Name: [1 pt]

Problem 1: Find the equation of the tangent line to the parametric curve $x(t) = \cos 2t$, $y(t) = \sin t$ at the point corresponding to $t = \pi$. [5 pts]

$\dfrac{dy}{dt} = \cos t$, $\dfrac{dx}{dt} = -2\sin 2t$ $\Rightarrow$ $\dfrac{dy}{dx} = \dfrac{\cos t}{-2\sin 2t}$

$y(\pi) = \sin\pi = 0$, $x(\pi) = \cos 2\pi = 1$

$\left.\dfrac{dy}{dx}\right|_{t=\pi} = \dfrac{\cos\pi}{-2\sin 2\pi} = \dfrac{-1}{-2\times 0} = \infty$

$\Rightarrow$ $y - y(\pi) = m\left(x - x(\pi)\right)$ $\Rightarrow$ $y - 0 = m\left(x-1\right)$

$\Rightarrow$ $y = m(x-1)$ and $m = \infty$ $\Rightarrow$ $\dfrac{1}{m} = 0$

$\Rightarrow$ $\dfrac{1}{m}y = x-1$ $\Rightarrow$ $0 \times y = x-1$ $\Rightarrow$ $x-1 = 0$ $\Rightarrow$ $\boxed{x = 1}$

Problem 2:

1. Find polar coordinates of the point $(-1, 1)$
2. Find Cartesian coordinates of the point $(2, -\pi/2)$.

[5 pts]

1) $(-1, 1)$ $\Rightarrow$ $r = \sqrt{(-1)^2 + (1)^2} = \sqrt{2}$

$\theta = \tan^{-1}\left(\dfrac{1}{-1}\right) = \pi - \tan^{-1} 1 = \pi - \dfrac{\pi}{4} = \dfrac{3\pi}{4}$

$\Rightarrow$ $(-1, 1) \equiv \left(\sqrt{2}, \dfrac{3\pi}{4}\right)$

2) $\left(2, -\dfrac{\pi}{2}\right)$ $\Rightarrow$ $x = r\cos\theta = 2\cos\left(\dfrac{-\pi}{2}\right) = 2 \times 0 = 0$

$y = r\sin\theta = 2\sin\left(\dfrac{-\pi}{2}\right) = -2\sin\dfrac{\pi}{2} = -2$

$\Rightarrow$ $\left(2, \dfrac{-\pi}{2}\right) \equiv \left(0, -2\right)$