Graphs of Linear Inequalities

ESSENTIALS

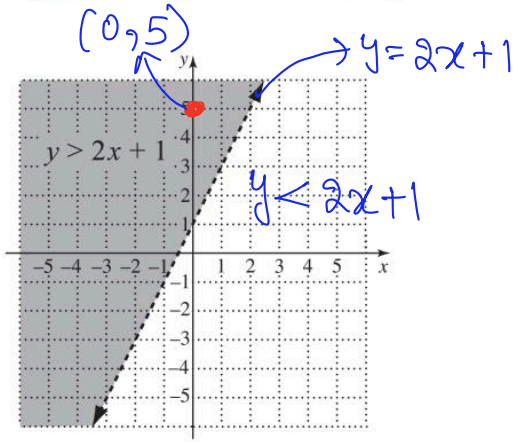
To graph a linear inequality:

1. Graph the boundary line using a dashed line for $<$ or $>$ or a solid line for $\leq$ or $\geq$.
2. Shade the appropriate half-plane. Use a test point to determine the appropriate half-plane or isolate y and shade below the boundary line for $y < mx+b$ or $y \leq mx+b$ or above it for $y > mx+b$ or $y \geq mx+b$.

Example

- Graph: $y > 2x+1$.

  We graph $y = 2x+1$ using a dashed line
  Then we shade above the line.



In $y > 2x+1$, Put $x=0$, $y=5$

$5 > 2(0)+1 \Rightarrow 5 > 1 \Rightarrow$ True.

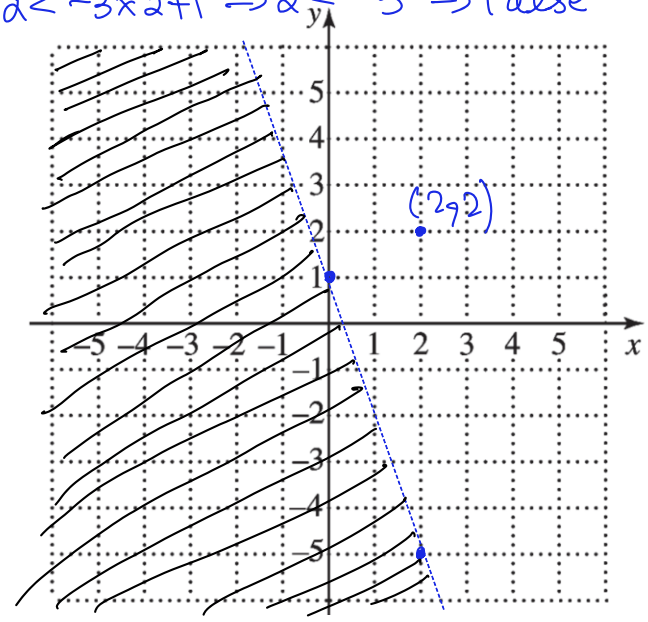
Example: Determine whether (0,-4) is a solution of $2x - 3y > 6$

$2(0) - 3(-4) > 6 \Rightarrow 0 + 12 > 6 \Rightarrow 12 > 6 \Rightarrow$ True

↳ Yes, $(0,-4)$ is a solution of $2x-3y>6$.

Example: Graph $y < -3x + 1$

$2 < -3\times 2+1 \Rightarrow 2 \not< -5 \Rightarrow$ False



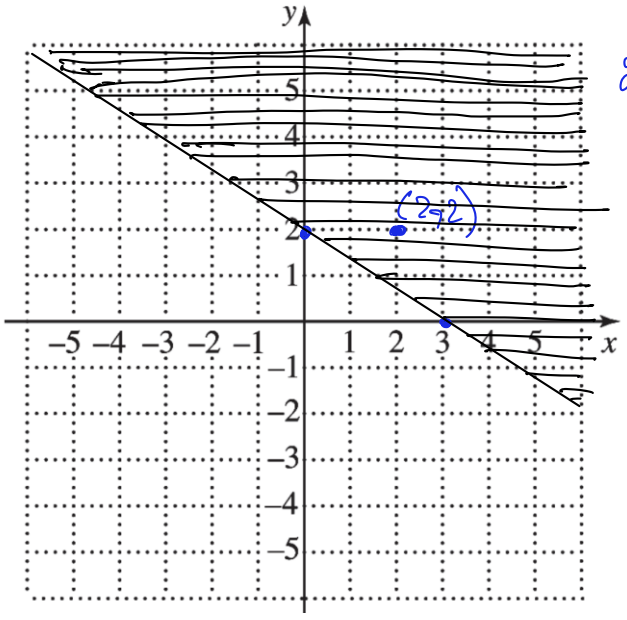
$y = -3x+1$

$x=0 \Rightarrow y=1 \quad (0,1)$

$x=2 \Rightarrow y = -3\times 2+1 = -5$

$(2,-5)$

Example: Graph $2x + 3y \geq 6$



$2(2)+3(2) \geq 6$

$4+6 \geq 6$

$10 \geq 6$

⇓

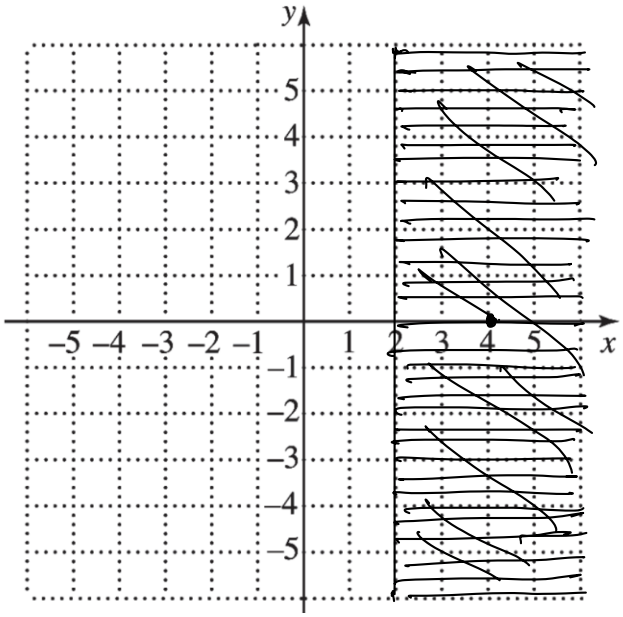
True

$2x+3y=6$

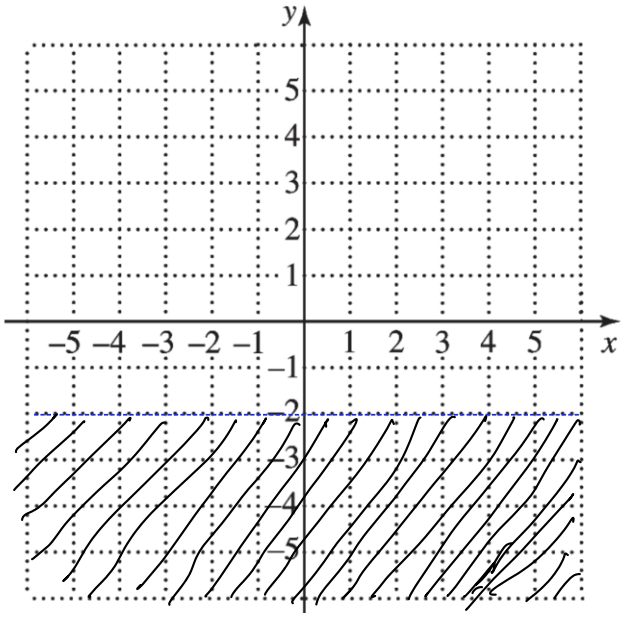
$x=0 \Rightarrow 3y=6 \Rightarrow y=2 \Rightarrow (0,2)$

$y=0 \Rightarrow 2x=6 \Rightarrow x=3 \Rightarrow (3,0)$

Example: Graph $x \geq 2$ $x=2$



Example: Graph $y < -2$ $y=-2$



Systems of Linear Inequalities

ESSENTIALS

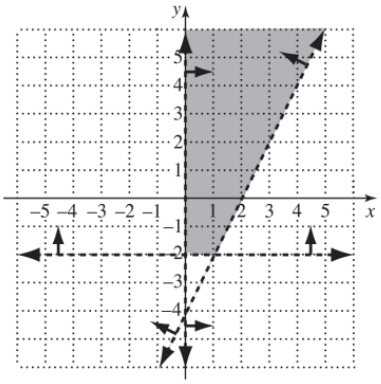
To graph a **system of linear inequalities**, graph the individual inequalities and then find the intersection of the graphs.

Example

- Graph the system: $2x - y < 4$,
  $x > 0$,
  $y > -2$.

We graph the individual inequalities, indicating which half-planes contain solutions by arrows near the ends of the boundary lines.

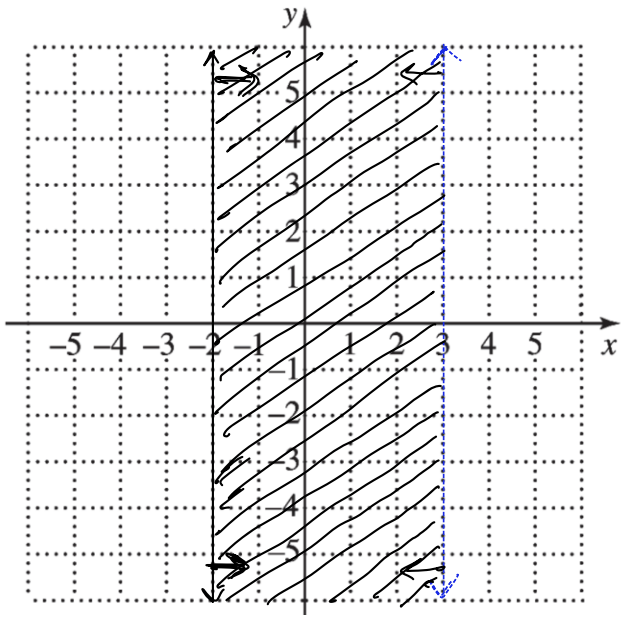
We shade the region of intersection of the half-planes that are solutions of the individual inequalities. This is the solution set of the system.



$x \geq -2$ and $x < 3$

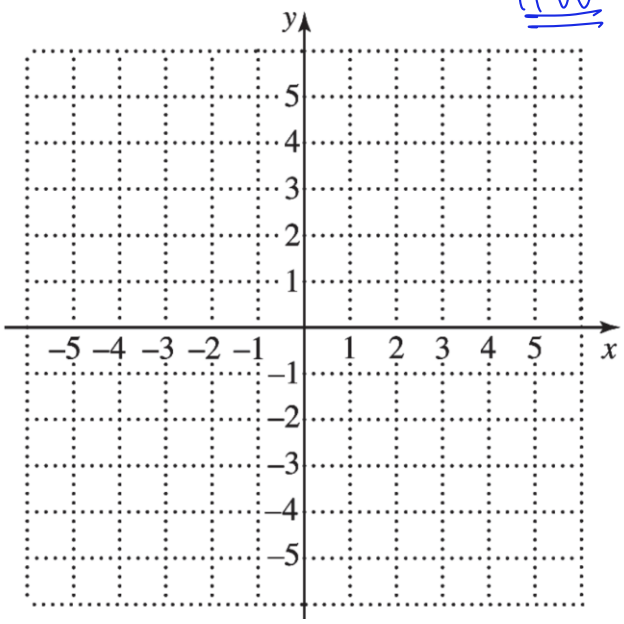
$-2 \leq x$ and $x < 3$

Example: Graph $-2 \leq x < 3$



Example: Graph $-3 \leq x < 4$

HW



Example: Graph the system. Find the coordinates of any vertices formed

$3x - 2y \geq 6$ → $3x - 2y = 6$

$y \geq -3$ ← $y + 3 \geq 0$ → $y + 3 = 0 \Rightarrow y = -3$

$x + y \leq 0$ → $x + y = 0$ $(5, -5)$, $(-5, 5)$

$3x - 2y = 6$

$x = 0 \Rightarrow -2y = 6$

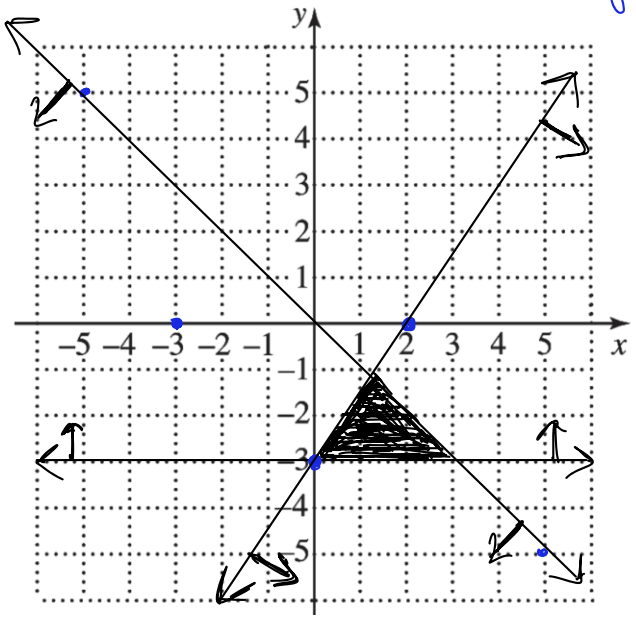
$\Rightarrow y = -3$

$(0, -3)$

$y = 0 \Rightarrow 3x = 6$

$\Rightarrow x = 2$

$(2, 0)$



$(-3, 0)$

$x + y \leq 0$

$-3 + 0 \leq 0$

$-3 \leq 0 \Rightarrow$ True

Pick a point

say $(0, 0)$

$3x - 2y \geq 6$ ← check

$3(0) - 2(0) \geq 6$

$0 \geq 6 \Rightarrow$ False

Practice Exercises

HW

Readiness Check

Complete the following sentences.

1. When graphing the boundary line for the graph of a linear inequality, if the symbol is $\geq$ or $\leq$ we draw the line _____ (dashed / solid).
2. When graphing an inequality in the form of $y < mx + b$, we shade _____ (above / below) the boundary line.
3. A linear inequality in two variables is graphed on _____ (a plane / the number line).
4. The solution set of a system of inequalities is the _____ (union / intersection) of the solution sets of the individual inequalities.

Graphs of Linear Inequalities

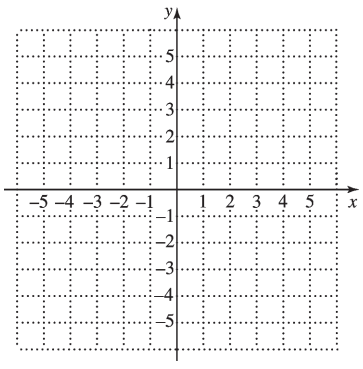
Determine whether each ordered pair is a solution of the given inequality.

5. $(1, -2)$; $-4x + y < -5$

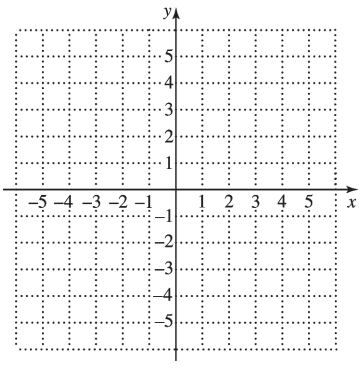
6. $(-5, 0)$; $3y - 4x \leq 0$

Graph.

7. $y \leq \frac{3}{2}x$



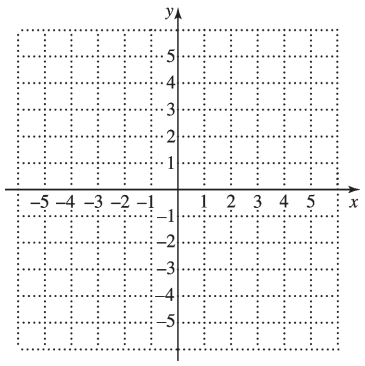
8. $2x - 5y > 10$



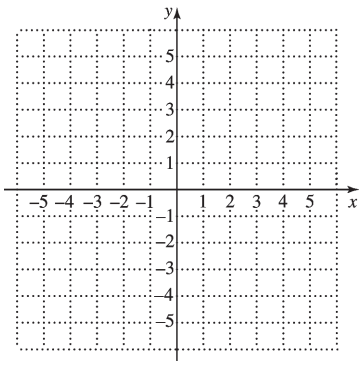
Systems of Linear Inequalities

Graph.

9. $-2 \le y < 4$



10. $y < x + 3,$
$x - 2y \le 4$



11. Graph the system of linear inequalities. Find the coordinates of any vertices formed.

$y \ge -2x + 1,$
$y - x \le -3,$
$x \le 3$