

Learning objectives:

1. To apply the methods learned for finding extremal values of functions.
2. Solve problems such as maximizing or minimizing areas, distances, volumes, profit etc.

Steps in solving optimization problems

1. Understand the problem: Figure out which function needs to be minimized or maximized (**objective function**) and what are the **constraints**.
2. Draw a **diagram** if possible to correctly arrive at the objective function.
3. Introduce notation: The objective function depends on certain **variables**. Figure out what are those variables.
4. Express the objective function in terms of these variables.
5. Find **relationships** among the variables and the **constraints** on them.
6. Use methods of Sections 3.1 (closed interval method) and 3.3 (local maxima or minima) to find the optimal value of the objective function.

First derivative test for absolute extremal values

Suppose that c is a critical number of a continuous function f defined on an interval.

1. If $f'(x) > 0$ for all $x < c$ and $f'(x) < 0$ for all $x > c$, then $f(c)$ is the absolute maximum value of f .
2. If $f'(x) < 0$ for all $x < c$ and $f'(x) > 0$ for all $x > c$, then $f(c)$ is the absolute minimum value of f .

Example 1. Suppose you have 2400 ft of fencing and want to fence off a rectangular field that borders a straight river. You need no fence along the river. What are the dimensions of the field that has the largest area?

Example 2. A cylindrical can is to be made to hold 1 liter of oil. Find the dimensions that will minimize the cost of the metal to manufacture the can.

Example 3. Find the point on the parabola $y^2 = 2x$ that is closest to the point $(1, 4)$.

Example 4. Suppose you launch your boat from point A on a bank of a straight river, 3 km wide, and want to reach point B, 8 km downstream on the opposite bank, as quickly as possible. You could row your boat directly across the river to point C and then run to B, or you could row directly to B, or you could row to some point D between C and B and then run to B. If you can row 6 km/h and run 8 km/h, where should you land to reach B as soon as possible? (We assume that the speed of the water is negligible compared with the speed at which you row.)

Example 5. Find the area of the largest rectangle that can be inscribed in a semicircle of radius r .

Let $p(x)$ be the price function of a quantity where x is units of the quantity.

Then the revenue function is given by $R(x) = xp(x)$.

If the cost function is $C(x)$, then profit is given by $R(x) - C(x)$.

Example 6. A store has been selling 200 flat-screen TVs a week at \$350 each. A market survey indicates that for each \$10 rebate offered to buyers, the number of TVs sold will increase by 20 a week. Find the demand function (assuming it is linear) and the revenue function. How large a rebate should the store offer to maximize its revenue?