Math 110
3.1,3.2,3.3,3.8 Notes

3.1, 3.2, 3.3, 3.8

~~3.4/3.5~~

Systems of Equations:

more than one equation.

Ex. $\left.\begin{array}{l} x+y=0 \\ x+2y=1 \end{array}\right\}$ a system of 2 linear equations in 2 unknowns (x, y)

Example 1: In 2012, there were 92 species of birds in the United States that were considered threatened or endangered. The number of species considered threatened was three less than one-fourth the number considered endangered. Write a system of equations that models the number of U.S. bird species considered endangered or threatened. Translate to a system of equations.

Let the number of endangered bird species be x
Let the number of threatened bird species be y.

$$x+y=92, \quad y=\frac{1}{4}x-3$$

Example 2: Star Bright Jewelry Design purchased 80 beads for a total of $39 to make a necklace. Some of the beads were sterling silver beads costing 0.40 each and the rest were gemstone beads costing 0.65 each. How many of each type were bought? Translate to a system of equations.

Let the number of sterling silver beads be x
Let the number of gemstone beads be y.

$$x+y=80, \quad 0.4x+0.65y=39$$

Identifying Solutions:

↳ Put the values of x and y in both equations and see if they are satisfied.

Determine whether $(-4,7)$ is a solution of the system

$x+y=3$
$5x-y=-27$

↳ $-4+7 \stackrel{?}{=} 3 \Rightarrow 3=3 \rightarrow$ True $\Rightarrow$ First eqn. is satisfied.

↳ $5(-4)-7 \stackrel{?}{=} -27 \Rightarrow -20-7 \stackrel{?}{=} -27 \Rightarrow -27=-27 \rightarrow$ True

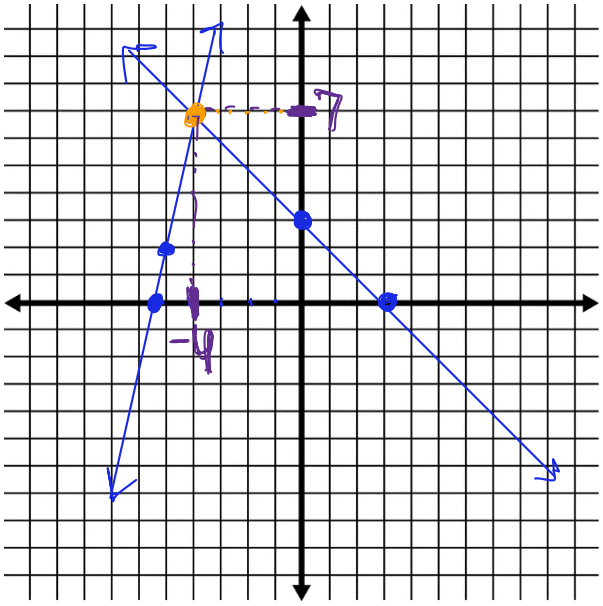
$\Rightarrow$ Second eqn. is satisfied.

$\Rightarrow (-4,7)$ is a solution of the given system.

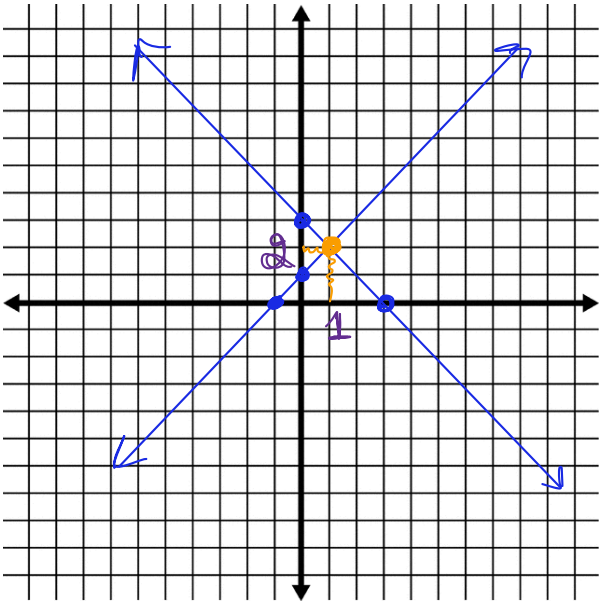
YES

Solving Systems Graphically:

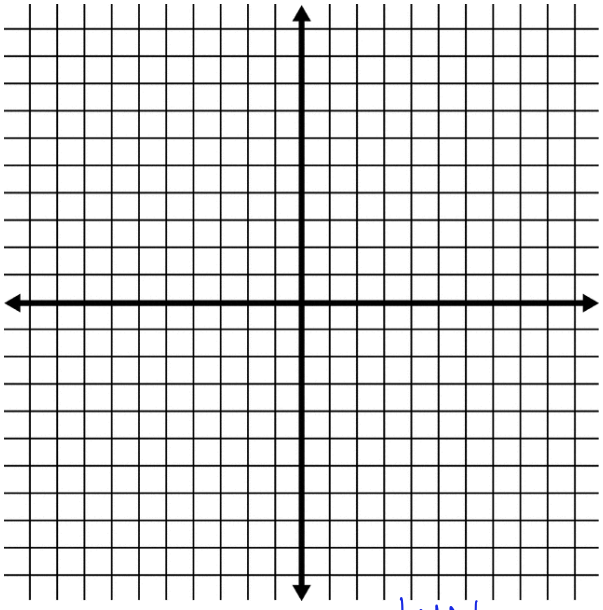
1. $x + y = 3$
   $5x - y = -27$



2. $y - x = 1$
   $y + x = 3$

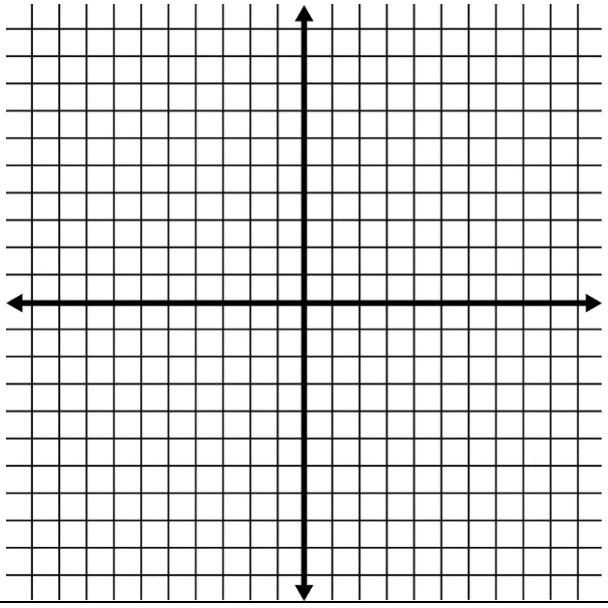


3. $y = -3x + 5$
   $y = -3x - 2$



HW

4. $3y - 2x = 6$
   $-12y + 8x = -24$



HW

3.2 Solving Systems by Substitution or Elimination
THE SUBSTITUTION METHOD

Steps for solving a systems by substitution

1. Use one of the equation to find x or y.
2. Substitute x or y found in the other equation.
3. Solve the other equation for x or y
4. Use the expression in Step 1 to find the other variable.

Example 1: Solve the system

$x + y = 4$
$x = y + 1$

1. $x = y + 1$
2. $(y+1) + y = 4$
3. $y + 1 + y = 4 \Rightarrow 2y + 1 = 4 \Rightarrow 2y = 4 - 1 \Rightarrow 2y = 3 \Rightarrow y = 3/2$
4. $x = \frac{3}{2} + 1 = \frac{5}{2} \Rightarrow x = \frac{5}{2}$

$\left(\frac{5}{2}, \frac{3}{2}\right)$

Example 2: Solve the system

$2x + y = 6$
$3x + 4y = 4$

1.) $2x + y = 6 \Rightarrow y = 6 - 2x$
2.) $3x + 4(6 - 2x) = 4$
3.) $3x + 24 - 8x = 4 \Rightarrow -5x + 24 = 4 \Rightarrow -5x = 4 - 24$
$\Rightarrow -5x = -20 \Rightarrow x = \frac{-20}{-5} \Rightarrow x = 4$
4.) $y = 6 - 2(4) \Rightarrow y = 6 - 8 \Rightarrow y = -2$

$(4, -2)$

Example 3: Solve the system

$x + 2y = 4$
$2x + 3y = 1$

1.) $x = 4 - 2y$
2.) $2(4 - 2y) + 3y = 1 \Rightarrow 8 - 4y + 3y = 1 \Rightarrow 8 - y = 1$
3.) $\Rightarrow -y = 1 - 8 \Rightarrow -y = -7 \Rightarrow y = 7.$
4.) $x = 4 - 2(7) \Rightarrow x = 4 - 14 \Rightarrow x = -10$

$(-10, 7)$

Example 5: Solve the system

$y = -3x + 5$
$y = -3x - 2$

1.) $y = -3x + 5$

2.) $-3x + 5 = -3x - 2$
$+3x \qquad +3x$

3.) $5 = -2 \rightarrow$ Not Possible

Parallel Lines

↑

NO SOLUTION

THE ELIMINATION METHOD

1. Multiply both equations with some constants so that coeff. of x or y in both equation is same but with opposite sign.
2. Add the two equations now and solve for x or y.

Example 6: Solve the system

$2x - 3y = 0$
$-4x + 3y = -1$

1.) Done

2.) $\left.\begin{aligned} 2x - 3y &= 0 \\ -4x + 3y &= -1 \end{aligned}\right\} \Rightarrow 2x - 4x = 0 - 1$

$\Rightarrow -2x = -1 \Rightarrow x = \frac{-1}{-2} \Rightarrow x = \frac{1}{2}$

$\rightarrow 2\left(\frac{1}{2}\right) - 3y = 0 \Rightarrow 1 - 3y = 0 \Rightarrow -3y = -1 \Rightarrow y = \frac{-1}{-3} \Rightarrow y = \frac{1}{3}$

$\left(\frac{1}{2}, \frac{1}{3}\right)$

Example 7: Solve the system

$5x + 4y = 22 \quad \times 2 \Rightarrow 10x + 8y = 44$
$-3x + 8y = 18 \quad \times -1 \Rightarrow 3x - 8y = -18$ } Add

$\Rightarrow 10x + 3x = 44 - 18$

$\Rightarrow 13x = 26 \Rightarrow x = \frac{26}{13} \Rightarrow x = 2$

$\rightarrow 5(2) + 4y = 22 \Rightarrow 10 + 4y = 22 \Rightarrow 4y = 22 - 10 \Rightarrow 4y = 12$

$\Rightarrow y = \frac{12}{4} \Rightarrow y = 3$ $\boxed{(2,3)}$

Example 8: Solve the system

$2x - 3y = 8$ $\times 3$

$6x + 5y = 4$ $\times -1$

HW.

Example 9: Solve the system

$3y - 2x = 6$ $\times 4$

$-12y + 8x = -24$ $\times 1$

$\Rightarrow 12y - 8x = 24$

$\Rightarrow -12y + 8x = -24$ } Add

$0 = 0$ (identity)

$\boxed{\text{coinciding lines}} \leftarrow \boxed{\text{INFINITE SOLUTIONS}}$

Example 10: Solve the system

$0.2x + 0.3y = 1.7$ $\times 5$

$\frac{1}{7}x + \frac{1}{5}y = \frac{29}{35}$ $\times -7$

HW.

The steps in each algebraic method for solving systems of two equations are given below. Note that in both methods, we find the value of one variable and then substitute to find the corresponding value of the other variable.

TO SOLVE A SYSTEM USING SUBSTITUTION

1. Isolate a variable in one of the equations (unless one is already isolated).
2. Substitute for that variable in the other equation, using parentheses.
3. Solve the equation in which the substitution was made.
4. Substitute the solution from step (3) in any of the equations, and solve for the other variable.
5. Form an ordered pair and check in the original equations.

TO SOLVE A SYSTEM USING ELIMINATION

1. Write both equations in standard form.
2. Multiply both sides of one or both equations by a constant, if necessary, so that the coefficients of one of the variables are opposites.
3. Add the left sides and the right sides of the resulting equations. One variable should be eliminated in the sum.
4. Solve for the remaining variable.
5. Substitute the value of the second variable in any of the equations, and solve for the other variable.
6. Form an ordered pair and check in the original equations.

Quiz 5

① Find the equation of line through $(1,1)$ and parallel to the line $2x+y=1$.

(x_1, y_1) and slope m.

$y-y_1 = m(x-x_1) \Rightarrow y-1 = m(x-1)$

$2x+y=1 \Rightarrow y = -2x+1$ $\quad [y = mx+b]$

$m = -2$

$y-1 = -2(x-1) \Rightarrow y-1 = -2x+2 \Rightarrow y = -2x+3$

$\Rightarrow 2x+y=3$

② Find the eqn. of line through $(1,0)$ and $(2,1)$.

$y-0 = m(x-1)$

$m = \frac{y_2-y_1}{x_2-x_1} = \frac{1-0}{2-1} = \frac{1}{1} = 1$

$y = 1(x-1)$

$y = x-1$

Test 1

1.1 to 1.6, 2.1 to 2.6, 3.1, 3.2, 3.3, 3.8