The Derivative at a Point

The derivative of a function $f(x)$ at $x = a$, denoted by $f'(a)$, is defined to be the limit

$$f'(a) := \lim_{x \to a} \frac{f(x) - f(a)}{x - a}.$$

Problem 1 Evaluate the limit
$$\lim_{x \to 1} \frac{f(x) - f(1)}{x - 1}$$
for the following functions:-

1. $f(x) = x^2$

2. $f(x) = x^3$

3. $f(x) = \frac{1}{x}$

4. $f(x) = \frac{1}{x^2}$

5. $f(x) = \sqrt{x}$

Your answers would be the value of derivative of the given function f at $x = 1$.

The Derivative as a Function

The derivative of a function $f(x)$, denoted by $f'(x)$, is defined to be the function whose value at a given input x is the limit

$$f'(x) := \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Problem 2 Evaluate the limit
$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

for the following functions:-

1. $f(x) = x^2$

2. $f(x) = x^3$

3. $f(x) = \frac{1}{x}$

4. $f(x) = \frac{1}{x^2}$

5. $f(x) = \sqrt{x}$

Your answers would be the derivative of the given function f.

Try to see the pattern in your answers and find the derivative of $f(x) = x^n$.