Math 110

5.3 – Common Factors and Factoring by Grouping

Terms with common factors

When factoring a polynomial we always look for common factor often called the greatest common factor.

Example 1: Factor out the common Factor

a) $6y^2 - 18$

$= 6y^2 - 6 \times 3 = 6(y^2 - 3)$

b) $5x^2 - 30$

$= 5(x^2 - 6)$

c) $30x^4 + 20x^5$

$= 10x^4(3 + 2x)$

d) $-4x - 24$

$= -4(x + 6)$

$-2x^3 = -2x \times x^2$

or

$\frac{-2x^3}{-2x} = x^{3-1} = x^2$

$\frac{6x^2}{-2x} = -3x$

$\frac{-2x}{-2x} = 1$

e) $-2x^3 + 6x^2 - 2x$

$= -2x(x^2 - 3x + 1)$

f) $-3x^4 - 6x^2 + 3a$

$= -3(x^4 + 2x^2 - a)$

Example 2: Suppose that a baseball is thrown upward with an initial velocity of 64 ft/sec. Its height in feet, h(t), after t seconds id given by

$h(t) = -16t^2 + 64t$ $\quad h(t) = -16t^2 + 64t$

Find an equivalent expression for h(t) by factoring out a common factor. $= -16t(t-4)$

Factor by grouping

a) $x(x+3) + 5(x+3)$

$(x+3)(x+5) = (x+5)(x+3)$

b) $y^3 + 3y^2 + 4y + 12$

$= y^2(y+3) + 4(y+3)$

$= (y^2+4)(y+3)$

Multiplication

Factoring

c) $4x^3 - 15 + 20x^2 - 3x$

$= 4x^3 + 20x^2 - 3x - 15$

$= 4x^2(x+5) - 3(x+5) = (4x^2-3)(x+5)$

d) $x^3 + 5x^2 + 6x + 30$

$= x^2(x+5) + 6(x+5) = (x^2+6)(x+5)$

Many polynomials with four terms, like $x^3 + x^2 + 3x - 3$, are prime. Not only is there no common monomial factor, but no matter how we group terms, there is no common binomial factor.

$x^3 + x^2 + 3x - 3$

$= x^2(x+1) + 3(x-1)$

Not same

$x^3 + 3x + x^2 - 3$

$= x(x^2+3) + 1(x^2-3)$

Not same

Not Factorizable

Practice

1. $-5x - 40$	2. $-5x - 35$
3. $-16x^2 + 96$	4. $-16x^2 + 128$
5. $x^6 + 3x^4 - x^3$	6. $-2x^2 + 4x - 12$
7. $a(b-5) + c(b-5)$	8. $r(t-3) - s(t-3)$
9. $(a+5)(a-2) + (a+5)(a+1)$	10. $a^2(x-y) + 5(y-x)$
11. $5x^2(x-6) + 2(6-x)$	12. $x^3 - x^2 + 3x - 3$
13. $x^3 + 6x^2 - 2x - 12$	14. $x^3 - 3x^2 + 6 - 2x$

5.4-Factoring Trinomials

$c>0$ ⇙ ⇘ $c<0$

$b>0$ $b<0$

Factoring Trinomials of the Type $x^2 + bx + c$

Example 1: Write an equivalent expression by factoring

a) $x^2 + 9x + 8$

Factor 8 : $1\times 8 \rightarrow 1+8=9$
$2\times 4 \rightarrow 2+4=6$

$= x^2 + (1+8)x + 8$

$= x^2 + x + 8x + 8 = x(x+1) + 8(x+1) = (x+8)(x+1)$

b) $x^2 + 5x + 6$

Factor 6 : $1\times 6 \rightarrow 1+6=7$
$2\times 3 \rightarrow 2+3=5$

$= x^2 + (2+3)x + 6$

$= x^2 + 2x + 3x + 6 = x(x+2) + 3(x+2) = (x+3)(x+2)$

c) $x^2 - 9x + 20$

Factor 20 : $-1\times -20 \rightarrow -1-20=-21$
$-2\times -10 \rightarrow -2-10=-12$
$-4\times -5 \rightarrow -4-5=-9$

$= x^2 + (-4-5)x + 20$

$= x^2 - 4x - 5x + 20 = x(x-4) - 5(x-4) = (x-5)(x-4)$

d) $x^2 - 7x + 12$ HW

11

$(x-3)(x-4)$

$12 = -1\times -12$
-2×-6
$-3\times -4 \rightarrow -3-4=-7$

e) $x^3 - x^2 - 30x$ $c<0$

$= x(x^2 - x - 30)$

$= x[x^2 + (5-6)x - 30]$

$30 = 1\times 30$
2×15
3×10
5×6

$-30 = -1\times 30 = 1\times -30$
$= -2\times 15 = 2\times -15$
$= -3\times 10 = 3\times -10$
$= -5\times 6 = 5\times -6$

$5-6=-1$

$= x[x^2 + 5x - 6x - 30]$

$= x[x(x+5) - 6(x+5)] = x(x-6)(x+5)$

$= x(x+5)(x-6)$

f) $2x^2 + 34x - 220$

$= 2(x^2 + 17x - 110)$

$= 2[x^2 + (-5+22)x - 110]$

$= 2[x^2 - 5x + 22x - 110]$

$-110 = -1\times 110 = 1\times -110$
$= -2\times 55 = 2\times -55$
$= -5\times 22 = 5\times -22$

$= 2[x(x-5)+22(x-5)]$ $= -10x11 = 10x-11$

$= 2(x-5)(x+22)$

g) $5x^2 + 35x - 150$

$5(x^2+7x-30)$ Fill in steps $-30 = -1x30 = 1x-30$

$= -2x15 = 2x-15$

$= -3x10 = 3x-10$

$= -5x6 = 5x-6$

$= 5(x-3)(x+10)$

h) $x^2 - x + 7$

NOT FACTORIZABLE $7 = -1x-7$, $-1-7 = -8 \neq -1$

$\Rightarrow x^2-x+7$ is Prime or irreducible.

i) $x^2 + 2x + 5$

$5 = 1x5$, $1+5 = 6 \neq 2$

↓

Prime

Factoring trinomials of the type $ax^2 + bx + c$

ac

a) $3x^2 - 2x - 5$

$3x-5 = -15 = -1x15 = 1x-15$

$-3x5 = 3x-5$

$3-5 = -2$

$3x^2 + (3-5)x - 5$

$= 3x^2 + 3x - 5x - 5$

$= 3x(x+1) - 5(x+1) = (3x-5)(x+1)$

b) $2x^2 + 3x - 9$

$2x-9 = -18 = -1x18 = 18x-1$

$= -2x9 = 2x-9$

$= -3x6 = 3x-6$

$2x^2 + (-3+6)x - 9$

$= 2x^2 - 3x + 6x - 9$

$= x(2x-3) + 3(2x-3) = (x+3)(2x-3)$

c) $3x^2 - 8x + 4$

$3x4 = 12 = -1x-12$

$= -2x-6$

$= -3x-4$

$= 3x^2 + (-2-6)x + 4$

$= 3x^2 - 2x - 6x + 4$

$= x(3x-2) - 2(3x-2)$

$= (x-2)(3x-2)$

d) $5x^2 + 19x + 12$

HW.

$6 \times 10 = 60 = -1x - 60$
$= -2x - 30$
$= -3x - 20$
$= -4x - 15$
$= -5x + 12$
$= -6x - 10$

e) $6x^6 - 19x^5 + 10x^4$

$= x^4(6x^2 - 19x + 10)$

$= x^4[6x^2 - 4x - 15x + 10]$

$= x^4[2x(3x-2) - 5(3x-2)] = x^4(2x-5)(3x-2)$

$\frac{6x^6}{x^4} = 6x^{6-4} = 6x^2$

f) $3x^2 - 4x - 4$

HW.

$6x - 15 = -90 = -1 \times 90 = 1x - 90$
$= -2 \times 45 = 2x - 45$
$= -3 \times 30 = 3x - 30$
$= -5 \times 18 = 5x - 18$
$= -6 \times 15 = 6x - 15$
$= -9 \times 10 = 9x - 10$

g) $6x^2 + x - 15$

$6x^2 - 9x + 10x - 15$

$= 3x(2x-3) + 5(2x-3)$

$= (3x+5)(2x-3)$

5.5

Factoring a Perfect Square Binomial

$a^2 - b^2 = (a-b)(a+b)$

a) $16x^2 - 9$

$(4x)^2 - 3^2 = (4x-3)(4x+3)$

b) $4x^2 - 25$

$(2x)^2 - 5^2 = (2x-5)(2x+5)$

c) $9x^2 - 1$

$(3x)^2 - 1^2 = (3x-1)(3x+1)$

d) $n^2 - 25$

$n^2 - 5^2 = (n-5)(n+5)$

Factoring a Perfect Square trinomial

$a^2+b^2+2ab = (a+b)^2$ or $a^2+b^2-2ab = (a-b)^2$

$= a^2 - 2ab + b^2$

e) $16x^2 - 40x + 25$

$(4x)^2 + 5^2 - 40x$

a b $2ab$

$(4x-5)^2$

$a = 4x,\ b = 5 \Rightarrow 2ab = 2 \times 4x \times 5 = 40x$

f) $4x^2 - 4x + 1$

$(2x)^2 + 1^2 - 4x$

a b

$= (2x-1)^2$

$2ab = 2 \times 2x \times 1 = 4x$

g) $3x^2 + 6x + 3$

$3(x^2 + 2x + 1)$

$= 3[x^2 + 1^2 + 2x]$

$= 3(x+1)^2$

$a = x,\ b = 1$

$2ab = 2 \times x \times 1 = 2x$

$x^2 + 4x + 1 = x^2 + 1^2 + 4x \neq (x+1)^2$

$2ab = 2 \times x \times 1 = 2x \neq 4x$

Quiz 9

$x=0 \Rightarrow y=1$

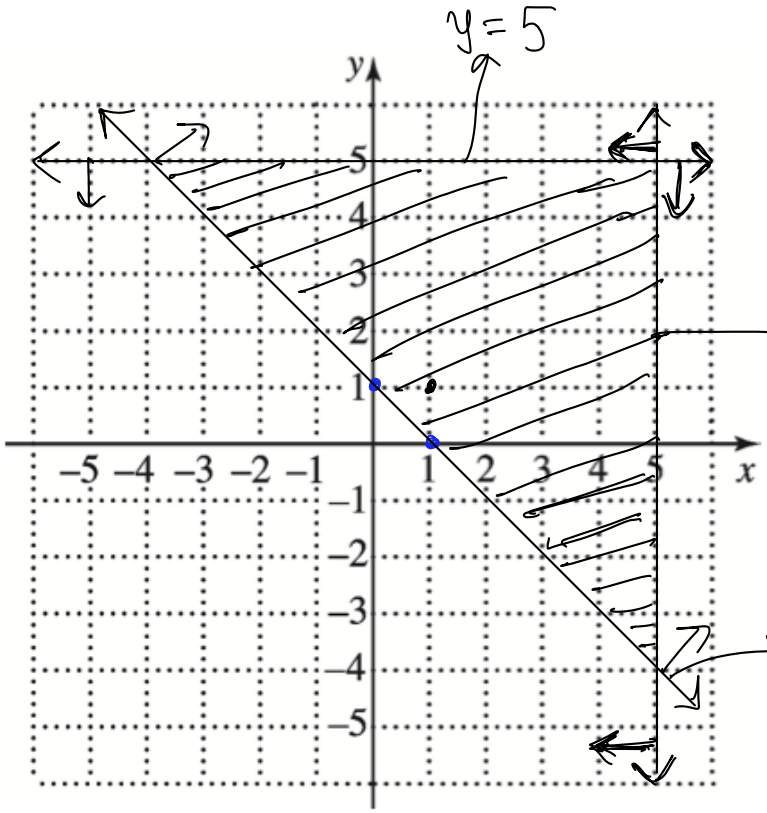
$y=0 \Rightarrow x=1$

$(0,1), (1,0)$

$x+y=1$

① Graph the inequalities $x+y \geq 1$, $x \leq 5$, $y \leq 5$.

$x=5$ $\quad$ $y=5$



$x+y \geq 1$

$1+1 \geq 1$

$2 \geq 1 \Rightarrow$ True

② (a) $(4x^3+x-1)+(x^4-2x^2-x+1)$

$= 4x^3+x-1+x^4-2x^2-x+1$

$= x^4+4x^3-2x^2$

(b) $(x^2+2x+1)-(2x-x^2)$

$= x^2+2x+1-2x+x^2$

$= 2x^2+1$