

Learning objectives:

1. Increasing and decreasing functions, and their relation to derivative.
2. The first derivative test for local extremal values.
3. What is concavity and convexity?
4. What are inflection points?
5. The second derivative test.

What is meant by being increasing/decreasing?**Increasing/Decreasing Test**

1. If $f'(x) > 0$ on an interval, then f is increasing on that interval.
2. If $f'(x) < 0$ on an interval, then f is decreasing on that interval.

Example 1. Find where the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ is increasing and where it is decreasing.

The First Derivative Test

Suppose that c is a critical number of a continuous function f .

1. If f' changes sign from positive to negative at c , then f has a local max at c .
2. If f' changes sign from negative to positive at c , then f has a local min at c .
3. If f' does not change sign at c , then f has neither local max nor min at c .

Example 2. Find the local minimum and maximum values of the function f in Example 1.

Example 3. Find the local maximum and minimum values of $f(x) = \sin x + \cos x$, $0 \leq x \leq 2\pi$.

Example 4. Find the local maximum and minimum values of $f(x) = \frac{x^2}{x-1}$.

Concavity and Convexity

If the graph of f lies above all of its tangent lines on an interval I , then it is called concave upward or convex on I . If the graph of f lies below all of its tangent lines on I , it is called concave downward on I .

Concavity Test

1. If $f''(x) > 0$ on an interval I , then f is concave upward on I .
2. If $f''(x) < 0$ on an interval I , then f is concave downward on I .

Example 5. Find where the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ is concave upward and where it is concave downward.

Inflection Point

A point P on a curve $y = f(x)$ is called an inflection point if f is continuous there and the curve changes from concave upward to concave downward or from concave downward to concave upward at P .

Example 6. Find the inflection points of $f(x) = \sin x + \cos x$, $0 \leq x \leq 2\pi$.

The second derivative test

Suppose f'' is continuous near c .

1. If $f'(c) = 0$ and $f''(c) > 0$ then f has a local minimum at c .
2. If $f'(c) = 0$ and $f''(c) < 0$ then f has a local maximum at c .

Example 7. Let $y = x^4 - 4x^3$. Find the intervals of concavity, points of inflections, and local maximum and minimum points of the given curve. Use this information to sketch the curve.

Example 8. Sketch the graph of the function $f(x) = x^{2/3}(6 - x)^{1/3}$.

Example 9. Sketch the graph of the function $f(\theta) = 2 \cos \theta + \cos^2 \theta$, $0 \leq \theta \leq 2\pi$.