Learning objectives:

1. The definition of indefinite integral
2. Apply the fundamental theorem to find derivatives of certain functions.
3. Apply the fundamental theorem to compute definite integrals.

Indefinite integral

$$\int f(x)\,dx \quad \text{means} \quad F'(x) = f(x)$$

Therefore, we have the following

$$\int cf(x)\,dx = c\int f(x)\,dx\,, \quad \int (f(x) \pm g(x))\,dx = \int f(x)\,dx \pm \int g(x)\,dx\,,$$

$$\int k\,dx = kx + c\,, \quad \int x^n\,dx = \frac{x^{n+1}}{n+1} + c\ (n \neq -1)\,,$$

$$\int \sin x\,dx = -\cos x + c\,, \quad \int \cos x\,dx = \sin x + c\,,$$

$$\int \sec^2 x\,dx = \tan x + c\,, \quad \int \csc^2 x\,dx = -\cot x + c\,,$$

$$\int \sec x\,\tan x\,dx = \sec x + c\,, \quad \int \csc x\,\cot x\,dx = -\csc x + c\,.$$

Example 1. Evaluate the indefinite integral $\int (10x^4 - 2\sec^2 x)\,dx$.

Example 2. Evaluate $\displaystyle\int \frac{\cos\theta}{\sin^2\theta}\, d\theta$.

Example 3. Evaluate $\int_0^3 (x^3 - 6x)\, dx$.

Example 4. Evaluate $\int_0^{12} (x - 12\sin x)\,dx$.

Example 5. Evaluate $\displaystyle\int_1^9 \frac{2t^2 + t^2\sqrt{t} - 1}{t^2}\,dt$.

The net change theorem

The integral of a rate of change is the net change, that is,

$$\int_a^b F'(x)\,dx = F(b) - F(a)\,.$$

Example 6. A particle moves along a line with velocity at time t, $v(t) = t^2 - t - 6$ (measured in meters per second).

1. Find the displacement of the particle during the time period $1 \le t \le 4$.
2. Find the distance traveled during this time period.