 \operatorname{Re}(c_n) \quad (8)$$

$$= \frac{2}{T} \int_0^T x(t) \cos(2\pi n f t) dt \quad (9)$$

$$b_n = -2 \operatorname{Im}(c_n) \quad (10)$$

$$= \frac{2}{T} \int_0^T x(t) \sin(2\pi n f t) dt \quad (11)$$