$\boxed{x=0}$ → $x=a$ (in general)

$\frac{1}{m}=0$

$m=\infty$

$y=mx+b$

$m=0$

$\Rightarrow y=b$

Graphing Horizontal Lines and Vertical Lines

ESSENTIALS

The graph of $y=b$ is a horizontal line with y-intercept $(0,b)$. Its slope is 0.

The graph of $x=a$ is a vertical line with x-intercept $(a,0)$. Its slope is undefined.

Examples

- If possible, find the slope of $x=-4$.
  The slope is undefined.
- If possible, find the slope of $y=10$.
  The slope is 0.

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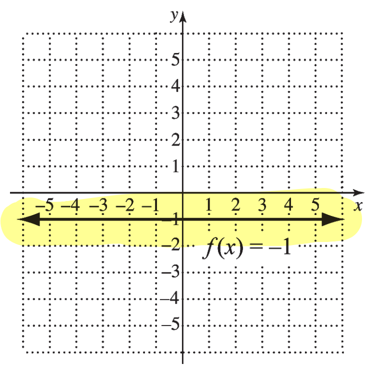
EXAMPLE 1

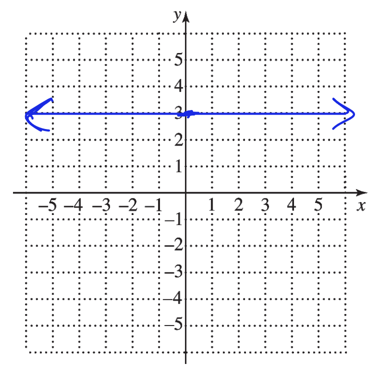
Graph: $f(x)=-1$.



The function can be written in slope-intercept form as $f(x)=0\cdot x+(-1)$. We see that the y-intercept is $\boxed{(0,-1)}$ and the slope is $\boxed{0}$.

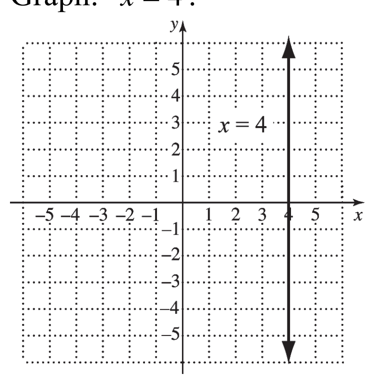
YOUR TURN 1

Graph: $f(x)=3$.



EXAMPLE 2

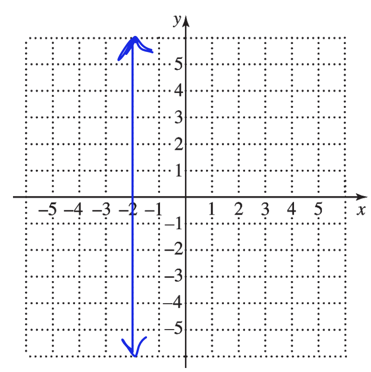
Graph: $x=4$.



The graph is a vertical line.
vertical / horizontal

YOUR TURN 2

Graph: $x=-2$.



EXAMPLE 3	YOUR TURN 3
Find the slope of the line $x+1=-5$. If the slope is undefined, state this. $x+1=-5$ $x=-5-1$ $x=\boxed{-6}$ $x=-6$ The graph is a ___vertical___ line. (vertical / horizontal) The slope is ___undefined___. (0 / undefined)	Find the slope of the line $3x=9$. If the slope is undefined, state this. $3x=9$ $x=\frac{9}{3} \Rightarrow x=3$ Undefined
EXAMPLE 4	**YOUR TURN 4**
Find the slope of the line $2y-4=7$. If the slope is undefined, state this. $2y-4=7$ $2y=\boxed{11}$ $y=\boxed{\frac{11}{2}}$ The graph is a ___horizontal___ line. (vertical / horizontal) The slope is ___0___. (0 / undefined)	Find the slope of the line $\frac{1}{2}-y=1$. If the slope is undefined, state this. $-y=1-\frac{1}{2} \Rightarrow -y=\frac{1}{2}$ $\Rightarrow y=-\frac{1}{2}$ slope = 0

YOUR NOTES Write your questions and additional notes.

Parallel Lines and Perpendicular Lines

ESSENTIALS

Two lines are parallel if they have different y-intercepts and the same slope or if they are both vertical.

Two lines are perpendicular if the product of their slopes is -1 or if one line is vertical and the other is horizontal.

Example

- Determine whether $y = 3x + 5$ and $y = -\frac{1}{3}x + 2$ are parallel, perpendicular, or neither.

  $\rightarrow m_1 = 3$, $m_2 = \frac{-1}{3}$

  The slope of $y = 3x + 5$ is 3.

  The slope of $y = -\frac{1}{3}x + 2$ is $-\frac{1}{3}$.

  We know they are perpendicular because $3\left(-\frac{1}{3}\right) = -1$.

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EXAMPLE 1

Determine whether the line given by $f(x) = 2x - 5$ is parallel to the line given by $2y - 4x = 3$.

The slope of $f(x) = 2x - 5$ is $\boxed{2}$.

We find the slope of $2y - 4x = 3$ by first writing the equation in slope-intercept form.

$$2y - 4x = 3$$
$$2y = 4x + 3$$
$$y = \boxed{2}x + \frac{3}{2}$$

The slope is $\boxed{2}$.

Because the slopes are (are / are not) equal, the lines ~~are~~ are (are / are not) parallel.

YOUR TURN 1

Determine whether the line given by $f(x) = -4x + 2$ is parallel to the line given by $4y - x = 5$.

$\rightarrow m_1 = -4$

$$4y - x = 5$$
$$4y = x + 5$$
$$y = \frac{1}{4}x + \frac{5}{4}$$
$$m_2 = \frac{1}{4}$$

$\Rightarrow$ Not Parallel

EXAMPLE 2	YOUR TURN 2
Determine whether the graphs of $3x - y = 7$ and $y = -\frac{1}{3}x + 3$ are perpendicular. The slope of $y = -\frac{1}{3}x + 3$ is $\boxed{}$. We find the slope of $3x - y = 7$ by first writing the equation in slope-intercept form. $3x - y = 7$ $-y = -3x + 7$ $y = \boxed{}$ HW The slope is $\boxed{}$. We find the product of the slopes: $\left(\boxed{}\right)\left(\boxed{}\right) = \boxed{}$ Since the product of the slopes $\underset{\text{is / is not}}{\underline{\qquad\qquad}}$ -1, the lines $\underset{\text{are / are not}}{\underline{\qquad\qquad}}$ perpendicular.	Determine whether the graphs of $5x - 6y = 30$ and $5y + 6x = 0$ are perpendicular. HW.

YOUR NOTES Write your questions and additional notes.

Graphing Using Intercepts

ESSENTIALS

The x-intercept of a graph is $(a, 0)$. To find a, let $y = 0$ and solve for x.

The y-intercept of a graph is $(0, b)$. To find b, let $x = 0$ and solve for y.

Example

- Find the intercepts of $5x - 4y = 20$.

$5x - 4 \cdot 0 = 20$ Letting $y = 0$
$5x = 20$
$x = 4$

The x-intercept is $(4, 0)$.

$5 \cdot 0 - 4y = 20$ Letting $x = 0$
$-4y = 20$
$y = -5$

The y-intercept is $(0, -5)$.

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EXAMPLE 1	YOUR TURN 1
Graph $3x - 4y = -12$ by using intercepts.	Graph $-2x + 5y = 10$ by using intercepts.

Graph $3x - 4y = -12$ by using intercepts.

$3x - 4 \cdot 0 = -12$ Letting $y = 0$

$3x = -12$

$x = \boxed{-4}$ Dividing both sides by 3

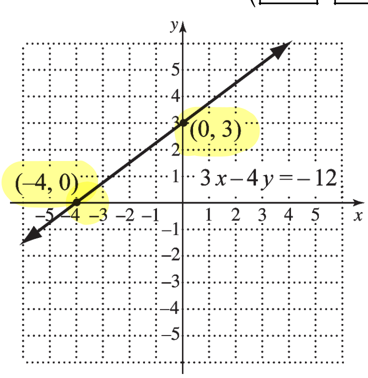
The x-intercept is $(\boxed{-4}, \boxed{0})$.

$3 \cdot 0 - 4y = -12$ Letting $x = 0$

$\boxed{-4y} = -12$

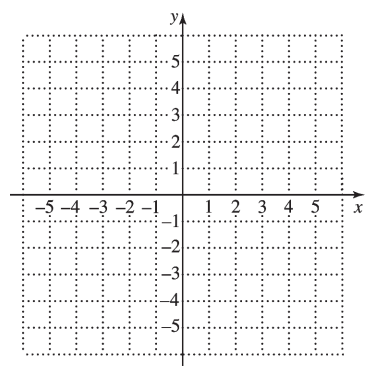
$y = \boxed{3}$ Dividing both sides by -4

The y-intercept is $(\boxed{0}, \boxed{3})$.



YOUR TURN 1

Graph $-2x + 5y = 10$ by using intercepts.



HW

EXAMPLE 2	YOUR TURN 2
Graph $f(x)=3x-4$ by using intercepts.	Graph $f(x)=2x-3$ by using intercepts.

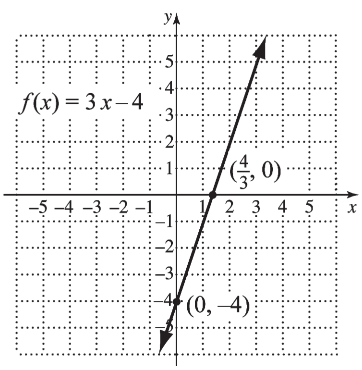
Graph $f(x)=3x-4$ by using intercepts.

The y-intercept is $(0, \boxed{})$.

Replace $f(x)$ with 0 and solve for x.

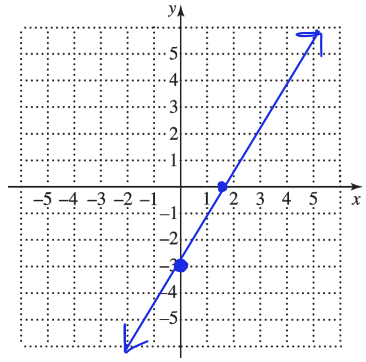
$$f(x)=3x-4$$
$$0=3x-4$$
$$4=\boxed{} \quad \text{Adding 4 to both sides}$$
$$\boxed{}=x \quad \text{Dividing both sides by 3}$$

The x-intercept is $\left(\frac{4}{3}, 0\right)$.



HW

Graph $f(x)=2x-3$ by using intercepts.



$y=2x-3$

x-intercept.

$y=0$

$\Rightarrow 2x-3=0$

$\Rightarrow 2x=3 \Rightarrow x=\frac{3}{2}$

$\left(\frac{3}{2}, 0\right)$

y-intercept

$x=0$

$y=2(0)-3 \Rightarrow y=-3$

$(0,-3)$

YOUR NOTES Write your questions and additional notes.

Solving Equations Graphically

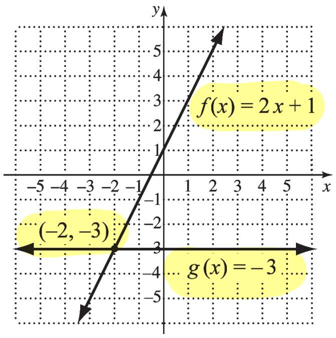
ESSENTIALS

To solve $f(x)=g(x)$ graphically, graph f and g on the same set of axes. The solutions are the x-coordinates of the points of intersection.

Example

- Find the solution of $2x+1=-3$.

  The solution is -2.



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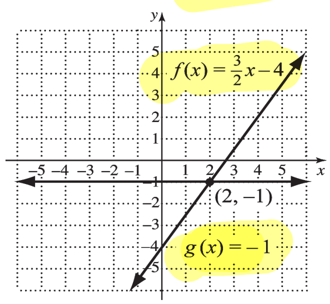
EXAMPLE 1	YOUR TURN 1
Solve graphically: $\frac{3}{2}x-4=-1$.	Solve graphically: $2x-1=-3$.

EXAMPLE 1

Solve graphically: $\frac{3}{2}x-4=-1$.

Graph $f(x)=\frac{3}{2}x-4$ and $g(x)=-1$.



The point of intersection appears to be $(\boxed{2}, \boxed{-1})$, so the solution appears to be $\boxed{2}$.

Check:

$$\frac{3}{2}x-4=-1$$

$\frac{3}{2}(\boxed{2})-4$	-1
$\boxed{3}-4$	

$$-1\overset{?}{=}-1 \qquad \text{TRUE}$$

The solution is $\boxed{\checkmark}$.

YOUR TURN 1

Solve graphically: $2x-1=-3$.

$y=2(0)-1$
$=0-1$
$=-1$

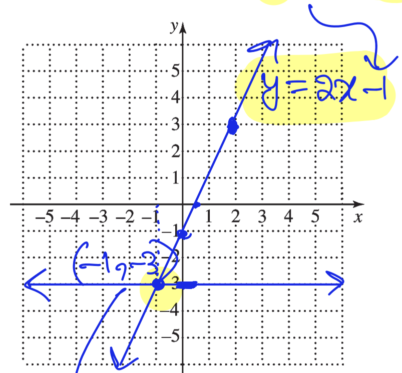
$y=2x-1$

$(0,-1)$

$0=2x-1$

$2x=1 \Rightarrow x=\frac{1}{2}$

$(\frac{1}{2}, 0)$



$y=-3$

intersect at $(-1,-3)$

$\Rightarrow$ Solution is $\boxed{x=-1}$

$\left\{ \begin{array}{l} x=0 \\ x=4 \end{array} \right\}$

EXAMPLE 2

Gwen's gym charges a \$60 enrollment fee plus \$35 per month for use of the gym. For how long has Gwen been a member if she paid a total of \$270? Use a graph to estimate the solution.

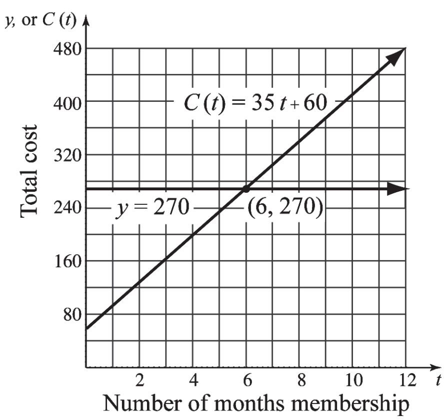
1., 2. Familiarize and Translate.

Let $C(t)$ represent the cost, in dollars, for t months of membership. We have

$C(t) = 35t + \boxed{}$.

3. Carry out.

Since we wish to find the time when the total amount paid is \$270, we graph $C(t) = 35t + 60$ and $y = 270$.



The point of intersection appears to be $\left(\boxed{}, \boxed{}\right)$ so the solution appears to be $\boxed{}$ months.

4. Check.

$C\left(\boxed{}\right) = 35 \cdot \boxed{} + 60 = \boxed{} + 60 = \boxed{}$

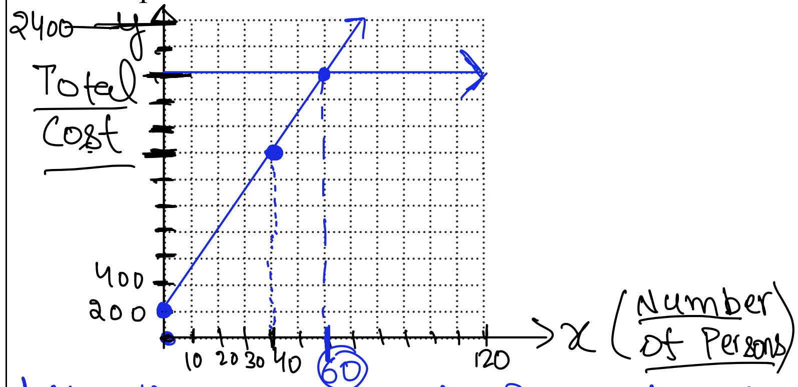
The answer checks.

5. State.

Gwen has been a member $\boxed{}$ months.

YOUR TURN 2

Fine Linen Catering charges a \$200 set-up fee plus \$30 per person for parties under 120 persons. Pat pays a total of \$2000 for catering for a party. Use a graph to estimate the number of persons who attended the party. Assume fewer than 120 persons attended.



Let the number of Persons be x.

$\underbrace{200 + 30x}_{y = 200 + 30x} = \underbrace{2000}_{y = 2000}$

$x = 0,\ y = 200$

$(0, 200)$

$x = 40,\ y = 200 + 30 \times 40$

$= 200 + 1200 = 1400$

$(40, 1400)$

Intersect at $(60, 2000)$

$x = 60$

YOUR NOTES Write your questions and additional notes.

Recognizing Linear Equations

ESSENTIALS

Any equation of the form $Ax + By = C$, where A, B, and C are real numbers and A and B are not both 0, is a linear equation in *standard form* and has a graph that is a straight line.

Examples

- $-2x + 6y = 18$ is linear.
- $8x - 5 = 0$ is linear.
- $x^2 - 2x + 4 = 0$ is not linear. → quadratic
- $xy = 8$ is not linear. → quadratic

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EXAMPLE 1

Determine whether $y = 3x + 2$ is linear. Find the slope if it is a nonvertical line.

We attempt to put the equation in standard form.

$$y = 3x + 2$$
$$-3x + y = 2$$

The equation ____________ linear.
is / is not

The slope is $\boxed{}$.

YOUR TURN 1

Determine whether $y = -6x - 4$ is linear. Find the slope if it is a nonvertical line.

$y = -6x - 4 \Rightarrow 6x + y = -4$

linear.

slope $= -6$

EXAMPLE 2

Determine whether $3x - 2f(x) = 4$ is linear. Find the slope if it is a nonvertical line.

The equation is in standard form: $3x - 2f(x) = 4$, or $3x - 2y = 4$. It ____________ linear.
is / is not

We put the equation into slope-intercept form to find the slope.

$$3x - 2f(x) = 4$$
$$3x - 4 = 2f(x)$$
$$\frac{3}{2}x - 2 = f(x)$$

The slope is $\boxed{}$.

YOUR TURN 2

Determine whether $x + f(x) = -2$ is linear. Find the slope if it is a nonvertical line.

$x + y = -2 \Rightarrow$ linear function

$y = -x - 2$

slope $= -1$

EXAMPLE 3	YOUR TURN 3
Determine whether $y = 3x^2 - 2$ is linear. We attempt to put the equation in standard form. $y = 3x^2 - 2$ $-3x^2 + y = -2$ This equation __________ linear because it is / is not has a(n) x^2-term .	Determine whether $\frac{5}{x} = y$ is linear. Not linear $xy = 5$
EXAMPLE 4	**YOUR TURN 4**
Determine whether $x - 40 = 0$ is linear. Attempting to put the equation in standard form, we have $1x + 0y = \boxed{}$. The equation is linear and the line is ______________. vertical / horizontal	Determine whether $5y + 20 = 0$ is linear. Find the slope if it is a nonvertical line. $5y + 20 = 0$ Linear $5y = -20 \Rightarrow y = -4$ Horizontal Slope $= 0$

YOUR NOTES Write your questions and additional notes.

Point-Slope Form

ESSENTIALS

Any equation of the form $y - y_1 = m(x - x_1)$ is said to be written in **point-slope form** and has a graph that is a straight line. The slope is m, and the line passes through (x_1, y_1). When we know a line's slope and a point on the line, we can draw the graph.

Examples

- $y-5=2(x+3)$, or $y-5=2[x-(-3)]$, passes through $(-3, 5)$ and has slope 2.

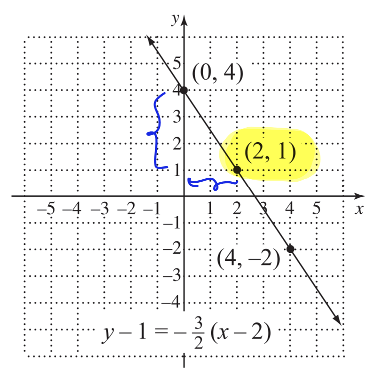
$y-5=2x+6 \Rightarrow y=2x+11$

- Graph: $y-1=-\frac{3}{2}(x-2)$.

  The line has slope $-\frac{3}{2}$, or $\frac{-3}{2}$, and passes through $(2,1)$.

  We plot $(2,1)$ and then find a second point by moving *down* 3 units and *to the right* 2 units.
  Then we draw the line.

  We could also think of the slope as $\frac{3}{-2}$. Then we could start at $(2,1)$ and move *up* 3 units and *to the left* 2 units to find another point.



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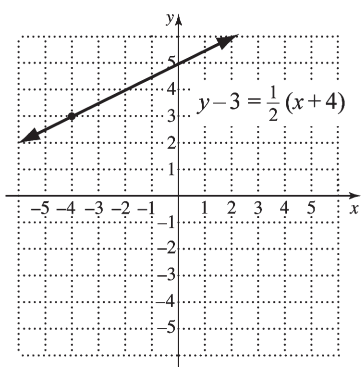
EXAMPLE 1

Graph: $y-3=\frac{1}{2}(x+4)$.

$$y-y_1=m(x-x_1)$$
$$y-3=\frac{1}{2}(x+4)$$
$$y-3=\frac{1}{2}[x-(-4)]$$

The line has slope $\boxed{}$ and passes through $(\boxed{}, \boxed{})$.

We plot $(-4, 3)$, count off a slope of $\frac{1}{2}$, and draw the line.

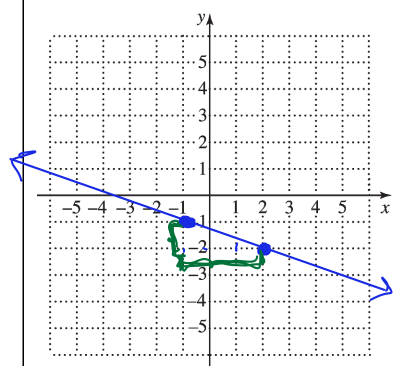


YOUR TURN 1

Graph: $y+2=-\frac{1}{3}(x-2)$. $\rightarrow [y-(-2)]=-\frac{1}{3}(x-2)$

$(2, -2)$

slope $=\frac{-1}{3}=\frac{1}{-3}$



$-2+1=-1 \leftarrow y$

$2-3=-1 \leftarrow x$

EXAMPLE 2

Use point-slope form to find an equation of the line with slope -4 that passes through $(-2, 5)$.

Substitute into point-slope form.

$$y-y_1=m(x-x_1)$$
$$y-\boxed{}=-4(x-(\boxed{}))$$

YOUR TURN 2

Use point-slope form to find an equation of the line with slope $\frac{2}{3}$ that passes through $(4, -9)$.

$[y-(-9)]=\frac{2}{3}(x-4)$

$(y+9)=\frac{2}{3}(x-4)$

YOUR NOTES Write your questions and additional notes.

$3(y+9)=2(x-4)$

$3y+27=2x-8$

$-3y \qquad -3y$

$2x-8-3y=27$

$\boxed{2x - 3y = 35}$

$2x - 3y - 8 - 27 = 0$

$2x - 3y - 35 = 0$

Finding the Equation of a Line

ESSENTIALS

Knowing the slope of a line and its y-intercept or the slope and a point on the line or two points on the line, we can find an equation of the line.

Example

- Find an equation for the line passing through $(-1, 4)$ which is parallel to $y = 3x - 5$.

  $y - y_1 = m(x - x_1)$ slope $= 3$

  $y - 4 = 3[x - (-1)]$, or $y = 3x + 7$

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EXAMPLE 1	YOUR TURN 1
Find an equation for the line parallel to $3x - 4y = 8$ with y-intercept $(0, 5)$. First, find the equation of the given line in slope-intercept form: $3x - 4y = 8$ $-4y = -3x + 8$ $y = \frac{3}{4}x - 2$ The slope of the given line is $\boxed{}$. The slope of a line parallel to it is also $\boxed{}$. Using y-intercept $(0, 5)$ and slope $\frac{3}{4}$, we have: $y = mx + b$ $y = \boxed{}x + \boxed{}$.	Find an equation for the line parallel to $2x + 5y = -3$ with y-intercept $(0, 5)$. $y = mx + b$ $b = 5$ $m =$ slope of the line $2x + 5y = -3$ $5y = -2x - 3$ $y = -\frac{2}{5}x - \frac{3}{5}$ $m = -\frac{2}{5}$ $y = -\frac{2}{5}x + 5$

$5y = -2x + 25$

$2x + 5y = 25$

EXAMPLE 2

Find an equation for the line perpendicular to $3x = 2y + 5$ and passing through $(4, -2)$.

First, find the equation of the given line in slope-intercept form:

$$3x = 2y + 5$$
$$-2y = -3x + 5$$
$$y = \frac{3}{2}x - \frac{5}{2}.$$

The slope of the given line is $\boxed{}$. The slope of a line perpendicular to it is the opposite of the reciprocal of $\frac{3}{2}$ or $\boxed{}$.

Substituting into point-slope form, we have:

$$y - y_1 = m(x - x_1)$$
$$y - (\boxed{}) = \boxed{}(x - \boxed{})$$

YOUR TURN 2

Find an equation for the line perpendicular to $-2x = 3 - 4y$ and passing through $\underline{(-5, 1)}$.

$$y - 1 = m(x - (-5))$$
$$\Rightarrow y - 1 = \underline{m}(x + 5)$$

want to find.

Slope of $-2x = 3 - 4y$

$$4y - 2x = 3$$
$$4y = 2x + 3$$
$$y = \frac{2}{4}x + \frac{3}{4}$$

$\parallel$

$\frac{2}{4} = \frac{1}{2}$

$$m \times \frac{1}{2} = -1 \Rightarrow m = -2$$
$$y - 1 = -2(x + 5)$$
$$y - 1 = -2x - 10$$
$$y = -2x - 10 + 1$$
$$\boxed{y = -2x - 9}$$

EXAMPLE 3

Find a linear function that has a graph passing through $(4,-1)$ and $(-2,-6)$.

First, we find the slope of the line:

$$\frac{y_2-y_1}{x_2-x_1}=\frac{-1-(-6)}{4-(-2)}=\frac{5}{6}.$$

Using slope-intercept form, we have:

$$y=mx+b$$

$$y=\frac{5}{6}x+b$$

$$-1=\frac{5}{6}\left(\boxed{}\right)+b \qquad \text{Substituting 4 for } x \text{ and } -1 \text{ for } y$$

$$-1=\frac{10}{3}+b$$

$$-\frac{13}{3}=b \qquad \text{Solving for } b$$

The equation is:

$$y=\boxed{}\,x+\left(\boxed{}\right),\text{ or } y=\frac{5}{6}x-\frac{13}{3}$$

$$f(x)=\boxed{}\,x-\boxed{}. \qquad \text{Using function notation}$$

We could have also used point-slope form with the slope and either of the two given points to get an equation. We then would have to solve for y before converting to function notation.

YOUR TURN 3

Find a linear function that has a graph passing through $(1,4)$ and $(-2,7)$.

$y-4=m(x-1)$

$(1,4)$, $(-2,7)$

$m=\frac{7-4}{-2-1}=\frac{3}{-3}=-1$

$y-4=-1(x-1)$

$y-4=-x+1$

$y=-x+1+4$

$y=-x+5$

So, the linear function is

$f(x)=-x+5$

EXAMPLE 4	YOUR TURN 4
Find (a) the equation of the horizontal line that passes through $(-2, 3)$ and (b) the equation of the vertical line that passes through $(-2, 3)$. a) An equation of a horizontal line is of the form $y = b$. In order for $(-2, 3)$ to be a solution of $y = b$, we must have $b = 3$. Thus the equation of the line is $y = 3$. b) An equation of a vertical line is of the form $x = a$. In order for $(-2, 3)$ to be a solution of $x = a$, we must have $a = -2$. Thus the equation of the line is $x = -2$.	Find (a) the equation of the horizontal line that passes through $(5, -3)$ and (b) the equation of the vertical line that passes through $(5, -3)$. (a) $y = -3$ (b) $x = 5$

YOUR NOTES Write your questions and additional notes.

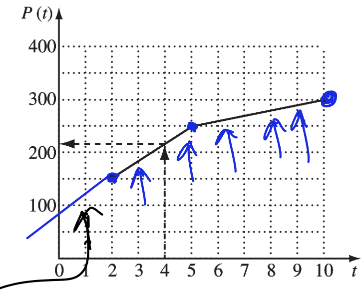
Interpolation and Extrapolation

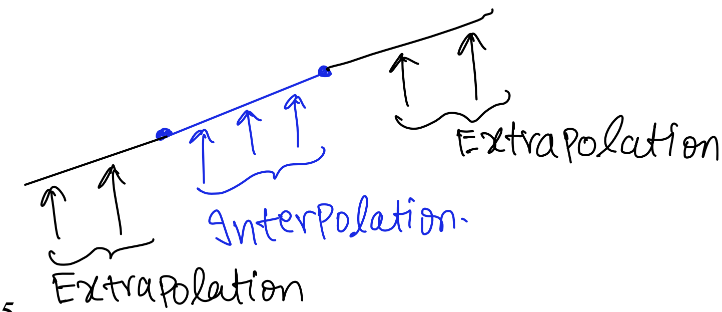
ESSENTIALS

Interpolation estimates values between known points.
Extrapolation predicts values beyond known points.

Example

- Estimate the value of the function when $t = 4$.





Extrapolation The value appears to be approximately 225.

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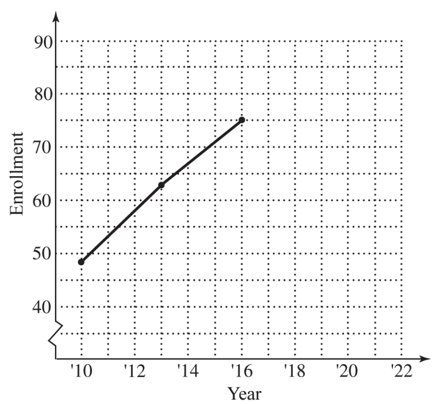
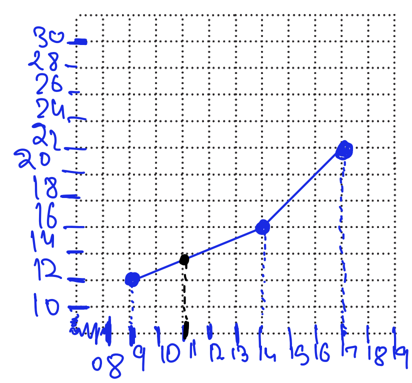
EXAMPLE 1	YOUR TURN 1
In 2010, The Dance Academy's enrollment was 48 students. In 2013, it was 63 students, and in 2016 it was 75 students. Estimate the number of students at the academy in 2012.	In 2009, 12% of the students at The Dance Academy were enrolled in a music class. In 2014, that number had risen to 16%, and in 2017 it had reached 22%. Use interpolation to estimate the percent of students enrolled in a music class in 2011.

We plot three points that represent the given information and connect them.



We estimate the number of students in 2012 by locating the point directly above the 2012 and estimating its second coordinate as $\boxed{}$.

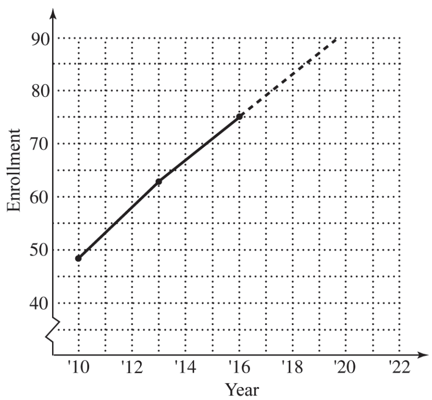
We estimate that there were $\boxed{}$ students at the dance academy in 2012.



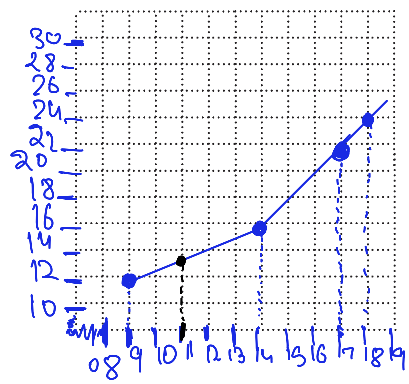
14 %

EXAMPLE 2	YOUR TURN 2
In 2010, The Dance Academy's enrollment was 48 students. In 2013, it was 63 students, and in 2016 it was 75 students. Predict the number of students at the dance academy in 2018.	In 2009, 12% of the students at The Dance Academy were enrolled in a music class. In 2014, that number had risen to 16%, and in 2017 it had reached 22%. Predict the percent of students enrolled in a music class in 2018.

We plot the points and connect them. Then we predict the number of students in 2018 by extending the graph and extrapolating. We locate the point directly above 2018 and estimate its second coordinate as $\boxed{}$.



We predict there will be $\boxed{}$ students at the dance academy in 2018.



~ 24 %

YOUR NOTES Write your questions and additional notes.

Linear Functions and Models

ESSENTIALS

Given two points, we can model data with a linear function.

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EXAMPLE 1

The average monthly revenue of Corp C Enterprise is shown in the table. Use the data from 2010 and 2012 to find a linear function that fits the data. Then use the function to estimate average monthly revenue in 2014.

Year	Average Monthly Revenue
2010	\$20,000
2011	20,000
2012	21,000

We let $t =$ the number of years since 2000 and $r =$ the average monthly revenue in tens of thousands of dollars. We find a linear function containing points $(10, \square)$ and $(\square, 21)$.

$$m = \frac{21 - \square}{\square - 10} = \frac{\square}{\square}$$

We use m and $(10, \square)$ to find an equation of the line.

$$r - \square = \square\,(t - 10)$$

$$r = \frac{1}{2}t + 15, \text{ or } r(t) = \frac{1}{2}t + 15$$

To estimate the average monthly revenue in 2014, we find $r(14)$.

$$r(14) = \frac{1}{2}(\square) + 15 = \square$$

Assuming constant growth, average monthly revenue in 2014 is expected to be \$$\square$.

YOUR TURN 1

The average monthly expenses of Corp C Enterprise are shown in the table. Use the data for 2009 and 2011 to find a linear function that fits the data. Let $t =$ the number of years since 2000 and $E =$ the average monthly expenses in tens of thousands of dollars. Then use the function to estimate average monthly expenses in 2014.

Year	Average Monthly Expenses	
2009 → $t=9$	\$20,000	→ $E = 2$
2010 → $t=10$	21,000	→ $E = 2.1$
2011 → $t=11$	19,000	→ $E = 1.9$
2012 → $t=12$	19,000	→ $E = 1.9$

$(9, 2)$, $(12, 1.9)$

Eqn. of st line.

$$m = \frac{1.9 - 2}{12 - 9} = \frac{-0.1}{3}$$

$$E - 2 = \frac{-0.1}{3}(t - 9)$$

$E(t)$

$$E(t) = \frac{-0.1}{3}t + \frac{0.1}{3} \times 9 + 2$$

$$E(t) = \frac{-0.1t}{3} + 2.3$$

$$E(14) = \frac{-0.1}{3} \times 14 + 2.3$$

$= 1.8\overline{3}$

⇒ Avg. monthly expense in 2014 was $18333.33

EXAMPLE 2

Suppose suppliers are willing to sell 100 handmade headbands when the price is $20 per headband and 60 handmade headbands when the price is $12 per headband. Find a linear function that expresses the number of headbands suppliers are willing to sell as a function of the price per headband. Use the function to predict how many headbands sellers would be willing to sell if the price were $15 per headband.

Let $p =$ the price and $h =$ the number of headbands. We find a linear function containing points $\left(20, \boxed{}\right)$ and $\left(\boxed{}, 60\right)$.

$$m = \frac{\boxed{} - 100}{12 - \boxed{}} = \frac{-40}{\boxed{}} = 5$$

We use m and $(20, 100)$ to find an equation of the line.

$$h - \boxed{} = \boxed{}(p - 20)$$

$$h = 5p, \text{ or } h(p) = 5p$$

To estimate the number of headbands sellers would be willing to sell if the price were $15, we find $h(15)$.

$$h(15) = 5 \cdot \boxed{} = \boxed{}$$

The suppliers would be willing to supply $\boxed{}$ headbands.

YOUR TURN 2

Suppose buyers are willing to buy 100 handmade headbands when the price is $10 per headband and 70 handmade headbands when the price is $12 per headband. Find a linear function that expresses the number of headbands buyers are willing to buy as a function of the price per headband. Let $p =$ the price and $h =$ the number of headbands. Use the function to predict how many headbands buyers would be willing to buy if the price were $15 per headband.

HW

YOUR NOTES Write your questions and additional notes.

The Sum, Difference, Product, or Quotient of Two Functions

ESSENTIALS

If f and g are functions and x is in the domain of both functions, then:

$(f+g)(x) = f(x) + g(x)$; $\quad (f-g)(x) = f(x) - g(x)$;

$(f \cdot g)(x) = f(x) \cdot g(x)$; $\quad (f/g)(x) = f(x)/g(x)$, provided $g(x) \neq 0$.

Example

- For $f(x) = x+2$ and $g(x) = x^2$, find $(f+g)(x)$ and $(f \cdot g)(3)$.

  $(f+g)(x) = (x+2) + (x^2) = x^2 + x + 2$

  $(f \cdot g)(3) = f(3) \cdot g(3) = (3+2)(3^2) = 5 \cdot 9 = 45$

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EXAMPLE 1	YOUR TURN 1
For $f(x) = x-3$ and $g(x) = x^2 - 2$, find $(f+g)(x)$. $(f+g)(x) = f(x) + g(x)$ $= \boxed{\quad\quad} + \boxed{\quad\quad}$ $= x^2 + x - \boxed{\quad}$	For $f(x) = x^2 - x$ and $g(x) = 5 + x$, find $(f+g)(x)$. $(f+g)(x) = f(x) + g(x)$ $= (x^2 - x) + (5 + x)$ $= x^2 - x + x + 5 = x^2 + 5$

EXAMPLE 2	YOUR TURN 2
For $f(x) = x-3$ and $g(x) = x^2 - 2$, find $(f-g)(5)$. $f(5) = 5 - 3 = \boxed{\quad}$ and $g(5) = 5^2 - 2 = \boxed{\quad}$. $(f-g)(5) = f(5) - g(5) = \boxed{\quad} - \boxed{\quad} = \boxed{\quad}$ Alternatively, $(f-g)(x) = f(x) - g(x)$ $= x - 3 - (x^2 - 2)$ $= x - 3 - x^2 + 2$ $= \boxed{\quad\quad\quad}$ So, $(f-g)(5) = -5^2 + 5 - 1 = \boxed{\quad}$. The two answers match.	For $f(x) = x^2 - x$ and $g(x) = 5 + x$, find $(g-f)(3)$. $(g-f)(3) = g(3) - f(3)$ $= (5+3) - (3^2 - 3)$ $= 8 - (9 - 3)$ $= 8 - 6 = 2$

EXAMPLE 3	YOUR TURN 3
For $f(x)=x-3$ and $g(x)=x^2-2$, find $(f\cdot g)(-4)$. $f(-4)=-4-3=-7$ and $g(-4)=(-4)^2-2=14$. Then, $(f\cdot g)(-4)=f(\boxed{})\cdot g(\boxed{})$ $=-7\cdot 14=-98$	For $f(x)=x^2-x$ and $g(x)=5+x$, find $(f\cdot g)(-1)$. $(f\cdot g)(-1) = f(-1)\cdot g(-1)$ $= [(-1)^2-(-1)]\cdot[5+(-1)]$ $= [1+1]\cdot[4] = 2\times 4 = \underline{\underline{8}}$

EXAMPLE 4	YOUR TURN 4
For $f(x)=x-3$ and $g(x)=x^2-2$, find $(f/g)(x)$. $(f/g)(x)=\dfrac{f(x)}{g(x)}=\dfrac{\boxed{}}{\boxed{}}$	For $f(x)=x^2-x$ and $g(x)=5+x$, find $(f/g)(x)$. $(f/g)(x) = \dfrac{f(x)}{g(x)} = \dfrac{x^2-x}{x+5}$

EXAMPLE 5	YOUR TURN 5
For $f(x)=x-3$ and $g(x)=x^2-2$, find $(g/f)(0)$. $(g/f)(0)=\dfrac{0^2-2}{0-3}=\boxed{}$.	For $f(x)=x^2-x$ and $g(x)=5+x$, find $(f/g)(1)$. $(f/g)(1) = \dfrac{f(1)}{g(1)} = \dfrac{1^2-1}{5+1} = \dfrac{0}{6} = \underline{\underline{0}}$

YOUR NOTES Write your questions and additional notes.

$$(f/g)(-5) = \frac{(-5)^2-(-5)}{\underbrace{5+(-5)}_{\to 0}} = \text{undefined.}$$

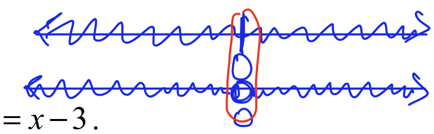
Domains and Graphs

ESSENTIALS

The domain of $f+g$, $f-g$, or $f \cdot g$ is the set of all values common to the domains of *f* and *g*.

The domain of f/g is the set of all values common to the domains of *f* and *g*, excluding any values for which $g(x)=0$.

Example



- Find the domain of $f+g$ and f/g when $f(x)=\frac{5}{x}$ and $g(x)=x-3$.

  The domain of $f+g=\{x \mid x \text{ is a real number } and\ x \neq 0\}$.

  The domain of $f/g=\{x \mid x \text{ is a real number } and\ x \neq 0\ and\ x \neq 3\}$.

$g(x) \neq 0$

$x-3 \neq 0 \Rightarrow \underline{x \neq 3}$

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EXAMPLE 1

For $f(x)=\frac{5}{x+1}$ and $g(x)=\frac{4+x}{x-3}$, find the domain of $f+g$, the domain of $f-g$, and the domain of $f \cdot g$.

Because division by 0 is undefined, we have

Domain of $f=\{x \mid x \text{ is a real number } and\ x \neq \boxed{}\}$ and

Domain of $g=\{x \mid x \text{ is a real number } and\ x \neq \boxed{}\}$.

The domain of $f+g$, $f-g$, and $f \cdot g$ is the set of all elements common to the domains of *f* and *g*. Thus, Domain of $f+g=$ Domain of $f-g=$ Domain of $f \cdot g=\{x \mid x \text{ is a real number } and\ x \neq \boxed{}\ and\ x \neq \boxed{}\}$.

YOUR TURN 1

For $f(x)=\frac{1}{x+2}$ and $g(x)=-\frac{3}{x}$, find the domain of $f+g$, the domain of $f-g$, and the domain of $f \cdot g$.

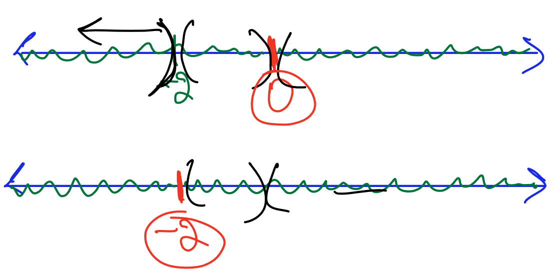
$x+2=0$

$\underline{x=-2}$

$Df=\{x \mid x \text{ is a real number and } x \neq -2\}$

$Dg=\{x \mid x \text{ is a real number and } x \neq 0\}$

$D(f+g)=D(f-g)=D(f \cdot g)=\{x \mid x \text{ is real number and } x \neq -2 \text{ and } x \neq 0\}$



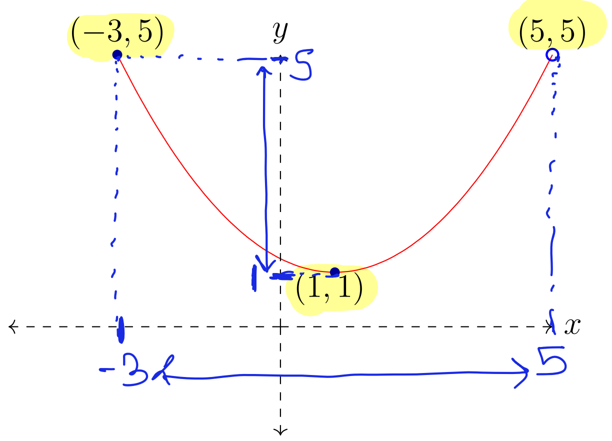
EXAMPLE 2	YOUR TURN 2
For $f(x)=\dfrac{4}{x-5}$ and $g(x)=3x-2$, find the domain of f/g. The domain of *f* is $\{x \mid x \text{ is a real number } \textit{and } x \neq \boxed{}\}$. The domain of *g* is the set of all $\boxed{}$ numbers, or $\mathbb{R}$. The domain of f/g must also exclude values of *x* for which $g(x)=0$. $g(x)=0$ $3x-2=0$ $3x=2$ $x=\boxed{}$ Thus, the domain of f/g is $\{x \mid x \text{ is a real number } \textit{and } x \neq \boxed{} \textit{ and } x \neq \boxed{}\}$.	For $f(x)=\dfrac{x-1}{2x}$ and $g(x)=x+3$, find the domain of f/g. $2x=0$ $\Rightarrow x=0$ $Dg=\{x \mid x$ is a real number$\}$ $Df=\{x \mid x$ is a real number and $x\neq 0\}$ $g(x)=0 \Rightarrow x+3=0$ $\Rightarrow x=-3$ $D(f/g)=\{x \mid x$ is a real number and $x\neq 0$ and $x\neq -3\}$

YOUR NOTES Write your questions and additional notes.

Math11000 Section 3962 Quiz 4
Summer 2023, May 16

Name: [1 pt]

Problem 1:



For the function f whose graph is drawn above, find domain of f, range of f and $f(-3)$. Note that there is an open dot at the point $(5,5)$. [5 pts]

$f(-3)=5$

Domain $= \{x \mid x \geq -3 \text{ and } x < 5\}$

Range $= \{y \mid y \geq 1 \text{ and } y \leq 5\}$

Problem 2:

1. Line L_1 has slope 2 and y-intercept $(0,-1)$. Find the equation of L_1. [2 pts]
2. Line L_2 has equation $y = -\frac{1}{2}x + 1$. Find whether lines L_1, L_2 are parallel, perpendicular or neither. [2 pts]

1) $y = mx + b$ where m is slope and $(0,b)$ is the y-int.
$y = 2x - 1$ ← equation of L_1

2) If two lines are parallel, then their slopes are equal.
If two lines are perpendicular, then the product of their slopes is -1

slope of $L_2 = -\frac{1}{2}$, slope of $L_1 = 2$.

$-\frac{1}{2} \times 2 = -1 \Rightarrow$ Lines are <u>perpendicular</u>.