Assignment 3

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Question 12.13.5.1: A die is thrown 6 times. if 'getting an odd number' is a sucess, what is the probability of

1) 5 successes?
2) at least 5 successes?
3) at most 5 successes?

Solution: when a die is rolled the posssible outcomes are $\{1, 2, 3, 4, 5, 6\}$. so the probability of getting odd number is $p = 1/2$,similarly the probability of getting even number is $q = 1/2$.

Let X be the random varible denoting number of successes.

$$X \in \{0, 1, 2, 3, 4, 5, 6\}$$

probabilty of getting i times success out of 6 times is

$$\Pr(X = i) = {}^6C_i p^i q^{6-i} \tag{1}$$

Now

1)

$$\Pr(X = 5) = {}^6C_5 p^5 q^1 \tag{2}$$

$$= {}^6C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^1 \tag{3}$$

$$= \frac{6}{2^6} \tag{4}$$

$$= \frac{3}{32} \tag{5}$$

2)

$$\Pr(X \geq 5) = \Pr(X = 5) + \Pr(X = 6) \tag{6}$$

$$= {}^6C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^1 + {}^6C_6 \left(\frac{1}{2}\right)^6 \left(\frac{1}{2}\right)^0 \tag{7}$$

$$= \frac{6}{2^6} + \frac{1}{2^6} \tag{8}$$

$$= \frac{7}{64} \tag{9}$$

3)

$$\Pr(X \leq 5) = 1 - \Pr(X = 6) \tag{10}$$

$$= 1 - {}^6C_6 \left(\frac{1}{2}\right)^6 \left(\frac{1}{2}\right)^0 \tag{11}$$

$$= 1 - \frac{1}{2^6} \tag{12}$$

$$= \frac{63}{64} \tag{13}$$