

M16600 Lecture Notes

Section 7.1: Integration by Parts

The method of **Integration by Parts** corresponds to the Product Rule in differentiation.

There is one formula you need to remember

$$\boxed{\int u dv = uv - \int v du}$$

follows from
 $(uv)' = uv' + u'v$

We will learn how this formula works in examples

Example 1: Find $\int x \sin x dx$

Note: u -substitution will not work for this problem.

$$\begin{aligned} I &= \int \underbrace{x}_u \underbrace{\sin x dx}_{dv} \\ &= x(-\cos x) - \int (-\cos x) dx \\ &\text{(RHS)} \end{aligned}$$

$$= -x \cos x + \int \cos x dx$$

$$= -x \cos x + \sin x + C$$

$$\begin{aligned} u &= x \Rightarrow \frac{du}{dx} = 1 \Rightarrow du = dx \\ dv &= \sin x dx \\ v &= \int \sin x dx \\ &= -\cos x + \underbrace{C}_{C=0} \\ &\text{(Choose an antiderivative)} \end{aligned}$$

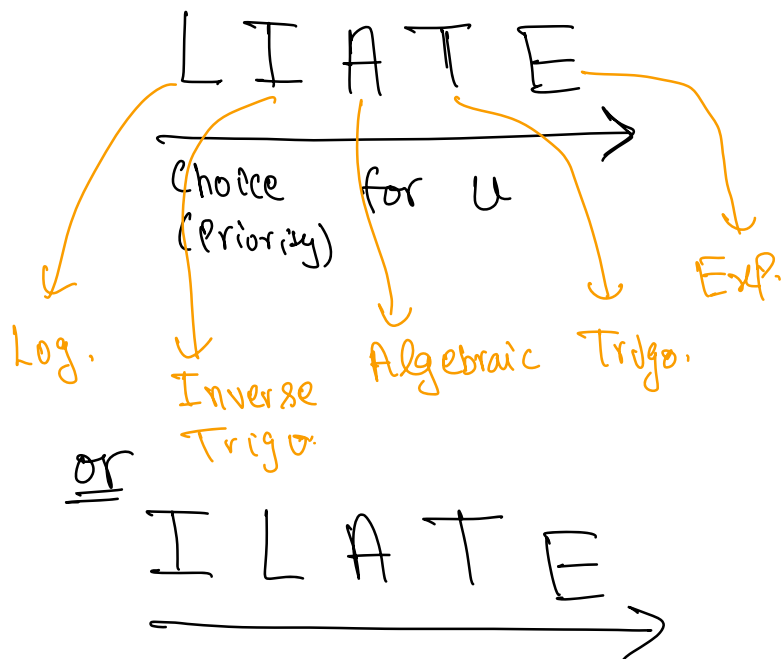
$$I = \int \underbrace{\sin x}_u \underbrace{x dx}_{dv}$$

$$= (\sin x) \left(\frac{x^2}{2} \right) - \int \frac{x^2}{2} \cos x dx$$

$$= \frac{1}{2} x^2 \sin x - \frac{1}{2} \int x^2 \cos x dx \quad \text{even more difficult.}$$

$$\begin{aligned} u &= \sin x \Rightarrow du = \cos x dx \\ dv &= x dx \Rightarrow v = \int x dx \\ &= \frac{x^2}{2} \end{aligned}$$

Example 2: Evaluate $\int 3x^3 \ln x \, dx$



Example 3: Find $\int t^2 e^t \, dt$

Example 4: Calculate $\int_0^1 \tan^{-1} x \, dx$

Example 5: Find $\int e^x \sin x \, dx$