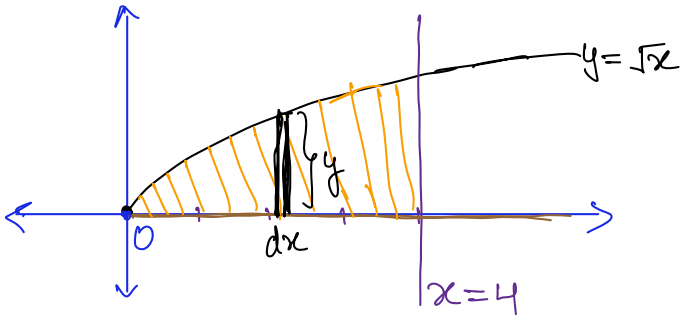
Example 1. Find area of the region bounded by $y = \sqrt{x}$, $x = 4$ and $y = 0$.



always increasing

$y' = \frac{1}{2\sqrt{x}} = \frac{1}{2} x^{-1/2} > 0$

$y'' = -\frac{1}{4} x^{-3/2} < 0$ (+ve)

always concave down.

$\int_a^b \sqrt{x}\, dx$ ← Area under $y = \sqrt{x}$ from $x = a$ to $x = b$.

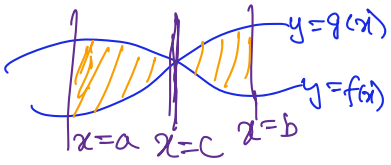
$$A = \int_0^4 \sqrt{x}\, dx$$

$$= \left. \frac{x^{1/2+1}}{\frac{1}{2}+1} \right|_0^4 = \left. \frac{x^{3/2}}{3/2} \right|_0^4 = \left. \frac{2}{3} x^{3/2} \right|_0^4$$

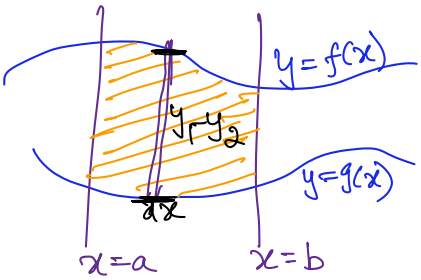
$$= \frac{2}{3} 4^{3/2} - \frac{2}{3} 0^{3/2}$$

$$= \frac{2}{3} (2)^3 - 0 = \frac{2}{3} \cdot 8 = \frac{16}{3}$$

Area between two curves: The area enclosed between two curves $y = f(x)$ and $y = g(x)$ from $x = a$ to $x = b$ is given by



$$A = \int_a^b |f(x) - g(x)|\, dx .$$



Note that

$$|f(x) - g(x)| = \begin{cases} f(x) - g(x) & \text{if } f(x) \geq g(x) \\ g(x) - f(x) & \text{if } g(x) \geq f(x) \end{cases} .$$

A property of definite integral: If f is continuous on $[a, b]$ and $a < c < b$, then

$$\int_a^b f(x)\, dx = \int_a^c f(x)\, dx + \int_c^b f(x)\, dx .$$

Example 2. Find area of the region bounded by $y = x^3$, $x = -1$, $x = 2$ and $y = 0$.

Example 3. Find area of the region bounded by the parabola $y^2 = 4x$ and the line $y = x$.