■ **Section 7.5** exercises, *page 547*: 1, 3, 5, 7, 9, 11, 13, 15, 21, 20, 2, 4, 6, 12, 16, 18, 37, 38, 8, 14, 17, 26, .

As we have seen, integration is more challenging than differentiation. No hard and fast rules can be given as to which integration method applies in a given situation, but you can think about these steps as a guideline.

- Do we need to use algebra or trigonometric identities to **rewrite** the integrand so that we can apply basic integration formulas?
- What about an obvious u-**substitution**?
- If the integrand is a *rational function* but the above two steps couldn't solve the integral, think about **integration by partial fractions** (section 7.4).
- If the integrand is a *product* of a polynomial with a transcendental function (such as a trigonometric function, exponential, or logarithmic function), then you can try **integration by parts**.
- If the integrand involves radicals couldn't be solved by an obvious u-sub, you can think about using **trigonometric substitution** (section 7.3).
- Try again.

Obviously, the first step of integration is to remember basic integral formulas. See next page for the **Table of Integration Formulas**.

Table of Integration Formulas

$$\int x^n\,dx = \left(\frac{1}{n+1}\right)x^{n+1} + C \quad (n \neq -1)$$

$$\int \sin(x)\,dx = -\cos(x) + C$$

$$\int \cos(x)\,dx = \sin(x) + C$$

$$\int \sec^2(x)\,dx = \tan(x) + C$$

$$\int \csc^2(x)\,dx = -\cot(x) + C$$

$$\int \sec(x)\tan(x)\,dx = \sec(x) + C$$

$$\int \csc(x)\cot(x)\,dx = -\csc(x) + C$$

$$\int e^x\,dx = e^x + C$$

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{x^2 + a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \sec(x)\,dx = \ln|\sec x + \tan x| + C$$

$$\int \tan(x)\,dx = \ln|\sec x| + C$$

$$\int \sinh x\,dx = \cosh x + C$$

$$\int \cosh x\,dx = \sinh x + C$$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}(x) + C$$

$$\int \frac{1}{\sqrt{5-x^2}}\,dx = \sin^{-1}\left(\frac{x}{\sqrt{5}}\right) + C$$

$a^2 = 5 \Rightarrow a = \sqrt{5}$

Example

$$\int \frac{1}{x^2-1}\,dx$$

$$a^2-b^2=(a-b)(a+b)$$
$$\downarrow \quad \downarrow$$
$$x^2 \quad 1^2$$

$$\frac{1}{x^2-1} = \frac{1}{(x-1)(x+1)} = \frac{a}{x-1} + \frac{b}{x+1} \Bigg] \times (x-1)(x+1)$$

$$1 = a(x+1) + b(x-1)$$

$$x=1 \Rightarrow 1 = a(1+1) \Rightarrow a = \frac{1}{2}$$

$$x=-1 \Rightarrow 1 = b(-1-1) \Rightarrow b = \frac{-1}{2}$$

$$\frac{1}{x^2-1} = \frac{1}{2}\,\frac{1}{x-1} - \frac{1}{2}\,\frac{1}{x+1}$$

$$\int \frac{1}{x^2-1}\,dx = \frac{1}{2}\int \frac{1}{x-1}\,dx - \frac{1}{2}\int \frac{1}{x+1}\,dx$$

$$= \frac{1}{2}\ln|x-1| - \frac{1}{2}\ln|x+1| + C$$

$$= \frac{1}{2}\Big[\ln|x-1| - \ln|x+1|\Big] + C$$

$$= \frac{1}{2}\ln\frac{|x-1|}{|x+1|} + C$$

$$\int \frac{1}{x^2-1}\,dx = \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| + C$$

Q

$$\int \frac{1}{x^2-4}\,dx = \frac{1}{2(2)}\ln\left|\frac{x-2}{x+2}\right| + C$$

$\underline{\underline{\theta}}$ $$\int \frac{1}{x^2-a^2}\,dx = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C$$

$$\frac{1}{x^2-a^2} = \frac{1}{2a}\left[\frac{1}{x-a} - \frac{1}{x+a}\right]$$

$$\int \frac{1}{x^2-7}\,dx = \frac{1}{2\sqrt{7}}\ln\left|\frac{x-\sqrt{7}}{x+\sqrt{7}}\right| + C$$

$\underline{\underline{\theta}}$ $$\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + C$$

Done

$x = a\sin\theta$

$\underline{\underline{\theta}}$ $$\int \sqrt{a^2+x^2}\,dx = \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2}\ln\left|x+\sqrt{a^2+x^2}\right| + C$$

$x = a\tan\theta$

HW

$\underline{\underline{\theta}}$ $$\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\ln\left|x+\sqrt{x^2-a^2}\right| + C$$

$x = a\sec\theta$

$\underline{\underline{\theta}}$ $$\int \frac{1}{\sqrt{a^2+x^2}}\,dx = \ln\left|x+\sqrt{a^2+x^2}\right| + C$$

$\underline{\underline{\theta}}$ $$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left|x+\sqrt{x^2-a^2}\right| + C$$

HW.
use Trigo. substitution