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A novel integration of the Fama–French and Black–Litterman models to enhance portfolio management

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A B S T R A C T

We propose a novel portfolio model integrating the Fama–French three-factor model into the Black–Litterman framework, enabling efficient investment strategies. The model surpasses traditional benchmarks, significantly increasing alpha, minimizing estimation error, and improving diversification. Performance improvements are shown by a tripled Sharpe ratio and doubled Certainty Equivalent Return compared to standard models. It maintains stability across different parameters and economic climates, leveraging improved weight adjustment to reduce estimation errors and withstand market volatility. It provides a new perspective for portfolio construction, leveraging long-term insights from asset pricing theory with significant implications.

1. Introduction

In the rapidly evolving field of finance, the construction of efficient portfolios that adequately address the issue of estimation error remains a central challenge. This study proposes an innovative approach to this enduring problem through the development of a novel Fama–French three-factor-based Black–Litterman (hereafter, B–L) portfolio model, marking a significant advancement in the field of portfolio management. Our study is primarily inspired by recent advancements in finance, especially those pertaining to portfolio management and optimization, with the aim of enhancing portfolio efficiency (Ayadi et al., 2023; Attig et al., 2023; Bacchetta et al., 2023; Hollstein and Prokopczuk, 2023; Anderson and Cheng, 2022; Kan and Wang, 2023; Zhang et al., 2023; Bartram et al., 2021; Zhang et al., 2020; Chen et al., 2021).

Building upon these recent advancements, our primary contributions are threefold. First, we propose a novel asset allocation framework that integrates the Fama–French three-factor model with the B–L portfolio model. This incorporation forges a crucial linkage between two dominant and overarching bodies of financial scholarship: asset pricing and portfolio construction. The Fama–French model provides a robust explanation of risk premiums in asset returns, paving the way for a more comprehensive understanding of return predictability and offering profound insights into asset return behavior. Despite its significance, the integration of this seminal theory within the B–L paradigm is markedly under-researched. Our proposed model addresses this gap by enabling accurate estimation of market view distributions, while concurrently laying a solid theoretical groundwork for our pioneering approach to view construction. This advancement effectively reconciles the divide between asset pricing and portfolio literature.

Second, our study contributes to the literature on B–L portfolio modeling (Punyaleadtip et al., 2024; Barua and Sharma, 2023; Gao et al., 2023; Han and Li, 2023; Barua and Sharma, 2022; Beach and Orlov, 2007; Bessler et al., 2017; Fernandes et al., 2018;

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Kara et al., 2019; Pyo and Lee, 2018; Ko et al., 2023a) by enhancing portfolio efficiency, resulting in demonstrably superior financial gains. We achieve this by conducting the most thorough comparative analysis to date, encompassing a wide array of B–L model variants and traditional methods. Our methodological innovation lies in the creation of a unique and sustainable approach to portfolio view construction, by integrating empirically substantiated insights from the Fama–French three-factor model into the B–L portfolio construction frameworks. Notably, the extensive comparative analysis of various existing and sophisticated B–L extensions has been largely neglected in the scholarly discourse on B–L models. By undertaking a comprehensive comparative analysis with both traditional and numerous advanced B–L extension portfolio models, we address this oversight. Consequently, our proposed framework is empirically proven to provide additional financial benefits in the realm of portfolio performance evaluation.

Finally, this study makes a substantive contribution to the literature on estimation error, a critical concern in portfolio management underscored by empirical research (DeMiguel et al., 2009; Simaan et al., 2018; Simaan, 1997, 2014; Dai and Kang, 2022; Lassance et al., 2023; Platanakis et al., 2021; Kan et al., 2022). It addresses the persistent and long-standing challenges associated with this issue in the broader context of portfolio management. This point of focus is pertinent, given that despite the model's original intent to address the estimation error of the Markowitz portfolio model, comprehensive exploration of this issue in existing B–L literature remains notably scarce. Our study fills this gap by examining portfolio results in light of estimation error from an empirical perspective. As we demonstrate empirically, our proposed model significantly curtails the detrimental effect of the estimation error of expected return.

What are the advantages of our Fama–French three-factor-based B–L portfolio model? First, from the perspective of portfolio allocation guided by implications from the asset pricing model, our proposed model addresses the limitations associated with the long-short portfolio derived from the Fama–French three-factor model. In general, to capitalize on the implications of the asset pricing model, one must construct a long-short portfolio by purchasing stocks expected to yield the highest returns and selling those anticipated to return the least, utilizing all stocks listed on a specific stock market, grouped into deciles for buying and selling. Nevertheless, certain circumstances may render this methodology unfeasible, owing to a multitude of influencing variables. For instance, short selling may be unfeasible depending on the specific market, national regulations, and prevailing market conditions. Additionally, smaller stocks are often illiquid, making it challenging to buy or sell in quantities sufficient to construct the desired portfolio, and buying or selling illiquid assets causes those prices to fluctuate irrationally, critically impacting potential profitability. Furthermore, the occurrence of short squeezes is one of the formidable issues difficult to address. Lastly, the number of assets required to construct a long-short portfolio can be overwhelming, particularly to sufficiently harness the implications associated with the identified market anomaly. However, by incorporating the principles of the Fama–French three-factor model into the view distribution of the B–L portfolio, these impracticality challenges can be mitigated. This is achieved by virtually reflecting the insights of the Fama–French three-factor model into the estimation of the return distribution in the construction process of the view distribution, thereby avoiding the need for actual shorting or buying of a potentially illiquid, expansive set of stocks that carry a likelihood of instigating a short squeeze.

Second, a notable hurdle in the generation of the B–L portfolio model is the absence of a solid convention to construct a view distribution for the future market. Traditionally, views are considered arbitrary and expected to be meticulously crafted by experts. Our proposed model, however, presents a solution to this challenge by automatically constructing a view distribution based on a methodology that infuses the insights of the Fama–French three-factor model into the B–L framework. Although several articles suggest the use of advanced expert systems to generate informed views rather than arbitrary ones (Barua and Sharma, 2022; Beach and Orlov, 2007; Bessler et al., 2017; Fernandes et al., 2018; Kara et al., 2019; Pyo and Lee, 2018), our methodology distinguishes itself by its simplicity and intuitiveness as well as superiority. Moreover, it offers a theoretically informed methodological innovation that yields more profound insights. The Fama–French three-factor model is a highly regarded and widely accepted asset pricing model that effectively captures real-world financial market phenomena. By leveraging our novel methodology, we systematically incorporate this robust financial theory into the construction of market views employing firm-level variables for the Fama–French factor model. Our approach facilitates the direct application of insights derived from asset pricing to the portfolio model, leading to improved portfolio performance in real-world data compared to benchmarks. Thus, our model stands as a significant advancement in portfolio management, offering a data-driven, theory-informed mechanism for view construction that enhances the performance and practicality of a portfolio.

Our research draws inspiration from two significant lines of literature. Initially, the advent of Markowitz (1952)'s mean–variance portfolio model gave rise to one of the primary fields of research in finance, specifically portfolio theory (Byun et al., 2023; Ko et al., 2022; Pedersen and Peskir, 2017; Maccheroni et al., 2013). Within this body of literature, a paramount concern pertains to the inherent estimation error in Markowitz's original portfolio model. Numerous scholars have endeavored to address this issue (DeMiguel et al., 2009; Simaan et al., 2018; Simaan, 1997, 2014; Dai and Kang, 2022; Lassance et al., 2023; Platanakis et al., 2021; Kan et al., 2022; Li et al., 2023). A pivotal contribution in this regard was made by Black and Litterman (1991), who devised a framework that utilizes a Bayesian updating process to derive a novel representation of estimated expected returns, thereby mitigating the problem of estimation error associated with the expected return as well as improving the portfolio efficiency.

In tandem with the evolution of portfolio theory, Sharpe (1964) and Lintner (1969) developed the Capital Asset Pricing Model (CAPM), building on the foundational Markowitz mean–variance model. This model heralded the beginning of asset pricing models. Expanding on CAPM, Fama and French (1992, 1993) later introduced the three-factor model, a ground-breaking development in asset pricing literature. Their model enhances CAPM by incorporating size (Banz, 1981) and book-to-market (Rosenberg et al., 1985) factors alongside the market factor. To the best of our knowledge, however, while the Fama–French three-factor model has greatly enriched our understanding of asset return predictability, no attempts have been made thus far to integrate this influential theory with the B–L framework. Consequently, in this study, we propose a B–L portfolio model that incorporates the Fama–French three-factor model. In doing so, we provide a novel perspective on portfolio construction in the finance field. Such a pioneering perspective is paramount to the creation of a more efficient portfolio.

This study aims to (1) propose a portfolio model that integrates the economic acumen of the Fama–French three-factor model into the B–L framework and to validate this integration across diverse parametric landscapes and economic scenarios, (2) scrutinize a wide array of existing B–L variants and conventional portfolio models cited in the extant literature by conducting a rigorous and comprehensive comparative analysis, and (3) undertake a quantitative evaluation of the proposed model, focusing particularly on its capacity for reducing estimation errors—a capability derived from our novel methodology for constructing views informed by the Fama–French three-factor model. Through a large-scale empirical analysis, this study examines the outcomes of a comparative assessment between our proposed model and ten benchmark models, utilizing data from all stocks listed on the U.S. stock market over a period of 65 years, spanning from 1957 to 2021. The fulfillment of our aims in this research holds considerable relevance, extending beyond the sphere of academia to the realm of real-world financial portfolio management. The potential influence of our study, therefore, encompasses scholarly discourse and pragmatic deployment within the finance industry, reinforcing the substantial import of our work.

Our research yields four primary findings. Firstly, a comparative evaluation between our proposed model and existing benchmarks elucidates the preeminence of our model. This implies that our innovative view construction method instigates a significant diversification impact relative to the traditional approaches, thereby fostering a more optimized portfolio. Particularly, our model consistently surpasses the market index, evidenced by a more than two-fold increase in the Sharpe Ratio (SR) and an improvement of the Certainty Equivalent Return (CER) by 50%. This reveals that our active strategy culminates in a more effective portfolio relative to a passive approach. Moreover, our model markedly outperforms the long-short portfolio derived from the Fama–French three-factor model, as illustrated by a more than 2.5 times increase in the SR. This underscores the pragmatic issue concerning the application of long-short portfolios based on Fama and French's asset pricing model. Intriguingly, while adhering to similar optimization-based portfolio principles yet adopting an enhanced strategy, our proposed model significantly eclipses the conventional Markowitz portfolio framework. This is evidenced by the model achieving approximately four times the SR and triple the CER annually, confirming its superior performance.

Secondly, we validate the economic significance and unique contribution of our innovative view construction methodology through an extensive comparative analysis using numerous B–L variant models identified in the existing literature. Specifically, we use basic B–L portfolio models, the CAPM equilibrium portfolio, as reference points, comparing all B–L benchmark models against this reference. Our rigorous investigation reveals that the naive application of sample mean returns for constructing the view distribution is ineffective. In contrast, we find that econometric methods, machine learning-based algorithms, and their hybrid versions perform commendably in estimating the expected return of the B–L model. Notably, B–L models that predict expected returns based on stock-level characteristics, such as the price-to-earnings (PE) ratio or return volatility, appear highly effective in constructing financially meaningful views within the B–L framework. Nevertheless, our proposed model, grounded in the seminal and foundational economic theory of Fama and French, proves to be the most effective in predicting market views, exhibiting the highest SR, CER, and alpha among all benchmarks. This suggests that our strategy is empirically reasonable and that the proposed view construction model can facilitate a more efficient and diversified portfolio strategy, thereby proving the economic value of our proposed methodology.

Thirdly, our examination of the proposed model concerns the reduction of estimation error, the primary motivation behind the B–L framework. We effectively demonstrate the superiority of our proposed model by comparing the in-sample performance of the mean–variance model, which serves as a benchmark for optimal performance, with the out-of-sample performance of each benchmark and our model. This approach is utilized to evaluate the estimation error, aligning with the methodologies used in portfolio literature to assess estimation errors. In other words, we utilize an optimal portfolio, conceived under the premise of possessing a comprehensive information set, as a standard for comparison. In a practical out-of-sample testing environment, we contrast our suggested model and the benchmark models against this established baseline. Strikingly, the empirical outcome illustrates that the difference between our model and the optimal model is significantly lowest among those allocated by the traditional models. Specifically, the disparity in SR and CER is considerably less than the differences between the traditional models and the optimal one. This suggests a noteworthy reduction of estimation error by our methodology, which can be attributed to the inaccuracies associated with estimating the expected return under the traditional frameworks. This benefit provided by the estimation error mitigation becomes more pronounced under increased asset quantities invested and extreme market conditions, with a difference in optimization premium and diversification effect, attesting to the consistent profitability of our proposed model facilitated by the substantial mitigation of estimation error.

Our study's final crucial finding concerns the robustness and reliability of our proposed model under variable parameter settings and assorted economic conditions. Specifically, by altering the parameters concerning the number of assets included in the portfolio, our model is shown to effectively harness the diversification effect offered by expanded asset pools. This successfully mitigates overfitting to in-sample data, resulting in a more appropriate fit for the expected return to out-of-sample data. This advantage is made possible through our innovative Fama–French three-factor-based view construction approach, unlike the shortcomings of the Markowitz framework in exploiting the diversification effect of expanded asset pools. Moreover, from an economic viewpoint, our model consistently surpasses the benchmark models even under highly volatile market conditions, emphasizing the model's reliability amidst extreme and turbulent market fluctuations. Additionally, our proposed model's performance superiority is maintained regardless of the investment periods, whether before or after a financial crisis. This underscores the model's sustainability over varying periods. We also confirm the reliability of the main argument under consideration of the transaction cost levels. Finally, our study robustly validates the proposed model, affirming its consistent effectiveness in recent periods and demonstrating its global applicability. Overall, these findings suggest a long-standing presence of the size and book-to-market effect and a persistent capacity to exploit efficiency benefits related to these effects, whose efficacy is attributed to our ground-breaking view construction methodology based on the Fama–French three-factor model.

Our study is structured as follows: Section 2 provides essential preliminaries necessary for comprehensive understanding. Section 3 introduces our proposed B–L portfolio model, grounded in the Fama–French three-factor approach, along with the innovative methodology for constructing this view distribution. Section 4 delves into the data and experimental design. Empirical findings are unveiled in Section 5. Finally, the paper is concluded in Section 6.

2. Preliminaries

2.1. Fama–French Three-Factor Model

The Fama–French Three-Factor Model is a seminal asset pricing model that extends the CAPM. Introduced by Fama and French (1992, 1993), this model seeks to improve the explanatory power of market returns by incorporating two additional factors beyond the market risk premium used in CAPM. Initially in CAPM, the asset excess return r_i can be described by the following explanatory model:

$$r_i = R_i - R_f = a_i + b_i(R_m - R_f). \tag{1}$$

where R_i signifies the return on the ith asset, while R_f indicates the risk-free rate. In contrast, the Fama–French model incorporates three factors: market risk, size effect, and value effect. The first factor is the market risk premium, which reflects the excess return of the market portfolio over the risk-free rate. The second factor captures the size effect, as measured by the difference in returns between small and large companies in market capitalization, represented by the 'SMB' (Small Minus Big) factor. Lastly, the value effect is captured through the 'HML' (High Minus Low) factor, representing the spread in returns between companies with high (value) and low (growth) book-to-market ratios. According to the Fama–French three-factor model, the asset return $r_{i,t}$ at a given time t can be described by the following explanatory model:

$$r_{i,t} = R_{i,t} - R_{f,t} = a_i + b_i(R_{m,t} - R_{f,t}) + s_i SMB_t + h_i HML_t. \tag{2}$$

where $R_{i,t}$ signifies the return of the ith asset at time t, while $R_{f,t}$ represents the risk-free rate at the same time point. The equation comprehensively incorporates three distinct risk factors. The first, known as the market risk premium, is the excess return of the market portfolio ($R_{m,t}$) over the risk-free rate ($R_{f,t}$), and is expressed as $R_{m,t} - R_{f,t}$. The second factor, which accounts for the size effect, is denoted by the 'SMB' (Small Minus Big) factor, reflecting the differential in returns between firms with small and large market capitalizations. The third factor, representing the value effect, is encapsulated by the 'HML' (High Minus Low) factor, which conveys the disparity in returns between companies with high and low book-to-market ratios, categorizing them as value and growth firms, respectively.

By introducing the size and value factors, the Fama–French model successfully addresses certain empirical anomalies that are not explained by the CAPM. In essence, it provides a more comprehensive and empirically robust framework for evaluating the expected returns of assets and has found wide adoption in both academic research and investment management (Grauer and Janmaat, 2010; Foye et al., 2013; Griffin, 2002).

2.2. Markowitz framework

The seminal study by Markowitz (1952), which forms the bedrock of Modern Portfolio Theory, suggests that a mean–variance framework can be acquired through the resolution of the following optimization problem:

$$\begin{aligned} \min_{\boldsymbol{w}} \quad & \frac{\lambda}{2}\boldsymbol{w}^\top \boldsymbol{\Sigma} \boldsymbol{w} - \boldsymbol{w}^\top \boldsymbol{\mu}, \\ \text{s.t.} \quad & \boldsymbol{w}^\top \mathbf{1} = 1 \end{aligned} \tag{3}$$

where $\boldsymbol{w} \in \Re^N$ denotes the portfolio's weight vector, $\boldsymbol{\Sigma} \in \Re^{N\times N}$ signifies the variance–covariance matrix of asset returns, $\boldsymbol{\mu} \in \Re^N$ designates the vector of expected returns, and $\lambda \in \Re^1$ corresponds to the risk aversion coefficient of the investor.

The analytical solution of Eq. (3) can be computed as per Merton (1972) in the following manner:

$$\boldsymbol{w}^* = \frac{\mathbf{1}^\top \boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}}{\lambda} \cdot \frac{\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}}{\mathbf{1}^\top \boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}} + (1 - \frac{\mathbf{1}^\top \boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}}{\lambda}) \cdot \frac{\boldsymbol{\Sigma}^{-1}\mathbf{1}}{\mathbf{1}^\top \boldsymbol{\Sigma}^{-1}\mathbf{1}} \tag{4}$$

The aforementioned symbols bear the same connotations as in the previous equation, with the addition of $\boldsymbol{w}^*$, which signifies the optimal weight vector of the portfolio.

The Markowitz portfolio optimization model, despite its utility and theoretical convenience, exhibits a significant limitation with respect to its practicality. The optimality of the resultant portfolio is highly susceptible to alterations in the input parameters, such as expected returns and the estimated covariance matrix (Best and Grauer, 1991). Even minuscule variations in these estimations can incite substantial shifts in the optimized portfolio's composition. This amplification effect, a product of the model's innate characteristic, magnifies the repercussions of estimation inaccuracies, often producing portfolios that defy intuitive investment strategies. Such portfolios, marked by their questionable structure, pose challenges for out-of-sample applications and, consequently, for actual implementation (DeMiguel et al., 2009).

2.3. BL framework

The susceptibility of optimization to errors in estimating expected returns remains a profound concern as aforementioned (Best and Grauer, 1991). Sole reliance on historical time-series data for estimation can precipitate suboptimal portfolio performance when applied to out-of-sample scenarios. Given that the sample mean presents considerable challenges for accurate estimation from historical data, enhancing the accuracy of estimations while operating within the constraints of a fixed time length necessitates a fundamental transformation in the architecture of the estimation methodology.

To address the problem of estimation errors in expected returns intrinsic to the traditional Markowitz mean–variance optimization model, Black and Litterman (1991) proposed an innovative approach. Their B–L model directly tackles the inherent shortcomings of the Markowitz framework, enhancing portfolio optimization by blending the equilibrium market returns with the views of investors to derive a set of posterior expected returns. This fusion effectively neutralizes the potential pitfalls of estimation errors, leading to the creation of more stable and rational portfolios. The B–L model commences its operation from an equilibrium state, which assumes no specific predilection for any particular security. Subsequently, it fine-tunes the portfolio weights incrementally, anchored in the strength of the investor views. This refined and subtle approach presents a more pragmatic and reliable model for portfolio optimization, exhibiting its distinctive resilience against estimation errors.

In the B–L framework, one must initially determine the conditional implied equilibrium expected return vectors of the CAPM market portfolio. This can be achieved using the following equation:

$$\boldsymbol{\pi}_t = \lambda \boldsymbol{\Sigma}_t \boldsymbol{w}_{\boldsymbol{mkt},t-\mathbf{1}}, \tag{5}$$

where $\boldsymbol{\pi}_t \in \Re^N$ represents the conditional implied equilibrium expected return vector, while λ symbolizes the risk aversion coefficient. Furthermore, $\boldsymbol{\Sigma}_t \in \Re^{N\times N}$ is indicative of the conditional historical covariance matrix of the excess return vector at time t. Lastly, $\boldsymbol{w}_{\boldsymbol{mkt},t-\mathbf{1}} \in \Re^N$ corresponds to the market capitalization weight of the assets at time $t-1$.

Assume that the prior distribution of the conditional expected excess return is denoted by $\boldsymbol{\mu}_t \sim \mathcal{N}(\boldsymbol{\pi}_t, \tau\boldsymbol{\Sigma}_t)$. Consequently, the conditional distribution of $\boldsymbol{q}_t$ is represented as $\boldsymbol{q}_t|\boldsymbol{\mu}_t \sim \mathcal{N}(\boldsymbol{P}_t\boldsymbol{\mu}_t, \boldsymbol{\Omega}_t)$, where $\boldsymbol{P}_t \in \Re^{K\times N}$ is the conditional matrix reflecting the experts' viewpoints, $\boldsymbol{q}_t \in \Re^K$ is the conditional expected excess return vector corresponding to those views, and $\boldsymbol{\Omega}_t \in \Re^{K\times K}$ is the diagonal covariance matrix of the views.

By employing a Bayesian framework, we can derive both the marginal distribution for vector $\boldsymbol{q}_t$ and the updated conditional expected excess returns, denoted by $\boldsymbol{\mu}_{\boldsymbol{posterior},t}$. This follows the methodology delineated by Pyo and Lee (2018)[1] where the process can be formulated as follows:

$$\begin{aligned} \boldsymbol{q}_t &\sim \mathcal{N}(\boldsymbol{P}_t\boldsymbol{\pi}_t, \boldsymbol{\Omega}_t + \boldsymbol{P}_t(\tau\boldsymbol{\Sigma}_t)\boldsymbol{P}_t^\top) \\ \boldsymbol{\mu}_{\boldsymbol{posterior},t} &\sim \mathcal{N}(\boldsymbol{\mu}_{\boldsymbol{BL},t}, \boldsymbol{\Sigma}_{\boldsymbol{BL},t}). \end{aligned} \tag{6}$$

In this context, the parameters denoted as $\boldsymbol{\Sigma}_{\boldsymbol{BL},t}$ and $\boldsymbol{\mu}_{\boldsymbol{BL},t}$ are hereby obtained as follows:

$$\boldsymbol{\Sigma}_{\boldsymbol{BL},t} = [(\tau\boldsymbol{\Sigma}_t)^{-1} + \boldsymbol{P}_t^\top\boldsymbol{\Omega}_t^{-1}\boldsymbol{P}_t]^{-1}, \tag{7}$$

$$\boldsymbol{\mu}_{\boldsymbol{BL},t} = \boldsymbol{\Sigma}_{\boldsymbol{BL},t}[(\tau\boldsymbol{\Sigma}_t)^{-1}\boldsymbol{\pi}_t + \boldsymbol{P}_t^\top\boldsymbol{\Omega}_t^{-1}\boldsymbol{q}_t]. \tag{8}$$

Upon ascertaining the value of $\boldsymbol{\mu}_{\boldsymbol{BL},t}$, the optimal weighting of the B–L portfolio can be calculated by addressing the following optimization problem, as prescribed by Idzorek (2007):

$$\min_{\boldsymbol{w}_t} \frac{\lambda}{2}\boldsymbol{w}_t^\top\boldsymbol{\Sigma}_t\boldsymbol{w}_t - \boldsymbol{w}_t^\top\boldsymbol{\mu}_{\boldsymbol{BL},t}. \tag{9}$$

Therefore, by adhering to the first-order condition,[2] the optimal portfolio is obtained:

$$\boldsymbol{w}_t^* = (\lambda\boldsymbol{\Sigma}_t)^{-1}\boldsymbol{\mu}_{\boldsymbol{BL},t}. \tag{10}$$

3. Proposed methodology

In this paper, we propose an innovative approach to configuring the view distribution within the B–L framework. This approach extends and adapts the well-established Fama–French three-factor model for asset pricing and applies its insights to defining view distribution such as view return $\boldsymbol{q}_t$ and view matrix $\boldsymbol{P}_t$. Our proposed methodology is not only clear and intuitive but also bears considerable similarity to the well-recognized method of portfolio grouping in the Fama–French three-factor model.

The foremost essential step prior to constructing the view distribution entails generating a grid of 25 portfolios predicated on the Fama–French three-factor model, which is typically delineated via a 5x5 matrix constructed based on firm size and the book-to-market ratio. The outcome of this process is a detailed grid comprising 25 portfolios, each distinctly classified according to these two determining characteristics. In the original implementation of the Fama–French three-factor model, a specific procedure was employed to construct the 5x5 portfolio grid. Specifically, breakpoints for size and book-to-market ratio, drawn from NYSE stocks,

[1] For additional information, consult Equation (A1) in Appendix A.

[2] Expanding upon the research of He and Litterman (2002), we confirm that the sum of elements in $\boldsymbol{w}_t^*$ equals to 1.

are employed to allocate stocks from NYSE, AMEX, and NASDAQ to five size quintiles and five book-to-market ratio quintiles. Subsequently, 25 portfolios were constructed from the intersections of these size and book-to-market ratio quintiles.

However, our approach diverges slightly from conventional procedures in adapting to the B–L portfolio framework. This deviation arises from the potential limitation presented when the asset pool is insufficient.[3] This deficit could result in a situation where one group within the five-by-five portfolio does not contain a minimum of one asset. To rectify this problem, we propose an amendment to the Fama–French methodology that ensures each category houses at least one asset. This resolution is achieved by first allocating assets into groups based on their predicted sizes. Subsequently, within each of these size-determined groups, assets are further classified according to their predicted book-to-market ratios. This two-step grouping procedure ensures a more equitably distributed assortment of assets across the portfolio and accords with the Fama–French model's stress on size and book-to-market ratios.[4]

In light of the aforementioned procedure, we subsequently introduce the theoretical underpinning of view return, represented by $\boldsymbol{q}_t$. The view return, within the scope of our model, is conceived to delineate the concept provided by the yield of a long-short portfolio with equal weighting. The return of this portfolio is hinged upon forthcoming asset returns, as projected by the Fama–French three-factor model.

As a second point, we introduce the methodology to construct the view matrix $\boldsymbol{P}_t$, a crucial component of our model. For coherence with the rationale underpinning the established view return $\boldsymbol{q}_t$, we suggest assigning positive exposure weights to the first group of portfolios and negative exposure weights to the final group of portfolios. These become the elements of the view matrix $\boldsymbol{P}_t$, a 1 by N matrix, with N representing the number of assets. Specifically, the elements corresponding to the first group of portfolios are assigned a value of $1/|D_1|$, where $|D_1|$ denotes the cardinality of the first group. Conversely, the last group is assigned a value of $-1/|D_{25}|$, where $|D_{25}|$ signifies the cardinality of the final group. The remaining assets are allocated a value of 0. This strategy embodies the long-short portfolio concept inherent in the Fama–French three-factor model, which suggests a long position on assets in the first group and a short position on assets in the last group. When distributing the weights to the equities in the first and last groups, we employ $|D_1|$ and $|D_{25}|$ as denominators for the first and final group, respectively, for each stock within each group. This ensures that the weight sum of the first group is one and the weight sum of the last group is -1. Consequently, this procedure guarantees that the sum of all weights amounts to zero. As a result, the view matrix $\boldsymbol{P}_t$, as per our proposed methodology, can be articulated as follows:

$$\boldsymbol{P}_t = \begin{bmatrix} \frac{1}{|D_1|} & \dots & 0 & \dots & -\frac{1}{|D_{25}|} \end{bmatrix} \tag{11}$$

Thirdly, we progress to define $\hat{\boldsymbol{r}}_t = [\hat{r}_{1,t}, \dots, \hat{r}_{N,t}]$, aligning this concept with the pre-established notion of $\boldsymbol{q}_t$. Traditionally, the asset return $r_{i,t}$ at a given time t can be described via Eq. (2).

However, contemplating the implications of 'view' within the B–L framework, the estimation of $r_{i,t}$, denoted by $\hat{r}_{i,t}$, entails a degree of foresight into the future trajectory of the market. Therefore, to harmonize with the B–L framework's anticipatory concept of future market 'view,' we advocate for a predictive variant derived from the Fama–French three-factor model to estimate $\hat{r}_{i,t}$, in line with the spirit of Fama and French. This estimation can be formulated as follows:

$$\hat{r}_{i,t} = a_i + b_i(\overline{R}_{m,t-1} - \overline{R}_{f,t-1}) + s_i\overline{SMB}_{t-1} + h_i\overline{HML}_{t-1}, \tag{12}$$

where $\overline{R}_{m,t-1}$, $\overline{R}_{f,t-1}$, $\overline{SMB}_{t-1}$, $\overline{HML}_{t-1}$ denote the prevalent sample averages of R_m, R_f, SMB, and HML until time $t-1$, respectively.

The fourth point to consider involves the determination of $\boldsymbol{P}_t$ and $\hat{\boldsymbol{r}}_t$, which allows us to define $\boldsymbol{q}_t$ through the ensuing equation:

$$\boldsymbol{q}_t = \boldsymbol{P}_t\hat{\boldsymbol{r}}_t \tag{13}$$

$$= \left[\frac{1}{|D_1|}\Sigma_{i\in D_1}\hat{r}_{i,t} - \frac{1}{|D_{25}|}\Sigma_{i\in D_{25}}\hat{r}_{i,t}\right] \tag{14}$$

$$= \left[E_{i\in D_1}[\hat{r}_{i,t}] - E_{i\in D_{25}}[\hat{r}_{i,t}]\right], \tag{15}$$

where $\boldsymbol{q}_t$, one by one matrix, signifies the view return at time t. D_l represents the lth group within the set of 25 portfolios, and $\hat{r}_{i,t}$ denotes the estimated asset return at time t. By defining a value to $\boldsymbol{q}_t$ as outlined in Eq. (13), we encapsulate, in essence, our investment view. This perspective posits that assets within the first portfolio group will yield higher expected returns than those in the 25th portfolio group, under the premise that assets are distributed into five by five groups according to the size and the book-to-market ratio, which is underpinned by the Fama–French three-factor model.

Subsequent to the determination of $\boldsymbol{q}_t$, we naturally establish $\boldsymbol{\Omega}_t$, a 1 by 1 matrix, through the equation:

$$\boldsymbol{\Omega}_t = \left[\tau \boldsymbol{p}^1_{\cdot,t}\boldsymbol{\Sigma}_t\boldsymbol{p}^{1'}_{\cdot,t}\right], \tag{16}$$

where $\boldsymbol{p}^1_{\cdot,t}$ is representative of the first row in the view matrix $\boldsymbol{P}_t$.

[3] Portfolio models based on optimization, such as the B–L framework, are capable of functioning only in scenarios where there is a constraint on the maximum number of assets. This is primarily due to the model's handling of inverse computations pertaining to covariance, which is distinguished from factor-based asset pricing models.

[4] The use of "predicted" variables offers flexibility and represents a significant advantage of the proposed framework. This feature aligns with the implication of "view" inherent in the B–L framework, facilitating the versatile incorporation of diverse predictive methods, such as the approaches employed by Pyo and Lee (2018) or ensemble methods. In the present study, we adopt an ensemble-based approach.

To elucidate the contributions of our suggested approach, we progress beyond Eq. (10) towards a more intuitive expression of $\boldsymbol{w}_t^*$, adopting the lens of our proposed methodology. This presentation aims to enhance the clarity of the theoretical foundations of our approach and ensure that it is logically coherent and easily comprehensible. This elaboration is encapsulated in Eq. (17) as per Proposition 1. While Eq. (17) draws upon the research conducted by Jones et al. (2007), this manuscript does not contain a concrete demonstration for this equation. Hence, we undertake the task of providing detailed proof to supplement our discussion.

Proposition 1. *Consider the matrices $\boldsymbol{P}_t \in \Re^{K\times N}$, $\boldsymbol{\Sigma}_t \in \Re^{N\times N}$, and $\boldsymbol{\Omega}_t \in \Re^{K\times K}$, and vectors $\boldsymbol{q}_t \in \Re^K$, and $\boldsymbol{\pi}_t \in \Re^N$. Let $\boldsymbol{P}_t(\tau\boldsymbol{\Sigma}_t)\boldsymbol{P}_t^\top + \boldsymbol{\Omega}_t$ be invertible. Then, $\boldsymbol{w}_t^*$ can be expressed as follows:*

$$\boldsymbol{w}_t^* = \boldsymbol{w}_{\boldsymbol{mkt},\boldsymbol{t-1}} + \boldsymbol{P}_t^\top \boldsymbol{v}_t, \tag{17}$$

where $\boldsymbol{v}_t \in \Re^K$.

Proof. Substituting Eqs. (7) and (8) into Eq. (10), we have:

$$\boldsymbol{w}_t^* = (\lambda\boldsymbol{\Sigma}_t)^{-1}[(\tau\boldsymbol{\Sigma}_t)^{-1} + \boldsymbol{P}_t^\top\boldsymbol{\Omega}_t^{-1}\boldsymbol{P}_t]^{-1}[(\tau\boldsymbol{\Sigma}_t)^{-1}\boldsymbol{\pi}_t + \boldsymbol{P}_t^\top\boldsymbol{\Omega}_t^{-1}\boldsymbol{q}_t]. \tag{18}$$

Assume that $\boldsymbol{P}_t(\tau\boldsymbol{\Sigma}_t)\boldsymbol{P}_t^\top + \boldsymbol{\Omega}_t$ is invertible.[5] Then, the application of the Sherman–Morrison–Woodbury formula[6] results in the following expression derived from Eq. (18):

$$\begin{aligned}\boldsymbol{w}_t^* &= (\lambda\boldsymbol{\Sigma}_t)^{-1}[\boldsymbol{\pi}_t + \tau\boldsymbol{\Sigma}_t\boldsymbol{P}_t^\top(\boldsymbol{P}_t(\tau\boldsymbol{\Sigma}_t)\boldsymbol{P}_t^\top + \boldsymbol{\Omega}_t)^{-1}(\boldsymbol{q}_t - \boldsymbol{P}_t\boldsymbol{\pi}_t)]\\ &= (\lambda\boldsymbol{\Sigma}_t)^{-1}\boldsymbol{\pi}_t + (\tau/\lambda)\boldsymbol{P}_t^\top(\boldsymbol{P}_t(\tau\boldsymbol{\Sigma}_t)\boldsymbol{P}_t^\top + \boldsymbol{\Omega}_t)^{-1}(\boldsymbol{q}_t - \boldsymbol{P}_t\boldsymbol{\pi}_t).\end{aligned} \tag{19}$$

From Eq. (5), where $\boldsymbol{w}_{\boldsymbol{mkt},\boldsymbol{t-1}} = (\lambda\boldsymbol{\Sigma}_t)^{-1}\boldsymbol{\pi}_t$, Eq. (19) can be rewritten as follows:

$$\boldsymbol{w}_t^* = \boldsymbol{w}_{\boldsymbol{mkt},\boldsymbol{t-1}} + \boldsymbol{P}_t^\top[(\tau/\lambda)(\boldsymbol{P}_t(\tau\boldsymbol{\Sigma}_t)\boldsymbol{P}_t^\top + \boldsymbol{\Omega}_t)^{-1}(\boldsymbol{q}_t - \boldsymbol{P}_t\boldsymbol{\pi}_t)]. \tag{20}$$

Given that $\boldsymbol{P}_t \in \Re^{K\times N}$, $\boldsymbol{\Sigma}_t \in \Re^{N\times N}$, $\boldsymbol{\Omega}_t \in \Re^{K\times K}$, $\boldsymbol{q}_t \in \Re^K$, and $\boldsymbol{\pi}_t \in \Re^N$, we define $\boldsymbol{v}_t = (\tau/\lambda)(\boldsymbol{P}_t(\tau\boldsymbol{\Sigma}_t)\boldsymbol{P}_t^\top + \boldsymbol{\Omega}_t)^{-1}(\boldsymbol{q}_t - \boldsymbol{P}_t\boldsymbol{\pi}_t) \in \Re^K$, thereby validating Eq. (17). □

Drawing upon Proposition 1, the optimal weight vector $\boldsymbol{w}_t^*$ can be reformulated as illustrated in Eq. (17). This reframing clarifies the intuition and conceptual underpinnings for our proposed model. In this context, $\boldsymbol{P}_t$ represents the view matrix, signifying how weights in relation to each view are apportioned to each asset, while $\boldsymbol{v}_t$, determined by the view distribution, designates the weight vector assigned to each view.

In our proposed model, $\boldsymbol{P}_t \in \Re^{1\times N}$ is a view matrix structured by five-by-five portfolios based on the Fama–French three-factor model, reflecting salient information about size and book-to-market ratio characteristics. Concurrently, $\boldsymbol{v}_t \in \Re^1$ symbolizes the view weight informed by the implication of the Fama–French three-factor asset pricing model, indicating a magnitude of shift from the baseline weights.

Consequently, the optimal weight $\boldsymbol{w}_t^*$ represents the updated weight derived from the CAPM weight $\boldsymbol{w}_{\boldsymbol{mkt},\boldsymbol{t-1}}$, adjusted by the additional Fama–French three-factor view weight, $\boldsymbol{P}_t^\top\boldsymbol{v}_t$. In shifting $\boldsymbol{w}_{\boldsymbol{mkt},\boldsymbol{t-1}}$ by $\boldsymbol{P}_t^\top\boldsymbol{v}_t$, assets (predicted to be) with small size and higher book-to-market ratio are attuned to receive greater weight than those predicted by CAPM, while assets (predicted to be) with large size and lower book-to-market ratio are attuned to receive less weight. By employing Eq. (17) within our proposed model, we derive a more intuitive and efficient portfolio. This portfolio effectively and systematically capitalizes on the enduring implications observable in financial markets, as elucidated by the Fama–French three-factor asset pricing model.

4. Data and experimental design

4.1. Data

The methodology applied in the present investigation is underpinned by an extensive corpus of empirical data spanning a period of 65 years from 1957 to 2021. The focus of the investigation is a thorough analysis of individual asset returns, sourced on a monthly basis from the Center for Research in Security Prices (CRSP) for all firms indexed on the AMEX, NASDAQ, and NYSE. The parameters of size and book-to-market ratios employed in the study are appropriated from the dataset proposed by Gu et al. (2020). As a surrogate for the risk-free rate, the study utilizes the one-month Treasury bill rate, echoing the methodology applied by Fama and French (1993). To compute individual excess returns, we extract simple return data from the Wharton Research Data Services (WRDS) database, which was subsequently merged with the Treasury bill rate.

In terms of comparative indices, the S&P 500 index, procured from Yahoo Finance,[7] serves as the market index. The secondary reference point is a long-short portfolio structured in accordance with the Fama–French three-factor model. This necessitated the

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[5] In our proposed model, we consider $\boldsymbol{P}_t \in \Re^{1\times N}$. In such a context, the invertibility of $\boldsymbol{P}_t(\tau\boldsymbol{\Sigma}_t)\boldsymbol{P}_t^\top + \boldsymbol{\Omega}_t$ is reliant upon the validation that $\tau\boldsymbol{p}^1_{\cdot,t}\boldsymbol{\Sigma}_t\boldsymbol{p}^{1'}_{\cdot,t} \in \Re^1$ is not equal to zero. To affirm the invertibility of this latter expression, we have conducted empirical validation for all simulation scenarios, thereby confirming our postulation.

[6] Let $\boldsymbol{A} \in \Re^{N\times N}$ and $\boldsymbol{C} \in \Re^{K\times K}$ be square invertible matrices, and let $\boldsymbol{U} \in \Re^{N\times K}$ and $\boldsymbol{V} \in \Re^{K\times N}$. Let $\boldsymbol{C}^{-1} + \boldsymbol{V}\boldsymbol{A}^{-1}\boldsymbol{U}$ be invertible. Then, $(\boldsymbol{A} + \boldsymbol{U}\boldsymbol{C}\boldsymbol{V})^{-1} = \boldsymbol{A}^{-1} - \boldsymbol{A}^{-1}\boldsymbol{U}(\boldsymbol{C}^{-1} + \boldsymbol{V}\boldsymbol{A}^{-1}\boldsymbol{U})^{-1}\boldsymbol{V}\boldsymbol{A}^{-1}$ holds.

[7] https://finance.yahoo.com

acquisition of market, SMB (small minus big), and HML (high minus low) factor returns, which were accessed from the data repository available on Ken French's website.[8]

4.2. Experimental design

To compute the covariance matrix, $\boldsymbol{\Sigma}_t$, in Eq. (10), we draw on 468 months of historical returns leading up to time $t-1$[9] for our estimation. One of the primary objectives of our study is to scrutinize the estimation error of expected return, as provided by our proposed variant of the B–L framework. Hence, by determining an adequate and sufficient window size for estimating the $\boldsymbol{\Sigma}_t$, we aim to minimize the potential adverse impact of the covariance matrix's estimation error.

In addition, in accordance with the work of Idzorek (2007), the risk aversion coefficient (λ) of the US market is roughly 3.07. Consequently, we utilize this value as the value of λ when obtaining $\boldsymbol{\pi}_t$ in our study.

In a manner akin to the original Fama–French approach, we formulate a five-by-five matrix adhering to the established setting. In June of each year t, all assets included in our investment portfolio are sorted in ascending order by predicted size and subsequently divided into quintile groups. Within each group, we then sort the assets in descending order by their predicted book-to-market ratio, establishing quintile sub-groups based on this ratio within each size group. The book-to-market ratio is calculated using the book value for the fiscal year ending in calendar year $t-1$ and the market value at the close of December of $t-1$. After generating the five-by-five matrix, we can deduce the $\boldsymbol{P}_t$ as depicted in Eq. (11).

To estimate the parameters a_i, b_i, s_i, and h_i in Eq. (12), we employ 468 months of historical returns for all i, along with the factor-mimicking returns from Ken French up until time $t-1$. This is done through performing a regression as depicted in Eq. (2). Based on obtained coefficients as aforementioned, in order to acquire $\hat{r}_{i,t}$ as shown in Eq. (12), we also utilize the mean of each factor returns over the 468-month historical dataset up until time $t-1$.

For the value of τ in Eq. (16), we adhere to the precedent established by Pyo and Lee (2018) and set it to 0.1 accordingly.

Given the number of assets N included in the investment basket, we derive $\boldsymbol{\Sigma}_t$, $\boldsymbol{\pi}_t$, $\boldsymbol{P}_t$, $\hat{\boldsymbol{r}}_t$, $\boldsymbol{q}_t$, and $\boldsymbol{\Omega}_t$. This allows us to obtain $\boldsymbol{\mu}_{BL}$ and subsequently solve the optimization problem to derive the portfolio weight $\boldsymbol{w}_t^*$ as per Eq. (10). Once the optimal portfolio weight $\boldsymbol{w}_t^*$ is derived, we apply this weight for one-year-based out-of-sample testing at time t. In each backtesting, monthly portfolio returns are computed from July of year t to June of $t+1$.[10] Our approach utilizes a rolling-window framework for out-of-sample testing to maintain the temporal order of the data, with annual portfolio weight rebalancing. As a result, we conducted a total of 26-year (312-month) out-of-sample backtests.

We randomly select N stocks from the investment universe without replacement. This selection process aligns with the settings of optimization-based portfolio frameworks such as Markowitz or the B–L portfolio. To ensure the robustness of our experiments, we repeat the 26-year out-of-sample portfolio simulation 10,000 times, using a distinct set of N assets randomly drawn for each trial.

5. Empirical results

5.1. Benchmarks and performance evaluation

5.1.1. Benchmarks

In order to comprehensively assess the efficiency of our proposed model, we conduct a comparative analysis by contrasting it with several prominent benchmark models. The benchmark models under consideration are classified into two distinct categories. The first category encompasses general benchmarks, which comprise the market index, the Fama–French three-factor-based long-short portfolio approach, and the optimization-based mean–variance portfolio model. The market index is specifically represented by the S&P 500 index, which serves as a widely recognized proxy for stock market performance. This market index holds substantial significance for two key reasons. Firstly, if we assume an efficient market, the market index represents an optimal investment strategy. Secondly, it serves as the baseline for passive investment approaches, wherein investors opt for straightforward ETF investments without employing sophisticated, active strategies.

We include the Fama–French three-factor-based long-short portfolio as one of the primary comparative models, given that our proposed model is a portfolio model that incorporates the Fama–French three-factor-based view construction methodology. Both our proposed model and this benchmark model share the common feature of utilizing the implications provided by the Fama–French three-factor model. However, the main distinction lies in the approach taken to implement these implications. In the benchmark model, direct buying and selling of assets are required based on the ranking determined by the asset pricing model. In contrast, our model does not necessitate such actions. Instead, our model subtly adjusts the original portfolio weights to align with the implications of the asset pricing model in an abstract manner. Lastly, we use the mean–variance portfolio model as the third

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[8] https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

[9] The selection of the window size for estimating the covariance matrix represents a balancing act between minimizing the estimation error of covariance and maximizing the quantity of out-of-sample data for testing. Our choice of a 468-month estimation period is the result of this equilibrium. However, we have also conducted tests under various parameters, including windows ranging from 120 to 456 months, with incremental steps of 12 months. The findings from these experiments are fundamentally consistent with the results derived from the 468-month period. Complete results are available upon request.

[10] This is consistent with Fama and French (1993), who confirmed they calculate returns starting in July of year t to ensure the book equity for the year $t-1$ is known.

Table 1
List of various benchmark models considered.

#	Model	Author	Abbreviation
Passive strategy			
1.	S&P 500 index for market proxy	–	s&p
Classical approach			
2.	Fama–French three-factor model-based long-short portfolio	Fama and French (1992, 1993)	FF3-ls
3.	Markowitz's mean–variance portfolio (tangency portfolio)	Markowitz (1952)	mvp
B–L reference			
4.	B–L with neutral view (CAPM equilibrium)	Black and Litterman (1991)	BL-capm
B–L variants			
5.	B–L with CNN-BiLSTM view	Barua and Sharma (2022)	BL-cb
6.	B–L with EGARCH-in-mean view	Beach and Orlov (2007)	BL-em
7.	B–L with sample mean return views	Bessler et al. (2017)	BL-sm
8.	B–L with PE ratio-based return forecasting view	Fernandes et al. (2018)	BL-prf
9.	B–L with ARMA-GARCH-SVR view	Kara et al. (2019)	BL-ags
10.	B–L with ANN-based volatility prediction view	Pyo and Lee (2018)	BL-avp

Notes. This table enumerates the diverse conventional benchmarks and Black–Litterman (B–L) extensions under examination. The first column of the table lists the names of various investment models, while the second column provides the respective references for each. The final column of the table provides abbreviations for each strategy, facilitating reference in subsequent tables where we juxtapose the performance of these benchmark strategies with that of the proposed model. The term 'Passive strategy' refers to an investment approach that involves passively mimicking the S&P 500 market index. The 'Classical approach' encompasses traditional investment models, such as the Fama–French Three-Factor Model which focuses on constructing long-short portfolios, and the Mean-Variance Model developed by Markowitz. 'B–L Reference' denotes the foundational strategy within the B–L framework, which aligns with the Capital Asset Pricing Model equilibrium portfolio. Finally, 'B–L variants' pertain to six sophisticated versions of the B–L model, each employing a unique method for the formulation of views.

primary comparative methodology, employing an optimization-based approach. Addressing the issue of estimation error inherent in the traditional Markowitz portfolio model is one of the key objectives of our study. As a result, it is essential for us to accurately demonstrate the outcomes of our mitigation efforts through our proposed model by comparison to the traditional mean–variance model.

The second category encompasses a range of B–L model variants as identified in contemporary research. Specifically, our study utilizes the CAPM market equilibrium portfolio, representing the foundational B–L model with a neutral view. This model serves as a reference within the B–L framework and is essential for highlighting the unique contributions derived from integrating the Fama–French three-factor model in the construction of the investor view. Furthermore, this research incorporates six additional B–L variants, each offering a distinct philosophy for constructing views. These include the B–L model augmented with views based on a Convolutional Neural Network and Bidirectional Long Short-Term Memory (CNN-BiLSTM) approach (Barua and Sharma, 2022), the B–L model with views derived from an Exponential Generalized Autoregressive Conditional Heteroskedasticity-in-mean (EGARCH-in-mean) methodology (Beach and Orlov, 2007; Duqi et al., 2014), the B–L model utilizing sample mean returns as views (Bessler et al., 2017, 2021), the B–L model incorporating return forecasting based on the PE ratio (Fernandes et al., 2018), the B–L model employing a hybrid approach that integrates Autoregressive Moving-Average with Generalized Autoregressive Conditional Heteroskedasticity and Support Vector Regression (ARMA-GARCH-SVR) (Kara et al., 2019), and the B–L model with views based on volatility prediction using Artificial Neural Networks (ANN) (Pyo and Lee, 2018). Through this comprehensive and detailed comparative analysis, the study aims to demonstrate the relative superiority of the proposed model among various B–L extensions in the existing literature, thereby empirically substantiating the distinctive contribution of our research. All benchmarks under consideration, along with their respective succinct explanations, are delineated in Table 1.

5.1.2. Performance evaluation

In order to compare the performance of all portfolio models, we utilize several key measures. These include the annualized expected return of the portfolio, the annualized standard deviation of portfolio returns as risk indicators, and the alpha, which denotes the portfolio's excess return over the market benchmark.[11] Additionally, we focus on two prominent risk-adjusted measures: the SR and the CER. The SR is calculated as the excess return of the portfolio over the risk-free rate divided by the portfolio's standard deviation. Mathematically, it can be expressed as $(E(r_p) - r_f)/\sigma_p$, where $E(r_p)$ represents the expected return of the portfolio, r_f denotes the risk-free rate, and σ_p signifies the risk associated with the portfolio. On the other hand, the CER reflects the return that investors would accept with certainty rather than taking a chance on a gamble with a higher expected return but also higher risk. CER is calculated as $E(r_p) - \frac{\lambda}{2} \cdot \sigma_p^2$, where λ represents the risk aversion of investors. In this research, to maintain consistency and coherence in calculating the CER, we adopt the same value of 3.07 as the risk aversion coefficient (λ), aligning with the value used for deriving $\boldsymbol{\pi}$.

[11] Notably, the S&P 500 index serves as the proxy for market returns in this context. Consequently, the calculation of alpha is based on the comparative performance against the returns of the S&P 500 index.

Table 2
Comparative performance metrics of the proposed and different benchmark portfolio models.

Portfolio Models	Mean Ret.	Std.	Skew.	Kurto.	SR	CER	α
Panel A: Classical models							
s&p	0.090	0.177	−1.39	1.915	0.395	0.074	–
FF3-ls	0.075	0.157	0.352	0.873	0.360	0.063	0.043
mvp	0.057	0.162	−1.669	3.242	0.231	0.043	0.035
mvp (in-sample)	0.285	0.041	0.190	−1.398	6.641	0.284	0.288
Panel B: B–L models							
BL-capm	0.102	0.137	−1.246	1.760	0.599	0.092	0.040
BL-cb	0.099	0.125	−1.052	1.179	0.631	0.091	0.043
BL-em	0.119	0.155	−1.304	1.780	0.641	0.107	0.049
BL-sm	0.102	0.136	−1.249	1.768	0.600	0.092	0.040
BL-prf	0.106	0.120	−1.412	2.694	0.717	0.099	0.053
BL-ags	0.102	0.128	−1.018	1.474	0.643	0.094	0.045
BL-avp	0.110	0.129	−0.944	1.701	0.699	0.101	0.055
Proposed (FF3 view)	0.137	0.130	−0.903	1.817	**0.902**	**0.128**	**0.099**

Notes. This table compares the performance of the proposed and benchmark models using various metrics. 'Mean Ret.' represents annualized average returns, while 'Std.' is the annualized standard deviation, indicating portfolio risk. 'Skew.' and 'Kurto.' show the skewness and kurtosis of portfolio returns, respectively. 'SR' measures risk-adjusted return (Sharpe ratio), with higher values signifying better performance. 'CER' is the Certainty Equivalent Return, reflecting the return investors would accept instead of a riskier option. 'alpha' is the excess return over a market index. Panel A examines traditional models like the market index ('s&p'), a Fama–French three-factor-based long-short portfolio ('FF3-ls'), a Markowitz mean–variance portfolio ('mvp'), and its in-sample evaluation ('mvp (in-sample)'). Panel B looks at the foundational Black–Litterman (B–L) model ('BL-capm') and its six variants, plus a proposed B–L model incorporating Fama–French three-factor methodology ('Proposed (FF3 view)').

5.2. Comparison of the proposed model with the benchmarks

Table 2 offers a detailed examination of the performance metrics for both the proposed portfolio model and various benchmark models. In Panel A, we explore traditional portfolio models: these include the market index (labeled as 's&p'), a long-short portfolio based on the Fama–French three-factor model which considers market, size, and value factors (denoted as 'FF3-ls'), the Markowitz mean–variance (tangency) portfolio ('mvp'), and the same portfolio evaluated in-sample ('mvp (in-sample)'). Panel B delves into the basic B–L model with a neutral view reflecting the CAPM equilibrium ('BL-capm'), along with six other B–L variants, and the proposed B–L model that integrates a view construction methodology based on the Fama–French three-factor model ('Proposed (FF3 view)'). With the exception of the 'mvp (in-sample)', all benchmarks and the proposed portfolio are assessed out-of-sample. The table columns comprehensively display a range of performance measures for each portfolio, including the annualized mean return, annualized risk, skewness, kurtosis, SR, CER, and annualized alpha.

It is crucial to highlight that in order to ensure a fair comparison, all portfolio model results, with the exception of the S&P 500 market index, have been derived under the premise of a 90-asset investment basket. For optimization-based portfolio models such as the Markowitz and B–L frameworks, the application of a fixed number of assets (e.g., represented as $N = 90$), is commonplace. Contrarily, asset pricing models that underpin general long-short portfolios typically encompass the entire universe of listed assets.

Considering the fact that optimization-based portfolios maintain a fixed asset count, and to further guarantee experimental reliability, they employ 10,000 random simulations; in order to uphold the integrity of comparison, a similar approach was employed in simulations involving long-short portfolios grounded on the Fama–French three-factor model. Therefore, a fixed asset count of 90 was used to obtain the long-short portfolios, along with 10,000 random trials, to maintain consistency in our comparative analysis.

In fact, the formulation of a long-short portfolio employing a fixed number of assets instead of utilizing the entire asset universe listed in the markets is not customary. Nevertheless, under the circumstances previously delineated, it is incumbent upon us to adopt the strategy of utilizing a set number of assets and conducting random trials. This testing methodology is employed with the objective of facilitating a judicious comparison with the optimization-based portfolio model. To corroborate the dependability of our model relative to the prevalent norms for formulating long-short portfolios, we executed a test in order to be consistent with the setup utilizing the entire asset universe. The results of this examination are comprehensively detailed in the subsequent subsection.

Additionally, in the fourth row of Panel A in Table 2, labeled 'mvp (in-sample)', the performances of the Markowitz mean–variance strategy are presented for the in-sample scenario, wherein estimation error is absent. By design, these values represent the apex of performances (e.g., SR and CER) across all evaluated models. Consequently, it is of particular significance to observe the disparity in SR and CER between the in-sample and out-of-sample contexts for the 'mvp' model. This discrepancy serves as an indicator of the reduction in optimal diversification effectiveness due to the presence of estimation error, as opposed to scenarios devoid of such error. A primary objective of this research is to mitigate the aforementioned disparity by proposing a novel portfolio model. Thus, we will scrutinize the extent of this disparity through a comparative analysis with other established benchmark models.

In Panel A, the 's&p' demonstrates notable performance, particularly considering its passive nature compared to other actively managed portfolio models. Specifically, this benchmark exhibits a significantly high annualized average return of 0.090, positioning it as the highest one among all classical models except for 'mvp (in-sample)'. The standard deviation of the 's&p' reflects a level of risk in its returns, with a value of 0.177, implying a trade-off between the high expected return and associated risk. The SR of 0.395 provides insight into the risk-adjusted performance of the 's&p', with a relatively high value indicating a favorable risk-adjusted return among classical models. Furthermore, the CER of 0.074 of the ‘s&p’ attains the highest value among the traditional models with the exception of ‘mvp (in-sample)’.

We also report the performance of ‘FF3-ls’, the Fama–French three-factor long-short portfolio. The ‘FF3-ls’ demonstrates an annualized average return of 0.075, which is comparable to that of the market index but slightly lower. The standard deviation of 0.157 indicates the risk associated with the returns of ‘FF3-ls’, which is relatively lower than that of the market index. Assessing the combined aspects of return and risk, the SR of 0.360 reflects the risk-adjusted performance of the ‘FF3-ls’. This value ranks second among all classical models considered except for ‘mvp (in-sample)’ but falls short of the SR achieved by the ‘s&p’. This observation highlights the challenges encountered in constructing an active portfolio by extracting insights from asset pricing models through the direct long-short approach, particularly from a practical perspective.[12] Furthermore, the CER of 0.063 is also lower than that of the market index, further indicating a diminished level of profitability for the long-short portfolio.

We also delineate the performance metrics of ‘mvp’, the traditional Markowitz mean–variance portfolio model, which embodies a customary strategy for portfolio construction predicated on optimizing both expected returns and associated risks. Remarkably, the portfolio optimized under the Markowitz framework registers an annualized average return of 0.057, the lowest amongst all classical models under scrutiny. Furthermore, the standard deviation, at 0.162, suggests a heightened degree of volatility relative to the other portfolios under review. Surprisingly, the Markowitz portfolio exhibits the lowest SR (0.231) and CER (0.043), both of which insinuate the weakest risk-adjusted performance among the portfolios studied. These observations suggest that, despite optimization, the benefits of diversification have not materialized in the Markowitz model. This evident inadequacy highlights the model’s limitations under practical conditions, such as the necessity for a diversified asset base in the investment portfolio. This finding accentuates the urgency for the development and implementation of novel methodologies that can effectively navigate these inherent limitations.

In Panel B, the performance metrics of the CAPM equilibrium model (‘BL-capm’), a basic reference point in the B–L framework without any specific view, are presented. Remarkably, the risk-adjusted returns, including the SR at 0.599 and the CER at 0.092, are unexpectedly high, even when compared to optimization models (‘mvp’), long-short portfolios (‘FF3-ls’), and the market index (‘s&p’). This observation suggests that the basic B–L portfolio, even in the absence of any specific views, outperforms traditional models. Therefore, incorporating the ‘BL-capm’ as a benchmark in the comparative analysis of our proposed model against other established models is imperative. This approach will ensure a comprehensive evaluation and underscore the unique contribution and validity of our proposed model.

Additionally, we also evaluate the performance of six additional BL-based benchmark models, namely ‘BL-cb’, ‘BL-em’, ‘BL-sm’, ‘BL-prf’, ‘BL-ags’, and ‘BL-avp’. It is observed that the ‘BL-sm’, which is based on sample mean return for view construction, demonstrates negligible improvement. This suggests that relying on simple mean return for estimating expected return does not significantly reduce estimation error, nor does it enhance the out-of-sample risk-adjusted portfolio performance. Contrastingly, models such as ‘BL-cb’, ‘BL-em’, and ‘BL-ags’ exhibit moderate improvements in SR, CER, and alpha compared to ‘BL-capm’. With the exception of the CER of ‘BL-cb’, these three models surpass ‘BL-capm’ in performance to a moderate degree. This indicates that constructing view distributions based on econometric methods, machine learning techniques, and a hybrid of these approaches can offer substantive benefits in estimating expected returns, resulting in more efficient portfolios relative to both ‘BL-capm’ and traditional models.

Notably, the ‘BL-prf’ and ‘BL-avp’ models demonstrate decent performance enhancements across SR, CER, and alpha. Specifically, the SR values for ‘BL-prf’ and ‘BL-avp’ are recorded at 0.717 and 0.699, respectively, representing an approximate 20% and 17% increase over the SR of ‘BL-capm’, and a more than threefold improvement compared to the ‘mvp’ model. The CER for both models also exhibits notable enhancements, with values of 0.099 and 0.101, respectively. Furthermore, both models achieve the highest alpha among all benchmarks, exceeding 0.5, a distinction unique to these two models. The notable performance of these two benchmarks implies that reflecting external variables, not just asset returns, stock-level characteristics such as PE ratio or volatility when properly processed in constructing view distribution, can be economically implicative in generating BL-based portfolio models thereby resulting in a more efficient portfolio.

In the last row of Panel B, we present the demonstration of performance metrics for the Fama–French three-factor-based B–L portfolio model we propose, utilizing a novel view construction methodology. This model is central to our study, devised with the intention of enhancing portfolio performance. The model delivers an annualized average return of 0.137, notably the highest return among all portfolios reviewed in this study except for ‘mvp (in-sample)’. The standard deviation, valued at 0.130, implies a relatively subdued volatility level when juxtaposed with all classical models. Interestingly, the alpha metric exhibits the most remarkable performance, registering the highest value of 0.099 among all benchmarks. This figure is notably more than double that of the ‘FF3-ls’ and ‘mvp’ models, and 1.8 times higher than the second-best performing benchmarks, encompassing both classical models and B–L variants.

The superior risk-adjusted performance of this model is demonstrated by its SR of 0.902 and CER of 0.128, which significantly surpass those of all other portfolios in this study. First, the model’s performance exceeds that of the market index, illustrating that a highly efficient portfolio can be achieved through a finely-tuned active strategy, thus outperforming passive market index investments. Second, the model surpasses the ‘FF3-ls’, signifying the greater effectiveness of constructing portfolios based on the insights of the Fama–French three-factor asset pricing model, compared to the direct construction of long and short positions, which

[12] The observed underperformance of the long-short portfolio, as characterized by the Fama–French three-factor model, aligns with the results documented in Gu et al. (2021).

is shown to be less feasible due to real-world constraints. This finding further provides empirical evidence that the issues of reduced profitability, often encountered from a practical standpoint in the long-short portfolio, are effectively mitigated. This is achieved by our approach of abstracting the implications of the Fama–French three-factor model, which strategically adjusts allocation weights. Third, our model outshines the 'mvp', a seminal optimization-based model, by successfully mitigating estimation errors in expected returns, resulting in a more efficient portfolio than the traditional mean–variance portfolio. More specifically, the model's SR is over three times higher than that of the 'mvp', and its CER more than doubles the CER of the 'mvp'. Fourth, the model outperforms the 'BL-capm', suggesting that incorporating the Fama–French three-factor model – which posits that asset returns are more accurately explained by size and book-to-market ratios, alongside the market return of the CAPM – into the B–L framework allows investors to more effectively capitalize on investment profitability. Lastly, the proposed model outstrips other B–L variants reported in existing literature, underscoring our study's novel and unique contribution to the field.

In our analysis of estimation error, we delve into a more detailed examination. Initially, a significant difference is observed between the 'mvp' and 'mvp (in-sample)' for both SR and CER. For instance, the 'mvp (in-sample)' exhibits an SR of 6.641 and a CER of 0.284, representing a scenario devoid of estimation error. In contrast, the 'mvp', which represents a scenario with estimation error, shows an SR of 0.231 and a CER of 0.043. This notable difference illustrates the impact of estimation error, highlighting the challenges investors and researchers face over the long term in the portfolio literature. Notably, the 'mvp' exhibits the greatest disparity among all models, suggesting that the impact of estimation error is substantial enough to nullify the benefits of optimal diversification. These comparisons of SR and CER underscore the well-documented risks associated with relying on traditional sample-based estimates of the expected return in implementing Markowitz's mean–variance portfolios.

Similarly, by comparing the in-sample SR and CER of the mean–variance strategy with the out-of-sample SR and CER of other benchmarks, we can assess the degree to which these benchmarks mitigate estimation error. Our analysis reveals that the B–L framework, when views are properly formulated using appropriate methodologies, appears to be highly effective in addressing estimation errors. Specifically, contrary to the 'mvp', B–L-based models, introduced with the primary objective of directly mitigating the estimation error in expected returns, exhibit a lower disparity. Even a B–L model with neutral views, implying a portfolio weighted according to the CAPM equilibrium, shows reasonable effectiveness.

When 'BL-capm' is established as the equitable reference portfolio against which other B–L models, including our proposed portfolio, an examination of the disparity demonstrated by 'BL-sm' suggests that naively using the sample mean is not particularly effective in mitigating estimation error. Encouragingly, the disparities observed in 'BL-cb', 'BL-em', and 'BL-ags' indicate that econometric and machine learning methodologies can be somewhat effective in reducing estimation error. Furthermore, it is noteworthy that incorporating characteristic information into the estimation of expected returns appears advantageous, as evidenced by the differences observed in 'BL-prf' and 'BL-avp'.

Most strikingly, the disparity shown by our proposed model is significantly reduced. This finding implies that integrating the insights of the Fama–French three-factor model with CAPM equilibrium reference weights in the estimation of expected returns can be the most effective approach to minimize estimation error.

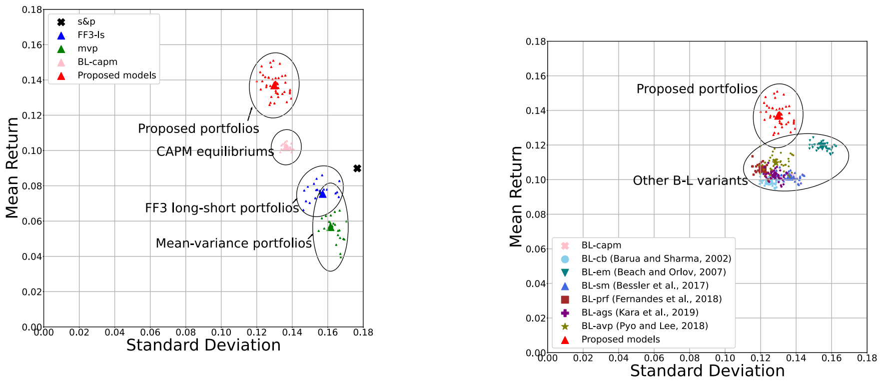
The above findings suggest that our methodology successfully leverages Fama–French's asset pricing model by employing a novel view construction process. This highlighted performance is achieved by adjusting the weight allocation to align well with the asset's out-of-sample return distribution, thereby accurately estimating the expected return of assets. Thus, our proposed approach successfully bridges the insights of the Fama–French three-factor model and B–L portfolio theory. This, as demonstrated by our risk-adjusted return performance, particularly when compared to the 'mvp (in-sample)', leads to superior portfolio performance and significant mitigation of estimation errors traditionally seen in portfolio modeling.

In summary, Table 2 provides a comprehensive comparison of the performance metrics for the proposed portfolio model and benchmark models. The proposed model, incorporating the Fama–French three-factor view construction methodology within the B–L framework, demonstrates superior performance in terms of average return, SR, CER, and alpha. These results highlight the significant value and improved efficiency of the proposed model in portfolio management. Additionally, our findings illustrate a significant reduction in the discrepancy between the outcomes of the optimal portfolio and those of our proposed model. This achievement represents the most substantial mitigation of issues traditionally encountered by conventional models across all benchmarks assessed.

The conclusions drawn in Table 2 are consistently reaffirmed in the risk-return profile plot depicted in Fig. 1. This plot offers a perspective on the risk-return characteristics of the proposed model's portfolios and classical models with 'BL-capm' (Panel (a)), and the proposed models and all B–L variants (Panel (b)). The benchmarks in Panel (a) include the 's&p' index, 'FF3-ls', 'mvp', and 'BL-capm', all of which were established based on out-of-sample performance. Meanwhile, the benchmarks in Panel (b) encompass a comprehensive range of B–L extensions, including 'BL-capm', 'BL-cb', 'BL-em', 'BL-sm', 'BL-prf', 'BL-ags', and 'BL-avp'. To ascertain the expected returns and associated risks of all portfolios, we averaged the returns and risks of 10,000 portfolios, as previously described[13] (except for the market index).

In Panel (a), the portfolios from the proposed model are illustrated in two distinct manners: the average of these portfolios is marked by a large, bold red triangle, while individual portfolios are marked by smaller red triangles. In a similar fashion, for the 'FF3-ls', 'mvp', and 'BL-capm', the average of all portfolios is represented by large, bold triangles in blue, green, and pink colors, respectively, and individual portfolios are represented by smaller triangles in corresponding colors. Additionally, the 's&p' market index is denoted by a large black 'X' mark.

[13] In the interest of readability, we present a randomly selected subset of portfolios from the total of 10,000.



(a) Classical models and BL-capm vs. the proposed model

(b) All B-L variants vs. the proposed model

Fig. 1. Risk return profile plot of the proposed model and all benchmarks. This chart compares the risk-return profiles of the proposed and traditional models (Panel (a)) and Black–Litterman (B–L) variants (Panel (b)). In Panel (a), benchmarks like ‘s&p’, ‘FF3-ls’, ‘mvp’, and ‘BL-capm’ are presented. The proposed model’s portfolios are shown as red triangles, with averages of ‘FF3-ls’, ‘mvp’, and ‘BL-CAPM’ in blue, green, and pink triangles, respectively, and the ‘s&p’ index as a black ‘X’. Panel (b) features averages of B–L models (‘BL-capm’, ‘BL-cb’, ‘BL-em’, ‘BL-sm’, ‘BL-prf’, ‘BL-ags’, ‘BL-avp’, and the proposed model) as distinct, bold symbols, with individual portfolios represented by smaller, matching symbols. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

In Panel (b), the representation of the ‘BL-capm’, ‘BL-cb’, ‘BL-em’, ‘BL-sm’, ‘BL-prf’, ‘BL-ags’, ‘BL-avp’, and the proposed portfolio is as follows: The average of all portfolios for each model is indicated by distinct large, bold symbols - a pink ‘X’ for ‘BL-capm’, a sky-blue circle for ‘BL-cb’, a teal lower triangle for ‘BL-em’, a royal blue upper triangle for ‘BL-sm’, a brown square for ‘BL-prf’, a purple cross for ‘BL-ags’, an olive star for ‘BL-avp’, and a red upper triangle for the proposed portfolio. Correspondingly, individual portfolios within each model are denoted by smaller symbols of the same color and shape.

Upon detailed inspection of Panel (a), it is evident that the portfolios of the proposed model predominantly occupy the upper-left quadrant, indicative of a high-return, low-risk positioning. The average of these portfolios is situated further upwards and to the left when compared to all benchmarks. This demonstrates the superior performance of our proposed model relative to the benchmarks, despite exhibiting a riskier yet more profitable investment profile.

In stark contrast, the portfolios derived from the conventional mean–variance model are congregated in the bottom-right quadrant, implying a high-risk, low-return scenario. Their positioning demonstrates performance that is substantially inferior to not only our proposed model but also to the long-short portfolios and ‘BL-capm’ portfolios. This reinforces the notion that this traditional optimization-based method falls short in comparison to other strategies under consideration.

Overall, the conclusions drawn from the figure are unequivocal. Assessing the risk-return outlook of the proposed model alongside the benchmarks, we find that our model achieves superior portfolio efficiency compared to all other models, suggesting that the proposed model’s tangency is the highest among the alternatives.

This conclusion is further supported in Panel (b), where the proposed model is compared with other advanced B–L variant models. The portfolios derived from the proposed model tend to cluster in regions indicative of higher mean returns, suggesting that it may offer a more favorable risk-return trade-off compared to the other B–L variants. In contrast, the benchmark portfolios are positioned lower than the proposed model, particularly the ‘BL-capm’, ‘BL-sm’, and ‘BL-em’ models, which are situated to the right of the proposed model. This positioning indicates that these three models yield lower returns at higher risk levels, thereby highlighting their underperformance. Although the ‘BL-prf’, ‘BL-cb’, ‘BL-ags’, and ‘BL-avp’ models exhibit lower risks compared to the proposed model, the tangency of the portfolios in the figure shows that the proposed portfolio is more efficient and dominant over these benchmarks, indicating a tendency towards riskier but reasonable investment strategies.

Thus far, our examination has juxtaposed the model we propose with standard benchmark models, highlighting the efficiency of our proposed model under the specifics of set parameters. Moving forward, we plan to initiate a robustness test.[14] aimed at assessing the reliability of the proposed model. This will entail the consideration of a comprehensive spectrum of possible parameters linked to the model, which is a crucial element for sound portfolio modeling.

Our study is principally focused on demonstrating both the statistical significance and robustness of our central arguments. For clarity and to prevent any potential confusion, we opt to omit the results pertaining to the market index in the subsequent sections. This decision is informed by the fact that our statistical analysis is conducted on portfolio strategies across 10,000 trials, whereas the market index does not constitute a portfolio strategy subjected to random trials. To test the robustness of our model, we will vary

[14] Given the constraints of our manuscript’s length, we have detailed the robustness examination outcomes with respect to the d by d portfolio grid dimensions in the Appendix. We undertake this measure to maintain focus within the main body of the article whilst ensuring these essential secondary analyses are comprehensively presented for the reader’s perusal.

Table 3
Robustness analysis: Comparative performance metrics across the different number of assets (N).

Model	SR			CER		
	N=60	N=75	N=90	N=60	N=75	N=90
FF3-ls	0.232 (−7.1***)	0.363 (−5.8***)	0.360 (−9.3***)	0.040 (−8.8***)	0.061 (−5.9***)	0.063 (−12.9***)
mvp	0.373 (−7.0***)	0.326 (−9.0***)	0.231 (−13.2***)	0.063 (−10.0***)	0.057 (−11.9***)	0.043 (−14.2***)
BL-capm	0.600 (0.0)	0.603 (0.0)	0.599 (0.0)	0.093 (0.0)	0.094 (0.0)	0.092 (0.0)
BL-cb	0.615 (0.5)	0.624 (0.7)	0.631 (2.1*)	0.088 (−2.0*)	0.090 (−1.8*)	0.091 (−1.4)
BL-em	0.629 (0.9)	0.639 (1.3)	0.641 (2.6**)	0.103 (3.7***)	0.106 (4.9***)	0.107 (12.4***)
BL-sm	0.600 (0.0)	0.604 (0.0)	0.600 (0.0)	0.093 (0.0)	0.094 (0.0)	0.092 (−0.1)
BL-prf	0.647 (1.1)	0.677 (1.8*)	0.717 (5.5***)	0.093 (0.1)	0.097 (0.9)	0.099 (4.5***)
BL-ags	0.628 (0.8)	0.638 (1.1)	0.643 (2.5**)	0.094 (0.3)	0.095 (0.5)	0.094 (1.3)
BL-avp	0.695 (1.9*)	0.720 (2.5**)	0.699 (4.8***)	0.105 (2.5**)	0.105 (2.9***)	0.101 (5.4***)
Proposed	**0.717** (2.3**)	**0.769** (3.1***)	**0.902** (11.3***)	**0.112** (3.1***)	**0.117** (4.0***)	**0.128** (15.9***)

Notes. This table presents a comparative analysis between the proposed model and all benchmarks, varying the number of assets (N) in the investment pool. Performance metrics, specifically the SR and CER, are examined for all models. The t-tests for SR (t_{SR}) and CER (t_{CER}), as indicated within the parentheses, highlight the statistical significance of the performance difference between the models. *, **, and, *** denotes statistical significance at the 10, 5, and 1% level, respectively.

the parameters under scrutiny in these comparisons. Additionally, to concentrate on risk-adjusted returns, which serve as proxies for the diversification effect, we limit our report to the SR and CER of the portfolios under investigation.

To ascertain statistical significance, we will conduct a t-test, focusing specifically on the differences in performance metrics between each model and a reference model. For this purpose, we have selected 'BL-capm' as the reference model. This choice is predicated on ensuring a reasonable standard for comparison, given that 'BL-capm' represents a reference portfolio within the B–L framework. If a B–L variant fails to surpass this foundational portfolio, it suggests that the methodology employed to construct views is no longer effective. Consequently, we will examine the disparities in the means of the SRs and CERs between the target strategy and the 'BL-capm' strategy.

5.3. Robustness check of the proposed model

5.3.1. The number of assets

Our initial robustness check pertains to the number of assets, denoted as N. Table 3 provides a robustness analysis via a comparative exploration of all benchmark models and our proposed model. This comparison spans different asset pool sizes (N) to evaluate how each model performs under a variety of portfolio configurations.

Upon jointly reviewing the SRs of all benchmark models and the proposed portfolio model, it becomes evident that the SR of the proposed model consistently outperforms those of all benchmark models, irrespective of the number of assets. This suggests that our proposed model achieves greater efficiency when the B–L model is employed in conjunction with our innovative approach to constructing views based on the Fama–French model, irrespective of the number of assets invested.

Furthermore, examining the 'mvp', the SR reveals a decreasing trend as the number of assets increases. However, it is commonly anticipated that an increase in the number of assets will provide enhanced portfolio diversification, attributable to the broadening of the investment spectrum, thereby elevating the SR. The rationale for this premise is that diversification, a derivative of an augmented asset pool, mitigates portfolio risk. This mitigation results from the beneficial influence of negatively correlated or uncorrelated asset returns, thereby enhancing the stability of the portfolio. The advantage conferred by diversification is encapsulated within the problem-solving architecture of mean–variance optimization, which seeks to maximize risk-adjusted returns. However, this optimization process can exhibit instability, potentially yielding unsuitable results when applied to out-of-sample data. This instability is primarily attributed to estimation errors, specifically the inability to accurately predict the precise expected return. In light of these observations, a reduction in the SR, as depicted by the mean–variance model, suggests a potential insufficiency of that model in attaining optimal diversification commensurate with the expansion of the asset base. As such, these findings illuminate the estimation errors inherent in the traditional Markowitz portfolio model. This aligns with the results of DeMiguel et al. (2009),[15] wherein an increase in the number of assets leads to a deterioration in portfolio performance, given a fixed window size for the estimation of expected returns. In short, under conventional frameworks, the exact estimation of expected return often falls short as N increases, thereby failing to achieve an efficient portfolio. This inadequacy tends to offset the diversification benefits brought about by an increased number of assets, which is attributed to estimation errors associated with a larger asset pool.

Contrastingly, our proposed model produces markedly different results. Intriguingly, as displayed in the 'proposed', the SR indicates an improvement in portfolio performance as the number of assets increases.[16] This observed trend of increasing SR, along

[15] The authors conveyed their findings by stating, "Our results suggest that the gain from optimal diversification is more than offset by estimation error when applied out-of-sample".

[16] Our findings align with those of DeMiguel et al. (2009), who established in their research that employing data regarding the cross-sectional characteristics of assets, as opposed to merely using statistical information about the moments of asset returns, results in enhanced SRs. Furthermore, they noted that when the range of investable assets is extensive, the improvement in SRs is more pronounced compared to a smaller set of assets, particularly when applying the method proposed by Brandt et al. (2009).

Table 4
Robustness analysis: Comparative performance metrics across the market conditions.

	SR			CER		
Model	Low volatility	High volatility	Total	Low volatility	High volatility	Total
FF3-ls	0.743 (−5.8***)	−0.438 (−8.4***)	0.360 (−9.3***)	0.105 (−1.2)	−0.038 (−9.0***)	0.063 (−12.9***)
mvp	1.105 (−1.7)	−0.181 (−10.1***)	0.231 (−13.2***)	0.095 (−6.5***)	−0.047 (−9.3***)	0.043 (−14.2***)
BL-capm	1.193 (0.0)	0.419 (0.0)	0.599 (0.0)	0.115 (0.0)	0.073 (0.0)	0.092 (0.0)
BL-cb	1.291 (2.9**)	0.379 (−3.0***)	0.631 (2.1*)	0.123 (4.8***)	0.065 (−5.3***)	0.091 (−1.4)
BL-em	1.276 (4.3***)	0.443 (2.2**)	0.641 (2.6**)	**0.132** (10.8***)	0.081 (5.5***)	0.107 (12.4***)
BL-sm	1.208 (0.7)	0.415 (−0.4)	0.600 (0.0)	0.116 (0.3)	0.072 (−0.6)	0.092 (−0.1)
BL-prf	1.458 (8.4***)	0.626 (7.5***)	0.717 (5.5***)	0.118 (2.2**)	0.087 (7.6***)	0.099 (4.5***)
BL-ags	1.408 (6.6***)	0.419 (0.0)	0.643 (2.5**)	0.121 (4.2***)	0.071 (−1.1)	0.094 (1.3)
BL-avp	1.375 (4.2***)	0.551 (2.8**)	0.699 (4.8***)	0.109 (−2.8**)	0.083 (1.9*)	0.101 (5.4***)
Proposed	**1.461** (3.1***)	**0.956** (28.8***)	**0.902** (11.3***)	0.124 (3.0***)	**0.151** (27.6***)	**0.128** (15.9***)

Notes. This table presents a comparative analysis between the proposed model and all benchmarks, under different market conditions. Performance metrics, specifically the SR and CER, are examined for all models. The t-tests for SR (t_{SR}) and CER (t_{CER}), as indicated within the parentheses, highlight the statistical significance of the performance difference between the models. *, **, and, *** denotes statistical significance at the 10, 5, and 1% level, respectively.

with the highest SRs recorded across all benchmarks, including classical models and other advanced B–L models, suggests that our model effectively harnesses the diversification benefits provided by the increased number of assets, sufficiently overcoming the negative impacts of estimation errors stemming from the optimization process based on the expanded asset pool. These findings indicate that our proposed model successfully extracts valuable insights from the view structure, thereby reducing the estimation errors intrinsic to traditional approaches. Overall, these results pertaining to the number of assets strongly support and corroborate the evidence of estimation error mitigation, as delineated in Table 2.

With regard to the CER, the patterns largely mirror those of the SR, regardless of the model considered. Notably, the CER of the mean–variance model declines as the number of assets escalates, whereas the CER of our proposed model increases. The overall CER values for our proposed model exceed those of the mean–variance model. These suggest a more diversified portfolio enabled by our unique approach, which successfully mitigates the estimation errors inherent in the traditional model.

Regarding both the SR and the CER, the proposed model consistently surpasses all other B–L models, including 'BL-capm', across various asset quantities, achieving this with significant margins. This outcome indicates that the distinctive contribution of our proposed model remains stable regardless of the size of the asset pool, thereby reinforcing the robustness of the primary findings presented in Table 2.

The final analysis involves t-tests comparing the SRs and CERs of each model with 'BL-capm'. Initially, classical methods such as 'FF3-ls' and 'mvp' demonstrate statistically significant lower SR and CER, as indicated by negative t-values. Among other B–L variants, at $N = 60$, only 'BL-avp' exhibits statistically significant improvement in SR. This list expands to include 'BL-prf' when $N = 75$. Notably, when N increases to 90, all B–L variants, with the exception of 'BL-sm', show statistical significance with positive t-values in SR. The situation is more challenging with CER; for instance, at $N = 60$ and $N = 75$, only two B–L models show significance, and this number increases to only three when $N = 90$. In contrast, for all asset quantities (N), both the SR and CER of the proposed model significantly outperform those of 'BL-capm', with all t-values achieving statistical significance at the 5% level for $N = 60$ and at the 1% level for $N = 75$ and $N = 90$. These findings underscore the reliability of the proposed model, indicating its ability to deliver markedly superior risk-adjusted returns across diverse portfolio compositions with statistical significance.

5.3.2. *The market conditions*

To enrich our analysis with an economic perspective, we examine the empirical results across varying market conditions. Contrary to the full sample results, where market conditions are not known a priori to the investor at the inception, this investigation clearly constitutes an ex-post analysis, given our preexisting knowledge of the market volatility. Despite the retrospective nature of this exercise, it adds value to our study by elucidating the impact of market conditions on portfolio construction and, consequently, the performance of our proposed model.

The samples classified under low and high volatility are comprised of the lowest and highest 30%, respectively, of out-of-sample volatility. We showcase these two groups because our proposed model strives to sustain robust performance under both standard and highly volatile conditions where there is significant potential for downside risk. Thus, an optimal portfolio strategy should demonstrate minimal performance degradation, even under conditions of high market volatility.

Table 4 provides a rigorous comparison of distinct performance metrics under varying market conditions, specifically under low and high volatility and full sample scenarios. Initially, within the scenario of low market volatility, all evaluated models exhibit an enhancement in SR compared to 'Total'. For instance, the 'mvp' model displays an SR of 1.105, which is a substantial increase compared to the SR of 0.231 observed under conditions encompassing a comprehensive sample. This significant shift underscores the mitigation of difficulty associated with constructing a robust portfolio due to a low-volatility market environment. Parallel to this, the proposed model echoes similar trends, demonstrating an enhanced SR of 1.461 under low volatility conditions. This represents the highest SR performance among all benchmarks, encompassing both classical and B–L models.

In contrast, under volatile market conditions, most benchmark models exhibit a decreased SR compared to their performance under comprehensive sample conditions. For instance, models such as 'FF3-ls' and 'mvp' transition from a positive SR in the total sample scenario to a negative SR. This signifies the inherent challenges involved in portfolio construction amidst heightened market fluctuations. However, the degree of these reductions diverges significantly between the benchmark models and our proposed model. The traditional model exhibits relatively significant declines, whereas the proposed model shows an improvement, with an SR of 0.956 compared to 0.902 in the total sample scenario. This discrepancy highlights the resilience of our proposed model, emphasizing its robust performance even amidst extreme market turbulence.

Most remarkably, under all circumstances – from comprehensive sample conditions to low and even high market volatility scenarios – the SR of the proposed model consistently surpasses those of the benchmark models. These findings imply that our model exhibits enhanced steadfastness in terms of mitigating estimation error, proving its superiority over conventional models across various market conditions.

In terms of the CER, the overall trend aligns with the SR results, with 'BL-em' under low volatility conditions being the sole exception. Specifically, in a low-volatility environment, 'BL-em' registers the highest CER. This suggests that incorporating heteroskedasticity and volatility clustering concepts into view construction has a meaningful impact on B–L portfolio performance in terms of CER under low-volatility conditions. Nonetheless, the CER of the proposed model ranks as the second highest among all benchmarks, indicating its commendable outperformance in low-volatility market conditions. Furthermore, similar to the SR results, under conditions of high market volatility and in the total sample case, the proposed model consistently exhibits superior CER performance compared to all benchmark models. Notably, the CER of the proposed model is approximately twice as high as that of other benchmarks, particularly under high-volatility market conditions, a trait of desirable portfolio strategy highly sought after by investors. In essence, our model demonstrates the capacity to offer investors reliable and efficient portfolio opportunities regardless of market fluctuations.

Further substantiating this observation are the t-tests for both the SR and CER of the proposed model, which register statistical significance at the 1% level across all market conditions, symbolized by ***. This significant result indicates that the superior performance of the proposed model over the 'BL-capm' across different market conditions is not merely coincidental, thereby affirming the model's unwaveringness and dominance.

5.3.3. The different investment periods

The efficacy of a portfolio strategy often hinges upon the specific timeframe during which it is applied. This sensitivity arises from the dynamic nature of market conditions, leading to variations in market behavior characteristics. Especially, events of substantial magnitudes, such as a financial crisis, can notably influence the applicability and effectiveness of a newly proposed portfolio strategy due to the significant shock and impact they exert on the market.

To examine the resilience and sustainability of the proposed model across varying investment periods, we perform a subsample analysis. We achieve this by segregating the full period into two distinct sub-periods: the pre and post-financial crisis of 2007–2009. The pre-crisis period spans from 1996 to 2006, whereas the post-crisis period extends from 2007 to 2021. To ensure that the post-crisis period encapsulates the eventual respite from the financial crisis's effects, we set 2007 as the cut-off year.[17] Our decision to utilize the 2007–2009 global financial crisis as the delineating event is twofold. Firstly, it represents the most influential financial occurrence during the out-of-sample period. Secondly, it allows for a balanced distribution of test samples before and after the event.[18]

The subsample analysis, differentiated based on the periods relative to the 2007–2009 financial crisis, is exhibited in Table 5. Upon meticulous examination of the table, it is quite surprising to note that only two models demonstrate statistically significant improvement over 'BL-capm' with positive t-values across all periods, including pre- and post-crisis, as well as the full period for both SR and CER. These models are 'BL-avp' and the proposed model. Notably, the performance metrics of the proposed model consistently exceed those of all benchmark models by substantial margins, even when compared to the second-best performing model. This holds true across the full sample and both subdivided periods, pre- and post-financial crisis. While minor variations in values are observed, the tendencies for the subsample results from our proposed model closely align with those from the full sample.

These findings carry significant implications. Regardless of the specific timeframe during which our proposed model is employed, it demonstrates robust superiority over the traditional models. This strongly suggests that our proposed methodology for constructing views can efficaciously distill meaningful information reflective of the insights offered by the Fama–French three-factor model. This consistency emanates from the persistence of the anomalies utilized by this asset pricing model, namely size and value effect, which continuously have played a crucial role in the financial stock market during the pre and post-financial crisis and the whole time. Lastly, these observations are statistically corroborated by the substantial t-values obtained from the t-tests for these performance metrics, underscoring their significance.

[17] To enhance the reliability of our findings, we conducted additional tests using cut-off years of +- 1, 2, and 3 from 2007. The results were fundamentally analogous to those derived using 2007 as the cut-off year. Complete results are available upon request.

[18] We also performed the same analysis using other significant financial events, such as the COVID-19 pandemic. However, these results were not reported due to the limited number of test samples available post-COVID-19, thereby restricting an adequate demonstration of the proposed model's reliability and superiority. Notably, these results were comparable to those derived from the 2007–2009 financial crisis.

Table 5
Robustness analysis: Comparative performance metrics across different investment periods.

	SR			CER		
Model	Before crisis	After crisis	Full sample	Before crisis	After crisis	Full sample
FF3-ls	0.291 (−5.5***)	0.435 (−3.3***)	0.360 (−9.3***)	0.060 (−7.8***)	0.065 (−3.2***)	0.063 (−12.9***)
mvp	0.246 (−7.8***)	0.228 (−12.1***)	0.231 (−13.2***)	0.061 (−8.0***)	0.031 (−12.2***)	0.043 (−14.2***)
BL-capm	0.624 (0.0)	0.582 (0.0)	0.599 (0.0)	0.111 (0.0)	0.079 (0.0)	0.092 (0.0)
BL-cb	0.612 (−0.8)	0.651 (3.3***)	0.631 (2.1*)	0.110 (−1.0)	0.077 (−0.7)	0.091 (−1.4)
BL-em	0.649 (1.6)	0.638 (2.8**)	0.641 (2.6**)	0.120 (8.6***)	0.097 (9.2***)	0.107 (12.4***)
BL-sm	0.624 (−0.0)	0.583 (0.0)	0.600 (0.0)	0.111 (−0.1)	0.079 (−0.0)	0.092 (−0.1)
BL-prf	0.913 (9.6***)	0.607 (1.1)	0.717 (5.5***)	0.128 (12.0***)	0.077 (−0.8)	0.099 (4.5***)
BL-ags	0.661 (2.5**)	0.631 (2.1*)	0.643 (2.5**)	0.115 (3.2***)	0.079 (0.1)	0.094 (1.3)
BL-avp	0.828 (5.7***)	0.630 (1.9*)	0.699 (4.8***)	0.124 (5.0***)	0.085 (2.2**)	0.101 (5.4***)
Proposed	**1.103** (20.4***)	**0.780** (5.6***)	**0.902** (11.3***)	**0.170** (23.3***)	**0.098** (6.9***)	**0.128** (15.9***)

Notes. This table presents a comparative analysis between the proposed model and all benchmarks, under different investment periods. Performance metrics, specifically the SR and CER, are examined for all models. The t-tests for SR (t_{SR}) and CER (t_{CER}), as indicated within the parentheses, highlight the statistical significance of the performance difference between the models. *, **, and, *** denotes statistical significance at the 10, 5, and 1% level, respectively.

Table 6
Robustness analysis: Comparative performance metrics varying transaction costs.

	SR			CER		
Model	0 bps	15 bps	30 bps	0 bps	15 bps	30 bps
FF3-ls	0.360 (−9.3***)	0.350 (−9.4***)	0.341 (−9.6***)	0.063 (−12.9***)	0.061 (−12.9***)	0.060 (−12.9***)
mvp	0.231 (−13.2***)	0.221 (−13.3***)	0.212 (−13.3***)	0.043 (−14.2***)	0.042 (−14.2***)	0.040 (−14.2***)
BL-capm	0.599 (0.0)	0.588 (0.0)	0.577 (0.0)	0.092 (0.0)	0.091 (0.0)	0.089 (0.0)
BL-cb	0.631 (2.1*)	0.619 (2.0*)	0.607 (2.0*)	0.091 (−1.4)	0.090 (−1.4)	0.088 (−1.4)
BL-em	0.641 (2.6**)	0.631 (2.7**)	0.621 (2.8**)	0.107 (12.4***)	0.106 (12.4***)	0.104 (12.4***)
BL-sm	0.600 (0.0)	0.589 (0.0)	0.578 (0.0)	0.092 (−0.1)	0.091 (−0.1)	0.089 (−0.1)
BL-prf	0.717 (5.5***)	0.705 (5.5***)	0.692 (5.5***)	0.099 (4.5***)	0.097 (4.5***)	0.096 (4.5***)
BL-ags	0.643 (2.5**)	0.631 (2.5**)	0.620 (2.5**)	0.094 (1.3)	0.093 (1.3)	0.091 (1.3)
BL-avp	0.699 (4.8***)	0.688 (4.8***)	0.676 (4.8***)	0.101 (5.4***)	0.100 (5.4***)	0.098 (5.4***)
Proposed	**0.902** (11.3***)	**0.890** (11.4***)	**0.878** (11.5***)	**0.128** (15.9***)	**0.127** (15.9***)	**0.125** (15.9***)

Notes. This table presents a comparative analysis between the proposed model and all benchmarks, under different transaction costs. Performance metrics, specifically the SR and CER, are examined for all models. The t-tests for SR (t_{SR}) and CER (t_{CER}), as indicated within the parentheses, highlight the statistical significance of the performance difference between the models. *, **, and, *** denotes statistical significance at the 10, 5, and 1% level, respectively.

5.3.4. The transaction costs

In this study, we delve deeper into the influence of transaction costs on portfolio performance. To achieve this, we examine a range of transaction costs, extending from 0 to 30 basis points. The findings from this examination are consolidated and presented in Table 6. While the comprehensive nature of transaction costs, including elements like market microstructure and liquidity, is critical to understand, our research is specifically concentrated on the overarching impact of transaction costs on portfolio performance. Therefore, we adopt a prudent approach in assessing portfolio performance, treating the range of transaction costs as an accurate depiction of the total transactional expenses incurred.

The table illustrates that with the escalation of transaction costs, there is a corresponding decrease in the SR for both traditional and proposed models. This highlights the substantial influence of high transaction costs on portfolio performance. Notwithstanding this decline, the overall trend aligns with the scenario devoid of transaction costs. To put it succinctly, regardless of the magnitude of transaction costs, the proposed model consistently outperforms the benchmark models in terms of SR. This suggests the proposed model's economic viability, even under severe transaction cost conditions. Displaying an SR greater than 0.87, our model continues to demonstrate impressive performance in absolute terms, even when transaction costs reach 30 basis points. Hence, our analysis implies that the general trend identified in Table 2 persists, even under cautious assumptions regarding the extent of transaction costs. Furthermore, our proposed model, which carries the beneficial economic implications of Fama–French's factor models, exhibits economically meaningful superiority under such conditions.

With regard to the CER, the general pattern largely mirrors previous observations, further highlighting the superior performance of the proposed model in comparison to the traditional models. This remains the case even when considering the implications of transaction costs, attesting to the robustness of our proposed model. Additionally, the results derived from the t-test provide statistical confirmation of these findings.

5.4. The single portfolio model comparison

In general, the long-short portfolio based on the Fama–French three-factor model is a single portfolio utilizing the entirety of the asset universe. To preserve this perspective, we introduce an alternative approach to ensure a fair comparison. This approach involves comparing a proposed portfolio model that uses the entire asset universe to both the long-short portfolio and the market

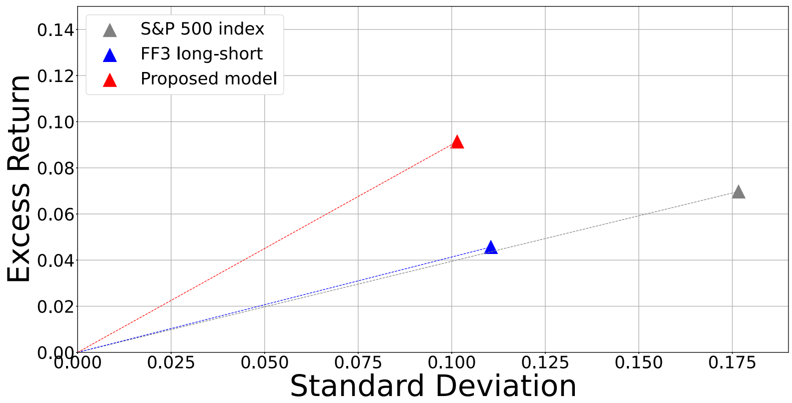


Fig. 2. Excess return vs. risk for all 'single' portfolios. The single portfolio of the proposed model, constructed as an equal-weighted composite of 10,000 weight vectors that span the entire asset universe, is represented by a red triangle. In comparison, the S&P 500 index, replicated here by a single Exchange-Traded Fund (ETF), and the single long-short portfolio generated from the Fama–French three-factor model, which similarly capitalizes on the full asset universe, are denoted by gray and blue triangles, respectively. Given that the y-axis reflects excess returns, the points at which the dotted lines meet the portfolio indicate the efficiency of each portfolio. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

index.[19] To execute this, we generate 10,000 proposed portfolio weights from 10,000 random trials, keeping the asset count fixed at N = 90. Subsequently, for all assets that are not included in the set of 90 assets of each weight vector, we assign a weight of zero. In an effort to circumvent potential optimization errors, we then aggregate these weight vectors in a straightforward fashion, giving each vector equal weight. Ultimately, we obtain a final, singular portfolio weight based on the proposed model. The findings from this approach are largely in harmony with our main results.

Fig. 2 provides a visualization of risk in relation to excess return for all 'single' portfolios, based on the proposed model's single portfolio version and two 'single' portfolio models (Fama–French three-factor long-short portfolio and S&P 500 index). The proposed model's single portfolio, derived from an equal-weighted average of 10,000 weight vectors (thereby utilizing the entirety of the asset universe), is indicated by a red triangle. In a similar vein, the S&P 500 index (a single ETF) and the single long-short portfolio (which also utilizes the entire asset universe) derived from the Fama–French three-factor model are symbolized by gray and blue triangles, respectively. Considering that the y-axis indicates the excess return, the tangency points of the dotted lines represent the efficiency of each portfolio.

Broadly speaking, the conclusions derived from this figure align seamlessly with our main argument. By evaluating the excess risk-return prospects and tangency points of the proposed model in comparison to the two benchmarks, it becomes apparent that our model demonstrates superior portfolio efficiency.

In summary, the superiority of the proposed model is evident even when we adopt a different experimental approach for fair comparison, such as the proposed model and the long-short portfolio employing the entire asset universe. This conclusion is substantiated by the fact that the tangency of the proposed model is the highest among all alternatives considered.

5.5. Recent periods analysis and global market performance

To validate the robustness and adaptability of our proposed model concerning recent periods and various stock markets, our empirical study has been extended to encompass recent data and incorporate data from additional international stock markets. This expansion aims to bolster the wide-ranging applicability of our findings.

5.5.1. Recent periods analysis

In this subsection, our objective is to examine the relevance and practicality of our model in recent periods. This exploration is motivated by the recognition that while utilizing the original, long-term data length enhances the robustness of our analysis for long-term investing, such a dataset may amalgamate disparate economic realities, potentially rendering it unrepresentative of current conditions. Consequently, we have conducted fresh evaluations under various scenarios, as outlined in Table 7, focusing on both the recent 5-year and 10-year datasets.[20] For these assessments, the input parameters for our model are calculated based on two different historical periods: 15 years and 20 years, respectively, prior to the testing intervals. The methodology and settings for these experiments remain consistent with those employed in the original analysis. Panel A presents the findings from the out-of-sample test over the recent 5 years, whereas Panel B details the results from the out-of-sample test spanning the recent 10 years.

[19] We have omitted the results of other benchmarks as these models generally rely on a fixed number of assets, rather than encompassing the entire asset universe.

[20] In this study, the terminal year for the U.S. stock market data is 2021. Consequently, we utilize datasets spanning the most recent 5-year and 10-year periods from 2021 for our analysis.

Table 7
Comparative performance metrics of the proposed and different benchmark portfolio models over the recent 5 and 10-year periods.

Portfolio Models	Mean Ret.	Std.	Skew.	Kurto.	SR	CER	α
Panel A: Recent 5 years							
s&p	0.163	0.130	−1.165	−0.178	1.177	0.079	–
FF3-ls	0.046	0.136	0.232	−0.998	0.200	−0.055	0.030
mvp	0.014	0.016	−0.288	−1.190	0.324	0.013	0.026
mvp (in-sample)	0.231	0.013	−0.409	−0.600	19.087	0.230	0.226
BL-capm	0.102	0.099	0.277	−0.976	0.924	0.053	−0.001
BL-cb	0.110	0.083	0.201	−1.092	1.214	0.075	0.034
BL-em	0.129	0.121	0.179	−1.122	0.983	0.056	0.001
BL-sm	0.102	0.099	0.280	−0.998	0.929	0.053	0.000
BL-prf	0.104	0.085	0.424	−1.035	1.117	0.068	0.025
BL-ags	0.109	0.094	0.831	−0.623	1.057	0.065	0.035
BL-avp	0.107	0.115	0.615	−0.757	0.849	0.040	−0.001
Proposed (FF3 view)	0.125	0.092	−0.196	−1.589	**1.246**	**0.082**	**0.037**
Panel B: Recent 10 years							
s&p	0.139	0.110	−0.408	−0.931	1.209	0.097	–
FF3-ls	0.077	0.136	0.071	−0.772	0.529	0.011	0.057
mvp	0.027	0.065	−0.618	0.252	0.380	0.011	0.056
mvp (in-sample)	0.227	0.012	0.010	−0.811	19.414	0.226	0.220
BL-capm	0.104	0.085	−0.121	−0.622	1.167	0.079	0.030
BL-cb	0.103	0.078	−0.384	−0.401	1.258	0.082	0.035
BL-em	0.133	0.104	−0.227	−0.737	1.232	0.096	0.040
BL-sm	0.105	0.085	−0.121	−0.645	1.172	0.080	0.031
BL-prf	0.107	0.077	−0.172	−0.809	1.321	0.086	0.043
BL-ags	0.109	0.079	0.350	−0.148	1.310	0.087	0.048
BL-avp	0.110	0.087	0.561	0.463	1.221	0.083	0.042
Proposed (FF3 view)	0.119	0.069	−0.087	−0.715	**1.658**	**0.102**	**0.079**

Notes. This table compares the performance of the proposed and benchmark models using various metrics. ‘Mean Ret.’ represents annualized average returns, while ‘Std.’ is the annualized standard deviation, indicating portfolio risk. ‘Skew.’ and ‘Kurto.’ show the skewness and kurtosis of portfolio returns, respectively. ‘SR’ measures risk-adjusted return (Sharpe ratio), with higher values signifying better performance. ‘CER’ is the Certainty Equivalent Return, reflecting the return investors would accept instead of a riskier option. ‘alpha’ is the excess return over a market index. Panel A explores the outcomes over the recent 5 years, and Panel B examines the results over the recent 10 years.

In Panel A, a salient observation is the proposed model’s highest SR of 1.246, surpassing that of other benchmark models. This is particularly noteworthy given the relatively stronger performance of the market index compared to longer-term periods, reflecting the recent ascendancy of the US stock market index. Among all the benchmark models evaluated, only the ‘BL-cb’ and the proposed model demonstrate outperformance relative to this passive index investment in terms of SR. In addition to SR, the CER at 0.082 and alpha at 0.037 of the proposed model also exceed those of all other benchmarks. This suggests that our model adeptly adjusts to recent timeframes, consistently delivering superior risk-adjusted returns.

Panel B reveals results that are broadly similar to those in Panel A. Here, the proposed model notably outstrips all other benchmarks in terms of SR (1.658), CER (0.102), and alpha (0.079). This period demonstrates a particularly significant margin between the metrics of the proposed model and those of other models. This finding emphasizes the enhanced reliability and effectiveness of employing our proposed model in investment practices over a 10-year timespan.

Upon comparing the outcomes of the recent periods with those of the main long-term period setting, there is a variation in the magnitude of each value, yet the overarching trend remains consistent. This trend persists across different temporal settings, whether long-term or recent periods, highlighting the unvarying superiority of our proposed model. This consistency underscores the effectiveness of a Fama–French three-factor model-based BL strategy, particularly one that leverages size and value effects. In essence, our empirical analysis confirms the applicability of our proposed model to more recent periods. This insight is likely to be useful for investors who typically focus on shorter investment horizons.

5.5.2. Global market performance

To ensure a comprehensive evaluation of our model’s performance, we carefully select two additional, globally significant stock markets: the European and Japanese markets. These markets are chosen for their distinctive significance, which offers a diverse testing ground for our model. The required data for applying our proposed strategy are acquired from the Datastream and Worldscope databases.

The European stock market, characterized by a diverse spectrum of major companies within the Eurozone, is selected to evaluate our model’s performance in a varied and influential European context. We utilize a dataset spanning from 2012 to 2021 for the out-of-sample test, comprising all firms listed on the Eurostoxx 50 index. This selection allows for in-depth analysis across different periods of economic stability and volatility in the Eurozone, with the Eurostoxx 50 index representing the European market. Similarly, the Japanese stock market, encompassing a range of major companies and exemplifying a key Asian market, is chosen to examine our model’s effectiveness in a diverse and significant Japanese setting. The dataset for this market, aligned with the same test period

Table 8
Comparative performance metrics of the proposed model and various benchmark portfolio models for the European and Japanese stock markets.

Portfolio Models	Mean Ret.	Std.	Skew.	Kurto.	SR	CER	α
Panel A: Europe							
Eurostoxx 50	0.071	0.114	−0.323	−0.436	0.570	0.022	–
FF3-ls	−0.049	0.082	0.451	−0.208	−0.700	−0.075	−0.067
mvp	0.040	0.051	−0.116	−0.536	0.724	0.029	0.040
mvp (in-sample)	0.203	0.025	1.194	1.025	8.433	0.201	0.210
BL-capm	0.110	0.104	−0.898	−0.356	1.012	0.070	0.057
BL-cb	0.112	0.105	−0.885	−0.354	1.016	0.070	0.050
BL-em	0.135	0.151	−0.217	0.607	0.961	0.022	0.081
BL-sm	0.133	0.115	−1.024	−0.278	1.120	0.083	0.079
BL-prf	0.110	0.103	−0.887	−0.371	1.011	0.069	0.057
BL-ags	0.127	0.108	−0.915	−0.219	1.117	0.083	0.080
BL-avp	0.106	0.085	−0.320	−0.542	1.199	0.078	0.063
Proposed (FF3 view)	0.172	0.138	−0.609	−0.255	**1.302**	**0.096**	**0.109**
Panel B: Japan							
Nikkei 225	0.031	0.230	−0.209	−0.316	0.043	0.005	–
FF3-ls	0.035	0.202	−0.069	1.006	0.068	0.015	0.024
mvp	0.032	0.338	−0.297	1.463	0.050	−0.028	0.013
mvp (in-sample)	0.114	0.022	−0.183	−0.827	4.319	0.114	0.116
BL-capm	0.042	0.225	−0.494	0.624	0.096	0.016	0.014
BL-cb	0.040	0.198	−0.507	0.653	0.099	0.020	0.016
BL-em	0.039	0.213	−0.513	0.667	0.087	0.016	0.013
BL-sm	0.041	0.225	−0.496	0.628	0.095	0.016	0.014
BL-prf	0.042	0.201	−0.461	0.609	0.109	0.022	0.018
BL-ags	0.039	0.204	−0.486	0.657	0.092	0.018	0.014
BL-avp	0.041	0.221	−0.508	0.642	0.095	0.017	0.014
Proposed (FF3 view)	0.054	0.213	−0.409	0.262	**0.161**	**0.031**	**0.030**

Notes. This table compares the performance of the proposed and benchmark models using various metrics. 'Mean Ret.' represents annualized average returns, while 'Std.' is the annualized standard deviation, indicating portfolio risk. 'Skew.' and 'Kurto.' show the skewness and kurtosis of portfolio returns, respectively. 'SR' measures risk-adjusted return (Sharpe ratio), with higher values signifying better performance. 'CER' is the Certainty Equivalent Return, reflecting the return investors would accept instead of a riskier option. 'alpha' is the excess return over a market index. Panel A focuses on the European stock market, while Panel B analyzes the Japanese stock market.

as our primary US datasets, includes all firms from the Nikkei 225 index. This facilitates a thorough analysis of various economic phases in Japan, with the Nikkei 225 index representing the Japanese market. To ensure the rigor of our experiments, we conducted 10,000 repetitions of the out-of-sample portfolio simulation, parallel to our main analysis of US data. Each trial involved a unique set of 25 randomly chosen assets.[21]

Table 8 presents a comparative analysis of our model's performance against established benchmarks in both the European (Panel A) and Japanese (Panel B) stock markets. A recurring theme in all examined markets is the superior performance of our model, as evidenced by key indicators such as the SR, CER, and alpha. In the European market, our model achieves an SR of 1.302, surpassing other models in the analysis. This trend is consistent for CER and alpha, with our model outperforming not only the Eurostoxx 50 market index and traditional models like 'FF3-ls' and 'mvp', but also the basic BL model and their variants.

These results are significant in the context of the European market's unique economic dynamics. The model's adaptability and performance in this market underscore its applicability and robustness beyond the US stock market. While the European market ostensibly diverges from its US counterpart, the underlying factors influencing asset returns in Europe exhibit parallels to those in the United States. This analysis of the European market is consistent with prior research, which underscores the presence of similar fundamental determinants of asset returns, as corroborated by a multitude of studies. For instance, Bauer et al. (2010) and Liao et al. (2019) have explored the presence of size and value effects, respectively, in Europe. Ammann et al. (2012) has further affirmed the comparability of European factors to those in the US. Our findings in the European market corroborate these empirical results, evidencing the viability and profitability of the Fama–French three-factor model in this region, similar to the findings reported in the works of Rossi (2012), Kousenidis et al. (2000), Bhatnagar and Ramlogan (2010) and Amel-Zadeh (2011).

In the Japanese market, our model also demonstrates the highest SR (0.161), CER (0.031), and alpha (0.030), indicating its effectiveness in a developed Asian market. This suggests its potential for investors interested in diversified portfolios across Asian economies. The model's success in adapting to the unique market dynamics of Japan further highlights its resilience and wider applicability. Research in the Japanese market, including studies by Hearn (2011), Chan et al. (1993, 1991), and Walid (2009), has shown the presence of size and value effects, supporting the effectiveness of the Fama–French three-factor model in this region (Pham

[21] This decision to utilize 25 assets, fewer than in our main US analysis, stems from the relatively smaller asset universes of the Eurostoxx 50 and Nikkei 225. This approach ensures a consistent and suitable number of assets for investment in our illustrative model.

and Long, 2007). Our results in the Japanese market are in line with these studies, demonstrating the model's proficiency and relevance in varied global contexts.

We provide a thorough and global perspective on the effectiveness of our proposed model, by integrating these additional data series into our empirical analysis. This model demonstrates optimal performance compared to all benchmarks across various stock markets. Its consistent outperformance in diverse markets, from European to Asian economies, underscores the model's adaptability and efficiency. The model's superiority is supported by its alignment with the economic principles established in numerous asset pricing studies with an international focus.

Our empirical results for both markets are congruent with the findings of Walkshäusl and Lobe (2014), who affirm the explanatory power of the Fama–French three-factor model across international stock markets, encompassing European nations, the Japanese market, and the US stock market. In light of Walkshäusl and Lobe (2014)'s studies, our findings illustrate the potential of our model as a universally applicable portfolio management strategy, capable of navigating varying economic contexts and market dynamics. The resilience of our model in these varied conditions indicates its value as a strategic tool for global investors aiming to optimize their portfolios amid the complexities of international financial markets.

Overall, our comprehensive empirical analysis not only tackles practical challenges but also markedly enhances the robustness of our research, particularly in terms of temporal scope and global relevance. The results underscore the effectiveness of incorporating established asset pricing theory into the BL model. This integration emerges as a significant tool for asset allocation in international portfolio management, indicating considerable potential in this domain for both long- and short-term investments.

6. Conclusion

In this study, we have embarked on an ambitious endeavor to unite two major schools of thought in finance, the B–L framework and the Fama–French three-factor model. Through rigorous analysis and quantitative modeling, we have developed a hybrid portfolio model that merges these two significant concepts.

Our research has produced four principal findings. First, our innovative portfolio model demonstrated superiority over conventional benchmarks, exhibiting improved diversification and overall performance. This was characterized by a marked increase in the SR, CER, and alpha suggesting that our model delivers a more efficient portfolio than passive strategies and outperforms even long-short portfolios based on Fama and French's asset pricing model. In addition, our proposed model remarkably outperforms the traditional optimization-based approach, the mean–variance model, by a huge margin.

Second, we affirmed the economic value and innovation of our view construction methodology by extensively comparing our proposed model with other various B–L models including basic B–L. We found that econometric, machine learning methods, and their hybrids seem to be effective while the use of sample mean returns for view distribution is less effective. Among all the benchmarks we considered, B–L models using stock-level characteristics are more productive. Most importantly, we found that our model, based on Fama and French's theory, outperforms others in market prediction, showing the highest SR, CER, and alpha. This underscores our method's empirical validity and its potential to enhance portfolio efficiency and diversification.

The third salient finding relates to the reduction of estimation error, a critical objective in portfolio modeling. Our proposed model showcased significant advancements in this area, with the SR and CER of our model being appreciably closer to the optimal values than those of traditional methods. It effectively harnessed the diversification effect offered by larger asset pools, thereby reducing overfitting and mitigating the estimation error issue of the traditional model.

Lastly, the model demonstrated notable robustness and reliability across a variety of economic conditions and diverse parameter configurations. The model maintained consistent performance across portfolios with varying asset quantities, during periods of heightened market volatility, through different phases of financial crises, and when considering transaction costs. This consistency extended to recent periods and various global investment environments. Remarkably, these observations affirm the model's effectiveness, suggesting that the improved return efficiency, attributable to the augmented optimization premium offered by our proposed model, is economically solid.

In this study, we make a substantial contribution to portfolio management literature, particularly in light of recent advancements in the field (Ayadi et al., 2023; Attig et al., 2023; Bacchetta et al., 2023; Hollstein and Prokopczuk, 2023; Anderson and Cheng, 2022; Kan and Wang, 2023; Zhang et al., 2023; Bartram et al., 2021; Zhang et al., 2020; Chen et al., 2021) by proposing a novel asset allocation methodology. Our methodological advancements, rooted in the comparative analysis of various B–L model variants, resonate with and extend recent efforts in enhancing portfolio efficiency (Punyaleadtip et al., 2024; Barua and Sharma, 2023; Gao et al., 2023; Han and Li, 2023; Barua and Sharma, 2022; Beach and Orlov, 2007; Bessler et al., 2017; Fernandes et al., 2018; Kara et al., 2019; Pyo and Lee, 2018; Ko et al., 2023a). Moreover, by addressing the critical issue of estimation error, our work not only aligns with but also advances key recent findings in finance (DeMiguel et al., 2009; Simaan et al., 2018; Simaan, 1997, 2014; Dai and Kang, 2022; Lassance et al., 2023; Platanakis et al., 2021; Kan et al., 2022). This approach not only reaffirms the significance of existing theories but also paves the way for future research in effective portfolio management.

Our findings have far-reaching implications for academia, as they present a novel perspective on portfolio construction and bridge two critical theories, thus opening up new avenues for theoretical development and empirical testing for a new B–L portfolio with the asset pricing model. This integration can be an appropriate response to the call for more comprehensive models in financial portfolio management. For investment professionals and institutional investors, the demonstrated performance robust superiority of our model provides a pragmatic solution to the perennial problem of portfolio optimization and risk management, aligning with recent studies emphasizing the importance of adapting to stylized facts in the financial markets (Fernandes et al., 2018; Pyo and Lee, 2018; Karmous et al., 2021).

In future research, we proposed to extend the application of the model to optimize different credit assets, such as fixed-income portfolios and Credit Default Swaps (CDS). Previous studies, particularly by Fama and French (1993) and Pereira et al. (2015), have shown the Fama–French model's applicability in bond and CDS markets. Our research will aim to further explore these asset markets, adapting the model to include specific risk factors and market dynamics, thereby broadening the understanding of the Fama–French three-factor-based BL portfolio model's utility in diverse financial contexts. Additionally, it is important to recognize that the Fama–French three-factor model has faced various criticisms due to its inherent limitations within the classical asset pricing literature. Numerous scholars have proposed more advanced models, such as Carhart (1997)'s four-factor model, Fama and French (2015)'s five-factor model, and Hou et al. (2019)'s q5-factor model. Consequently, it would be wise to further explore the possibilities of integrating these alternative factor models into our framework in future research. Additionally, exploring alternative methods for financial view construction, particularly machine learning-based models for clustering (Lee et al., 2021; Ben-Hur et al., 2001), prediction (Ko et al., 2019; Niu et al., 2023), and generation (Park et al., 2023; Ko and Lee, 2023b), could be highly beneficial. The integration of the proposed model into investment strategies related to retirement portfolio management (Ko et al., 2023b, 2024a) and hedging (Ko and Lee, 2023a; Black and Jones, 1987; Ko et al., 2024b; Rubinstein, 1985) may also offer novel insights into the field.

In conclusion, our study provides a substantial contribution to financial portfolio theory by integrating the Fama–French three-factor model into the B–L framework, thereby offering a more robust and efficient portfolio management tool. The implications of our findings are significant and wide-ranging, promising to influence both academic research and practical applications in financial portfolio management.

CRediT authorship contribution statement

Hyungjin Ko: Writing – original draft, Visualization, Validation, Software, Resources, Project administration, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Bumho Son:** Writing – review & editing, Validation, Data curation. **Jaewook Lee:** Writing – review & editing, Validation, Supervision, Funding acquisition.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

The authors do not have permission to share data.

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Appendix A. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.intfin.2024.101949.

References

Amel-Zadeh, A., 2011. The return of the size anomaly: Evidence from the german stock market. Eur. Financial Manag. 17 (1), 145–182.
Ammann, M., Odoni, S., Oesch, D., 2012. An alternative three-factor model for international markets: Evidence from the European monetary union. J. Bank. Financ. 36 (7), 1857–1864.
Anderson, E., Cheng, A.-r., 2022. Portfolio choices with many big models. Manage. Sci. 68 (1), 690–715.
Attig, N., Guedhami, O., Nazaire, G., Sy, O., 2023. What explains the benefits of international portfolio diversification? J. Int. Financial Markets, Inst. Money 83, 101729.
Ayadi, A., Gana, M., Goutte, S., Guesmi, K., 2023. Optimizing portfolios for the BREXIT: An equity-commodity analysis of US, European and BRICS markets. J. Int. Financial Markets, Inst. Money 89, 101863.
Bacchetta, P., Tieche, S., Van Wincoop, E., 2023. International portfolio choice with frictions: Evidence from mutual funds. Rev. Financ. Stud. 36 (10), 4233–4270.
Banz, R.W., 1981. The relationship between return and market value of common stocks. J. Financial Econ. 9 (1), 3–18.
Bartram, S.M., Branke, J., De Rossi, G., Motahari, M., 2021. Machine learning for active portfolio management. The Journal of Financial Data Science 3 (3), 9–30.
Barua, R., Sharma, A.K., 2022. Dynamic black litterman portfolios with views derived via CNN-BiLSTM predictions. Finance Res. Lett. 49, 103111.
Barua, R., Sharma, A.K., 2023. Using fear, greed and machine learning for optimizing global portfolios: A Black-Litterman approach. Finance Res. Lett. 58, 104515.
Bauer, R., Cosemans, M., Schotman, P.C., 2010. Conditional asset pricing and stock market anomalies in europe. Eur. Financial Manag. 16 (2), 165–190.
Beach, S.L., Orlov, A.G., 2007. An application of the Black–Litterman model with EGARCH-M-derived views for international portfolio management. Financial Mark. Portfolio Manag. 21 (2), 147–166.
Ben-Hur, A., Horn, D., Siegelmann, H.T., Vapnik, V., 2001. Support vector clustering. J. Mach. Learn. Res. 2 (Dec), 125–137.

Bessler, W., Opfer, H., Wolff, D., 2017. Multi-asset portfolio optimization and out-of-sample performance: an evaluation of Black–Litterman, mean-variance, and naïve diversification approaches. Eur. J. Finance 23 (1), 1–30.
Bessler, W., Taushanov, G., Wolff, D., 2021. Optimal asset allocation strategies for international equity portfolios: A comparison of country versus industry optimization. J. Int. Financial Markets, Inst. Money 72, 101343.
Best, M.J., Grauer, R.R., 1991. On the sensitivity of mean-variance-efficient portfolios to changes in asset means: some analytical and computational results. Rev. Financial Stud. 4 (2), 315–342.
Bhatnagar, C.S., Ramlogan, R., 2010. The capital asset pricing model versus the three factor model: A united kingdom perspective. Int. J. Bus. Soc. Res. (IJBSR) 2 (1), 11–23.
Black, F., Jones, R., 1987. Simplifying portfolio insurance. J. Portfolio Manag. 14 (1), 48.
Black, F., Litterman, R.B., 1991. Asset allocation: Combining investor views with market equilibrium. J. Fixed Income 1 (2), 7–18.
Brandt, M.W., Santa-Clara, P., Valkanov, R., 2009. Parametric portfolio policies: Exploiting characteristics in the cross-section of equity returns. Rev. Financ. Stud. 22 (9), 3411–3447.
Byun, J., Ko, H., Lee, J., 2023. A privacy-preserving mean–variance optimal portfolio. Finance Res. Lett. 54, 103794.
Carhart, M.M., 1997. On persistence in mutual fund performance. J. Finance 52 (1), 57–82.
Chan, L.K., Hamao, Y., Lakonishok, J., 1991. Fundamentals and stock returns in Japan. J. Finance 46 (5), 1739–1764.
Chan, L.K., Hamao, Y., Lakonishok, J., 1993. Can fundamentals predict Japanese stock returns? Financ. Anal. J. 63–69.
Chen, W., Zhang, H., Mehlawat, M.K., Jia, L., 2021. Mean–variance portfolio optimization using machine learning-based stock price prediction. Appl. Soft Comput. 100, 106943.
Dai, Z., Kang, J., 2022. Some new efficient mean–variance portfolio selection models. Int. J. Finance Econ. 27 (4), 4784–4796.
DeMiguel, V., Garlappi, L., Uppal, R., 2009. Optimal versus naive diversification: How inefficient is the 1/N portfolio strategy? Rev. Financial Stud. 22 (5), 1915–1953.
Duqi, A., Franci, L., Torluccio, G., 2014. The Black–Litterman model: the definition of views based on volatility forecasts. Appl. Financial Econ. 24 (19), 1285–1296.
Fama, E.F., French, K.R., 1992. The cross-section of expected stock returns. J. Finance 47 (2), 427–465.
Fama, E.F., French, K.R., 1993. Common risk factors in the returns on stocks and bonds. J. Financial Econ. 33 (1), 3–56.
Fama, E.F., French, K.R., 2015. A five-factor asset pricing model. J. Financial Econ. 116 (1), 1–22.
Fernandes, B., Street, A., Fernandes, C., Valladão, D., 2018. On an adaptive Black–Litterman investment strategy using conditional fundamentalist information: A Brazilian case study. Finance Res. Lett. 27, 201–207.
Foye, J., Mramor, D., Pahor, M., 2013. A respecified fama french three-factor model for the new European union member states. J. Int. Financial Manag. Account. 24 (1), 3–25.
Gao, J., Wang, J., Zhou, Y., Lv, M., Wei, D., 2023. Enhancing investment performance of Black-Litterman model with AI hybrid system: Can it be done? Expert Syst. Appl. 122924.
Grauer, R.R., Janmaat, J.A., 2010. Cross-sectional tests of the CAPM and Fama–French three-factor model. J. Bank. Finance 34 (2), 457–470.
Griffin, J.M., 2002. Are the Fama and French factors global or country specific? Rev. Financ. Stud. 15 (3), 783–803.
Gu, S., Kelly, B., Xiu, D., 2020. Empirical asset pricing via machine learning. Rev. Financ. Stud. 33 (5), 2223–2273.
Gu, S., Kelly, B., Xiu, D., 2021. Autoencoder asset pricing models. J. Econometrics 222 (1), 429–450.
Han, Y., Li, J., 2023. The impact of global economic policy uncertainty on portfolio optimization: A Black–Litterman approach. Int. Rev. Financ. Anal. 86, 102476.
He, G., Litterman, R., 2002. The intuition behind Black-Litterman model portfolios. 334304, Available at SSRN.
Hearn, B., 2011. Size and liquidity effects in Japanese regional stock markets. J. Japan. Int. Econ. 25 (2), 157–181.
Hollstein, F., Prokopczuk, M., 2023. Managing the market portfolio. Manage. Sci. 69 (6), 3675–3696.
Hou, K., Mo, H., Xue, C., Zhang, L., 2019. Which factors? Rev. Finance 23 (1), 1–35.
Idzorek, T., 2007. A step-by-step guide to the Black-Litterman model: Incorporating user-specified confidence levels. In: Forecasting Expected Returns in the Financial Markets. Elsevier, pp. 17–38.
Jones, R.C., Lim, T., Zangari, P.J., 2007. The Black-Litterman model for structured equity portfolios. J. Portfolio Manag. 33 (2), 24–33.
Kan, R., Wang, X., 2023. Optimal portfolio choice with unknown benchmark efficiency. Manage. Sci..
Kan, R., Wang, X., Zhou, G., 2022. Optimal portfolio choice with estimation risk: No risk-free asset case. Manage. Sci. 68 (3), 2047–2068.
Kara, M., Ulucan, A., Atici, K.B., 2019. A hybrid approach for generating investor views in Black–Litterman model. Expert Syst. Appl. 128, 256–270.
Karmous, A., Boubaker, H., Belkacem, L., 2021. Forecasting volatility for an optimal portfolio with stylized facts using copulas. Comput. Econ. 58 (2), 461–482.
Ko, H., Byun, J., Lee, J., 2023a. A privacy-preserving robo-advisory system with the Black-Litterman portfolio model: A new framework and insights into investor behavior. J. Int. Financial Markets, Inst. Money 89, 101873.
Ko, H., Lee, J., 2023a. Non-fungible tokens: A hedge or a safe haven? Appl. Econ. Lett. 1–8.
Ko, H., Lee, J., 2023b. Can chatgpt improve investment decision? From a portfolio management perspective. 4390529, Available at SSRN.
Ko, H., Lee, J., Byun, J., Son, B., Park, S., 2019. Loss-driven adversarial ensemble deep learning for on-line time series analysis. Sustainability 11 (12), 3489.
Ko, H., Lee, S., Lee, J., 2023b. Dynamic investment strategy to safeguard retirement portfolio and insights into retiree behavior. 4613707, Available at SSRN.
Ko, H., Lee, S., Lee, J., 2024a. Sequence and longevity risks of South Korean retirees: Insights and potential remedies. Pac.-Basin Finance J. 83, 102263.
Ko, H., Son, B., Lee, J., 2024b. Portfolio insurance strategy in the cryptocurrency market. Res. Int. Bus. Finance 67, 102135.
Ko, H., Son, B., Lee, Y., Jang, H., Lee, J., 2022. The economic value of NFT: Evidence from a portfolio analysis using mean–variance framework. Finance Res. Lett. 47, 102784.
Kousenidis, D.V., Negakis, C.I., Floropoulos, I.N., 2000. Size and book-to-market factors in the relationship between average stock returns and average book returns: Some evidence from an emerging market. Eur. Account. Rev. 9 (2), 225–243.
Lassance, N., Vanderveken, R., Vrins, F., 2023. On the combination of naive and mean-variance portfolio strategies. J. Bus. Econom. Statist. 1–15.
Lee, W., Ko, H., Byun, J., Yoon, T., Lee, J., 2021. Fair clustering with fair correspondence distribution. Inform. Sci. 581, 155–178.
Li, D., Zhang, Z., Cerrato, M., 2023. Factor investing and currency portfolio management. Int. Rev. Financ. Anal. 87, 102626.
Liao, T.L., Tsai, L.-C., Ke, M.-C., Chiang, Y.-C., Hsu, C.-H., 2019. Financial crisis and market efficiency: evidence from European stock markets. Eur. J. Finance 25 (13), 1194–1210.
Lintner, J., 1969. The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets: A reply. Rev. Econ. Stat. 222–224.
Maccheroni, F., Marinacci, M., Ruffino, D., 2013. Alpha as ambiguity: Robust mean-variance portfolio analysis. Econometrica 81 (3), 1075–1113.
Markowitz, H., 1952. Portfolio selection. J. Finance 7 (1), 77–91.
Merton, R.C., 1972. An analytic derivation of the efficient portfolio frontier. J. Financial Quant. Anal. 7 (4), 1851–1872.
Niu, Z., Demirer, R., Suleman, M.T., Zhang, H., Zhu, X., 2023. Do industries predict stock market volatility? Evidence from machine learning models. J. Int. Financial Markets, Inst. Money 101903.
Park, J., Ko, H., Lee, J., 2023. Exploring generative AI for modeling the dynamics of asset price process. 4491342, Available at SSRN.
Pedersen, J.L., Peskir, G., 2017. Optimal mean-variance portfolio selection. Math. Financial Econ. 11, 137–160.

Pereira, J., Sorwar, G., Nurullah, M., 2015. Application of the Fama and French factor models to the credit default swaps (CDS) market. 2704883, Available at SSRN.

Pham, V.T.L., Long, T., 2007. Constructing fama-french factors from style indexes: Japanese evidence. Econ. Bull. 7 (7), 1–10.

Platanakis, E., Sutcliffe, C., Ye, X., 2021. Horses for courses: Mean-variance for asset allocation and 1/N for stock selection. European J. Oper. Res. 288 (1), 302–317.

Punyaleadtip, K., Kantavat, P., Nimmanunta, K., Kijsirikul, B., 2024. Black–Litterman portfolio management using the investor's views generated by recurrent neural networks and support vector regression. J. Financial Data Sci. 5 (4).

Pyo, S., Lee, J., 2018. Exploiting the low-risk anomaly using machine learning to enhance the Black–Litterman framework: Evidence from South Korea. Pac.-Basin Finance J. 51, 1–12.

Rosenberg, B., Reid, K., Lanstein, R., 1985. Persuasive evidence of market inefficiency. J. Portfolio Manag. 11 (3), 9–16.

Rossi, F., 2012. The three-factor model: Evidence from the Italian stock market. Res. J. Finance Account. 3 (9), 151–160.

Rubinstein, M., 1985. Alternative paths to portfolio insurance. Financial Anal. J. 42–52.

Sharpe, W.F., 1964. Capital asset prices: A theory of market equilibrium under conditions of risk. J. Finance 19 (3), 425–442.

Simaan, Y., 1997. Estimation risk in portfolio selection: the mean variance model versus the mean absolute deviation model. Manage. Sci. 43 (10), 1437–1446.

Simaan, Y., 2014. The opportunity cost of mean–variance choice under estimation risk. European J. Oper. Res. 234 (2), 382–391.

Simaan, M., Simaan, Y., Tang, Y., 2018. Estimation error in mean returns and the mean-variance efficient frontier. Int. Rev. Econ. Finance 56, 109–124.

Walid, E.M., 2009. New evidence on risk factors, characteristics and the cross-sectional variation of Japanese stock returns. Asia-Pac. Financial Markets 16, 33–50.

Walkshäusl, C., Lobe, S., 2014. The alternative three-factor model: An alternative beyond US markets? Eur. Financial Manag. 20 (1), 33–70.

Zhang, C., Gong, X., Zhang, J., Chen, Z., 2023. Dynamic portfolio allocation for financial markets: A perspective of competitive-cum-compensatory strategy. J. Int. Financial Markets, Inst. Money 84, 101737.

Zhang, Z., Zohren, S., Roberts, S., 2020. Deep learning for portfolio optimization. J. Financial Data Sci. 2 (4), 8–20.