Problem 1. Differentiate the following functions:

1.
$$y = x^2 \sqrt{x^2 - 1}$$
.

2.
$$y = 2(x^6 + 1)^{10}$$
.

3.
$$y = x \sqrt{x}$$
.

$$4. \ \ y = \frac{x^3 + 2x}{x^2 - 8} \, .$$

5.
$$P = \frac{t-2}{t^2+4}$$
.

6.
$$y = \frac{(4+x^4)^5}{5}$$
.

7.
$$y = \frac{2}{\sqrt{x^3 - 3x}}$$
.

Problem 2. Find dy/dx implicitly.

1.
$$x^4y^4 - 3y^2 + 5x = 6$$
.

2.
$$x + x^2y^2 - y = 1$$
.

Problem 3. Find the equations of tangent and normal lines to each of the following curves at the given point.

1.
$$x^2 + 4y^2 = 5$$
 at $(1, -1)$.

2.
$$y^2 - x^2 = 5$$
 at $(2, -3)$.

3.
$$y^2 + 12x = 0$$
 at $(-3, 6)$.

Answers to problem 1. (1) $\frac{dy}{dx} = \frac{3x^3 - 2x}{\sqrt{x^2 - 1}}$. (2) $\frac{dy}{dx} = 120x^5(x^6 + 1)^9$. (3) $\frac{dy}{dx} = \frac{3}{2}\sqrt{x}$.

$$(4) \frac{dy}{dx} = \frac{x^4 - 26x^2 - 16}{(x^2 - 8)^2} \cdot (5) \frac{dP}{dt} = \frac{-t^2 + 4t + 4}{(t^2 + 4)^2} \cdot (6) \frac{dy}{dx} = 4x^3(x^4 + 4)^4.$$

$$(7) \frac{dy}{dx} = -3(x^2 - 1)(x^3 - 3x)^{-3/2}.$$

Answers to problem 2. (1) $\frac{dy}{dx} = \frac{-(4x^3y^4 + 5)}{4x^4y^3 - 6y}$. (2) $\frac{dy}{dx} = \frac{-(2xy^2 + 1)}{2x^2y - 1}$.

Answers to problem 3.

- 1. Equation of tangent is x 4y 5 = 0. Equation of normal is 4x + y 3 = 0.
- 2. Equation of tangent is 2x + 3y + 5 = 0. Equation of normal is 3x 2y 12 = 0.
- 3. Equation of tangent is x + y 3 = 0. Equation of normal is x y + 9 = 0.