

Math-I 110 3.6 Notes

Determine the dimensions of the matrix

rows \times # columns

$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ 2×2 or $2, 2$	$\begin{bmatrix} 1 & 2 & 5 \\ 3 & 4 & 6 \end{bmatrix}$ 2×3
$\begin{bmatrix} 1 & 2 \\ 5 & 6 \\ 3 & 4 \end{bmatrix}$ 3×2	$\begin{bmatrix} 1 & 2 & 7 \\ 5 & 6 & 8 \\ 3 & 4 & 9 \end{bmatrix}$ 3×3

Given the following matrices

You can only add and subtract matrices with like dimensions

same # of rows
AND same # of columns

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ 2×2	$B = \begin{bmatrix} -1 & -3 \\ -2 & -4 \end{bmatrix}$ 2×2	$C = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$ 2×1	$D = \begin{bmatrix} 3 & 6 \end{bmatrix}$ 1×2
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Find the following if possible

1. $A + B$

✓

$$A + B = \begin{bmatrix} 1 + (-1) & 2 + (-3) \\ 3 + (-2) & 4 + (-4) \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

2. $B + D$

✗

Not Possible

3. $A - B$

✓

$$A - B = \begin{bmatrix} 1 - (-1) & 2 - (-3) \\ 3 - (-2) & 4 - (-4) \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 5 \\ 5 & 8 \end{bmatrix}$$

4. **B-A**

$$B-A = \begin{bmatrix} -1-1 & -3-2 \\ -2-3 & -4-4 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -5 \\ -5 & -8 \end{bmatrix}$$

5. **C+D**

Not possible.

Scalar Multiplication

Given the following matrices

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$B = \begin{bmatrix} -1 & -3 \\ -2 & -4 \end{bmatrix}$$

$$C = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

$$D = \begin{bmatrix} 3 & 6 \end{bmatrix}$$

Find each of the following if possible

1. **3B**

$$3B = \begin{bmatrix} 3(-1) & 3(-3) \\ 3(-2) & 3(-4) \end{bmatrix}$$

$$= \begin{bmatrix} -3 & -9 \\ -6 & -12 \end{bmatrix}$$

2. **(-1)B**

$$(-1)B = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

3. **(-1)C**

$$(-1)C = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$B = \begin{bmatrix} -1 & -3 \\ -2 & -4 \end{bmatrix}$$

$$C = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

$$D = [3 \quad 6]$$

4. $5D$

$$5D = [15 \quad 30]$$

5. $2A+B$

$$2A = \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix}$$

$$2A+B = \begin{bmatrix} 2+(-1) & 4+(-3) \\ 6+(-2) & 8+(-4) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 4 & 4 \end{bmatrix}$$

6. $A+2B$

$$2B = \begin{bmatrix} -2 & -6 \\ -4 & -8 \end{bmatrix}$$

$$A+2B = \begin{bmatrix} 1-2 & 2-6 \\ 3-4 & 4-8 \end{bmatrix} = \begin{bmatrix} -1 & -4 \\ -1 & -4 \end{bmatrix}$$

Matrix Multiplication

You can only multiply two matrices where the number of columns in the first matrix equals the number of rows in the second.

Rule For Matrix Multiplication



$$\begin{array}{c} A \cdot B = AB \\ m \times n \quad n \times p \quad m \times p \\ \downarrow \quad \downarrow \quad \downarrow \\ \text{Dimensions of } AB \end{array}$$

$$AB \neq BA$$

Given the following Matrices

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ 2×2	$B = \begin{bmatrix} -1 & -3 \\ -2 & -4 \end{bmatrix}$ 2×2	$C = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$ 2×1	$D = [3 \quad 6]$ 1×2
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Find the following if possible

1. AC

\rightarrow Yes

$$2 \times 2 \cdot 2 \times 1 \Rightarrow 2 \times 1$$

$$\begin{bmatrix} * \\ * \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

$$\begin{array}{l} \rightarrow .2 \\ \quad + \\ 2.(-2) \end{array}$$

$$= \begin{bmatrix} 1 \cdot 2 + 2(-2) \\ 3 \cdot 2 + 4(-2) \end{bmatrix} = \begin{bmatrix} -2 \\ -2 \end{bmatrix}_{2 \times 1}$$

$$= 2 + (-4) \\ = -2$$

2. **BD**

$$2 \times 2 \cdot 1 \times 2$$

Not Possible

3. **CA**

$$2 \times 1 \cdot 2 \times 2$$

Not Possible

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2 \times 2}$$

$$B = \begin{bmatrix} -1 & -3 \\ -2 & -4 \end{bmatrix}_{2 \times 2}$$

$$C = \begin{bmatrix} 2 \\ -2 \end{bmatrix}_{2 \times 1}$$

$$D = \begin{bmatrix} 3 & 6 \end{bmatrix}_{1 \times 2}$$

4. **DB**

$$(1 \times 2) \cdot (2 \times 2) = 1 \times 2$$

$$DB = \begin{bmatrix} 3 & 6 \end{bmatrix} \begin{bmatrix} -1 & -3 \\ -2 & -4 \end{bmatrix} = \begin{bmatrix} 3(-1) + 6(-2) & 3(-3) + 6(-4) \\ -3 - 12 & -9 - 24 \end{bmatrix}$$

5. **CD**

$$(2 \times 1) \cdot (1 \times 2) = 2 \times 2$$

$$= \begin{bmatrix} -15 & -33 \end{bmatrix}$$

$$CD = \begin{bmatrix} 2 \\ -2 \end{bmatrix} \begin{bmatrix} 3 & 6 \end{bmatrix} = \begin{bmatrix} 2(3) & 2(6) \\ -2(3) & -2(6) \end{bmatrix}$$

6. **DC**

$$(1 \times 2) \cdot (2 \times 1) = (1 \times 1) = \begin{bmatrix} 6 & 12 \\ -6 & -12 \end{bmatrix}$$

$$DC = \begin{bmatrix} 3 & 6 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \end{bmatrix} = \begin{bmatrix} 3(2) + 6(-2) \\ -6 \end{bmatrix} = \begin{bmatrix} -6 \end{bmatrix}$$

i^{th} row j^{th} column element in the product is obtained by multiplying i^{th} row of first factor with j^{th} column of second factor.

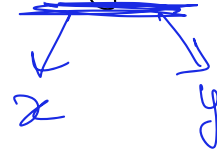
In-class Quiz 4

Sep 23

①
$$\begin{cases} 2x + y = 3 \\ y = 2x - 1 \end{cases}$$
 solve the system for x and y
7 pts

②
$$\begin{cases} 3x + 4y = 7 \\ 4x - 3y = 1 \end{cases}$$
 solve the system for x and y .
8 pts

③ There are two ^{Sum is 90°} complementary angles which differ by 2 degrees. Find the angles.
5 pts



$$x + y = 90$$

$$x - y = 2$$

$$\begin{array}{r} x + y = 90 \\ x - y = 2 \\ \hline 2x = 92 \end{array} \Rightarrow \begin{aligned} x &= 46 \\ y &= 44 \end{aligned}$$