State space formulation

October 20, 2018

Equations from the paper

$y_t = \mu_t + \gamma_t + z_t^a o_t + (1 - z_t^a)\varepsilon_t$

$\mu_t = \mu_{t-1} + \delta_{t-1} + z_t^c r_t + (1 - z_t^c) u_t$

$\delta_t = \delta_{t-1} + v_t$

$\gamma_t = -\sum_{s=1}^{S} \gamma_{t-s} + w_t$

State space equations

$y_t = Z_t \alpha_t + a_t o_t + (1 - a_t)\varepsilon_t$

$\alpha_{t+1} = T_t \alpha_t + R_t c_t \eta_t + R_t (1 - c_t)\ \xi_t$

$o_t \sim \mathcal{N}(0, H_t^o)$

$\varepsilon_t \sim \mathcal{N}(0, H_t^\varepsilon)$

$\eta_t \sim \mathcal{N}(0, Q_t^\eta)$

$\xi_t \sim \mathcal{N}(0, Q_t^\xi)$

Where

$\alpha_t = (\mu_t, \delta_t, \gamma_t, \gamma_{t-1}, \dots, \gamma_{t-S+2})^T$

$a_t = z_t^a \sim \text{Bernoulli}(p_a)$

$c_t = z_t^c \sim \text{Bernoulli}(p_c)$

$\eta_t = (r_t, v_t, w_t)$

$\xi_t = (u_t, v_t, w_t)$

$Z_t = (1, 0, 1, 0, \dots, 0)$

$T_t = \text{diag}(T_\mu, T_\gamma)$

$T_\mu = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

$$T_\gamma = \begin{pmatrix} -1 & -1 & \dots & -1 & -1 \\ 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & 0 \end{pmatrix}$$

$$R = \mathrm{diag}(R_\mu, R_\gamma)$$

$$R_\mu = I_2$$

$$R_\gamma = (1, 0, \dots, 0)^T$$

$$H_t^o = \sigma_o^2$$

$$H^\varepsilon = \sigma_\varepsilon^2$$

$$Q^\eta = \mathrm{diag}(\sigma_r^2, \sigma_v^2, \sigma_w^2)$$

$$Q^\xi = \mathrm{diag}(\sigma_u^2, \sigma_v^2, \sigma_w^2)$$