Maths Problem Set- Inner Product Spaces

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Exercise 1

- Using distributive properties of inner products and noting the fact that it is a real inner product space (so that conjugates return the same inner product), we get

$$\begin{aligned}
\frac{1}{4}\left(||x+y)||^2 - ||x-y||^2\right) &= \frac{1}{4}\left(\langle x+y, x+y\rangle - \langle x-y, x-y\rangle\right) \\
&= \frac{1}{4}\left(\langle x, x+y\rangle + \langle y, x+y\rangle - \langle x, x-y\rangle + \langle y, x-y\rangle\right) \\
&= \frac{1}{4}\left(\langle x, x\rangle + \langle x, y\rangle + \langle y, x\rangle + \langle y, y\rangle - \langle x, x\rangle\right. \\
&\left. + \langle x, y\rangle + \langle y, x\rangle - \langle y, y\rangle\right) \\
&= \langle x, y\rangle
\end{aligned}$$

- Again using distributive properties we have

$$\begin{aligned}
\frac{1}{2}\left(||x+y||^2 + ||x-y||^2\right) &= \frac{1}{2}\left(\langle x+y, x+y\rangle + \langle x-y, x-y\rangle\right) \\
&= \frac{1}{2}\left(\langle x, x+y\rangle + \langle y, x+y\rangle + \langle x, x-y\rangle - \langle y, x-y\rangle\right) \\
&= \frac{1}{2}\left(\langle x, x\rangle + \langle x, y\rangle + \langle y, x\rangle + \langle y, y\rangle\right. \\
&\left. + \langle x, x\rangle - \langle x, y\rangle - \langle y, x\rangle + \langle y, y\rangle\right) \\
&= \left(||x||^2 + ||y||^2\right)
\end{aligned}$$