

The product rule:

$$\frac{d(uv)}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}.$$

Example 1. Find the derivative of $y = (x^3 + 2x^2 - 3x)(x^3 - 4x)$.

The quotient rule:

$$\frac{d}{dx} \left(\frac{u}{v} \right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}.$$

Example 2. Find the derivative of $y = \frac{x^3 - 3x}{x + 1}$.

Chain rule:

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}.$$

The generalized power rule:

$$\frac{du^n}{dx} = nu^{n-1} \frac{du}{dx}.$$

Example 3. Differentiate $f(x) = \sqrt{x^2 + 1}$.

Example 4. Let $y = x \sqrt{x^2 - 2x}$. Find y' .

Example 5. Find the derivative of $y = \frac{x}{\sqrt{x^2 + 1}}$.

Note. To find the derivative of functions of the form

$$y = \frac{k}{g(x)},$$

write $y = k[g(x)]^{-1}$ and use generalized power rule (instead of quotient rule).

Example 6. Differentiate $y = \frac{4}{\sqrt[3]{x^3 + x}}$.