

Math16600 Section 23715 Quiz 5

Fall 2023, October 03

Name: Solutions

[1 pt]

Problem 1: Evaluate the integral

$$\int \tan^3 x \sec^6 x \, dx$$

let $u = \sec x \Rightarrow du = \sec x \tan x \, dx$

[5 pts]

$$\Rightarrow I = \int \tan^3 x \sec^6 x \frac{du}{\sec x \tan x} = \int \tan^2 x \sec^5 x \, du$$

$$= \int (\sec^2 x - 1) \sec^5 x \, du = \int (u^2 - 1) u^5 \, du$$

$$= \int u^7 \, du - \int u^5 \, du = \frac{u^8}{8} - \frac{u^6}{6} + C = \frac{\sec^8 x}{8} - \frac{\sec^6 x}{6} + C$$

Alternatively, let $u = \tan x \Rightarrow du = \sec^2 x \, dx$

$$\Rightarrow I = \int u^3 \sec^4 x \, du = \int u^3 (1 + u^2)^2 \, du = \int u^3 (u^4 + 2u^2 + 1) \, du$$

$$= \int u^7 \, du + \int 2u^5 \, du + \int u^3 \, du = \frac{u^8}{8} + \frac{2u^6}{6} + \frac{u^4}{4} + C = \frac{\tan^8 x}{8} + \frac{\tan^6 x}{3} + \frac{\tan^4 x}{4} + C$$

Problem 2: Evaluate the integral:

$$\int \sqrt{1-x^2} \, dx$$

let $x = \sin \theta \Rightarrow dx = \cos \theta \, d\theta$

[5 pts]

$$\Rightarrow I = \int \sqrt{1-\sin^2 \theta} \cos \theta \, d\theta = \int \cos \theta \cos \theta \, d\theta$$

$$= \int \cos^2 \theta \, d\theta = \int \frac{1 + \cos 2\theta}{2} \, d\theta = \frac{1}{2} \int d\theta + \frac{1}{2} \int \cos 2\theta \, d\theta$$

$$= \frac{1}{2} \theta + \frac{1}{2} \frac{\sin 2\theta}{2} + C = \frac{1}{2} \theta + \frac{1}{2} \sin \theta \cos \theta + C$$

$$\sin \theta = x \Rightarrow \cos \theta = \sqrt{1-\sin^2 \theta} = \sqrt{1-x^2}$$

\downarrow

$$\theta = \sin^{-1} x$$

$$\Rightarrow I = \frac{1}{2} \sin^{-1} x + \frac{1}{2} x \sqrt{1-x^2} + C$$