## THE PRODUCT RULE FOR RADICALS

For any real numbers $\sqrt[n]{a}$ and $\sqrt[n]{b}$,

$$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{a \cdot b}.$$

$\frac{2}{4} = \frac{1}{2}$

To multiply radicals they need to have the same index otherwise they cannot be multiplied.

Example 1: Multiply the following radicals

1. $\sqrt{2}\sqrt{7}$ $= \sqrt{2 \times 7} = \sqrt{14}$
2. $\sqrt{5}\sqrt{10}$ $= \sqrt{50} = \sqrt{5 \times 5 \times 2} = \sqrt{5 \times 5} \times \sqrt{2} = \sqrt{5^2} \times \sqrt{2} = 5\sqrt{2}$
3. $\sqrt{7}\sqrt{7}$ $= \sqrt{49} = 7$
4. $\sqrt[3]{7}\,\sqrt[3]{7}$ $= \sqrt[3]{49}$
5. $\sqrt[3]{7}\,\sqrt[3]{7}\,\sqrt[3]{7}$ $= \sqrt[3]{7 \times 7 \times 7} = \sqrt[3]{343} = \sqrt[3]{7^3} = 7$

Simplifying radicals: when multiplying radicals you should always simplify them in order to get to simplest form

1. $\sqrt{12}$ $= \sqrt{4 \times 3} = \sqrt{4}\,\sqrt{3} = 2\sqrt{3}$
2. $\sqrt{300}$ $= \sqrt{2 \times 2 \times 75} = \sqrt{2 \times 2} \times \sqrt{75} = 2\sqrt{75} = 2\sqrt{5 \times 5 \times 3} = 2 \times 5\sqrt{3}$ $= 10\sqrt{3}$
3. $\sqrt{45}$ $= \sqrt{3 \times 100} = \sqrt{100}\sqrt{3} = 10\sqrt{3}$
4. $\sqrt{27}$ $= \sqrt{9 \times 5} = \sqrt{9}\,\sqrt{5} = 3\sqrt{5}$
5. $\sqrt{8}$ $= \sqrt{9 \times 3} = \sqrt{9}\,\sqrt{3} = 3\sqrt{3}$
6. $\sqrt{12}$ $= \sqrt{2 \times 4} = \sqrt{4}\,\sqrt{2} = 2\sqrt{2}$
7. $\sqrt{75}$ $= \sqrt{25 \times 3} = \sqrt{25}\,\sqrt{3} = 5\sqrt{3}$
8. $\sqrt{120}$ $= \sqrt{4 \times 30} = \sqrt{4}\,\sqrt{30} = 2\sqrt{30} = 2\sqrt{2 \times 3 \times 5}$
9. $\sqrt{350}$ $= \sqrt{7 \times 50} = \sqrt{7} \times \sqrt{50} = \sqrt{7} \times \sqrt{25 \times 2} = \sqrt{25}\,\sqrt{7}\,\sqrt{2}$
$= 5\sqrt{7}\,\sqrt{2}$
$= 5\sqrt{14}$

Multiplying and Simplifying

1. $\sqrt{5}\sqrt{10}$ $= \sqrt{5\times10} = \sqrt{50} = \sqrt{25\times2} = \sqrt{25}\sqrt{2} = 5\sqrt{2}$

2. $3\sqrt{12} * \sqrt{6}$ $= 3\sqrt{12}\times\sqrt{6} = 3\sqrt{12\times6} = 3\sqrt{2\times\underbrace{6\times6}} = 3\times6\sqrt{2} = 18\sqrt{2}$

3. $\sqrt{5} * -4\sqrt{20}$ $= \sqrt{5}\times-4\times\sqrt{20} = -4\sqrt{5\times20} = -4\sqrt{5\times5\times4} = -4\sqrt{\underbrace{5\times5}\times\underbrace{2\times2}} = -4\times5\times2 = -40$

$-4\sqrt{100} = -4\times10$

4. $-4\sqrt{15} * -\sqrt{3}$ $= -4\times(-1)\times\sqrt{15}\times\sqrt{3} = 4\sqrt{15\times3} = 4\sqrt{5\times\underbrace{3\times3}} = 4\times3\times\sqrt{5} = 12\sqrt{5}$

5. $\sqrt[3]{3}\sqrt[3]{9}$ $= \sqrt[3]{3\times9} = \sqrt[3]{27} = \sqrt[3]{\underbrace{3\times3\times3}} = 3$

$\sqrt[3]{3^3} = 3$

6. $\sqrt[3]{7}\sqrt[3]{5}$ $= \sqrt[3]{7\times5} = \sqrt[3]{35}$

**THE QUOTIENT RULE FOR RADICALS**

For any real numbers $\sqrt[n]{a}$ and $\sqrt[n]{b}$, $b \neq 0$,

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}.$$

Divide, and if possible simplify

1. $\frac{\sqrt{80}}{\sqrt{5}}$ $= \sqrt{\frac{\cancel{80}\,16}{\cancel{5}}} = \sqrt{16} = 4$

2. $\frac{\sqrt{72}}{\sqrt{8}}$ $= \sqrt{\frac{72}{8}} = \sqrt{9} = 3$

3. $\frac{\sqrt{15}}{\sqrt{12}}$ $= \sqrt{\frac{\cancel{15}\,5}{\cancel{12}\,4}} = \sqrt{\frac{5}{4}} = \frac{\sqrt{5}}{\sqrt{4}} = \frac{\sqrt{5}}{2} = \frac{1}{2}\sqrt{5}$

We do not like to keep radicals in the denominator so… we do something called rationalizing the denominator. Let's use the last example.

$$\frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{a}}{\sqrt{b}}\times\frac{\sqrt{b}}{\sqrt{b}} = \frac{\sqrt{a}\times\sqrt{b}}{\sqrt{b}\times\sqrt{b}} = \frac{\sqrt{ab}}{b}$$

1. $\frac{\sqrt{15}}{\sqrt{12}}$ $= \frac{\sqrt{15}\times\sqrt{12}}{\sqrt{12}\times\sqrt{12}} = \frac{\sqrt{15\times12}}{12} = \frac{\sqrt{5\times3\times3\times4}}{12} = \frac{\sqrt{5}\times\sqrt{9}\times\sqrt{4}}{12} = \frac{\sqrt{5}\times\cancel{3}\times\cancel{2}}{\cancel{12}\,\cancel{4}\,2} = \frac{\sqrt{5}}{2}$

2. $\frac{\sqrt{4}}{2\sqrt{20}}$ $= \frac{\sqrt{4}\times\sqrt{20}}{2\sqrt{20}\times\sqrt{20}} = \frac{\sqrt{80}}{2\times20} = \frac{\sqrt{16}\sqrt{5}}{40} = \frac{\cancel{4}\sqrt{5}}{\cancel{40}\,10} = \frac{\sqrt{5}}{10}$

$\boxed{* \; 80 = 4\times20 = 4\times4\times5 = 16\times5}$

3. $\frac{\sqrt{4}}{4\sqrt{5}}$ $= \frac{2}{2\,\cancel{4}\sqrt{5}} = \frac{1}{2\sqrt{5}}$

**THE NUMBER *i***

*i* is the unique number for which $i = \sqrt{-1}$ and $i^2 = -1$.

$\frac{1}{2\sqrt{5}} = \frac{1}{2\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}} = \frac{1\times\sqrt{5}}{2\sqrt{5}\times\sqrt{5}} = \frac{\sqrt{5}}{2\times5} = \frac{\sqrt{5}}{10}$

# Quiz 11

1. Factorize $2x^2+x-1$ $\qquad 2x \cdot -1 = -2 = 1 \times -2 = -1 \times 2$

$$= \underbrace{2x^2 - x} + \underbrace{2x - 1}$$
$$= x(2x-1) + 1(2x-1) = (x+1)(2x-1)$$

2. $a^2 - b^2 = (a+b)(a-b)$. Factorize $9x^2 - 4y^2$

$$9x^2 - 4y^2 = (3x)^2 - (2y)^2 = (3x+2y)(3x-2y)$$

SIMPLIFYING RADICALS MAZE

Write each radical in simplest form.