1.5 Formulas, Models, and Geometry

- A formula is an equation that uses letters to represent a relationship between two or more quantities.

Solving Literal Equations/ Formulas

$$3x+5=7$$

Example: Solve $V = lwh$ for h

$$\frac{lwh}{l} = \frac{V}{l} \quad \Rightarrow \quad wh = \frac{V}{l} \quad \Rightarrow \quad h = \frac{V}{lw}$$

Example: $C = \frac{5}{9}(F - 32)$ for F

$$\underbrace{\frac{9}{5} \cdot \frac{5}{9}}_{=1}(F-32) = \frac{9}{5} \cdot C \quad \Rightarrow \quad F - 32 = \frac{9}{5}C \quad \Rightarrow \quad F = \frac{9}{5}C + 32$$

Example: $C = 2\pi r$ for r

$$2\pi r = C \quad \Rightarrow \quad \pi r = \frac{C}{2} \quad \Rightarrow \quad r = \frac{C}{2\pi}$$

Divide by 2

Divide by π

You Try!

Solve the formula $a = lw$ for l	Solve the formula $I = Prt$ for t
$l = \dfrac{a}{w}$	$t = \dfrac{I}{Pr}$

To Solve a Formula for a Specified Letter

1. Get all the terms with the specified letter on one side of the equation and all the other terms on the other side, using the addition principle. To do this may require removing parentheses.
   - To remove parentheses, either divide both sides by the multiplier in front of the parentheses or use the distributive law.
2. When all the terms with the specified letter are on the same side, factor (if necessary) so that the variable is written only once.
3. Solve for the letter in question by dividing both sides by the multiplier of that letter.

Example: Solve $\frac{x}{a} + \frac{y}{b} = 1$ for x

$$\frac{x}{a} + \frac{y}{b} = 1 \quad\Rightarrow\quad \frac{x}{a} = 1 - \frac{y}{b}$$

$$-\frac{y}{b} \qquad -\frac{y}{b}$$

$$= \frac{b-y}{b}$$

$$\frac{1}{1} - \frac{y}{b}$$

$$= \frac{b-y}{b}$$

$$\Rightarrow \frac{x}{a} = \frac{b-y}{b}$$

$$\Rightarrow x = a \cdot \frac{b-y}{b}$$

$$\Rightarrow \boxed{x = \frac{a(b-y)}{b}}$$

Example: Solve $y + mx_2 = mx_1$ for m

$$-mx_2 \quad -mx_2$$

$$y = mx_1 - mx_2 \Rightarrow mx_1 - mx_2 = y$$

$$\Rightarrow m(x_1 - x_2) = y$$

$$\frac{m(x_1-x_2)}{(x_1-x_2)} \quad \frac{y}{(x_1-x_2)}$$

$$\Rightarrow \boxed{m = \frac{y}{x_1 - x_2}}$$

Example: Solve $pr + pq = m$ for p

$$P(r + q) = m$$

$$\Rightarrow \boxed{P = \frac{m}{r+q}}$$

Example: Solve $abc + b = 2a$ for b

$$\Rightarrow b(ac + 1) = 2a$$

$$\Rightarrow \boxed{b = \frac{2a}{ac+1}}$$

You Try!

Solve the formula $S = 2WH + 2WL$ for H	Solve the formula $x = ya + yz$ for y
$-2WL \qquad -2WL$ $2WH = S - 2WL$ $H = \dfrac{S - 2WL}{2W}$	$y = \dfrac{x}{a+z}$

FORMULAS AS MODELS

Example: Eliana has $2600 to invest for 6 months. If she needs the money to earn $104 in that time, at what rate of simple interest must Eliana invest. Note: Simple Interest is $I = Prt$

$$I = Prt$$

$104 \qquad 2600$

6 months $= \frac{1}{2}$ year

to find
(yearly)

$$\Rightarrow 104 = (2600)\, r\left(\tfrac{1}{2}\right)$$

$$\Rightarrow r = \frac{(104)\,2}{2600}$$

$$\Rightarrow r = \frac{208}{2600} \times 100\,\%$$

$$= 8\,\%$$
Per annum

Example: The area of a parallelogram is $96\ cm^2$. The base of the figure is 6 cm. What is the height?

$$A = bh \qquad \Rightarrow 6h = 96$$

$96 \qquad 6$

$$\Rightarrow h = 16\ cm$$

Example: A gallon of paint covers about $400\ cm^2$ Jenn wants to paint a solid wood fence that is 8 ft tall. What is the longest fence that she can paint with one gallon of paint

$$A = \ell h \qquad \Rightarrow \ell = \frac{400\ cm^2}{8\ ft} = \frac{400}{8(12)(2.54)} = \,?$$

$400 \qquad 8ft$

$$1ft = 12(2.54)$$

$$1\,inch = 2.54cm$$