Name: [1 pt]

Problem 1: Let $f(x) = \dfrac{1}{x^2}$. Evaluate the limit

$$\lim_{x \to 1} \frac{f(x) - f(1)}{x - 1}.$$

[5 pts]

$$\lim_{x \to 1} \frac{\frac{1}{x^2} - \frac{1}{1^2}}{x-1} = \lim_{x \to 1} \frac{\frac{1}{x^2} - 1}{x-1} = \lim_{x \to 1} \frac{\frac{1-x^2}{x^2}}{x-1}$$

$$= \lim_{x \to 1} \frac{1-x^2}{x^2(x-1)} = \lim_{x \to 1} \frac{(1-x)(1+x)}{x^2(x-1)} = \lim_{x \to 1} \frac{-(x-1)(1+x)}{x^2(x-1)}$$

$$= \lim_{x \to 1} \frac{-(1+x)}{x^2} = -\frac{(1+1)}{1^2} = -2$$

Problem 2: Let $f(x) = x^3$. Evaluate the limit

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

$$\lim_{h \to 0} \frac{(x+h)^3 - x^3}{h} \qquad (x+h)^3 = x^3 + 3x^2 h + 3xh^2 + h^3 \quad \text{[4 pts]}.$$

$$= \lim_{h \to 0} \frac{x^3 + 3x^2 h + 3xh^2 + h^3 - x^3}{h} = \lim_{h \to 0} \frac{3x^2 h + 3xh^2 + h^3}{h}$$

$$= \lim_{h \to 0} \frac{h(3x^2 + 3xh + h^2)}{h} = 3x^2 + 3x(0) + 0^2$$

$$= 3x^2$$