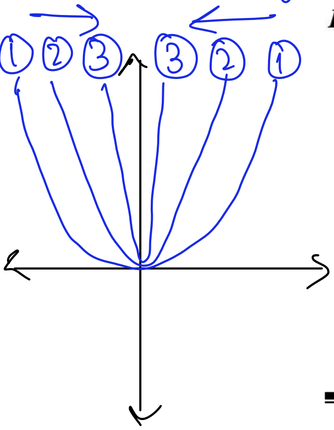
The Graph of $f(x) = ax^2$

ESSENTIALS

The graph of $f(x) = ax^2$ is a parabola with $x = 0$ as its axis of symmetry and vertex at $(0,0)$.

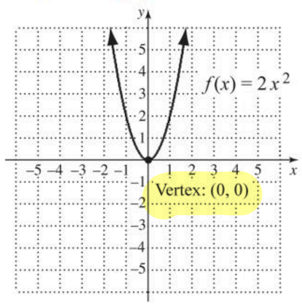
For $a > 0$, the parabola opens upward, and for $a < 0$ it opens downward.

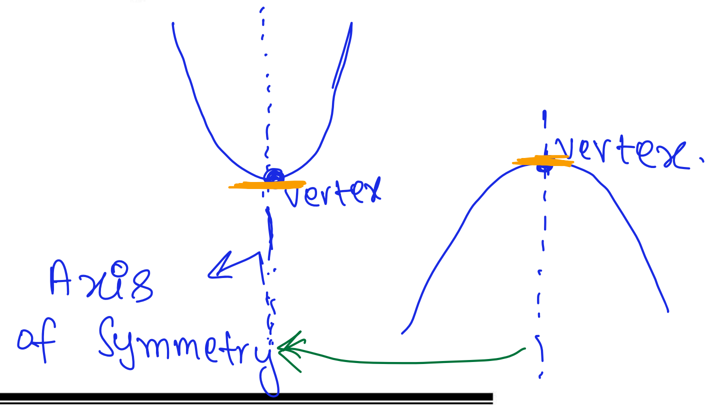
a increasing



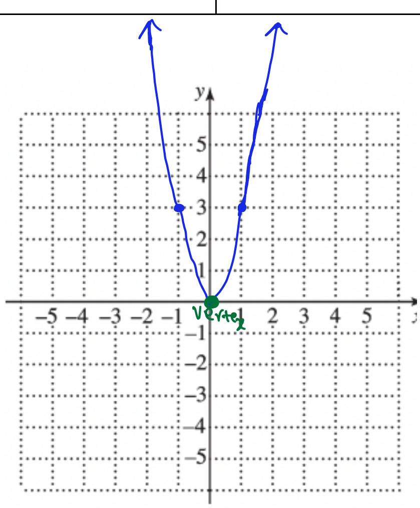
Example

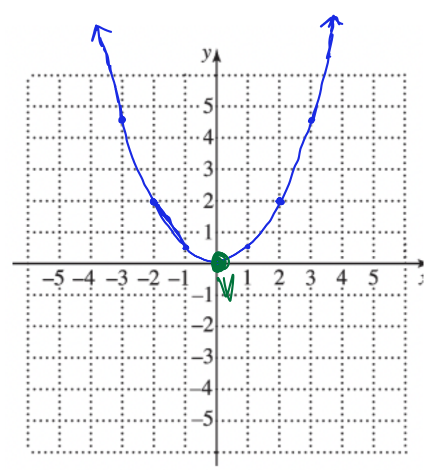
- Graph: $f(x) = 2x^2$.





Graph: $f(x) = 3x^2$	Graph: $f(x) = \frac{1}{2}x^2$
1. Find Vertex: $(0,0)$	1. Find Vertex: $(0,0)$
2. Use Rule of 1,3,5,	2. Use Rule of 1,3,5,





$x=1 \Rightarrow y = 3x^2 = 3(1)^2 = 3$

$x=2 \Rightarrow y = 3(2)^2 = 12$

$x=0 \Rightarrow y = 3(0)^2 = 0$

$x=-1 \Rightarrow y = 3(-1)^2 = 3$

$x=-2 \Rightarrow y = 3(-2)^2 = 12$

$x=1 \Rightarrow y = \frac{1}{2}(1)^2 = 0.5 \leftarrow x=-1$

$x=2 \Rightarrow y = \frac{1}{2}(2)^2 = 2 \leftarrow x=-2$

$x=3 \Rightarrow y = \frac{1}{2}(3)^2 = \frac{9}{2} = 4.5$

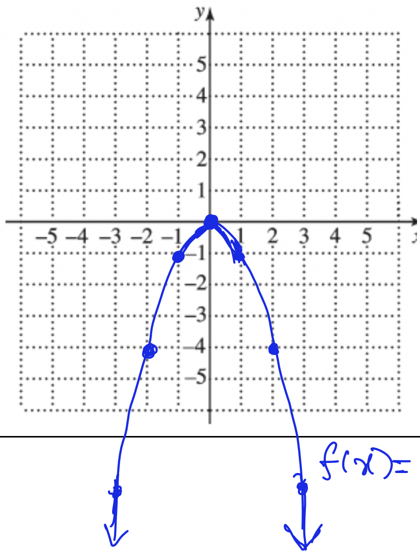
$x=0 \Rightarrow y = \frac{1}{2}(0)^2 = 0$

$x=-3$

$x = 1 \Rightarrow y = -(1)^2 = -1$

Graph: $f(x) = -x^2$

1. Find Vertex	2. ~~Use Rule of 1,3,5,~~
$(0, 0)$	$x = -1$ $y = -(-1)^2 = -1$

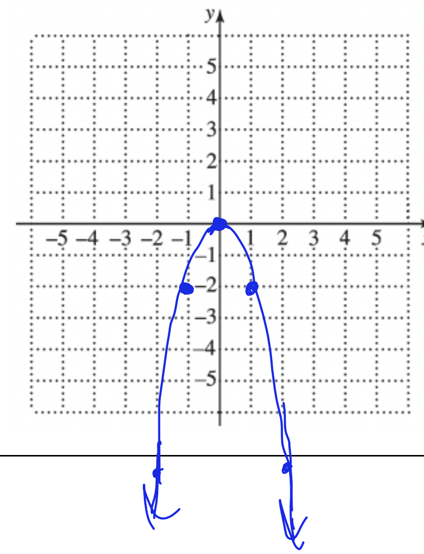


$f(x) = -x^2$

Graph: $f(x) = -2x^2$

$f(0) = 0$

1. Find Vertex	2. ~~Use Rule of 1,3,5,~~
$(0, 0)$	$f(1) = -2 = f(-1)$ $f(2) = -8 = f(-2)$



The Graph of $f(x) = a(x-h)^2$

(*) Vertex of $f(x) = ax^2$ is $(0,0)$

(*) Axis of Symmetry of $f(x) = ax^2$ is $x = 0$

ESSENTIALS

The graph of $f(x) = a(x-h)^2$ has the same shape as the graph of $y = ax^2$.

If h is positive, the graph of $y = ax^2$ is shifted h units to the right.

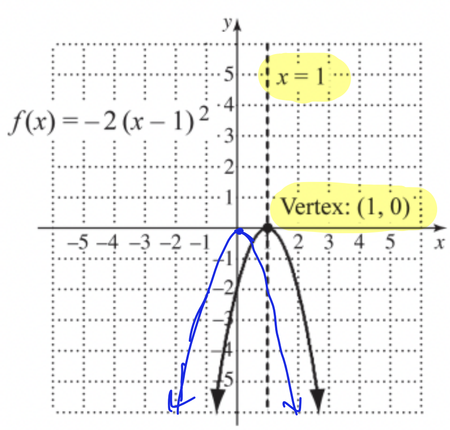
If h is negative, the graph of $y = ax^2$ is shifted $|h|$ units to the left.

The vertex is $(h, 0)$ and the axis of symmetry is $x = h$.

Example

- Graph: $f(x) = -2(x-1)^2$.

$h = 1$



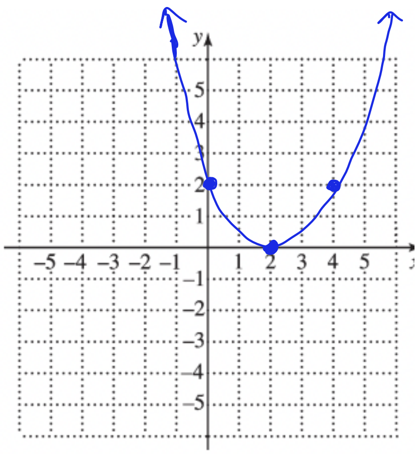
$f(x) = -2x^2$

$f(x) = -3(x+2)^2$
$= -3(x-(-2))^2$
$h = -2$

$x - h = x - 2 \Rightarrow -h = -2 \Rightarrow h = 2$

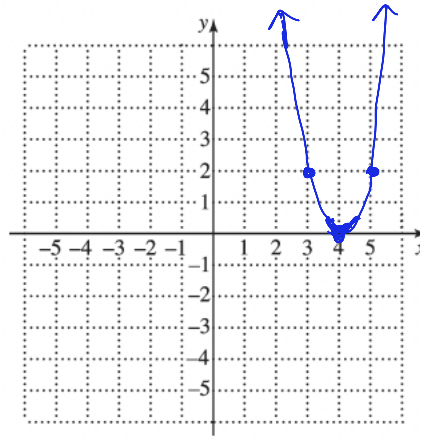
Graph: $f(x) = \frac{1}{2}(x-2)^2$ $h = 2$

1. Find Vertex	2. Use Rule of 1,3,5,
$(2, 0)$ $x = 2$	



Graph: $f(x) = 2(x-4)^2$ $h = 4$

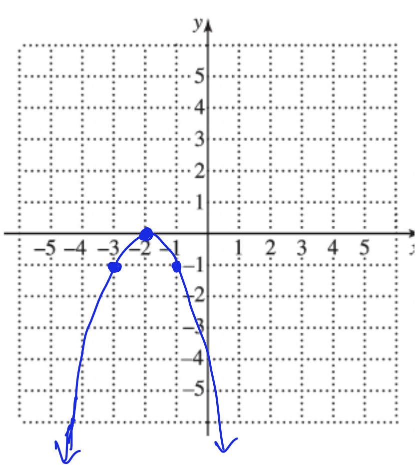
1. Find Vertex	2. Use Rule of 1,3,5,
$(4, 0)$ $x = 4$	



$x - h = x + 2 \Rightarrow -h = 2 \Rightarrow h = -2$

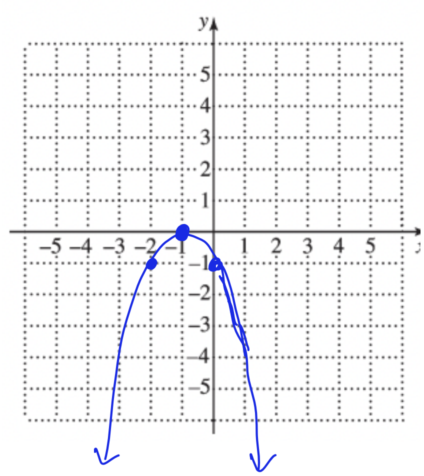
Graph: $f(x) = -(x+2)^2$ $h = -2$

3. Find Vertex	4. Use Rule of 1,3,5,
$(-2, 0)$ $x = -2$	



Graph: $f(x) = -(x+1)^2$ $h = -1$

3. Find Vertex	4. Use Rule of 1,3,5,
$(-1, 0)$ $x = -1$	



The Graph of $f(x) = a(x-h)^2 + k$

ESSENTIALS

The graph of $f(x) = a(x-h)^2 + k$ has the same shape as the graph of $y = a(x-h)^2$.

If k is positive, the graph of $y = a(x-h)^2$ is shifted k units up.

If k is negative, the graph of $y = a(x-h)^2$ is shifted $|k|$ units down.

The vertex is (h, k) and the axis of symmetry is $x = h$.

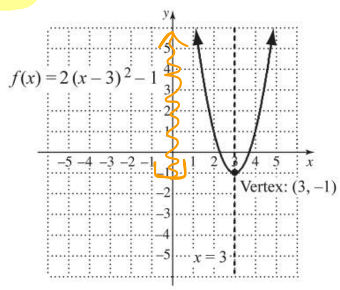
The domain of f is $(-\infty, \infty)$.

For $a > 0$, the minimum value is k, and the range is $[k, \infty)$. $y \geq k$ For $a < 0$, the maximum value is k, and the range is $(-\infty, k]$. $y \leq k$

Example

- Graph: $f(x) = 2(x-3)^2 - 1$.

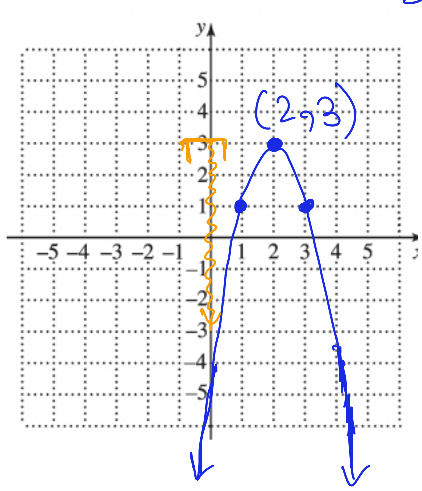
$a = 2$
$h = 3$
$k = -1$

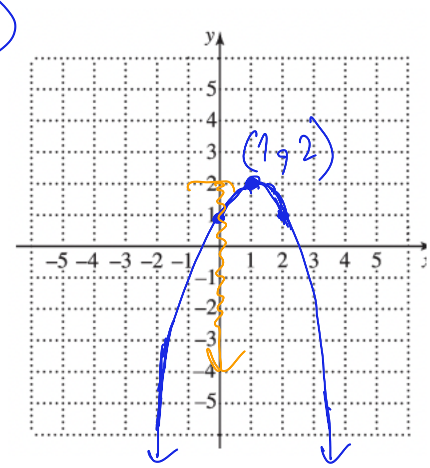


Range $= [-1, \infty)$

Graph: $f(x) = -2(x-2)^2 + 3$		Graph: $f(x) = -(x-1)^2 + 2$	
5. Find Vertex	6. ~~Use Rule of 1,3,5,~~	5. Find Vertex	6. ~~Use Rule of 1,3,5,~~
$(2, 3)$ $x = 2$	$a = -2$ $h = 2$ $k = 3$	$(1, 2)$ $x = 1$	$a = -1$ $h = 1$ $k = 2$

Domain $= (-\infty, \infty)$





Range $= (-\infty, 3]$

Range $= (-\infty, 2]$

Math 110- 8.7 Notes

Graphing $f(x) = ax^2 + bx + c$

ESSENTIALS

The graph of $f(x) = ax^2 + bx + c$ is a parabola.

Vertex: $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$, or $\left(-\frac{b}{2a}, \frac{4ac-b^2}{4a}\right)$

The x-coordinate of the vertex is $-\frac{b}{2a}$, and the axis of symmetry is $x = -\frac{b}{2a}$.

The second coordinate of the vertex is most commonly found by computing $f\left(-\frac{b}{2a}\right)$.

It is the maximum or minimum function value.

$f(x) = ax^2 + bx + c$ can be converted to the form $f(x) = a(x-h)^2 + k$ by completing the square.

Graph $f(x) = x^2 - 8x + 18$

1. Find Vertex

$\frac{-b}{2a} = \frac{-(-8)}{2(1)} = 4$

$f(4) = 4^2 - 8 \times 4 + 18$
$= 16 - 32 + 18$
$= 2$

$(4, 2)$

2. ~~Use Rule of 1,3,5~~

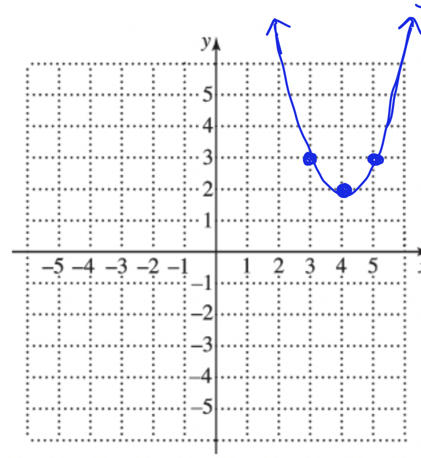
$a = 1$
$b = -8$
$c = 18$

Vertex: $(4, 2)$

Axis of Symmetry: $x = 4$

$f(5) = 5^2 - 8 \times 5 + 18 = 25 - 40 + 18$
$= 3$

$f(3) = 3^2 - 8 \times 3 + 18 = 9 - 24 + 18$
$= 3$



Domain $= (-\infty, \infty)$

Range $= [2, \infty)$

Graph $f(x) = -2x^2 - 6x + 1$

1. Find Vertex	2. ~~Use Rule of 1,3,5~~
$\frac{-b}{2a} = \frac{-(-6)}{2(-2)} = \frac{6}{-4} = -\frac{3}{2}$ $f\left(\frac{-3}{2}\right) = -2\left(\frac{-3}{2}\right)^2 - 6\left(\frac{-3}{2}\right) + 1$ $= -2 \times \frac{9}{4} + \frac{18}{2} + 1$	$a = -2$ $b = -6$ $c = 1$

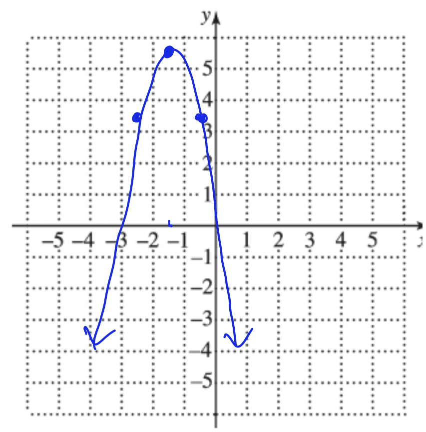
$= \frac{-9}{2} + 9 + 1 = \frac{9}{2} + 1 = \frac{11}{2}$

Vertex: $\left(\frac{-3}{2}, \frac{11}{2}\right)$

Axis of Symmetry: $x = \frac{-3}{2}$

Domain $= (-\infty, \infty)$

Range $= \left(-\infty, \frac{11}{2}\right]$

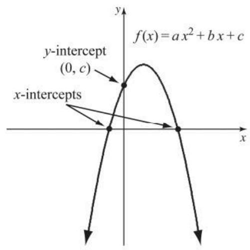


Finding Intercepts

ESSENTIALS

The *y*-intercept of the graph of $f(x)=ax^2+bx+c$ is $(0, f(0))$, or $(0, c)$.

To find the *x*-intercepts of the graph of $f(x)=ax^2+bx+c$, solve $f(x)=0$.



Example

- Find any *x*-intercepts and the *y*-intercept of $f(x)=x^2-x-6$.

  $f(0)=0^2-0-6=-6$, so the *y*-intercept is $(0,-6)$.

  Solving $f(x)=0$, we have

  $$x^2-x-6=0$$
  $$(x-3)(x+2)=0$$
  $$x=3 \text{ or } x=-2.$$

  The *x*-intercepts are $(3,0)$ and $(-2,0)$.

$0^2-6(0)+2=2$

Find any x-intercepts and the y-intercepts of the graph of $f(x) = x^2 - 6x + 2$

y-intercept : $(0, 2)$

$x^2-6x+2=0$

$x = \dfrac{-(-6)\pm\sqrt{(-6)^2-4\times 2}}{2(1)}$

$x = \dfrac{6\pm\sqrt{36-8}}{2} = \dfrac{6\pm\sqrt{28}}{2}$

$\sqrt{28} = \sqrt{4\times 7} = \sqrt{4}\sqrt{7} = 2\sqrt{7}$

$x = \dfrac{6\pm 2\sqrt{7}}{2} = \dfrac{2(3\pm\sqrt{7})}{2}$

$x = 3\pm\sqrt{7}$

x-intercepts : $(3+\sqrt{7}, 0)$
$(3-\sqrt{7}, 0)$

Find any x-intercepts and the y-intercepts of the graph of $f(x) = x^2 - 3x - 4$

y-intercept : $(0, -4)$

$x^2-3x-4=0$

$-4 = 1x-4$

$x^2+x-4x-4=0$

$x(x+1)-4(x+1)=0$

$(x-4)(x+1)=0$

$x-4=0$ or $x+1=0$

$x=4$ or $x=-1$

x-intercepts : $(4, 0)$
$(-1, 0)$

$2 = -1x - 2$
$= -2x - 1$

$b^2-4ac = (-3)^2-4(1)(-4) = 9+16=25$

Quiz 14

① $x^2 - 4x + 2 = 0$

$ax^2 + bx + c = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a = 1,\ b = -4,\ c = 2$

$x = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 1 \times 2}}{2(1)} = \dfrac{4 \pm \sqrt{16 - 8}}{2}$

$= \dfrac{4 \pm \sqrt{8}}{2} = \dfrac{4 \pm 2\sqrt{2}}{2}$

$= \dfrac{2(2 \pm \sqrt{2})}{2} = \boxed{2 \pm \sqrt{2}}$

$* \sqrt{8} = \sqrt{4 \times 2} = \sqrt{4}\sqrt{2}$
$= 2\sqrt{2}$

$* \dfrac{2\sqrt{2}}{2} = \sqrt{2}$

$* \sqrt{2} = 1.414$

$* \dfrac{4 \pm 2\sqrt{2}}{2} \neq 2 \pm 2\sqrt{2}$

② $x^2 - 2x + 2 = 0$

$a = 1,\ b = -2,\ c = 2$

$x = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4 \times 1 \times 2}}{2(1)} = \dfrac{2 \pm \sqrt{4 - 8}}{2} = \dfrac{2 \pm \sqrt{-4}}{2}$

$x = \dfrac{2 \pm 2i}{2} = \dfrac{2(1 \pm i)}{2}$

$\Rightarrow x = 1 \pm i$

$* \sqrt{-4} = \sqrt{4}\sqrt{-1}$
$= 2i$

$* 1 + i \neq 2i$
$\neq 2$

$* \dfrac{2i}{2} = i$