Math16500 Section 24246 Quiz 6
Fall 2022, September 26

Name: [1 pt]

Problem 1: Differentiate $f(x) = x^3 \tan x$. [3 pts]

Use product rule :-

$$\Rightarrow f'(x) = \frac{d}{dx}(x^3 \tan x) = x^3 \frac{d}{dx}(\tan x) + \frac{d}{dx}(x^3)\tan x$$

$$= x^3 \sec^2 x + 3x^2 \tan x$$

$$= \frac{x^3}{\cos^2 x} + \frac{3x^2 \sin x}{\cos x}$$

$$= \frac{x^3 + 3x^2 \sin x \cos x}{\cos^2 x}$$

any of the three is acceptable

Problem 2: Differentiate $f(x) = \dfrac{\cos x}{x}$. [3 pts]

use quotient rule :-

$$f'(x) = \frac{x \frac{d}{dx}(\cos x) - \cos x \frac{d}{dx}(x)}{x^2}$$

$$= \frac{x(-\sin x) - \cos x}{x^2} = \frac{-x \sin x - \cos x}{x^2}$$

Problem 3: Differentiate $f(x) = \sin(\sqrt{x}\,)$. [3 pts]

use chain rule :-

$$f'(x) = \frac{d}{dx}\left(\sin(\sqrt{x})\right)$$

let $z = \sqrt{x}$

$$\Rightarrow \frac{dz}{dx} = \frac{1}{2\sqrt{x}}$$

$$\Rightarrow f'(x) = \frac{d}{dx}(\sin z) = \frac{d}{dz}(\sin z)\frac{dz}{dx}$$

$$= \cos z \times \frac{1}{2\sqrt{x}}$$

$$= \frac{\cos(\sqrt{x})}{2\sqrt{x}}$$

Bonus Problem: Differentiate $f(x) = \sqrt{\csc x}$. [2 pts]

use chain rule :-

$$f'(x) = \frac{d}{dx}\left(\sqrt{\csc x}\right) = \frac{d}{dz}\left(\sqrt{z}\right)\frac{dz}{dx} = \frac{1}{2\sqrt{z}} \times (-\csc x \cot x)$$

$$= \frac{-\csc x \cot x}{2\sqrt{\csc x}}$$

$$= -\frac{1}{2}\sqrt{\csc x}\cot x.$$

let $z = \csc x$

$$\Rightarrow \frac{dz}{dx} = -\csc x \cot x$$