**Improper integrals of type 1**: When the upper or lower or both limits of the integral are infinite. In such cases we have

$$\int_a^{\infty} f(x)\,dx = \lim_{b\to\infty} \int_a^b f(x)\,dx\,.$$

$$\int_{-\infty}^{b} f(x)\,dx = \lim_{a\to-\infty} \int_a^b f(x)\,dx\,.$$

If the limit converges to some finite number we say the given improper **integral converges**, otherwise we say the given improper **integral diverges**.

When both limits are infinite we choose some point $c$ on the real line and write the integral as a sum of two improper integrals. Then we have

$$\int_{-\infty}^{\infty} f(x)\,dx = \lim_{a\to-\infty} \int_a^c f(x)\,dx + \lim_{b\to\infty} \int_c^b f(x)\,dx\,.$$

Such an integral is said to converge if both the limits in the sum converge. Otherwise, we say the improper integral diverges.

**Example 1**. Evaluate the integral $\displaystyle\int_1^{\infty} \frac{dx}{x^2}$ .

**Example 2**. Evaluate the integral $\displaystyle\int_0^\infty \frac{dx}{(x+2)^{3/2}}$ .

**Example 3**. Evaluate the integral $\displaystyle\int_{-\infty}^1 \frac{dx}{(3-x)^{5/3}}$ .

**Example 4**. Find the area bounded by the curve $y = 1/\sqrt{x}$, $x = 1$ and the coordinate axes.

**Example 5**. Evaluate the integral $\displaystyle\int_1^\infty \frac{dx}{\sqrt{x}}$.

**Improper integrals of type 2**. When the integrand is discontinuous at some point in the interval of integration.

Suppose $f(x)$ is discontinuous at $x = a$. Then

$$\int_a^b f(x)\,dx = \lim_{t \to a^+} \int_t^b f(x)\,dx\,.$$

Suppose $f(x)$ is discontinuous at $x = b$. Then

$$\int_a^b f(x)\,dx = \lim_{t \to b^-} \int_a^t f(x)\,dx\,.$$

Suppose $f(x)$ is discontinuous at $x = c$ where $a < c < b$. Then

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx = \lim_{t \to c^-} \int_a^t f(x)\,dx + \lim_{t \to c^+} \int_t^b f(x)\,dx$$

**Example 6**. Evaluate the integral $\displaystyle\int_{-1}^{2} \frac{dx}{(x-1)^2}\,dx$ .