Math 110

1.1 Basics of Algebra

- Algebraic Expression: Consists of variables and/ or numerals, often with operation signs and grouping symbols. Some examples of algebraic expressions are

$$x+4+y+7\div 7 \quad , \quad 48-t+9$$

$$4\times 8$$

- When an equal sign is placed between two expressions, an **equation** is formed. We often solve equations.

$$\underbrace{x+4+y+7\div 7}=\underbrace{10} \quad , \quad \underbrace{48-t+9}=\underbrace{0}$$

$$x+4+y+7\div 7=48-t+9 \quad , \quad n=n+1$$

Translating Algebraic Expressions

Addition	Subtraction	Multiplication	Division
$x+4$ $\downarrow$ Add 7 $=7+x+4$ $x+4+7$ $=x+11$	$x+4$ $\downarrow$ Subtract 4 $x+\underbrace{4-4}$ $=x+0=x$	$4x$ $\downarrow$ multiply 5 $4x\times 5$ or $5\times 4x$ $=4\times\underbrace{x\times 5}=4\times 5\times x$ $=20x$	$4x$ $\downarrow$ Divide by 4 $4x\div 4=4\times x\div 4$ $=x\times 4\div 4$ $=x\times 1=1\times x=1x=x$

CAUTION: $4-x+4$ (wrong)

Phrase	Algebraic Expression
Five *more* than $\underbrace{\text{some number}}_{x}$ → y	$x+5$ or $y+5$
Half *of* $\underbrace{\text{a number}}_{a}$ of = multiplication	$\frac{1}{2}\times a=\frac{1}{2}a=a\times\frac{1}{2}=\frac{a}{2}$
Five *more than* three *times* $\underbrace{\text{some number}}_{b}$	$3\times b+5=3b+5$
The difference of $\underbrace{\text{two number}}$ x and y	$x-y$
Six led than the product of $\underbrace{\text{two numbers}}$ less a and b	$a\times b-6=ab-6$
Seventy –six percent $\overset{\downarrow}{\text{of}}$ some number → x multiplication	$\frac{76}{100}\times x=\frac{76}{100}x=0.76x$

↙ divide by 100

Example 1: Five less than forty-three percent of the quotient of two numbers $\rightarrow$ a and b

$$\frac{43}{100} \times \frac{a}{b} - 5 = 0.43\frac{a}{b} - 5$$

Exponential Notation

- The expression, a^n, in which n, is a counting number, means $\rightarrow \underbrace{a \times a \times a \times \cdots \times a}_{n \text{ times}}$

  $n=3$, then $a^3 = a \times a \times a$

- In a^n, a is called the base and n is the exponent. When no exponent appears, the exponent is assumed to be 1. $a^1 = a$

$$2^4 = 2 \times 2 \times 2 \times 2 = 4 \times 2 \times 2 = 8 \times 2 = 16$$

Example 2: a^n is read : a to the Power n or a to the n.

Example 3: We read s^2 as : s square / squared.

s^3 : s cube / cubed.

Evaluating ~~Exponential~~ Expressions

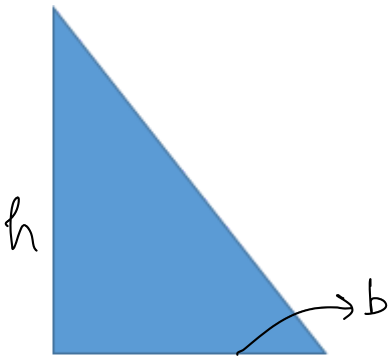
- When we replace a variable with a number, we say that we are substituting for the variable. The calculations that follows the substitution is called evaluating the expression.

Example 4: The base of a triangular sail is 3.1 m and the height is 4m. Find the area of the sail.

Area of a triangle: $\frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2}bh$

$b = 3.1$, $h = 4$

$$\frac{1}{2} \times 3.1 \times 4 = 3.1 \times \frac{1}{2} \times 4 = 3.1 \times 2$$
$$= \underline{\underline{6.2}}$$



<u>Example</u> : Evaluate the expression $x+5$ for $x = \underline{1}$.

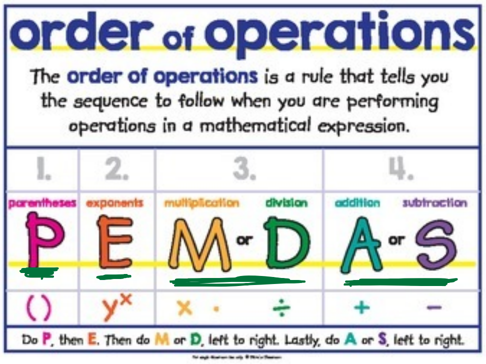
Substitution: $1+5 = 6$

<u>Example</u> : Evaluate the expression x^3-7 for $x=3$.

Substitute : $3^3-7 = 3\times3\times3-7 = 9\times3-7 = 27-7 = \underline{20}$

Order of Operations

1. Simplify within any grouping symbols such as (), [], or {}, working the innermost symbol first
2. Simplify all exponential expressions
3. Perform all multiplication and division, working from left to right
4. Perform all addition and subtraction, working from left to right



order of operations

The order of operations is a rule that tells you the sequence to follow when you are performing operations in a mathematical expression.

1.	2.	3.		4.	
parentheses	exponents	multiplication	division	addition	subtraction
P	E	M or	D	A or	S
( )	y^x	× ·	÷	+	−

Do P, then E. Then do M or D, left to right. Lastly, do A or S, left to right.

Example 5: Evaluate $5 + 2(a - 1)^2$ $for\ a = 4$

Substitute: $5+2(4-1)^2 = 5+2(3)^2 = 5+2\times 9$

$3^2 = 3\times 3 = 9$

$= 5+18$

$= 23$

Example 6: Evaluate $9 - x^3 + 6\div 2y^2$ $for\ x = 2\ and\ y = 5$

Substitute: $9-2^3+6\div 2\times 5^2 = 9-8+\underbrace{6\div 2}\times 25$

$5^2 = 25$

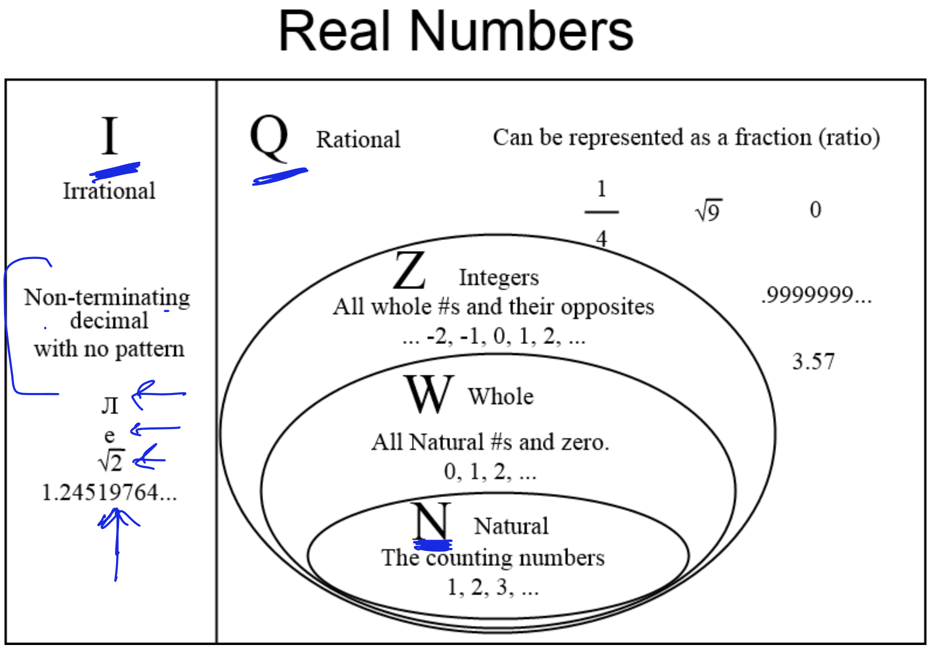
$2^3 = 8$

$= 9-8+\underbrace{3\times 25} = \underbrace{9-8}+75$

$= \underbrace{1+75}$

$= 76$

Sets of Numbers



$\sqrt{9} = 3 = \frac{3}{1}$

$Z = \{\dots, -2, -1, 0, 1, 2, 3, \dots\}$

$W = \{0, 1, 2, 3, \dots\}$

$\mathbb{N} = \{1, 2, 3, \dots\}$

Example 7: The set containing the numbers -2, 1,3 and three can be written as: $\{a, b, c, \dots, z\}$

$\{-2, 1, \underbrace{3, 3}\} \rightarrow \{-2, 1, 3\}$

This is called roster notation

$\rightarrow \{x \mid x \text{ is an alphabet}\}$

Example 8: To describe sets containing the numbers between integers, we use a second type of set notation called set builder notation

Describe a set containing numbers between integers -1 and 4.

$\{x \mid x \text{ is a number between } -1 \text{ and } 4\}$

$x = 4.66666\dots \Rightarrow 10x = 46.6666\dots \Rightarrow 9x = 42$

$x = 4.66666\dots \Rightarrow x = 42/9$

Example 9: Which numbers in the following list are **(a) whole numbers? (b) integers? (c)Rational numbers? (d) Irrational (e) real numbers?**

4.66666.....

4.12121212.....

$\underbrace{3.8245}_{\text{Rational}} = \frac{38245}{10000}$

$\sqrt{49} = \sqrt{7^2}$

$= 7$

$-29, \ -\frac{7}{4}, \ 0, \ 2, \ \underline{3.9}, \ \sqrt{42}, \ 78$

(a) Whole: $0, 2, 78$

(b) Integers: $-29, 0, 2, 78$

(c)Rational: $-29, 0, 2, 78, -\frac{7}{4}, 3.9$

(d)Irrational $\sqrt{42}$

(e)Real $-29, 0, 2, 78, -\frac{7}{4}, 3.9, \sqrt{42}$

$3.9 = \frac{39}{10}$

$\parallel$

$3 + 0.9 = 3 + \frac{9}{10}$

0.43

$\parallel$

$\frac{43}{100}$

$0.4 = \frac{4}{10}$

$0.004 = \frac{4}{1000}$

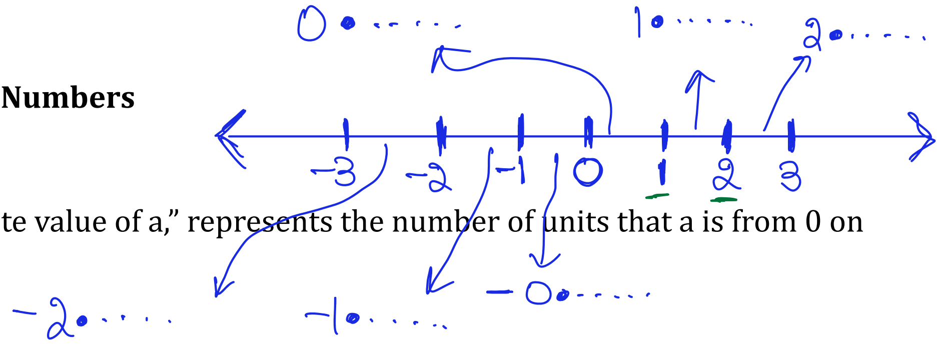
$0.04 = \frac{4}{100}$

Your Turn:

1. Translate to an algebraic expression: Half of the difference of two numbers
2. The base of a triangle is 5 ft and the height is 3ft. FInd the area of the triangle
3. Evaluate $2(x + 1)^2 =- 10$ for $x = 5$
4. Evaluate $8a^2 \div 5b - 4 + a$ for $a = 5 \ and \ b = 2$
5. Using both roster and set builder notation. represent the set of all multiples of 5 between 1 and 21
6. Classify the statement $\frac{1}{2}\epsilon\{x|\ x\ is\ a\ whole\ number\}$ as either true of false

1.2 Operations and Properties of Real Numbers

Absolute Value



- The notation |a| , read "the absolute value of a," represents the number of units that a is from 0 on the number line.

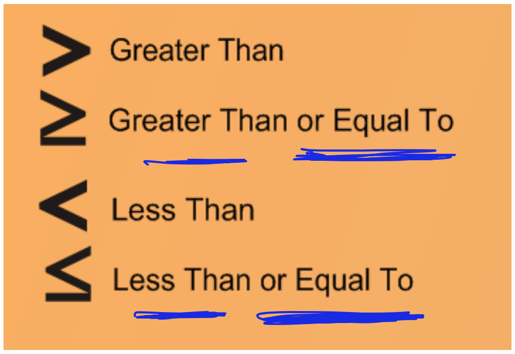
Example 1: Find the absolute value of

A. |-4| = 4

B. |2.5| = 2.5

C. |0| = 0

Inequalities



$>$ Greater Than

$\geq$ Greater Than or Equal To

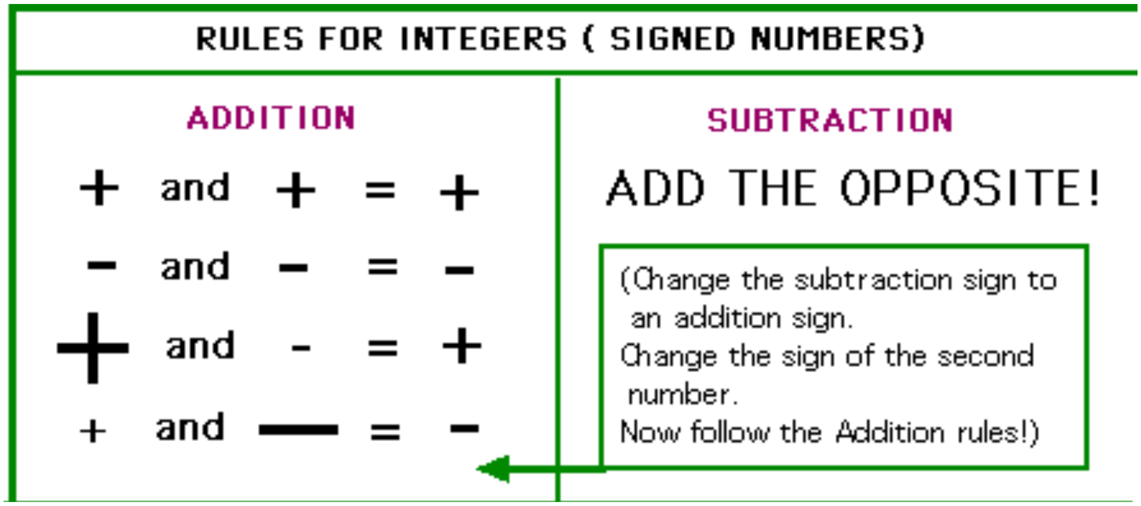
$<$ Less Than

$\leq$ Less Than or Equal To

Example 2: write out the meaning of each inequality and determine whether it is a true statement

a. − 7 < 2 → -7 is less than 2 → True	b. − 3≥ − 2 → -3 is greater than or equal to -2. → False
c. 5≤6 → 5 is less than or equal to 6 → True	d. 6≤6 → 6 is less than or equal to 6 → True

Addition Subtraction and Opposite



RULES FOR INTEGERS (SIGNED NUMBERS)

ADDITION	SUBTRACTION
+ and + = + − and − = − + and − = + + and − = −	ADD THE OPPOSITE! (Change the subtraction sign to an addition sign. Change the sign of the second number. Now follow the Addition rules!)

Reminder: Show students how to use calculator for fractions!

Example 3: Add

a. $-9+(-5)$ $\lvert-9\rvert=9$, $\lvert-5\rvert=5$ $=-(9+5)$ $=-14$	b. $-3.24+8.7$ $\lvert-3.24\rvert=3.24$ $\lvert 8.7\rvert=8.7$ $+(8.7-3.24)$ $=5.46$	c. $-\frac{3}{4}+\frac{1}{3}$ $-\frac{3\times3}{4\times3}+\frac{1\times4}{3\times4}$ $-\frac{9}{12}+\frac{4}{12}=\frac{-9+4}{12}$ $\lvert-9\rvert=9$, $\lvert4\rvert=4$ $=\frac{-(9-4)}{12}$

$=-\frac{5}{12}$

The Law of Opposites

For any two numbers *a* and -*a*

$$a+(-a)=0$$

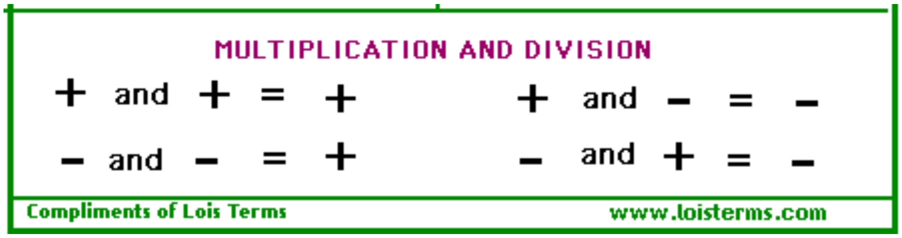
Example 4: Opposites

a. -17.5 $-(-17.5)=17.5$	b. $\frac{4}{5}$ $-\frac{4}{5}$

Example 5: Subtract

a. $5 - \underline{9}$ 5 + (opposite of 9) $5 + (-9)$ $\lvert -9 \rvert = 9$, $\lvert 5 \rvert = 5$ $-(9-5) = -4$	b. $-1.2 - (-3.7)$ -1.2 + (opposite of -3.7) $-1.2 + 3.7$ $\lvert -1.2 \rvert = 1.2$, $\lvert 3.7 \rvert = 3.7$ $+(3.7 - 1.2)$ $= +2.5$	c. $-\frac{5}{4} - \frac{2}{3}$ $\frac{-5}{4}$ + (opposite of $\frac{2}{3}$) $-\frac{5}{4} + (\frac{-2}{3})$ $\frac{-5\times3}{4\times3} + (\frac{-2\times4}{3\times4})$ $= \frac{-15}{12} + \frac{-8}{12} = \frac{-15+(-8)}{12}$ $= \frac{-(15+8)}{12}$ $= -\frac{23}{12}$

Multiplication, Division, and Reciprocals



MULTIPLICATION AND DIVISION

+	and	+	=	+	+	and	−	=	−
−	and	−	=	+	−	and	+	=	−

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Example 6: Multiply or Divide as indicated

a. $(-4)9$ -36	b. $(-\frac{2}{3})(-\frac{3}{8})$ $\frac{2\times3}{3\times8} = \frac{\cancel{6}^{1}}{\cancel{24}_{4}} = \frac{1}{4}$
c. $20 \div (-4)$ $\frac{20}{1} \times \frac{1}{-4} = -\frac{\cancel{20}^{5}}{\cancel{4}_{1}} = -5$	d. $\frac{-45}{-15}$ $\frac{\cancel{45}^{9}}{\cancel{15}_{3}} = \frac{\cancel{9}^{3}}{\cancel{3}_{1}} = 3$

The Law of Reciprocals

For any two numbers *a* and $\frac{1}{a}$ (a≠0)

$$a * \frac{1}{a} = 1$$

$4 = \frac{4}{1}$

↓Reciprocal

$\frac{1}{4}$

Example 6: Find the reciprocal

a. $\frac{7}{8}$ $\frac{8}{7}$	b. $-8 = -\frac{8}{1}$ $\frac{1}{-8} = -\frac{1}{8}$

Example 7: Divide

Keep Change Flip

a. $-\frac{1}{4} \div \frac{3}{5}$

Note: You cannot divide by 0, $\frac{number}{0}$ *is undefined*

Note 2: Order of operations apply to all real numbers

Example 8: Simplify

a. $\underline{(-5)}^2$ $-5 \times -5 = 25$	b. -5^2 $-(5 \times 5) = -25$

Example 9: Simplify

$$7 - 5^2 + 6 \div 2\underbrace{(-5)^2}$$

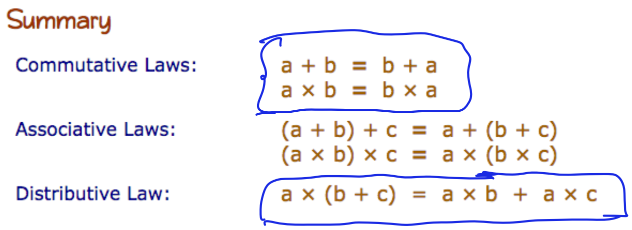
$7 - 5^2 + 6 \div 2 \times 25 = 7 - 25 + 6 \div 2 \times 25$

$= 7 - 25 + 3 \times 25 = \underbrace{7 - 25 + 75} = -18 + 75$

$= \underbrace{7 + (-25)} + 75 = \underline{\underline{57}}$

$= 7 + (-25 + 75) = 7 + 50 = 57$

The Commutative, Associative, and Distributive Law



Summary

Commutative Laws:	a + b = b + a a × b = b × a
Associative Laws:	(a + b) + c = a + (b + c) (a × b) × c = a × (b × c)
Distributive Law:	a × (b + c) = a × b + a × c

$a + b + c$

$(a+b) + c = a + (b + c)$

$a \times b \times c$

$(a \times b) \times c = a \times (b \times c)$

Example 10: Use the distributive property to simplify

$a \times (b + c)$

$\|$

$a \times b + a \times c$

$$5x(y + 4)$$

$5x \times y + 5x \times 4 = \boxed{5xy + 20x}$

$5 \times x \times 4 = 5 \times 4 \times x = 20 \times x$

$12 \times 103 = 12(100 + 3) = 12 \times 100 + 12 \times 3 = 1200 + 36 = 1236$

Example 11: Obtain an expression equivalent the following by factoring

~~$3(x - 6)$~~ $3x - 6$ $\qquad 6 = 3 \times 2$

$3 \times x - 3 \times 2 = 3(x - 2)$

Your Turn:

1. Find the absolute value of |-237|
2. Write out the meaning of $-4 \leq -3$
3. Add 4.2 + (-12)
4. Find the opposite of -13
5. Find -x fo $x =- 12$
6. Subtract $6 - (-13)$
7. Multiply $(-16)(-0.1)$
8. Find the reciprocal of $-\frac{1}{9}$
9. Divide $12 \div (-\frac{2}{3})$
10. Simplify -8^2 and $(-8)^2$
11. Simplify $24 \div (-3) \bullet (-2)^2 - 3(-6)$
12. Calculate $\frac{6-4+5-2^2}{2-|35-6^2|}$
13. Use commutative law to write and expression equivalent to $3 + mn$
14. Write an expression equivalent to 2x(y) using the associative law of multiplication
15. Obtain an expression equivalent to -3(x+7)
16. Obtain and expression equivalent to 5x+5y+5 by factoring

1.3 Solving Equations

Equivalent Equations- Equations that have the same solution.

$5x = 10$ → A solution would be value of x for which LHS = RHS

⇒ $\boxed{x=2}$

Example 1: Determine whether 4x=12 and 10x=30 are equivalent equations.

$x = 3$ $\qquad$ $x=3$

⇒ Equivalent

Example 2: Determine whether $3x = 4x$ and $3/x = 4/x$ are equivalent equations.

⇒ NOT Equivalent

$3x - 3x = 4x - 3x = (4-3)x$

$0 = x$ ⇒ $\boxed{x=0}$ → $\frac{3}{0} = \frac{4}{0}$

undefined $\qquad$ undefined.

The addition and multiplication principles

For any real numbers a, b, c:

1. **a=b is equivalent to a+c = b+c** → can add / same real number on both the sides.
2. **a=b is equivalent to a*c=b*c** → can multiply same NON-ZERO real number on both sides.

Why?

Solving one step equations

Addition	**Subtraction**
$y + 4.7 = 13.9$ $y+4.7-4.7 = 13.9-4.7$ $y+0 = 9.2$	$y - 4.7 = 13.9$ $y-4.7+4.7 = 13.9+4.7$ $y+0 = 18.6$

⇒ $\boxed{y=9.2}$ $\qquad$ ⇒ $\boxed{y=18.6}$

Multiplication	**Division**
$\frac{1}{4.7} \neq 0$ $4.7y = 13.9$ $\frac{1}{\cancel{4.7}} \times \cancel{4.7}y = \frac{1}{4.7} \times 13.9$ $\boxed{y = \frac{13.9}{4.7}}$	$4.7 \neq 0$ $\frac{y}{4.7} = 13.9$ $4.7 \times \frac{y}{4.7} = 13.9 \times 4.7$ $\boxed{y = 13.9 \times 4.7}$

$= 65.33$

FRACTIONS!!!

Solve: $\frac{2}{5}x = -\frac{9}{10}$ $\Rightarrow 5 \times \frac{2}{5}x = -\frac{9}{\cancel{10}_{2}} \times \cancel{5} \Rightarrow 2x = -\frac{9}{2}$

$\Rightarrow \frac{1}{2} \times 2x = -\frac{9}{2} \times \frac{1}{2}$

$\Rightarrow \boxed{x = -\frac{9}{4}}$

COMBINING LIKE TERMS

TERM:

LIKE OR SIMILAR TERMS:

COMBING OR COLLECTING LIKE TERMS:

Example: Simplify the following expressions

$\underline{3a} + \underline{5a^2} - \underline{7a} + \underline{a^2}$ $3a - 7a + 5a^2 + a^2$ $= (3-7)a + (5+1)a^2$ $= \underline{-4a + 6a^2}$	$-10y$ $\parallel$ $5 \times (-2) \times y$ $3x + 2[4 + 5(x - 2y)]$ $3x + 2[4 + 5x + 5(-2y)]$ $3x + 2[4 + 5x - 10y]$ $3x + 2 \times 4 + 2 \times 5x - 2 \times 10y$ $\underline{3x} + 8 + \underline{10x} - 20y$ $= 8 + 3x + 10x - 20y$ $= \underline{8 + 13x - 20y}$

$1 \cdot x = x$

The Property of -1: -1(x)=-x $\quad -1 \cdot x = -x$

Example: Simplify $-(a-b)$ $= -1(a-b) = -1 \times a + (-1)(-b) = -a+b$

$= b + (-a)$

$= b - a$

The opposite of a-b is -a+b or b-a

Example: Simplify

Equation

$9x - 5y - (\underbrace{5x + y}_{a} - \underbrace{7}_{b})$ $9x - 5y - (5x+y) + 7$ $9x - 5y - 1(5x+y) + 7$ $9x - 5y + (-1) \times 5x + (-1) \times y + 7$ $9x - 5y - 5x - y + 7$ $9x - 5x - 5y - y + 7$ $(9-5)x + (-5-1)y + 7$	$5x - 2(x-5) = 7x - 2$ $5x + (-2)x + (-2)(-5) = 7x - 2$ $(5-2)x + 10 = 7x - 2$ $3x + 10 = 7x - 2$ $-7x \qquad -7x$ $3x - 7x + 10 = 7x - 7x - 2$ $-4x + 10 = -2 - 10$ -10 $-4x = -12$

$= 4x - 6y + 7$

$\frac{-4x}{-4} = \frac{-12}{-4} \Rightarrow x = 3$

Type of Equation	Definition	Example	Solution Set
Identity	An equation that is true for all replacements	$x = x$, $x + 2y = x + 2y$ $0 = 0$, $(a+b)(a-b) = a^2 - b^2$	Every real number
Contradiction	An equation that is never true	$1 = 0$, $n = n + 1$	Empty
Conditional	An equation that is sometimes true and sometimes false, depending on the replacement fot the variabele	$5x - 2(x-5) = 7x - 2$ $3x = 4x$ $4x = 8$	$\rightarrow x = 3$ $\rightarrow x = 0$ $\rightarrow x = 2$

Example: Solve each equation. Then Classify the equation as either an identity, a contradiction Or a conditional equation.

a. $2x + 7 = 7\boxed{(x +)} - 5x$

$7(x+1)-5x$

$2x+7 = \underline{7x}+7-\underline{5x}$

$2x+7 = 2x+7$ $\Rightarrow$ Identity

x can be any real number.

b. $3x - 5 = 3(x - 2) + 4$

$3x-5 = 3x-6+4$

$\underline{3x}-5 = \underline{3x}-2$

$-3x \qquad -3x$

$\Rightarrow -5 = -2 \Rightarrow$ Solution set is empty $\Rightarrow$ Contradiction

c. $3 - 8x = 5 - 7x$

$3-8x = 5-7x$

$+7x \qquad +7x$

$3+(-8+7)x = 5$

$(-1)x$

↑

$3-x = 5 \Rightarrow -x = 5-3 \Rightarrow -x = 2$

$-3 \qquad -3$

$\Rightarrow \frac{-x}{-1} = \frac{2}{-1} \Rightarrow \boxed{x = -2}$

$\Rightarrow$ Conditional