## M16600 Lecture Notes

Section 11.5: Alternating Series

**Section 11.5** textbook exercises, page 776:  $\# \underline{4}$ , 5, 7, 9, 6, 14.

**DEFINITION.** An *alternating series* is a series whose terms are alternately positive and negative.

E.g., 
$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n}$$

As a convention, we write an alternating series as  $\sum (-1)^n b_n$ , where  $b_n > 0$  for all n.

For the example above,  $b_n = \frac{1}{100}$ 

## Convergence/Divergence for Alternating Series $\sum (-1)^n b_n$

- Alternating Series Test (AST): The alternating series  $\sum (-1)^n b_n$  converges if these two conditions are satisfied:
  - (i)  $\lim_{n\to\infty} b_n = 0$
  - (ii)  $b_{n+1} \leq b_n$  (the terms  $b_n$  are decreasing)
- The alternating series  $\sum (-1)^n b_n$  diverges if  $\lim_{n\to\infty} b_n \neq 0$ .

Example 1: Use the Alternating Series Test to show that the alternating series  $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n}$ 

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n}$$

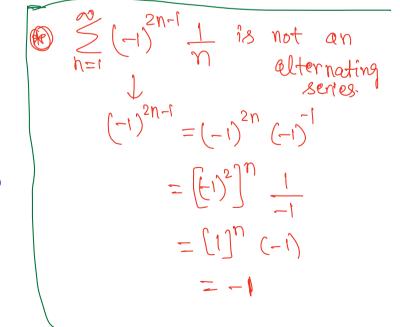
converges.

$$\sum_{n=1}^{\infty} \left(-1\right)^{n-1} \frac{1}{n}$$

$$b_{\eta} = \frac{L}{N}$$

$$\underbrace{\text{lii}}_{n+1} > n \Rightarrow \frac{1}{n+1} < \frac{1}{n}$$

$$\Rightarrow b_{n+1} < b_n \Rightarrow b_n s$$



are dec, => By AST, the

## Example 2: Test the series for convergence or divergence

**Hint:** The first step in determining convergence or divergence for an **alternating series** is to compute  $\lim_{n\to\infty} b_n = 0$ .

(a) 
$$\sum_{n=1}^{\infty} \frac{(-1)^n}{2\sqrt{n}+5}$$

$$\Rightarrow b_n = \frac{1}{2 \sqrt{n+5}} \Rightarrow \lim_{n \to \infty} \frac{1}{2 \sqrt{n+5}} = \frac{1}{6 \sqrt{n+5}} = 0$$

$$b_{n+1} = \frac{1}{2J_{n+1}+5} < \frac{1}{2J_{n+5}} = b_n$$

$$\sqrt{n+1} > \sqrt{n} = 2\sqrt{n+1} > 2\sqrt{n} = 2\sqrt{n+1} + 5 > 2\sqrt{n} + 5$$

$$\frac{1}{2\sqrt{n+1} + 5} < \frac{1}{2\sqrt{n+5}} = 2\sqrt{n} + 5 > 2\sqrt{n} + 5$$

$$\frac{1}{2\sqrt{n+1} + 5} < \frac{1}{2\sqrt{n+5}} = 2\sqrt{n} + 5 > 2\sqrt{n} + 5$$

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$$\frac{1}{2\sqrt{n+5}} = 2\sqrt{n} + 5$$

(b) 
$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{3n^4 + n}{4n^4 - n^3 + n^2 + 2}$$

$$b_{n} = \frac{3n^{4} + n}{4n^{4} - n^{3} + n^{2} + 2} \Rightarrow \lim_{n \to \infty} b_{n} = \lim_{n \to \infty} \frac{3n^{4}}{4n^{4}} = \lim_{n \to \infty} \frac{3}{4}$$

$$= \frac{3}{4}$$

$$\Rightarrow \lim_{n \to \infty} b_n = \frac{3}{4} \neq 0$$