Math16600 Section 23715 Quiz 1
Fall 2023, August 29

Name: Solutions [1 pt]

Problem 1: Let $f(x) = 3x^3 + 4x^2 + 6x + 5$. Find $\left(f^{-1}\right)'(5)$. [5 pts]

$$(f^{-1})'(5) = \frac{1}{f'(f^{-1}(5))}$$

let $x = f^{-1}(5) \Rightarrow f(x) = 5 \Rightarrow 3x^3+4x^2+6x+5 = 0$

$\Rightarrow 3x^3+4x^2+6x = 0 \Rightarrow x(3x^2+4x+6) = 0 \Rightarrow x = 0$

$\Rightarrow f^{-1}(5) = 0 \Rightarrow (f^{-1})'(5) = \frac{1}{f'(0)}$

$f'(x) = 9x^2 + 8x + 6 \Rightarrow f'(0) = 6 \Rightarrow \boxed{(f^{-1})'(5) = \frac{1}{6}}$

Problem 2: Differentiate

$$f(x) = \frac{x^2 e^x}{x^2 + e^x}$$

[5 pts]

$$f'(x) = \frac{(x^2+e^x)(x^2e^x)' - x^2e^x(x^2+e^x)'}{(x^2+e^x)^2}$$

$$= \frac{(x^2+e^x)(2xe^x + x^2e^x) - x^2e^x(2x+e^x)}{(x^2+e^x)^2}$$

$$= \frac{(x^2+e^x)xe^x(2+x) - x^2e^x(2x+e^x)}{(x^2+e^x)^2}$$

$$= \frac{xe^x\left[2x^2 + 2e^x + x^3 + xe^x - 2x^2 - xe^x\right]}{(x^2+e^x)^2}$$

$$\boxed{= \frac{xe^x(x^3+2e^x)}{(x^2+e^x)^2}}$$