The derivative as slope of tangent (section 2.3)

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

or

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

The Four Step Process (section 2.4)

1. Replace x by $x + h$ in the equation $y = f(x)$ to obtain $y + \Delta y = f(x + h)$.
2. Subtract $y = f(x)$ from both sides to obtain $\Delta y = f(x + h) - f(x)$.
3. Divide both sides by $\Delta x = h$ to obtain $\dfrac{\Delta y}{\Delta x} = \dfrac{f(x + h) - f(x)}{h}$.
4. Take the limit $\Delta x = h \to 0$ on both sides to obtain

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} .$$

Note: The derivative $f'(x)$ gives us the instantaneous rate of change of one variable with respect to another.

Example 1.
Find the derivative of the following functions:

1. $y = \dfrac{1}{x - 1}$.
2. $y = \sqrt{x + 2}$.

Derivatives of polynomials (section 2.5)

1. The constant rule: $\frac{dc}{dx} = 0$ where c is a constant.
2. The derivative of x: $\frac{dx}{dx} = 1$.
3. The derivative of x^n, $n > 0$: $\frac{dx^n}{dx} = nx^{n-1}$.
4. The sum rule: $\frac{d}{dx}(u + v) = \frac{du}{dx} + \frac{dv}{dx}$.
5. The constant multiple rule: $\frac{d}{dx}(cu) = c\frac{du}{dx}$.

Example 2. Differentiate the following:

1. $y = 3x^4$.
2. $y = 4x^5 + 5x^3 - x^2 + 1$.