Math16600 Section 23715 Quiz 11
Fall 2023, November 21

Name: [1 pt]

Problem 1: Find the Taylor series of the function $f(x) = \ln x$ about the point $x = 2$. [5 pts]

Taylor series: $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(2)}{n!}(x-2)^n$ about $a=2$

$f(x) = \ln x \Rightarrow f(2) = \ln 2$

$f'(x) = \frac{1}{x} \Rightarrow f'(2) = \frac{1}{2} = (-1)^0 \frac{0!}{2^1}$ $\Rightarrow$ For $n \geq 1$,

$f''(x) = \frac{-1}{x^2} \Rightarrow f''(2) = \frac{-1}{2^2} = (-1)^1 \frac{1!}{2^2}$ $\quad f^{(n)}(2) = (-1)^{n-1} \frac{(n-1)!}{2^n}$

$f'''(x) = \frac{2}{x^3} \Rightarrow f'''(2) = \frac{2}{2^3} = (-1)^2 \frac{2!}{2^3}$

$f^{(iv)}(x) = -\frac{2\cdot 3}{x^4} \Rightarrow f^{(iv)}(2) = -\frac{2\cdot 3}{2^4} = (-1)^3 \frac{3!}{2^4}$

$\vdots$

$\Rightarrow \ln x = \ln 2 + \sum_{n=1}^{\infty} (-1)^{n-1} \frac{(n-1)!}{2^n \, n!}(x-2)^n = \ln 2 + \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} \frac{(x-2)^n}{2^n}$

Problem 2: Find the MacLaurin series of the function $f(x) = \cos x$. [5 pts]

$f(x)$	$\cos x$	$f(0) = 1$
$f'(x)$	$-\sin x$	$f'(0) = 0$
$f''(x)$	$-\cos x$	$f''(0) = -1$
$f'''(x)$	$\sin x$	$f'''(0) = 0$
$f^{(iv)}(x)$	$\cos x$	$f^{iv}(0) = 1$
$\vdots$		$\vdots$

$\Rightarrow f^{(n)}(0) = \begin{cases} 0 & \text{if } n \text{ is odd} \\ (-1)^{\frac{n}{2}} & \text{if } n \text{ is even} \end{cases}$

$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$

$\Rightarrow \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} \mp \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$