Learning objectives:

1. To differentiate an equation of the form $f(x, y) = 0$ with respect to x.
2. Apply this to find equations of tangents and/or normals.

What is implicit differentiation?

When we do not have an explicit dependence of y on x like $y = f(x)$ for some function f but instead we have an equation involving both x and y. For example:

$$x^2 + y^2 + xy = 1 \ .$$

In such cases one can differentiate with respect to x to find dy/dx in terms of both x and y.

Example 1. Differentiate the following with respect to x:

1. y.
2. y^2.
3. y^3.
4. y^n.

Example 2. Differentiate the following with respect to x:

1. xy.
2. xy^2.
3. xy^3.
4. x^2y.
5. x^4y^6.

Example 3.

1. If $x^2 + y^2 = 25$, find dy/dx.
2. Find an equation of the tangent to the circle $x^2 + y^2 = 25$ at the point $(3, 4)$.

Example 4.

1. Find y' if $x^3 + y^3 = 6xy$.
2. Find the equation tangent to the given curve at the point $(3, 3)$.
3. At what point in the first quadrant is the tangent line horizontal?
4. Find the equation of normal to the given curve at $(3, 3)$.

Example 5.

1. Find y' if $\sin(x + y) = y^2 \cos x - \pi^2$.
2. Find equation of tangent and normal lines to the given curve at $(0, \pi)$.

Example 6. Find y'' if $x^4 + y^4 = 16$.

Example 7. Find y'' if $\sin y + \cos x = 1$.