MATH 16600 Practice Final Exam, *Version 4*

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1 Given a one-to-one function $f(x) = 2x^3 + \ln x$, $x > 0$. Find $(f^{-1})'(2)$.

2 The common inhabitant of human intestines is the bacterium *Escherichia coli*, named after the German pediatrician Theodor Escherich, who identified it in 1885. A cell of this bacterium in a nutrient-broth medium divideds into two cells every 20 minutes. The initial population of a culture is 50 cells.

(a) Find the number of cells after 6 hours.

(b) When will the population reach a million cells?

3 Find the limit $\lim_{x \to 0} \dfrac{\ln(1+x)}{\cos x + e^x - 2}$.

4 Let $f(x) = \ln\left[\dfrac{\sqrt[4]{x^2+1}}{(x+1)^5}\right]$. Use the properties of logarithmic functions to decompose $f(x)$ completely. Using the decomposition you have obtained, find $f'(x)$.

5 Evaluate the integral $\int (3x - 5) \sin x \, dx$.

6 Evaluate the integral $\int \frac{x - 4}{x^2 - 5x + 6} \, dx$.

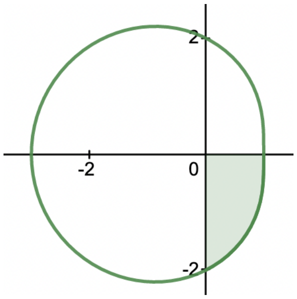
7 Evaluate the integral $\displaystyle\int \frac{1}{x^2\sqrt{x^2+1}}\,dx$.

8 Set up, but do not evaluate an integral that represents the area of the surface obtained by rotating the curve $y = 2x - \dfrac{1}{x^2}$, $1 \le x \le 4$ about the x-axis.

9 Determine whether the improper integral $\displaystyle\int_e^{\infty} \frac{1}{x(\ln x)^2}\, dx$ is convergent or divergent. Evaluate the integral if it is convergent.

10 Find an equation of the tangent line to the parametric curve $x = e^{3t-3}, \quad y = t^3 + 3t$ at the point where $t = 1$.

11 Find the area of the shaded region.



$$r = 2 - \cos\theta$$

12 Determine whether the series $\sum_{n=1}^{\infty}(-1)^n\frac{1}{\sqrt[4]{n}}$ is absolutely convergent, conditionally convergent, or divergent.

13 Determine whether the series $\sum_{n=1}^{\infty}\frac{n^2+2}{n^2+2n+2}$ is convergent or divergent.

14 Use the definition of Taylor series to find the first **four** nonzero terms of the series for $f(x) = \sin x$ centered at $a = \pi$.

15 Determine whether the geometric series is convergent or divergent. If it is convergent, find its sum $\displaystyle\sum_{n=0}^{\infty} \frac{(-3)^n}{4^{n+1}}$.

16 Find the radius of convergence and interval of convergence of the series $\sum_{n=1}^{\infty} \dfrac{x^n}{n^9\, 9^n}$.

17 Find the exact length of the curve $y = \ln(\sec x)$, $0 \leq x \leq \pi/4$.

18 Determine whether the series $\sum_{n=1}^{\infty} \frac{1+\cos n}{6^n}$ is convergent or divergent.