Problem 1: Simplify and have all the exponents positive.

1.
$$\left(\frac{2}{x}\right)^{-2}$$

2.
$$\frac{x^{-3}\,y^2}{z^{-1}\,y^6}$$

3.
$$\left(\frac{x^{-2}\,y^5}{4z^{-3}\,y^8}\right)^{-3}$$

$$\left\{\begin{array}{l} (a^m)^n = a^{mn} \\ a^m a^n = a^{m+n} \\ \dfrac{a^m}{a^n} = a^{m-n} \\ \left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m} \\ (ab)^m = a^m b^m \end{array}\right.$$

1. $$\left(\frac{2}{x}\right)^{-2} = \left(\frac{x}{2}\right)^2 = \frac{x^2}{2^2} = \frac{x^2}{4}$$

2. $$x^{-3} \times \frac{1}{z^{-1}} \times \frac{y^2}{y^6} = x^{-3} \times \frac{1}{z^{-1}} \times y^{2-6} = x^{-3} \times \frac{1}{z^{-1}} \times y^{-4}$$

$$= \left(\frac{1}{x}\right)^3 \times \left(\frac{1}{z}\right)^{-1} \times \left(\frac{1}{y}\right)^4 = \left(\frac{1}{x}\right)^3 \times z^1 \times \left(\frac{1}{y}\right)^4$$

$$= \frac{1}{x^3} \times z \times \frac{1}{y^4} = \frac{z}{x^3 y^4}$$

3. $$\left(\frac{x^{-2}\,y^5}{4z^{-3}\,y^8}\right)^{-3} = \left(\frac{4z^{-3}y^8}{x^{-2}y^5}\right)^3 = \left(\frac{4z^{-3}\,y^{8-5}}{x^{-2}}\right)^3$$

$$= \left(\frac{4z^{-3}y^3}{x^{-2}}\right)^3 = \left(\frac{4\,x^2y^3}{z^3}\right)^3 = \frac{(4x^2y^3)^3}{(z^3)^3}$$

$$= \frac{(4)^3(x^2)^3(y^3)^3}{(z^3)^3} = \frac{64\,x^6y^9}{z^9}$$

Problem 2: Three numbers are such that second is one more than twice the first and third is 5 more than the second. If their sum is 22 then find all the three numbers.

Let the first number be x

Second number $= 1+2x$

Third number $= 5+(1+2x) = 6+2x$

$$x + 1+2x + 6+2x = 22$$

$$\Rightarrow 5x+7 = 22$$

$$\Rightarrow 5x = 22-7$$

$$\Rightarrow 5x = 15$$

$$\Rightarrow x = \frac{15}{5} \quad \Rightarrow x=3$$

First number is 3

Second number is $1+2\times 3 = 7$

Third number is $5+7 = 12$

Problem 3: Let

$$f(x) = \begin{cases} 2x+1 & \text{if } x \geq 2 \\ x-2 & \text{if } x < 2 \end{cases}.$$

Then find

1. $f(1)$
2. $f(2)$
3. $f(3)$

① $f(1)$: $1 < 2 \Rightarrow 2^{nd}$ defn. applies.

$\Rightarrow f(1) = 1-2 \Rightarrow f(1) = -1$

② $f(2)$: $2 = 2 \Rightarrow 1^{st}$ defn applies

$\Rightarrow f(2) = 2(2)+1 = 5 \Rightarrow f(2) = 5$

③ $f(3)$: $3 > 2 \Rightarrow 1^{st}$ defn. applies

$\Rightarrow f(3) = 2(3)+1 = 7 \Rightarrow f(3) = 7$

Problem 4: Find the domain of the following functions:

1.
$$f(x) = \frac{2-x}{x-3}$$

2.
$$f(x) = \sqrt{5-x}$$

* Cannot divide by 0.

* Expressions inside even n^{th} roots cannot be negative

(1) $D_f = \{x \mid \text{Denominator of } f \neq 0\}$

$= \{x \mid x-3 \neq 0\} = \{x \mid x \neq 3\}$

= All real numbers except 3.

$= (-\infty, 3) \cup (3, \infty)$

(2) $5-x \geq 0 \Rightarrow -x \geq -5$

$\Rightarrow \frac{-x}{-1} \leq \frac{-5}{-1}$

$\Rightarrow x \leq 5 \Rightarrow D_f = (-\infty, 5]$

Problem 5: Find the slope, x-intercept and y-intercept of the straight line whose equation is $4x + 2y = 6$.

Slope-intercept form

$$y = \underset{\substack{\uparrow \\ \text{slope}}}{m}x + \underset{\substack{\uparrow \\ y\text{-intercept}}}{b}$$

$$4x + 2y = 6.$$

$$\Rightarrow 2y = -4x + 6$$

$$\Rightarrow \frac{2y}{2} = \frac{-4x+6}{2}$$

$$\Rightarrow y = \frac{-4x}{2} + \frac{6}{2}$$

$$\Rightarrow y = -2x + 3$$

$m = -2$, y-intercept $= (0, 3)$

For x-intercept :-

Put $y = 0$ and solve for x.

$$\Rightarrow 0 = -2x + 3 \Rightarrow 2x = 3 \Rightarrow x = \frac{3}{2}$$

$$\Rightarrow x\text{-intercept} = \left(\frac{3}{2}, 0\right)$$

Problem 6: Let L be a line whose equation is $6x - 3y = 9$. Find the equation of the following lines:

1. The line parallel to L and passing through $(1, 1)$.
2. The line perpendicular to L and passing through $(2, 3)$.
3. The line passing through $(1, 1)$ and $(2, 3)$.

Slope-point form :-

$$y - y_1 = \underset{\uparrow \text{ slope}}{m}(x - x_1)$$

① $y-1 = m(x-1)$. m = slope of L

Express $6x-3y=9$ in slope-intercept form (solve for y)

$-3y = -6x + 9$

$\frac{-3y}{-3} = \frac{-6x+9}{-3} \Rightarrow y = \frac{-6x}{-3} + \frac{9}{-3}$

$\Rightarrow y = 2x - 3 \Rightarrow m_L = 2$

$y-1 = 2(x-1) \Rightarrow y-1 = 2x-2 \Rightarrow y = 2x-2+1$

$\Rightarrow y = 2x - 1$

② $y-3 = m(x-2)$. Line is $\perp$ to L

$m_L = 2 \Rightarrow m \times 2 = -1 \Rightarrow m = -\frac{1}{2}$

$y-3 = \frac{-1}{2}(x-2) \Rightarrow 2(y-3) = -1(x-2)$

$\Rightarrow 2y - 6 = -x + 2 \Rightarrow 2y = -x + 2 + 6$

$\Rightarrow 2y = -x + 8 \Rightarrow x + 2y = 8$

③ $y-1 = m(x-1)$.

$(1,1) \longleftrightarrow (2,3)$

$m = \frac{3-1}{2-1} = \frac{2}{1} = 2$

$y-1 = 2(x-1)$

$\Rightarrow y-1 = 2x-2 \Rightarrow y = 2x-1$

Problem 7: Let $f(x) = \dfrac{2}{x-2}$ and $g(x) = \dfrac{x}{5-x}$. Find the domain of the functions:

1. $f+g$
2. $f-g$
3. $f\,g$

$\left.\begin{array}{l} 1. \\ 2. \\ 3. \end{array}\right\}$ same domains. $= D_f \cap D_g$

4. $\dfrac{f}{g} \longrightarrow D_f \cap D_g \cap \{x \mid g(x) \neq 0\}$

$D_f = \{x \mid \text{Denominator is not zero}\}$
$= \{x \mid x-2 \neq 0\} = \{x \mid x \neq 2\}$

$D_g = \{x \mid \text{Denominator is not zero}\}$
$= \{x \mid 5-x \neq 0\} = \{x \mid x \neq 5\}$

1, 2, 3 $\quad D_f \cap D_g = \{x \mid x \neq 2 \text{ and } x \neq 5\}$

$(-\infty, 2) \cup (2, 5) \cup (5, \infty)$
$= D(f+g) = D(f-g) = D(fg)$

4 $\quad \{x \mid g(x) \neq 0\} = \{x \mid \frac{x}{5-x} \neq 0\}$
$= \{x \mid x \neq 0\}$

$D(f/g) = D_f \cap D_g \cap \{x \mid x \neq 0\}$
$= \{x \mid x \neq 2 \text{ and } x \neq 5 \text{ and } x \neq 0\}$

$= (-\infty, 0) \cup (0, 2) \cup (2, 5) \cup (5, \infty)$

Problem 8: Find the point of intersection of the following pair of lines

OR

Solve the given linear system.

1. $3x + y = 5$ and $2x + 3y = 1$ Substitution
2. $4x + 3y = 7$ and $4x - 3y = 1$ Elimination

① Solve for y in eqn. (i)

$3x + y = 5 \Rightarrow y = -3x + 5$

Substitute this value of y in eqn. (2)

$2x + 3(-3x + 5) = 1$

Solve for x :-

$2x - 9x + 15 = 1 \Rightarrow -7x + 15 = 1$

$\Rightarrow -7x = 1 - 15 \Rightarrow -7x = -14 \Rightarrow x = \frac{-14}{-7} = 2$

$\Rightarrow x = 2$. Using value of x, find y.

$y = -3x + 5 = -3(2) + 5 = -6 + 5 = -1$

$(x, y) = (2, -1)$

② $4x + \cancel{3y} = 7$

$4x \cancel{- 3y} = 1$

Add

$4x + 4x = 7 + 1 \Rightarrow 8x = 8 \Rightarrow x = 1$

Put $x = 1$ in any of the two eqns. and solve for y.

$4(1) + 3y = 7 \Rightarrow 4 + 3y = 7 \Rightarrow 3y = 7 - 4$

$\Rightarrow 3y = 3 \Rightarrow y = \frac{3}{3} \Rightarrow y = 1$

$(x, y) = (1, 1)$

Problem 9: 1000 dollars are invested, part of it at 8% interest rate and part of it at 12% interest rate. After a year, the total yield is \$104. How much was invested at each rate.

Let \$$x$ be invested at 8 %

and \$$y$ be invested at 12 %

Interest = Principal × Rate × Time

1yr.

$x \times \frac{8}{100} \times 1$ $\qquad$ $y \times \frac{12}{100} \times 1$

$$\frac{8x}{100} + \frac{12y}{100} = 104$$

$$\Rightarrow 8x + 12y = 10400$$

$$x + y = 1000 \;]\times -8$$

$$\cancel{8x} + 12y = 10400$$

$$\cancel{-8x} - 8y = -8000$$

Add

$$12y - 8y = 10400 - 8000 \Rightarrow 4y = 2400$$

$$\Rightarrow y = \frac{2400}{4} \Rightarrow y = \$600$$

$$x + 600 = 1000 \Rightarrow x = 1000 - 600 = 400$$

$\Rightarrow$ \$400 were invested at 8 %

\$600 were invested at 12 %

Problem 10: Solve the following linear inequalities and write your answer in interval notation:

1. $-2x + 3 > 5$ and $5 + 2x < 9$
2. $3 - x < -1$ or $7x - 9 > 5$

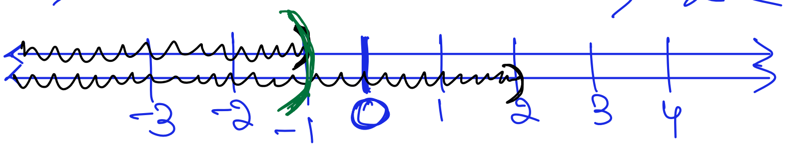
① $-2x+3>5$ and $5+2x<9$

$\Rightarrow -2x > 5-3 \qquad \Rightarrow 2x < 9-5$

$\Rightarrow -2x > 2 \qquad \Rightarrow 2x < 4$

$\Rightarrow \frac{-2x}{-2} < \frac{2}{-2} \qquad \Rightarrow \frac{2x}{2} < \frac{4}{2}$

$\Rightarrow x < -1$ and $\Rightarrow x < 2$



$(-\infty, -1)$

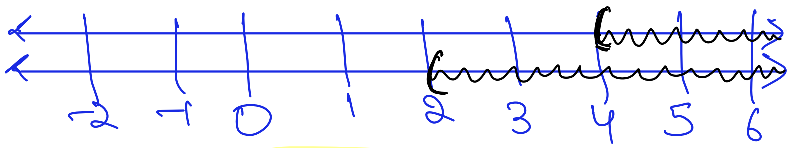
② $3-x<-1$ or $7x-9>5$

$\Rightarrow -x < -1-3 \qquad \Rightarrow 7x > 9+5$

$\Rightarrow -x < -4 \qquad \Rightarrow 7x > 14$

$\Rightarrow \frac{-x}{-1} > \frac{-4}{-1} \qquad \Rightarrow \frac{7x}{7} > \frac{14}{7}$

$\Rightarrow x > 4$ or $\Rightarrow x > 2$



$(2, \infty)$

Problem 11: Let $p(x) = 5 - x + 2x^2$ and $q(x) = -x^2 + 2x - 4$. Then find:

1. $2p(x) + q(x)$
2. $p(x) - 2q(x)$
3. $p(x) \,.\, q(x)$

① $2p(x) + q(x) = 2(5 - x + 2x^2) + (-x^2 + 2x - 4)$

$= 10 - 2x + 4x^2 - x^2 + 2x - 4$

$= 3x^2 + 6$

② $p(x) - 2q(x) = 5 - x + 2x^2 - 2(-x^2 + 2x - 4)$

$= 5 - x + 2x^2 + 2x^2 - 4x + 8$

$= 4x^2 - 5x + 13$

③ $p(x) \cdot q(x) = (5 - x + 2x^2) \cdot (-x^2 + 2x - 4)$

$= 5(-x^2 + 2x - 4) - x(-x^2 + 2x - 4) + 2x^2(-x^2 + 2x - 4)$

$= -5x^2 + 10x - 20 + x^3 - 2x^2 + 4x - 2x^4 + 4x^3 - 8x^2$

$= -2x^4 + 5x^3 - 15x^2 + 14x - 20$

Problem 12: Use algebraic identities to simplify the following expressions:

1. $(3x - y)^2$
2. $(x + 2y)(x - 2y)$
3. $\left(\dfrac{x}{3} + \dfrac{y}{2}\right)^2$

① $(3x-y)^2 \leftarrow$ Use $(a-b)^2 = a^2 - 2ab + b^2$

$= (3x)^2 - 2 \cdot 3x \cdot y + (y)^2$

$= 9x^2 - 6xy + y^2$

② $(x+2y)(x-2y) \leftarrow$ Use $(a+b)(a-b) = a^2 - b^2$

$= (x)^2 - (2y)^2 = x^2 - 4y^2$

Problem 13: Factor the following polynomials:

1. $x^3 - x^2 + 3x - 3$
2. $6x^2 - 5x + 1$
3. $4x^2 - \dfrac{y^2}{9}$
4. $9x^2 + 6x + 1$

① $\underbrace{x^3 - x^2} + \underbrace{3x - 3}$

$= x^2(x-1) + 3(x-1) = (x^2+3)(x-1)$

② $6x^2 - 5x + 1$ $\qquad 6 \times 1 = 6 = -1 \times -6$
$\qquad \underbrace{-2 \times -3}$

$= 6x^2 - 3x - 2x + 1$

$= 3x(2x-1) - 1(2x-1) = (2x-1)(3x-1)$

③ $4x^2 - \dfrac{y^2}{9}$

$= (2x)^2 - \left(\dfrac{y}{3}\right)^2 \longleftarrow$ Use $a^2 - b^2 = (a-b)(a+b)$

$= \left(2x - \dfrac{y}{3}\right)\left(2x + \dfrac{y}{3}\right)$

④ $9x^2 + 6x + 1$

$= (3x)^2 + 2 \cdot 3x \cdot 1 + (1)^2 \longleftarrow$ Use $a^2 + 2ab + b^2 = (a+b)^2$

$= (3x+1)^2$

Problem 14: Simplify. Do not use exponents that are fractions in your final answer.

1. $$\sqrt[3]{125(3y-4)^3}$$

2. $$\sqrt{49y^{10}}$$

3. $$\sqrt[3]{\sqrt[5]{x^3}}$$

4. $$\left(\sqrt[3]{x^3y^2z}\right)^9$$

① $\sqrt[3]{125(3y-4)^3} = \left[125(3y-4)^3\right]^{1/3}$

$= (125)^{1/3}\left((3y-4)^3\right)^{1/3} = 5\ (3y-4)^{3\times\frac{1}{3}}$

$= 5(3y-4)$

② $\sqrt{49y^{10}} = \sqrt{(7y^5)^2} = |7y^5| = 7|y|^5$

(if y is assumed to be +ve) $= (49y^{10})^{1/2} = (49)^{1/2}(y^{10})^{1/2} = 7\ y^{10\times\frac{1}{2}} = 7y^5$

③ $\sqrt[3]{\sqrt[5]{x^3}} = \sqrt[3]{(x^3)^{1/5}} = \sqrt[3]{x^{3/5}}$

$= (x^{3/5})^{1/3} = x^{\frac{3}{5}\times\frac{1}{3}} = x^{\frac{1}{5}} = \sqrt[5]{x}$

④ $\left(\sqrt[3]{x^3y^2z}\right)^9 = \left((x^3y^2z)^{1/3}\right)^9$

$= (x^3y^2z)^{\frac{1}{3}\times 9} = (x^3y^2z)^3$

$= (x^3)^3(y^2)^3(z)^3 = x^9y^6z^3$

Problem 15: Solve the following quadratic equations:

1. $x^2 + 9x - 10 = 0$
2. $(x+2)^2 = 2$
3. $x^2 + 4x + 2 = 0$

① $x^2 + 9x - 10 = 0 \Rightarrow x^2 + 10x - x - 10 = 0$

$\Rightarrow x(x+10) - 1(x+10) = 0$

$\Rightarrow (x-1)(x+10) = 0$

$\Rightarrow x - 1 = 0$ or $x + 10 = 0$

$\Rightarrow x = 1$ or $x = -10$

② $(x+2)^2 = 2 \Rightarrow x + 2 = \pm\sqrt{2}$

$\Rightarrow x = -2 \pm \sqrt{2}$

③ $x^2 + 4x + 2 = 0$ → does not factorize
⇓
use quadratic formula.

$a = 1,\ b = 4,\ c = 2$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(2)}}{2} = \frac{-4 \pm \sqrt{16 - 8}}{2}$$

$$\Rightarrow x = \frac{-4 \pm \sqrt{8}}{2} \Rightarrow x = \frac{-4 \pm 2\sqrt{2}}{2}$$

$$\Rightarrow x = \frac{2(-2 \pm \sqrt{2})}{2} \Rightarrow x = -2 \pm \sqrt{2}$$

Problem 16: Consider the quadratic function $f(x) = x^2 + x - 2$.

1. Find the vertex of the graph of f.
2. Find the range of f.
3. Find the x and y-intercepts of the graph of f.

① $x = \frac{-b}{2a}$ $\qquad a=1,\ b=1,\ c=-2$

$\Rightarrow x = \frac{-1}{2(1)} = \frac{-1}{2}$

$\Rightarrow y = f\left(\frac{-1}{2}\right) = \left(\frac{-1}{2}\right)^2 + \left(\frac{-1}{2}\right) - 2 = \frac{1}{4} - \frac{1}{2} - 2$

$\Rightarrow y = \frac{1}{4} - \frac{2}{4} - \frac{8}{4} = \frac{1-2-8}{4} = \frac{-9}{4}$

Vertex $= \left(\frac{-1}{2}, \frac{-9}{4}\right)$

② $a = 1 > 0 \Rightarrow$ Parabola opens upwards.
$\Rightarrow f$ has $\underbrace{\text{minimum value}}_{\frac{-9}{4}}$

Range of $f = \left[\frac{-9}{4}, \infty\right)$

③ $\underbrace{f(x)}_{y} = x^2 + x - 2$

y-intercept : Put $x=0 \Rightarrow y = 0^2 + 0 - 2 \Rightarrow y = -2$
$= (0, -2)$

x-intercept : Put $y=0$ and solve for x

$x^2 + x - 2 = 0 \Rightarrow x^2 - x + 2x - 2 = 0$

$-2 = -1 \times 2$ $\qquad \Rightarrow x(x-1) + 2(x-1) = 0$

1×-2

$\Rightarrow (x+2)(x-1) = 0$

$\Rightarrow x+2=0$ or $x-1=0 \Rightarrow x = -2$ or 1

$\Rightarrow$ x-intercepts $=$ $(-2, 0), (1, 0)$

Problem 17: Solve for x:

1. $\log_3 x = 2$.
2. $\log_x 125 = 3$.
3. $\log x = 3$
4. $\ln x = 2$
5. $\log_x 7 = \frac{1}{2}$

$\log_a b = c$

$\equiv \; b = a^c$

① $\log_3 x = 2 \Rightarrow x = 3^2 \Rightarrow x = 9$

② $\log_x 125 = 3 \Rightarrow 125 = x^3 \Rightarrow x^3 = 5^3 \Rightarrow x = 5$

③ $\log x = 3 \Rightarrow x = 10^3 \Rightarrow x = 1000$

④ $\ln x = 2 \Rightarrow x = e^2 \simeq 7.29$

⑤ $\log_x 7 = \frac{1}{2} \Rightarrow 7 = x^{1/2} \Rightarrow 7^2 = (x^{1/2})^2$

$\Rightarrow 49 = x^{1/2 \times 2} \Rightarrow x = 49$

Problem 18:

1. Express the following expression as a single logarithm:

$$\frac{1}{2}\log x + 5\log\sqrt[5]{y^2} + 2\log z$$

2. Express the following expression in terms of individual logarithms of x, y and z:

$$\log\left(\frac{x^3\sqrt{y}}{z^3}\right)$$

(1) $\frac{1}{2}\log x + 5\log\sqrt[5]{y^2} + 2\log z$

$= \log x^{1/2} + \log\left(\sqrt[5]{y^2}\right)^5 + \log z^2$

$= \log\sqrt{x} + \log\left(y^2\right)^{\frac{1}{5}\times 5} + \log z^2$

$= \log\sqrt{x} + \log y^2 + \log z^2$

$= \log\left(\sqrt{x}\,y^2 z^2\right)$

(2) $\log\left(\frac{x^3\sqrt{y}}{z^3}\right) = \log\left(x^3\sqrt{y}\right) - \log z^3$

$= \log x^3 + \log\sqrt{y} - \log z^3$

$= \log x^3 + \log y^{1/2} - \log z^3$

$= 3\log x + \frac{1}{2}\log y - 3\log z$

Problem 19: Let $f(x) = (x+1)^2$ and $g(x) = 2(x+2)$.

1. Find $(f \circ g)(x)$
2. Find $(g \circ f)(1)$
3. Find the inverse of g.

① $(f \circ g)(x) = f(g(x)) = f(2(x+2))$

$= (2(x+2)+1)^2 = (2x+4+1)^2$

$= (2x+5)^2 = (2x)^2 + 2 \cdot 2x \cdot 5 + (5)^2$

$= 4x^2 + 20x + 25$

② $(g \circ f)(1) = g(f(1))$

$f(1) = (1+1)^2 = 2^2 = 4$

$\Rightarrow (g \circ f)(1) = g(f(1)) = g(4) = 2(4+2) = 12$

③ $y = 2(x+2) \leftarrow$ Replace $g(x)$ with y

$x = 2(y+2) \leftarrow$ Interchange x and y.

Solve for y :—

$2(y+2) = x \Rightarrow y + 2 = \frac{x}{2} \Rightarrow y = \frac{x}{2} - 2$

Replace y with $f^{-1}(x)$:—

$f^{-1}(x) = \frac{x}{2} - 2.$

Problem 20: Find the validity of the following argument:

If orange is red then red is orange. Red is orange but orange is not red. Therefore, orange is red or red is orange.

q and $\sim p$

p: Orange is red

q: Red is orange

$$\begin{array}{c} p \to q \\ \sim p \wedge q \\ \hline p \vee q \end{array}$$

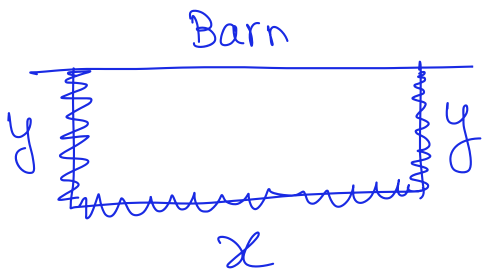
$$r = \big((p \to q) \wedge (\sim p \wedge q)\big) \to (p \vee q)$$

p	q	$\sim p$	$p \to q$	$\sim p \wedge q$	$(p \to q) \wedge (\sim p \wedge q)$	$p \vee q$	r
T	T	F	T	F	F	T	T
T	F	F	F	F	F	T	T
F	T	T	T	T	T	T	T
F	F	T	T	F	F	F	T

r is Tautology

$\Rightarrow$ The argument is valid.

MLM HW 8.8



$$86 = y + x + y = x + 2y$$

$$\Rightarrow x + 2y = 86$$

Maximize the area.

$x + 2y = 86$

$x = 86 - 2y$

$$A = xy \Rightarrow A = (86 - 2y)\,y$$

$$\Rightarrow A(y) = -2y^2 + 86y$$

$a = -2, \; b = 86$

$$y = \frac{-b}{2a} = \frac{-86}{2(-2)} = \frac{-86}{-4} = \frac{43}{2}$$

$$y = \frac{43}{2} \Rightarrow x + 2\left(\frac{43}{2}\right) = 86 \Rightarrow x + 43 = 86 \Rightarrow x = 86 - 43 = 43$$

Dimensions that maximize the area are:

$x = 43$ ft and $y = \frac{43}{2} = 21.5$ ft.

Composite Functions

$g(2) = 2^3$

$f(x) = 3x, \quad g(x) = x^3$

$f(1) = 3 \times 1 = 3$

$f(2) = 3 \times 2 = 6$

$f(a) = 3a$

$f(a^3) = 3a^3$

$$(f \circ g)(a) = f(g(a)) = f(a^3)$$

$$= 3a^3$$

$$(f \circ g)(x) = 3x^3$$

Logic

① Check if the argument is valid or not?

$p \vee q$ } Premises
$p \wedge q$
___
$p \to q$ } Conclusion

$$((p \vee q) \wedge (p \wedge q)) \to (p \to q)$$

p	q	$p \vee q$	$p \wedge q$	$(p \vee q) \wedge (p \wedge q)$	$p \to q$	$((p \vee q) \wedge (p \wedge q)) \to (p \to q)$
T	T	T	T	T	T	T
T	F	T	F	F	F	T
F	T	T	F	F	T	T
F	F	F	F	F	T	T

Valid ✓

② If he drives fast, he will crash.
He drive fast.
___
He will crash.

p: He drives fast q: He will crash.

$p \to q$
p
___
q

$$((p \to q) \wedge p) \to q$$

p	q	$p \to q$	$(p\to q)\wedge p$	$((p\to q)\wedge p)\to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

VALID

(3) State the contrapositive, converse and inverse of the following statement:

$\underbrace{\text{I will help you}}_{q}$ if $\underbrace{\text{you will help me.}}_{p}$

Given $p \to q$, its

Contrapositive is $\sim q \to \sim p$

Converse is $q \to p$

inverse is $\sim p \to \sim q$.

Contrapositive: You will not help me if I will not help you.

$|||$

If I will not help you then you will not help me

Converse: You will help me if I will help you

$|||$

If I will help you then you will help me.

Inverse: I will not help you if you will not help me.

$|||$

If you will not help me, then I will not help you.