M16600 Lecture Notes

Section 10.1: Curves Defined by Parametric Equations

■ **Section 10.1** textbook exercises, *page 685*: #5, 7, <u>8</u>.

Equations such as

$$y(x) = 3e^x + x^3 \qquad \text{or} \qquad x(y) = y^2 - 1$$

describe some curves in the xy-plane.

$y = x + 2$

In this section, we have ANOTHER way to describe curves in the xy-plane, called ***parametric equations***:

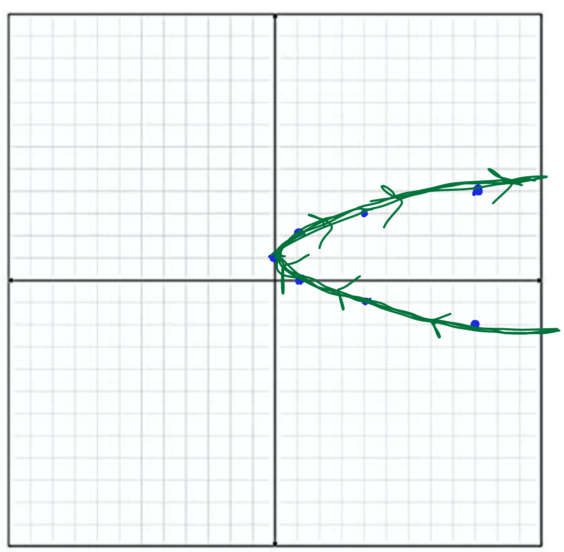
$$x = x(t) \qquad \underline{\text{and}} \qquad y = y(t)$$

Here, t is the <u>*parameter*</u>.

Example 1: (a) Sketch the given ***parametric curves*** (i.e. curves given by *parametric equations*). Indicate with an arrow the direction in which the curve is traced as t increases. (b) Eliminate the parameter to find a ***Cartesian equation*** (equation with only x and y) of the curve

(1) $x = t^2$ and $y = t + 1$

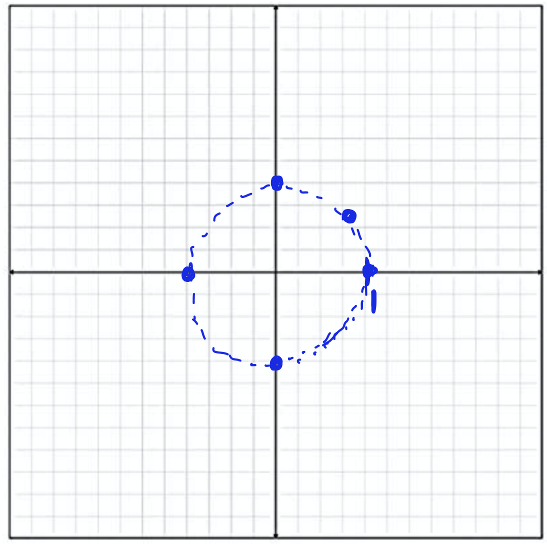
t	x ($= t^2$)	$y = t+1$
-3	9	-2 ✓
-2	4	-1 ✓
-1	1	0 ✓
0	0	1 ✓
1	1	2 ✓
2	4	3 ✓
3	9	4 ✓



(b) $x = t^2$, $y = t + 1 \Rightarrow t = y - 1 \Rightarrow x = (y-1)^2$

(2) $x = \cos t$ and $y = \sin t$, where $0 \le t \le 2\pi$.

t	x	y
0	1	0
$\pi/4$	0.7	0.7
$\pi/2$	0	1
π	-1	0
$3\pi/2$	0	-1
2π	1	0



$x = \cos t, \quad y = \sin t$

$\cos^2 t + \sin^2 t = 1 \Rightarrow x^2 + y^2 = 1$

Example 2: Let $\mathcal{C}$ be the parametric curve given by $x = t^2$ and $y = t^3 - 3t$.

(a) Find the point on the curve $\mathcal{C}$ when $t = 3$.

$x = 3^2 = 9$

$y = (3)^3 - 3(3) = 27 - 9 = 18$

$\Rightarrow (9, 18)$

(b) Find t at the point $(1, 2)$.

$(x, y) = (1, 2)$

$\Rightarrow x = 1$ and $y = 2$

$t^2 = 1$ and $\underbrace{t^3 - 3t}_{\text{LHS}} = \underbrace{2}_{\text{RHS}}$

$\downarrow$

$t = \pm 1 \Rightarrow$ Consider $t = 1$, $(1)^3 - 3(1) = 1 - 3 = -2 \neq 2$

$\Rightarrow t=1$ is not a solution

Consider $t=-1$, $LHS = (-1)^3 - 3(-1) = -1+3 = 2 = RHS$

$\Rightarrow t=-1$ is a solution.

The Point $(1,2)$ corresponds to $t=-1$