Learning objectives:

1. Express areas under curves as limit of a sum.
2. Apply this to calculating distance.

Example 1. Find the area under the curve $y = x^2$ for $0 \leq x \leq 1$.

Area as limit of a sum

The area A of the region S that lies under the graph of a continuous function f is the limit of the sum of the areas of approximating rectangles.

$$A = \lim_{n\to\infty} R_n = \lim_{n\to\infty} \left(f(x_1)\Delta x + f(x_2)\Delta x + \cdots + f(x_n)\Delta x\right) = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x \,.$$

Distance

Distance is the area under the graph of the velocity function.

Example 2. An object starts to move at $t = 0$ with a velocity that varies with time as $v(t) = t^3$. Find the distance covered up to time $t = 4$ seconds.