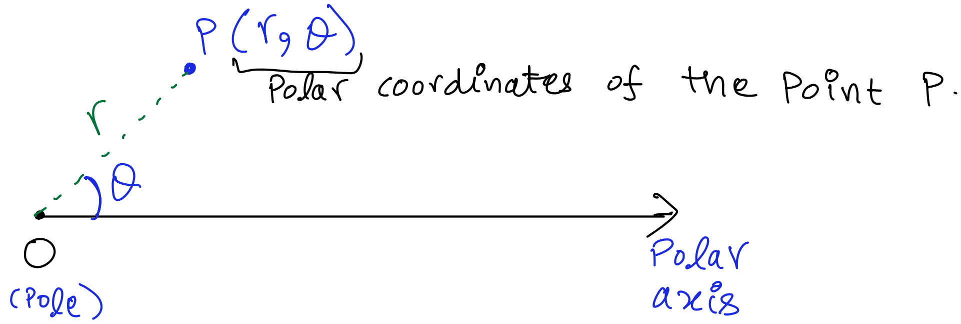
■ **Section 10.3** textbook exercises, *page 706*: #1, 3, 5, 21, 25, 29, 31

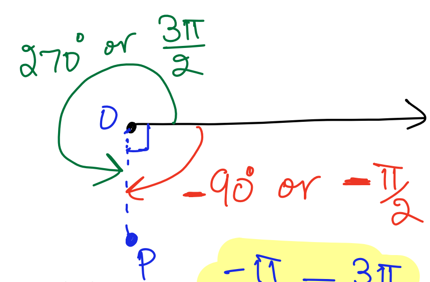
Polar Coordinates:

r = length of OP ($r > 0$)

θ = angle b/w OP and the Polar axis



Conventions:



- $\theta > 0$ if θ is measured in <u>counterclockwise</u> direction.
- $\theta < 0$ if θ is measured in <u>clockwise</u> direction.
- The polar coordinates for the pole is $(0, \theta)$ for any values of θ.

$-\frac{\pi}{2} = \frac{3\pi}{2}$

- For $r > 0$, to plot a point $(-r, \theta)$, i.e., a point with a negative radius, we plot the corresponding point of positive radius (r, θ) then reflect it about the pole.

$r = 0 \Rightarrow$ length of $OP = 0 \Rightarrow$ P is at the Pole (for any value of the angle θ)

$\underbrace{(0, \frac{\pi}{3}), (0, \pi), (0, \frac{\pi}{2})}_{\text{the same Point that is the Pole}}$

$(-r, \theta) = (r, \theta + \pi)$

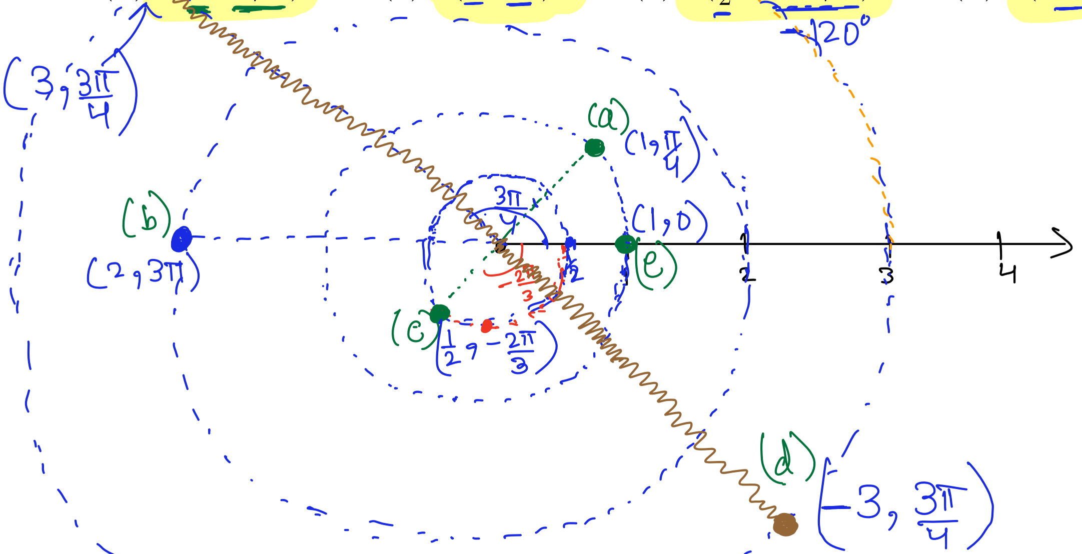
Example 1: Plot the points whose polar coordinates are given

(a) $(1, \pi/4)$ (b) $(2, 3\pi)$ (c) $(\frac{1}{2}, -2\pi/3)$ (d) $(-3, 3\pi/4)$ (e) $(1, 0)$

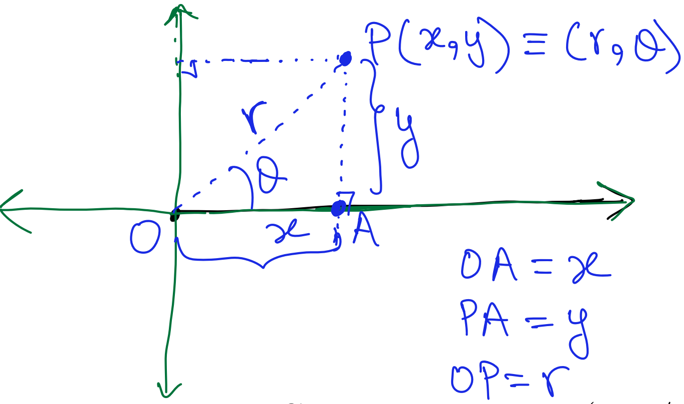
$-120°$

Plot $(3, \frac{3\pi}{4})$ then reflect about the Pole.

$(3, \frac{3\pi}{4})$



The Connection between Polar and Cartesian Coordinates:



$OA = x$

$PA = y$

$OP = r$

In ΔOAP :—

$\sin\theta = \dfrac{y}{r}$, $\cos\theta = \dfrac{x}{r}$

$\Rightarrow \boxed{x = r\cos\theta, \; y = r\sin\theta}$

Given Polar Coord, it gives us Cartesian Coord

$r = \sqrt{x^2+y^2}$

$\theta_0 = \operatorname{Tan}^{-1} \dfrac{|y|}{|x|}$.

$$\theta = \begin{cases} \theta_0 & \text{if } P \text{ is in } 1^{st} \text{ quad} \\ \pi - \theta_0 & \text{if } P \text{ in } 2^{nd} \text{ quad.} \\ \pi + \theta_0 & \text{if } P \text{ in } 3^{rd} \text{ quad.} \\ 2\pi - \theta_0 & \text{if } P \text{ in } 4^{th} \text{ quad.} \end{cases}$$

Given Cartesian coord, it gives us Polar coord

Example 2: Convert the point $(2, \pi/3)$ from polar to Cartesian coordinates

$x = 2\cos\frac{\pi}{3}$, $y = 2\sin\frac{\pi}{3}$

$= 2\left(\frac{1}{2}\right) = 1$, $= 2\left(\frac{\sqrt{3}}{2}\right) = \sqrt{3}$

$(x, y) = (1, \sqrt{3})$

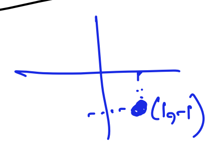
Example 3: Convert the point to polar coordinates.

(a) $(1, -1)$ $\quad x = 1, \; y = -1$

$r = \sqrt{(1)^2 + (-1)^2} = \sqrt{1+1} = \sqrt{2}$

$\theta_0 = \operatorname{Tan}^{-1}\frac{1}{1}$

$= \operatorname{Tan}^{-1} 1 = \pi/4$



$P \equiv (1, -1)$ is in the 4^{th} quad

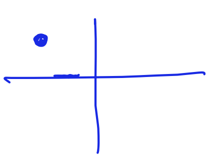
$\theta = 2\pi - \theta_0 = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4} \Rightarrow (r, \theta) = \left(\sqrt{2}, \frac{7\pi}{4}\right)$

(b) $(-\sqrt{3}, 1)$

$x = -\sqrt{3}, \; y = 1$

$r = \sqrt{(-\sqrt{3})^2 + (1)^2} = \sqrt{3+1} = \sqrt{4} = 2$

$\theta_0 = \operatorname{Tan}^{-1}\frac{1}{\sqrt{3}} = \frac{\pi}{6} \Rightarrow \theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$



$\Rightarrow (r, \theta) = \left(2, \frac{5\pi}{6}\right)$

Example 4: Find a polar equation for the curve represented by the Cartesian equation $x^2 + 2x + y^2 = 0$.

in terms of x and y $\Rightarrow$ we want it in terms of r and θ

$$\boxed{\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \end{aligned}} \Rightarrow (r\cos\theta)^2 + 2(r\cos\theta) + (r\sin\theta)^2 = 0$$

$$\Rightarrow r^2\cos^2\theta + 2r\cos\theta + r^2\sin^2\theta = 0$$

$$\Rightarrow \underbrace{r^2\cos^2\theta + r^2\sin^2\theta} + 2r\cos\theta = 0$$

$$\Rightarrow r^2(\cos^2\theta + \sin^2\theta) + 2r\cos\theta = 0$$

$$\Rightarrow \boxed{r^2 + 2r\cos\theta = 0} \Rightarrow r(r + 2\cos\theta) = 0$$

$$\Rightarrow r = 0 \text{ or } \boxed{r + 2\cos\theta = 0}$$

$$\boxed{r = -2\cos\theta}$$

rejected because $r=0$ corresponds to pole, and pole any satisfies

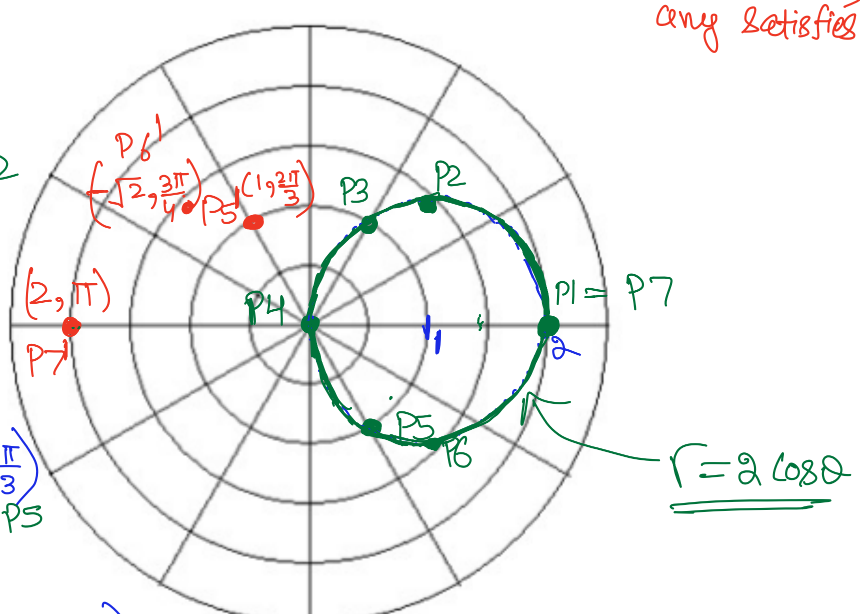
a set of points satisfying some equation in r and θ

$r = f(\theta)$

A Polar Curve is the **Graph of a Polar Equation** $r = r(\theta)$.

Example 5: Sketch the polar curve $r = 2\cos\theta$

θ	$r = 2\cos\theta$
0	$2\cos 0 = 2 \equiv (2, 0)$ P1
$\pi/4$	$2\cos\frac{\pi}{4} = \sqrt{2} \equiv \left(\sqrt{2}, \frac{\pi}{4}\right)$ P2
$\pi/3$	$2\cos\frac{\pi}{3} = 1 \equiv \left(1, \frac{\pi}{3}\right)$ P3
$\pi/2$	$2\cos\frac{\pi}{2} = 0 \equiv \left(0, \frac{\pi}{2}\right)$ P4
$2\pi/3$	$2\cos\frac{2\pi}{3} = 2\left(-\frac{1}{2}\right) = -1 \equiv \left(-1, \frac{2\pi}{3}\right)$ P5
$3\pi/4$	$2\cos\frac{3\pi}{4} = 2\left(-\frac{1}{\sqrt{2}}\right) = -\sqrt{2} \equiv \left(-\sqrt{2}, \frac{3\pi}{4}\right)$ P6
π	$2\cos\pi = -2 \equiv (-2, \pi)$ P7



$$\cos\frac{2\pi}{3} = \cos\left(\pi - \frac{\pi}{3}\right) = -\cos\frac{\pi}{3} = -\frac{1}{2}$$

sin +ve Cos, Tan −ve	All ratios +ve
Tan +ve sin, cos −ve	Cos +ve sin, Tan −ve

$\frac{3\pi}{4} = \pi - \frac{\pi}{4}$

Example 6: Sketch the polar curve $r = 1 + \sin\theta$

θ	$r = 1 + \sin\theta$
0	$1 + \sin 0 = 1$
$\frac{\pi}{6}$	$1 + \sin\pi/6 = 1.5$
$\frac{\pi}{3}$	$1 + \sin\frac{\pi}{3} = 1.732$
$\pi/2$	$1 + \sin\pi/2 = 2$
$2\pi/3$	$1 + \sin\frac{2\pi}{3} = 1.732$
$5\pi/6$	$1 + \sin\frac{5\pi}{6} = 1.5$
π	$1 + \sin\pi = 1$
$\frac{7\pi}{6}$	$1 + \sin\frac{7\pi}{6} = 1 - \sin\frac{\pi}{6} = 0.5$
$\frac{4\pi}{3}$	$1 + \sin 4\pi/3 = 1 - 0.732 = 0.268$
$\frac{3\pi}{2}$	$1 + \sin\frac{3\pi}{2} = 1 - 1 = 0$
$-\pi/3$	$1 + \sin(-\pi/3) = 1 - \sin\pi/3 = 0.268$
$-\pi/6$	$1 + \sin(-\pi/6) = 1 - \sin\pi/6 = 0.5$
0	$1 + \sin 0 = 1$

$0.732 = \frac{\sqrt{3}}{2}$