Name:

Critical Points: Given a function $y = f(x)$, the critical points of f are those points in the domain of f, for which either $f'(x) = 0$ or $f'(x)$ does not exist.

Example: Find the critical points of the function $f(x) = x^{3/5}(4 - x)$.

Solution: $f'(x) = x^{3/5}\dfrac{d}{dx}(4 - x) + (4 - x)\dfrac{d}{dx}(x^{3/5}) = x^{3/5}(-1) + (4 - x)\frac{3}{5}x^{-2/5}$

$\Rightarrow f'(x) = -x^{3/5} + \dfrac{12 - 3x}{5x^{2/5}} = \dfrac{-5x + 12 - 3x}{5x^{2/5}} = \dfrac{12 - 8x}{5x^{2/5}}$.

Thus, $f'(x) = 0 \Rightarrow \dfrac{12 - 8x}{5x^{2/5}} = 0 \Rightarrow 12 - 8x = 0 \Rightarrow x = \dfrac{12}{8} = \dfrac{3}{2}$.

We also see that at $x = 0$, the denominator goes to 0 and $f'(x)$ does not exist.

Therefore, the critical points of the given function f are 0 and $\dfrac{3}{2}$.

Problem 1: Find the critical points of the function $f(x) = 2x^3 + 3x^2 - 12x + 7$.

Problem 2: Find the critical points of the function $f(x) = \dfrac{x^2}{x + 2}$.

Note that -2 is not in the domain of f and hence cannot be a critical point.

Problem 3: Find the critical points of the following two functions:-

1. $f(x) = x + \sqrt{x}$
2. $g(x) = x - \sqrt{x}$

Problem 4: For $f(x) = 2x^3 + 3x^2 - 12x + 7$ (as in problem 1), find the intervals where $f'(x) > 0$ and the intervals where $f'(x) < 0$.

Inflection Point: A point a in the domain of a function $y = f(x)$ is called an inflection point if f is continuous at a and $f''(a) = 0$ or does not exist.

Problem 5: Find inflection points for $f(x) = x^2 + \dfrac{1}{x}$.

Problem 6: Find inflection points for $f(x) = x^3$.