

Name:

[1 pt]

Problem 1. If $x = \cot \sqrt{t}$, find $\frac{dx}{dt}$.

[5 pts]

$$\begin{aligned}\frac{dx}{dt} &= \frac{d}{dt} (\cot \sqrt{t}) \\ &= -(\csc^2 \sqrt{t}) \cdot \frac{d}{dt} (\sqrt{t}) \\ &= -(\csc^2 \sqrt{t}) \cdot \frac{1}{2\sqrt{t}} \\ &= -\frac{\csc^2 \sqrt{t}}{2\sqrt{t}}\end{aligned}$$

Problem 2. Find the second derivative of $y = x \tan x$.

[5 pts]

$$\begin{aligned}y' &= [x]' \tan x + x [\tan x]' \quad [\text{Product rule}] \\ &= \tan x + x \sec^2 x \\ \Rightarrow y'' &= [\tan x]' + \underbrace{[x \sec^2 x]'}_{\text{use Product rule again.}} \\ &= \sec^2 x + [x]' \sec^2 x + x [\sec^2 x]' \\ &= \sec^2 x + \sec^2 x + x (2 \sec x \cdot [\sec x]') \quad \text{Chain rule} \\ &= 2 \sec^2 x + 2x \sec x (\sec x \tan x) \\ &= 2 \sec^2 x + 2x \sec^2 x \tan x \\ &= 2 \sec^2 x (1 + x \tan x)\end{aligned}$$