Math 110- 8.4, 5.8, 8.8 Notes

Solving Formulas

ESSENTIALS

To solve a formula for a certain letter, we use the principles for solving equations to get that letter alone on one side.

We can often assume letters represent nonnegative quantities.

Example

- Solve $s=\frac{1}{2}gt^2$ for t. Assume $t\geq 0$.

$$s=\frac{1}{2}gt^2$$

$$\frac{2}{g}\cdot s=\frac{2}{g}\cdot\frac{1}{2}gt^2 \quad \text{Multiplying by } \frac{2}{g}$$

$$\frac{2s}{g}=t^2$$

$$\sqrt{\frac{2s}{g}}=t$$

$2s=gt^2$

$\frac{2}{g}s=t^2$

$t^2=\frac{2s}{g}$

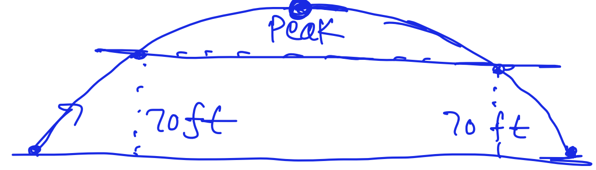
$t=\pm\sqrt{\frac{2s}{g}}$

rejected because $t\geq 0$

$m=\sqrt{\frac{5}{x}}$	$x=\sqrt{\frac{1}{yz}}$
Solve for x :-	Solve for y :-
$m^2=\left(\sqrt{\frac{5}{x}}\right)^2$	$x^2=\frac{1}{yz}$
$m^2=\frac{5}{x}$	Multiply by y :-
Divide by 5 :-	$x^2y=\frac{1}{z}$
$\frac{m^2}{5}=\frac{1}{x}$	Divide by x^2 :-
multiply by $x \rightarrow \frac{m^2}{5}x=1$	$y=\frac{1}{x^2z}$
multiply by $5 \rightarrow m^2x=5$	
$x=\frac{5}{m^2}$	

Sections 8.4 and 5.8

Problem Solving



Example 1: During intermission at sporting events, it has become common for team mascots to use a powerful slingshot to launch tightly rolled t-shirts into the sands. The height h(t), in feet, of an airborne tee shirt t seconds after being launched can be approximated by

$$h(t) = -15t^2 + 75t + 10$$

After peaking, a rolled- up tee shirt is caught by a fan 70 ft above ground level. How long was the t-shirt in the air?

$h(t) = 70$. Find t

$4 = -1 \times -4$

-2×-2

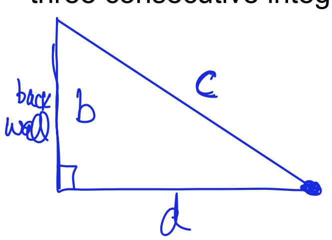
$-15t^2 + 75t + 10 = 70 \Rightarrow -15t^2 + 75t - 60 = 0$

$\Rightarrow -15(t^2 - 5t + 4) = 0 \Rightarrow t^2 - 5t + 4 = 0$

$\Rightarrow t^2 - t - 4t + 4 = 0 \Rightarrow t(t-1) - 4(t-1) = 0 \Rightarrow (t-1)(t-4) = 0 \Rightarrow t = 1 \text{ or } 4$

$\Rightarrow t = 4\,s.$

Example 2: In order to build a deck at a right angle to their house, Lucinda and Felipe decide to hammer a stake in the ground a precise distance from the back wall of their house. This stake will combine with two marks on the house to form a right triangle. From a course in geometry, Lucinda remembers that there are three consecutive integers that can work as sides of a right triangle. Find the measurements of that triangle.



$c^2 = b^2 + d^2$ (c: 17 ft, b: 12 ft) $\Rightarrow 17^2 = 12^2 + d^2 \Rightarrow d^2 = 17^2 - 12^2$

$= 289 - 144$

$= (17-12)(17+12)$

$= 5 \times 29 = 145$

$\Rightarrow d^2 = 145$

$\Rightarrow d = \pm\sqrt{145} = 12.04$ ft.

reject

Example 3: A sports card is 4 cm wide and 5 cm long. The card is to be encased by Lucite that is 5.5 times the area of the card. Find the dimensions of the Lucite that will ensure a uniform border around the card.

$(2x+4)(2x+5) = 5.5 \times \underbrace{4 \times 5}$

$= 110$

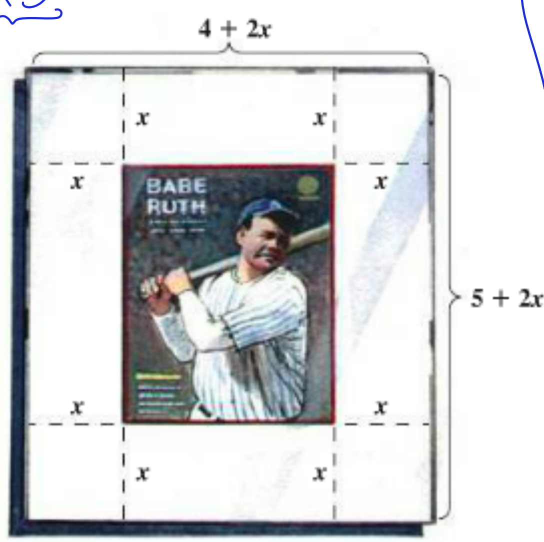
$\Rightarrow 4x^2 + 8x + 10x + 20 = 110$

$\Rightarrow 4x^2 + 18x + 20 = 110$

$\Rightarrow 4x^2 + 18x + 20 - 110 = 0$

$\Rightarrow 4x^2 + 18x - 90 = 0$

$\Rightarrow 2(2x^2 + 9x - 45) = 0$



$2x^2 + 9x - 45 = 0$

$2 \times -45 = -90$

$= 1 \times -90 = -1 \times 90$

$= 2 \times -45 = -2 \times 45$

$= 3 \times -30 = -3 \times 30$

$= 5 \times -18 = 5 \times 18$

$= 6 \times -15 = -6 \times 15$

$2x^2 + 15x - 6x - 45 = 0$

$\Rightarrow x(2x+15) - 3(2x+15) = 0$

$\Rightarrow (x-3)(2x+15) = 0$

$\Rightarrow x - 3 = 0 \text{ or } 2x + 15 = 0 \Rightarrow x = 3 \text{ or } x = -\frac{15}{2}$

$\Rightarrow x = 3 \Rightarrow$ dimension of Lucite $= 3$ cm. reject.

Example 4: An envelope is 6 cm longer than it is wide. The area is 72 cm^2Find the length and the width.

Let x cm be its width $\Rightarrow$ length $= x+6$

$-72 = -1\times 72 = -2\times 36 = -3\times 24 = -4\times 18 = -6\times 12$

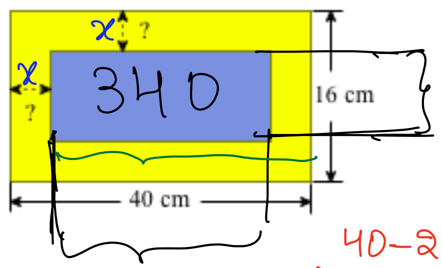
$x(x+6) = 72 \Rightarrow x^2+6x = 72 \Rightarrow x^2+6x-72=0$

$\Rightarrow x^2+12x-6x-72=0 \Rightarrow x(x+12)-6(x+12)=0$

$\Rightarrow (x-6)(x+12)=0 \Rightarrow x-6=0$ or $x+12=0 \Rightarrow x=6$ or $\cancel{x=-12}$

width $= 6$, length $= 12$

Example 5: A picture frame measures 16 cm by 40 cm on the outside, and 340 cm^2 of a picture shows. Find the uniform width of the frame around the picture.



$16-x-x = 16-2x$

$40-x-x = 40-2x$

$40-2(25) = -10$

$75 = -1\times -75 = -3\times -25$

$(40-2x)(16-2x)=340$

$640-32x-80x+4x^2=340$

$\Rightarrow 4x^2-112x+640=340$

$\Rightarrow 4x^2-112x+300=0$

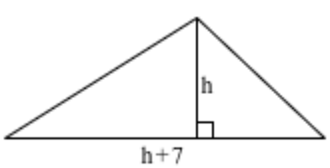
$\Rightarrow 4(x^2-28x+75)=0$

$\Rightarrow x^2-28x+75=0 \Rightarrow x^2-3x-25x+75=0$

$\Rightarrow x(x-3)-25(x-3)=0 \Rightarrow (x-25)(x-3)=0$

$\Rightarrow x=3$ or $\cancel{x=25}$ $\Rightarrow$ uniform width $= 3$ cm

Example 6: the base of a triangle is 7 cm greater than the height. The area is 22 cm^2. Find the height and the length of the base.



Area of a triangle $= \frac{1}{2}\times$ base $\times$ height

$\Rightarrow \frac{1}{2}h(h+7) = 22$

$\Rightarrow h(h+7) = 44 \Rightarrow h^2+7h=44$

$-44 = -1\times 44 = -2\times 22 = -4\times 11$

$\Rightarrow h^2+7h-44=0 \Rightarrow h^2+11h-4h-44=0$

$\Rightarrow h(h+11)-4(h+11)=0 \Rightarrow (h-4)(h+11)=0$

$\Rightarrow h-4=0$ or $h+11=0 \Rightarrow h=4$ or $h=-11$

$\Rightarrow$ Height $= 4$ cm

Base $= 4+7 = 11$ cm

More Solving of Quadratic Equations

a) Solve $x^2 = 25$

$x = \pm 5$

b) Solve $64 = x^2$

$x = \pm 8$

c) Solve $3x^2 = 6$

$x^2 = 2 \Rightarrow x = \pm\sqrt{2} = \pm 1.414$

d) Solve $x^2 = 10$

$\Rightarrow x = \pm\sqrt{10} = \pm 3.16$

e) Solve $-5x^2 + 2 = 0$

$\Rightarrow -5x^2 = -2 \Rightarrow x^2 = \frac{-2}{-5} \Rightarrow x^2 = \frac{2}{5} \Rightarrow x = \pm\sqrt{\frac{2}{5}}$

$\Rightarrow x = \pm\sqrt{0.4} = \pm 0.632$

f) Solve $3x^2 = 1$

$\Rightarrow x^2 = \frac{1}{3} \Rightarrow x = \pm\sqrt{\frac{1}{3}} = \pm\frac{1}{\sqrt{3}} = \pm\frac{\sqrt{3}}{3} = \pm\frac{1.732}{3} =$

$= \pm 0.577$

g) Solve $4x^2 + 9 = 0$

$4x^2 = -9 \Rightarrow x^2 = \frac{-9}{4} \Rightarrow x = \pm\sqrt{\frac{-9}{4}} = \pm\sqrt{\frac{9}{4}}\sqrt{-1}$

$\Rightarrow x = \pm\frac{3}{2}i$

h) Solve $2x^2 + 200 = 0$

$2x^2 = -200 \Rightarrow x^2 = -100 \Rightarrow x = \pm 10i$

i) Let $f(x) = (x+5)^2$ find the values for which $f(x) = 3$

Find x for which $(x+5)^2 = 3 \Rightarrow x + 5 = \pm\sqrt{3}$

$f(3)$

$\Rightarrow x = -5 \pm \sqrt{3}$

$2\cdot x\cdot 3$ $\quad 3^2$ $\quad\Rightarrow x^2+6x+7=0 \quad 7=1\times 7$

$\rightarrow$ Quadratic formula

j) Solve $x^2+6x+9=2$

$$(x+3)^2=2 \Rightarrow x+3=\pm\sqrt{2} \Rightarrow x=-3\pm\sqrt{2}$$

k) Solve $x^2-10x+25=3$

$2\times x\times 5 \quad 5^2$

$$(x-5)^2=3 \Rightarrow x-5=\pm\sqrt{3} \Rightarrow x=5\pm\sqrt{3}$$

The formula $S=16t^2$is used to approximate the distance S, in feet, that an object falls freely from rest in t seconds. The height of a building is 1358 Feet. How long would it take for an object to fall from the top?

$$\Rightarrow 16t^2=1358 \Rightarrow t^2=\frac{1358}{16}$$

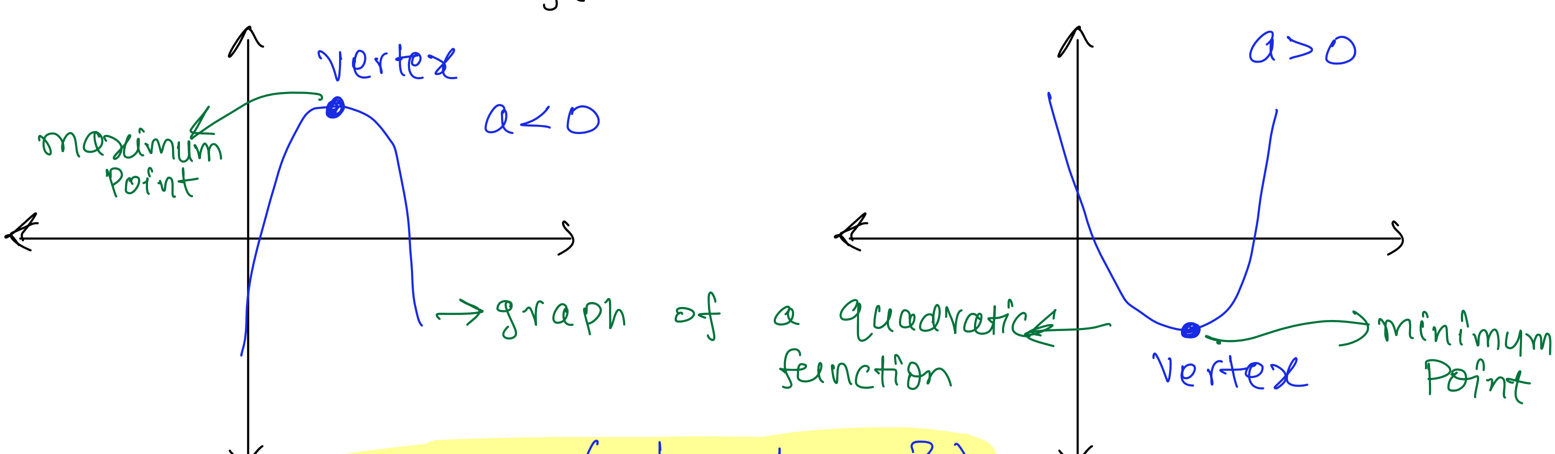
$$\Rightarrow t=\pm\sqrt{\frac{1358}{16}} \Rightarrow t=\sqrt{\frac{1358}{16}}=9.2128.$$

reject

The height of the building id 1383 feet. How long would it take an object to fall to the ground form the top? Use the formula $S=16t^2$where s is the distance in feet traveled by an object falling freely from rest in t seconds.

\ $$16t^2=1383 \Rightarrow t^2=\frac{1383}{16} \Rightarrow t=\sqrt{\frac{1383}{16}}=9.298.$$

$$f(x)=ax^2+bx+c$$



$$\text{Vertex}=\left(\frac{-b}{2a},\ \frac{4ac-b^2}{4a}\right)$$

$$\frac{4ac-b^2}{4a}=f\left(\frac{-b}{2a}\right)$$

Section 8.8

Maximum and Minimum Problems

ESSENTIALS

If a problem in which a quantity must be maximized or minimized can be modeled with a quadratic function, then the problem can often be solved by finding the coordinates of the vertex of the function.

$a=1,\ b=-8,\ c=50$

Example

- The value in dollars of a share of a certain stock can be represented by $V(x)=x^2-8x+50$, where x is the number of months after its first day of trading. What is the lowest value it reached and when did that occur?

  $\frac{-b}{2a}=\frac{-(-8)}{2(1)}=\frac{8}{2}=4$ ✓

  The vertex of $V(x)=x^2-8x+50$ is $(4,34)$. A minimum value of \$34 occurs 4 months after the stock's first day of trading.

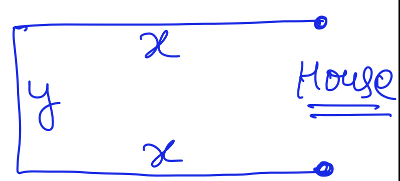
  $V(4)=4^2-8\times4+50=16-32+50=34$ ✓

Mount Sterling Furniture has determined that when x hundred bookcases are produced, the average cost per bookcase can be estimated by , where is in hundreds of dollars. What is the minimum average cost per bookcase and how many bookcases should be built in order to achieve that minimum?

↓ next class

Tish and Ben are fencing in a rectangular play area for their dogs. They have 120 ft of fence and their house will form one side of the rectangle. What is the maximum area they can enclose? What should the dimensions be to yield this area?

120 ft of fence $\Rightarrow 2x+y=120$ ft.



Maximize area. Area $= xy$

$2x+y=120 \Rightarrow y=120-2x$

$\Rightarrow A(x)=x(120-2x) \leftarrow$ maximize.

Find the maximum of $A(x)=-2x^2+120x$

$a=-2,\ b=120,\ c=0$

$x=\frac{-b}{2a}=\frac{-(120)}{2(-2)}=\frac{-120}{-4}$

$\Rightarrow x=30$ $\Rightarrow y=120-2(30)=120-60=60$

↑ gives maximum area

$\boxed{\Rightarrow \text{Dimensions are } 60 \text{ ft and } 30 \text{ ft.}}$

Quiz 13

① $(x+2)^2 = 7$

$\Rightarrow x+2 = \pm\sqrt{7} \Rightarrow x = -2 \pm \sqrt{7}$

② $x^2 - 3x + 2 = 0$

$2 = -1 \times -2$

$\Rightarrow x^2 - x - 2x + 2 = 0$

$\Rightarrow x(x-1) - 2(x-1) = 0 \Rightarrow (x-2)(x-1) = 0$

$x - 2 = 0$ or $x - 1 = 0$

$\Rightarrow x = 2$ or $x = 1 \Rightarrow x = 1$ or 2

ALTERNATIVELY,

$ax^2 + bx + c = 0 \Rightarrow x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a = 1,\ b = -3,\ c = 2$

$\Rightarrow x = \dfrac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(2)}}{2(1)} = \dfrac{3 \pm \sqrt{9-8}}{2}$

$= \dfrac{3 \pm 1}{2} = \dfrac{4}{2}$ or $\dfrac{2}{2}$

$\Rightarrow x = 2$ or 1