M16600 Lecture Notes

Section 10.1: Curves Defined by Parametric Equations

■ **Section 10.1** textbook exercises, *page 685*: #5, 7, 8.

Equations such as

$$y(x) = 3e^x + x^3 \qquad \text{or} \qquad x(y) = y^2 - 1$$

$x^2+y^2=4$

$y = \begin{cases} \sqrt{4-x^2} \\ -\sqrt{4-x^2} \end{cases}$

describe some curves in the xy-plane.

In this section, we have ANOTHER way to describe curves in the xy-plane, called ***parametric equations***:

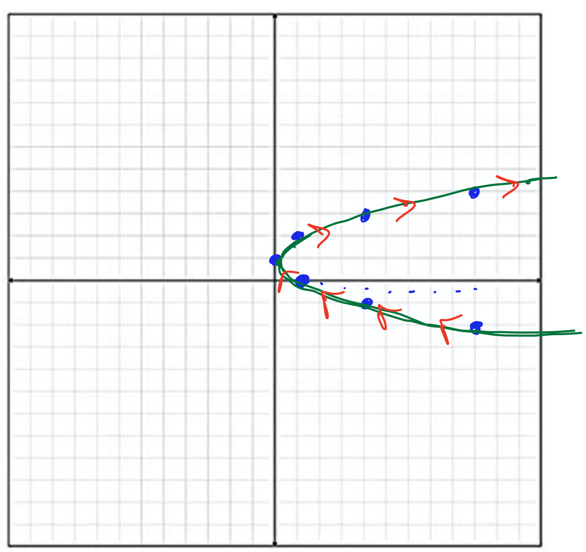
$$x = x(t) \qquad \underline{\text{and}} \qquad y = y(t)$$

Here, t is the *parameter*.

Example 1: (a) Sketch the given ***parametric curves*** (i.e. curves given by *parametric equations*). Indicate with an arrow the direction in which the curve is traced as t increases.
(b) Eliminate the parameter to find a ***Cartesian equation*** (equation with only x and y) of the curve

(1) $x = t^2$ and $y = t + 1$

t	x	y
-3	9	-2
-2	4	-1
-1	1	0
0	0	1
1	1	2
2	4	3
3	9	4



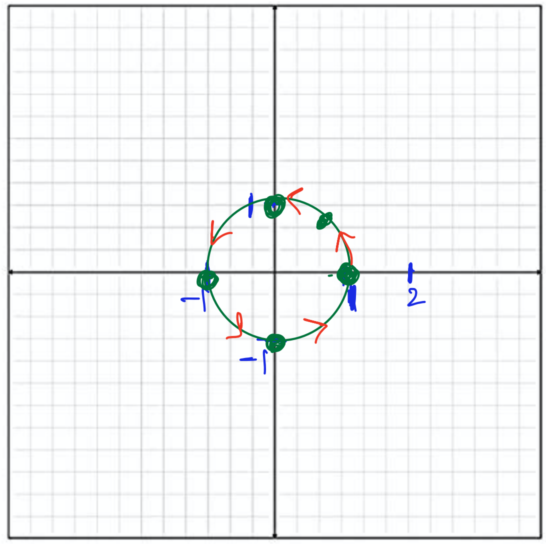
(b) $x = t^2$, $y = t+1$ } → want just one eqn. involving x and y.

$\Downarrow$

$t = y-1 \Rightarrow \boxed{x = (y-1)^2}$

(2) $x = \cos t$ and $y = \sin t$, where $0 \le t \le 2\pi$.

t	x	y
0	$\cos 0 = 1$	$0 = \sin 0$
$\pi/4$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
$\pi/2$	0	1
π	-1	0
$3\pi/2$	0	-1
2π	1	0



b) Find Cartesian eqn. $\Rightarrow$ eliminate t

$$\cos^2 t + \sin^2 t = 1 \Rightarrow x^2 + y^2 = 1$$

Example 2: Let $\mathcal{C}$ be the parametric curve given by $x = t^2$ and $y = t^3 - 3t$.

(a) Find the point on the curve $\mathcal{C}$ when $t = 3$.

$$x = 3^2 = 9$$
$$y = 3^3 - 3(3) = 27 - 9 = 18$$
$$\Rightarrow (x, y) = (9, 18)$$

(b) Find t at the point $(1, 2)$.

$$\left.\begin{array}{l} x = 1 \\ y = 2 \end{array}\right\} \Rightarrow \begin{array}{l} t^2 = 1 \Rightarrow t = \pm 1 \\ \text{and} \\ t^3 - 3t = 2 \end{array}$$

check if these satisfy $t^3 - 3t = 2$

not a soln.

$t = 1$, $1^3 - 3(1) = 1 - 3 = -2 \neq 2$

$t = -1$, $(-1)^3 - 3(-1) = -1 + 3 = 2 \Rightarrow$ For $t = -1$ we have $t^2 = 1$, $t^3 - 3t = 2$

$\Rightarrow t = -1$