Learning objectives:

1. Chain rule and its use in computing derivatives.

The Chain Rule

If g is differentiable at x and f is differentiable at $g(x)$, then the composition $F = f \circ g$ is differentiable at x, and F' is given by

$$F'(x) = f'\big(g(x)\big)\, g'(x)\ .$$

In other words, if $y = f(u)$ and $u = g(x)$ are both differentiable functions, then

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}\ .$$

Example 1. Find $F'(x)$ if $F(x) = \sqrt{x^2 + 1}$.

Example 2. Differentiate

1. $y = \sin(x^2)$.
2. $y = \sin^2 x$.

The power rule combined with the chain rule

If n is any real number and $u = g(x)$, then

$$\frac{d}{dx}(u^n) = nu^{n-1}\frac{du}{dx}.$$

Alternatively,

$$\frac{d}{dx}[g(x)]^n = n[g(x)]^{n-1}\,g'(x).$$

Example 3. Differentiate $y = (x^2 - 1)^{100}$.

Example 4. Find $f'(x)$ if $f(x) = \dfrac{1}{\sqrt[3]{x^2 + x + 1}}$.

Example 5. Find the derivative of the function $g(t) = \left(\frac{t-2}{2t+1}\right)^9$.

Example 6. Differentiate $y = (2x+1)^5\,(x^3 - x + 1)^4$.

Example 7. If $f(x) = \sin(\cos(\tan x))$, then find $f'(x)$.

Example 8. Differentiate $y = \cos\sqrt{\sin(\tan \pi x)}$.

Example 9. Differentiate $y = [x + (x + \sin^2 x)^3]^4$.

Example 10. Differentiate $y = \sqrt{x + \sqrt{x + \sqrt{x}}}$.