Problem 1. Find the approximate value of the given integrals by using Trapezoidal rule with the specified value of n.

1. $\displaystyle\int_0^5 \sqrt{x}\,dx$, $n = 5$.
2. $\displaystyle\int_{-2}^2 \frac{x^2}{x^2+1}\,dx$, $n = 8$.

Problem 2. Find the approximate value of the given integrals by using Simpson's rule with the specified value of n.

1. $\displaystyle\int_1^3 \sqrt{x^2-1}\,dx$, $n = 4$.
2. $\displaystyle\int_1^7 \frac{\sqrt{x}}{x+1}\,dx$, $n = 6$.

Problem 3. Find the volume of the solid obtained by revolving the region bounded by the given curves about x-axis. Use disk-washer method.

1. $y = 2x$, $x = 1$, $x = 4$, x-axis.
2. $y = 1 + x^3$, $x = 0$, $x = 1$, $y = 1$.

Problem 4. Find the volume of the solid obtained by revolving the region bounded by the given curves about y-axis. Use shell method.

1. $y = x^2 - 2x$ and x-axis.
2. $y = x + 2$, $y = x^2$ (first quadrant).

Hints to problem 1.

1. $\Delta x = 1$, $x_i = i \Rightarrow \displaystyle\int_0^5 \sqrt{x}\,dx \approx \frac{1}{2}(\sqrt{0} + 2\sqrt{1} + 2\sqrt{2} + 2\sqrt{3} + 2\sqrt{4} + \sqrt{5})$.
2. $\Delta x = \dfrac{1}{2}$, $x_i = -2 + \dfrac{i}{2}$,

$$\int_{-2}^2 \frac{x^2}{x^2+1}\,dx \approx \frac{1}{4}\Bigg(\frac{(-2)^2}{(-2)^2+1} + 2\frac{(-1.5)^2}{(-1.5)^2+1} + 2\frac{(-1)^2}{(-1)^2+1} + 2\frac{(-0.5)^2}{(-0.5)^2+1} + 2\frac{(0)^2}{(0)^2+1} + 2\frac{(0.5)^2}{(0.5)^2+1} + 2\frac{(1)^2}{(1)^2+1} + 2\frac{(1.5)^2}{(1.5)^2+1} + \frac{(2)^2}{(2)^2+1}\Bigg).$$

Hints to problem 2.

1. $\Delta x = \dfrac{1}{2}$, $x_i = 1 + \dfrac{i}{2}$,

$$\int_1^3 \sqrt{x^2-1}\,dx \approx \frac{1}{6}\Big(\sqrt{(1)^2-1} + 4\sqrt{(1.5)^2-1} + 2\sqrt{(2)^2-1} + 4\sqrt{(2.5)^2-1} + \sqrt{(3)^2-1}\Big).$$

2. $\Delta x = 1$, $x_i = 1 + i$,

$$\int_1^7 \frac{\sqrt{x}}{x+1}\,dx \approx \frac{1}{3}\Big(\frac{\sqrt{1}}{1+1} + 4\frac{\sqrt{2}}{2+1} + 2\frac{\sqrt{3}}{3+1} + 4\frac{\sqrt{4}}{4+1} + 2\frac{\sqrt{5}}{5+1} + 4\frac{\sqrt{6}}{6+1} + \frac{\sqrt{7}}{7+1}\Big).$$

Hints to problem 3.

1. Volume $= \pi \displaystyle\int_1^4 (2x)^2\,dx = 252\pi/3.$

2. Volume $= \pi \displaystyle\int_0^1 [(1+x^3)^2 - (1)^2]\,dx = 9\pi/14.$

Hints to problem 4.

1. Volume $= 2\pi \displaystyle\int_0^2 x(2x - x^2)\,dx = 8\pi/3.$

2. Volume $= 2\pi \displaystyle\int_0^2 x(2 + x - x^2)\,dx = 16\pi/3.$