

Name:

[1 pt]

**Problem 1.** For the function  $f(x) = x^3 - 3x$ ,

1. Find all the relative extremal and inflection points. [5 pts]

2. Sketch the graph of  $f$ . [5 pts]

$$f'(x) = 3x^2 - 3 = 3(x^2 - 1)$$

$$f''(x) = 6x$$

$$f'(x) = 0 \Rightarrow 3(x^2 - 1) = 0 \Rightarrow x^2 - 1 = 0 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$$

$$f''(1) = 6 > 0 \Rightarrow x = 1 \text{ is min pt.}, f(1) = -2$$

$$f''(-1) = -6 < 0 \Rightarrow x = -1 \text{ is max pt.}, f(-1) = 2$$

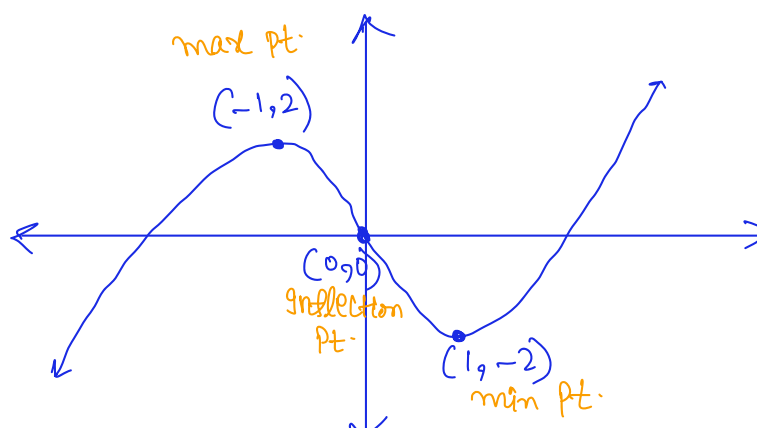
3 pts.

$$f''(x) = 0 \Rightarrow 6x = 0 \Rightarrow x = 0$$

$\leftarrow \begin{array}{c} - \\ \text{Concave} \\ \text{down} \end{array} \quad \begin{array}{c} + \\ \text{Concave} \\ \text{up} \end{array} \rightarrow \Rightarrow x = 0 \text{ is an inflection pt.}$

2 pts

$$f(0) = 0^3 - 3(0) = 0 \Rightarrow (0, 0) \text{ is y-int.}$$



5 pts.