Station: 4 Scratch Paper: /

DIVISION OF DIVERSITY, EQUITY & INCLUSION
ACCESSIBLE EDUCATIONAL SERVICES
Indianapolis

AES Testing Record

Student: Ethan Aldrich Winnett -(Ethan) **Must Stop At:** 1:55pm

Test Date: February 19, 2025 **Test Time:** 12:00 pm **Location:** AES Testing Lab (UL 3135H -Lib 3rd fl)

Student Status:

Course Title: ANALYTIC GEOMETRY & CALCULUS I **Code:** MATH-I 165 30129

Instructor: Keshav Dahiya

Test Type: Exam **Test Format:** Paper **Name/Number:** Test 1

Accommodations: Distraction-Reduced Environment; Extended Time on Quizzes and Exams (150%)

Instructor's Directions: Closed book/notes. No calc. No scratch paper.

Time Allotted: 113min **Start Time:** 12:02pm **Ending Time:** 1:55

Breaks Taken:

Proctors: Lily, Karla, Jill

Proctor Notes:

Delivery Preference: Scan/email test, then keep in AES Office UC100

Delivery Log (Please contact AES for delivery records)

Emailed By: Date: Time:

Delivered By: Date: Time: Location:

Received By: Date: Time:

Attempted By: Date: Time: Location:

Explanation:

Attempted By: Date: Time: Location:

Explanation:

Indiana University, Indianapolis

Spring 2025 Math-I 165

Test 1

Instructor: Keshav Dahiya

Name: Ethan Wilmett

Instructions:

- No cell phones, calculators, watches, technology, hats - stow all in your bags.
- **Write your name** on this cover page.
- This test is **closed book and closed notes.**
- All work must be clearly shown for partial credit.
- If you wish for something not to be graded, please strike it out neatly.
- Box, circle, or otherwise clearly indicate your final answer.
- When you finish, return your test to the proctor, and leave the classroom.
- There are a total of **12 problems including 2 bonus problems**.
  - Problems 1-10 are each worth 10 points.
  - The bonus problems are each worth 5 points.
- You can score a **maximum of 110 points out of 100**.
- There are a total of **7 pages** including the cover page.

Problem 1. Evaluate the limit: $\lim_{t\to 0} \frac{\sqrt{t^2+9}-3}{t^2}$. [10 pts]

$$\lim_{t\to 0} \frac{\sqrt{t^2+9}-3}{t^2} \frac{(\sqrt{t^2+9}+3)}{(\sqrt{t^2+9}+3)} \to \lim_{t\to 0} \frac{t^2+9-9}{t^2(\sqrt{t^2+9}+3)}$$

$$\lim_{t\to 0} \frac{\cancel{t^2}}{\cancel{t^2}(\sqrt{t^2+9}+3)} \quad \lim_{t\to 0} \frac{1}{\sqrt{t^2+9}+3} \quad \lim_{t\to 0} \frac{1}{\sqrt{0+9}+3} \to \frac{1}{\sqrt{9}+3} \to \frac{1}{3+3}$$

$$\frac{1}{6}$$

Problem 2. Let $f(x) = \begin{cases} \frac{x^2-x-2}{x-2} & x \neq 2, \\ 1 & x = 2. \end{cases}$ Is f continuous? Explain why/why not.

$$f(x) = \frac{x^2-x-2}{x-2} \to \frac{\cancel{(x-2)}(x+1)}{\cancel{x-2}} \to (x+1) \quad x \neq 2$$

$$2+1=3$$

$f(x) = 1 \quad x=2 \qquad 1 \neq 3$

Its discontinuous because both numbers do not equal. They aren't even attatched on the graph to each other

Problem 3. Use the limit definition $f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$ of derivative to find $f'(3)$ if $f(x) = \frac{3}{x}$.
[10 pts]

$$f'(x) = \frac{\frac{3}{x} - \frac{3}{a}}{x-a} \to \frac{\frac{3a}{xa} - \frac{3x}{xa}}{x-a} \quad \frac{3a-3x}{(x)(a)(x-a)}$$

$$\frac{-3\cancel{(x-a)}}{(x)(a)\cancel{(x-a)}} \to \frac{-3}{(x)(a)} \to \frac{-3}{ax}$$

Problem 4 . If $h(x) = x\,g(x)$ and $g(3) = 5$, $g'(3) = 2$, then find $h'(3)$. [10 pts]

Hint: Use product rule to differentiate $h(x)$.

$h(3) = 3g(3) \to h(3) = 3(5)$ $h(3) = 15$
$h(3) = 0$

$h'(x) = (x)'(g(x)) + x(g(x))'$ $h'(x) = (1)(g(x)) + x(g(x)')(1)$

$h'(x) = (g(x)) + (x\,g(x)'\frac{d}{dx}$ $h'(3) = (g(3)) + 3(g'(3))\frac{d}{dx}$

$h'(3) = 5 + 3(2)$ $h'(3) = 5 + 6$ $h'(3) = 11$

Problem 5. Find the second derivative of $\csc x$. [10 pts]

$$f'(x) = -\csc_x \cot_x$$

$$f''(x) = (-\csc_x)'(\cot_x) + (-\csc_x)(\cot_x)'$$

$$f''(x) = (\csc x \cot x)(\cot x) + (-\csc_x)(-\csc_x^2)$$

$$f''(x) = \frac{\cos^2}{\sin^3} + \frac{1}{\sin^3} \qquad f''(x) = \frac{\cos^2 + 1}{\sin^3}$$

Problem 6. Use chain rule to find the derivative of $f(x) = \dfrac{1}{\sqrt[3]{x^2 + x + 1}}$. [10 pts]

$$\frac{1}{\sqrt[3]{9}} \qquad f'(x) = \frac{1}{\sqrt[3]{x^2+x+1}}(x^2+x+1)(2x+1)$$

$$f'(x) = \frac{2x^3+2x^2+2x+x^2+x+1}{\sqrt[3]{x^2+x+1}} \qquad f'(x) = \frac{2x^3+3x^2+3x+1}{\sqrt[3]{x^2+x+1}}$$

$f'(x)$

Problem 7. Use implicit differentiation to find the equation of normal to the curve $x^3 + y^3 = 6xy$ at the point $(3, 3)$. [10 pts]

$$f(x) = x^3 + y^3 - 6xy \qquad f(x)' = 3x^2 + 3y^2 - ((6x)'(y) + (6x)(y)')$$

$$f'(x) = 3x^2 + 3y^2\frac{dy}{dx} - \left(6y + 6x\frac{dy}{dx}\right) \qquad f'(x) = 3x^2 + 3y^2\frac{dy}{dx} - 6y - 6x\frac{dy}{dx}$$

$$f'(x) = 3y^2\frac{dy}{dx} - 6x\frac{dy}{dx} = -3x^2 + 6y$$

$$f'(x) = \frac{dy}{dx}(3y^2 - 6x) = -3x^2 + 6y$$

$$f'(x) = \frac{dy}{dx} = \frac{-3x^2 + 6y}{3y^2 - 6x} \rightarrow \frac{dy}{dx} = \frac{-3(x^2 - 2y)}{3(y^2 - 2x)}$$

$$\frac{dy}{dx} = \frac{-(x^2 - 2y)}{(y^2 - 2x)}$$

Problem 8. The position of a particle is given by the equation $s(t) = t^3 - 6t^2 + 9t$ where t is measured in seconds and s is measured in meters. Find the total distance traveled by the particle in the first five seconds. [10 pts]

$(5)^3 - 6(5)^2 + 9(5)$

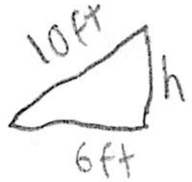
$125 + 150 + 45 = 20$

$$\frac{dd}{dt} \quad 3t^2 - 12t + 9$$

$$3(5)^2 - 12(5) + 9 \quad 75 - 60 + 9 \quad 15 + 9 = 24$$

Distance after the first 5 seconds is 24 m/s

Problem 9. A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 ft/s, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6 ft from the wall? [10 pts]



$10^2 = 6^2 + B^2$ $\quad 100 = 36 + B^2$

$B^2 = 64$ $\quad B = \pm 8$ $\quad B = 8$

$\frac{dr}{dx} = 1 \text{ ft/s}$

$\frac{dr}{dx} = \frac{1}{2}(6)(8)$ $\quad \frac{dr}{dx} = \frac{1}{2}(48)$ $\quad \frac{dr}{dx} = 24 \text{ ft/s}$

Problem 10. The area of a circle was measured and it was found that the measured value has a relative error of 1%. If we compute radius of the circle using this value of area, what would be the relative error in the radius of the circle? [10 pts]

Hint: $A = \pi r^2$.

$A' = 2\pi r$ $\quad \frac{2\pi r}{\pi r^2} = \frac{R}{r}\frac{dr}{dx}$

$2\pi\left(\frac{1}{100}\right)\left(\frac{1}{2}\right)$ $\quad dr = 1\%$ error

Bonus Problem 1. At what points on the hyperbola $xy = 12$ is the tangent line parallel to the line $3x + y = 0$? [5 pts]

$y = \frac{12}{x}$

$F'(x) = (x)'(y) + (x)(y)' = 0$

$\frac{dy}{dx} = \frac{(12)'(x) - (x)'(12)}{x^2}$

$f'(x) = y + x = 0 \quad y = -x$

$\frac{dy}{dx} = \frac{x - 12}{x^2}$

$3x - x = 0 \quad 2x = 0 \quad x = 0$

points are (0,0)

Bonus Problem 2. Find $\frac{d^2y}{dx^2}$ if $x^4 + y^4 = 16$. [5 pts]

$f'(x) = 4x^3 + 4y^3 = 0$

$f''(x) = 12x^2 + 12y^2 = 0$

$f''(x) = 12x^2 = -12y^2$

$f''(x) = x^2 = -y^2$

$\frac{d^2y}{dx^2} = \frac{-y^2}{x^2}$