Math16500 Section 24246 Quiz 15+16

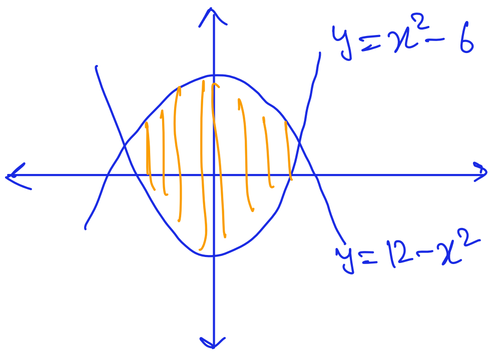
Fall 2022, November 21

Name: [1 pt]

Problem 1: Find the area enclosed between the following curves:-

1. $y = 12 - x^2$ and $y = x^2 - 6$. [5 pts]
2. $y = x^2$ and $y = 4x - x^2$. [5 pts]

1)



$$A = \int_{x_1}^{x_2} \left[(12-x^2) - (x^2-6) \right] dx$$

To find x_1, x_2 we have :-

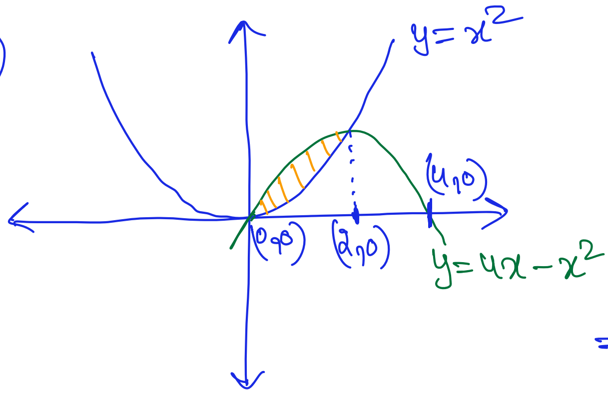
$$x^2 - 6 = 12 - x^2 \Rightarrow 2x^2 = 18 \Rightarrow x^2 = 9$$
$$\Rightarrow x = \pm 3$$

$$\Rightarrow A = \int_{-3}^{3} (18 - 2x^2)\, dx \Rightarrow A = 2\int_0^3 (18-2x^2)\, dx$$

even function

$$\Rightarrow A = 2\left[18x \Big|_0^3 - 2\frac{x^3}{3}\Big|_0^3 \right] = 2\left[54 - \frac{54}{3}\right]$$
$$= 2[54 - 18] = 72 \text{ sq. units.}$$

2)

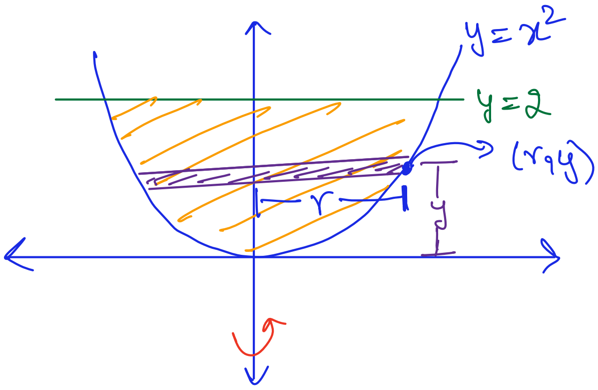


To find points of intersection :-

$$x^2 = 4x - x^2 \Rightarrow 2x^2 = 4x$$
$$\Rightarrow x^2 = 2x \Rightarrow x = 0 \text{ or } 2.$$

$$\Rightarrow A = \int_0^2 \left[(4x - x^2) - x^2\right] dx$$
$$= \int_0^2 (4x - 2x^2)\, dx$$
$$= \frac{4x^2}{2}\Big|_0^2 - \frac{2x^3}{3}\Big|_0^2 = 8 - \frac{16}{3} = \frac{8}{3}$$

Problem 2: Find the volume of the solid obtained by revolving the area enclosed between $y = x^2$ and $y = 2$ about the y-axis. [5 pts]



$$V = \int_0^2 dV$$

$$dV = \pi r^2 \, dy$$

(r,y) lies on $y = x^2$

$\Rightarrow y = r^2 \Rightarrow r = \sqrt{y}$

$$\Rightarrow V = \int_0^2 \pi (\sqrt{y})^2 \, dy = \int_0^2 \pi y \, dy$$

$$= \pi \frac{y^2}{2} \Big|_0^2 = \underline{\underline{2\pi}}$$

Problem 3: Evaluate the definite integral $\int_0^1 \sin(3\pi t)\, dt$. [4 pts]

$$I = \int_0^1 \sin(3\pi t)\, dt.$$

Substitute $y = 3\pi t$

$\Rightarrow dy = 3\pi \, dt$

$$\Rightarrow I = \int_0^{3\pi} \sin y \frac{dy}{3\pi} = \frac{1}{3\pi} \int_0^{3\pi} \sin y \, dy$$

$$= \frac{1}{3\pi} (-\cos y) \Big|_0^{3\pi} = \frac{1}{3\pi} \left[-\cos 3\pi - (-\cos 0) \right]$$

$$= \frac{1}{3\pi} \left[-(-1) - (-1) \right] = \underline{\underline{\frac{2}{3\pi}}}$$

Bonus Problem: Evaluate the definite integal $\int_{-1}^1 x^2 \sin x \, dx$. [2 pts]

Hint: Use Symmetry.

$$I = \int_{-1}^1 x^2 \sin x \, dx$$

$f(-x) = (-x)^2 \sin(-x) = -x^2 \sin x = -f(x)$

$\Rightarrow f$ is an odd function

$\Rightarrow I = 0$