CO2 Estimation Models

Delta Altitude Reward Models

Force Model

Theory

Given a vehicle moving in a flat surface, there are majorly two forces acting, with other elementary forces:

$$F_T = (F_H + F_V) \cdot J \tag{1}$$

$$\begin{gathered} F_T = \text{Total force exerted} \\ F_H = \text{Horizontal Forces; dependent on load momentum} \\ F_V = \text{Vertical Forces; load and gravity only} \end{gathered}$$

$$F_H = m \cdot \frac{d^2V}{dT^2} \tag{2}$$

$$F_V = \frac{G \cdot M \cdot m}{r^2} \tag{3}$$

$$J - \text{Calibratable factor calculation, typically 1} \tag{4}$$

Forces exerted due to F_H are typically overpowered by the truck's engines, which essentially creates the movement and control of the freight in entirety. F_V is balanced by the structural integrity of the engine.

The demand of the engine is (usually defined in terms of torque requirement), defined as T_{dem}. In a normalised measurement scenario, the value of T_{dem} is 1 on flat roads.

However during ascent, the load m is shifted from F_V to F_H partly, effectively increasing net F_H. This increases $T_{dem} \geq 1$ finally. The converse is valid; during descent, the F_H is reduced since F_H is moved to F_V.

From a single body propulsion point of view, torque demand is directly proportional to the force exerted by the vehicle to counteract (1):

$$T_{dem} \propto F_T \tag{5}$$

This can be converted to a linear form:

$$F_T = k \cdot T_{dem} \tag{6}$$

These two different simple models complement each other from different directions.

Effects

From vehicle combustion point of view, torque demand is directly proportional to the fuel consumed per kilometre:

$$T_{dem} \propto Q_{km} \tag{7}$$

And similarly creating a linear form:

$$Q_{km} = k \cdot T_{dem} \tag{8}$$

With known quantities of average fuel consumption of vehicle based on class $Q_{km,avg}$, torque class T_{eng} to derive T_{dem} based on average mass in vehicle; it is possible to solve (6) to get k.

Total consumed Q_{km} can be calculated from (8) with known k and T_{dem}.

On average, CO2 production from diesel is defined as:

$$CO_{2,avg} = 3.17\text{g per g of diesel} \tag{9}$$

This is the maximum theoretical value derived from combustion of pure diesel, vehicles produce way less that aforementioned value, attributed by the after-treatment parts and active emission control devices, for example EGRs.

This lesser production can be a factor defined similar to k, since all equations $(6), (7) \text{ and } (9)$ are linear proportional values.

Known Approximations

1. Very simple force body model is used throughout the analysis: that is, a vehicle is considered a uni-body. Interacting forces are not considered, for example: momentum imbalance between load and drive-train areas of a truck, sway and other road impacts.
2. The emission systems are dependent to vary between high and low load scenarios, for example: cold-start conditions, any effects due to AECD, full-load operation and limp-mode situations.
3. The factor k is considered uniform, and approximated heavily. However this can be tuned using factor j that is controlled by the user, creating a closed loop system that does not affect the calculation chain in the middle, causing and unintended effects.
4. Many external forces are neglected: wind drag, aerodynamics of the vehicle, traffic situations, road smoothness.

Combustion Model

Theory

Diesel Engines and all combustion engines depend on considerable amount of O2 for the proper function and torque delivery. Diesel engines rely on a stoichiometric AFR

(Air-to-fuel ratio) of $14.5:1$. Essentially this would mean that to combust 1g of diesel, the engines require a minimum of 14.5g of air (in composition, all gases including H2 and N2 along with O2). This theory uses the mass-flow model to determine rate of change of consumption of air, resulting in proportional change of fuel consumption based on altitude and temperature.

Rewriting the above text mathematically,

$$MF_{air} \propto Q_{act} \tag{10}$$

$$Q_{act} = k \cdot MF_{air}, \text{where k} = 14.5. \tag{11}$$

From (9), we know that $1\text{g of } Q_{act} = 3.17\text{g of CO2}$,

$$\implies \frac{dm_{CO_2}}{dQ_{act}} = 3.17 g/g \tag{12}$$

Estimated mass flow can be calculated empirically from ideal gas equation:

$$MF_{in} = \frac{P_{env} \cdot V_{engine}}{R \cdot T_{env}} \tag{13}$$

Lapse rate due to altitude change also has a relation:

$$L = -\frac{dT}{dH} \text{ degC/m}$$

This lapse rate is known for every geographic region and is uniform over continents. This can be used to determine dT:

$$dT = -L \cdot dH \tag{14}$$

Considering adiabatic compression scenario, (13) can also be written as:

$$P \cdot dV = -\frac{R \cdot dP}{\gamma}; \quad \gamma = c_p/c_v \tag{15}$$

Combine all equations and simplify from (13) to (15) to get:

$$MF_{air} = \frac{M \cdot C_p \cdot dV_{engine}}{R} \tag{16}$$

All values are known, except for C_p. This can be evaluated using Sutherland's law for mass flow and compression:

$$C_p = \frac{A + B \cdot \left(\frac{\frac{C}{T_{env}}}{sinh\left(\frac{C}{T_{env}}\right)}\right)^2 + D \cdot \left(\frac{\frac{E}{T_{env}}}{cosh\left(\frac{E}{T_{env}}\right)}\right)^2}{MW} \tag{17}$$

$$
\begin{gathered}
\mathrm{A} = 28958 \\
\mathrm{B} = 9390 \\
\mathrm{C} = 3012 \\
\mathrm{D} = 7580 \\
\mathrm{E} = 1484 \\
\mathrm{MW} = 28.951 \ \mathrm{kg/mol}
\end{gathered}
$$

This estimated mass flow from (16) can be applied to calculate required fuel change for combustion in (11), then re-applied to (12) to determine change of CO2 emissions.

Known Approximations

1. The following environmental and vehicle conditions are assumed:
    1. Ideal Gas Conditions:
        1. Temperature = 21 degC
        2. Pressure = 1014 hPa
        3. Volume = 1 litre of O2
2. The vehicle is assumed to have one turbo-charger, and is assumed primarily in adiabatic conditions in full load.

References

1. Specific Heat Capacities: https://www.grc.nasa.gov/www/k-12/BGP/specheat.html
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3. Sutherland Law and corresponding values: https://ubitutors.com/thermal-properties-of-air-at-a-given-temperature/
4. Sutherland Law in terms of Conductivity: https://doc.comsol.com/5.5/doc/com.comsol.help.cfd/cfd_ug_fluidflow_high_mach.08.27.html