1

Assignment 3

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Question 12.13.5.1: A die is thrown 6 times. if 'getting an odd number' is a sucess, what is the probability of

- 1) 5 successes?
- 2) at least 5 successes?
- 3) at most 5 successes?

Solution: when a die is rolled the posssible outcomes are $\{1, 2, 3, 4, 5, 6\}$. so the probability of getting an odd number is p = 1/2, similarly the probability of getting even number is q = 1/2.

| parameter | value | Description |
|-----------|-------|--------------------------------------|
| n | 6 | number of throws |
| p | 0.5 | probability of success in each throw |
| q | 0.5 | probability of failure in each throw |

TABLE 3: Given data

Let X be the random varible denoting number of successes.

Now

Binomial distribution of X is

$$Pr(X = i) = p_X(i) = {}^{6}C_i p^i q^{6-i}$$
(1)

1)

$$\Pr(X=5) = {}^{6}C_{5}p^{5}q^{1} \tag{2}$$

$$={}^{6}C_{5}\left(\frac{1}{2}\right)^{5}\left(\frac{1}{2}\right)^{1}\tag{3}$$

$$=\frac{6}{2^6}\tag{4}$$

$$=\frac{3}{32}\tag{5}$$

2)

$$\Pr(X \ge 5) = 1 - F_X(4)$$
 (6)

$$= F_X(6) - F_X(4) \tag{7}$$

$$= p_X(5) + p_X(6) (8)$$

$$= {}^{6}C_{5} \left(\frac{1}{2}\right)^{5} \left(\frac{1}{2}\right)^{1} + {}^{6}C_{6} \left(\frac{1}{2}\right)^{6} \left(\frac{1}{2}\right)^{0} \tag{9}$$

$$=\frac{6}{2^6}+\frac{1}{2^6}\tag{10}$$

$$=\frac{7}{64}\tag{11}$$

$$\Pr\left(X \le 5\right) = F_X(5) \tag{12}$$

$$=1-p_X(6) (13)$$

$$\begin{aligned}
& = 1 - p_X(5) & \text{(12)} \\
& = 1 - p_X(6) & \text{(13)} \\
& = 1 - {}^{6}C_{6} \left(\frac{1}{2}\right)^{6} \left(\frac{1}{2}\right)^{0} & \text{(14)} \\
& = 1 - \frac{1}{2^{6}} & \text{(15)} \\
& = \frac{63}{64} & \text{(16)}
\end{aligned}$$

$$=1-\frac{1}{2^{6}}\tag{15}$$

$$=\frac{63}{64}$$
 (16)