Learning objectives:

1. Derivatives of power functions: The power rule
2. Taking derivatives of combinations of functions: the constant multiple rule, the sum and difference rules, the product rule, the quotient rule

Derivative of a constant function

$$\frac{d}{dx}(c) = 0 \, .$$

Derivative of a power function

$$\frac{d}{dx}(x^n) = x^{n-1} \, .$$

Here n can be any real number.

Example 1. Find derivative of the following functions.

1. $f(x) = x^{600}$.
2. $f(x) = 1/x^2$.
3. $f(x) = x^{\pi}$.
4. $f(x) = \sqrt[100]{x^3}$.

The constant multiple rule

$$\frac{d}{dx}\Big(cf(x)\Big) = c\frac{d}{dx}f(x)\ .$$

Example 2. Find derivative of the following functions.

1. $f(x) = 3x^4$.
2. $f(x) = -x$.

The sum rule

$$\frac{d}{dx}\Big(f(x) + g(x)\Big) = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)\ .$$

The difference rule

$$\frac{d}{dx}\Big(f(x) - g(x)\Big) = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)\ .$$

Example 3. Find the derivative of $f(x) = x^8 + 12x^5 - 4x^4 + 10x^3 - 6x + 5$.

Example 4. Find the points of the curve $y = x^4 - 6x^2 + 4$ where the tangent line is horizontal.

Example 5. The position function of a particle is $s(t) = 2t^3 - 5t^2 + 3t + 4$ where s is measured in meters and t is measured in seconds. Find the time instants where the particle is at rest. Find the the acceleration as a function of time. What is the acceleration after 2 seconds?

The product rule

$$\frac{d}{dx}\Big(f(x)g(x)\Big) = f(x)\frac{d}{dx}g(x) + g(x)\frac{d}{dx}f(x)\ .$$

The quotient rule

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{\Big(g(x)\Big)^2}\ .$$