Linear Programming

ESSENTIALS

The Corner Principle: The maximum value or the minimum value of an **objective function** $F = ax + by + c$, subject to **contraints** on x and y, will be found at a vertex of the **feasible region** formed by the system of linear inequalities consisting of the constraints.

Example

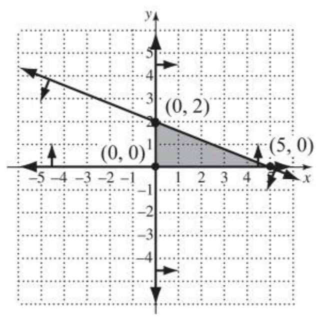
- Find the maximum and the minimum values of the objective function F and the values of x and y at which they occur.

  $F = 3x - 4y$, subject to

  $2x + 5y \le 10$,

  $x \ge 0$,

  $y \ge 0$



The vertices of the shaded region are $(0, 0)$, $(0, 2)$, and $(5, 0)$.

Evaluate F at each vertex.

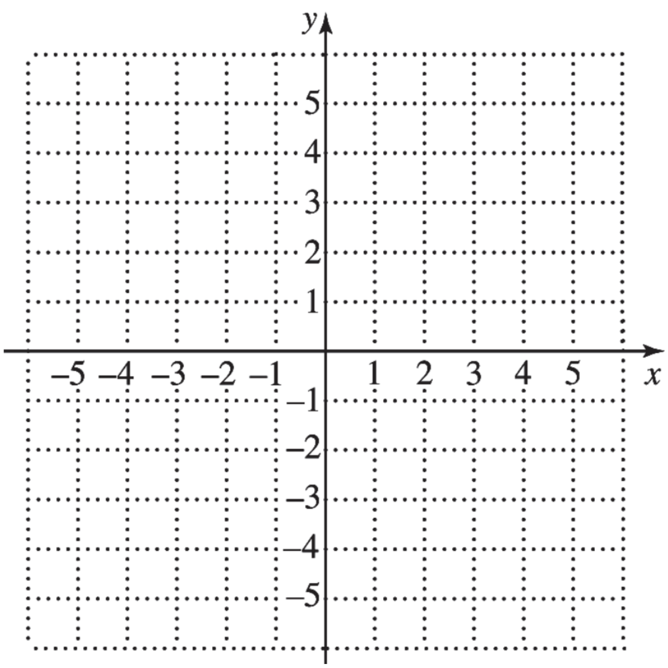
Vertex	$F = 3x - 4y$
$(0, 0)$	$3(0) - 4(0) = 0$
$(0, 2)$	$3(0) - 4(2) = -8$
$(5, 0)$	$3(5) - 4(0) = 15$

The maximum value of F, 15, occurs at $(5, 0)$.

The minimum value of F, -8, occurs at $(0, 2)$.

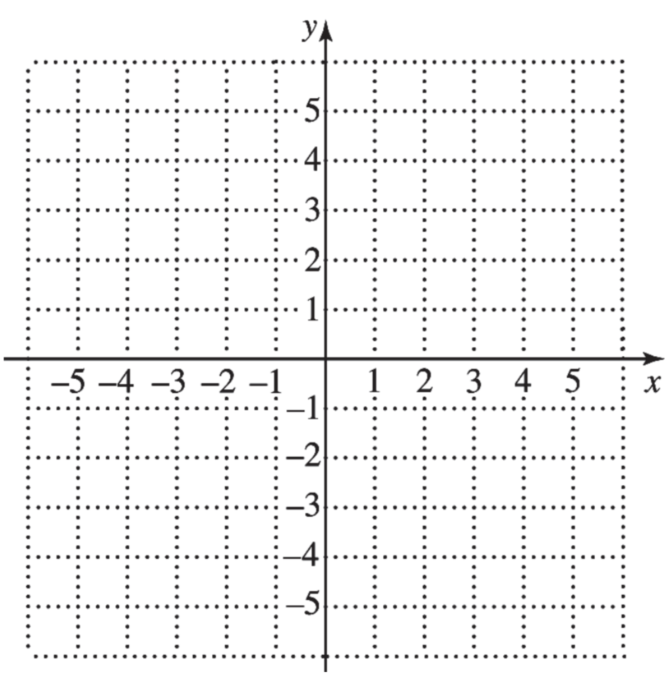
Find the maximum and minimum values of the objective function F and the values of x and y at which they occur

$F = 2x - 5y$ subject to $x + y \leq 2$, $2x - y \geq -4$, $y \geq 0$

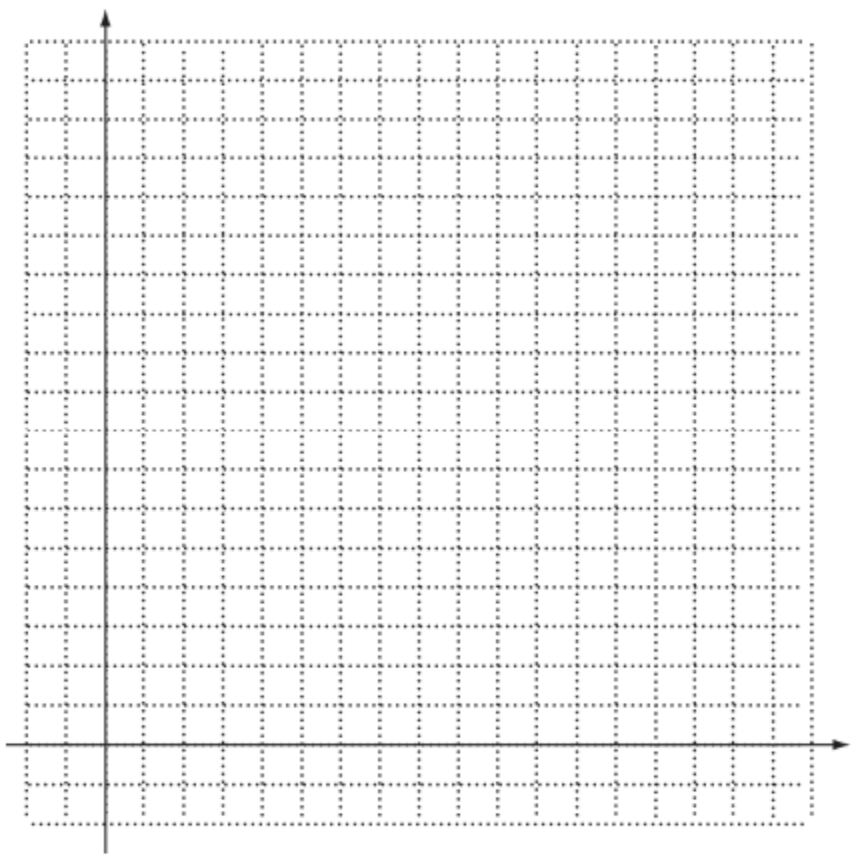


Find the maximum and minimum values of the objective function F and the values of x and y at which they occur

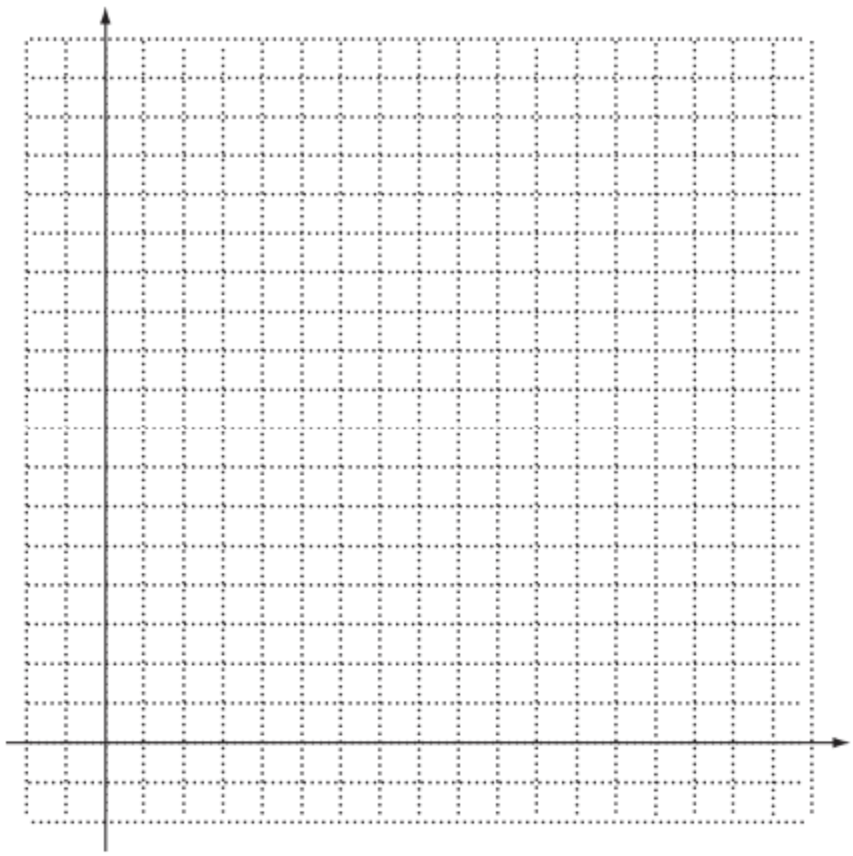
$F = 3x + 2y$, subject to $x + y \leq 3$, $2x + y \leq 4$, $x \geq 0$, $y \geq 0$



Joe's Cup o' Joe has 960 lb of Brazilian coffee beans and 600 lb of Kenyan coffee beans. A batch of house blend requires 15 lb of Brazilian beans and 5 lb of Kenyan beans and yields a profit of $80. A batch of supreme blend requires 12 lb of Brazilian beans and 8 lb of Kenyan beans and yields a profit of $60. Find the maximum profit and the number of batches of each kind that should be made to maximize profit.



Joe's Cup o' Joe has 400 lb of Columbian coffee beans and 200 lb of Java coffee beans. A batch of Lucky Joe's blend requires 8 lb of Columbian beans and 2 lb of Java beans and yields a profit of $30. A batch of Good Morning blend requires 5 lb of Columbian beans and 5 lb of Java beans and yields a profit of $40. Find the maximum profit and the number of batches of each kind that should be made to maximize profit.



Practice Exercises

Readiness Check

Choose the expression from the following list that best completes the sentence. Expressions may be used more than once or not at all.

constraint objective function vertex origin feasible region

1. In linear programming, the function we seek to minimize or maximize is called a(n) ________________________.

2. In linear programming, ordered pairs that satisfy all constraints are in the ________________.

3. In linear programming, a demand placed on an objective function is called a(n) ________________.

4. The corner principle tells us that, if it exists, a maximum or minimum value of the objective function is found at a(n) ________________ of the feasible region.

Linear Programming

Find the maximum and the minimum values of each objective function and the values of x and y at which they occur.

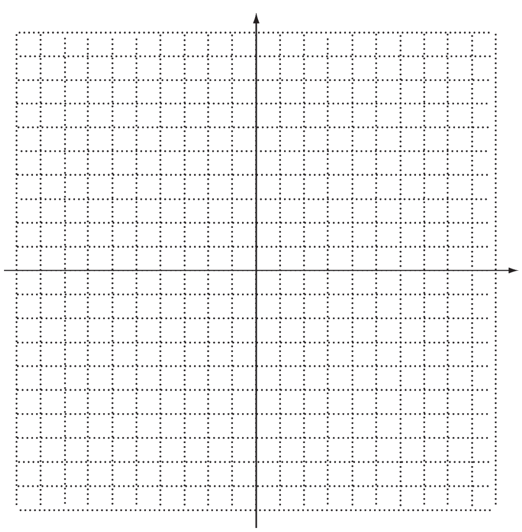
5. $F = 3x - 5y$, subject to

$5x + 10y \le 60,$

$2x + 8y \le 40,$

$x \ge 0,$

$y \ge 0$



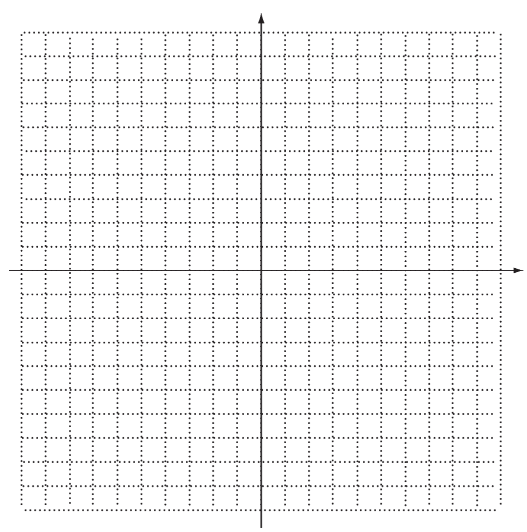
6. $P = 8x + 12y - 50$, subject to

$3x + 6y \le 36,$

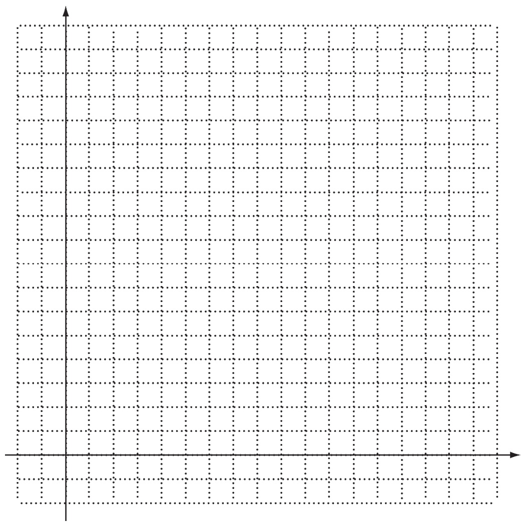
$8x + 4y \le 36,$

$x \ge 0,$

$y \ge 0$



7. Ken is investing up to \$50,000 in corporate bonds or in municipal bonds, or both. He must invest from \$5000 to \$30,000 in corporate bonds, and will not invest more than \$25,000 in municipal bonds. The interest on corporate bonds is 4.5% and on municipal bonds is 3.5%. This is simple interest for one year. How much should Ken invest in each type of bond in order to earn the most interest? What is the maximum interest?



8. Great Valley Vineyard has 50 acres upon which to plant chardonnay and cabernet sauvignon grapes. Profit per acre of chardonnay is \$360 and profit per acre of cabernet sauvignon is \$500. The number of hours of labor available is 3800. Each acre of chardonnay requires 70 hr of labor and each acre of cabernet sauvignon requires 100 hr of labor. Determine how the land should be divided between chardonnay and cabernet sauvignon in order to maximize profit.