Fama French and Black Litterman Model

A novel integration of the Fama–French and Black–Litterman models to enhance portfolio management .

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1) Abstract Summary

The paper proposes a novel integration of the **Fama–French three-factor model** and the **Black–Litterman (B–L) framework** to enhance portfolio management. This integrated model aims to address estimation errors, reduce market volatility sensitivity, and optimize portfolio returns. Through empirical analysis, the model demonstrates superior performance compared to traditional approaches like the Markowitz model, showing increased **Sharpe Ratios (SR)**, reduced estimation errors, and enhanced diversification. This method automates the construction of market views by embedding the insights from asset pricing theories into the B–L framework, improving investment strategies.

2) Keywords and Definitions

1) **Black–Litterman portfolio model**: A Bayesian-based portfolio optimization framework that blends market equilibrium with investor views to enhance asset allocations.

2) **Fama–French three-factor model**: An asset pricing model incorporating three factors—market risk, size effect (Small Minus Big, SMB), and value effect (High Minus Low, HML)—to explain asset returns.

3) **Mean-variance portfolio model**: Introduced by Markowitz, it aims to optimize a portfolio's return against its risk by considering variances and covariances of asset returns.

4) **Estimation error**: Errors that arise from predicting asset returns based on historical data, impacting portfolio optimization models.

5) **Asset pricing**: The method used to determine the fair value of an asset based on expected returns and risk factors.

3) Mathematical Formulae and Logic

- **Fama–French Three-Factor Model**:
  - $r_i, t = R_i, t - R_f, t = a_i + b_i(R_m, t - R_f, t) + s_i SMB_t + h_i HML_t$
  - Here, r_i, t is the excess return, R_f, t is the risk-free rate, and SMB, HML capture size and value effects, respectively.

- **Markowitz Mean–Variance Portfolio Optimization**:
  - Objective function:

$$\min_w \left(\frac{\lambda}{2} w^T \Sigma w - w^T \mu \right), \quad s.t. \quad w^T 1 = 1$$

  - Where w is the weight vector, Σ is the covariance matrix, μ is the expected return vector, and λ is the risk aversion parameter.

- **Black–Litterman (B–L) Model**:
  - **Implied Equilibrium Return**:

$$\pi_t = \lambda \Sigma_t w_{mkt,t}$$

  - **Posterior Mean of Expected Returns**:

$$\mu_{posterior,t} = \Sigma_{BL,t} \left[(\tau \Sigma_t)^{-1} \pi_t + P_t^T \Omega_t^{-1} q_t \right]$$

  - Where P_t is the matrix representing views, Ω_t is the view covariance matrix, and q_t is the vector of expected returns based on investor views.
  - The **Optimal Portfolio Weights** are derived as:

$$w_t^* = (\lambda \Sigma_t)^{-1} \mu_{BL,t}$$

- **Certainty Equivalent Return (CER)**:

$$CER = E(r_p) - \frac{\lambda}{2} \sigma_p^2$$

  - This reflects the return an investor would accept instead of taking a risk with higher returns.

- **Sharpe Ratio (SR)**:

$$SR = \frac{E(r_p) - r_f}{\sigma_p}$$

  - Measures the excess return per unit of risk.

Detailed Definitions of certain Concepts

1) Capital Asset Pricing Model (CAPM)

1) The **Capital Asset Pricing Model (CAPM)** is a financial model that describes the relationship between the expected return of an investment and its risk. It is widely used in finance to assess the expected return on assets, particularly stocks, given the risk relative to the market.

Formula Overview

{ E(Ri)=Rf+βi ⋅ (E(Rm)−Rf) }

$$E(R_i) = R_f + \beta_i \cdot (E(R_m) - R_f)$$

1. **Expected Return (E(Ri))**: This is the return an investor expects from an asset.
2. **Risk-Free Rate (Rf)**: This represents the return on a risk-free investment, typically government bonds. It's the baseline return that investors can earn with no risk.
3. **Beta (βi)**: Beta measures the sensitivity of an asset's return relative to the return of the overall market.
   - If $\beta=1$, the asset's price moves in line with the market.
   - If $\beta>1$, the asset is more volatile than the market (higher risk).
   - If $\beta<1$, the asset is less volatile than the market (lower risk).
4. **Market Risk Premium (E(Rm)−Rf)**: The difference between the expected return of the market (E(Rm)) and the risk-free rate. It represents the extra return investors expect to earn by taking on market risk.

2) Markowitz's Mean-Variance Optimization (MVO)

1) **Markowitz's Mean-Variance Optimization (MVO)** is a mathematical framework which forms the foundation of modern portfolio theory. The goal of MVO is to construct an optimal portfolio by maximizing expected return for a given level of risk or, equivalently, minimizing risk for a given expected return. The model uses the **expected returns**, **variances**, and **covariances** of assets to balance risk and return.

Key Concepts

1. **Expected Return**:
    - This is the weighted average of the expected returns of individual assets in the portfolio. Each asset's return is weighted according to the proportion of the portfolio invested in that asset.
2. **Risk (Variance/Standard Deviation)**:
    - The risk of a portfolio is measured by its **variance** or **standard deviation**, which reflects how much the returns deviate from the expected value. MVO seeks to minimize this risk for a given expected return.
3. **Covariance**:
    - Covariance between two assets measures how the returns of those assets move in relation to each other. If two assets have a low or negative covariance, combining them can reduce the overall risk of the portfolio.
4. **Efficient Frontier**:
    - The result of mean-variance optimization is the **efficient frontier**, a curve that shows the set of portfolios that have the highest expected return for each level of risk. Investors should ideally choose a portfolio on this curve based on their risk tolerance.
5. **Risk-Return Trade-Off**:
    - In Markowitz's framework, investors make decisions based on the trade-off between risk and return. Rational investors seek to maximize returns while minimizing risk, which is formalized in the mean-variance optimization process.

Steps of MVO

To perform MVO, the investor needs three inputs:

1. **Expected returns** for each asset.
2. **Variance and covariance** of the asset returns (which form the covariance matrix).
3. **Portfolio weights** for each asset.

Using these inputs, MVO minimizes the portfolio's variance subject to a constraint on the expected return or maximizes the return for a given level of risk. The optimization can be written as:

Formula Overview

- **Minimize portfolio variance**:
  $\sigma_p^2 = w^T \Sigma w$ where:
  - σ_p^2 is the portfolio variance,
  - w is the vector of portfolio weights,
  - Σ is the covariance matrix of asset returns.
- **Subject to an expected return constraint**: $E(r_p) = w^T E(r)$ where:
  - $E(r_p)$ is the expected portfolio return,
  - w is the vector of portfolio weights,
  - $E(r)$ is the vector of expected asset returns.

The result is a set of portfolio weights www that provide the best trade-off between return and risk.

3) Black Litterman Model

The **Black–Litterman model** is a sophisticated asset allocation framework that combines the **Capital Asset Pricing Model (CAPM)** market equilibrium with the investor's subjective views to create a more robust portfolio optimization method. It was developed by **Fischer Black** and **Robert Litterman** at Goldman Sachs in 1990 to address some of the limitations of traditional portfolio optimization models, such as **Markowitz's mean-variance optimization**.

Key Concepts

1. **CAPM (Market Equilibrium)**:
   - CAPM is a widely-used model that describes the relationship between expected return and risk in a market portfolio. It assumes that the market is in equilibrium, meaning all investors hold the market portfolio, and the prices reflect all available information.
   - In the Black–Litterman model, CAPM provides the equilibrium returns (or the "market-implied returns") for each asset in a portfolio, often called the **prior** in Bayesian terminology.

2. **Investor Views**:
    - The key innovation of the Black–Litterman model is its ability to incorporate **subjective views** of the investor about future returns. These views are usually related to specific assets or asset classes and may involve an expected return different from the market equilibrium return.
    - These views can be **absolute** (an investor believes a particular asset will have a return of 10%) or **relative** (an investor believes Asset A will outperform Asset B by 5%).
3. **Bayesian Framework**:
    - The model applies **Bayesian inference** to combine the CAPM equilibrium returns (prior) with the investor's views (new information) to generate a **posterior distribution** of expected returns.
    - This results in adjusted returns that account for both the market equilibrium and the investor's subjective expectations, weighted by the investor's confidence in their views.
4. **Reverse Optimization**:
    - Instead of starting with expected returns (as in traditional mean-variance optimization), the Black–Litterman model starts by inferring the expected returns implied by the market (market equilibrium returns) using the **market capitalization weights**.
    - This process is called **reverse optimization**, where the equilibrium returns are derived by assuming that the market portfolio is mean-variance efficient.

Steps of BL- Model

1. **Market Equilibrium**:
    - Use the global market portfolio (e.g., stocks, bonds) and the covariance matrix of returns to compute the **implied excess equilibrium returns**. These are based on market capitalization weights and can be thought of as the market's expected returns if no investor had any special information or views.
2. **Formulate Views**:
    - The investor introduces their views on one or more assets or asset classes, either in absolute terms (e.g., "I expect Asset X to return 7%") or in relative terms (e.g., "Asset A will outperform Asset B by 3%").
    - Each view is accompanied by a **confidence level**. This confidence determines how much weight the model should give to the investor's view versus the market equilibrium.
3. **Blend Views and Equilibrium Returns**:
    - The investor's views are combined with the market equilibrium returns using Bayesian techniques. The model adjusts the equilibrium returns by incorporating the investor's views while taking into account the confidence levels. If the investor is highly confident, the views will have a larger influence. If the investor is less confident, the market equilibrium returns will dominate.
4. **Posterior Returns**:

    - The output is a new set of expected returns for each asset in the portfolio, called **posterior returns**. These are a blend of the equilibrium market returns and the investor's views.
5. **Optimize Portfolio**:
    - Once the posterior returns are obtained, they are used in the traditional **mean-variance optimization** framework to construct the optimal portfolio.
    - The resulting portfolio is adjusted for the investor's views but still remains close to the efficient frontier due to the initial reliance on market equilibrium.

Advantages of the Black–Litterman Model

1. **Stability**:
    - Traditional mean-variance optimization is highly sensitive to small changes in expected returns, often leading to extreme and unrealistic portfolio weights. The Black–Litterman model smooths out this sensitivity by anchoring on market equilibrium returns.
2. **Incorporates Investor Views**:
    - It allows investors to systematically incorporate their views into the portfolio optimization process, giving them more control over the outcome without completely abandoning the market information.
3. **Reduced Estimation Error**:
    - Traditional models rely on historical returns to estimate expected returns, which can be error-prone. The Black–Litterman model reduces estimation errors by incorporating market equilibrium information, which is generally considered more reliable.
4. **Balanced Portfolio**:
    - The model avoids extreme portfolios, providing more reasonable asset allocations by maintaining a balance between market consensus and individual views.

Formula Overview

Formula Overview

The key mathematical components of the Black–Litterman model include:

- **Implied Equilibrium Returns**:
  $\Pi = \lambda \Sigma w_m$
  where:
  - Π is the vector of implied equilibrium excess returns,
  - λ is the risk aversion coefficient,
  - Σ is the covariance matrix of returns,
  - w_m is the market capitalization weight vector of assets.
- **Posterior Returns** (after incorporating views): $\mu_{BL} = \left((\tau\Sigma)^{-1} + P^T\Omega^{-1}P\right)^{-1}\left((\tau\Sigma)^{-1}\Pi + P^T\Omega^{-1}Q\right)$ where:
  - μ_{BL} is the posterior expected returns,
  - τ is a scaling factor (usually a small positive number),
  - P is the matrix of views,
  - Ω is the covariance matrix of the views,
  - Q is the vector of view returns.

The **Black–Litterman model** is a powerful tool for portfolio construction, blending market equilibrium with individual investor views in a systematic and coherent way. It improves the stability of portfolio optimization by relying on a solid foundation of market data, while still allowing flexibility for subjective opinions on expected returns.

4) Fama French Model

The **Fama-French Three-Factor Model** is an asset pricing model. It extends the traditional **Capital Asset Pricing Model (CAPM)** by adding two additional factors—**size** and **value**—to explain stock returns more effectively than using market risk alone. The model helps to capture patterns in asset returns that CAPM fails to account for.

The Three Factors in the Model

1. **Market Risk (Market Excess Return - $R_m - R_f$)**:
   - This is the same risk factor as in the CAPM, which measures the excess return of the market over the risk-free rate (usually a treasury bond rate). It represents the **systematic risk** that affects all stocks.
   - Formula:

     $$R_m - R_f$$

     where:

     - R_m is the return on the market portfolio (e.g., a broad stock market index),
     - R_f is the risk-free rate (e.g., return on treasury bonds).

2. **Size Effect (Small Minus Big - SMB)**:
   - This factor captures the **size effect**, which is the tendency for smaller companies (in terms of market capitalization) to outperform larger companies. Historically, small-cap stocks have been shown to generate higher returns than large-cap stocks.
   - **SMB** is calculated by subtracting the returns of large-cap stocks from the returns of small-cap stocks:

     $$\text{SMB} = R_{\text{small}} - R_{\text{big}}$$

     where:

     - R_{small} is the average return of small-cap stocks,
     - R_{big} is the average return of large-cap stocks.

3. **Value Effect (High Minus Low - HML)**:
   - This factor captures the **value effect**, which is the tendency for **value stocks** (stocks with a high book-to-market ratio) to outperform **growth stocks** (stocks with a low book-to-market ratio). Historically, companies with higher book value relative to their market value (value stocks) tend to offer higher returns than those with low book-to-market ratios (growth stocks).
   - **HML** is calculated by subtracting the returns of growth stocks from value stocks:

     $$\text{HML} = R_{\text{high}} - R_{\text{low}}$$

     where:

     - R_{high} is the average return of value stocks (high book-to-market ratio),
     - R_{low} is the average return of growth stocks (low book-to-market ratio).

Formula Overview

The expected return of a portfolio or stock under the Fama-French model is given by the following equation:

$$R_p - R_f = \alpha + \beta_m(R_m - R_f) + \beta_{SMB} \cdot \mathrm{SMB} + \beta_{HML} \cdot \mathrm{HML} + \epsilon$$

Where:

- R_p is the expected return of the portfolio or asset.
- R_f is the risk-free rate.
- α is the **alpha** or **intercept**, representing any excess returns not explained by the model's factors (skill or anomaly).
- $R_m - R_f$ is the **market risk premium**, the excess return of the market over the risk-free rate.
- β_m is the portfolio's or asset's sensitivity to market risk (similar to CAPM's beta).
- **SMB** is the size factor, the return difference between small-cap and large-cap stocks.
- β_{SMB} is the portfolio's or asset's sensitivity to the size factor.
- **HML** is the value factor, the return difference between value and growth stocks.
- β_{HML} is the portfolio's or asset's sensitivity to the value factor.
- ϵ is the **error term** representing unexplained variability in the asset's return.

Intuition Behind the Model

- The **Market Risk Premium** Rm−Rf represents the compensation investors require for taking on the risk of investing in the market as opposed to holding a risk-free asset.
- **SMB** (Small Minus Big) captures the idea that smaller companies tend to be riskier but offer higher returns. This is likely due to smaller firms being less established and more volatile, thus requiring a premium to entice investors.
- **HML** (High Minus Low) reflects the tendency for value stocks to outperform growth stocks. Value stocks are typically those that are undervalued or underappreciated by the market, potentially offering greater returns when their true value is recognized.

Why the Fama-French Model Was Developed

- **CAPM's Limitations**:
    - CAPM only uses the **market risk premium** to explain asset returns. However, empirical evidence suggests that this single factor doesn't fully explain the variations in stock returns. CAPM often underestimates the returns of small-cap and value stocks.
- **Fama and French's Research**:
    - Fama and French conducted extensive research using historical stock returns and found that both **size** and **value** factors had significant explanatory power for stock returns. These two factors were not accounted for by CAPM, leading to the development of their multi-factor model.

Empirical Findings Supporting the Fama-French Model

1. **Small-Cap Outperformance**:
    - Historically, smaller companies have tended to deliver higher average returns than larger companies, although with higher risk. The size factor (SMB) accounts for this phenomenon.
2. **Value Premium**:
    - Stocks with high book-to-market ratios (value stocks) have been found to outperform low book-to-market stocks (growth stocks) over the long run. The value factor (HML) captures this effect.
3. **Diversification of Risk**:
    - By adding two additional factors, the Fama-French model better explains the **cross-section of stock returns**, especially those of small-cap and value stocks that were underrepresented in CAPM.

Applications of the Fama-French Model

- **Portfolio Management**:
    - The model helps investors better understand the sources of risk and return in their portfolios by breaking down returns into market risk, size, and value effects.
    - It also aids in constructing more diversified portfolios by balancing exposure to these different factors.
- **Performance Evaluation**:
    - The Fama-French model can be used to evaluate the performance of mutual funds or hedge funds. If a fund outperforms the market, the model can help determine whether that outperformance is due to skill (alpha) or simply exposure to the size and value factors.
- **Asset Pricing**:

   - Academics and practitioners use the model to explain differences in stock returns across various time periods and to understand which factors drive those differences.

Limitations of the Fama-French Model

1. **Other Potential Factors**:
   - While the three-factor model improves on CAPM, it still doesn't capture all aspects of stock returns. Other factors, such as **momentum** (the tendency for stocks that have performed well recently to continue performing well), are not included.
2. **Model's Static Nature**:
   - The Fama-French model is based on historical data and is static in its application. It does not account for changing market conditions or dynamic factors that might influence stock returns over time.
3. **Not Universal**:
   - While the Fama-French model explains a significant portion of stock returns, it doesn't explain everything. There are still anomalies and deviations that the model cannot capture, especially in sectors with unique characteristics like technology or healthcare.
4. **Factor Correlations**:
   - There can be some correlation between the factors (e.g., small stocks might also tend to be value stocks), which might make it difficult to completely separate their effects in certain market environments.

The **Fama-French Three-Factor Model** provides a more comprehensive framework than CAPM for understanding stock returns by incorporating size and value effects alongside market risk. While it addresses many of CAPM's shortcomings, it also has limitations, leading to further developments in asset pricing models, such as the **Fama-French Five-Factor Model** and other multi-factor approaches. Nonetheless, it remains a widely-used tool in finance for analyzing and explaining asset returns.

5) Understanding of the Paper

The integration of the **Fama–French three-factor model** with the **Black–Litterman framework** is a significant advancement for portfolio optimization. By embedding factor insights (like size and value effects) into the B–L model, this approach provides an automated method for constructing market views, reducing the reliance on expert views and mitigating estimation errors. It directly addresses the limitations of traditional models, such as the impracticalities of long-short portfolios and estimation inaccuracies inherent in the **Markowitz mean–variance model**. This hybrid model achieves superior risk-adjusted returns, enhanced diversification, and robustness across different economic conditions.

6) Conclusion

The paper presents a great integration of the **Fama–French three-factor model** with the **Black–Litterman framework**, effectively addressing key challenges in portfolio management such as estimation errors and market volatility sensitivity. The proposed model outperforms traditional benchmarks, significantly increasing the Sharpe ratio and providing more stable and diversified portfolios. This research contributes a novel methodology for view construction in the B–L framework, offering a practical and theoretically sound tool for real-world financial portfolio management.