

Problem 1. Use the substitution rule to evaluate the following integrals.

1. $\int x^2 \sqrt{x^3 + 1} dx.$

2. $\int_0^{\pi/2} \sin^2 \theta \cos \theta d\theta.$

3. $\int \sin t \sqrt{1 + \cos t} dt.$

4. $\int \sin x \sin(\cos x) dx.$

5. $\int \frac{\sin \sqrt{x}}{\sqrt{x}} dx.$

6. $\int_0^{\pi/6} \frac{\sin t}{\cos^2 t} dt.$

7. $\int_0^{13} \frac{dx}{\sqrt[3]{(1 + 2x)^2}} dx.$

8. $\int_{-\pi/3}^{\pi/3} x^4 \sin x dx.$

Problem 2. Sketch the region enclosed by the following curves and find its area.

1. $y = 12 - x^2$, $y = x^2 - 6.$

2. $x = 2y^2$, $x = 4 + y^2.$

3. $y = \sec^2 x$, $y = 8 \cos x$, $x = -\pi/3$, $x = \pi/3.$

4. $y = \cos \pi x$, $y = 4x^2 - 1.$

5. $x = y^4$, $y = \sqrt{2 - x}$, $y = 0.$

6. $y = \cos x$, $y = 1 - \cos x$, $x = 0$, $x = \pi.$

7. $y = x^4$, $y = 2 - |x|.$

8. $4x + y^2 = 12$, $x = y.$