MATH 16600 Practice Final Exam, *Version 3*

1 Given a one-to-one function $f(x) = 1 + 4x + \sin x$, $-\infty < x < \infty$. find $f^{-1}(1)$ and $(f^{-1})'(1)$.

2 The mass of a radio-active material is reduced to 30% of the original quantity in 30 years. What is the half-life?

3 Find the limit. $\lim_{x \to \pi/4} \frac{x - \pi/4}{\sin x - \cos x}$.

4 Evaluate the integral $\int \frac{x+1}{x(x-2)} dx$

5 Evaluate the integral. $\int 2xe^x \, dx$.

6 Let $f(x) = \ln \sqrt{\dfrac{x+1}{x^2+1}}$. Use the properties of logarithmic functions to decompose $f(x)$ completely then find $f'(x)$.

7 Evaluate the integral. $\int x\sqrt{x^2 - 9}\, dx$.

8 Set up an integral that represents the length of the curve $y = x + \dfrac{1}{x}$, $1/2 \leq x \leq 2$.

9 Determine whether the improper integral $\int_0^5 \frac{1}{(5-x)^3}\,dx$ is convergent or divergent. Evaluate the integral if it is convergent.

$$\int_0^5 \frac{1}{(5-x)^3}\,dx = \lim_{t \to 5^-} \int_0^t (5-x)^{-3}\,dx$$

not continuous at $x=5$

$$\int (5-x)^{-3}\,dx = \left[\frac{(5-x)^{-2}}{-2}\right]\frac{1}{(-1)}\Bigg|_0^t = \frac{(5-t)^{-2}}{2} - \frac{(5-0)^{-2}}{2}$$

Alternatively $u = 5-x$

$$= \frac{1}{2(5-t)^2} - \frac{1}{50}$$

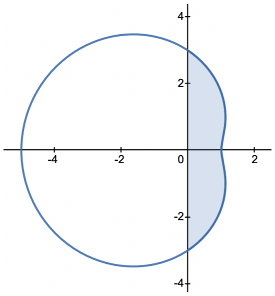
$$\lim_{t \to 5^-}\left[\frac{1}{2(5-t)^2} - \frac{1}{50}\right] = \frac{1}{2(\to 0)^2} - \frac{1}{50}$$

$$= \infty$$

$\Rightarrow$ The integral diverges.

10 Find an equation of the tangent line to the curve at the point corresponding to the given value of the parameter. $x = 2\sin t$, $y = \cos t$, $t = \pi/4$.

11 Find the area of the shaded region



$$r = 3 - 2\cos\theta$$

12 Determine whether the series $\displaystyle\sum_{n=1}^{\infty}(-1)^n \frac{1}{n^2+1}$ is convergent or divergent.

13 Determine whether the series $\sum_{n=1}^{\infty} \frac{4^n}{2^n + 3^n}$ is convergent or divergent.

14 Determine whether the series $\sum_{n=1}^{\infty} \frac{2^n}{n!}$ is convergent or divergent.

15 Find the area of the surface obtained by rotating the curve $y = x^3$, $0 \le x \le 2$, about the x-axis. **Note:** You need to compute the definite integral for this problem. Your answer should be a real number.

16 Use the definition of Taylor series to find the first **four** nonzero terms of the series for $f(x) = \sin(x)$ centered at $a = \frac{\pi}{2}$.

17 Find the radius of convergence and interval of convergence of the series $\sum_{n=0}^{\infty}(n+1)\frac{x^n}{3^n}$.

18 Determine whether the series $\sum_{n=1}^{\infty}(-1)^n\frac{1}{\sqrt{n}}$ is absolutely convergent, conditionally convergent, or divergent.