Concavity and Inflection Points: For every x lying in an interval $[a, b]$, the graph of a function f is:

1. concave up if $f''(x) > 0$,
2. concave down if $f''(x) < 0$.

A point on the graph at which the concavity changes is called an inflection point.

Example 1. Find the inflection points and graph the function $y = x^3 - 3x + 2$ of Example 2 from the previous lecture.

Second Derivative Test:

1. Find the critical numbers of f. Suppose c is a critical number.
2. Find $f''(c)$.
   (a) If $f''(c) > 0$ then $f(c)$ is a minimum value and $(c, f(c))$ is a minimum point.
   (b) If $f''(c) < 0$, then $f(c)$ is a maximum value.
   (c) If $f''(c) = 0$, then the test fails and $(c, f(c))$ may be a relative minimum or a relative maximum or neither.

Example 2. Test the function $f(x) = x^3 - 3x$ for extreme values.

Procedure for Curve Sketching:

1. Find all critical numbers.
2. Test the critical numbers for relative extremal points.
   (a) Use the second derivative test.
   (b) If the second derivative test fails, use the first derivative test.
3. Use the second derivative test to determine intervals where graph of f is concave up and where it is concave down.
4. Determine the points of inflection.
5. Find any easily determined intercepts.
6. Plot the critical points, inflection points and intercepts.
7. Sketch an approximate curve.

Example 3. Sketch the curve $y = x^3 - 3x$.

Example 4. Sketch the curve $y = x^4 + (4/3)x^3$.

Example 5. Sketch the curve $y = \dfrac{x}{\sqrt{x-1}}$.

Example 6. Sketch the curve $y = (x-2)^{2/3}$.

Example 7. Sketch the curve $y = x + \dfrac{1}{x}$.