4.1 Notes

ESSENTIALS

An **inequality** is any sentence containing $<, >, \leq, \geq$, or $\neq$.

A **solution** of an inequality is a value of the variable that makes the inequality true.

The **solution set** of an inequality is the set of all solutions of the inequality.

Example

- Determine whether -5 is a solution of $x+2>-2$.

$$\begin{array}{r|l} x+2>-2 & \\ \hline -5+2 & -2 \\ & ? \\ & -3>-2 \quad \text{FALSE} \end{array}$$

-3 is not less than -2, so -5 is not a solution of the inequality.

Example : Determine whether –1 is a solution of $2x - 4 = -5$

$2(-1)-4 \stackrel{?}{=} -5$

$-2-4 \stackrel{?}{=} -5$

$-6 \stackrel{?}{=} -5 \Rightarrow$ False $\Rightarrow -1$ is not a solution.

Example: Determine whether 2 is a solutions of $3x + 4 \leq 10$

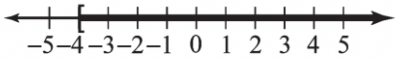
$3(2)+4 \leq 10 \Rightarrow 10 \leq 10 \Rightarrow$ True

$\Rightarrow 2$ is a solution of $3x+4 \leq 10$

Interval Notation

Graph $y \geq -4$ on the number line and write the solution using set-builder and interval notations.

We shade all numbers to the right of -4 and use a bracket to indicate that -4 is also a solution.



Set-builder notation: $\{y|y \geq -4\}$

Interval notation: $[-4, \infty)$

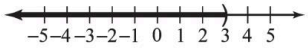
$y > -4$

$(-4, \infty)$

Graph $y < 3$ on the number line and write the solution using set-builder and interval notations.

We shade all numbers to the left of 3 (right / left)

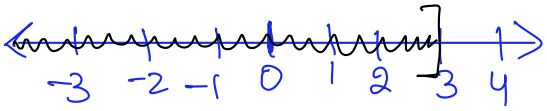
and use a parenthesis to indicate that 3 is not part of the solution. (parenthesis / bracket)



Set-builder notation: $\{y|y < 3\}$

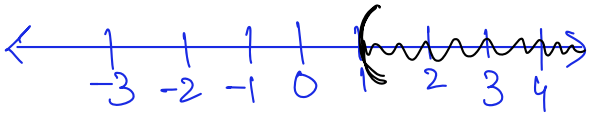
Interval notation: $(-\infty, 3)$

Example: Graph $y \leq 3$ on a number line and write the solution in interval notation



$(-\infty, 3]$

Example: Graph $y > 1$ on the number line and write the solution in interval notation

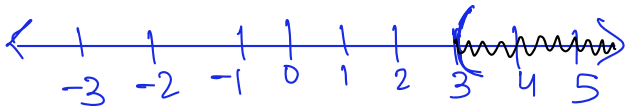


$(1, \infty)$

Solving Inequalities

Example: Solve and Graph $m + 2 > 5$

$m+2>5 \Rightarrow m>5-2 \Rightarrow m>3$

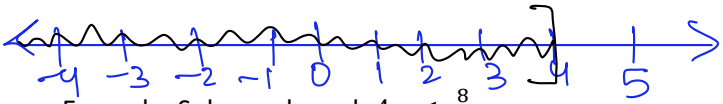


$(3, \infty)$

Example: Solve and graph $6x - 2 \leq 5x + 2$

$6x-2 \leq 5x+2 \Rightarrow 6x \leq 5x+2+2 \Rightarrow 6x \leq 5x+4$

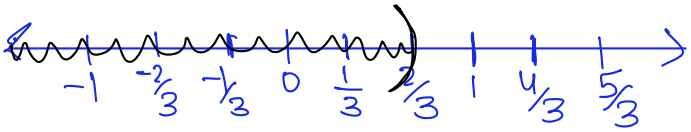
$\Rightarrow 6x-5x \leq 4 \Rightarrow x \leq 4$



$(-\infty, 4]$

Example: Solve and graph $4x < \frac{8}{3}$

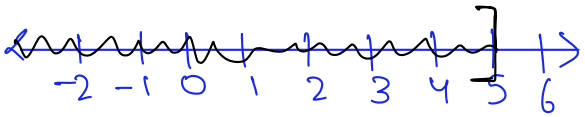
$4x < \frac{8}{3} \Rightarrow \frac{4x}{4} < \frac{1}{4} \times \frac{8}{3}^{2} \Rightarrow x < \frac{2}{3}$



$(-\infty, \frac{2}{3})$

Example: Solve and graph $-4x \geq -20$

$-4x \geq -20 \Rightarrow \frac{-4x}{-4} \leq \frac{-20^{5}}{-4_{1}} \Rightarrow x \leq 5$



$(-\infty, 5]$

Example: Solve and write solution in interval notation $6 \geq -5x + 8$

$6 \geq -5x+8 \Rightarrow 5x+6 \geq 5x-5x+8 \Rightarrow 5x+6 \geq 8$

$5x \geq 8-6 \Rightarrow 5x \geq 2 \Rightarrow x \geq \frac{2}{5}$

$\Rightarrow \left[\frac{2}{5}, \infty\right)$

Example: Solve and write solution in interval notation $3 - 5x < -12x + 10$

$3-5x < -12x+10 \Rightarrow 12x+3-5x < 12x-12x+10$

$\Rightarrow 7x+3 < 10 \Rightarrow 7x < 10-3 \Rightarrow 7x < 7$

$\Rightarrow x < \frac{7}{7} \Rightarrow x < 1 \Rightarrow (-\infty, 1)$

Problem Solving

Phrase	Symbol
"is less than"	$<$
"is more than"	$>$
"is at most"	$\leq$
"is at least"	$\geq$
"no less than"	$\geq$
"no more than"	$\leq$

Corinne's Cakes charges $225, plus $25 per design, for cupcakes for Quinn's birthday party. Sweet Confections charges $275, plus $10 per additional design after the first, for cupcakes for Quinn's birthday party. For how many designs is Corinne's Cakes the less expensive option for cupcakes for Quinn's birthday party?

Let the number of designs be x

Total charges for Corinne's Cakes $= 225 + 25x$

Total charges for Sweet Confections $= 275 + 10(x-1)$

$225 + 25x < 275 + 10(x-1)$

$225 + 25x < 275 + 10x - 10$

$\Rightarrow 225 + 25x < 265 + 10x$

$\Rightarrow 225 + 25x - 10x < 265 \Rightarrow 225 + 15x < 265$

$\Rightarrow 15x < 265 - 225 \Rightarrow 15x < 40$

$\Rightarrow x < \frac{\cancel{40}^{8}}{\cancel{15}_{3}} \Rightarrow x < \frac{8}{3}$

$\Rightarrow x < 2\frac{2}{3}$ or $x < 2.67$

$\Rightarrow x$ is 1 or 2. $\Rightarrow \{1, 2\}$