Name:

Problem 1: Evaluate the following definite integrals

1. $\int_0^1 x\,dx$
2. $\int_0^1 x^2\,dx$
3. $\int_0^1 x^3\,dx$

as limit of the right Riemann sums, that is, using the formula

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \frac{b-a}{n} \sum_{i=1}^{n} f\left(a + i\frac{b-a}{n}\right)$$

Problem 2: Using properties of definite integrals and the results of problem 1, evaluate

$$\int_1^0 (4x^3 - 6x^2 - 2x + 1)\, dx$$

.

Problem 3: Use midpoint rule with $n = 5$ to approximate the integral

$$\int_0^2 \frac{x}{x+1}\, dx.$$

Problem 4: Use Fundamental Theorem of Calculus to find the following derivatives:-

1. $\displaystyle\int_2^{1/x} \sin^4 u \, du$
2. $\displaystyle\int_{\sin x}^{1} \sqrt{1+t^2} \, dt$
3. $\displaystyle\int_{x^2}^{\tan x} \frac{1}{\sqrt{2+u^4}} du$
4. $\displaystyle\int_{\sqrt{x}}^{x^2} \cos(t^2) \, dt$

Problem 5: Evaluate the following indefinite integrals (use substitution if needed):-

1. $\displaystyle\int \frac{1-\sin^3 t}{\sin^2 t}\,dt$
2. $\displaystyle\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$
3. $\displaystyle\int \frac{z^2}{\sqrt[3]{1+z^3}}\,dz$
4. $\displaystyle\int \frac{dt}{\cos^2 t\sqrt{1+\tan t}}\,dx$

Problem 6: Evaluate the following definite integrals:

1. $\displaystyle\int_1^8 \frac{2+t}{\sqrt[3]{t^2}}\,dt$
2. $\displaystyle\int_0^{3\pi/2} |\sin x|\,dx$
3. $\displaystyle\int_{-1}^{2} \big(x - 2|x|\big)\,dx$
4. $\displaystyle\int_0^{\pi} f(x)\,dx$ where $f(x) = \begin{cases} \sin x & 0 \le x \le \pi/2 \\ \cos x & \pi/2 \le x \le \pi \end{cases}$.

Problem 7: Evaluate the following definite integrals using substitution and/or symmetry:

1. $\displaystyle\int_{-\pi/3}^{\pi/3} x^4 \sin x \, dx$
2. $\displaystyle\int_0^1 x\sqrt{1-x}\, dx$
3. $\displaystyle\int_0^{\pi/2} \cos x \sin(\sin x)\, dx$
4. $\displaystyle\int_0^1 \frac{dx}{(1+\sqrt{x})^4}$