Maths Problem Set- Measure Theory

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Problem 1

- Consider $A \in \mathcal{G}_1 \implies A$ open on $\mathbb{R} \implies A^c$ is either closed on $\mathbb{R}$ or semi-open on $\mathbb{R}$. So $A^c \notin \mathcal{G}_1$ as it is not a purely open interval. Hence $\mathcal{G}_1$ is not a σ- algebra nor an algebra.
- Consider $A_n \in \mathcal{G}_2, n \in \mathbb{N}$. Then $\bigcup_{n=1}^{\infty} A_n \notin \mathcal{G}_2$ since $\mathcal{G}_2$ contains only sets which are finite unions of intervals of the form $(a, b], (-\infty, b], (a, \infty)$. Thus $\mathcal{G}_2$ is not a σ- algebra. Now we check whether $\mathcal{G}_2$ is an algebra. It is clear that $\phi \in \mathcal{G}_2$. Now consider any interval of the form $(a, b]$. Then it's complement is of the form $(-\infty, a] \cup (b, \infty)$ which $\in \mathcal{G}_2$. Similarly for any interval of the form $(-\infty, b]$, its complement is of the form (b, ∞) which $\in \mathcal{G}_2$. Thus, for all $A \in \mathcal{G}_2$, $A^c \in \mathcal{G}_2$

  $\bigcup_{n=1}^{\mathbb{N}} A_n$
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