Name:

Problem 1: Find the equation of tangent line to the hyperbola $y = \dfrac{5}{x}$ at the point $(1, 5)$.

Problem 2: Let $f(x) = \begin{cases} -x & \text{if } x \leq 0 \\ x^2 & \text{if } 0 < x < 1 \\ 2x - 1 & \text{if } x \geq 1 \end{cases}$.

Is f continuous everywhere on $\mathbb{R}$? If not, then find the points where f is discontinuous.

Is f differentiable everywhere on $\mathbb{R}$? If not, then find the points where f is not differentiable.

Problem 3: Let $f(x) = \begin{cases} \sin x + 2\cos x & \text{if } x \leq \dfrac{\pi}{4} \\ \cos x + 2\sin x & \text{if } x > \dfrac{\pi}{4} \end{cases}$.

Is f continuous everywhere on $\mathbb{R}$? If not, then find the points where f is discontinuous.

Is f differentiable everywhere on $\mathbb{R}$? If not, then find the points where f is not differentiable.

Note that $\sin(\frac{\pi}{4}) = \cos(\frac{\pi}{4}) = \frac{1}{\sqrt{2}}$.

Problem 4: A particle starts to move along x-axis at time $t = 0$ with its position varying with time as $x(t) = t^3 - 27t + 7$.

1. Find the velocity of the particle as a function of time.
2. At what time instant was the particle at rest?
3. Find the time interval for which the particle was moving backwards.
4. Find the acceleration of the particle as a function of time.
5. When was the particle speeding up? When was it slowing down?

Problem 5: Let $y = \sin x + \cos x$. Find $\dfrac{d^2y}{dx^2}$ and $\dfrac{d^4y}{dx^4}$.

Problem 6: If f is a differentiable function, find an expression for the derivative of

$$y = \frac{1 + xf(x)}{\sqrt{x}}$$

in terms of $f'(x)$.

Problem 7: Suppose $x \sin y + y \sin x = 1$. Find $\dfrac{dy}{dx}$ by implicit differentiation.

Problem 8: Let $\sin y + \cos x = 1$. Find $\dfrac{d^2y}{dx^2}$ by implicit differentiation.

Problem 9: Find the equation of tangent line to the "devil's curve" $y^2(y^2 - 4) = x^2(x^2 - 5)$ at the point $(0, -2)$.

Problem 10: A ladder 15 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 2 ft/s, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 12 ft from the wall?

Problem 11: Find the linearization $L(x)$ of the function $f(x) = \dfrac{2}{\sqrt{x^2 - 5}}$ at the point $x = 3$.

Note that $L(x) = f(a) + f'(a)(x - a)$ at $x = a$.

Problem 12: The radius of a sphere was measured and found to be 10 cm with a possible error of 10^{-3} cm. What is the maximum error in using this value of the radius to compute the volume of the sphere? Note that volume V of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$.