Station: 4 Scratch Paper: ______/______

DIVISION OF DIVERSITY, EQUITY & INCLUSION
ACCESSIBLE EDUCATIONAL SERVICES
Indianapolis

AES Testing Record

Student: Ethan Aldrich Winnett -(Ethan) **Must Stop At:** 1:56 Pm

Test Date: March 26, 2025 **Test Time:** 12:00 pm **Location:** AES Testing Lab (UL 3135H -Lib 3rd fl)

Student Status:

Course Title: ANALYTIC GEOMETRY & CALCULUS I **Code:** MATH-I 165 30129

Instructor: Keshav Dahiya

Test Type: Exam **Test Format:** Paper **Name/Number:** Test 2

Accommodations: Distraction-Reduced Environment; Extended Time on Quizzes and Exams (150%)

Instructor's Directions: Closed book/notes. No calculator. No scratch paper.

Time Allotted: 113min **Start Time:** 12:03 Pm **Ending Time:** 1:56 PM

Breaks Taken:

Proctors: Jimmie, Ahalya

Proctor Notes:

Delivery Preference: Scan/email test, then keep in AES Office UC100

Delivery Log (Please contact AES for delivery records)

Emailed By: Date: Time:

Delivered By: Date: Time: Location:

Received By: Date: Time:

Attempted By: Date: Time: Location:

Explanation:

Attempted By: Date: Time: Location:

Explanation:

Indiana University, Indianapolis

Spring 2025 Math-I 165

Test 2

March 26, 2025

Instructor: Keshav Dahiya

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Name: Ethan Winnett

Instructions:

- No cell phones, calculators, watches, technology, hats - stow all in your bags.
- **Write your name** on this cover page.
- This test is **closed book and closed notes**.
- All work must be clearly shown for partial credit.
- If you wish for something not to be graded, please strike it out neatly.
- Box, circle, or otherwise clearly indicate your final answer.
- When you finish, return your test to the proctor, and leave the classroom.
- There are a total of **12 problems including 2 bonus problems**.
  - Problems 1-10 are each worth 10 points.
  - The bonus problems are each worth 5 points.
- You can score a **maximum of 110 points out of 100**.
- There are a total of **7 pages** including the cover page.

Problem 1. Let $f(x) = \sin x + \cos x$ be defined on the interval $[0, 2\pi]$. Find the critical numbers of f in the given interval. Use the closed interval method to find the absolute maximum and minimum values of f on the given interval. [6+4 pts]

$f'(x) = \cos x - \sin x \longrightarrow x = \frac{\pi}{4}, \frac{5\pi}{4}$ $f'(\frac{\pi}{2}) = 0 - 1 = -1$

$f'(0) = 1 - 0 = 1$ $f'(\frac{\pi}{2}) = 0 - 1$

$f(\pi) = -1 + 0 = -1$

$f(\frac{\pi}{4})$ $\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

$f'(\frac{5\pi}{4}) = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} = -\frac{2\sqrt{2}}{2} = -\sqrt{2}$

$f'(\frac{3\pi}{2}) =$

abs max $= (\frac{\pi}{4}, \sqrt{2})$

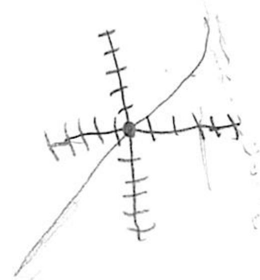
abs min $(0,$

5

Problem 2. Let f be an everywhere continuous and everywhere differentiable function. Suppose $f(0) = 0$ and $f'(x) \le 5$ for all values of x. Find the largest possible value of $f(3)$. [10 pts]

$\frac{f(b) - f(a)}{b - a} \le 5$

Put $b = 3, a = 0$



4

Problem 3. Find all the local extremal values of $f(x) = x - 2\sin x$ for $0 \le x \le 2\pi$. [10 pts]

$f'(x) = 1 - 2\cos x = 0$ $\quad \frac{-2\cos x}{-2} = \frac{-1}{-2}$ $\quad \cos x = \frac{1}{2}$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

$+ \mid - \mid +$
$0 \quad \frac{\pi}{3} \quad \frac{5\pi}{3}$

$f'(\frac{\pi}{2}) = \frac{\pi}{2} - 2\sin(\frac{\pi}{2}) = \frac{3.14}{2} - 2 = -.86$

$f'(\frac{\pi}{6}) = \frac{\pi}{6} - 2\sin(\frac{\pi}{6}) = \frac{3.14}{6} - 1 = -2.86$

$f'(\frac{-5\pi}{6}) = \frac{-\pi}{6} - 2\sin(\frac{-5\pi}{6}) = -\frac{\pi}{6} - 2(-\frac{1}{2})$

$-\frac{3.14}{6} + 1 = -2.86$

$+ \mid - \mid +$
$0 \quad \frac{5\pi}{3}$

local max at 0 or 2π $\quad \pi/3$

local min at $\frac{5\pi}{3}$

$+ \quad \frac{\pi}{3} \quad - \quad \frac{5\pi}{3} \quad +$

8

Problem 4. Find the intervals of concavity and points of inflection of $f(x) = x^4 - 4x^3$. [10 pts]

$f'(x) = 4x^3 - 12x^2$ $\quad 4x^2(x-3)$ $\quad x = 3, 0$

$+ \mid - \mid +$
$0 \quad 3$

$f(2) = -$
$f(4) = +$
$f(-1) = +$

increases on $(-\infty, 0) \cup (3, \infty)$
decreases on $(0, 3)$

$f''(x) = 12x^2 - 24x$

$12x(x-2)$ $\quad x = 0, 2$

$+ \mid - \mid +$
$0 \quad 2$

inflection points are 0 and 2

concave up at $(-\infty, 0) \cup (2, \infty)$
concave down at $(0, 2)$

$f''(1) = -12$
$f''(-1) = 36$
$f''(3) = 36$

10

Problem 5. Find all the horizontal asymptotes to the graph of $f(x) = \dfrac{\sqrt{4x^2+1}}{3x-5}$. [10 pts]

$\frac{x}{x}\left(\frac{\sqrt{4x^2+1}}{3x-5}\right)$ $\frac{\sqrt{4}}{3} \to \pm\frac{2}{3}$ = horizontal asy

10

horizontal asy = $\frac{2}{3}, -\frac{2}{3}$ ✓

$\frac{\sqrt{4x^2+1}}{3x-5} = 0$

$\sqrt{4x^2+1} = 0$ $4x^2+1=0$ $4x^2=-1$ $x^2 = -\frac{1}{4}$

$\sqrt{x} = \sqrt{-\frac{1}{4}}$ doesn't exist

Problem 6. Find all the points of local maxima and minima of the function $f(x) = \dfrac{x^2}{x^2-1}$. [10 pts]

$f'(x) = \frac{(x^2)'(x^2-1) - (x^2)(x^2-1)'}{(x^2-1)^2} \to \frac{2x(x^2-1) - x^2(2x)}{(x^2-1)^2}$

$\frac{2x^3-2x-2x^3}{(x^2-1)^2} \to \frac{-2x}{(x^2-1)^2}$ ✓ $-2x=0$ $x=0$

$x = \pm 1$

+ + − −
−1 0 1 ✓

$f'(-\frac{1}{2}) = \frac{-2(-\frac{1}{2})}{((\frac{-1}{2})^2-1)^2} \to \frac{1}{(\frac{1}{4}-1)^2}$ $\frac{1}{(\frac{-3}{4})^2} \to \frac{1}{\frac{9}{16}} \to \frac{16}{9}$

$f'(\frac{1}{2}) = -$

$f'(2) = -$

$f'(-2) = +$

$x = 0$ is l. max. pt.

9

Problem 7. For the function $f(x) = \dfrac{x^2}{x^2 - 1}$ from Problem 6, find the intervals of concavity and points of inflection. [10 pts]

$\frac{x^2}{x^2-1}$ $(x^2)'(x^2-1) - (x^2)(x^2-1)'$

$f'(x) = \frac{2x^3 - 2x - 2x^3}{(x^2-1)^2}$ $\frac{-2x}{(x^2-1)^2}$

$f''(x)$ $\frac{(-2x)'(x^2-1)^2 - (-2x)(x^2-1)^{2'}}{(x^2-1)^3} \to -2(x^2-1)^2 - (-2x)(2)(x^2-1)(2x)$

$\frac{-2(x^4-2x^2+1) - (-8x^4+8x^2)}{(x^2-1)^3} \to \frac{-2x^4+4x^2-2+8x^4-8x^2}{(x^2-1)^3}$ ✓

$\frac{6x^4-4x^2-2}{(x^2-1)^3}$ $\frac{2(3x^4-2x^2-1)}{(x^2-1)^3} \to (3x^2+1)(x^2-1)$: $3x^2=-1$ X $x^2-1=\pm1$

$x=\pm1$ increased rate but then rate decreases

8

no points of inflection

concave up $(-\infty, 0)$ always

$f'(x)$ + + − −

$f''(x)$ + + + +

−1 0 1

+ − +

−1 1

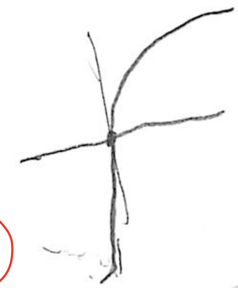
Problem 8. For the function $f(x) = \dfrac{x^2}{x^2 - 1}$ from Problem 6, find all the horizontal and vertical asymptotes. [10 pts]

$\frac{x^2}{x^2-1} = 0$ $x=0$ $\frac{x^2}{x^2}\left(\frac{x^2}{x^2-1}\right) = 1$ HA = 1, 0 $y=1$ only

$(x^2-1) \to x = 1, -1$ VA = 1, −1 ✓

9

Problem 9. Let $f(x) = \dfrac{x^2}{x^2-1}$. Find the domain of f. Find the x-intercepts and y-intercept of $y = f(x)$. Use this information along with information obtained in problems 6, 7, 8 to sketch the curve $y = f(x)$. [10 pts]

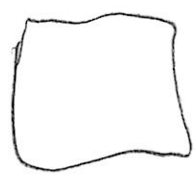


Domain=?
x, y int.

It concaves up forever but from $(0, \infty)$ the rate of it as it concaves up decreases

2

Problem 10. You have to choose to buy a rectangular farm having fixed area of 10,000 square feet. Find the dimensions that you should choose so that the cost of fencing this farm is as minimum as possible. [10 pts]



$A = 10{,}000$ $100 \cdot 100 = 10{,}000$?

$A = XY$

$A' = (X)'(Y) + (X)(Y)'$

$A' = Y + Xy'$ $100 + 100 = 200$

3

Bonus Problem 1. The cost function of a firm is $C(x) = 1000 + 40x - x^2$. If the demand function is given by $p(x) = 100 - 4x$, find the production level that maximizes the profit. [5 pts]

$-x^2 + 40x + 1000 = -4x + 100$

$-2x + 40 \to x = 20$

1

Bonus Problem 2. For what values of the constants a and b is $(1, 3)$ a point of inflection of the curve $y = ax^3 + bx^2$? [5 pts]

$x^2(ax + b)$

$x + 3 = 0 \quad x = -3$

$\frac{\cancel{f(b)} - f(a)}{b - a}$

$y = 1(-3)^3 + 3(-3)^2$

$y = -27 + 27 \quad y = 0$

1

1 3

$f(-3)$