Math16500 Section 24246 Quiz 03
Fall 2022, September 07

Name: [1 pt]

Problem 1: Express the following function as a composition $f \circ g$:

$$h(x) = \frac{\cos x}{1 + \cos^3 x}$$

Note that you are not allowed to choose $f(x) = x$ or $g(x) = x$. [4 pts].

h is a rational function in $\cos(x)$ so we can let $\boxed{g(x) = \cos x}$ $\Rightarrow h(x) = \frac{g(x)}{1+[g(x)]^3} \Rightarrow \boxed{f(x) = \frac{x}{1+x^3}}$

Check that $(f \circ g)(x) = f(g(x)) = f(\cos x) = \frac{\cos x}{1 + \cos^3 x} = h(x)$

$\cos^3 x \to (\cos x)^3$

Problem 2: Evaluate the difference quotient

$$\frac{f(2+h) - f(2)}{h}$$

if $f(x) = 4 - x^2$. What happens when h tends to 0 but is not exactly equal to 0? [5 pts].

$f(2+h) = 4 - (2+h)^2 = 4 - (4 + h^2 + 2 \times 2 \times h) = \cancel{4} - \cancel{4} - h^2 - 4h$

$\Rightarrow f(2+h) = -h^2 - 4h$

$f(2) = 4 - 2^2 = 4 - 4 = 0$

$\Rightarrow \boxed{\frac{f(2+h) - f(2)}{h} = \frac{-h^2 - 4h}{h}}$

Note that we cannot cancel by h at this point since we do not know whether $h \neq 0$.

when $h \to 0$ but $h \neq 0$, we have

$\frac{f(2+h) - f(2)}{h} = \frac{-h^2 - 4h}{h} = \frac{h(-h-4)}{h}$

$= -h - 4 \longrightarrow -4$ as $h \to 0$

Bonus Problem: Given the graph of a function $y = f(x)$, describe how we may obtain the graph of the equation $y = f(|x|)$. [2 pts].

Given the graph of $y = f(x)$, the graph of $y = f(|x|)$ is obtained by replacing the portion of $y = f(x)$ in the left half plane (that is portion over $-ve$ x-axis) with the mirror image of the portion of $y = f(x)$ in the right half plane. The mirror image is taken by considering the y-axis as a mirror.