Sigma Notation

1. $\displaystyle\sum_{i=1}^{n} i = 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$.

2. $\displaystyle\sum_{i=1}^{n} i^2 = 1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$.

3. $\displaystyle\sum_{i=1}^{n} i^3 = 1^3 + 2^3 + 3^3 + \cdots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$.

The Area Problem: Find the area enclosed between the curve $y = f(x)$ and the x-axis from $x = a$ to $x = b$.

The area is $\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x_i$ which is denoted by $\int_a^b f(x)\,dx$ and is called the definite integral of f from a to b.

The x here is a dummy variable so we have

$$\int_a^b f(x)\,dx = \int_a^b f(y)\,dy = \int_a^b f(z)\,dz = \int_a^b f(w)\,dw.$$

Example 1. Evaluate $\int_0^3 x^2\,dx$ using the definition of definite integral.