

Intersections of Sets and Conjunctions of Sentences

ESSENTIALS

The **intersection** of two sets A and B is the set of all elements that are common to both A and B . The word "and" corresponds to "intersection" and to the symbol " \cap ". When two or more sentences are joined by the word *and*, we have a **conjunction**.

Examples

- Find the intersection of $\{2, 3, 5, 7\}$ and $\{1, 2, 3, 4, 5\}$.

The numbers 2, 3, and 5 are common to both sets, so the intersection is $\{2, 3, 5\}$.

- Solve: $-2 \leq 3x + 4 \leq 7$.

$$-2 \leq 3x + 4 \text{ and } 3x + 4 \leq 7$$

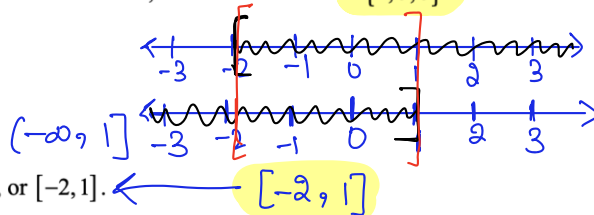
$$-6 \leq 3x \text{ and } 3x \leq 3$$

$$-2 \leq x \text{ and } x \leq 1$$

The solution set is $\{x | -2 \leq x \leq 1\}$, or $[-2, 1]$.

$$[-2, \infty)$$

$$x \geq -2$$



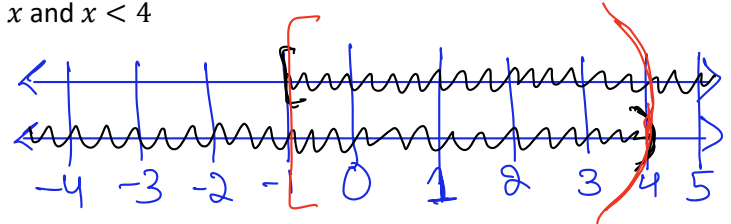
Find the intersection $\{-2, 0, 2, 4\} \cap \{2, 4, 6, 8\}$

$$\{2, 4\}$$

Graph and write interval notation for the conjunction $-1 \leq x$ and $x < 4$

$$x \geq -1 \text{ and } x < 4$$

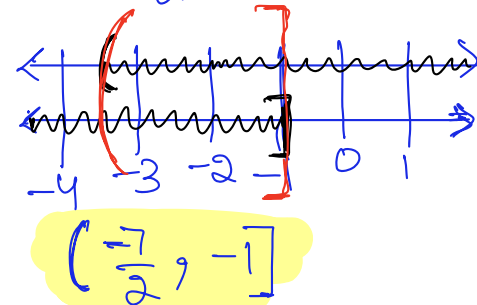
$$[-1, 4)$$



Example: Solve and write the solution interval notation $-4 < 2x + 3 \leq 1$

$$\begin{aligned} -4 < 2x + 3 & \text{ and } 2x + 3 \leq 1 \\ -4 - 2x < 3 & \Rightarrow 2x \leq 1 - 3 \\ -2x < 3 + 4 & \Rightarrow 2x \leq -2 \Rightarrow x \leq -1 \\ -2x < 7 \Rightarrow x > \frac{7}{-2} \Rightarrow x > -\frac{7}{2} \end{aligned}$$

$$x > -\frac{7}{2} \text{ and } x \leq -1$$



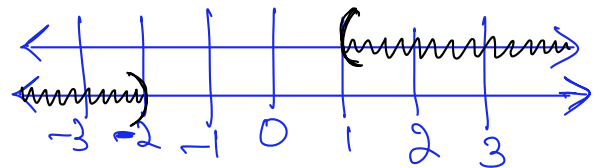
Solve, Graph and write solution in interval notation $-4x + 3 < -1$ and $15x < -30$

$$-4x + 3 < -1 \text{ and } 15x < -30$$

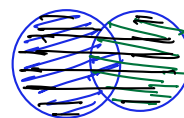
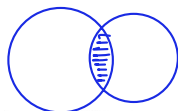
$$-4x < -1 - 3 \Rightarrow x < \frac{-30}{15}$$

$$-4x < -4 \Rightarrow x < -1$$

$$\frac{-4x}{-4} > \frac{-4}{-4} \Rightarrow x > 1$$



NO SOLUTION



Unions of Sets and Disjunctions of Sentences

ESSENTIALS

The **union** of two sets A and B is the collection of elements belonging to A and/or B .
The word "or" corresponds to "union" and to the symbol " \cup ".

When two or more sentences are joined by *or*, we have a **disjunction**. Any solution of a disjunction must make at least one part of the disjunction true.

Examples

- Find the union of $\{2, 3, 5, 7\}$ and $\{1, 2, 3, 4, 5\}$.

The numbers 1, 2, 3, 4, 5, and 7 are in either or both sets so the union is $\{1, 2, 3, 4, 5, 7\}$.

$$\{2, 3, 5, 7, 1, 4\}$$

- Solve: $3x + 4 \leq -2$ or $3x + 4 > 7$.

$$3x + 4 \leq -2 \quad \text{or} \quad 3x + 4 > 7$$

$$3x \leq -6 \quad \text{or} \quad 3x > 3$$

$$x \leq -2 \quad \text{or} \quad x > 1$$

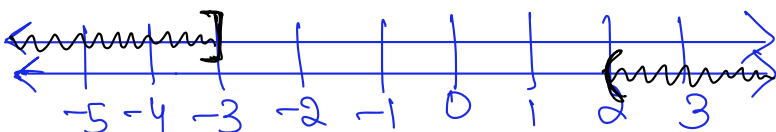


The solution set is $\{x | x \leq -2 \text{ or } x > 1\}$, or $(-\infty, -2] \cup (1, \infty)$.

Find the union of the set $\{a, e, i, o, u, y\} \cup \{b, a, y\}$

$$\{a, e, i, o, u, y, b\}$$

Graph and write interval notation for the disjunction $x \leq -3$ or $x > 2$



$$(-\infty, -3] \cup (2, \infty)$$

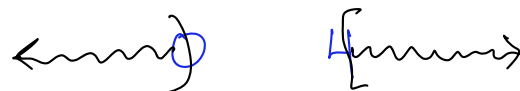
Solve and write the solution in interval notation $3 - 2x > 3$ or $x + 4 \geq 8$

$$3 - 2x > 3 \quad \text{or} \quad x + 4 \geq 8$$

$$-2x > 3 - 3 \quad x > 8 - 4 \Rightarrow x \geq 4$$

$$-2x > 0$$

$$\frac{-2x}{-2} < \frac{0}{-2} \Rightarrow x < 0$$



$$(-\infty, 0) \cup [4, \infty)$$

Solve, graph, and write the solution in interval notation $-3x + 1 < 7$ or $x + 2 > 3$

$$-3x + 1 < 7 \quad \text{or} \quad x + 2 > 3$$

$$-3x < 7 - 1$$

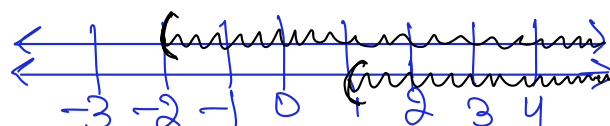
$$-3x < 6$$

$$\frac{-3x}{-3} > \frac{6}{-3}$$

$$x > -2$$

$$x > 3 - 2$$

$$x > 1$$



$$(-2, \infty)$$

$$\{a, a, b\} = \{a, b\}$$

Interval Notation and Domain

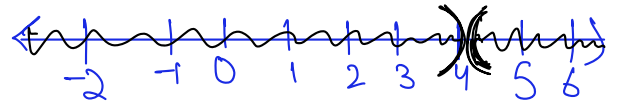
Use Interval notation to write the domain of the function

$$g(x) = \frac{8}{x-4}$$

$$Dg = \{x \mid x \neq 4\}$$

$$= (-\infty, 4) \cup (4, \infty)$$

$$x-4 \neq 0 \Rightarrow x \neq 4$$



$$g(x) = -\frac{2}{x+3}$$

$$Dg = \{x \mid x \neq -3\}$$

$$= (-\infty, -3) \cup (-3, \infty)$$

$$x+3 \neq 0 \Rightarrow x \neq -3$$

$$f(x) = \frac{x+2}{2x-6}$$

$$Df = \{x \mid x \neq 3\}$$

$$= (-\infty, 3) \cup (3, \infty)$$

$$2x-6 \neq 0 \Rightarrow 2x \neq 6$$

$$\Rightarrow x \neq \frac{6}{2} \Rightarrow x \neq 3$$

$$h(x) = \sqrt{2x+8}$$

$$\begin{aligned} \sqrt{-1} &\rightarrow \text{Not} \\ \sqrt{-2} &\rightarrow \text{a real number} \end{aligned}$$

Anything inside a radical sign should be nonnegative positive or zero.

$$2x+8 \geq 0 \Rightarrow 2x \geq 0-8 \Rightarrow 2x \geq -8$$

$$\Rightarrow x \geq \frac{-8}{2} \Rightarrow x \geq -4$$

$$\Rightarrow [-4, \infty)$$

$$\Rightarrow \text{Domain of } h = [-4, \infty)$$

Math11000 Section 3962 Quiz 7

Summer 2023, May 22

Name:

[1 pt]

Problem 1: The perimeter of a rectangle is 200. The length is 20 more than the width. Find the dimensions of the rectangle. [5 pts]

Let length be l and the width be w

$$2l + 2w = 200, \quad l = 20 + w$$

$$2(20 + w) + 2w = 200$$

$$40 + 2w + 2w = 200 \Rightarrow 40 + 4w = 200$$

$$\Rightarrow 4w = 200 - 40 \Rightarrow 4w = 160 \Rightarrow w = \frac{160}{4} = 40$$

$$\Rightarrow l = 20 + 40 = 60$$

Thus, length is 60 and width is 40.

Problem 2: The cost function for a certain company is $C(x) = 5x + 1000$. The revenue function is $R(x) = 15x$. Find the profit function and the breakeven point. [4 pts]

$$P(x) = R(x) - C(x)$$

$$= 15x - (5x + 1000)$$

$$= 15x - 5x - 1000 \Rightarrow P(x) = 10x - 1000$$

$$P(x) = 0 \Rightarrow 10x - 1000 = 0 \Rightarrow 10x = 1000$$

$$\Rightarrow x = \frac{1000}{10} \Rightarrow x = 100$$