Math16600 Section 23715 Quiz 4
Fall 2023, September 26

Name: Solutions [1 pt]

Problem 1: Evaluate the integral

$$\int \underbrace{\left(\cos^{-1} x\right)}_{u} \underbrace{dx}_{dv}$$

[5 pts]

$u = \cos^{-1} x \Rightarrow du = \frac{-1}{\sqrt{1-x^2}}\,dx$

$dv = dx \Rightarrow v = x$

$\Rightarrow \int \cos^{-1} x \, dx = x\cos^{-1} x - \int x \frac{-1}{\sqrt{1-x^2}}\,dx$

$= x\cos^{-1} x + \underbrace{\int \frac{x}{\sqrt{1-x^2}}\,dx}$ Let $y = 1-x^2 \Rightarrow dy = -2x\,dx$

$\Rightarrow x\,dx = -\frac{dy}{2}$

$= x\cos^{-1}x + \int \frac{1}{\sqrt{y}} \frac{-dy}{2} = x\cos^{-1}x - \int \frac{1}{2\sqrt{y}}\,dy = x\cos^{-1}x - 2\sqrt{y} + C$

$= x\cos^{-1}x - 2\sqrt{1-x^2} + C$

Problem 2: Evaluate the integral:

$$\int \overbrace{e^{2x}}^{dv} \underbrace{\sin 3x}_{u}\, dx$$

[5 pts]

$u = \sin 3x \Rightarrow du = 3\cos 3x\,dx$

$dv = e^{2x}\,dx \Rightarrow v = \frac{e^{2x}}{2}$

$\Rightarrow I = \frac{e^{2x}}{2}\sin 3x - \int \frac{e^{2x}}{2} 3\cos 3x\,dx$

$\Rightarrow I = \frac{e^{2x}}{2}\sin 3x - \frac{3}{2}\boxed{\int e^{2x}\cos 3x\,dx} \rightarrow I'$

$I' = \int \overbrace{e^{2x}\,dx}^{dv} \underbrace{\cos 3x}_{u}$ $u = \cos 3x \Rightarrow du = -3\sin 3x\,dx$

$dv = e^{2x}\,dx \Rightarrow v = \frac{e^{2x}}{2}$

$\Rightarrow I' = \frac{e^{2x}}{2}\cos 3x - \int \frac{e^{2x}}{2} (-3\sin 3x)\,dx = \frac{e^{2x}}{2}\cos 3x + \frac{3}{2}\underbrace{\int e^{2x}\sin 3x\,dx}_{I}$

So we have, $I = \frac{e^{2x}}{2}\sin 3x - \frac{3}{2}I'$ and $I' = \underbrace{\frac{e^{2x}}{2}\cos 3x + \frac{3}{2}I}$

$\Rightarrow I = \frac{e^{2x}}{2}\sin 3x - \frac{3}{2}\left[\frac{e^{2x}}{2}\cos 3x + \frac{3}{2}I\right] = \frac{e^{2x}}{2}\sin 3x - \frac{3}{4}e^{2x}\cos 3x - \frac{9}{4}I$

$\Rightarrow I + \frac{9}{4}I = \frac{e^{2x}}{2}\sin 3x - \frac{3}{4}e^{2x}\cos 3x \Rightarrow I = \frac{e^{2x}}{13}\left[2\sin 3x - 3\cos 3x\right] + C$