1.4 Introduction to Problem Solving

Five Steps for Problem Solving with Algebra
1. *Familiarize* yourself with the problem.
2. *Translate* to mathematical language.
3. *Carry out* some mathematical manipulation.
4. *Check* your possible answer in the original problem.
5. *State* the answer clearly.

Example: The sum of two numbers is 57. If one of the numbers is 6 more than twice the other, what are the numbers?

Let the numbers be x and y.

$$\begin{cases} x + y = 57 \quad \text{---} ① \\ x = 6 + 2y \quad \text{---} ② \end{cases}$$

Substitute the value of x (given) in ②, inside ①.

$$6 + 2y + y = 57 \Rightarrow 6 + 3y = 57 \Rightarrow 3y = 51 \Rightarrow y = \frac{51}{3}$$
$$\Rightarrow y = 17$$

Substitute y in ②, $x = 6 + 2 \times 17 = 6 + 34 = 40$

Example: The faculty discount at a bookstore is 15%. If a sweatshirt after the discount was $32.30. What was the original price of the sweatshirt?

Let the original price be x.

Discount $= 0.15 \times x = 0.15x$

$$x - 0.15x = 32.30 \Rightarrow (1 - 0.15)x = 32.30$$
$$\Rightarrow \frac{0.85}{0.85} x = \frac{32.30}{0.85} \Rightarrow x = \frac{32.30}{0.85} \Rightarrow \boxed{x = 38}$$

Example: One angle of a triangle has the same measure as a second angle. The third angle measures 10° more than three times the measure of the first angle. Find the measures of the angles.

Let the first angle be x degrees. 2nd angle $= x$

3rd angle $= 10 + 3x$. Angle Sum Property: The sum of the three angles of a triangle is 180°.

$x + x + 10 + 3x = 180$

$\Rightarrow 5x + 10 = 180 \Rightarrow 5x = 170$

$-10 \quad -10$

1st and 2nd angles

$\Rightarrow x = \frac{170}{5} = 34°$ • 3rd angle $= 10 + 3(34) = 112°$

You Try!

a. Cheyenne pays $157.94 for a cordless headset. If the price paid includes a 6% sales tax, what is the price of the headset itself?

b. In an effort to make their home more energy efficient, Jess and Drew Purchased 200in. of 3M Press-In-Place window glazing. This will be just enough to outline two square skylights. If the length of the sides of the larger skylight is 1.5 times the length of the sides of the smaller one, how should the glazing be cut?

c. Three numbers are such that the second is 6 less than three times the first, and the third is 2 more than two thirds the first. The sum of the three numbers is 150. Find the largest of the 3 numbers.

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Let the first number be x.

Then second number is $3x-6$ {6 less than 3 times the first}

Third number is $\frac{2}{3}x+2$ {2 more than $\frac{2}{3}$rd of first}

Sum of all three is 150.

$\Rightarrow x + 3x - 6 + \frac{2}{3}x + 2 = 150$

$\Rightarrow 4x + \frac{2}{3}x - 6 + 2 = 150$

$\Rightarrow \frac{12+2}{3}x - 4 = 150 \Rightarrow \frac{14x}{3} = 154$

$\Rightarrow x = \frac{3 \times 154^{11}}{\cancel{14}} = 33 \rightarrow$ first number

$\Rightarrow 2^{nd}$ number $= 3x-6 = 3 \times 33 - 6 = 93$

$\Rightarrow 3^{rd}$ number $= \frac{2}{3}x+2 = \frac{2}{3} \times 33 + 2 = 22+2 = 24$

So, the largest of the three numbers is 93.

A headset was sold at 212 dollars. If the price included a 6% sales tax, what was the price of headset itself.

Let the original price be x.

Sales Tax $= \frac{6}{100} \times x$

$\Rightarrow x + \frac{6}{100}x = 212$

$\Rightarrow \frac{106}{100}x = 212 \Rightarrow x = \frac{212 \times 100}{106} = 200$

Thus, price of headset itself was 200 dollars.

Sum of two numbers is 20. If one number is 2 more than half of the other, find the two numbers.

Let the two numbers be x and y.

Then $x + y = 20$.

$y = 2 + \frac{1}{2}x$ (2 more than half of the other)

$\Rightarrow x + 2 + \frac{1}{2}x = 20 \Rightarrow x + \frac{1}{2}x = 18$

$\Rightarrow \frac{3}{2}x = 18 \Rightarrow x = \frac{18 \times 2}{3}$

$\Rightarrow x = 12$

$\Rightarrow y = 2 + \frac{1}{2} \times 12 = 8$

Thus, the two numbers are 12 and 8.

1.5 Formulas, Models, and Geometry

- A formula is an equation that uses letters to represent a relationship between two or more quantities.
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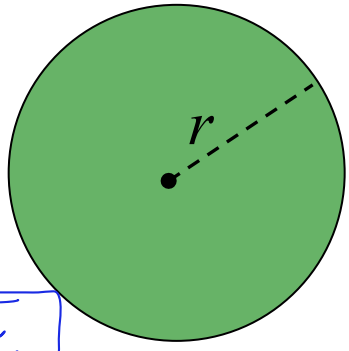
- The area of a circle of radius *r* is given by the formula:

$$A = \pi r^2$$

formula



→ Solve for the radius.

$\Rightarrow r^2 = \frac{A}{\pi} \Rightarrow \boxed{r = \sqrt{\frac{A}{\pi}}}$

$\frac{\pi r^2}{\pi} = r^2$

Solving Literal Equations/ Formulas

WRONG: $\frac{l+w+h}{w+h} = l$

Example: Solve $V = lwh$ for length l.

$lwh = V \Rightarrow \frac{lwh}{wh} = \frac{V}{wh} \Rightarrow \frac{l \times w \times h}{w \times h} = \frac{V}{wh}$

$\Rightarrow \boxed{l = \frac{V}{wh}}$

To Solve a Formula for a Specified Letter

1. Get all the terms with the specified letter on one side of the equation and all the other terms on the other side, using the addition principle. To do this may require removing parentheses.
   - To remove parentheses, either divide both sides by the multiplier in front of the parentheses or use the distributive law.
2. When all the terms with the specified letter are on the same side, factor (if necessary) so that the variable is written only once.
3. Solve for the letter in question by dividing both sides by the multiplier of that letter.

Example: Solve $\frac{x}{a} + \frac{y}{b} = 1$ for x.

$-\frac{y}{b} \quad -\frac{y}{b}$

$\Rightarrow \frac{x}{a} = \frac{1}{1} - \frac{y}{b} = \frac{1 \times b}{1 \times b} - \frac{y \times 1}{b \times 1} = \frac{b}{b} - \frac{y}{b} = \frac{b-y}{b}$

$\Rightarrow \frac{x}{a} = \frac{b-y}{b} \Rightarrow a \times \frac{x}{a} = a \times \frac{b-y}{b} \Rightarrow \boxed{x = \frac{a(b-y)}{b}}$

$\times a \quad \times a$

$\Rightarrow \boxed{x = \frac{ab - ay}{b}}$

Example: Solve $y + mx_2 = mx_1$ for x_2. $(m \neq 0)$

$-y \quad -y$

$$mx_2 = mx_1 - y \Rightarrow \frac{mx_2}{m} = \frac{mx_1 - y}{m}$$

$$\Rightarrow \boxed{x_2 = \frac{mx_1 - y}{m}}$$

You Try!

Solve the formula $a = l * w$ for l	Solve the formula $I = Prt$ for t

Formulas as models

Example: Oprah is 5 ft 7 in. tall and if she has a body mass index of approximately 23.5. What is her weight?

$$I = \frac{704.5W}{H^2}$$

I: BMI; W: weight; H: Height

$$\frac{704.5\,W}{H^2} = I \Rightarrow H^2 \times \frac{704.5\,W}{H^2} = H^2 \times I$$

$$\Rightarrow \frac{704.5\,W}{704.5} = \frac{H^2 \times I}{704.5} \Rightarrow W = \frac{H^2 \times I}{704.5}$$

5 ft 7 in

$\downarrow$

$12 \times 5 + 7$ in

67 in

$$\Rightarrow W = \frac{67^2 \times 23.5}{704.5}$$

1.6 Properties of Exponents

Product of Powers	$a^m \cdot a^n = a^{m+n}$
Power of a Power	$(a^m)^n = a^{mn}$
Power of a Product	$(ab)^m = a^m b^m$
Negative Exponents	$a^{-m} = \frac{1}{a^m}, a \neq 0$
Quotient of Powers	$\frac{a^m}{a^n} = a^{m-n}, a \neq 0$
Power of a Quotient	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$

$\frac{1}{8} = \frac{1}{2^3} = 2^{-3} = \left(\frac{1}{2}\right)^3 = \frac{1^3}{2^3}$

Example 1: Multiply and simplify

a. $m^5 * m^7$

$= m^{5+7} = m^{12}$

b. $(5ab^3)(3a^4b^5)$

$5 \times a \times b^3 \times 3 \times a^4 \times b^5$

$= 5 \times a \times 3 \times b^3 \times a^4 \times b^5$

$= 5 \times 3 \times a^1 \times a^4 \times b^3 \times b^5$

$= 15\, a^{1+4}\, b^{3+5} = 15\, a^5 b^8$

Example 2: Divide and simplify

a. $\frac{r^9}{r^3}$

$= r^{9-3} = r^6$

b. $\frac{-10x^{11}y^5}{-2x^4y^3}$

$\frac{-10^5}{-2} \times \frac{x^{11}}{x^4} \times \frac{y^5}{y^3} = 5x^{11-4}y^{5-3}$

$= 5x^7y^2$

ANYTHING TO THE ZERO POWER IS 1 $a \neq 0$ then $a^0 = 1$

$\frac{6^3}{6^3} = 1$ → $6^{3-3} = 1$

$6^0 = 1$

BECAUSE

Example 3: Evaluate each of the following for x= 2.9

a. x^0	b. $-x^0$	c. $(-x^0)$ $(-x)^0$
$= 2.9^0 = 1$	$= -2.9^0 = -1$	$= (-2.9)^0 = 1$

CAUTION : $x^{-2} \neq x^{1/2}$, $\frac{1}{x^{-2}} \neq x^{1/2}$

$\frac{1}{a^m} = a^m$ ← $a^{-m} = \frac{1}{a^m}$

Example 4: Express each of the following without negative exponents and, if possible simplify. FLIP THE BASE, CHANGE SIGN OF EXPONENT.

a. 7^{-2}	b. $5x^{-4}y^3$	c. $\frac{vx^{-2}y^{-5}}{z^{-4}w^{-3}}$
$= 7^{-2} = \frac{1}{7^2}$	$= 5 \times x^{-4} \times y^3$ $= 5 \times \frac{1}{x^4} \times y^3$ $= \frac{5y^3}{x^4}$	$= v \times x^{-2} \times y^{-5} \times \frac{1}{z^{-4}} \times \frac{1}{w^{-3}}$ $= v \times \frac{1}{x^2} \times \frac{1}{y^5} \times z^4 \times w^3$ $= \frac{vz^4w^3}{x^2y^5}$

Example 8: Simplify

a. $(3^4)^5$	b. $(y^{-5})^{-7}$	c. $(-3x^5y^{-1})^{-4}$
$= 3^{4 \times 5}$ $= 3^{20}$	$= y^{-5 \times (-7)}$ $= y^{35}$	$= (-3 \times x^5 \times y^{-1})^{-4}$ $= (-3)^{-4} \times (x^5)^{-4} \times (y^{-1})^{-4}$ $= \left(\frac{1}{-3}\right)^4 \times x^{-20} \times y^4$ $= \frac{1^4}{(-3)^4} \times \left(\frac{1}{x}\right)^{20} \times y^4$ $= \frac{1}{81} \times \frac{1}{x^{20}} \times y^4 = \frac{y^4}{81x^{20}}$

$(-3)^4 = (-3) \times (-3) \times (-3) \times (-3) = 9 \times 9 = 81$

$(a^m)^n = a^{mn}$, $(ab)^m = a^m b^m$

$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

Example 9: Simplify

a. $\left(\frac{x^2}{2}\right)^4$

$= \frac{(x^2)^4}{2^4} = \frac{x^{2\times 4}}{2\times 2\times 2\times 2}$

$= \frac{x^8}{16}$

b. $\left(\frac{y^2z^3}{5}\right)^{-3}$

$= \left(\frac{5}{y^2z^3}\right)^3 = \frac{5^3}{(y^2z^3)^3}$

$= \frac{125}{(y^2)^3 \times (z^3)^3} = \frac{125}{y^6z^9}$

(1) $8(a+1)^2 \div 5b - 4 + a$ for $a=4, b=2$

$8(4+1)^2 \div 5\times 2 - 4 + 4$

$8\times 5^2 \div 5\times 2 - 4 + 4$

$8\times 25 \div 5\times 2 - 4 + 4$

$200 \div 5\times 2 - 4 + 4$

$40 \times 2 - 4 + 4$

$80 - 4 + 4 = 76 + 4 = 80$

(2) 1 less than 50% of the diff. of two numbers.

x and y

$\frac{50}{100} \times (x-y) - 1 = 0.5(x-y) - 1$

(3) roster and set builder notation to represent all multiples of 3 between 1 and 14.

$\ldots, -6, -3, 0, 3, 6, 9, 12, 15, 18, \ldots$

$\longrightarrow \{3, 6, 9, 12\}$ → roster

$\longrightarrow \{x \mid x$ is a multiple of 3 b/w 1 and $14\}$

set builder

Quiz 2 [10 Pts total]

Name [1 Pt]

(1) $3x = 4(x+2) - 5x$ [4+1 Pts.]

Solve the above equation.

Find whether its an identity or conditional or contradiction.

(2) Sum of two numbers is 10. If one of the numbers x is 1 more than twice of the other, then find the two numbers. [4 Pts.]

(1) $3x = 4x + 8 - 5x$

$3x = -x + 8 \Rightarrow 4x = 8 \Rightarrow x = \frac{8}{4} \Rightarrow x = 2$

Conditional

(2) Let the two numbers be x and y.

$$\left.\begin{aligned} x + y &= 10 \\ x &= 1 + 2y \end{aligned}\right\} \Rightarrow 1 + 2y + y = 10 \Rightarrow 1 + 3y = 10$$

$$\Rightarrow 3y = 9 \Rightarrow y = \frac{9}{3}$$

$$\Rightarrow \boxed{y = 3}$$

$$x = 1 + 2 \times 3 = 7 \Rightarrow \boxed{x = 7}$$

(7, 3)