

## 5.2- Multiplication of Polynomials

$$a^m \cdot a^n = a^{m+n}$$

Example 1: Multiply and Simplify

<p>a) <math>(-8x^4y^7)(5x^3y^2)</math></p> $= -8 \cdot 5 \cdot x^4 \cdot x^3 \cdot y^7 \cdot y^2$ $= -40x^{4+3}y^{7+2}$ $= -40x^7y^9$	<p>b) <math>(-3a^5bc^6)(-4a^2b^5c^8)</math></p> $= -3(-4) \cdot a^5 \cdot a^2 \cdot b \cdot b^5 \cdot c^6 \cdot c^8$ $= 12a^{5+2}b^{1+5}c^{6+8}$ $= 12a^7b^6c^{14}$	<p>c) <math>(6nm^8)(-n^2m^3)</math></p> $= -6n^3m^{11}$
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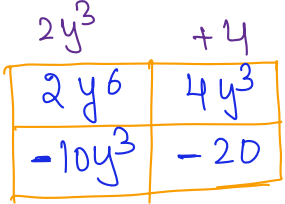
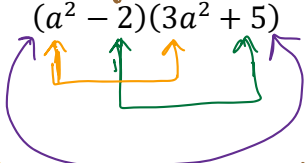

### Multiplying Monomials and Binomials

Example 2: Multiply

<p>a) <math>2t(3t - 5)</math></p> $= (2t)(3t) + (2t)(-5)$ $= 6t^2 - 10t$	<p>b) <math>3a^2b(a^2 - b^2)</math></p> $= (3a^2b)(a^2) + (3a^2b)(-b^2)$ $= 3a^4b - 3a^2b^3$	<p>c) <math>5x^2y^3(3x - 4y^2)</math></p> $= (5x^2y^3)(3x) + (5x^2y^3)(-4y^2)$ $= 15x^3y^3 - 20x^2y^5$
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### Example 3: Multiplying a Binomial and a Binomial

First Outer Inner Last  
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$(y^3 - 5)(2y^3 + 4)$  <table><tr><td><math>y^3</math></td><td><math>2y^3</math></td><td><math>+4</math></td></tr><tr><td><math>-5</math></td><td><math>2y^3</math></td><td><math>4y^3</math></td></tr><tr><td></td><td><math>-10y^3</math></td><td><math>-20</math></td></tr></table> $= 2y^6 + 4y^3 - 10y^3 - 20$ $= 2y^6 - 6y^3 - 20$	$y^3$	$2y^3$	$+4$	$-5$	$2y^3$	$4y^3$		$-10y^3$	$-20$	$(a^2 - 2)(3a^2 + 5)$  $a^2 \cdot (3a^2) + a^2(5) + (-2)(3a^2) + (-2)(5)$ $= 3a^4 + 5a^2 - 6a^2 - 10$ $= 3a^4 - a^2 - 10$
$y^3$	$2y^3$	$+4$								
$-5$	$2y^3$	$4y^3$								
	$-10y^3$	$-20$								
$(x - 4)(x - 8)$  $= x(x - 8) - 4(x - 8)$ $= x(x) - 8x - 4x + (-4)(-8)$ $= x^2 - 8x - 4x + 32$ $= x^2 - 12x + 32$	$(2x + 3y)(x - 4y)$ $= 2x(x - 4y) + 3y(x - 4y)$ $= 2x(x) + (2x)(-4y) + (3y)(x) + 3y(-4y)$ $= 2x^2 - 8xy + 3xy - 12y^2$ $= 2x^2 - 5xy - 12y^2$									

### Example 4: Multiplying any two Polynomials

a)  $(p + 2)(p^4 - 2p^3 + 3)$

	$p^4$	$-2p^3$	$+3$
$p$	$p(p^4) = p^5$	$p(-2p^3) = -2p^4$	$p(3) = 3p$
$+2$	$+2(p^4) = 2p^4$	$+2(-2p^3) = -4p^3$	$+2(3) = 6$

$$= p^5 - 2p^4 + 3p + 2p^4 - 4p^3 + 6$$

$$= p^5 - 4p^3 + 3p + 6$$

b)  $(x + 3)(x^3 - 5x - 1)$

	$x^3$	$-5x$	$-1$
$x$	$x^4$	$-5x^2$	$-x$
$3$	$3x^3$	$-15x$	$-3$

$$= x^4 - 5x^2 - x + 3x^3 - 15x - 3 = x^4 + 3x^3 - 5x^2 - 16x - 3$$

c)  $(5x^3 + x - 4)(-2x^2 + 3x + 6)$

	$-2x^2$	$3x$	$+6$
$5x^3$	$-10x^5$	$15x^4$	$30x^3$
$+x$	$-2x^3$	$3x^2$	$6x$
$-4$	$8x^2$	$-12x$	$-24$

$$= -10x^5 + 15x^4 + 30x^3 - 2x^3 + 3x^2 + 6x + 8x^2 - 12x - 24$$

d)  $(2x^2 + 8x - 7)(x^2 + x - 4)$

	$x^2$	$+x$	$-4$
$2x^2$	$2x^4$	$2x^3$	$-8x^2$
$+8x$	$8x^3$	$8x^2$	$-32x$
$-7$	$-7x^2$	$-7x$	$28$

$$= -10x^5 + 15x^4 + 28x^3 + 11x^2 - 6x - 24$$

$$= 2x^4 + 8x^3 + 2x^3 - 7x^2 + 8x^2 - 8x^2 - 7x - 32x + 28$$

$$= 2x^4 + 10x^3 - 7x^2 - 39x + 28$$

e)  $(t+2)(t-4)(t+5)$

$$= (t+2)(t^2+t-20)$$

	$t^2$	$t$	$-20$
$t$	$t^3$	$t^2$	$-20t$
$2$	$2t^2$	$2t$	$-40$

$$= t^3 + 2t^2 + t^2 + 2t - 20t - 40 = t^3 + 3t^2 - 18t - 40$$

$$\begin{aligned}(t-4)(t+5) &= t(t+5) - 4(t+5) \\ &= t^2 + 5t - 4t - 20 \\ &= t^2 + t - 20\end{aligned}$$

Example 5: Squaring a binomial.  $(a+b)^2 = a^2 + 2ab + b^2$

$$(a+b)^2 = a^2 + b^2$$

a)  $(y-5)^2 \equiv (a+b)^2$   
 $\uparrow \quad \downarrow$   
 $a=y \quad b=-5$

$$\begin{aligned}&= (y)^2 + 2(y)(-5) + (-5)^2 \\ &= y^2 - 10y + 25\end{aligned}$$

b)  $(2x+3y)^2$   
 $\underbrace{\quad}_a \quad \underbrace{\quad}_b$

$$\begin{aligned}&= (2x)^2 + 2(2x)(3y) + (3y)^2 \\ &= 4x^2 + 12xy + 9y^2\end{aligned}$$

c)  $(\frac{1}{2}x - 3y^4)^2$   
 $\underbrace{\quad}_a \quad \underbrace{\quad}_b$

$$\begin{aligned}&= (\frac{1}{2}x)^2 + 2(\frac{1}{2}x)(-3y^4) + (-3y^4)^2 \\ &= \frac{1}{4}x^2 - 3xy^4 + 9y^8\end{aligned}$$

Example 6: Products of Sum and Differences

$$(a+b)(a-b) = (a-b)(a+b) = a^2 - b^2$$

a)  $(t+5)(t-5) \equiv (a+b)(a-b)$   
 $\uparrow \quad \uparrow$   
 $a \quad b$

$$\begin{aligned}&= (t)^2 - (5)^2 \\ &= t^2 - 25\end{aligned}$$

b)  $(2xy^2 + 3x)(2xy^2 - 3x)$   
 $\underbrace{\quad}_a \quad \underbrace{\quad}_b$

$$\begin{aligned}&= (2xy^2)^2 - (3x)^2 \\ &= 4x^2y^4 - 9x^2\end{aligned}$$

Example 7: Given  $f(x) = x^2 - 4x + 5$ , find and simplify each of the following

a)  $f(a) + 3$

$$= \underbrace{a^2 - 4a + 5} + 3 = a^2 - 4a + 8$$

b)  $f(a + 3)$

$$= \underbrace{(a+3)^2} - \underbrace{4(a+3)} + 5$$

$$= \underbrace{a^2 + 2(a)(3) + (3)^2}_{\text{identity}} - \underbrace{4a - 12}_{\text{distribute } -4} + 5$$

$$= a^2 + \underbrace{6a} + \underbrace{9} - \underbrace{4a} - \underbrace{12} + 5 = a^2 + 2a + 2$$

c)  $f(a + h) - f(a)$

$$= \underbrace{(a+h)^2 - 4(a+h) + 5}_{f(a+h)} - \underbrace{(a^2 - 4a + 5)}_{f(a)}$$

$$= \underbrace{a^2 + 2ah + h^2}_{\text{identity}} - \underbrace{4a - 4h + 5}_{\text{distribute } -4} - \underbrace{a^2 + 4a - 5}_{\text{distribute } -1}$$

$$= \cancel{a^2} + 2ah + h^2 - \cancel{4a} - 4h + \cancel{5} - \cancel{a^2} + \cancel{4a} - \cancel{5}$$

$$= h^2 + 2ah - 4h$$