

Trapezoidal rule:

$$\int_a^b f(x) dx \approx \frac{b-a}{2n} (f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n))$$

where $x_i = a + \frac{i(b-a)}{n}$, $i = 0, 1, 2, \dots, n$.

Example 1. Use the trapezoidal rule with $n = 3$ to approximate the integral

$$\int_{-1}^2 \frac{2}{\sqrt{x^2 + 1}} dx. \text{ Given that } \sqrt{5} \approx 2.236 \text{ and } \sqrt{2} \approx 1.414.$$

Simpson's rule:

$$\int_a^b f(x) dx \approx \frac{b-a}{3n} (f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n))$$

where n is an even integer and $x_i = a + \frac{i(b-a)}{n}$, $i = 0, 1, 2, \dots, n$.

Example 2. Use the Simpson's rule with $n = 4$ to approximate the integral

$$\int_{-2}^2 \frac{dx}{x^2 + 1}.$$