

Math16500 Section 24246 Quiz 13+14
Fall 2022, November 14

Name:

[1 pt]

Problem 1: Evaluate the indefinite integral $\int (3 \sin^2 \theta \cos \theta + \cos \theta) d\theta$.

[5 pts]

Hint: Factor out $\cos \theta$ and use substitution $x = \sin \theta$.

$$I = \int (3 \sin^2 \theta + 1) \cos \theta d\theta.$$

$$\text{Put } x = \sin \theta \Rightarrow \frac{dx}{d\theta} = \cos \theta \Rightarrow dx = \cos \theta d\theta$$

$$\begin{aligned} \Rightarrow I &= \int (3x^2 + 1) dx = \int 3x^2 dx + \int 1 dx \\ &= 3 \frac{x^3}{3} + x + C = x^3 + x + C \end{aligned}$$

$$\Rightarrow \boxed{I = \sin^3 \theta + \sin \theta + C}$$

Problem 2: Evaluate the definite integral $\int_0^{\pi} |\cos \theta| d\theta$.

[5 pts]

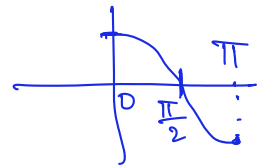
$$\int_0^{\pi} |\cos \theta| d\theta = \int_0^{\pi/2} |\cos \theta| d\theta + \int_{\pi/2}^{\pi} |\cos \theta| d\theta$$

$$= \int_0^{\pi/2} \cos \theta d\theta + \int_{\pi/2}^{\pi} -\cos \theta d\theta$$

$$= \sin \theta \Big|_0^{\pi/2} - \sin \theta \Big|_{\pi/2}^{\pi}$$

$$= \left[\sin \frac{\pi}{2} - \sin 0 \right] - \left[\sin \pi - \sin \frac{\pi}{2} \right]$$

$$= [1 - 0] - [0 - 1] = 1 - (-1) = \underline{\underline{2}}$$



Problem 3: Evaluate the indefinite integral $\int \frac{\sin \theta}{\cos^2 \theta} d\theta$.

[4 pts]

$$\int \frac{\sin \theta}{\cos^2 \theta} d\theta = \int \frac{1}{\cos \theta} \frac{\sin \theta}{\cos \theta} d\theta = \int \sec \theta \tan \theta d\theta = \sec \theta + C$$

Alternatively

$$\text{Substitute } u = \cos \theta \Rightarrow \frac{du}{d\theta} = -\sin \theta \Rightarrow du = -\sin \theta d\theta$$

$$\Rightarrow I = \int \frac{-du}{u^2} = -\frac{u^{-2+1}}{-2+1} + C = -\frac{u^{-1}}{-1} + C = \frac{1}{u} + C$$

$$\Rightarrow I = \frac{1}{\cos \theta} + C = \sec \theta + C$$

Problem 4: Evaluate the definite integral $\int_0^1 x(1-x) dx$.

[5 pts]

$$\begin{aligned} I &= \int_0^1 x(1-x) dx = \int_0^1 (x - x^2) dx = \int_0^1 x dx - \int_0^1 x^2 dx \\ &= \left. \frac{x^2}{2} \right|_0^1 - \left. \frac{x^3}{3} \right|_0^1 = \left[\frac{1^2}{2} - \frac{0^2}{2} \right] - \left[\frac{1^3}{3} - \frac{0^3}{3} \right] \\ &= \frac{1}{2} - \frac{1}{3} = \frac{1}{6} \end{aligned}$$

$$\Rightarrow \boxed{I = \frac{1}{6}}$$

Bonus Problem: Let $\int_0^1 f(x) dx = 4$ and $\int_2^1 f(x) dx = 3$. Find $\int_0^2 f(x) dx$.

[2 pts]

$$\begin{aligned} \int_0^2 f(x) dx &= \int_0^1 f(x) dx + \int_1^2 f(x) dx = \int_0^1 f(x) dx - \int_2^1 f(x) dx \\ &= 4 - 3 = \underline{\underline{1}} \end{aligned}$$