Math 110

5.1-Introduction to Polynomials and Polynomial Functions

Terms and Polynomials

Some Examples of Algebraic Expressions are:

1) x^2+5x+3 2) $5x^3-x^2-4x-1$

3) x^2y-2xy^2+x+5y

A ___term___ is a number or variable raised to a power or a product of numbers and /or variables raised to powers.

Example 1: Identify the terms of the Polynomial.

1) x^2, $5x$, 3 2) $5x^3$, $-x^2$, $-4x$, -1

3) x^2y, $-2xy^2$, x, $5y$

When all variables in a term are raised to $\underbrace{\text{whole number}}_{\{0,1,2,3,\ldots\}}$ powers, the term is a ___monomial.___

Monomial Examples:

1) x^2, x, x^0 $(3=3\times1=3x^0)$

2) x^3, x^2, x, x^0 3) x^2y, xy^2, x, y

A ___Polynomial___ is a monomial or a sum of monomials

A polynomial with two terms is called ___binomial___, and those with three terms are called ___trinomial.___

The ___degree___ of a monomial is the sum of the exponents of the variables.

Degree of $x^3=3$, Degree of $1=0$,

Degree of $x^2y=3$, Degree of $xy=2$

Example 2: Determine the degree of each term a) $8x$^4 b)$3x$ c) 7 d) $9x^2yz^4$

a) 4
b) 1
c) 0
d) 7

$\to -2x^0$

The degree of a constant, such as -2, is 0, since there are no variable Factors.

The part of a term that is a constant factor is the Coefficient

Example 3: Identify the coefficient of each term in the polynomial

$$4x^2\text{-}7x^2y + x\text{-}8$$

4   -7 1 -8

leading term
$-7x^2y$
Leading coeff. $= -7$
degree $=3$

The leading term of a polynomial is the term of highest degree

Its coefficient is called the leading coefficient and its degree is referred to as the degree of the polynomial.

leading term $= x^3$

LC $=1$, deg $=3$

We usually Arrange polynomials in one variable so that exponents *decrease* from left to right. This is called descending order.

$x^3 + 4x + 1$

Example 4: Arrange the following polynomial in descending order

$$12 + 2x^3\text{-}7x + x^2$$

$2x^3 + x^2 - 7x + 12$

Polynomial Functions

Example 6:

a) For the polynomial function $P(x) = -x^2 + 4x - 1$, Find P(5) and P(-5)

$P(5) = -(5)^2 + 4(5) - 1 = -25 + 4 \times 5 - 1$

$= -25 + 20 - 1 = -5 - 1 = -6$

$P(-5) = -(-5)^2 + 4(-5) - 1 = -25 - 20 - 1 = -46$

b) For the polynomial function $P(x) = x - 2x^2$ Find P(3) and P(-3)

$P(3) = 3 - 2(3)^2 = 3 - 2 \times 9 = 3 - 18 = -15$

$P(-3) =$ HW (-21)

Adding Polynomials

Example 7: Combine like terms

a) $3x^2 - 4y + 2x^2$

$3x^2 + 2x^2 - 4y = 5x^2 - 4y$

b) $4t^3 - 6t - 8t^2 + t^3 + 9t^2$

$4t^3 + t^3 - 8t^2 + 9t^2 - 6t$

$= 5t^3 + t^2 - 6t$

c) $3x^2y + 5xy^2 - 3x^2y - xy^2$

$3x^2y - 3x^2y + 5xy^2 - xy^2$

$= 0 + 4xy^2 = 4xy^2$

$-1 - 1 = -2$

d) $3n - n^3 + 2n + 5 - n^3 + 6$

$3n + 2n - n^3 - n^3 + 5 + 6 = 5n - 2n^3 + 11$

We add polynomials by combining like terms

Example 8: Add

a) $(-3x^3 + 2x - 4) + (4x^3 + 3x^2 + 2)$

$= -3x^3 + 2x - 4 + 4x^3 + 3x^2 + 2$

$= -3x^3 + 4x^3 + 2x + 3x^2 - 4 + 2$

$= x^3 + 3x^2 + 2x - 2$

b) $(y^2 - 2y + 3) + (y^2 + 2y - 7)$

$= y^2 - 2y + 3 + y^2 + 2y - 7 = y^2 + y^2 - 2y + 2y + 3 - 7$

$= 2y^2 + 0 - 4 = 2y^2 - 4$

We can also ass using columns, we write the polynomials one under the other, listing like terms under one another and leaving spaces for any missing terms.

c) $(4n^3 + 4n - 5\ ) + (-n^3 + 7n^2 - 2)$

$$\begin{array}{rrrr} 4n^3 & & +4n & -5 \\ -n^3 & +7n^2 & & -2 \\ \hline 3n^3 & +7n^2 & +4n & -7 \end{array}$$

d) $(2x^4 + x^2 + x) + (-3x^3 + 2x^2 - 7)$

$$\begin{array}{rrrrr} 2x^4 & & +x^2 & +x & \\ & -3x^3 & +2x^2 & & -7 \\ \hline 2x^4 & -3x^3 & +3x^2 & +x & -7 \end{array}$$

e) $(13x^3y + 3x^2y - 5y) + (x^3y + 4x^2y - 3xy)$

$$\begin{array}{rrrr} 13x^3y & +3x^2y & & -5y \\ x^3y & +4x^2y & -3xy & \\ \hline 14x^3y & +7x^2y & -3xy & -5y \end{array}$$

f) $(6c^2d - 8cd + d^2) + (3cd + 5cd^2 - d^2)$

$$\begin{array}{r} 6c^2d \qquad -8cd + d^2 \\ 5cd^2 + 3cd - d^2 \\ \hline 6c^2d + 5cd^2 - 5cd \end{array}$$

If the sum of two polynomials is 0**, the polynomials are** opposite **or** additive inverses **of each other**

For Example: $5x^2 - x + 1$ and $-5x^2 + x - 1$

$(5x^2 - x + 1) + (-5x^2 + x - 1) = 5x^2 - 5x^2 - x + x + 1 - 1 = 0$

Example 9: Write two equivalent expressions for the opposite of the following polynomials

a) $7xy^2 - 6xy - 4y + 3$

opposite $= -(7xy^2 - 6xy - 4y + 3)$

$= -7xy^2 + 6xy + 4y - 3$

Subtracting Polynomials

To subtract **a polynomial, we** add **its opposite**

Example 10: Subtract

a) $(-3x^2 + 4xy) - (2x^2 - 5xy + 7y^2)$

$= -3x^2 + 4xy - 2x^2 + 5xy - 7y^2$

$= -5x^2 + 9xy - 7y^2$

b) $(x^2 - x + 1) - (3x^2 - 2x - 7)$

$= x^2 - x + 1 - 3x^2 + 2x + 7$

$= -2x^2 + x + 8$

c) $(3x^4 - 2x^3 + 6x - 1) - (3x^4 - 9x^3 - x^2 + 7)$

$= 3x^4 - 2x^3 + 6x - 1 - 3x^4 + 9x^3 + x^2 - 7$

$= 7x^3 + x^2 + 6x - 8$

d) $(-2n^3 - n^2 - 6n) - (3n^3 - n^2 + 5)$

$= -2n^3 - n^2 - 6n - 3n^3 + n^2 - 5$

$= -5n^3 - 6n - 5$

5.2- Multiplication of Polynomials

Example 1: Multiply and Simplify

a) $(-8x^4y^7)(5x^3y^2)$

$= -8 \times 5 \times x^4 \times x^3 \times y^7 \times y^2$

$= -40x^7y^9$

b) $(-3a^5bc^6)(-4a^2b^5c^8)$

$= -3 \times (-4) \times a^5 \times a^2 \times b \times b^5 \times c^6 \times c^8$

$= 12a^7b^6c^{14}$

c) $(6nm^8)(-n^2m^3)$

$= -6n^3m^{11}$

Multiplying Monomials and Binomials

Example 2: Multiply

a) $2t(3t-5)$

$= (2t)(3t) + (2t)(-5)$

$= 6t^2 - 10t$

b) $3a^2b(a^2-b^2)$

$= (3a^2b)(a^2) + (3a^2b)(-b^2)$

$= 3a^4b - 3a^2b^3$

c) $5x^2y^3(3x-4y^2)$

$= (5x^2y^3)(3x) + (5x^2y^3)(-4y^2)$

$= 15x^3y^3 - 20x^2y^5$

First Outer Inner Last

FOIL

$(A+B)(C+D)$

$= AC + AD + BC + BD$

Example 3: Multiplying a Binomial and a Binomial

a) $(y^3 - 5)(2y^3 + 4)$

$= y^3(2y^3+4) - 5(2y^3+4)$

$= (y^3)(2y^3) + y^3(4) - 5(2y^3) - 5(4)$

$= 2y^6 + 4y^3 - 10y^3 - 20 = 2y^6 - 6y^3 - 20$

b) $(a^2 - 2)(3a^2 + 5)$

$= a^2(3a^2+5) - 2(3a^2+5)$

$= 3a^4 + 5a^2 - 6a^2 - 10 = 3a^4 - a^2 - 10$

c) $(x - 4)(x - 8)$

$= x(x-8) - 4(x-8)$

$= x^2 - 8x - 4x + 32 = x^2 - 12x + 32$

d) $(2x + 3y)(x - 4y)$

$= 2x(x-4y) + 3y(x-4y)$

$= 2x^2 - 8xy + 3xy - 12y^2 = 2x^2 - 5xy - 12y^2$

Example 4: Multiplying any two Polynomials

a) $(p + 2)(p^4 - 2p^3 + 3)$

p^5	$\cancel{-2p^4}$	$3p$
$\cancel{2p^4}$	$-4p^3$	6

$p(p^4 - 2p^3 + 3)$

$+2(p^4 - 2p^3 + 3)$

$= p^5 + 3p - 4p^3 + 6$

$= p^5 - 4p^3 + 3p + 6$

b) $(x+3)(x^3-5x-1)$

x^4	$-5x^2$	$-x$	$x(x^3-5x-1)$
$3x^3$	$-15x$	-3	$3(x^3-5x-1)$

$= x^4+3x^3-5x^2-16x-3$

c) $(5x^3+x-4)(-2x^2+3x+6)$

$-10x^5$	$15x^4$	$30x^3$	$5x^3(-2x^2+3x+6)$
$-2x^3$	$3x^2$	$6x$	$x(-2x^2+3x+6)$
$8x^2$	$-12x$	-24	$-4(-2x^2+3x+6)$

$= -10x^5+15x^4+28x^3+11x^2-6x-24$

d) $(2x^2+8x-7)(x^2+x-4)$

• $2x^3+8x^3=10x^3$

$2x^4$	$2x^3$	$-8x^2$	$2x^2(x^2+x-4)$
$8x^3$	$8x^2$	$-32x$	$8x(x^2+x-4)$
$-7x^2$	$-7x$	28	$-7(x^2+x-4)$

$= 2x^4+10x^3-7x^2-39x+28$

e) $(t+2)\underbrace{(t-4)(t+5)}$

$(t-4)(t+5) = t(t+5) - 4(t+5)$

$= t^2 + 5t - 4t - 20 = t^2 + t - 20$

$(t+2)\underbrace{(t-4)(t+5)} = (t+2)(t^2+t-20)$

t^3	t^2	$-20t$
$2t^2$	$2t$	-40

$t(t^2+t-20)$

$2(t^2+t-20)$

$= t^3 + 3t^2 - 18t - 40$

Example 5: Squaring a binomial

$y + (-5)$

a) $(y-5)^2 = (y-5)(y-5)$

$y^2 + 5^2 - 2 \times y \times 5$

$= y^2 + 25 - 10y$

$(A+B)^2 = A^2 + B^2 + 2AB$

$\hookrightarrow (A+B)(A+B) = A(A+B) + B(A+B)$

$= A^2 + \underline{AB} + \underline{BA} + B^2$

$= A^2 + 2AB + B^2$

$(A-B)^2 = A^2 + B^2 - 2AB$

$\hookrightarrow (A-B)(A-B) = A(A-B) - B(A-B)$

$= A^2 - \underline{AB} - \underline{BA} + B^2$

$= A^2 - 2AB + B^2$

b) $(2x+3y)^2$

$= (2x)^2 + (3y)^2 + 2 \times 2x \times 3y$

$= 4x^2 + 9y^2 + 12xy$

c) $\left(\frac{1}{2}x - 3y^4\right)^2$

$= \left(\frac{1}{2}x\right)^2 + (3y^4)^2 - 2 \times \frac{1}{2}x \times 3y^4$

$= \frac{1}{4}x^2 + 9y^8 - 3xy^4$

$(a^m)^n = a^{mn}$

$\boxed{\text{CAUTION}: (A+B)^2 \neq A^2 + B^2}$

Example 6: Products of Sum and Differences

a) $(t+5)(t-5)$

$= t^2 - 5^2 = t^2 - 25$

	a	b
a	a^2	ab
b	ab	b^2

b) $(2xy^2+3x)(2xy^2-3x)$

$= (2xy^2)^2 - (3x)^2$

$= 4x^2y^4 - 9x^2$

$(A+B)(A-B) = A^2 - B^2$

$A(A-B) + B(A-B) = A^2 - \underline{AB} + \underline{BA} - B^2$

c) $(0.2t - 1.4m)(0.2t + 1.4m)$

$= (0.2t)^2 - (1.4m)^2$

$= 0.04t^2 - 1.96m^2$

d) $\left(\frac{2}{3}n - m^3\right)\left(\frac{2}{3}n + m^3\right) = \left(\frac{2}{3}n\right)^2 - (m^3)^2 = \frac{4}{9}n^2 - m^6$

Example 7: Given $f(x) = x^2 - 4x + 5$, find and simplify each of the following

a) $f(a) + 3$

$f(a) = a^2 - 4a + 5$

$f(a) + 3 = a^2 - 4a + 5 + 3 = a^2 - 4a + 8$

$(A+B)^2 = A^2 + B^2 + 2AB$

b) $f(a + 3)$

$= (a+3)^2 - 4(a+3) + 5$

$= \underbrace{a^2 + 3^2 + 2(a)(3)} - 4a - 12 + 5 = a^2 + 9 + 6a - 4a - 7$

$= a^2 + 2a + 2$

c) $f(a + h) - f(a)$

$f(a+h) = (a+h)^2 - 4(a+h) + 5$

$f(a) = a^2 - 4a + 5$

$f(a+h) - f(a) = (a+h)^2 - 4(a+h) + 5 - (a^2 - 4a + 5)$

$= \underbrace{a^2 + h^2 + 2ah} - 4a - 4h + 5 - a^2 + 4a - 5$

$= h^2 + 2ah + \cancel{a^2} - \cancel{a^2} - \cancel{4a} + \cancel{4a} - 4h + \cancel{5} - \cancel{5}$

$= h^2 + 2ah - 4h$

Quiz 8

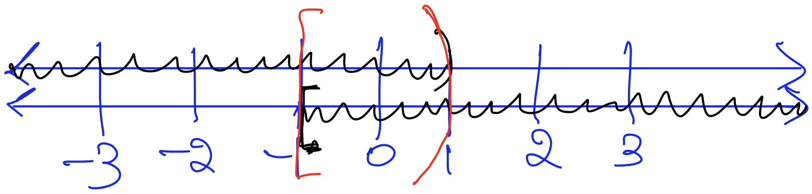
① $7x+4 < 11$ and $3-2x \le 5$. write solution in interval notation.

$\Rightarrow 7x < 11-4$ $\qquad$ $\Rightarrow -2x \le 5-3$

$\Rightarrow 7x < 7$ $\qquad$ $\Rightarrow -2x \le 2$

$\Rightarrow x < \frac{7}{7}$ $\qquad$ $\Rightarrow \frac{-2x}{-2} \ge \frac{2}{-2}$

$\Rightarrow x < 1$ $\qquad$ $\Rightarrow x \ge -1$



$[-1, 1)$

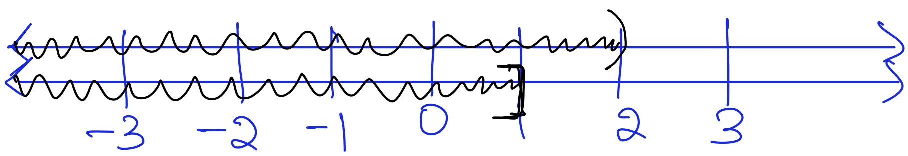
② $5x+6<16$ or $4x+8 \le 12$

$\Rightarrow 5x < 16-6$ $\qquad$ $\Rightarrow 4x \le 12-8$

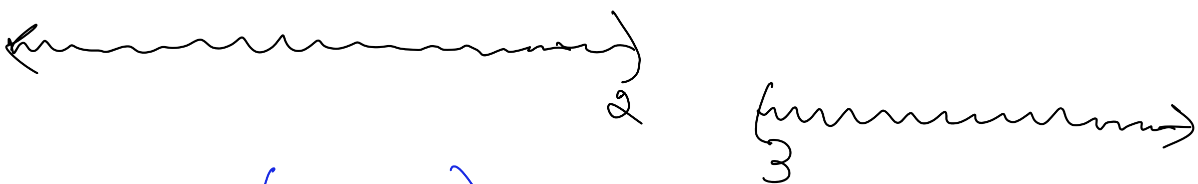
$\Rightarrow 5x < 10$ $\qquad$ $\Rightarrow 4x \le 4$

$\Rightarrow x < \frac{10}{5}$ $\qquad$ $\Rightarrow x \le \frac{4}{4}$

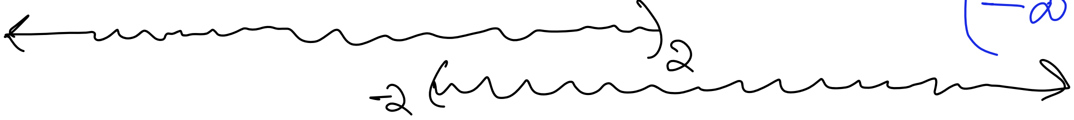
$\Rightarrow x < 2$ $\qquad$ $\Rightarrow x \le 1$



$(-\infty, 2) \cup (-\infty, 1] = (-\infty, 2)$



$(-\infty, 2) \cup (3, \infty)$



$(-\infty, 2) \cup (-2, \infty)$
$= (-\infty, \infty)$