Math 110

2.1-2.3 Notes

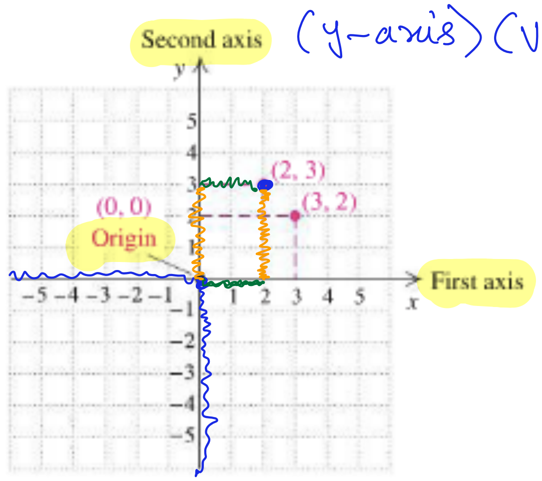
2.1 Graphs
Points and Ordered Pairs

- On a number line, each point corresponds to a **number**
- On a plane, each point corresponds to an **ordered pair**
- We use two perpendicular number lines, called **axes** to identify points on a plane
- The variable x usually represented by on the horizontal axis and the variable y on the vertical axis, so we often call such a plane an **x,y coordinate system.**
- to label a point on the x, y coordinate system, we use a pair of numbers in the form (x,y). the number in the pair are called **coordinates**

(y-axis) (vertical)

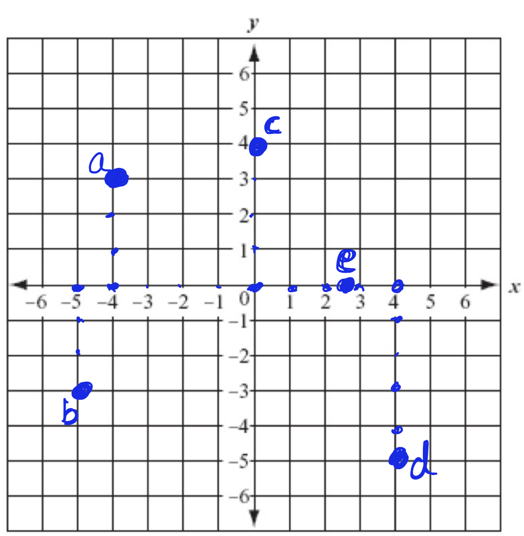
(2, 3)

(3, 2)

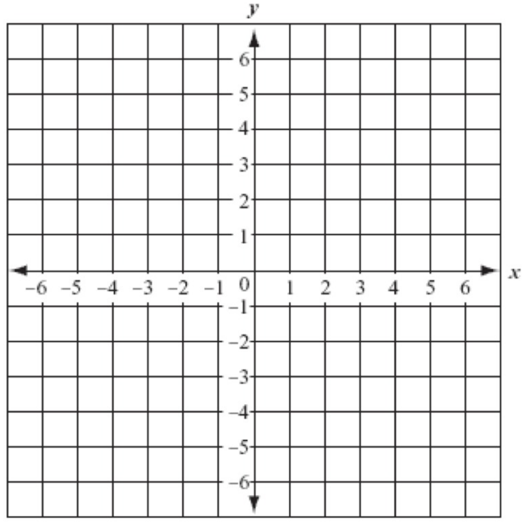


(x-axis)
(horizontal)

Example 1: Plot the points (-4,3) , (-5,-3), (0,4),(4,-5) and (2.5,0)
a b c d e



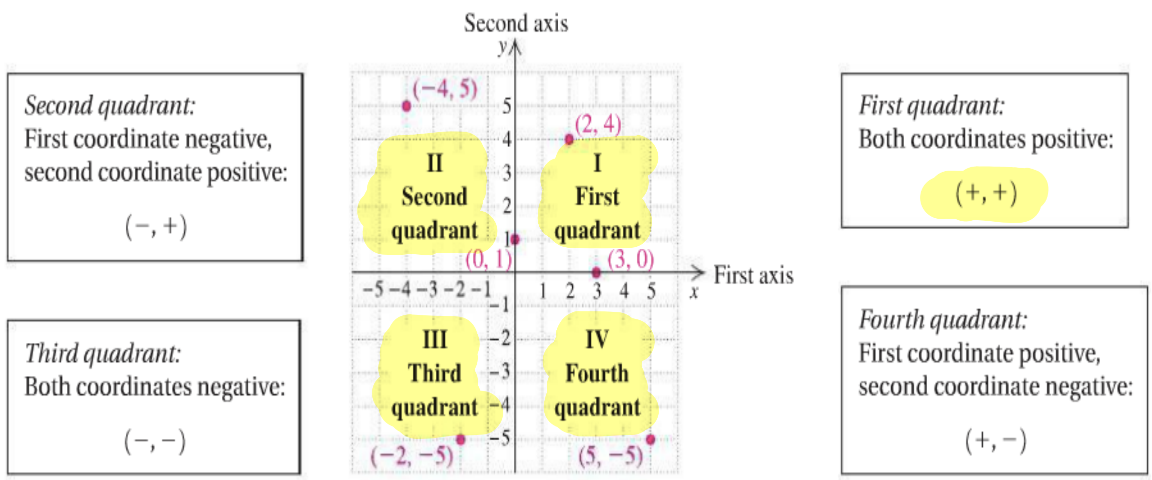
Example 2: Plots the points (-2,5), (3,-1),(0,-1), (-2,-4), and (4,0)



Homework

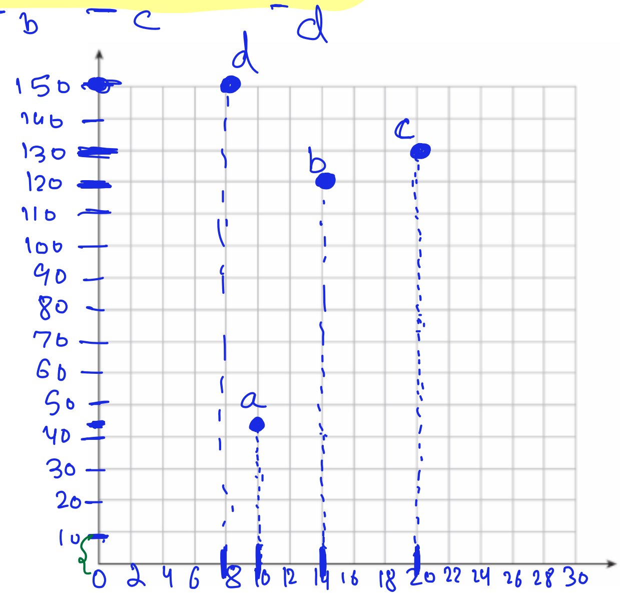
Quadrants and Scales

- The horizontal axis and the vertical axis divide the plane into four regions, or **quadrants**.



Example 3: Plots (10,44),(14,120),(20,130), and (8,150)

a b c d



Solution of Equations

Example 4: Determine whether (4,2), (-1,-4), and (2,5) are solutions of y=3x-1

$(4,2) \Rightarrow x=4, y=2$ In $y=3x-1$, Put $y=2, x=4$

$\Rightarrow 2=3(4)-1 \Rightarrow 2=12-1 \Rightarrow 2=11 \Rightarrow$ contradiction

$\Rightarrow (4,2)$ is not a solution of $y=3x-1$

$(-1,-4) \Rightarrow -4=3(-1)-1 \Rightarrow -4=-3-1 \Rightarrow -4=-4 \Rightarrow$ True

$\Rightarrow (-1,-4)$ is a solution of $y=3x-1$

$(2,5) \Rightarrow 5=3(2)-1 \Rightarrow 5=6-1 \Rightarrow 5=5 \Rightarrow (2,5)$ is a solution.

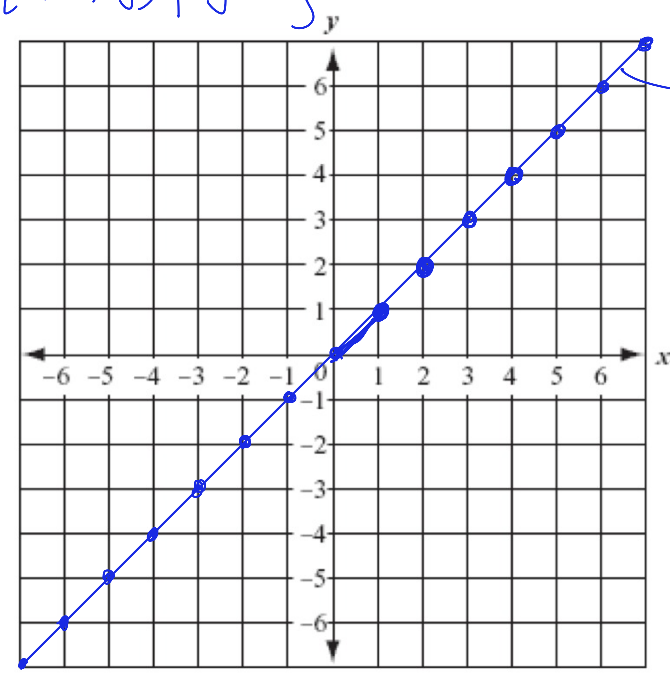
Example 5: Determine whether (7,-1) is a solution of x-y=6

Homework

Graph of $y=x$ is the set of all solutions of the equation $y=x$.

Example 6: Graph y=x

$\{(x,y) \mid y=x\}$



This line is the Graph of $y=x$

Example 7: Graph y=x+1

$x = 6 \Rightarrow y = x+1$

$= 6+1$

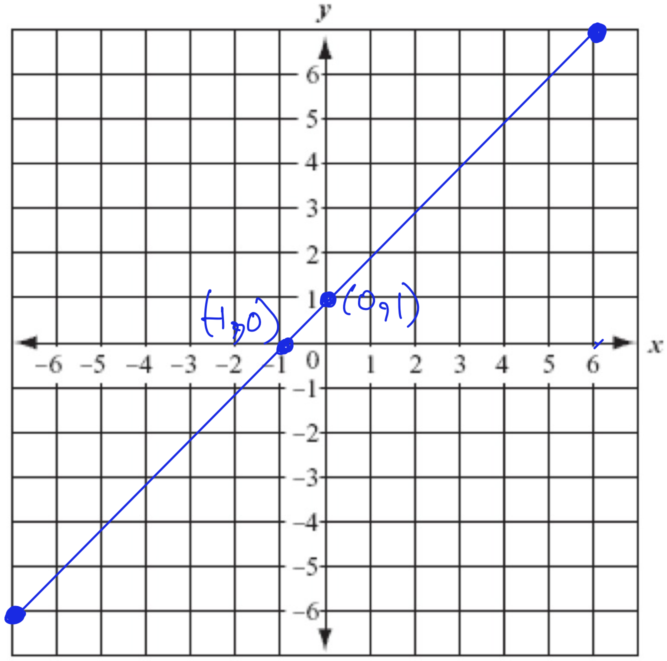
$= 7$

$(6,7)$

$x = -7 \Rightarrow y = -7+1$

$= -6$

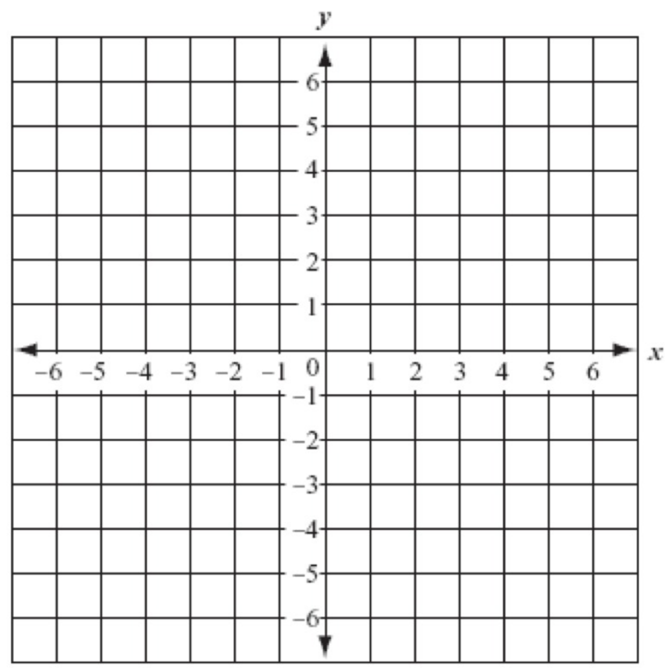
$(-7,-6)$



Example 8: Graph y=2x-1

$y = 2x-1$

Homework



Example 9: Graph $y = -\frac{1}{2}x$

$x = 6$

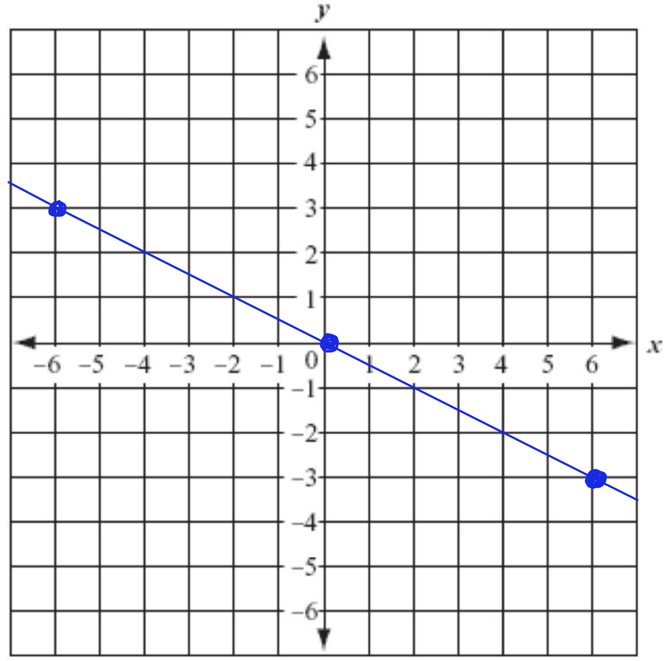
$\Rightarrow y = -\frac{1}{2} \times 6 = -3$

$(6, -3)$

$x = -6$

$\Rightarrow y = -\frac{1}{2} \times -6 = 3$

$(-6, 3)$

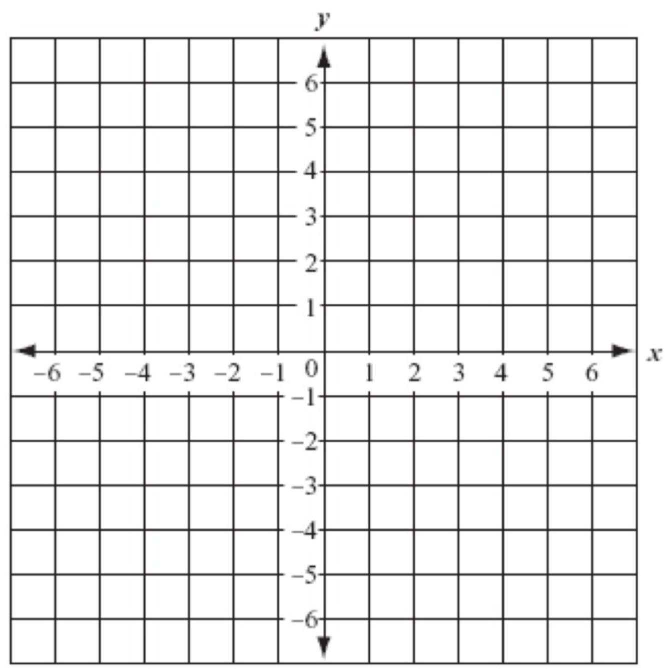


$\left.\begin{array}{l} x = 6 \\ x = -6 \end{array}\right\}$ your choices.

One can choose some diff values.

Example 10: Graph $y = -\frac{1}{3}x$

Homework



Nonlinear Equations

- For many equations, the graph is not a straight line. Graphing these **nonlinear equations** require plotting many points in order to see the general shape of the graph.

Example 11: Graph $y = |x|$

$x = 0 \Rightarrow y = |0| = 0$

$(0, 0)$

$x = 1 \Rightarrow y = |1| = 1$

$(1, 1)$

$x = -1,\ y = |-1| = 1$

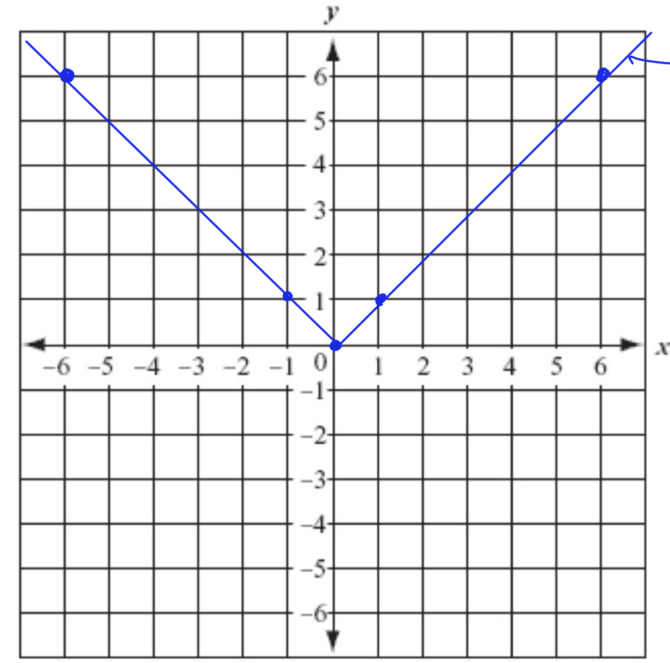
$(-1, 1)$

$x = 6,\ y = |6| = 6$

$(6, 6)$

$x = -6 \Rightarrow y = |-6| = 6$

$(-6, 6)$



Graph of $y = |x|$

Example 12: Graph $y = x^2 - 5$

$(-x)^2 = x^2 \Rightarrow (-x)^2 - 5 = x^2 - 5$

$x = 0 \Rightarrow y = 0^2 - 5 = -5$

$(0, -5)$

$x = 1 \Rightarrow y = 1^2 - 5 = -4$

$(1, -4)$

$x = 2 \Rightarrow y = 2^2 - 5 = -1$

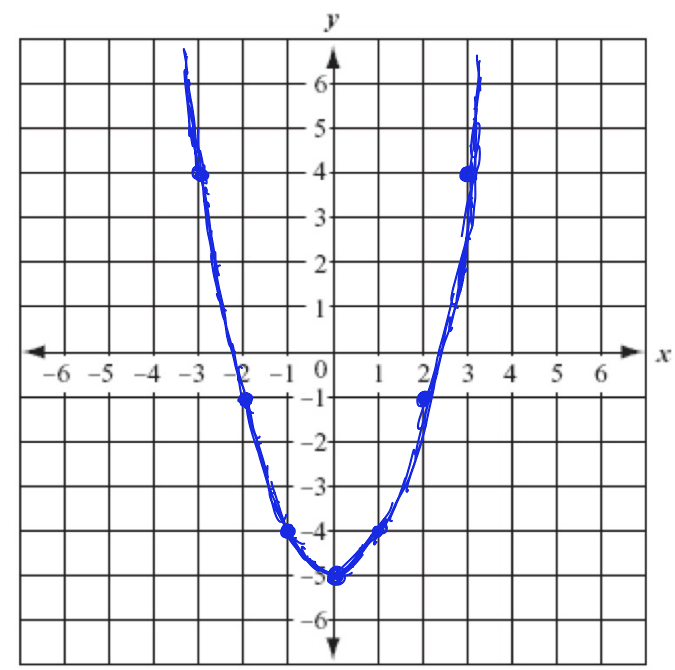
$(2, -1)$

$x = 3 \Rightarrow y = 3^2 - 5 = 4$

$(3, 4)$

$x = 4 \Rightarrow y = 4^2 - 5 = 11$

$(4, 11)$



$x = -1 \Rightarrow y = (-1)^2 - 5 = -4$

$(-1, -4)$

$x = -2 \Rightarrow y = (-2)^2 - 5 = -1$

$(-2, -1)$

$x = -3 \Rightarrow y = (-3)^2 - 5$

$= 9 - 5 = 4$

$(-3, 4)$

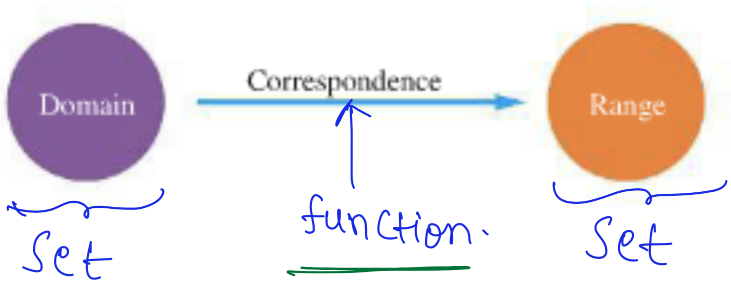
2.2 Functions

Domain and Range

A function is a special kind of correspondence between two sets. For example,

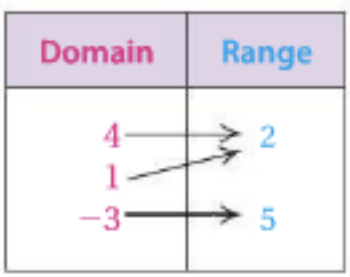
To each person in a class	there corresponds	a date of birth.
To each bar code in a store	there corresponds	a price.
To each real number	there corresponds	the cube of that number.

In each example, the first set is called the **domain**. The second set is called the **range**. For any member of the domain, there is *exactly one* member of the range to which it corresponds. This kind of correspondence is called a **function**.



Example 1: Determine whether each correspondence is a function

a)

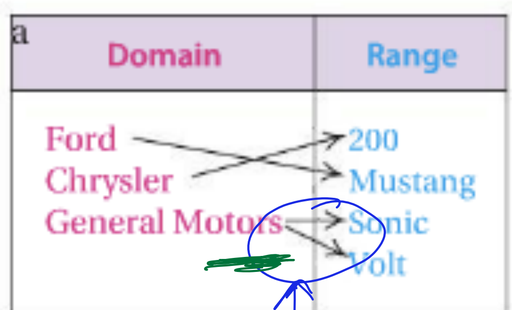


f

YES

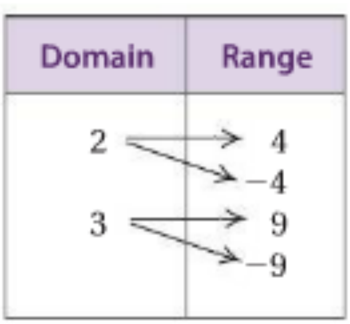
$f(4)=2$, $f(1)=2$, $f(-3)=5$

b)



NOT A FUNCTION

Example 2: Determine whether each correspondence is a function



NO

FUNCTION

A *function* is a correspondence between a first set, called the *domain*, and a second set, called the *range*, such that each member of the domain corresponds to *exactly one* member of the range.

Example 3: Determine whether each correspondence is a function.

a) The correspondence that assigns to a person his or her weight

YES

b) The correspondence that assigns to the numbers -2, -1, 0, 1, 2 each numbers square

$-2 \to 4$
$-1 \to 1$
$0 \to 0$
$1 \to 1$
$2 \to 4$

YES

c) The correspondence that assigns to a best- selling author the titles of the books written by that author

NO

Example 4: For the correspondence {(-6,7), (1,4), (-3,4),(4,-5)}

$$\left\{ \begin{array}{l} -6 \to 7 \\ 1 \to 4 \\ -3 \to 4 \\ 4 \to -5 \end{array} \right\}$$

a) Write the domain

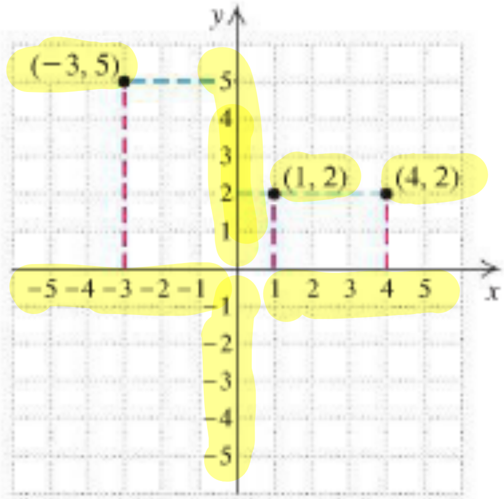
$\{-6, 1, -3, 4\}$

b) Write the range

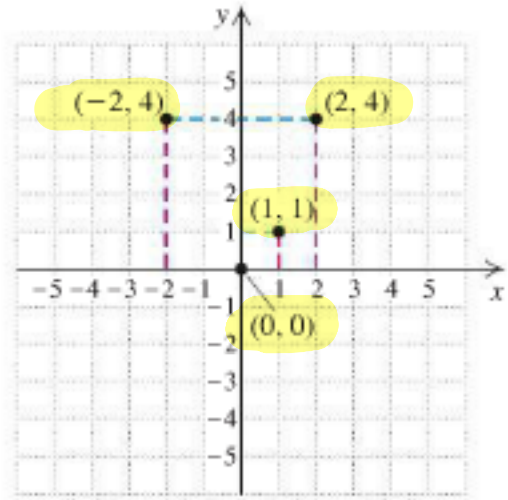
$\{7, 4, -5\}$

Functions and Graphs

The function in Example 1(a) can be written $\{(-3, 5), (1, 2), (4, 2)\}$ and the function in Example 2(b) $\{(-2, 4), (0, 0), (1, 1), (2, 4)\}$. We graph these functions in black as follows.



The function $\{(-3, 5), (1, 2), (4, 2)\}$
Domain is $\{-3, 1, 4\}$
Range is $\{5, 2\}$

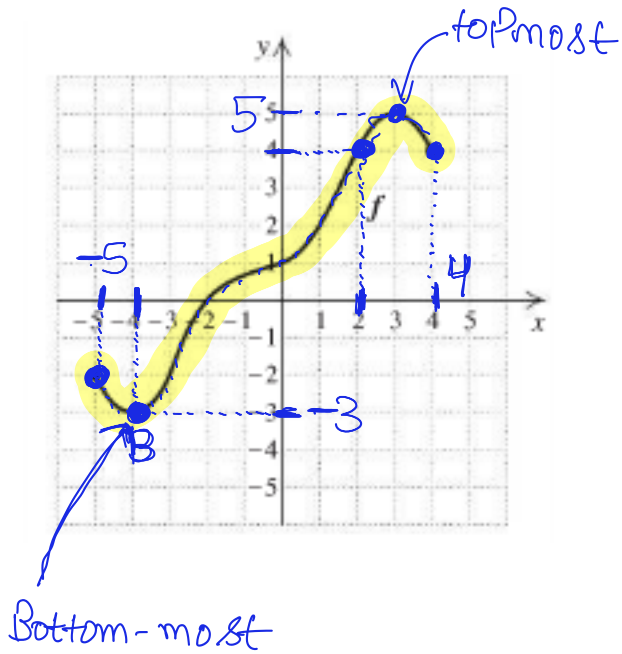


The function $\{(-2, 4), (0, 0), (1, 1), (2, 4)\}$
Domain is $\{-2, 0, 1, 2\}$
Range is $\{4, 0, 1\}$

Example 5: For the function f, determine each of the following.

a) the member of the range that is paired with 2

4



b) the domain of f

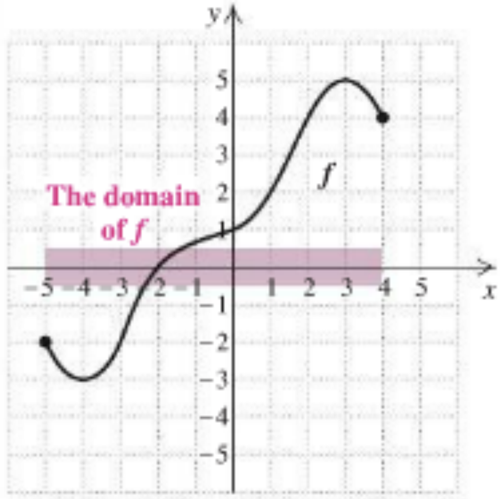
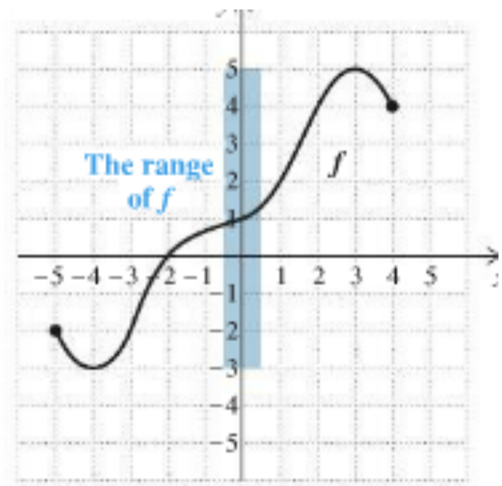
$\{x \mid x \geq -5 \text{ and } x \leq 4\}$

c) the member of the domain paired with -3

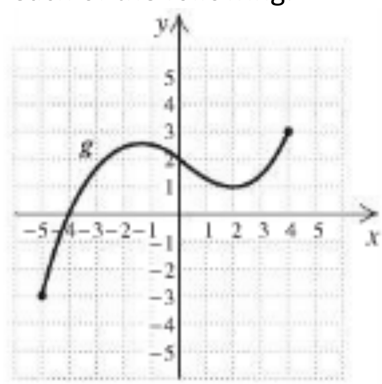
−4

d) the range of f

$\{y \mid y \geq -3 \text{ and } y \leq 5\}$

Domain	Range
	

Example 6: For the function f, determine each of the following.



a) the member of the range that is paired with 2

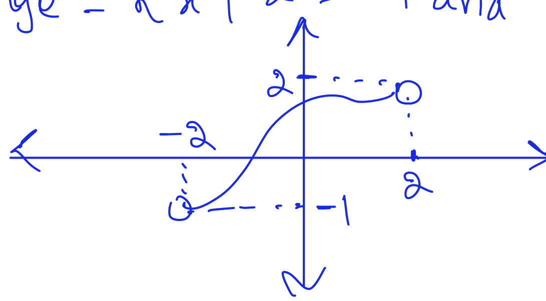
b) the domain of f

HW

c) the member of the domain paired with -3

Domain = $\{x \mid x > -2 \text{ and } x < 2\}$

Range = $\{x \mid x > -1 \text{ and } x < 2\}$



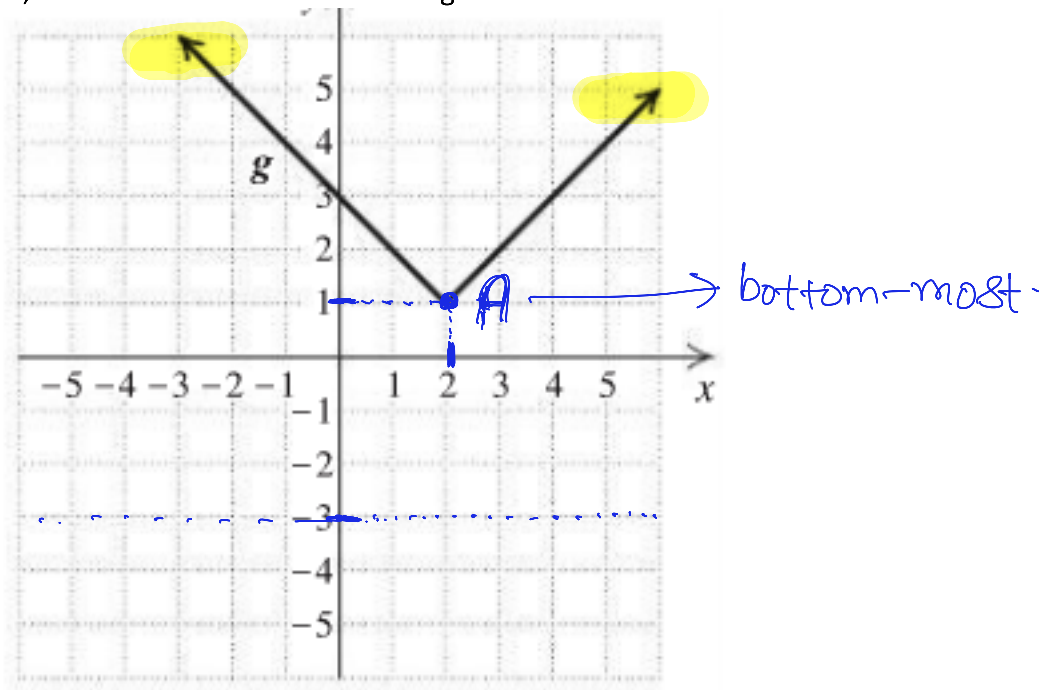
d) the range of f

Note:

A closed dot on a graph, such as in Example 4, indicates that the point is part of the function. An open dot indicates that the point is *not* part of the function.

The dots in Example 4 also indicate endpoints of the graph. A function may have a domain and/or a range that extends without bound toward positive infinity or negative infinity.

Example 7: For the function f, determine each of the following.



a) the member of the range that is paired with 2

1

b) the domain of f

$\{x \mid x \text{ is a real number}\} = \{x \mid x > -\infty \text{ and } x < \infty\}$

c) the member of the domain paired with -3

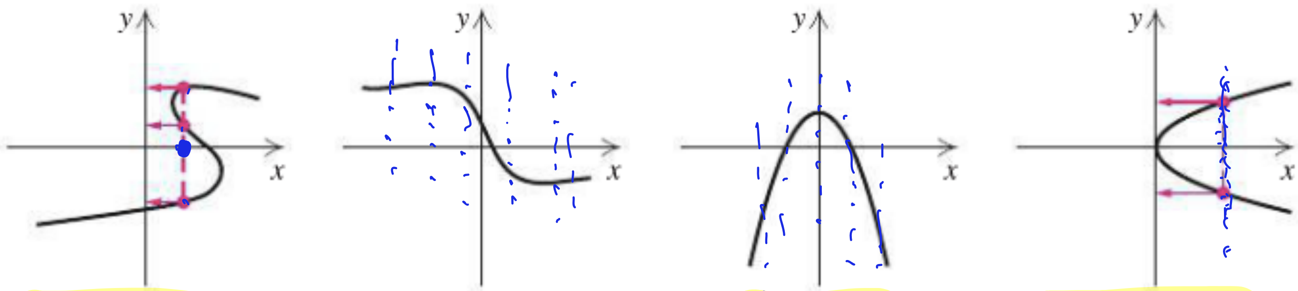
No member of domain is paired with -3.

d) the range of f

$\{y \mid y \geq 1\}$

THE VERTICAL-LINE TEST

If it is possible for a vertical line to cross a graph more than once, then the graph is not the graph of a function.



Not a function. Three y-values correspond to one x-value.

A function

A function

Not a function. Two y-values correspond to one x-value.

RELATION

A *relation* is a correspondence between a first set, called the *domain*, and a second set, called the *range*, such that each member of the domain corresponds to *at least one* member of the range.

Function Notation and Equations

- We often think of an element of the domain of a function as an **input** and its corresponding element of the range as an **output**

$$\boldsymbol{f = \{(\underline{-3, 1}), (1, \underline{-2}), (3, 0), (4, 5)\}}$$

$\boldsymbol{f(-3)} =$ 1 $\qquad$ $\boldsymbol{f(1)} =$ -2 $\qquad$ $\boldsymbol{f(3)} =$ 0 $\qquad$ $\boldsymbol{f(4)} =$ 5

CAUTION! *$f(x)$ does not mean f times x.*

- Most functions are described by equations.

$$f(x) = \underset{\text{Double}}{2}\overset{\text{Input}}{x} + \underbrace{3}_{\text{Add 3}}$$

$$f(4) = 2 \cdot 4 + 3$$
$$= 11. \longleftarrow \text{Output}$$

Example 7: Find each indicated function value

a) $f(5), for\ f(x) = 3x + 2$

$f(5) = 3\times 5 + 2 = 15 + 2 = \underline{\underline{17}}$

b) $h(4),\ for\ h(x) = 7$

$h(4) = 7$

c) $g(-2), for\ g(r) = 5r^2 + 3r$

$g(-2) = 5 \cdot (-2)^2 + 3 \cdot (-2) = 5 \cdot 4 + 3 \cdot (-2)$
$= 20 - 6 = \underline{\underline{14}}$

d) $F(a) + 1\ ,\ for\ F(x) = 3x + 2$

$F(a) + 1 = 3 \cdot a + 2 + 1 = \underline{\underline{3a + 3}}$

e) $F(a + 1), for\ F(x) = 3x + 2$

$F(a+1) = 3(a+1) + 2 = 3a + 5$

Example 8: Let $f(x) = 3x - 7$

a) what output corresponds to an input of 5?

$$f(5) = 3 \cdot 5 - 7 = 15 - 7 = \underline{\underline{8}}$$

b) what input corresponds to an output of 5?

Find x for which $f(x) = 5$

$3x - 7 = 5 \Rightarrow 3x = 5 + 7 \Rightarrow 3x = 12$

$\Rightarrow x = \frac{12}{3} \Rightarrow \underline{\underline{x = 4}}$

$f(4) = 3(4) - 7$
$= 12 - 7 = 5$

Example 9: For the equation, determine the domain of f

a) $f(x) = |x|$

$f(x) = \frac{|x|}{1} \Rightarrow$ defined for every real number.

$\Rightarrow$ Domain of $f = \{x \mid x \text{ is a real number}\}$

b) $f(x) = \frac{x}{2x-6}$

Find x for which denominator is 0.

$2x - 6 = 0 \Rightarrow 2x = 6 \Rightarrow x = \frac{6}{2} \Rightarrow \underline{\underline{x = 3}}$

Domain of $f = \underline{\{x \mid x \neq 3\}}$ or $\{x \mid x$ is any real number not equal to $3\}$

> **CAUTION!** The denominator cannot be 0, but the numerator can be any number.

Piecewise Defined Function

Example 10: Find each function value for the function given

$$\boldsymbol{f(x)} = \begin{cases} \boldsymbol{2x\,, if\ x < 3} \\ \boldsymbol{x+1, if\ x \geq 3} \end{cases}$$

a) $f(4)$ Check which part of domain x is in.?

$4>3 \Rightarrow$ 2nd Part

$f(4) = 4+1 = \underline{\underline{5}}$

b) $f(-10)$ See that $-10 < 3 \Longrightarrow 1^{st}$ part

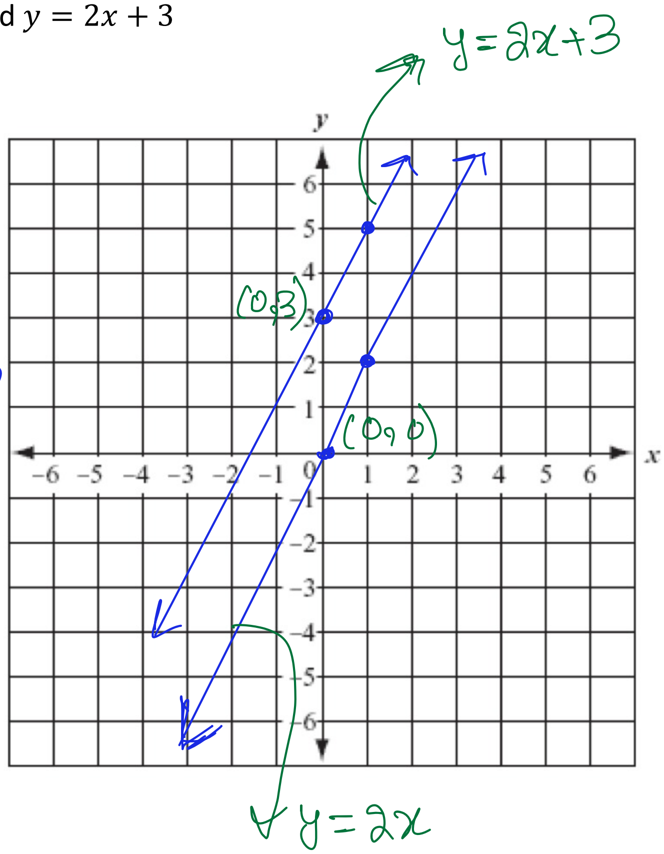
$f(-10) = 2(-10) = \underline{\underline{-20}}$

(*) Absolute value function : $f(x) = |x|$ is also Piecewise defined

$$f(x) = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

2.3 Slope Intercept Form

Example 1: Graph $y = 2x$ and $y = 2x + 3$



$x = 0 \Rightarrow y = 2(0) = 0$

$x = 1 \Rightarrow y = 2(1) = 2$

$(0,0)$, $(1,2)$

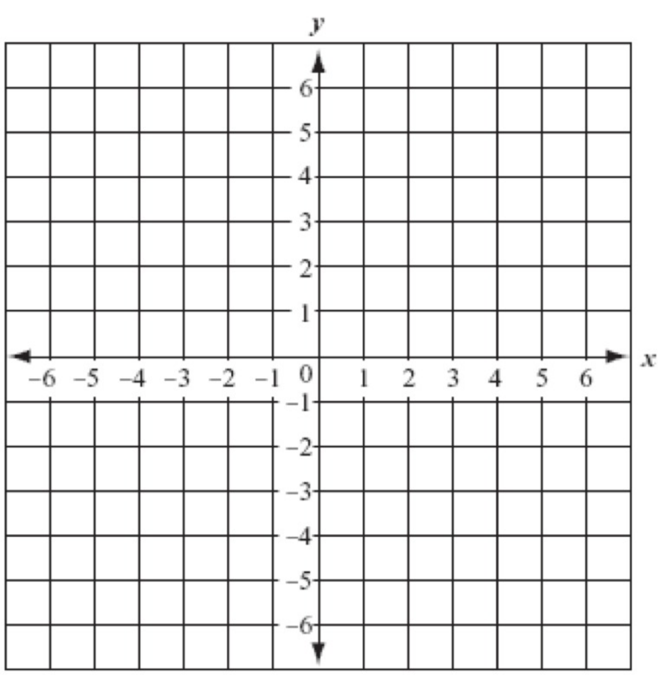
$x = 0 \Rightarrow y = 2(0)+3 = 3$

$x = 1 \Rightarrow y = 2(1)+3$

$= 2+3$

$= 5$

Example 2: Graph $f(x) = \frac{1}{3}x$ and $f(x) = \frac{1}{3}x - 2$



HW

Example 3: Find the y- intercept

a) $y = -5x + 7$

$x = 0$ and find corresponding y-value

$y = -5 \cdot 0 + 7 = 0 + 7 = 7 \Rightarrow$ y-intercept $= (0, 7)$

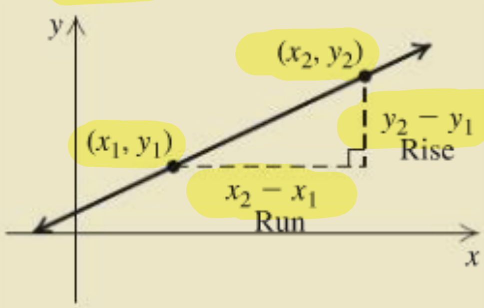
b) $f(x) = 5.3x - 12$

$x = 0$ Find $f(0)$

$f(0) = 5.3 \times 0 - 12 = 0 - 12 = \underline{\underline{-12}}$ $\Rightarrow$ y-intercept $= (0, -12)$

SLOPE

The *slope* of the line passing through (x_1, y_1) and (x_2, y_2) is given by



$$m = \frac{\text{rise}}{\text{run}} = \frac{\text{vertical change}}{\text{horizontal change}}$$

$$= \frac{\text{the difference in } y}{\text{the difference in } x}$$

$$= \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}.$$

In the definition above, (x_1, y_1) and (x_2, y_2)—read "x sub-one, y sub-one and x sub-two, y sub-two"—represent two different points on a line. It does not matter which point is considered (x_1, y_1) and which is considered (x_2, y_2) so long as coordinates are subtracted in the same order in both the numerator and the denominator.

SLOPE–INTERCEPT FORM

Any equation of the form $y = mx + b$ is said to be written in *slope–intercept* form and has a graph that is a straight line.

The slope of the line is m.

The y-intercept of the line is $(0, b)$.

Example 4: Determine the slope and the y-intercept of the line given by $y = -\frac{1}{3}x + 2$

$\Rightarrow b = 2 \Rightarrow$ y-intercept $= (0, 2)$

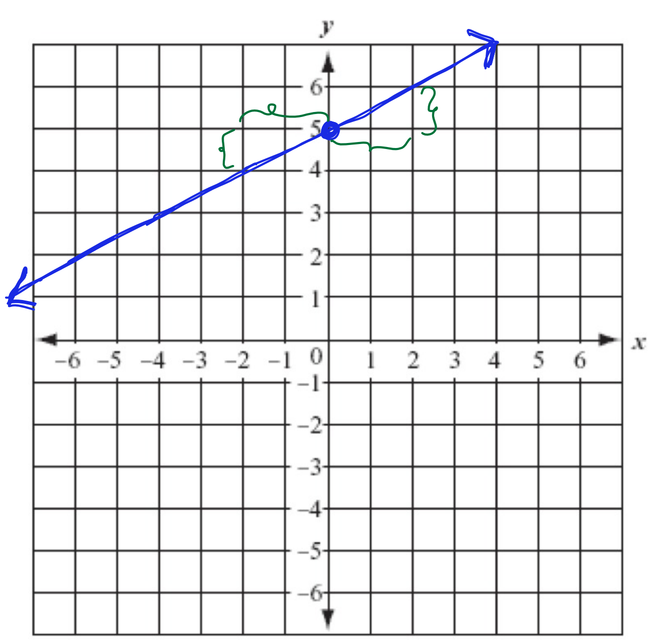
$\Rightarrow m = -\frac{1}{3} \Rightarrow$ slope $= -\frac{1}{3}$

Example 5: Find the equation of a line whose slope is 3 and y-intercept is (0,-1)

$m = 3$, $b = -1$

$\Rightarrow$ Equation of the line is $y = 3x - 1$

Example 6: Determine the slope and y- intercept of the line given by $f(x) = \frac{1}{2}x + 5$. Then draw the graph.



$\Rightarrow y = \frac{1}{2}x + 5$

$\Rightarrow$ slope $= \frac{1}{2}$

$\Rightarrow$ y-intercept $= (0, 5)$

Example 7: Determine the slope and the y-intercept for the equation $5x - 4y = 8$

$$\Rightarrow 5x - 4y = 8 \Rightarrow \begin{array}{c} 5x - 4y = 8 \\ -5x \qquad\quad -5x \end{array} \Rightarrow -4y = -5x + 8$$

$$\Rightarrow \frac{-4y}{-4} = \frac{-5x+8}{-4} \Rightarrow y = \frac{-5x}{-4} + \frac{8}{-4}$$

$$\Rightarrow y = \frac{5}{4}x - 2 \Rightarrow \text{slope} = \frac{5}{4},\ \text{y-intercept} = (0, -2)$$

Applications

Example 8: A computer recycling business uses the function given by $V(t) = -400t + 1200$ to determine the salvage value V(t), in dollard, of a particular computer t years after its purchase

a) what ~~is~~ do the numbers -400 and 1200 signify?

-400 ↓ slope, 1200 ↓ y-intercept.

-400 signifies the rate of depreciation of the value of a computer.

1200 signifies the initial value.

b) How long will it take the machine to completely depreciate?

Find t for which $V(t) = 0$

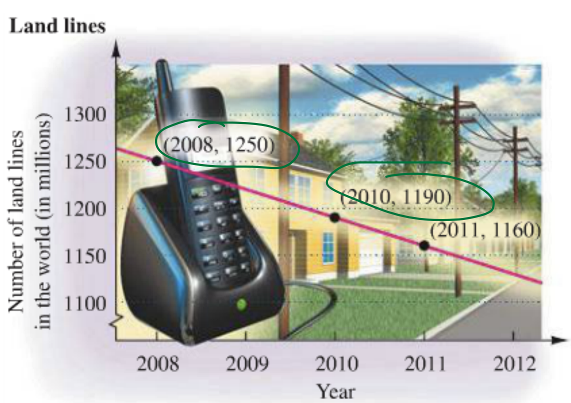
$$-400t + 1200 = 0 \Rightarrow -400t = -1200 \Rightarrow t = \frac{-1200}{-400} = \boxed{3 \text{ yrs.}}$$

Example 9: As more people use cell phones as their primary phone line, the number of ladg lines in the world has been changing, as show in the following graph. Use the graph to find the rate at which this number is changing.

rate at which this number is changing = slope

$(2008, 1250)$ → (x_1, y_1)

$(2010, 1190)$ → (x_2, y_2)



Source: International Telecommunication Union

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1190 - 1250}{2010 - 2008} = \frac{-60}{2} = \underline{\underline{-30}}$$

Quiz 3

① $\dfrac{8x^{-3}y^{5}}{2x^{4}y^{-2}}$ → Simplify so that there are no negative exponents.

$\dfrac{8}{2} \times \dfrac{x^{-3}}{x^{4}} \times \dfrac{y^{5}}{y^{-2}} = 4 \times x^{-3-4} \times y^{5-(-2)}$

$= 4x^{-7}y^{7}$

$= 4\left(\dfrac{1}{x}\right)^{7} y^{7} = 4 \times \dfrac{1}{x^{7}} \times y^{7}$

$= \dfrac{4y^{7}}{x^{7}}$

② A and B are two points on $y = 2x - 3$

1) x-coord. of A is 4. Find y-coord. of A.

$y = 2(4) - 3 = 8 - 3 = 5$ and $A = (4, 5)$

2) y-coord. of B is 1. Find x-coord. of B.

$1 = 2x - 3 \Rightarrow 1 + 3 = 2x \Rightarrow 2x = 4$

$\Rightarrow x = \dfrac{4}{2} \Rightarrow x = 2 \Rightarrow B = (2, 1)$