Problem 1. Find the numbers (values of x) at which f is discontinuous. At which of these numbers is f continuous from the right, from the left, or neither?

1. * $f(x) = \begin{cases} \dfrac{x^2 - x}{x^2 - 1} & \text{if } x \neq 1 , \\ 1 & \text{if } x = 1 . \end{cases}$

2. $f(x) = \begin{cases} \cos x & \text{if } x < 0 , \\ 0 & \text{if } x = 0 , \\ 1 - x^2 & \text{if } x > 0 . \end{cases}$

3. $f(x) = \begin{cases} \dfrac{2x^2 - 5x - 3}{x - 3} & \text{if } x \neq 3 , \\ 6 & \text{if } x = 3 . \end{cases}$

4. $f(x) = \begin{cases} x + 2 & \text{if } x < 0 , \\ 2x^2 & \text{if } 0 \leq x \leq 1 , \\ 2 - x & \text{if } x > 1 . \end{cases}$

5. $f(x) = \begin{cases} x^2 + 1 & \text{if } x \leq 1 , \\ 3 - x & \text{if } 1 < x \leq 4 , \\ \sqrt{x} & \text{if } x > 4 . \end{cases}$

Problem 2. Find whether the following functions are continuous on the given interval(s).

1. $f(x) = x + \sqrt{x - 4}$, on $[4, \infty)$.

2. $f(x) = \begin{cases} \dfrac{x - 1}{3x + 6} & \text{if } x < -2 , \\ 1 & \text{if } x = -2 , \\ |x| - 1 & \text{if } x > -2 , \end{cases}$ on $(-\infty, -2]$ and $[-2, 4]$.

Problem 3. Use continuity to evaluate the following limits.

1. * $\lim\limits_{x \to 2} x \sqrt{20 - x^2}$.

2. $\lim\limits_{x \to \pi} \sin (x + \sin x)$.

3. $\lim\limits_{x \to 2} \dfrac{x^3}{\sqrt{x^2 + x - 2}}$.

Problem 4. Use the intermediate value theorem to show that there is a root of the equation

$$x^4 + x - 3 = 0$$

in the interval $(1, 2)$.

Problem 5. Find the derivative of the following functions at the given point.

1. * $f(x) = 1 - x^2$ at $x = 1$.
2. $f(x) = 1/x^2$ at $x = 2$.

Problem 6. A particle moves along the x-axis with its displacement varying with time as $x(t)$. Find the velocity of the particle for various $x(t)$ at the specified time instant.

1. * $x(t) = t^2 - 2t + 2$ at $t = 0$ seconds.
2. $x(t) = t^3$ at $t = 1$ seconds.

Problem 7. A particle moves along the x-axis with its velocity varying with time as $v(t)$. Find the acceleration of the particle for various $v(t)$ at the specified time instant.

1. $v(t) = 2t - 2$ at $t = 0$ seconds.
2. $v(t) = 3t^2$ at $t = 1$ seconds.