Learning objectives:

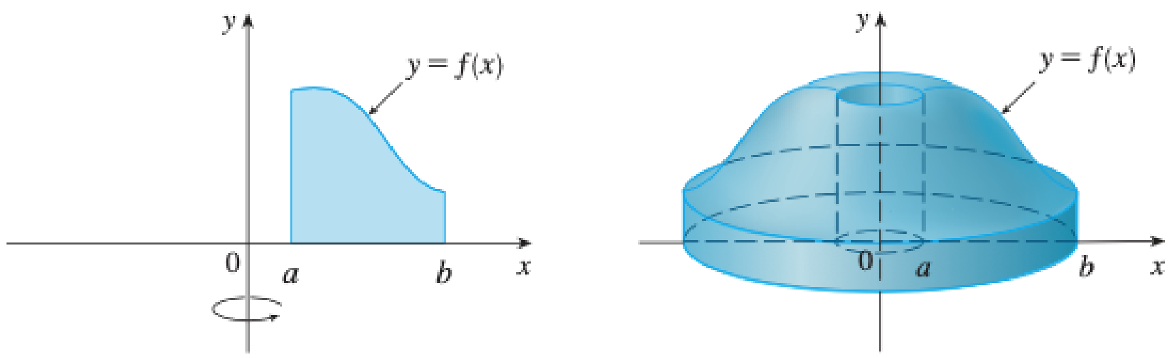
1. Find volumes of solids of revolution, obtained by revolving a region about a line called axis.
2. We divide the given solid into infinite cylinderical shells of infinitesimally small thickness.

The volume of a thin cylinderical shell of radius r and height h is given by

$$dV = 2\pi\, r\, h\, dr\,.$$

The volume of the solid shown in figure below, obtained by rotating the region on the left (region under $y = f(x)$ from a to b) about the y-axis, is

$$V = \int_a^b 2\pi\, x\, f(x)\, dx\,.$$



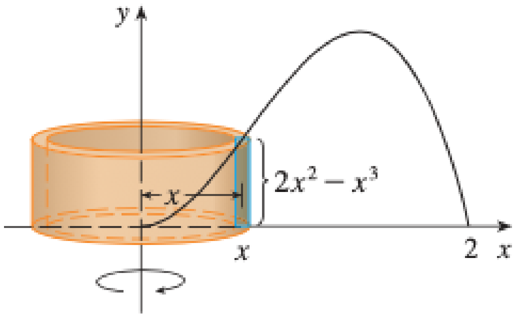
In general for a region bounded between $y = f(x)$ and $y = g(x)$ between $x = a$ to $x = b$, the volume of solid obtained by rotating it about the y-axis, is

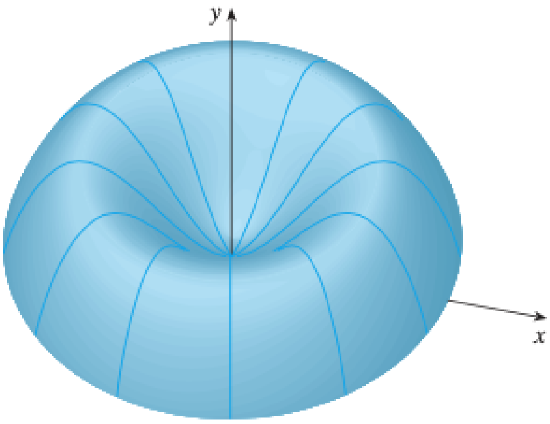
$$V = \int_a^b 2\pi\, x\, |f(x) - g(x)|\, dx\,.$$

For a region bounded between $x = f(y)$ and $x = g(y)$ between $y = a$ to $y = b$, the volume of solid obtained by rotating it about the x-axis, is

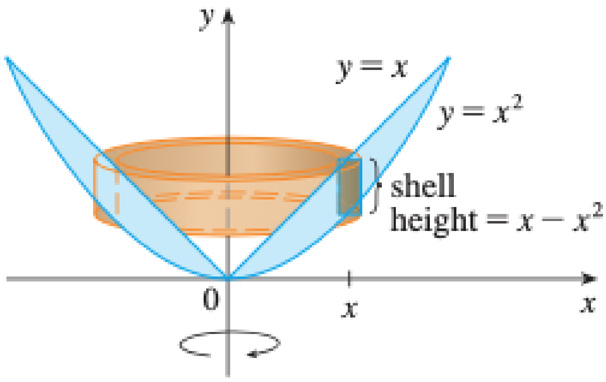
$$V = \int_a^b 2\pi\, y\, |f(y) - g(y)|\, dy\,.$$

Example 1. Find the volume of the solid obtained by rotating the region bounded by $y = 2x^2 - x^3$ and $y = 0$, about the y-axis.

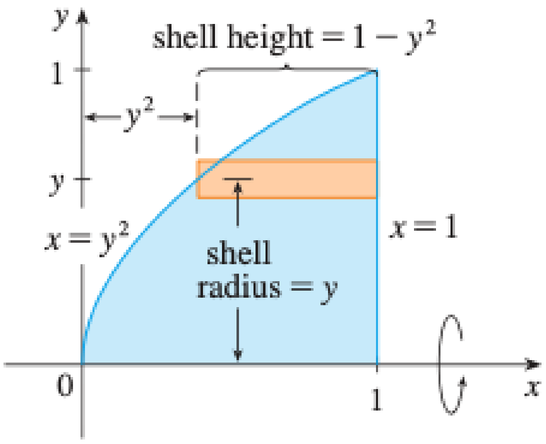




Example 2. Find the volume of the solid obtained by rotating about the y-axis the region between $y = x$ and $y = x^2$.



Example 3. The region R enclosed by the curves $y = \sqrt{x}$ and $y = 0$ is rotated about the x-axis. Find the volume of the resulting solid using cylinderical shell method.



Example 4. The region R enclosed by the curves $y = x - x^2$ and $y = 0$ is rotated about the $x = 2$ line. Find the volume of the resulting solid.