**Learning objectives**:

1. What are antiderivatives?
2. How to find antiderivatives of functions?
3. Applications to straight line motion.

### Antiderivative

A function $F$ is called an antiderivative of $f$ on an interval $I$ if $F'(x) = f(x)$ for all $x$ in $I$.

### Theorem

If $F$ is an antiderivative of $f$ on an interval $I$, then the most general antiderivative of $f$ on $I$ is

$$F(x) + c$$

where $c$ is an arbitrary constant.

**Example 1**. Find the most general antiderivatives of the following functions.

1. $f(x) = \sin x$.
2. $f(x) = x^2$.
3. $f(x) = x^{-3}$.

**Antiderivatives of sums and constant multiples**

1. If $F$ is an antiderivative of $f$ then $cF(x)$ is an antiderivative of $cf(x)$.
2. If $F$ and $G$ are antiderivative of $f$ and $g$ respectively then an antiderivative of $f(x) + g(x)$ is $F(x) + G(x)$.

**Antiderivatives of common functions**

| Function | Most general antiderivative |
|---|---|
| $x^n$ , $n \neq -1$ | $\dfrac{x^{n+1}}{n+1} + c$ |
| $\cos x$ | $\sin x + c$ |
| $\sin x$ | $-\cos x + c$ |
| $\sec^2 x$ | $\tan x + c$ |
| $\sec x \tan x$ | $\sec x + c$ |
| $\csc^2 x$ | $-\cot x + c$ |
| $\csc x \cot x$ | $-\csc x + c$ |

**Example 2**. Find the most general antiderivative of $g(x) = 4\sin x + \dfrac{2x^5 - \sqrt{x}}{x}$.

**Example 3**. Find $f$ if $f'(x) = x\sqrt{x}$ and $f(1) = 2$.

**Example 4**. Find $f$ if $f''(x) = 12x^2 + 6x - 4$, $f(0) = 4$ and $f(1) = 1$.

**Example 5**. A particle moves in a straight line and has acceleration given by $a(t) = (6t + 4)$ cm/s$^2$. Its initial velocity is $v(0) = -6$ cm/s and its initial displacement is $s(0) = 9$ cm. Find its position function $s(t)$.

**Example 6**. A ball is thrown upward with a speed of 48 ft/s from the edge of a cliff 432 ft above the ground. Find its height above the ground $t$ second later. When does it reach its maximum height? When does it hit the ground? Use the value of acceleration due to gravity to be $-32$ ft/s$^2$.