Example 1. Find area of the region bounded by $y = \sqrt{x}$, $x = 4$ and $y = 0$.

Area between two curves: The area enclosed between two curves $y = f(x)$ and $y = g(x)$ from $x = a$ to $x = b$ is given by

$$A = \int_a^b \big|f(x) - g(x)\big|\, dx\,.$$

Note that

$$\big|f(x) - g(x)\big| = \begin{cases} f(x) - g(x) & \text{if } f(x) \geq g(x) \\ g(x) - f(x) & \text{if } g(x) \geq f(x) \end{cases}.$$

A property of definite integral: If f is continuous on $[a, b]$ and $a < c < b$, then

$$\int_a^b f(x)\, dx = \int_a^c f(x)\, dx + \int_c^b f(x)\, dx\,.$$

Example 2. Find area of the region bounded by $y = x^3$, $x = -1$, $x = 2$ and $y = 0$.

Example 3. Find area of the region bounded by the parabola $y^2 = 4x$ and the line $y = x$.

Area along horizontal strips: The area bounded between the curves $x = f(y)$ and $x = g(y)$ from $y = c$ to $y = d$ is given by

$$A = \int_c^d \left| f(y) - g(y) \right| dy \, .$$

Example 4. Find the area of the region bounded by $x = 8y - 2y^2$ and y-axis.

Example 5. Find the area of the region bounded by $x = y^2 - y - 2$ and $y = x + 2$.