Learning objectives:

1. Find volumes of solids of revolution, obtained by revolving a region about a line called axis.
2. We divide the given solid into disks/washer by cutting it into infinite infinitesimally small cross-sections (region perpendicular to the axis of rotation).

Let S be a solid that lies between $x = a$ and $x = b$. If the cross-section area of S in the plain P_x, through x and perpendicular to the x-axis, is $A(x)$, where A is a continuous function, then the volume of S is

$$V = \int_a^b A(x)\,dx\,.$$

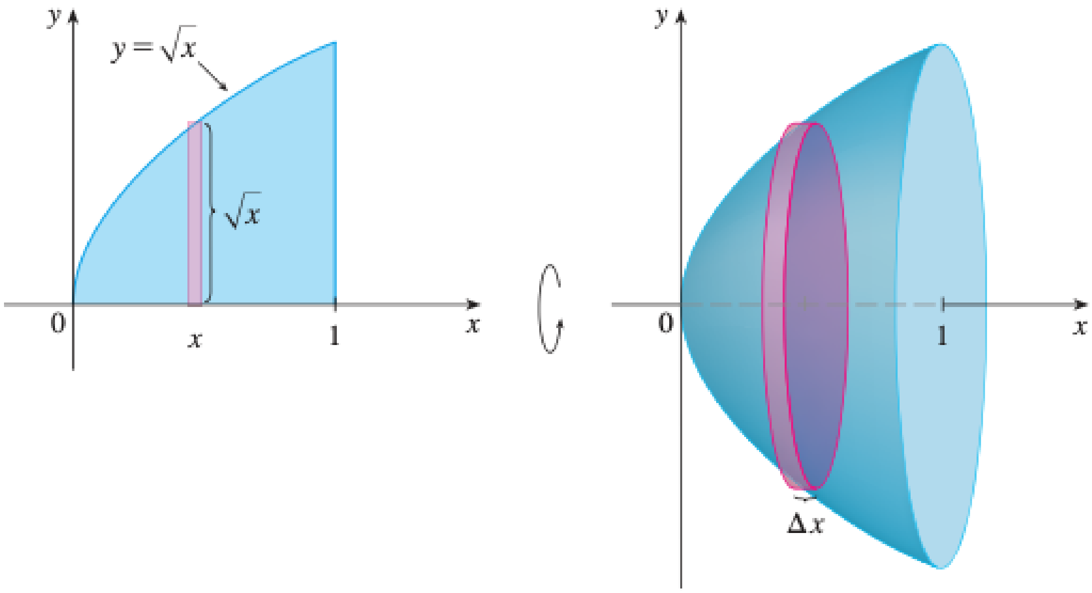
We use the above formula when a solid is obtained by rotating a region about an axis which is parallel to the x-axis.

Let S be a solid that lies between $y = a$ and $y = b$. If the cross-section area of S in the plain P_y, through y and perpendicular to the y-axis, is $A(y)$, where A is a continuous function, then the volume of S is

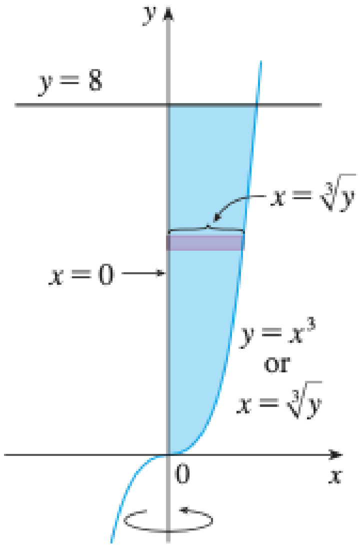
$$V = \int_a^b A(y)\,dy\,.$$

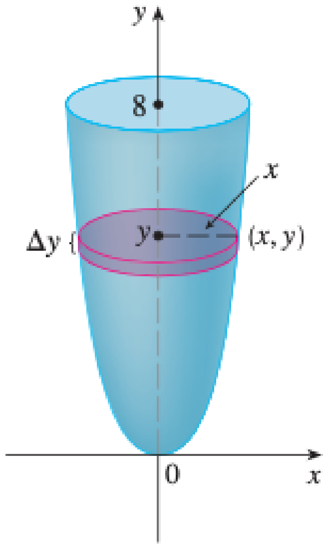
We use the above formula when a solid is obtained by rotating a region about an axis which is parallel to the y-axis.

Example 1. Find the volume of the solid obtained by rotating about the x-axis the region under the curve $y = \sqrt{x}$ from $x = 0$ to $x = 1$.

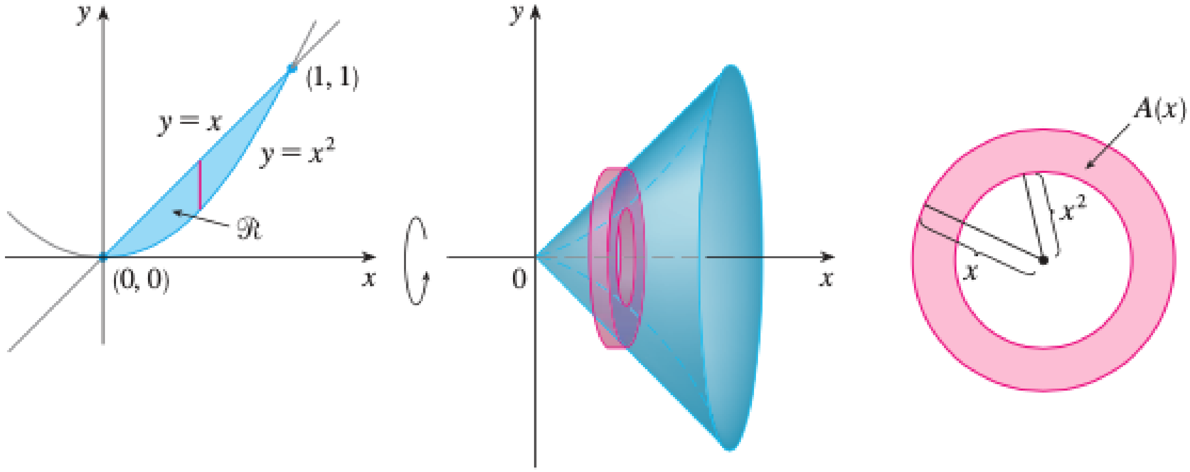


Example 2. Find the volume of the solid obtained by rotating the region bounded by $y = x^3$, $y = 8$ and $x = 0$ about the y-axis.

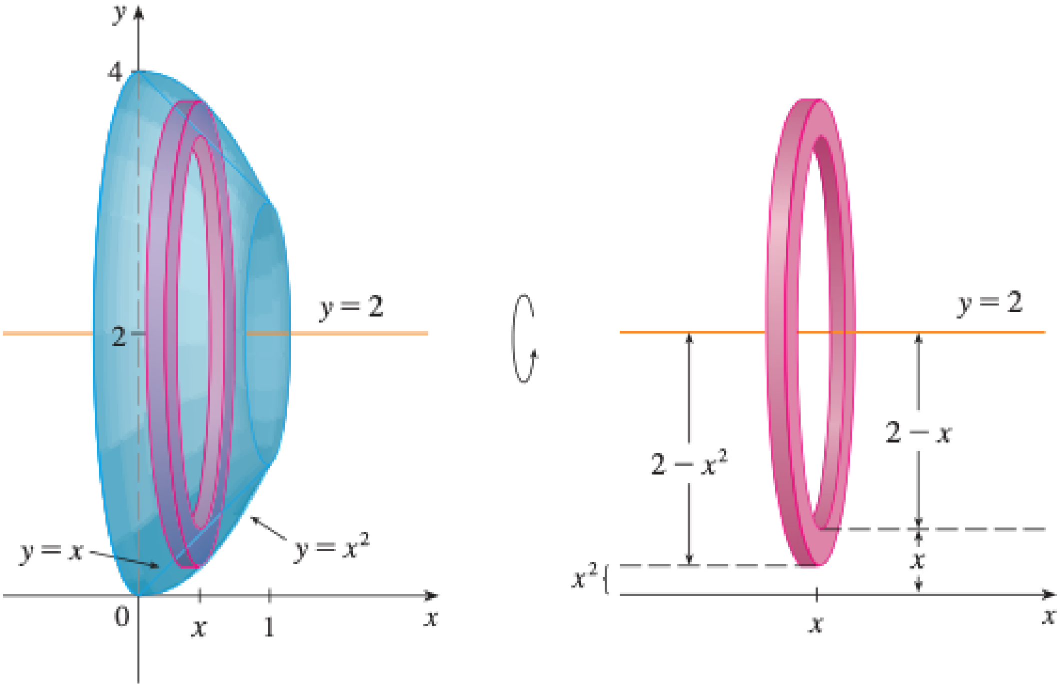




Example 3. The region R enclosed by the curves $y = x$ and $y = x^2$ is rotated about the x-axis. Find the volume of the resulting solid.



Example 4. The region R enclosed by the curves $y = x$ and $y = x^2$ is rotated about the $y = 2$ line. Find the volume of the resulting solid.



Example 5. The region R enclosed by the curves $y = x$ and $y = x^2$ is rotated about the $x = -1$ line. Find the volume of the resulting solid.