Name:

Differentiate the following functions:-

1. $f(x) = \underbrace{x^2}\underbrace{(2-x)}$ (Hint: Use Product Rue)

$$f'(x) = \frac{d}{dx}\left(x^2(2-x)\right) = \frac{d}{dx}(x^2)(2-x) + x^2\frac{d}{dx}(2-x)$$
$$= 2x(2-x) + x^2(-1)$$
$$= 4x - 2x^2 - x^2$$
$$= 4x - 3x^2$$

2. $f(x) = \dfrac{x^2+4x+3}{\sqrt{x}}$ (Hint: Use Quotient Rule)

$$f'(x) = \frac{\sqrt{x}\frac{d}{dx}(x^2+4x+3) - (x^2+4x+3)\frac{d}{dx}(\sqrt{x})}{(\sqrt{x})^2}$$

$$= \frac{\sqrt{x}(2x+4) - \frac{(x^2+4x+3)}{2\sqrt{x}}}{x} = \frac{2\sqrt{x}\times\sqrt{x}(2x+4) - (x^2+4x+3)}{2x\sqrt{x}}$$

$$= \frac{2x(2x+4) - (x^2+4x+3)}{2x\sqrt{x}} = \frac{4x^2+8x-x^2-4x-3}{2x\sqrt{x}} = \frac{3x^2+4x-3}{2x\sqrt{x}}$$

3. $f(x) = \dfrac{\sqrt{x}}{2+x}$ (Hint: Use Quotient Rule)

$$f'(x) = \frac{(2+x)\frac{d}{dx}(\sqrt{x}) - \sqrt{x}\frac{d}{dx}(2+x)}{(2+x)^2}$$

$$= \frac{(2+x)\frac{1}{2\sqrt{x}} - \sqrt{x}\times 1}{(2+x)^2} = \frac{2+x - \sqrt{x}\times 2\sqrt{x}}{2\sqrt{x}(2+x)^2}$$

$$= \frac{2+x-2x}{2\sqrt{x}(2+x)^2} = \frac{2-x}{2\sqrt{x}(2+x)^2}$$

4. $f(x) = \left(\dfrac{1+2x}{3-4x}\right)^{100}$ (Hint: Use Chain Rule and Quotient Rule)

Let $z = \dfrac{1+2x}{3-4x}$ $\Rightarrow$ $\dfrac{dz}{dx} = \dfrac{(3-4x)\frac{d}{dx}(1+2x) - (1+2x)\frac{d}{dx}(3-4x)}{(3-4x)^2}$

$= \dfrac{(3-4x)2 - (1+2x)(-4)}{(3-4x)^2}$

$= \dfrac{6-8x+4+8x}{(3-4x)^2} = \dfrac{10}{(3-4x)^2}$

$\Rightarrow f'(x) = \dfrac{d}{dx}(z^{100})$

$= \dfrac{d}{dz}(z^{100})\dfrac{dz}{dx}$

$= 100z^{99} \times \dfrac{10}{(3-4x)^2} = 100\left(\dfrac{1+2x}{3-4x}\right)^{99} \times \dfrac{10}{(3-4x)^2} = 1000\dfrac{(1+2x)^{99}}{(3-4x)^{101}}$

5. $f(x) = x\cos x + 2\cot x$ (Hint: Use Product Rule for $x\cos x$)

$f'(x) = \dfrac{d}{dx}(x\cos x) + \dfrac{d}{dx}(2\cot x)$

$= \left[x\dfrac{d}{dx}(\cos x) + \cos x\dfrac{d}{dx}(x)\right] + 2\dfrac{d}{dx}(\cot x)$

$= \left[x(-\sin x) + \cos x \times 1\right] + 2(-\csc^2 x)$

$= -x\sin x + \cos x - 2\csc^2 x$

6. $f(x) = \dfrac{x}{2-\tan x}$ (Hint: Use Quotient Rule)

$f'(x) = \dfrac{(2-\tan x)\frac{d}{dx}(x) - x\frac{d}{dx}(2-\tan x)}{(2-\tan x)^2}$

$= \dfrac{(2-\tan x)(1) - x(0-\sec^2 x)}{(2-\tan x)^2} = \dfrac{2-\tan x + x\sec^2 x}{(2-\tan x)^2}$

7. $f(x) = \left(\dfrac{\cos x}{1 - \sin x}\right)^{50}$ (Hint: Use Chain Rule and Quotient Rule)

Let $z = \dfrac{\cos x}{1 - \sin x} \Rightarrow \dfrac{dz}{dx} = \dfrac{(1-\sin x)\frac{d}{dx}(\cos x) - \cos x \frac{d}{dx}(1 - \sin x)}{(1-\sin x)^2}$

$$= \frac{(1-\sin x)(-\sin x) - \cos(-\cos x)}{(1-\sin x)^2}$$

$$= \frac{-\sin x + \sin^2 x + \cos^2 x}{(1-\sin x)^2} = \frac{1-\sin x}{(1-\sin x)^2}$$

$$= \frac{1}{(1-\sin x)}$$

$\Rightarrow f'(x) = \dfrac{d}{dx}(z^{50})$

$= \dfrac{d}{dz}(z^{50}) \dfrac{dz}{dx}$

$= 50 z^{49} \dfrac{dz}{dx} = 50 \left(\dfrac{\cos x}{1-\sin x}\right)^{49} \dfrac{1}{(1-\sin x)}$

$= \dfrac{50 \cos^{49} x}{(1-\sin x)^{50}}$

8. $f(x) = \underbrace{x^2 \sin x}\,\underbrace{\tan x}$ (Hint: Use Product Rule Twice)

$\Rightarrow f'(x) = x^2 \sin x \dfrac{d}{dx}(\tan x) + \dfrac{d}{dx}(x^2 \sin x) \tan x$

$= x^2 \sin x \sec^2 x + \underbrace{\dfrac{d}{dx}(x^2 \sin x)} \tan x$

$\dfrac{d}{dx}(x^2 \sin x) = x^2 \dfrac{d}{dx}(\sin x) + \dfrac{d}{dx}(x^2) \sin x$

$= x^2 \cos x + 2x \sin x$

$\Rightarrow f'(x) = x^2 \sin x \sec^2 x + (x^2 \cos x + 2x \sin x) \tan x$

$= x^2 \sin x \sec^2 x + x^2 \sin x + 2x \sin x \tan x$

9. $f(x) = \sqrt{\dfrac{\tan x - 1}{\sec x}}$ (Hint: Use Chain Rule and Quotient Rule)

Let $z = \dfrac{\tan x - 1}{\sec x} \Rightarrow \dfrac{dz}{dx} = \dfrac{\sec x \frac{d}{dx}(\tan x - 1) - (\tan x - 1)\frac{d}{dx}(\sec x)}{\sec^2 x}$

$$= \frac{\sec x(\sec^2 x) - (\tan x - 1)\sec x \tan x}{\sec^2 x}$$

$$= \frac{\sec^3 x - \sec x \tan^2 x + \sec x \tan x}{\sec^2 x}$$

Using $\tan^2 x = \sec^2 x - 1$

$$= \frac{\sec^3 x - \sec x(\sec^2 x - 1) + \sec x \tan x}{\sec^2 x}$$

$$= \frac{\cancel{\sec^3 x} - \cancel{\sec^3 x} + \sec x + \sec x \tan x}{\sec^2 x}$$

$$= \frac{\sec x(1 + \tan x)}{\sec^2 x} = \cos x(1 + \tan x)$$

$$= \cos x + \sin x$$

$\Rightarrow f'(x) = \dfrac{d}{dx}(\sqrt{z})$

$= \dfrac{d}{dz}(\sqrt{z}) \dfrac{dz}{dx}$

$= \dfrac{1}{2\sqrt{z}} \times (\cos x + \sin x)$

$= \dfrac{1}{2}\sqrt{\dfrac{\sec x}{\tan x - 1}} (\cos x + \sin x)$

10. $f(x) = \left(x + \sqrt{x}\right)^{100}$ (Hint: Use Chain Rule)

Let $z = x + \sqrt{x} \Rightarrow \frac{dz}{dx} = \frac{d}{dx}(x) + \frac{d}{dx}(\sqrt{x}) = 1 + \frac{1}{2\sqrt{x}}$

$\Rightarrow f'(x) = \frac{d}{dx}(z^{100}) = \frac{d}{dz}(z^{100})\frac{dz}{dx}$

$= 100z^{99}\left(1+\frac{1}{2\sqrt{x}}\right) = 100\left(x+\sqrt{x}\right)^{99}\left(1+\frac{1}{2\sqrt{x}}\right)$

11. $f(x) = \sin(x + \cos\sqrt{x})$ (Hint: Use Chain Rule Twice)

Let $z = x + \cos\sqrt{x} \Rightarrow \frac{dz}{dx} = \frac{d}{dx}(x) + \frac{d}{dx}(\cos\sqrt{x})$

$u = \sqrt{x}$

$= 1 + \frac{d}{dx}(\cos u) = 1 + \frac{d}{du}(\cos u)\frac{du}{dx}$

$= 1 - (\sin u)\left(\frac{1}{2\sqrt{x}}\right)$

$= 1 - \frac{\sin\sqrt{x}}{2\sqrt{x}}$

$\Rightarrow f'(x) = \frac{d}{dx}(\sin z)$

$= \frac{d}{dz}(\sin z)\frac{dz}{dx}$

$= \cos z\left(1 - \frac{\sin\sqrt{x}}{2\sqrt{x}}\right)$

$= \cos(x + \cos\sqrt{x})\left(1 - \frac{\sin\sqrt{x}}{2\sqrt{x}}\right)$

12. $f(x) = \sqrt{\dfrac{1+\sin x}{1+\cos x}}$ (Hint: Use Chain Rule and Quotient Rule)

Let $z = \frac{1+\sin x}{1+\cos x} \Rightarrow \frac{dz}{dx} = \frac{(1+\cos x)\frac{d}{dx}(1+\sin x) - (1+\sin x)\frac{d}{dx}(1+\cos x)}{(1+\cos x)^2}$

$= \frac{(1+\cos x)\cos x - (1+\sin x)(-\sin x)}{(1+\cos x)^2}$

$= \frac{\cos x + \cos^2 x + \sin x + \sin^2 x}{(1+\cos x)^2}$

$= \frac{\sin x + \cos x + 1}{(1+\cos x)^2}$

$\Rightarrow f'(z) = \frac{d}{dx}(\sqrt{z})$

$= \frac{d}{dz}(\sqrt{z})\frac{dz}{dx}$

$= \frac{1}{2\sqrt{z}} \times \frac{\sin x + \cos x + 1}{(1+\cos x)^2}$

$= \frac{1}{2}\sqrt{\frac{1+\cos x}{1+\sin x}}\;\frac{\sin x + \cos x + 1}{(1+\cos x)^2}$