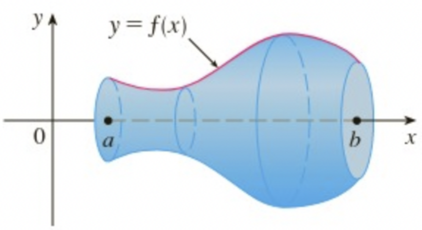
M16600 Lecture Notes

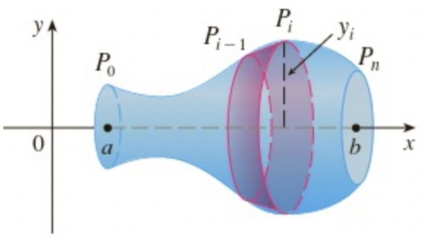
Section 8.2: Area of a Surface of Revolution

■ **Section 8.2** textbook exercises, *page 595*: # 1, 2, 3, 7.

A ***surface of revolution*** is formed when a curve is rotated about a line. How do we find the area of such a surface?



(a) Surface of revolution



(b) Approximating band

The area of the i band is $2\pi f(x_i^*)\sqrt{1+\left[f'(x_i^*)\right]^2}\Delta x$. See the discussion on page 591–592 of the textbook for more detail. Then an approximation of the surface area is

$$\sum_{i=1}^{n} 2\pi f(x_i^*)\sqrt{1+\left[f'(x_i^*)\right]^2}\Delta x$$

Thus, the surface area is

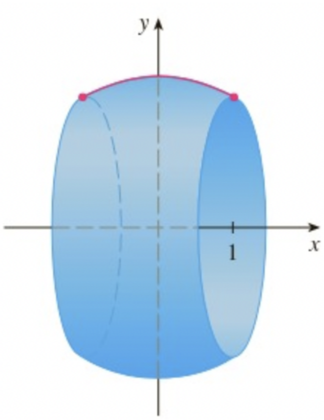
$$\lim_{n\to\infty}\sum_{i=1}^{n} 2\pi f(x_i^*)\sqrt{1+\left[f'(x_i^*)\right]^2}\Delta x$$

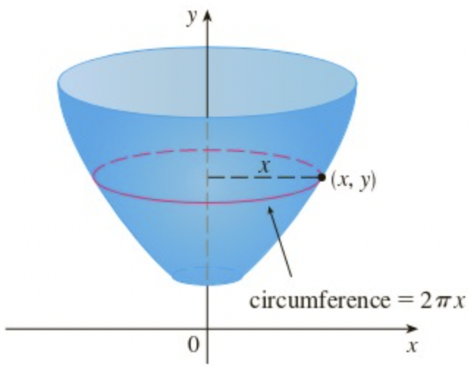
$$= \int_a^b 2\pi f(x)\sqrt{1+\left[f'(x)\right]^2}\,dx.$$

Area of a Surface of Revolution about the x-axis. The surface area of a surface obtained by rotating the curve $y = y(x)$, $a \le x \le b$, about the x-axis is

$$S = \int_a^b 2\pi y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

Example 1: The curve $y = \sqrt{4-x^2}$, $-1 \le x \le 1$, is an arc of the circle $x^2+y^2=4$. Find the area of the surface obtained by rotating this arc about the x-axis.





Area of a Surface of Revolution about the y-axis. The surface area of a surface obtained by rotating the curve $y = y(x)$, $a \leq x \leq b$, about the y-axis is

$$S = \int_a^b 2\pi x \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

Example 2: The arc of the parabola $y = x^2$ from $(1, 1)$ to $(2, 4)$ is rotated about the y-axis. Find the area of the resulting surface.