Math16500 Section 24246 Quiz 4
Fall 2022, September 12

Name: [1 pt]

Problem 1: Evaluate the limit

$$L = \lim_{x\to 1} \frac{x^2 - x}{x^2 - 1}.$$

$= \frac{1^2-1}{1^2-1} = \frac{\to 0}{\to 0}$

$\Rightarrow L = \lim_{x\to 1} \frac{x(x-1)}{(x-1)(x+1)} = \lim_{x\to 1} \frac{x}{x+1}$ [4 pts]

$= \frac{1}{1+1} = \frac{1}{2}$

$\frac{\to 0}{\to 0}$

Problem 2: Evaluate the limit

$$L = \lim_{x\to 0} \frac{\sqrt{2+x} - \sqrt{2-x}}{4x}.$$

$= \frac{\sqrt{2+0} - \sqrt{2+0}}{4(0)} = \frac{\sqrt{2}-\sqrt{2}}{\to 0}$

Rationalize the numerator here. [5 pts].

$\Rightarrow L = \lim_{x\to 0} \frac{\sqrt{2+x} - \sqrt{2-x}}{4x} \times \frac{\sqrt{2+x} + \sqrt{2-x}}{\sqrt{2+x} + \sqrt{2-x}}$

$= \lim_{x\to 0} \frac{(\sqrt{2+x})^2 - (\sqrt{2-x})^2}{4x(\sqrt{2+x} + \sqrt{2-x})} = \lim_{x\to 0} \frac{2+x-(2-x)}{4x(\sqrt{2+x} + \sqrt{2-x})}$

$= \lim_{x\to 0} \frac{2+x-2+x}{4x(\sqrt{2+x} + \sqrt{2-x})} = \lim_{x\to 0} \frac{2x}{4x(\sqrt{2+x} + \sqrt{2-x})}$

$= \frac{1}{2} \times \frac{1}{\sqrt{2+0} + \sqrt{2-0}} = \frac{1}{2} \times \frac{1}{2\sqrt{2}} = \frac{1}{4\sqrt{2}}$

Bonus Problem: Evaluate the limit

$$\lim_{x\to \pi} \frac{\sin x}{\cos x - 2}.$$

$= \frac{\sin \pi}{\cos \pi - 2} = \frac{0}{-1-2} = \frac{0}{-3} = 0$ [2 pts].