Station: 4 Scratch Paper: /

DIVISION OF DIVERSITY, EQUITY & INCLUSION
ACCESSIBLE EDUCATIONAL SERVICES
Indianapolis

AES Testing Record

Student: Ethan Aldrich Winnett -(Ethan) **Must Stop At:** 5:30 pm

Test Date: May 03, 2025 **Test Time:** 2:30 pm **Location:** AES Testing Lab (UL 3135H -Lib 3rd fl)

Student Status:

Course Title: ANALYTIC GEOMETRY & CALCULUS I **Code:** MATH-I 165 30129

Instructor: Keshav Dahiya

Test Type: Final Exam **Test Format:** Paper **Name/Number:** FINAL EXAM

Accommodations: Distraction-Reduced Environment; Extended Time on Quizzes and Exams (150%)

Instructor's Directions: Closed book/notes. No calculator. No scratch paper.

Time Allotted: 180min **Start Time:** 2:31pm **Ending Time:** 5:15PM

Breaks Taken:

Proctors: Ashley

Proctor Notes:

Delivery Preference: Scan/email test, then keep in AES Office UC100

Delivery Log (Please contact AES for delivery records)

Emailed By: Date: Time:

Delivered By: Date: Time: Location:

Received By: Date: Time:

Attempted By: Date: Time: Location:

Explanation:

Attempted By: Date: Time: Location:

Explanation:

Indiana University, Indianapolis

Spring 2025 Math-I 165

FINAL EXAM (May 03, 2025)

Instructor: Keshav Dahiya

Name: Ethan Wimmett 135

University ID: 2001263682

Instructions:

- No cell phones, calculators, watches, technology, hats - stow all in your bags.
- **Write your name** on this cover page.
- This test is **closed book and closed notes**.
- All work must be clearly shown for partial credit.
- If you wish for something not to be graded, please strike it out neatly.
- Box, circle, or otherwise clearly indicate your final answer.
- When you finish, return your test to the proctor, and leave the classroom.
- There are a total of **18 problems including 2 bonus problems**.
  - Problems 1-8 are each worth 10 points.
  - Problems 9-16 are each worth 15 points.
  - The bonus problems are each worth 8 points.
- You can score a **maximum of 216 points out of 200**.
- There are a total of **14 pages** including the cover page.

Problem 1. Differentiate the function $f(x) = \dfrac{x^2 - 1}{x^2 + 1}$. [10 pts]

$$\frac{(x^2-1)'(x^2+1) - (x^2-1)(x^2+1)'}{(x^2+1)^2}$$

$$\frac{(x-1)\cancel{(x+1)}}{\cancel{(x+1)}(x+1)}$$

$$\frac{x-1}{x+1}$$

$$2x(x^2+1) - (x^2-1)(2x)$$

$$2x^3+2x-(2x^3-2x)$$

$$2x^3+2x-2x^3+2x$$

$$4x^3+4x \quad 4x(x^2+1)$$

8

Problem 2. Differentiate the function $f(x) = x^2 \sin(x^2)$. [10 pts]

$$(x^2)'(\sin(x^2)) + (x^2)(\sin(x^2))'$$

$$2x(\sin(x^2)) + (x^2)(\cos(x^2)(2x))$$

$$2x\big((\sin(x^2)) + (x^2)\cos(x^2)\big)$$

10

Problem 3. Find the points of local maxima and minima for the function $f(x) = x^2(x-1)^2$. [10 pts]

$(x^2)'(x-1)^2 + (x^2)(x-1)^{2\prime}$

$2x(x-1)^2 + (x^2)2(x-1)(1)$

$2x(x^2-2x+1) + (x^2)(2x-2)$

$2x^3 - 4x^2 + 2x + 2x^3 - 2x^2$

$4x^3 - 6x^2 + 2x \quad 2x(2x^2-3x+1)$

$2x(2x-1)(x-1), \quad x = 1, \frac{1}{2}, 0$

$F(-1) \; 2(-1)(2(-1)^2-3(-1)+1) = -12$

$F(2) \; 2(2)(2(2)^2-3(2)+1) = -12$

$0 \quad \frac{1}{2} \quad 1$

9

F(0) is a local min at -12
F(2) is a local max at 12

Problem 4. A particle moves in a straight line with the position function $s(t) = \sin \pi t + \cos \pi t$. Find the velocity and acceleration of the particle at $t = 0$ seconds and $t = 1$ seconds. [10 pts]

$\int_0^1$

$s'(t) + -\cos \pi x + \sin \pi x$

$-\cos(\pi(1)) + \sin \pi(1) - (-\cos(\pi(0)) + \sin \pi(0))$

$-\cos(\pi) + \sin(\pi) + \cos(0) - \sin(0)$

$-1(-1) + 0 + 1 - 0 = 1 + 1 = 2$

velocity = 2 seconds

4

Problem 5. The relative error in the radius of a sphere is 0.2%. Find the relative error in the volume of the sphere. [10 pts]

$\frac{4}{3}\pi r^3 h \quad V(r)\ \frac{4}{3}\pi(.2)^3 \quad 3.\frac{4}{3}\pi(.2)^2$

$4\pi(.04) \to .16\pi$

3

Problem 6. An object moves along the x-axis with a velocity of $v(t) = t - 2$. Find the distance travelled in the first four seconds, that is, from $t = 0$ to $t = 4$. [10 pts]

$\int_0^4 \quad x(t)\ \frac{t^2}{2} - 2t \quad \frac{4-0}{h} \to \frac{4}{h}$

4

5

Problem 7. Find the work done when a force of magnitude $F(x) = \cos(\pi x/2)$ newtons is applied to move an object from $x = 1$ to $x = 2$ meters. [10 pts]

$\cos(\frac{\pi}{2}x)$ $\frac{2-1}{n} \to \frac{1}{n}$

$\int_1^2$ $\left(1 + \frac{1}{n}\right)$

1

Problem 8. Find a number c such that the average value of the function $f(x) = \sqrt{x}$ on the interval $[0, 4]$ equals $f(c)$. [10 pts]

$\int_0^4 \sqrt{x}$ $\int_0^4 x^{\frac{1}{2}}$

$\int_0^4 \frac{2}{3}x^{\frac{3}{2}}+C$ $\int_0^4 \frac{2}{3}(4)^{\frac{3}{2}}+C - \left(\frac{2}{3}(0)^{\frac{3}{2}}+C\right)$ $\frac{2}{3}(4)^{\frac{3}{2}} = \frac{16}{3}$

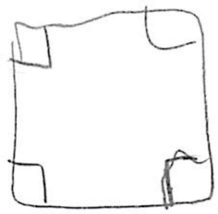
$\int_0^4 \frac{16}{3} + C = C = -\frac{16}{3}$

8

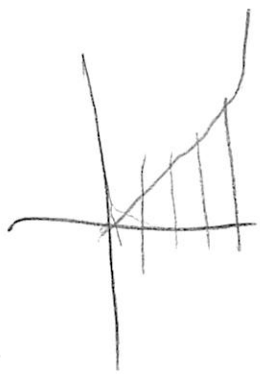
Problem 9. A rectangular (cuboid) tin container with a square base and open top has a volume of 32 m^3. Find the dimensions of the container that minimize the amount of tin used. [15 pts]

Volume = 32 m^3

$\frac{32}{3}$



L·W·H



3

Problem 10. Let $x^4 + y^4 = 16$.

1. Find the derivative $\dfrac{dy}{dx}$. [7 pts]

$4x^3+4y^3=0$

5

2. Find the second derivative $\dfrac{d^2y}{dx^2}$. [8 pts]

$12x^2+12y^2$

2

Problem 11. Evaluate the following limits:

1. $\lim_{x\to 0} \frac{\sin 4x}{\tan 3x}$. [5 pts]

$$\frac{(\sin(4x))'(\tan(3x)) - (\sin(4x))(\tan 3x)'}{(\tan(3x))^2}$$

$$\frac{\cos(4x)(\tan(3x)) - (\sin(4x))(\sec^2(3x))}{(\tan(3x))^2}$$

1

2. $\lim_{x\to 0} \frac{x^2 - x}{x^2 + x}$. [5 pts]

$$\frac{x(x-1)}{x(x+1)} \quad \frac{1.1(1.1-1)}{1.1(1.1+1)} \quad \frac{.1}{2.1} \quad \infty$$

2

3. $\lim_{x\to\infty} \frac{2x^2 + 4x + 5}{3x^2 - x - 1}$. [5 pts]

$$\frac{x^2}{x^2}\left(\frac{2x^2+4x+5}{3x^2-x-1}\right)$$

$$\frac{2+0+0}{3+0+0} \to \frac{2}{3}$$

5

Problem 12. Let $f(x) = x^3 - 12x + 6$.

1. Find the local maximum and minimum values of f. [5 pts]

$3x^2-12 \quad 3(x^2-4) \quad x=\pm 2$

$+ \quad - \quad +$

$-2 \quad 2$

$f(-2)=(-2)^3-12(-2)+6$

$-8+24+6 \Rightarrow 22$

$f(0)=3(0)^2-12=-12$

$f(-3)=3(-3)^2-12=15$

$f(3)=15$

local min is f(0) at -12

local max is f(3) at 15

4

2. Find the inflection points of f. [5 pts]

$6x \quad 6x=0 \quad x=0$

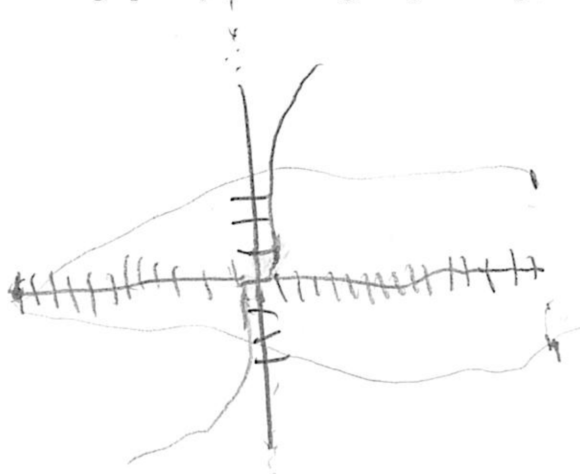
$- \quad +$

0

0 is an inflection point

5

3. Sketch the graph of f (indicating the y-intercept, local max/min and inflection points). [5 pts]



2

Problem 13. Consider the curve $x^4 + y^4 + 2xy = 4$.

1. Find dy/dx. [7 pts]

$4x^3 + 4y^3 y' + (2x)'(y) + (2x)(y)'$

$4x^3 + 2y + 4y^3\frac{dy}{dx} + 2x\frac{dy}{dx}$

$-4y^3\frac{dy}{dx} - 2x\frac{dy}{dx} = 4x^3 + 2y$

15

$\frac{dy}{dx}(-4y^3 - 2x) = 4x^3 + 2y$

$\frac{dy}{dx} = \frac{4x^3+2y}{-4y^3-2x}$

$\frac{dy}{dx} = 2\left(\frac{2x^3+y}{-2y^3-x}\right)$

2. Find the equation of tangent to the curve at the point (1, 1). [4 pts]

$2\left(\frac{2(1)^3+1}{-2(1)^3-(1)}\right)$

$2\left(\frac{3}{-3}\right) = \frac{-6}{6} = -1$ $\frac{y-1}{x-1} = \frac{-1}{1}$ $y-1 = -x+1$

$y = -x+2$

3. Find the equation of normal to the curve at the point (1, 1). [4 pts]

$(1)^4 + (1)^4 + 2(1)(1) = 4$

-1 reciprical $+1$

$\frac{y-1}{x-1} = 1$

$y-1 = x-1$

Do I put it equal to 4 for 2 and 3?

$y = x + 0$

$\frac{y-1}{x-1} = 4$

$y-1 = 4x-4$ $y = 4x-3$

Problem 14. Evaluate the following integrals:

1. $\int \sqrt{\sin x} \cos x \, dx.$ [9 pts]

$\cos x \to \sin x$

$\sin x^{\frac{1}{2}}$ $\int \sqrt{u} \sin x$ $\int u^{\frac{1}{2}} \sin x$ $\int \frac{2}{3} u^{\frac{3}{2}} dx \sin x$

$\int \sin x \sqrt{\sin x}$

8

2. $\int \frac{x^2 - 1}{x^4} dx.$ [6 pts]

$\frac{x^2}{x^4} - \frac{1}{x^4}$ $\frac{1}{x^2} - \frac{1}{x^4}$

$(x^{-2} - x^{-4}) dx$ $-x^{-1} + \frac{x^{-3}}{3}$

6

Problem 15. Find the area of the following regions:

1. The region enclosed by $x = y - y^2$ and the y-axis. [7 pts]

$-y^2 + y = x$

$-y(y-1) = x \quad y = 0, 1$

2

2. The region enclosed by $y = |\sin x|$ and the x-axis from $x = 0$ to $x = \pi$. [8 pts]

$\int_{\frac{\pi}{2}}^{\pi} |\sin(x)| \quad \int_{0}^{\frac{\pi}{2}} |\sin(x)|$

$|\sin(\pi)| - |\sin(\frac{\pi}{2})| + \sin(\frac{\pi}{2}) - \sin(0)$

$0 - 1 = -1 \quad 1 - 0 = 1$

$-1 + 1 = 0$ IDK

7

Problem 16. Consider the region bounded between the curves $y = \sin x$, the x-axis and the vertical line $x = \pi/2$. Set up (no need to evaluate) an integral for the volume of the solid obtained by rotating the given region about the following axes:

1. x-axis (Hint: use disk method). [5 pts]

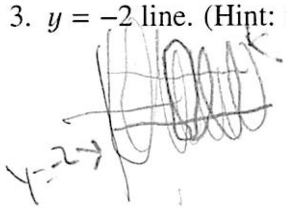
$\sin(\frac{\pi}{2})$

2. y-axis. (Hint: use shell method). [5 pts]

$\int \pi (r)^2 - (x)^2$

4

3. $y = -2$ line. (Hint: use washer method). [5 pts]



$-2 = \sin(\frac{\pi}{2})$

$-2 = -1 \quad -3$

$+2 \; +2$

Bonus Problem 1. Let $f(x) = \begin{cases} \dfrac{\sqrt{1+x^2}-1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$. Prove that f is continuous at $x = 0$. [8 pts]

$$\frac{\sqrt{1+x^2}-1}{x} \cdot \frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}+1} \qquad \frac{1+x^2-1}{x\sqrt{x^2+2}}$$

$$\frac{\cancel{x}^2}{\cancel{x}\sqrt{x^2+2}} \qquad \frac{x}{\sqrt{x^2+2}} \qquad \frac{0}{\sqrt{(0)^2+2}} = \frac{0}{\sqrt{2}} = 0$$

8

Bonus Problem 2. Use the closed interval method to find the absolute maximum value of the function $f(x) = x^3 - x^2$ on the interval $[-2, 2]$. [8 pts]

$x^2(x-1)$ $\quad x = 0, 1$

$f(-2) = (-2)^2(-2-1) = -12$

$-2 \quad 0 \quad 1 \quad 2$

$f(0) = (0)^2(0-1) = 0$

$F(-1) = (-1)^2(-1-1) = -2$

$F(1) = (1)^2(1-1) = 0$

$F(2) = (2)^2(2-1) = 4$

absolute maximum is $F(2)$ at 4

8