Learning objectives:

1. Understand the fundamental theorem of calculus.
2. Apply the fundamental theorem to find derivatives of certain functions.
3. Apply the fundamental theorem to compute definite integrals.

Fundamental theorem of calculus I.

Let f be continuous on $[a, b]$ and define the function g by

$$g(x) = \int_a^x f(t)\,dt\,, \quad a \le x \le b\,.$$

Then

1. g is continuous on $[a, b]$.
2. g is differentiable on (a, b).
3. $g'(x) = f(x)$.

Example 1. Find the derivative of the function $g(x) = \int_0^x \sqrt{1 + t^2}\,dt$.

Example 2. Find the derivative of the function $g(x) = \int_1^{x^4} \sec t \, dt$.

Example 3. Find the derivative of the function $\int_x^0 \sqrt{1 + \sec t} \, dt$.

Example 4. Find the derivative of the function $\int_{\cos x}^{\sin x} \sqrt{1-s^2}\, ds,\ 0 \le x \le \pi/2$.
Use it to compute the given integral in terms of x.

Fundamental theorem of calculus II.

If f is continuous on $[a, b]$, then

$$\int_a^b f(x)\, dx = F(b) - F(a)\ ,$$

where F is any antiderivative of f.

Example 5. Evaluate the integral $\int_{-2}^{1} x^3\,dx$.

Example 6. Find the area under the parabola $y = x^2$ from $x = 0$ to $x = 1$.

Example 7. Find the area under the cosine curve from $x = 0$ to $x = b$, where $0 \leq b \leq \pi/2$.

Example 8. What is wrong with the following calculation?

$$\int_{-1}^{2} \frac{1}{x^2}\, dx = \frac{x^{-1}}{-1}\bigg|_{-1}^{2} = -\frac{1}{2} - \frac{-1}{-1} = -\frac{3}{2}\,.$$