Learning objectives:

1. Find an expression for the average value of a function.
2. Understand the mean value theorem for integrals.

Average value of a function

Let f be a function defined on a closed interval $[a, b]$. Then the average value of f on the interval $[a, b]$ is given by

$$f_{av} = \frac{1}{b-a} \int_a^b f(x)\, dx \, .$$

Example 1. Find the average value of the function $f(x) = 1 + x^2$ on the interval $[-1, 2]$.

The mean value theorem for integrals. If f is continuous on $[a, b]$, then there exist a number c in $[a, b]$ such that

$$f(c) = f_{av} = \frac{1}{b-a} \int_a^b f(x)\, dx\ .$$

Example 2. Let $f(x) = 1 + x^2$ be as in Example 1. Find all possible numbers c for which $f(c) = f_{av}$.