$(\sqrt{x})^2 = x$, $\sqrt[3]{x^3} = x$

$y = a^x$ ← interchanged x and y.

The Meaning of Logarithms

$\log_a x = y$ if $a^y = x$

ESSENTIALS

The Meaning of $\log_a x$

For $x > 0$ and a a positive constant other than 1, $\log_a x$ is the exponent to which a must be raised in order to get x. Thus,

$$\log_a x = m \text{ means } a^m = x$$

or equivalently,

$$\log_a x \text{ is that unique exponent for which } a^{\log_a x} = x.$$

$$\left\{ \begin{array}{l} a^{\log_a x} = x \\ \log_a a^x = x \end{array} \right\}$$

Example

- Simplify: $\log_3 27. = m \Rightarrow 3^m = 27$

  $\log_3 27$ is the exponent to which we raise 3 to get 27. That exponent is 3.

  Thus, $\log_3 27 = 3$.

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EXAMPLE 1	YOUR TURN 1
Simplify: $\log_8 64. = m \Rightarrow 8^m = 64$ $\log_8 64$ is the exponent to which we raise 8 to get 64. That exponent is $\boxed{2}$. Thus, $\log_8 64 = \boxed{2}$.	Simplify: $\log_2 16. = m \Rightarrow 2^m = 16$ $\Rightarrow m = 4$ $\Rightarrow \log_2 16 = 4$
EXAMPLE 2	YOUR TURN 2
Simplify: $\log_5 \frac{1}{125}. = m \Rightarrow 5^m = \frac{1}{125}$ $\log_5 \frac{1}{125}$ is the exponent to which we raise 5 to get $\frac{1}{125}$. Since $5^{-3} = \frac{1}{125}$, we have $\log_5 \frac{1}{125} = \boxed{-3}$.	Simplify: $\log_4 \frac{1}{256}. = m \Rightarrow 4^m = \frac{1}{256}$ $256 = 16^2 = (4^2)^2 = 4^4$ $\frac{1}{256} = 4^{-4} \Rightarrow m = -4$ $\Rightarrow \log_4 \frac{1}{256} = -4$

$5^3 = 125 \Rightarrow \frac{1}{125} = \frac{1}{5^3} = \frac{5^0}{5^3} = 5^{-3}$

$5^m = 5^{-3}$

EXAMPLE 3	YOUR TURN 3
Simplify: $\log_{36} 6$. $= m$ $\log_{36} 6$ is the exponent to which 36 is raised to get 6. Since $36^{1/2} = 6$, we have $\log_{36} 6 = \boxed{\frac{1}{2}}$.	Simplify: $\log_{16} 2$. $= m$ $\Rightarrow 16^m = 2$ $\Rightarrow (2^4)^m = 2 \Rightarrow 2^{4m} = 2^1 \Rightarrow 4m = 1$ $\Rightarrow m = \frac{1}{4} \Rightarrow \log_{16} 2 = \frac{1}{4}$

EXAMPLE 4	YOUR TURN 4
Simplify: $15^{\log_{15} 19}$. Remember that $\log_{15} 19$ is the exponent to which 15 is raised to get 19. Raising 15 to that exponent, we have $15^{\log_{15} 19} = \boxed{19}$.	Simplify: $9^{\log_9 25}$. $9^{\log_9 25} = 25$

YOUR NOTES Write your questions and additional notes.

$36^m = 6$

$\Rightarrow (6^2)^m = 6 \Rightarrow 6^{2m} = 6^1$

$2m = 1 \Rightarrow m = \frac{1}{2}$

$36^{1/2} = 6$

⊛ $\log_{36} \frac{1}{6} = -\frac{1}{2}$

$\quad \shortparallel$

$\quad m \Rightarrow 36^m = \frac{1}{6}$

$\Rightarrow (6^2)^m = 6^{-1}$

$\Rightarrow 6^{2m} = 6^{-1} \Rightarrow 2m = -1$

$\Rightarrow m = -\frac{1}{2}$

Graphs of Logarithmic Functions

ESSENTIALS

To graph logarithmic functions recall that if $y = \log_a x$, then $a^y = x$.

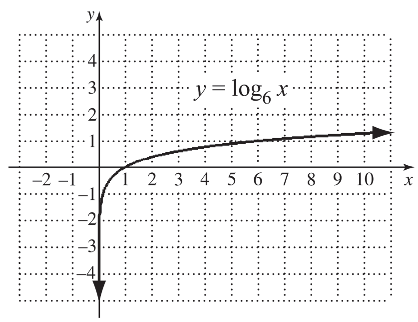
1. Choose values for y and compute the x-values.
2. Plot the points.
3. Connect the points with a smooth curve.

Example

- Graph: $y = f(x) = \log_6 x$.

$y = \log_6 x$ is equivalent to $6^y = x$. Choose y-values, compute x-values, plot the points, and connect them with a smooth curve.

x	y
1	0
6	1
36	2
$\frac{1}{6}$	−1
$\frac{1}{36}$	−2



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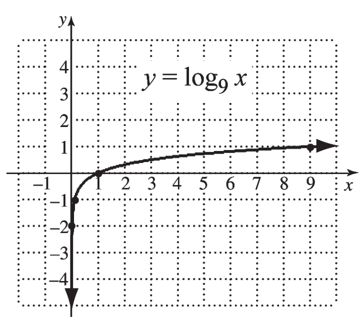
EXAMPLE 1	YOUR TURN 1

EXAMPLE 1

Graph: $y = \log_9 x$.

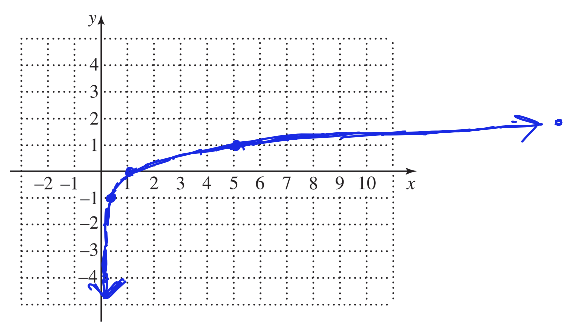
$y = \log_9 x$ is equivalent to $9^y = x$. Choose y-values, compute x-values, plot the points, and connect them with a smooth curve.

x	y
1	0
$\boxed{9}$	1
81	2
$\boxed{\frac{1}{9}}$	−1
$\frac{1}{81}$	−2



YOUR TURN 1

Graph: $y = \log_5 x$. $\Rightarrow x = 5^y$



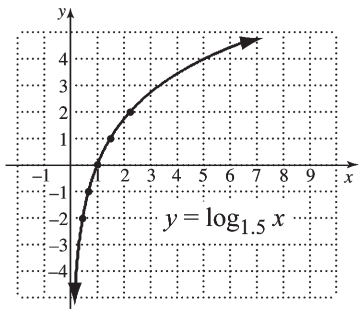
x	y
1	0
5	1
25	2
$0.2 = \frac{1}{5}$	−1
$0.04 = \frac{1}{25}$	−2

EXAMPLE 2

Graph: $y = \log_{1.5} x$.

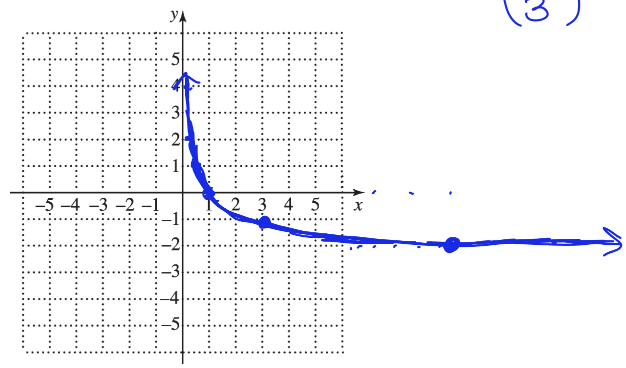
$y = \log_{1.5} x$ is equivalent to $1.5^y = x$, or $\left(\frac{3}{2}\right)^y = x$. Choose y-values, compute x-values, plot the points, and connect them with a smooth curve.

x	y
$\boxed{1}$	0
$\frac{3}{2}$	1
$\frac{9}{4}$	2
$\boxed{\frac{2}{3}}$	−1
$\frac{4}{9}$	−2



YOUR TURN 2

Graph: $y = \log_{1/3} x$. $\Rightarrow x = \left(\frac{1}{3}\right)^y$



x	y
1	0
$\frac{1}{3}$	1
$\frac{1}{9}$	2
3	−1
9	−2

YOUR NOTES Write your questions and additional notes.

$1.5^{-1} = \frac{1}{1.5} = \frac{2}{3}$

$\left(\frac{1}{3}\right)^{-1} = \frac{1}{1/3} = 3$

$\left(\frac{1}{3}\right)^{-2} = \left(\left(\frac{1}{3}\right)^{-1}\right)^2 = 3^2 = 9$

Equivalent Equations

ESSENTIALS

A *logarithmic equation* can be written as an *exponential equation*, or vice versa, by using the definition of a logarithm:

$m = \log_a x$ is equivalent to $a^m = x$.

Examples

- Rewrite $3 = \log_a 9$ as an equivalent exponential equation.

  $3 = \log_a 9$ is equivalent to $a^3 = 9$. The logarithm, 3, is the exponent. The base, *a*, remains the base.

- Rewrite $9 = 3^x$ as an equivalent logarithmic equation.

  $9 = 3^x$ is equivalent to $x = \log_3 9$. The exponent, *x*, is the logarithm. The base remains the base.

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EXAMPLE 1	YOUR TURN 1
Rewrite $y = \log_2 6$ as an equivalent exponential equation. $y = \log_2 6$ is equivalent to $2^y = \boxed{6}$.	Rewrite $y = \log_5 4$ as an equivalent exponential equation. $5^y = 4$
EXAMPLE 2	**YOUR TURN 2**
Rewrite $3 = \log_7 x$ as an equivalent exponential equation. $3 = \log_7 x$ is equivalent to $\boxed{7}^3 = \boxed{x}$.	Rewrite $4 = \log_2 x$ as an equivalent exponential equation. $2^4 = x$
EXAMPLE 3	**YOUR TURN 3**
Rewrite $y^{-2} = 8$ as an equivalent logarithmic equation. $y^{-2} = 8$ is equivalent to $\boxed{-2} = \log_y 8$.	Rewrite $y^6 = 15$ as an equivalent logarithmic equation. $\log_y 15 = 6$

EXAMPLE 4	YOUR TURN 4
Rewrite $\left(\frac{1}{2}\right)^{-3} = x$ as an equivalent logarithmic equation. $\left(\frac{1}{2}\right)^{-3} = x$ is equivalent to $\boxed{-3} = \log_{\boxed{1/2}} x$.	Rewrite $\left(\frac{1}{3}\right)^{-1} = x$ as an equivalent logarithmic equation. $\log_{1/3} x = -1$

YOUR NOTES Write your questions and additional notes.

Solving Certain Logarithmic Equations

ESSENTIALS

The Principle of Exponential Equality

For any real number b, where $b \neq -1$, 0, or 1,

$b^m = b^n$ is equivalent to $m = n$.

(Powers of the same base are equal if and only if the exponents are equal.)

$\mathbf{log_a 1}$ $= m \quad \Rightarrow a^m = 1 \Rightarrow m = 0$

The logarithm, base a, of 1 is 0: $\log_a 1 = 0$.

$\mathbf{log_a a}$ $= m \quad \Rightarrow a^m = a^1 \Rightarrow m = 1$

The logarithm, base a, of a is 1: $\log_a a = 1$.

Examples

- Solve: $\log_3 x = -2$.

  $\log_3 x = -2$

  $3^{-2} = x$ Rewriting as an exponential equation

  $\frac{1}{9} = x$ Computing 3^{-2}

  Check: $\log_3 x = -2$ is the exponent to which 3 must be raised to get $\frac{1}{9}$.

  Since that exponent is -2, the number $\frac{1}{9}$ checks.

- Solve: $\log_{10} 100 = x$.

  $\log_{10} 100 = x$

  $10^x = 100$ Rewriting as an exponential equation

  $10^x = 10^2$ Writing 100 as a power of 10

  $x = 2$ Equating exponents

  Check: $\log_{10} 100 = x$ is the exponent to which 10 must be raised to get 100.
  Since $10^2 = 100$, the solution is 2.

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EXAMPLE 1	YOUR TURN 1
Solve: $\log_2 x = -5$. $\log_2 x = -5$ $2^{\boxed{-5}} = x$ $\boxed{\frac{1}{32}} = x$ The solution is $\boxed{\frac{1}{32}}$.	Solve: $\log_2 x = -2$. $2^{-2} = x$ $\Rightarrow x = \frac{1}{2^2} = \frac{1}{4}$ $\Rightarrow x = \frac{1}{4}$

EXAMPLE 2	YOUR TURN 2
Solve: $\log_x 12 = \frac{1}{2}$. $\log_x 12 = \frac{1}{2}$ $x^{1/2} = 12$ $\left(x^{1/2}\right)^2 = \boxed{12}^2$ $x = \boxed{144}$ The solution is $\boxed{144}$.	Solve: $\log_x 2 = \frac{1}{5}$. $x^{\frac{1}{5}} = 2$ $\left(x^{\frac{1}{5}}\right)^5 = 2^5 \Rightarrow x^{\frac{1}{5}\times 5} = 32$ $\Rightarrow x = 32$

EXAMPLE 3	YOUR TURN 3
Solve: $\log_3 81 = x$. $\log_3 81 = x$ $3^x = 81$ $3^x = \boxed{3}^4$ $x = \boxed{4}$ The solution is $\boxed{4}$.	Solve: $\log_2 16 = x$. $2^x = 16$ $2^x = 2^4$ $\Rightarrow x = 4$

YOUR NOTES Write your questions and additional notes.

Practice Exercises

Readiness Check

Determine whether the statement is true or false.

1. The logarithm, base a, of 1 is 1.
2. A logarithmic function is the inverse of an exponential function.
3. A logarithm is an exponent.
4. The logarithm, base a, of a is 0.

The Meaning of Logarithms

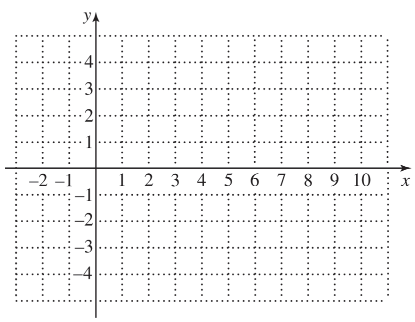
Simplify.

5. $\log_{10} 10,000$ **6.** $\log_{49} 7$ **7.** $\log_7 7$

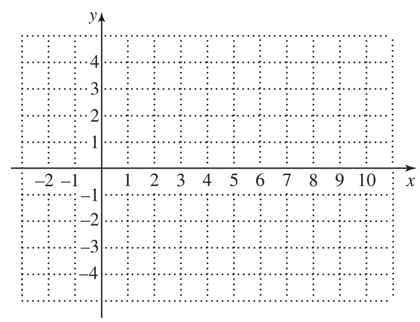
8. $\log_8 \dfrac{1}{64}$ **9.** $\log_{16} 64$ **10.** $5^{\log_5 24}$

Graphs of Logarithmic Equations

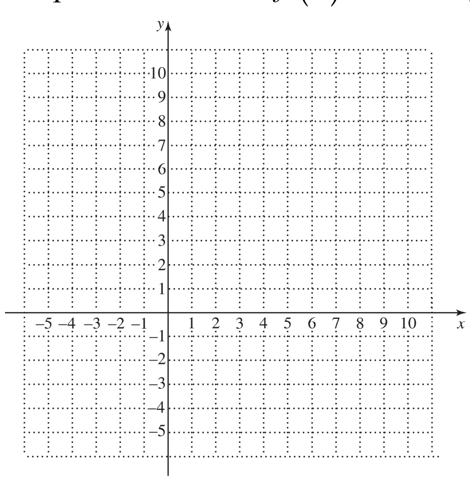
11. $y = \log_4 x$



12. $y = \log_{3.5} x$



13. Graph the functions $f(x) = 8^x$ and $f^{-1}(x) = \log_8 x$ using one set of axes.



Equivalent Equations

Rewrite each of the following as an equivalent exponential equation. Do not solve.

14. $x = \log_{20} 12$

15. $\log_a b = 6$

16. $\log_e 0.975 = -0.025$

Rewrite each of the following as an equivalent logarithmic equation. Do not solve.

17. $4^{-3} = \frac{1}{64}$

18. $p^t = 15$

19. $128^{1/7} = 2$

Solving Certain Logarithmic Equations

Solve.

20. $\log_7 x = 2$

21. $\log_5 125 = x$

22. $\log_x 18 = 1$

$x^1 = 18 \Rightarrow x = 18$

23. $\log_2 x = -5$

24. $\log_3 1 = x$

$3^x = 1$

$3^x = 3^0$

$x = 0$

25. $\log_{32} x = \frac{3}{5}$

$x = 32^{3/5}$

$= (2^5)^{3/5}$

$= 2^{5 \times \frac{3}{5}} = 2^3 = 8$

$\Rightarrow x = 8$