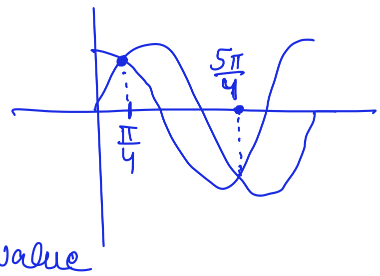
Math16500 Section 24246 Quiz 9

Fall 2022, October 19

Name: [1 pt]

Problem 1: Find the absolute maximum and minimum values of $f(x) = \sin x + \cos x$ defined on the interval $[0, 2\pi]$. [4 pts]

$f(x) = \sin x + \cos x \Rightarrow f'(x) = \cos x - \sin x$



$f'(x) = 0 \Rightarrow \cos x = \sin x \Rightarrow x = \frac{\pi}{4}, \frac{5\pi}{4}$

$f(\frac{\pi}{4}) = \sin\frac{\pi}{4} + \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2} \longrightarrow$ absolute max value

$f(\frac{5\pi}{4}) = \sin\frac{5\pi}{4} + \cos\frac{5\pi}{4} = \frac{-1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = -\sqrt{2} \longrightarrow$ absolute min value

$f(0) = \sin 0 + \cos 0 = 0 + 1 = 1$

$f(2\pi) = \sin 2\pi + \cos 2\pi = 0 + 1 = 1$

Problem 2: Find the points of local maxima and local minima for $f(x) = \dfrac{x}{x^2+1}$. [5 pts]

$$f'(x) = \frac{(x^2+1) - x(2x)}{(x^2+1)^2} = \frac{x^2+1-2x^2}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2} = \frac{-(x-1)(x+1)}{(x^2+1)^2}$$

$-$ $\;-1\;$ $+$ $\;1\;$ $-$

$\Rightarrow x = 1$ is a pt. of local maxima

$x = -1$ is a pt of local minima.

Bonus Problem: Find points of inflection for $f(x) = \frac{1}{2}x^4 - 3x^2 + 4$. [2 pts]

$$f'(x) = \frac{4}{2}x^3 - 6x = 2x^3 - 6x \Rightarrow f''(x) = 6x^2 - 6 = 6(x^2-1)$$
$$= 6(x-1)(x+1)$$

$+$ $\;-1\;$ $-$ $\;1\;$ $+$

$\Rightarrow x = 1, x = -1$ are points of inflection.

$\nVdash$

$$\nVdash_0 + \nVdash_4^{-1} \nvDash_2 + \nvDash_6^{-1} \nVDash_3 \nVDash_5 + \nVDash_7^{-1} \nVDash_3 \ntrianglerighteq_6 + \nvDash_4 \nVDash_5^{-1} \nVDash_7^{-1} \ntrianglerighteq_4 \ntrianglerighteq_6 + \nVDash_3 \ntrianglerighteq_8^{-1} + \nVdash_6 \nvDash_8^{-1} \ntrianglerighteq_4 \ntrianglerighteq_6 + \nvDash_4 \nVDash_5^{-1} \ntrianglerighteq_8^{-1} \ntrianglerighteq_4 + \nvDash_4 \ntrianglerighteq_6^{-1} \ntrianglerighteq_8$$