Name: [1 pt]

Problem 1: Evaluate the indefinite integral $\int \frac{x+2}{\sqrt{x^2+4x}}\,dx$. [4 pts]

$$I = \int \frac{x+2}{\sqrt{x^2+4x}}\,dx.$$

Substitute $y = x^2+4x$

$$\Rightarrow dy = (2x+4)\,dx = 2(x+2)\,dx$$

$$\Rightarrow (x+2)\,dx = \frac{dy}{2}$$

$$\Rightarrow I = \int \frac{1}{\sqrt{y}}\,\frac{dy}{2} = \frac{1}{2}\int y^{-1/2}\,dy = \frac{1}{2}\times 2\sqrt{y} + C$$

$$= \sqrt{y} + C$$

$$\Rightarrow I = \sqrt{x^2+4x} + C$$

Problem 2: A variable force $F(x) = x + \sin x$ is acting along the x-axis. Find the work done in moving an object from $x = 0$ to $x = \pi$. [5 pts]

$$W = \int_0^\pi (x+\sin x)\,dx = \int_0^\pi x\,dx + \int_0^\pi \sin x\,dx$$

$$= \frac{x^2}{2}\Big|_0^\pi + (-\cos x)\Big|_0^\pi$$

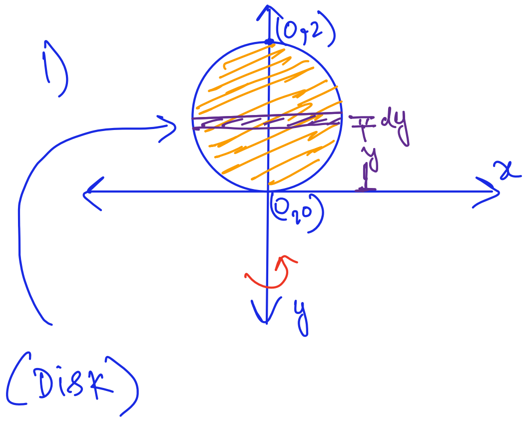
$$= \frac{\pi^2}{2} + \left[-\cos\pi - (-\cos 0)\right]$$

$$= \frac{\pi^2}{2} + \left[-(-1) - (-1)\right] = \frac{\pi^2}{2} + 2$$

Problem 3: The circle $x^2 + (y-1)^2 = 1$ is rotated about:-

1. y-axis. Find the volume of solid generated using disk-wahser method. [6 pts]
2. x-axis. Find the volume of solid generated using cylindrical shells method. [6 pts]

1)



$$V = \int_0^2 dV$$

$$dV = \pi r^2\, dy$$

$$r = x \Rightarrow r^2 = x^2 = 1-(y-1)^2 = 1-(y^2-2y+1) = 2y-y^2$$

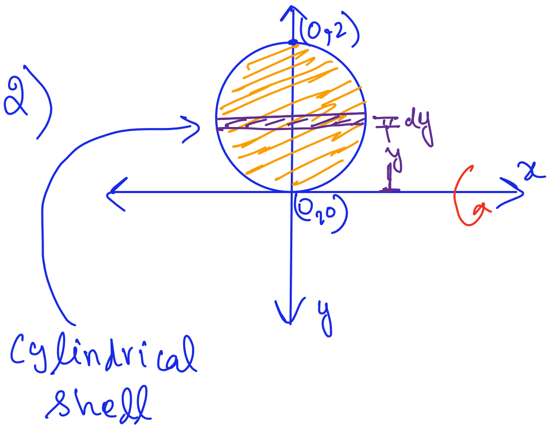
$$\Rightarrow V = \int_0^2 \pi(2y-y^2)\,dy = 2\pi\int_0^2 y\,dy - \pi\int_0^2 y^2\,dy$$

$$= 2\pi \left.\frac{y^2}{2}\right|_0^2 - \pi \left.\frac{y^3}{3}\right|_0^2$$

$$= 2\pi\left(\frac{2^2}{2}\right) - \pi\left(\frac{2^3}{3}\right) = 4\pi - \frac{8\pi}{3} = \frac{4\pi}{3}$$

$$\Rightarrow \boxed{V = \frac{4\pi}{3}}$$

2)



$$V = \int_0^2 dV$$

$$dV = 2\pi r h\, dy$$

$$r = y, \quad h = x_2 - x_1$$

where x_1, x_2 satisfy $x^2 + (y-1)^2 = 1$

$$\Rightarrow x^2 + y^2 - 2y + 1 = 1 \Rightarrow x^2 = 2y - y^2$$

$$\Rightarrow x = \pm\sqrt{2y-y^2} \Rightarrow h = x_2 - x_1 = 2\sqrt{2y-y^2}$$

$$\Rightarrow V = \int_0^2 2\pi y \times 2\sqrt{2y-y^2}\,dy = 4\pi\int_0^2 y\sqrt{2y-y^2}\,dy$$

Substitute $y = 1 + \sin\theta \Rightarrow dy = \cos\theta\, d\theta$

and $2y - y^2 = 2(1+\sin\theta) - (1+\sin\theta)^2 = 2 + 2\sin\theta - 1 - 2\sin\theta - \sin^2\theta$

$$= 1 - \sin^2\theta = \cos^2\theta$$

For $0 \le y \le 2$, $\sin\theta = y-1 \Rightarrow -1 \le \sin\theta \le 1$

$$\Rightarrow -\frac{\pi}{2} \le \theta \le \frac{\pi}{2} \Rightarrow \cos\theta > 0$$

$$\Rightarrow \sqrt{2y-y^2} = \sqrt{\cos^2\theta} = \cos\theta$$

$$\Rightarrow V = 4\pi \int_{-\pi/2}^{\pi/2} (1+\sin\theta)\cos\theta \cos\theta \, d\theta$$

$$= 4\pi \int_{-\pi/2}^{\pi/2} \underbrace{\cos^2\theta}_{\text{even function}} \, d\theta + 4\pi \int_{-\pi/2}^{\pi/2} \underbrace{\sin\theta \cos^2\theta}_{\text{an odd function}} \cdot d\theta$$

$$\Rightarrow V = 4\pi \int_0^{\pi/2} 2\cos^2\theta \, d\theta + 0$$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$\Rightarrow V = 4\pi \int_0^{\pi/2} 2\left(\frac{1+\cos 2\theta}{2}\right) d\theta = 4\pi \int_0^{\pi/2} (1+\cos 2\theta)\, d\theta$$

$$= 4\pi \int_0^{\pi/2} d\theta + 4\pi \int_0^{\pi/2} \cos 2\theta \, d\theta$$

$$= 4\pi \, \theta \Big|_0^{\pi/2} + 4\pi \frac{\sin 2\theta}{2} \Big|_0^{\pi/2} = 4\pi \times \frac{\pi}{2} + \frac{4\pi}{2}\left[\sin\pi - \sin 0\right]$$

$$= 2\pi^2 + \frac{4\pi}{2}[0-0]$$

$$\Rightarrow \boxed{V = 2\pi^2}$$