

Problem 1: Describe and sketch the surface in \mathbb{R}^3 represented by the following equations:-

1. $x + y = 2$
2. $x^2 + z^2 = 9$
3. $x^2 + y^2 + z^2 - 2x - 2z - 2 = 0$

Problem 2: Find the equation of a sphere centered at $(0, 0, 1)$ and passing through the origin.

Problem 3: Let $\vec{a} = 4\hat{i} + 3\hat{j} - \hat{k}$ and \vec{b} be the vector from $A(0, 3, 1)$ to $B(2, 3, -1)$.

1. Find the components of \vec{b} and write it in the form $x\hat{i} + y\hat{j} + z\hat{k}$.
2. Find $4\vec{a} - 3\vec{b}$ and $|\vec{a} - \vec{b}|$.
3. Find the vector that has the same direction as \vec{b} but has length 4.
4. Find the unit vector in the direction of $\vec{b} - \vec{a}$.

Problem 4: Let $\vec{a} = \hat{i} + \hat{j}$ and $\vec{b} = \hat{k} - \hat{j}$.

1. Compute $\vec{a} \cdot \vec{b}$ and find the angle between \vec{a} and \vec{b} .
2. Find the direction cosines and direction angles of the vector $\vec{a} - \vec{b}$.
3. Find the scalar and vector projections of $\vec{a} + \vec{b}$ onto \vec{b} .
4. Find the unit vector orthogonal to \vec{a} and parallel to \vec{b} .

Problem 5: Find the following vectors, without using determinant, but by using the properties of cross products.

1. $(\hat{i} \times \hat{j}) \times \hat{k}$
2. $(\hat{i} + 2\hat{j}) \times (\hat{i} - \hat{j} + 2\hat{k})$

Problem 6: Let $P(0, -2, 0)$, $Q(4, 1, -2)$, $R(5, 3, 1)$ be points in the 3-D space.

1. Find the area of the triangle PQR .
2. Find a nonzero vector orthogonal to the plane passing through points P , Q and R .

Problem 7: Find the volume of the parallelepiped determined by the vectors

$$\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\vec{b} = -\hat{i} + \hat{j} + 2\hat{k}$$

$$\vec{c} = 2\hat{i} + \hat{j} + 4\hat{k}$$