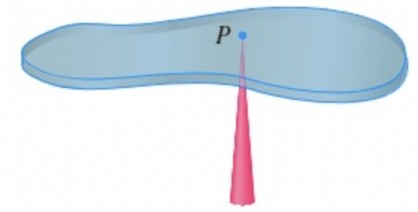
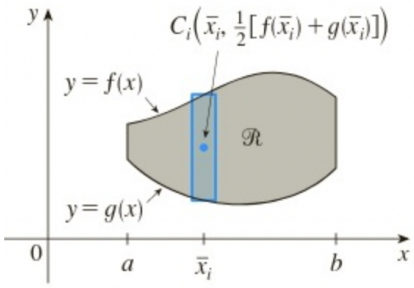
M16600 Lecture Notes
Section 8.3: Center of Mass of Centroid

■ **Section 8.3** textbook exercises, *page 595*: # 29, 30, 32.



Our main objective here is to find the point P on which a thin plate of any given shape balance horizontally as in the figure on the left. This point is called the ***center of mass*** of the plate (or the ***centroid*** of the plate).

We consider a flat plate (called *lamina*) that occupies a region $\mathcal{R}$ of the xy-plane.



The center of mass of the region $\mathcal{R}$ is located at the point $(\bar{x}, \bar{y})$, where

$$\bar{x} = \frac{1}{A}\int_a^b x\big[f(x)-g(x)\big]\,dx$$

$$\bar{y} = \frac{1}{A}\int_a^b \tfrac{1}{2}\Big(\big[f(x)\big]^2-\big[g(x)\big]^2\Big)\,dx$$

Here A is the <u>area</u> of the region $\mathcal{R}$.

See the discussion on page 600-603 of the textbook for more detail

Example: Find the centroid of the region bounded by the line $y = x$ and the parabola $y = x^2$.