Graphing Exponential Functions

ESSENTIALS

The function $f(x) = a^x$, where a is a positive constant different from 1, is called an **exponential function**, base a.

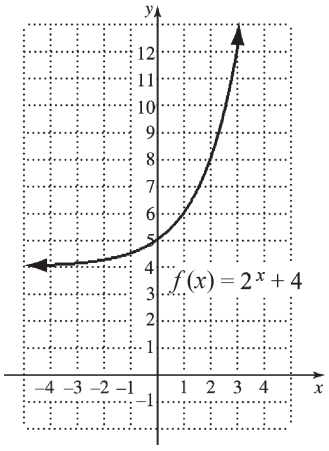
All functions $f(x) = a^x$ go through the point $(0,1)$. That is, the y-intercept is $(0,1)$.

Example $f(0) = a^0 = 1$

- Graph the exponential function $f(x) = 2^x + 4$.

  Construct a table of values. Plot the points and connect them with a smooth curve.

x	$f(x)$	
0	5	2^0+4
1	6	2^1+4
2	8	2^2+4
3	12	2^3+4
-1	$4\frac{1}{2}$	$2^{-1}+4$
-2	$4\frac{1}{4}$	$2^{-2}+4$
-3	$4\frac{1}{8}$	$2^{-3}+4$



$f(x) = 3^x$ or $f(x) = 4^{-x}$ or $f(x) = 2^{-x+4}$ or $f(x) = 2^{-x} + 4$

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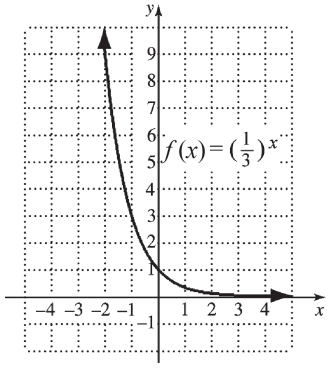
EXAMPLE 1

Graph: $f(x) = \left(\frac{1}{3}\right)^x = 3^{-x}$.

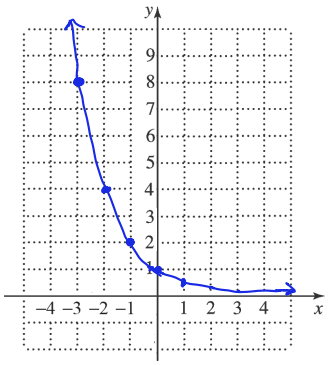
List the function values in a table, plot the points, and connect them with a smooth curve.

x	$f(x)$
0	1
1	$\frac{1}{3}$
2	
3	$\frac{1}{27}$
-1	
-2	9
-3	



YOUR TURN 1

Graph: $f(x) = \left(\frac{1}{2}\right)^x = 2^{-x}$.



$f(0) = 2^0 = 1$

$f(1) = 2^{-1} = \frac{1}{2}$

$f(2) = 2^{-2} = \frac{1}{4}$

$f(3) = 2^{-3} = \frac{1}{8}$

$f(-1) = 2^{-(-1)} = 2$

$f(-2) = 2^{-(-2)} = 4$

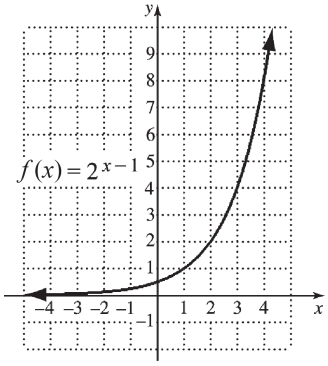
$f(-3) = 2^{-(-3)} = 8$

EXAMPLE 2

Graph: $f(x) = 2^{x-1}$.

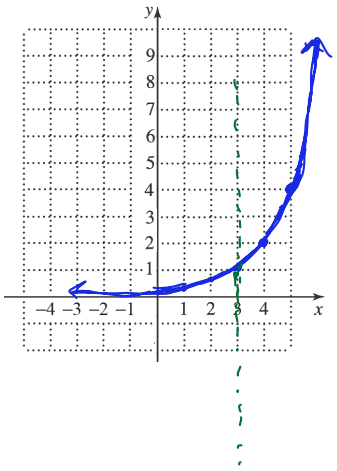
List function values in a table, plot the points, and connect them with a smooth curve.

x	$f(x)$
0	$\frac{1}{2}$
1	
2	2
3	4
−1	$\frac{1}{4}$
−2	
−3	$\frac{1}{16}$



YOUR TURN 2

Graph: $f(x) = 2^{x-3}$.



$f(0) = 2^{0-3} = \frac{1}{8}$

$f(1) = 2^{1-3} = \frac{1}{4}$

$f(2) = 2^{2-3} = \frac{1}{2}$

$f(3) = 2^{3-3} = 1$

$f(4) = 2^{4-3} = 2$

$f(5) = 2^{5-3} = 4$

YOUR NOTES Write your questions and additional notes.

Equations with *x* and *y* Interchanged

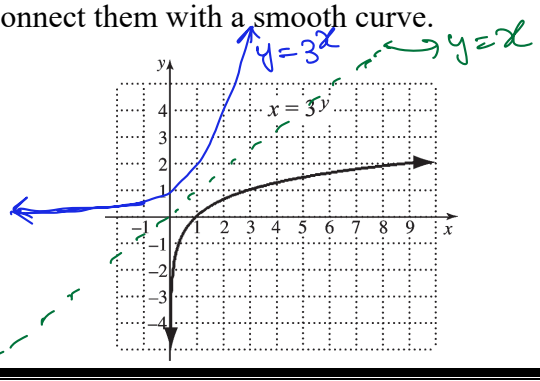
ESSENTIALS

Example

- Graph: $x = 3^y$.

  Choose values for y, compute values for x, and list the results in a table. Then plot the points and connect them with a smooth curve.

x	y
1	0
3	1
9	2
$\frac{1}{3}$	-1
$\frac{1}{9}$	-2



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EXAMPLE 1

Graph: $x = \left(\frac{1}{2}\right)^y$.

Note that $x = \left(\frac{1}{2}\right)^y = 2^{-y}$. Choose values for y and compute values for x. Then plot the points and connect them with a smooth curve.

For $y = 0$, $x = 2^{-0} = 1$.

For $y = 1$, $x = 2^{-1} = \frac{1}{2}$.

For $y = 2$, $x = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$.

For $y = 3$, $x = 2^{-3} = \frac{1}{\square} = \frac{1}{8}$.

For $y = -1$ $x = 2^{-(-1)} = 2^1 = 2$.

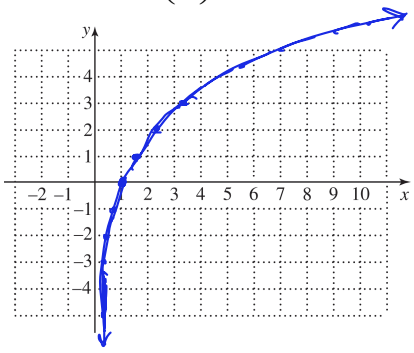
For $y = -2$, $x = 2^{-(-2)} = 2^{\square} = 4$

For $y = -3$, $x = 2^{-(-3)} = 2^3 = \square$.

(continued)

YOUR TURN 1

Graph: $x = \left(\frac{3}{2}\right)^y$.



x	y
1	0
1.5	1
2.25	2
3.375	3
0.67	−1
0.44	−2
0.296	−3

$\left(\frac{3}{2}\right)^0 = 1$

$\left(\frac{3}{2}\right)^1 = \frac{3}{2}$

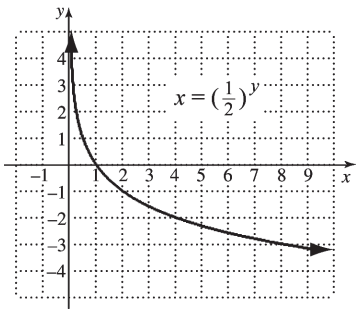
$\left(\frac{3}{2}\right)^2 = \frac{9}{4}$

$\left(\frac{3}{2}\right)^3 = \frac{27}{8}$

$\left(\frac{3}{2}\right)^{-1} = \frac{1}{\frac{3}{2}} = \frac{2}{3}$; $\left(\frac{3}{2}\right)^{-2} = \left(\left(\frac{3}{2}\right)^{-1}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$

$\left(\frac{3}{2}\right)^{-3} = 0.296$

x	y
	0
$\frac{1}{2}$	1
	2
$\frac{1}{8}$	3
2	−1
4	−2
	−3



YOUR NOTES Write your questions and additional notes.

Applications of Exponential Functions

ESSENTIALS

Example

- The amount of money A that a principal P will be worth after t years at an interest rate r, compounded annually, is given by $A = P(1+r)^t$. Suppose that \$5000 is invested at 5% interest, compounded annually.

  a) Find a function for the amount in the account after t years.
  b) Find the amount of money in the account at $t = 0$, $t = 3$, $t = 6$, and $t = 10$.
  c) Graph the function.

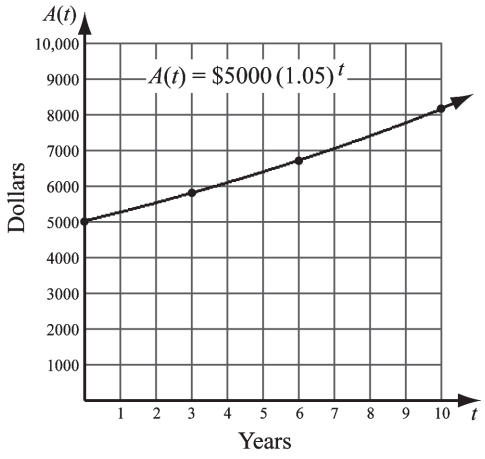
  a) If $P = \$5000$ and $r = 5\% = 0.05$, we can substitute these values into the formula to get the following function:

$$A(t) = \$5000(1+0.05)^t$$
$$= \$5000(1.05)^t.$$

  b) Find the function values.

$$A(0) = \$5000(1.05)^0 = \$5000(1) = \$5000;$$
$$A(3) = \$5000(1.05)^3 \approx \$5000(1.157625) \approx \$5788.13;$$
$$A(6) = \$5000(1.05)^6 \approx \$5000(1.340095641) \approx \$6700.48;$$
$$A(10) = \$5000(1.05)^{10} \approx \$5000(1.628894627) \approx \$8144.47$$

  c) We use the function values computed in part (a), and others if we wish, to draw the graph.



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EXAMPLE 1

Digital music sales, in billions of dollars, t years after 2010, can be approximated by $m(t) = 3.2(1.23)^t$.

a) Estimate the digital music sales in 2010, 2013, and 2015.

b) Graph the function.

a) The years 2010, 2013, and 2015 are represented by $t = 0$, $t = 3$, and $t = 5$, respectively.

$m(0) = 3.2(1.23)^0$

$= 3.2(1)$

$= 3.2$;

$m(3) = 3.2(1.23)^3$

$= 3.2(1.860867)$

≈ 6;

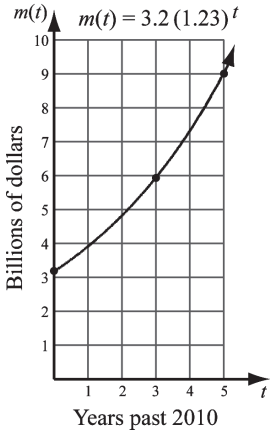
$m(5) = 3.2(1.23)^5$

$\approx 3.2(2.815305684)$

≈ 9.0

Digital music sales in 2010, 2013, and 2015 were \$3.2 billion, \$6 billion, and \$ 9 billion, respectively.

b)



YOUR TURN 1

The percentage of smokers P who receive telephone counseling to quit smoking and are still successful t months later can be approximated by $P(t) = 21.4(0.914)^t$.

a) Estimate the percentage of smokers receiving telephone counseling who are successful in quitting for 1 month, 4 months, and 10 months.

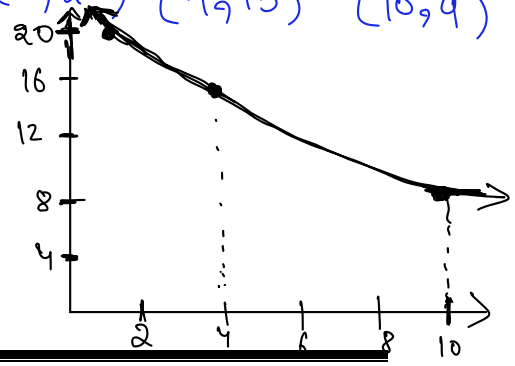
b) Graph the function.

$P(1) = 21.4(0.914)^1 = 19.56 \approx 20$

$P(4) = 21.4(0.914)^4 = 14.93 \approx 15$

$P(10) = 21.4(0.914)^{10} = 8.7 \approx 9$

(1, 20) (4, 15) (10, 9)



YOUR NOTES Write your questions and additional notes.

Practice Exercises

Readiness Check

Choose the word that best completes each sentence.

1. The graph of $f(x)=\left(\frac{1}{2}\right)^x+5$ looks just like the graph of $f(x)=\left(\frac{1}{2}\right)^x$, but it is translated 5 units _____.
up / down

2. The graph of $f(x)=\left(\frac{1}{2}\right)^x-5$ looks just like the graph of $f(x)=\left(\frac{1}{2}\right)^x$, but it is translated 5 units _____.
up / down

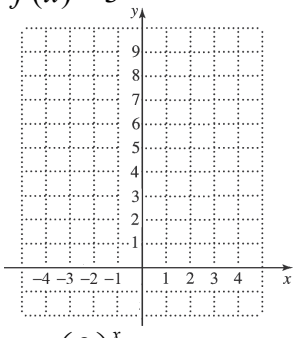
3. The graph of $f(x)=\left(\frac{1}{2}\right)^{(x+5)}$ looks just like the graph of $f(x)=\left(\frac{1}{2}\right)^x$, but it is translated 5 units to the _____.
left / right

4. The graph of $f(x)=\left(\frac{1}{2}\right)^{(x-5)}$ looks just like the graph of $f(x)=\left(\frac{1}{2}\right)^x$, but it is translated 5 units to the _____.
left / right

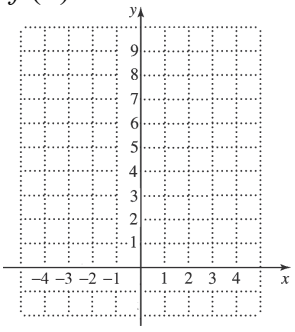
Graphing Exponential Functions

Graph.

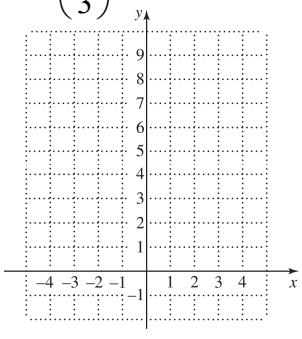
5. $f(x)=3^x$



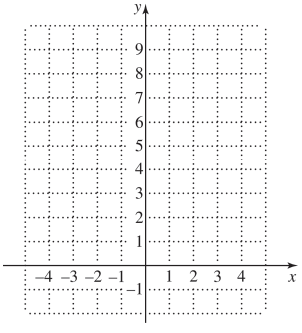
6. $f(x)=3^x+2$



7. $y=\left(\frac{2}{3}\right)^x$



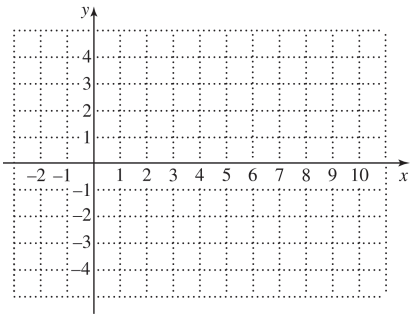
8. $y=2^{x+1}-2$



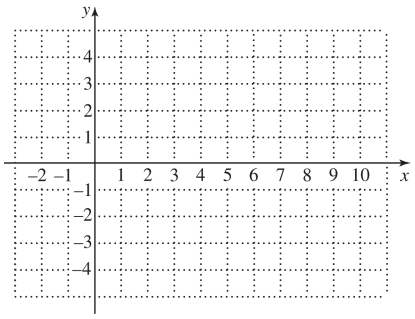
Equations with x and y Interchanged

Graph.

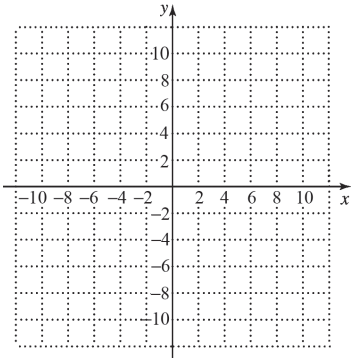
9. $x = 5^{-y}$



10. $x = \left(\frac{1}{3}\right)^{y}$



11. Graph the equations $y = 4^x$ and $x = 4^y$ on the same set of axes.

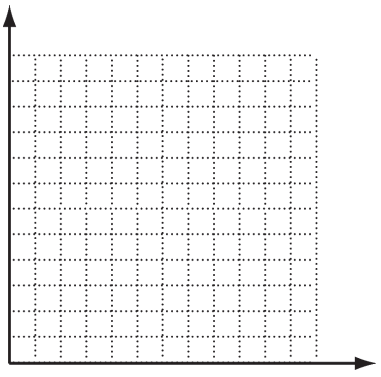


Applications of Exponential Functions

12. A laser printer is purchased for \$1400. Its value each year is about 60% of the value of the preceding year. Its value, in dollars, after t years is given by the exponential function $V(t) = 1400(0.6)^t$.

a) Find the value of the printer after 0 year, 1 year, 2 years, 5 years, and 10 years.

b) Graph the function.



Quiz 15

(1) Solve for x : $y = \sqrt{\frac{z}{x}}$

$$y^2 = \frac{z}{x} \Rightarrow y^2 x = z \Rightarrow \boxed{x = \frac{z}{y^2}}$$

(2) Rectangle with area 50, length 5 more than width. Find the length and the width

Let width be x. Then length is $x+5$.

$$x(x+5) = 50 \Rightarrow x^2 + 5x = 50 \Rightarrow x^2 + 5x - 50 = 0$$

$$\Rightarrow x^2 + 10x - 5x - 50 = 0 \Rightarrow x(x+10) - 5(x+10) = 0$$

$$\Rightarrow (x-5)(x+10) = 0 \Rightarrow x = 5 \text{ or } -10$$

-10 ← rejected

$\Rightarrow$ width $= 5$ and length $= 5 + 5 = 10$