Common Logarithms on a Calculator

ESSENTIALS

Common logarithms, abbreviated **log**, are logarithms with base 10. For example, $\log 68$ means $\log_{10} 68$.

Example

- Use a calculator to approximate each number to four decimal places
  a) $\log 26$
  b) $10^{4.213}$

a) Press LOG 2 6) ENTER.
$\log 26 \approx 1.4150$

b) Press 10^x 4 . 2 1 3) ENTER.
$10^{4.213} \approx 16,330.5195$

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EXAMPLE 1	YOUR TURN 1
Use a calculator to approximate $\log 0.216$ to four decimal places. Press LOG . 2 1 6) ENTER. $\log 0.216 \approx$ ☐	Use a calculator to approximate $\log 0.008$ to four decimal places.
EXAMPLE 2	**YOUR TURN 2**
Use a calculator to approximate $\frac{\log 5}{\log 8}$ to four decimal places. Note that we are dividing two separate logarithms. Press LOG 5) ☐ LOG 8) ENTER. $\frac{\log 5}{\log 8} \approx$ ☐	Use a calculator to approximate $\frac{\log 7}{\log 2}$ to four decimal places.

$\log 2 = 0.301$

EXAMPLE 3	YOUR TURN 3
Use a calculator to approximate $\log \frac{5}{8}$ to four decimal places. Note that we are finding the logarithm of a fraction. Press LOG 5 (8) ENTER. $\log \frac{5}{8} \approx$ [ ] -0.2041	Use a calculator to approximate $\log \frac{7}{2}$ to four decimal places. $\log \frac{7}{2} = 0.5441$
EXAMPLE 4	**YOUR TURN 4**
Use a calculator to approximate $10^{2.627}$ to four decimal places. Press 10^x 2 . 6 2 7) ENTER. $10^{2.627} \approx$ [ ] 423.6430	Use a calculator to approximate $10^{3.361}$ to four decimal places. $10^{3.361} = 2296.1486$

YOUR NOTES Write your questions and additional notes.

The Base *e* and Natural Logarithms on a Calculator

ESSENTIALS

$\log_e x = \ln x$

The Number *e*

Euler's number → $e \approx 2.7182818284\ldots$

Logarithms base e are called **natural logarithms**, or Napierian logarithms, and are abbreviated "ln." For example, $\ln 5$ means $\log_e 5$.

Examples

- Use a calculator to approximate $\ln 354$ to four decimal places.

  Press [LN] [3] [5] [4] [)] [ENTER].

  $\ln 354 \approx 5.8693$

- Use a calculator to approximate $e^{2.012}$ to four decimal places.

  Press [e^x] [2] [.] [0] [1] [2] [)] [ENTER].

  $e^{2.012} \approx 7.4783$

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EXAMPLE 1	YOUR TURN 1
Use a calculator to approximate $\ln 6 + 3$ to four decimal places. Be sure that only the 6 is in the parentheses. Press [LN] [6] [ ] [+] [3] [ENTER]. $\ln 6 + 3 \approx$ [ ]	Use a calculator to approximate $\ln 7 - 3$ to four decimal places. $\ln 7 - 3 = -1.0541$ ↳ 1.9459
EXAMPLE 2	**YOUR TURN 2**
Use a calculator to approximate $e^{-1/2}$ to four decimal places. Press [e^x] [(−)] [1] [/] [2] [)] [ENTER]. $e^{-1/2} \approx$ [ ]	Use a calculator to approximate $e^{-10/3}$ to four decimal places. $e^{-10/3} = 0.0356$

YOUR NOTES Write your questions and additional notes.

* $\log \quad 10^x$
* $\ln \quad e^x$

Changing Logarithmic Bases

ESSENTIALS

The Change-of-Base Formula

For any logarithmic bases a and b, and any positive number M,

$$\log_b M = \frac{\log_a M}{\log_a b}.$$

$= \frac{1}{\log_a b} \log_a M$ or $\log_a M = \log_b M \times \log_a b$

$\log_b x$ → want to change this base to a.

Example

- Find $\log_5 12$ using the change-of-base formula. Round to four decimal places.

  We use the change-of-base formula with $a = 10$, $b = 5$, and $M = 12$.

$$\log_5 12 = \frac{\log_{10} 12}{\log_{10} 5} \approx \frac{1.079181246}{0.6989700043} \approx 1.5440$$

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EXAMPLE 1

Find $\log_4 0.3$ using the change-of-base formula. Round to four decimal places.

Using common logarithms, we have $a = 10$, $b = 4$, and $M = 0.3$.

$$\log_4 0.3 = \frac{\log_{10} 0.3}{\log_{10} 4}$$

$$\approx \frac{-0.5228787453}{0.6020599913}$$

$$\approx \boxed{\quad} -0.868$$

YOUR TURN 1

Find $\log_3 0.1$ using the change-of-base formula and common logarithms. Round to four decimal places.

$0.1 = \frac{1}{10} = 10^{-1}$

$\log_{10} 0.1 = \log_{10} 10^{-1}$

$= -1 \log_{10} 10$

$\log_3 0.1 = \frac{\log_{10} 0.1}{\log_{10} 3} = \frac{-1}{\log_{10}(3)}$

$= \frac{-1}{0.477}$

$= -2.0959$

$10^3 = 1000$

$\log_{10} 3$

EXAMPLE 2

Find $\log_{2.5} 25$ using the change-of-base formula. Round to four decimal places.

We will use natural logarithms, so $a = e$, $b = 2.5$, and $M = 25$.

$$\log_{2.5} 25 = \frac{\log_e 25}{\boxed{\quad}}$$

$$\approx \frac{3.218875825}{0.9162907319}$$

$$\approx \boxed{\quad}$$

YOUR TURN 2

Find $\log_{0.15} 15$ using the change-of-base formula and natural logarithms. Round to four decimal places.

$\log_{0.15} 15 = \frac{\ln 15}{\ln 0.15}$

$= \frac{2.7081}{-1.8971} = -1.427$

$\log_{10} 10^{-3}$

$= -3$

$\log_{10} 10^3 = 3$

$\log_{10} 3$

10^3

YOUR NOTES Write your questions and additional notes.

Graphs of Exponential Functions and Logarithmic Functions, Base *e*

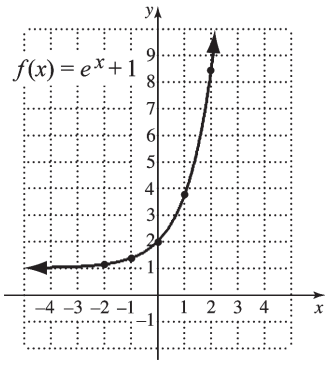
ESSENTIALS $\ln x$ and e^x

Examples

- Graph the following functions and state the domain and range of each.

  a) $f(x) = e^x + 1$ b) $f(x) = \ln x - 3$

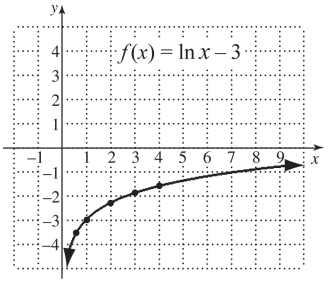
  a) Use a calculator to approximate function values, plot the points, and connect them with a smooth curve.



x	$f(x) = e^x + 1$
0	2
1	3.7
2	8.4
−1	1.4
−2	1.1

The domain is $\mathbb{R}$. The range is $(1, \infty)$.

  b) Use a calculator to approximate function values, plot the points, and connect them with a smooth curve.



x	$f(x) = \ln x - 3$
$\frac{1}{2}$	−3.7
1	−3
2	−2.3
3	−1.9
4	−1.6

The domain is $(0, \infty)$. The range is $\mathbb{R}$.

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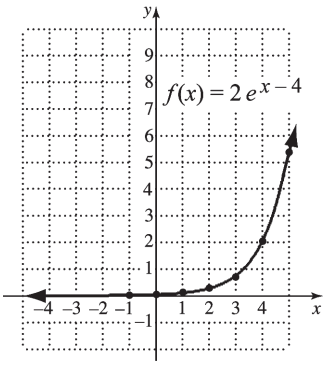
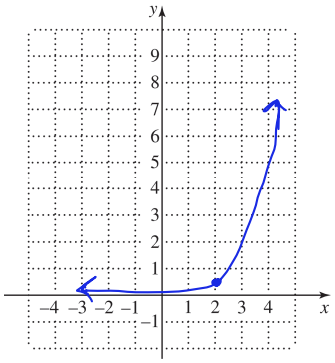
EXAMPLE 1	YOUR TURN 1
Graph $f(x) = 2e^{x-4}$ and state the domain and range. Approximate function values, plot the points, and connect them with a smooth curve. (continued)	Graph $f(x) = 0.5e^{x-2}$ and state the domain and range. $f(2) = 0.5\ e^{2-2} = 0.5 \times 1 = 0.5$ (continued)

$f(2) = 2e^{2-4} = 2e^{-2}$, $f(5) = 2e^{5-4} = 2e^{1}$

x	$f(x) = 2e^{x-4}$
0	0.04
1	0.10
2	0.27
3	0.74
4	2
5	5.4
−1	0.01



The domain is $(-\infty, \infty)$. The range is $(0, \infty)$.



Domain $= (-\infty, \infty) = \mathbb{R}$
Range $= (0, \infty)$

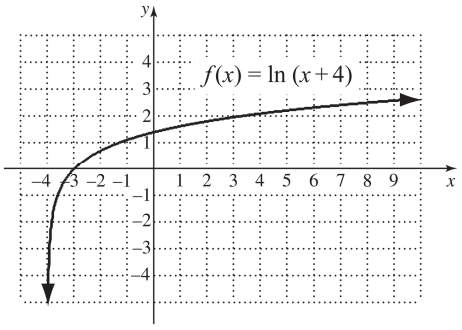
EXAMPLE 2

Graph $f(x) = \ln(x+4)$ and state the domain and range.

Approximate the function values, plot the points, and connect then with a smooth curve.

$x + 4 > 0$
$x > -4$

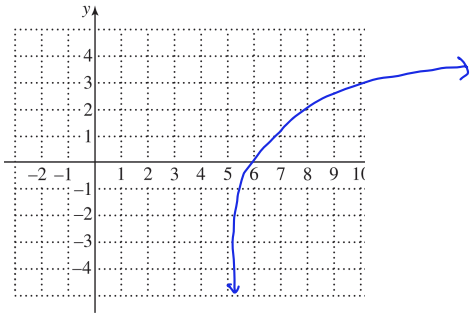
x	$f(x) = \ln(x+4)$
0	1.4
5	2.2
10	2.6
−1	1.1
−2	0.7
−3	0



The domain is $(-4, \infty)$. The range is $\mathbb{R}$.

YOUR TURN 2

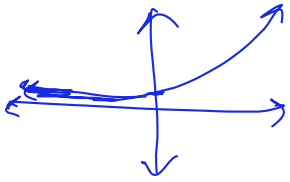
Graph $\ln(x-5)$ and state the domain and range.



$x - 5 > 0$
$x > 5$

Domain $= (5, \infty)$
Range $= \mathbb{R}$

YOUR NOTES Write your questions and additional notes.



Practice Exercises

$x+1>0 \Rightarrow x>-1$

$(-1,\infty)$

Readiness Check

Classify each of the following statements as true or false.

1. The domain of the function $f(x)=\ln(x+1)$ is $(-1,\infty)$. True.

Range of e^x is $(0,\infty)$

2. The range of the function $f(x)=5e^x$ is $[0,\infty)$. False

3. The base of a natural logarithm is 10. False

4. The expression $\log_e 5$ is equivalent to $\ln 5$. True

Common log → 10

natural log → e

ln → log e

log → $\log_{10}$

Logarithms on a Calculator

Use a calculator to find each of the following to four decimal places.

5. $\ln 19$

6. $\ln\frac{11}{9}$

7. $\log 0.629$

8. $\log\left(\frac{12}{13}\right)$

9. $10^{-3.415}$

10. $e^{3.2}$

11. $e^{-3.6}$

12. $\frac{\ln 1500}{\ln 0.06}$

13. $\frac{\log 1250}{\log 3}$

Changing Logarithmic Bases

Find each of the following logarithms using the change-of-base formula. Round answers to four decimal places.

14. $\log_3 16$

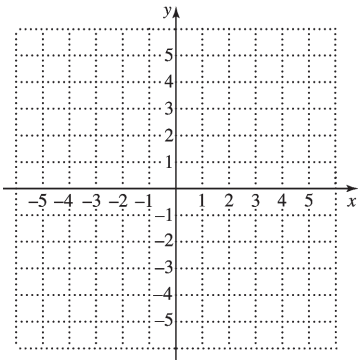
15. $\log_{0.2} 3$

16. $\log_\pi 1000$

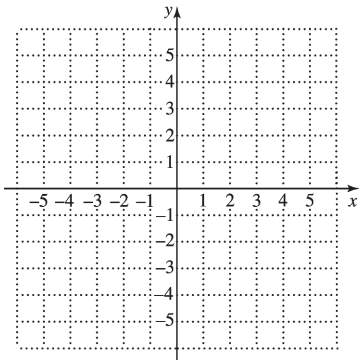
Graphs of Exponential Functions and Logarithmic Functions, Base *e*

Graph and state the domain and range of each function.

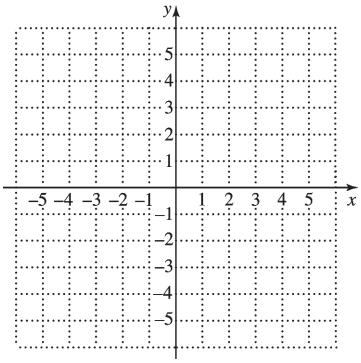
17. $f(x) = -e^x + 1$



18. $f(x) = e^{x+4}$



19. $g(x) = -3\ln x$



20. $g(x) = \ln(x-2)$