Math16600 Section 23715 Quiz 10
Fall 2023, November 14

Name: [1 pt]

Problem 1: Determine whether the following series is absolutely convergent, conditionally convergent or divergent:

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$$

[5 pts]

Absolute convergence: check convergence of $\sum_{n=2}^{\infty} \frac{1}{\ln(n)}$

$\hookrightarrow$ Ratio test fails.

$\Rightarrow$ use comparison test

$\ln(n) < n$ for all $n \geq 2$

$\Rightarrow \frac{1}{\ln(n)} > \frac{1}{n}$ for all $n \geq 2$ $\Rightarrow \sum_{n=2}^{\infty} \frac{1}{\ln(n)} > \sum_{n=2}^{\infty} \frac{1}{n}$ $\leftarrow$ diverges

$\Rightarrow \sum_{n=2}^{\infty} \frac{1}{\ln(n)}$ also diverges.

Conditional Convergence:

$\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$ is alternating series with $b_n = \frac{1}{\ln(n)}$

- $\ln(n+1) > \ln(n) \Rightarrow \frac{1}{\ln(n+1)} = b_{n+1} < b_n = \frac{1}{\ln(n)}$
- $\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{1}{\ln(n)} = \frac{1}{\infty} = 0$

$\Rightarrow$ By AST, the given series converges.

$\Rightarrow$ the given series is conditionally convergent

Problem 2: Find the radius of convergence and interval of convergence of the power series:

$$\sum_{n=0}^{\infty} \frac{(x-2)^n}{n^2+1} = \sum_{n=0}^{\infty} \frac{1}{n^2+1} (x-2)^n$$

[5 pts]

$\Downarrow$

$C_n = \frac{1}{n^2+1}$

$\frac{1}{R} = \lim_{n\to\infty} \left| \frac{C_{n+1}}{C_n} \right|$

$= \lim_{n\to\infty} \frac{1}{(n+1)^2+1} \times \frac{n^2+1}{1} = \lim_{n\to\infty} \frac{n^2+1}{(n+1)^2+1}$

$= \lim_{n\to\infty} \frac{n^2+1}{n^2+2n+2} = \lim_{n\to\infty} \frac{n^2}{n^2} = 1 \Rightarrow \frac{1}{R} = 1 \Rightarrow R = 1$

$\Rightarrow (a-R, a+R) = (2-1, 2+1) = (1, 3)$

$x=1 \Rightarrow \sum_{n=0}^{\infty} \frac{(-1)^n}{n^2+1} \rightarrow$ Alternating series with $b_n = \frac{1}{n^2+1}$

- $(n+1)^2+1 > n^2+1 \Rightarrow \frac{1}{(n+1)^2+1} = b_{n+1} < b_n = \frac{1}{n^2+1}$
- $\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{1}{n^2+1} = 0.$

By AST, the series converges

$x=3 \Rightarrow \sum_{n=1}^{\infty} \frac{1}{n^2+1}$ similar to $\sum_{n=1}^{\infty} \frac{1}{n^2}$ $\rightarrow$ convergent P-series $\Rightarrow \sum_{n=1}^{\infty} \frac{1}{n^2+1}$ converges

$\Rightarrow \sum_{n=0}^{\infty} \frac{1}{n^2+1}$ also converges.

$\rightarrow$ converges at both end-points.

Thus, interval of convergence is $[1, 3]$