Math16500 Section 24246 Quiz 8
Fall 2022, October 10

Name: [1 pt]

Problem 1: Differentiate $x^2y = y + \sin x$ implicitly to find $\dfrac{dy}{dx}$. [4 pts]

$$\frac{d}{dx}(x^2y) = \frac{d}{dx}(y) + \frac{d}{dx}(\sin x) \Rightarrow \frac{d}{dx}(x^2)\, y + x^2 \frac{dy}{dx} = \frac{dy}{dx} + \cos x$$

$$\Rightarrow 2xy + x^2\frac{dy}{dx} = \frac{dy}{dx} + \cos x$$

$$\Rightarrow (x^2-1)\frac{dy}{dx} = \cos x - 2xy$$

$$\Rightarrow \boxed{\frac{dy}{dx} = \frac{\cos x - 2xy}{x^2-1}}$$

Problem 2: Find equation of tangent to the parabola $y^2 = x + 3$ at the point $(1, 2)$. [5 pts]

$$\frac{d}{dx}(y^2) = \frac{d}{dx}(x+3) \Rightarrow 2y\frac{dy}{dx} = 1 \Rightarrow \frac{dy}{dx} = \frac{1}{2y}$$

Equation of tangent line is :-

$$\frac{y-2}{x-1} = \left.\frac{dy}{dx}\right|_{(1,2)} = \left.\frac{1}{2y}\right|_{y=2} = \frac{1}{4}$$

$$\Rightarrow 4(y-2) = 1(x-1)$$

$$\Rightarrow 4y - 8 = x - 1 \Rightarrow 4y = x - 1 + 8$$

$$\Rightarrow \boxed{4y = x + 7}$$

Bonus Problem: Find points where the function $f(x) = |x - 2|$ is not differentiable.
Is $f(x)$ continuous at those points? [2 pts]

$$f(x) = \begin{cases} x-2 & \text{if } x \geq 2 \\ 2-x & \text{if } x < 2 \end{cases} \Rightarrow 2 \text{ can be a possible point of non-differentiability.}$$

$$\underset{x=2}{LHD} = \lim_{h\to 0} \frac{f(2-h) - f(2)}{-h} = \lim_{h\to 0} \frac{|2-h-2| - 0}{-h} = \lim_{h\to 0}\frac{h}{-h} = -1$$

$$\underset{x=2}{RHD} = \lim_{h\to 0} \frac{f(2+h) - f(2)}{h} = \lim_{h\to 0} \frac{|2+h-2| - 0}{h} = \lim_{h\to 0}\frac{h}{h} = 1$$

$\Rightarrow \underset{x=2}{LHD} = -1 \neq 1 = \underset{x=2}{RHD} \Rightarrow$ $\underline{f(x) \text{ is not differentiable at } x=2}$

For continuity at $x=2$, check that $\underset{x=2}{LHL} = \lim_{h\to 0} |2-h-2| = \lim_{h\to 0} h$
$= 0$

and $\underset{x=2}{RHL} = \lim_{h\to 0} |2+h-2| = \lim_{h\to 0} h = 0$

Since, $\underset{x=2}{LHL} = 0 = \underset{x=2}{RHL}$, $\underline{f(x) \text{ is continuous at } x=2}$