Derivatives of logarithmic functions

$$\frac{d}{dx}(\ln u) = \frac{1}{u}\frac{du}{dx}\,.$$

$$\frac{d}{dx}(\log_b u) = \frac{1}{u\,\ln b}\frac{du}{dx}\,.$$

Example 1. Differentiate $y = \log_2 x^2$ with respect to x .

Example 2. Differentiate $T = \log_{10}(v^2 + v)$ with respect to v.

Example 3. Differentiate $y = \ln \sec x$ with respect to x.

Example 4. Find dy/dx if $y = \ln \sqrt[3]{x^2 + 1}$.

Example 5. Find the derivative of $y = \ln(\sin^2 x/x)$.

Logarithmic Differentiation: Differentiate $y = [f(x)]^{g(x)}$.

1. *Step 1*: Take ln on both sides so that $\ln y = g(x) \ln f(x)$.
2. *Step 2*: Simplify the RHS if possible.
3. *Step 3*: Differentiate both sides with respect to x.

   Note that the LHS always differentiates to $\dfrac{1}{y}\dfrac{dy}{dx}$.
4. *Step 4*: Multiply both sides with y to obtain $\dfrac{dy}{dx}$.

Example 6. Differentiate $y = x^x$.

Example 7. Differentiate $y = (\sin x)^{\cos x}$.