Math16600 Section 23715 Quiz 6
Fall 2023, October 10

Name: Solutions [1 pt]

Problem 1: Evaluate the integral

$$\int \frac{4\,x}{x^3+x^2+x+1}\,dx$$

[5 pts]

$\underbrace{x^3+x^2}+\underbrace{x+1}$

$= x^2(x+1)+(x+1) = (x+1)(x^2+1)$

$\frac{4x}{(x+1)(x^2+1)} = \frac{a}{x+1} + \frac{bx+c}{x^2+1} \Rightarrow 4x = a(x^2+1)+(bx+c)(x+1)$

$= ax^2+a+bx^2+bx+cx+c$

$\Rightarrow 4x = \underbrace{(a+b)x^2+(b+c)x+a+c}$

Comparing coefficients we have :

$a+b=0,\ b+c=4,\ a+c=0 \Rightarrow b=-a,\ c=-a \Rightarrow -a+(-a)=4 \Rightarrow a=-2$
$\Rightarrow b=2=c$

Thus, $\int \frac{4x}{x^3+x^2+x+1}dx = \int \frac{-2}{x+1}dx + \int \frac{2x+2}{x^2+1}dx$

$\Rightarrow I = -2\ln|x+1| + \int \frac{2x}{x^2+1}dx + 2\int \frac{1}{x^2+1}dx = -2\ln|x+1|+\ln|x^2+1| + 2\tan^{-1}x + C$

Problem 2: Evaluate the integral:

$$\int e^{\sqrt{x}}\,dx$$

Hint: Use the substitution ~~u~~ $z = \sqrt{x}$. [5 pts]

$z=\sqrt{x} \Rightarrow dz = \frac{1}{2\sqrt{x}}dx \Rightarrow dx = 2\sqrt{x}\,dz = 2z\,dz$

$\Rightarrow I = \int e^{\sqrt{x}}dx = \int e^z(2z\,dz) = 2\int \underbrace{z}_{u}\,\underbrace{e^z\,dz}_{dv}$ (By Parts)

$u=z \Rightarrow du=dz,\ dv=e^z dz \Rightarrow v=e^z$

Thus, $\int z e^z dz = ze^z - \int e^z dz = ze^z - e^z + C$

$\Rightarrow I = 2(ze^z - e^z)+C = 2e^z(z-1)+C$

$= 2e^{\sqrt{x}}(\sqrt{x}-1)+C$