

Math16600 Section 23715 Quiz 8

Fall 2023, October 31

Name:

[1 pt]

Problem 1: Determine whether the series

$$\sum_{n=1}^{\infty} \frac{4^{2n+1}}{(-2)^n}$$

converges or diverges. If it converges find the limit.

[5 pts]

$$a_n = \frac{4^{2n+1}}{(-2)^n} \Rightarrow a_{n+1} = \frac{4^{2(n+1)+1}}{(-2)^{n+1}} = \frac{4^{2n+3}}{(-2)^{n+1}}$$

$$r = \frac{a_{n+1}}{a_n} = \frac{4^{2n+3}}{(-2)^{n+1}} \times \frac{(-2)^n}{4^{2n+1}} = \frac{4^{2n+3-2n-1}}{(-2)^{n+1-n}} = \frac{4^2}{(-2)} = -8$$

r is independent of $n \Rightarrow$ the given series is geometric

$|r| = 8 > 1 \Rightarrow$ the given geometric series **diverges**.

Problem 2: Determine whether the series

$$\sum_{n=1}^{\infty} \frac{n^3 + 2n^2 + 1}{2n^3 + n - 1}$$

converges or diverges.

[5 pts]

$$a_n = \frac{n^3 + 2n^2 + 1}{2n^3 + n - 1}$$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n^3 + 2n^2 + 1}{2n^3 + n - 1} = \lim_{n \rightarrow \infty} \frac{n^3}{2n^3} = \frac{1}{2}$$

$$\Rightarrow \lim_{n \rightarrow \infty} a_n = \frac{1}{2} \neq 0$$

\Rightarrow By Test for divergence, the **given series diverges**.