

Math16500 Section 24246 Quiz 5

Fall 2022, September 19

Name:

[1 pt]

**Problem 1:** Let  $f(x) = \frac{1}{x^2}$ . Evaluate the limit

$$\lim_{x \rightarrow 1} \frac{f(x) - f(1)}{x - 1}.$$

[5 pts]

$$\begin{aligned} \lim_{x \rightarrow 1} \frac{\frac{1}{x^2} - \frac{1}{1^2}}{x - 1} &= \lim_{x \rightarrow 1} \frac{\frac{1}{x^2} - 1}{x - 1} = \lim_{x \rightarrow 1} \frac{\frac{1 - x^2}{x^2}}{x - 1} \\ &= \lim_{x \rightarrow 1} \frac{1 - x^2}{x^2(x - 1)} = \lim_{x \rightarrow 1} \frac{(1 - x)(1 + x)}{x^2(x - 1)} = \lim_{x \rightarrow 1} \frac{-\cancel{(x - 1)}(1 + x)}{x^2 \cancel{(x - 1)}} \\ &= \lim_{x \rightarrow 1} \frac{-(1 + x)}{x^2} = -\frac{(1 + 1)}{1^2} = \underline{\underline{-2}} \end{aligned}$$

**Problem 2:** Let  $f(x) = x^3$ . Evaluate the limit

$$\lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}.$$

$$\lim_{h \rightarrow 0} \frac{(x + h)^3 - x^3}{h} \quad (x + h)^3 = x^3 + 3x^2h + 3xh^2 + h^3 \quad [4 \text{ pts}].$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 - \cancel{x^3}}{h} = \lim_{h \rightarrow 0} \frac{3x^2h + 3xh^2 + h^3}{h}$$

$$\begin{aligned} &= \lim_{h \rightarrow 0} \frac{\cancel{h}(3x^2 + 3xh + h^2)}{\cancel{h}} = 3x^2 + 3x(0) + 0^2 \\ &= \underline{\underline{3x^2}} \end{aligned}$$