Math16500 Section 24246 Quiz 12
Fall 2022, November 07

Name: [1 pt]

Problem 1: If the acceleration of a particle is $a(t) = \sin t + \cos t$, the initial velocity is $v(0) = 4$ and the initial position is $x(0) = 3$, then find its position function $x(t)$. [6 pts]

$$v(t) = \int a(t)\,dt = \int (\sin t + \cos t)\,dt = \int \sin t\,dt + \int \cos t\,dt$$

$$\Rightarrow v(t) = -\cos t + \sin t + C$$

$$4 = v(0) = -\cos 0 + \sin 0 + C = -1 + 0 + C \Rightarrow 4 = -1 + C \Rightarrow C = 5$$

$$\Rightarrow v(t) = -\cos t + \sin t + 5$$

$$\Rightarrow a(t) = \int v(t)\,dt = \int (-\cos t + \sin t + 5)\,dt = -\int \cos t\,dt + \int \sin t\,dt + 5\int dt$$

$$= -\sin t - \cos t + 5t + d$$

$$3 = a(0) = -\sin 0 - \cos 0 + 5(0) + d = -1 + d \Rightarrow 3 = -1 + d \Rightarrow d = 4$$

$$\Rightarrow \boxed{a(t) = -\sin t - \cos t + 5t + 4}$$

Problem 2: Find the most general antiderivative of $f(x) = \dfrac{1+x}{\sqrt{x}}$. [3 pts]

$$F(x) = \int f(x)\,dx = \int \frac{1+x}{\sqrt{x}}\,dx = \int \left(\frac{1}{\sqrt{x}} + \frac{x}{\sqrt{x}}\right)dx = \int \left(x^{-\frac{1}{2}} + x^{\frac{1}{2}}\right)dx$$

$$= \int x^{-1/2}\,dx + \int x^{1/2}\,dx$$

$$= \frac{x^{-1/2+1}}{-\frac{1}{2}+1} + \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C = \frac{x^{1/2}}{1/2} + \frac{x^{3/2}}{3/2} + C = 2\sqrt{x} + \frac{2}{3}x^{3/2} + C$$

$$\boxed{F(x) = 2\sqrt{x} + \frac{2}{3}x^{3/2} + C}$$

Bonus Problem: Evaluate the integral $\displaystyle\int \frac{10}{x^9}\,dx$. [2 pts]

$$\int \frac{10}{x^9}\,dx = 10\int \frac{1}{x^9}\,dx = 10\int x^{-9}\,dx = 10\,\frac{x^{-9+1}}{-9+1} + C$$

$$= \frac{10}{-8}x^{-8} + C = -\frac{5}{4}x^{-8} + C = \boxed{\frac{-5}{4x^8} + C}$$