1) Evaluate $\int \frac{x^4}{x^2+1}\,dx$.

$$\int \frac{x^4}{x^2+1}\,dx = \int \frac{(x^4-1)+1}{x^2+1}\,dx$$

$$= \int \frac{x^4-1}{x^2+1}\,dx + \int \frac{1}{x^2+1}\,dx$$

$$= \int \frac{(x^2-1)\cancel{(x^2+1)}}{\cancel{(x^2+1)}} + \int \frac{1}{x^2+1}\,dx$$

$$= \int (x^2-1)\,dx + \arctan(x) + C$$

$$= \frac{x^3}{3} - x + \arctan(x) + C$$

2) Evaluate $\int \sec^3\theta\,d\theta$.

use integration by Parts.

$$\int \sec^3\theta\,d\theta = \int \underbrace{\sec\theta}_{u} \cdot \underbrace{\sec^2\theta\,d\theta}_{dv}$$

$u = \sec\theta \Rightarrow du = \sec\theta\tan\theta\,d\theta$

$dv = \sec^2\theta\,d\theta \Rightarrow v = \int \sec^2\theta\,d\theta$

$= \tan\theta$

$$\Rightarrow \int \sec^3\theta \, d\theta = \sec\theta \cdot \tan\theta - \int \tan\theta \cdot \sec\theta \tan\theta \, d\theta$$

$$= \sec\theta \tan\theta - \int \sec\theta \tan^2\theta \, d\theta$$

$$= \sec\theta \tan\theta - \int \sec\theta (\sec^2\theta - 1) \, d\theta$$

$$= \sec\theta \tan\theta - \int (\sec^3\theta - \sec\theta) \, d\theta$$

$$= \sec\theta \tan\theta - \int \sec^3\theta \, d\theta + \int \sec\theta \, d\theta$$

$$\int \sec^3\theta \, d\theta = \sec\theta \tan\theta - \int \sec^3\theta \, d\theta + \ln|\sec\theta + \tan\theta| + C$$

$$\Rightarrow 2\int \sec^3\theta \, d\theta = \sec\theta \tan\theta + \ln|\sec\theta + \tan\theta|$$

$$\Rightarrow \int \sec^3\theta \, d\theta = \frac{1}{2} \sec\theta \tan\theta + \frac{1}{2} \ln|\sec\theta + \tan\theta| + C$$

(3) Evaluate $\int \cot\theta \, d\theta$

$$\int \cot\theta \, d\theta = \int \frac{\cos\theta}{\sin\theta} \, d\theta = \int \underbrace{\frac{1}{\sin\theta}}_{u} \cdot \underbrace{\cos\theta \, d\theta}_{du}$$

Substitute $u = \sin\theta \Rightarrow du = \cos\theta \, d\theta$

$$\Rightarrow \int \cot\theta \, d\theta = \int \frac{1}{u} \, du = \ln|u| + C$$

$$= \ln|\sin\theta| + C$$