

Assignment 3

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Question 12.13.5.1: A die is thrown 6 times. if 'getting an odd number' is a success, what is the probability of

- 1) 5 successes?
- 2) at least 5 successes?
- 3) at most 5 successes?

Solution: when a die is rolled the possible outcomes are {1, 2, 3, 4, 5, 6}. so the probability of getting an odd number is $p = 1/2$, similarly the probability of getting even number is $q = 1/2$.

Let X be the random variable denoting number of successes.

Given:

$$n = 6 \quad (1)$$

$$p = \frac{1}{2} \quad (2)$$

$$q = \frac{1}{2} \quad (3)$$

where n is number of trials, p is probability of success in each trial and q is probability of failure in each trial.

Now

Binomial distribution of X is

$$\Pr(X = i) = {}^6C_i p^i q^{6-i} \quad (4)$$

1)

$$\Pr(X = 5) = {}^6C_5 p^5 q^1 \quad (5)$$

$$= {}^6C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^1 \quad (6)$$

$$= \frac{6}{2^6} \quad (7)$$

$$= \frac{3}{32} \quad (8)$$

2)

$$\Pr(X \geq 5) = \Pr(X = 5) + \Pr(X = 6) \quad (9)$$

$$= {}^6C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^1 + {}^6C_6 \left(\frac{1}{2}\right)^6 \left(\frac{1}{2}\right)^0 \quad (10)$$

$$= \frac{6}{2^6} + \frac{1}{2^6} \quad (11)$$

$$= \frac{7}{64} \quad (12)$$

3)

$$\Pr(X \leq 5) = 1 - \Pr(X = 6) \quad (13)$$

$$= 1 - {}^6C_6 \left(\frac{1}{2}\right)^6 \left(\frac{1}{2}\right)^0 \quad (14)$$

$$= 1 - \frac{1}{2^6} \quad (15)$$

$$= \frac{63}{64} \quad (16)$$