Increasing and Decreasing Functions:

1. A function f is increasing on an interval if, for any two numbers x_1 and x_2 in the interval,
$$x_1 < x_2 \text{ implies that } f(x_1) < f(x_2) \,.$$

2. A function f is decreasing on an interval if, for any two numbers x_1 and x_2 in the interval,
$$x_1 < x_2 \text{ implies that } f(x_1) > f(x_2) \,.$$

Example 1. Let $y = 1 - x^2$. Determine the intervals in which the function is increasing and decreasing.

Relative maximum and minimum:

1. A point is called a relative (or local) maximum if it has a larger y-value than any point near it.
2. A point is called a relative (or local) minimum if it has a smaller y-value than any point near it.
3. Either of maximum and minimum points/values are called extreme points/values.

Example 2. Determine the intervals on which the function $y = x^3 - 3x + 2$ is increasing and decreasing. From this information determine the maximum and minimum points.

Critical Points: A number c in the domain of f for which $f'(c) = 0$ is called a critical number of f. The points $(c, f(c))$ are called critical points.

First Derivative Test:

1. Find the critical numbers of f. Suppose c is a critical number.
2. Test the derivative with two values of x, one slightly less and the other slightly more than c.
   (a) If, as x increases, the sign of the derivative changes from $+$ to $-$, then $f(c)$ is a maximum value and $(c, f(c))$ is a maximum point.
   (b) If the sign changes from $-$ to $+$, then $f(c)$ is a minimum value.
   (c) If the sign does not change, then $(c, f(c))$ is neither a minimum nor a maximum.

Example 3. Test the function $f(x) = x^3$ for extreme values and sketch the graph.

Example 4. Find the extreme values of the function $y = (-2/3)x^3 + x^2 + 4x - 5$ and sketch the graph.