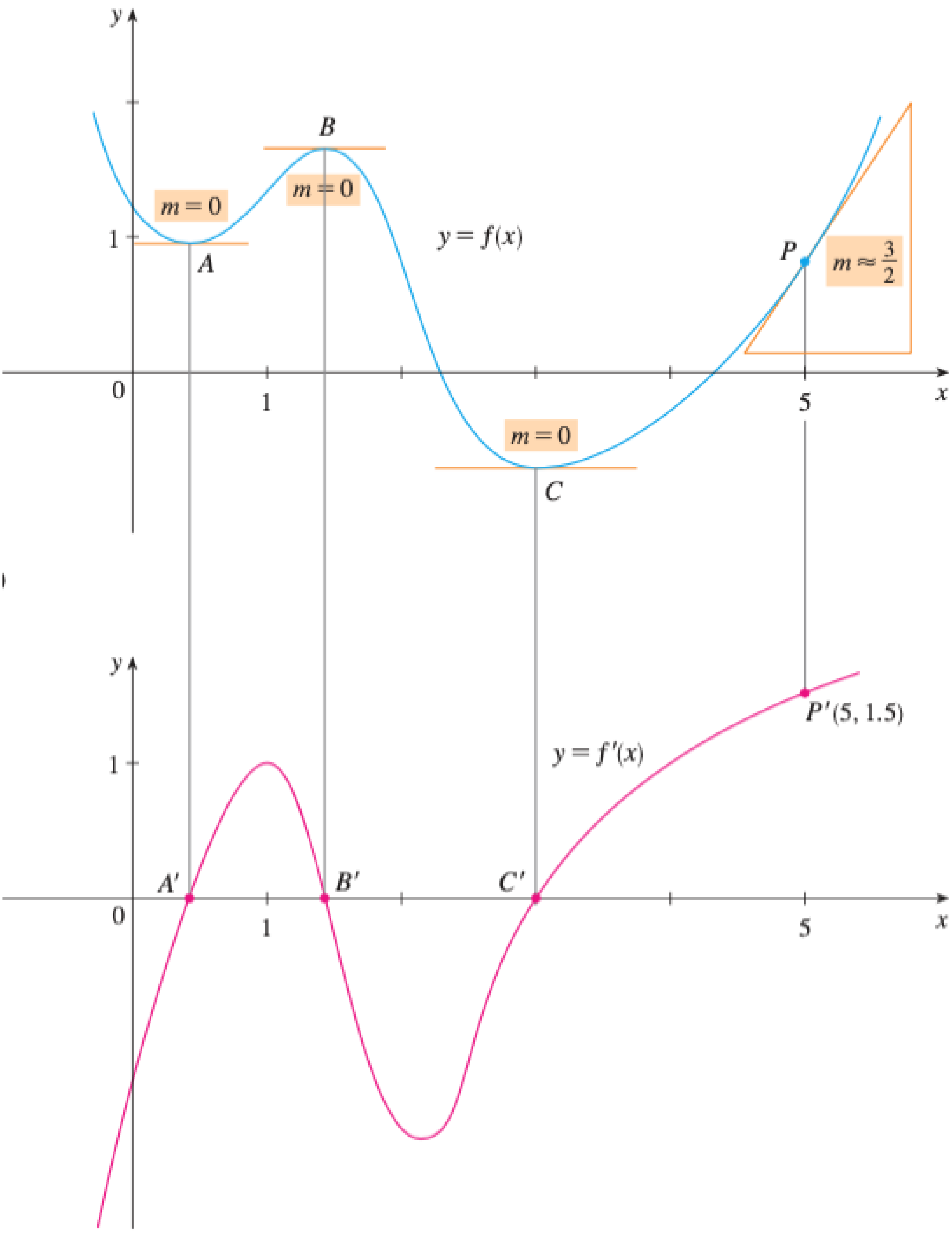
Learning objectives:

1. Define the derivative as a function.
2. The property of differentiability
3. When can a function fail to be differentiable?
4. Higher derivatives and their interpretation.

The derivative of a function $y = f(x)$ is a new function $f'(x)$ defined by

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} .$$



Example 1. If $f(x) = x^3 - x$, find a formula for $f'(x)$.

Example 2. Find $f'(x)$ if $f(x) = \dfrac{1-x}{2+x}$.

Other Notations for Derivative

$$f'(x) = y' = \frac{dy}{dx} = \frac{df}{dx} = \frac{d}{dx}f(x) = Df(x) = D_x f(x) \ .$$

The symbol D and d/dx are called the differentiation operators since they indicate the process of differentiation.

We often write $f'(a)$ as $\left.\frac{dy}{dx}\right|_{x=a}$.

Differentiability

A function f is said to be differentiable at a if $f'(a)$ exists. It is differentiable on an open interval if it is differentiable at every number in the interval.

Example of $y = \sqrt{x}$ from previous lecture:

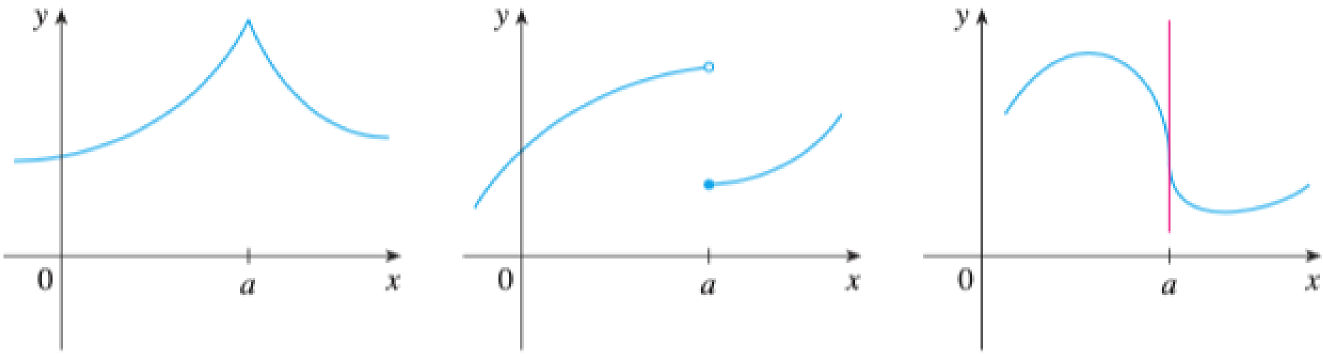
Example 3. Where is function $f(x) = |x|$ differentiable?

Differentiability implies continuity

If f is differentiable at a then f is continuous at a.

There exist functions that are continuous but not differentiable.

How can a function fail to be differentiable?



(a) A corner (b) A discontinuity (c) A vertical tangent

Higher Derivatives

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}\,.$$

Same as $f'' = (f')'$ we have $f^{(n)} = (f^{(n-1)})'$, that is, in general

$$\frac{d}{dx}\left(\frac{d^{n-1}y}{dx^{n-1}}\right) = \frac{d^ny}{dx^n}\,.$$

Position (function) $\xrightarrow{\text{derivative}}$ velocity $\xrightarrow{\text{derivative}}$ acceleration $\xrightarrow{\text{derivative}}$ jerk

Example 4. If $f(x) = x^3 - x$, find $f''(x)$, $f'''(x)$ and $f^{(4)}(x)$.