Math16600 Section 23715 Quiz 9
Fall 2023, November 07

Name: [1 pt]

Problem 1: Determine whether the following series is convergent or divergent:

$$\sum_{n=2}^{\infty} \frac{\ln n}{n^2}$$

Hint: Use the integral test. [5 pts]

$a_n = \frac{\ln n}{n^2} \Rightarrow f(x) = \frac{\ln x}{x^2}$

- f is positive and continuous
- $f = \frac{\text{slower growing function}}{\text{faster growing function}} \Rightarrow f$ is ultimately decreasing.

$$\int_2^{\infty} \frac{\ln x}{x^2}\,dx \underset{y=\ln x}{=} \int_{\ln 2}^{\infty} \frac{y}{e^y}\,dy = \int_{\ln 2}^{\infty} y e^{-y}\,dy = -e^{-y}(y+1)\Big|_{\ln 2}^{\infty}$$

$$= \lim_{y\to\infty} -\frac{(y+1)}{e^y} + e^{-\ln 2}(\ln 2+1)$$

$$= \lim_{y\to\infty} \frac{-1}{e^y} + \frac{1}{2}(\ln 2+1)$$

$$= 0 + \frac{1}{2}(\ln 2+1)$$

$\Rightarrow I = \frac{1}{2}(\ln 2+1) < \infty \Rightarrow$ integral converges

$\Rightarrow$ the series also converges.

Problem 2: Determine whether the following series is convergent or divergent:

$$\sum_{n=1}^{\infty} \frac{1}{n!}$$

Hint: Use comparison test to show that the series converges. [5 pts]

$n! > 2^n$ for $n \geq 4$

$\Rightarrow \frac{1}{n!} < \frac{1}{2^n}$ for $n \geq 4 \Rightarrow \sum_{n=4}^{\infty} \frac{1}{n!} < \sum_{n=4}^{\infty} \frac{1}{2^n} = \frac{1/2^4}{1-\frac{1}{2}} = \frac{1}{2^3} = \frac{1}{8}$

$\Rightarrow \sum_{n=4}^{\infty} \frac{1}{n!} < \frac{1}{8}$

$\Rightarrow \sum_{n=1}^{\infty} \frac{1}{n!} = \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{8} = 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{8} < \infty$

$\Rightarrow$ the given series converges.