

Assignment 3

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Question 12.13.5.1: A die is thrown 6 times. if 'getting an odd number' is a success, what is the probability of

- 1) 5 successes?
- 2) at least 5 successes?
- 3) at most 5 successes?

Solution: when a die is rolled the possible outcomes are $\{1, 2, 3, 4, 5, 6\}$. so the probability of getting an odd number is $p = 1/2$, similarly the probability of getting even number is $q = 1/2$.

parameter	value	Description
n	6	number of throws
p	0.5	probability of success in each throw
q	0.5	probability of failure in each throw

TABLE 3: Given data

Let X be the random variable denoting number of successes.

Now

Binomial distribution of X is

$$\Pr(X = i) = p_X(i) = {}^6C_i p^i q^{6-i} \quad (1)$$

1)

$$\Pr(X = 5) = {}^6C_5 p^5 q^1 \quad (2)$$

$$= {}^6C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^1 \quad (3)$$

$$= \frac{6}{2^6} \quad (4)$$

$$= \frac{3}{32} \quad (5)$$

2)

$$\Pr(X \geq 5) = 1 - F_X(4) \quad (6)$$

$$= F_X(6) - F_X(4) \quad (7)$$

$$= p_X(5) + p_X(6) \quad (8)$$

$$= {}^6C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^1 + {}^6C_6 \left(\frac{1}{2}\right)^6 \left(\frac{1}{2}\right)^0 \quad (9)$$

$$= \frac{6}{2^6} + \frac{1}{2^6} \quad (10)$$

$$= \frac{7}{64} \quad (11)$$

3)

$$\Pr(X \leq 5) = F_X(5) \quad (12)$$

$$= 1 - p_X(6) \quad (13)$$

$$= 1 - {}^6C_6 \left(\frac{1}{2}\right)^6 \left(\frac{1}{2}\right)^0 \quad (14)$$

$$= 1 - \frac{1}{2^6} \quad (15)$$

$$= \frac{63}{64} \quad (16)$$