3.3- Solving Applications

Example 1: The sum of two numbers is 46 The first number is $\frac{3}{20}$ of the second number. What are the numbers?

Let the two numbers be x and y.

$$\left.\begin{aligned} x+y&=46 \\ x&=\frac{3}{20}y \end{aligned}\right\} \Rightarrow \frac{3}{20}y+y=46$$

$$\Rightarrow \left(\frac{3}{20}+1\right)y=46 \Rightarrow \left(\frac{3+20}{20}\right)y=46$$

$$\Rightarrow \frac{23}{20}y=46 \Rightarrow y=\frac{20}{\cancel{23}}\times \cancel{46}^{2}=40$$

$$\Rightarrow y=40$$

$$x+40=46$$

$$x=46-40=6$$

$\Rightarrow$ $\boxed{\text{The two numbers are } 6, 40}$

Example 2: Two angles are supplementary. One angle is 3^0 less than twice the other. Find the measures of the angles.

supplementary $\downarrow$ sum is 180°

* Complementary means sum is 90°

Let the angles be x° and y°.

$$x+y=180, \quad x=2y-3$$

$$\Rightarrow 2y-3+y=180 \Rightarrow 3y-3=180 \Rightarrow 3y=183$$

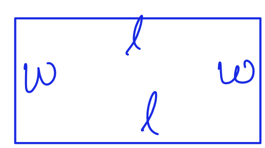
$$\Rightarrow y=61$$

$$x=2\times 61-3=122-3=119$$

Thus, the two angles have measures $\underline{119^\circ \text{ and } 61^\circ}$.

Example 3: The perimeter of a rectangle field is 320 yards. The length is 60 yards longer than the width. Find the dimensions.

Let the length be l and width be w.



$$2l+2w=320, \quad l=w+60$$

$$2(w+60)+2w=320 \Rightarrow 2w+120+2w=320$$

$$\Rightarrow 4w+120=320 \Rightarrow 4w=200 \Rightarrow w=50$$

$$l=50+60=110$$

Thus, the rectangle has length 110 yards and width 50 yards.

Example 4: Each course at Matrix College is either 2 or 3 credits. The members of the men's swim team are taking a total of 51 courses that are worth a total of 115 credits. How many 2-credit courses and how many 3-credit courses are being taken?

Let number of 2-cr. courses be x and of 3-cr. courses be y.

$$\left.\begin{aligned} x+y&=51 \\ 2x+3y&=115 \end{aligned}\right\} \begin{aligned} &\times -2 \\ &\times 1 \end{aligned} \Rightarrow \left.\begin{aligned} -\cancel{2x}-2y&=-102 \\ \cancel{2x}+3y&=115 \end{aligned}\right\} \text{Add}$$

$$3y-2y=115-102 \Rightarrow y=13$$

$$x+13=51 \Rightarrow x=51-13=38$$

Thus, $\boxed{\begin{aligned} \#\,2\text{ cr. courses} &= 38 \\ \#\,3\text{ cr. courses} &= 13 \end{aligned}}$

Example 5: A museum chargers \$15.50 for a one-day youth admission and \$19.50 for a one day adult admission. One Friday the museum collected \$1833 from a total of 110 youth and adults. How many admissions of each type were sold?

Let #youth adm. $= x$ and #adult adm. $= y$

$$\left.\begin{aligned} x+y&=110 \\ 15.50x+19.50y&=1833 \end{aligned}\right\} \begin{aligned} &\times -15.5 \\ &\times 1 \end{aligned} \Rightarrow \left.\begin{aligned} -\cancel{15.50x}-15.50y&=-1705 \\ \cancel{15.50x}+19.50y&=1833 \end{aligned}\right\} \text{Add}$$

$$19.5y-15.5y=1833-1705$$

$$\Rightarrow 4y=128 \Rightarrow y=32$$

$$x+32=110$$

$$\Rightarrow x=78$$

Thus, $\boxed{\#\text{ youth adm.} = 78 \text{ and } \#\text{ adults} = 32}$

Example 6: The Coffee Counter charges \$9.00 per pound for Kenyan French Roast coffee and \$11.00 per pound for Sumatran coffee. How much of each type should be used to make a 24 pound blend that sells for \$10.00 per pound?

Let Kenyan coffee be x pounds and Sumatran Coffee be y pounds

$$\left.\begin{aligned} x+y&=24. \\ 9x+11y&=240 \end{aligned}\right\} \begin{aligned} &\times -9 \\ &\times 1 \end{aligned}$$

sells for \$10 per pound

$\Rightarrow$ Total $= 24\times 10 = 240$ \$

$x=y=12$

Example 7: One canned juice drink is 30% orange juice; another is 5% orange juice. How many liters of each should be mixed together in order to get 25L that is 27% orange juice?

In the mixture, let there be x L of first and y L of second.

$$x + y = 25$$
$$\frac{30}{100}x + \frac{5}{100}y = \frac{27}{100} \times 25$$

$\Rightarrow$ $x + y = 25$ $\times -5$ $\Rightarrow$ $-5x - 5y = -125$

$30x + 5y = 27 \times 25$ $\times 1$ $\quad 30x + 5y = 675$

Add $\Rightarrow$ $30x - 5x = 675 - 125 \Rightarrow 25x = 550 \Rightarrow \boxed{x = 22}$

$\Rightarrow 22 + y = 25 \Rightarrow y = 25 - 22 \Rightarrow \boxed{y = 3}$

Example 8: $5900 is invested, part of it at 10% and part of it at 8%. Or a certain year the total yield is $534. How much was invested at each rate?

Let $\$x$ was invested at 10% and $\$y$ was invested at 8%.

$$x + y = 5900$$
$$\frac{10}{100}x + \frac{8}{100}y = 534$$

$x + y = 5900$ $\times -10$

$10x + 8y = 53400$ $\times 1$

$$\boxed{\begin{aligned} x &= 3100 \\ y &= 2800 \end{aligned}}$$

3.8- Business and Economics Applications

Break Even Analysis

$\rightarrow$ Point at which total Profit $= 0$

Example 1: Suppose that for a certain company $C(x) = 25x + 100{,}000$ represents the total cost function, and $R(x) = 65x$ represents the total revenue function.

a. Find the total-profit function

$P(x) = R(x) - C(x) \Rightarrow P(x) = 65x - (25x + 100{,}000)$

$\uparrow$ Profit function

$\Rightarrow P(x) = 65x - 25x - 100{,}000$

$\Rightarrow \boxed{P(x) = 40x - 100\,000}$

b. Find the break even point.

$P(x) = 0$

$\Rightarrow 40x - 100000 = 0$

$\Rightarrow 40x = 100000 \Rightarrow x = \frac{100\,000}{40} \Rightarrow \boxed{x = 2500}$

Example 2: Suppose that for a certain company, $C(x) = 15x + 3100$ represents the total cost function, and $R(x) = 40x$ represents the total revenue function

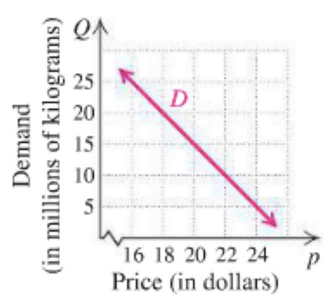
HW: Find total profit function.
Find the break-even point.

CAUTION! Do not confuse "cost" with "price." When we discuss the *cost* of an item, we are referring to what it costs to produce the item. The *price* of an item is what a consumer pays to purchase the item and is used when calculating revenue.

Supply and Demand

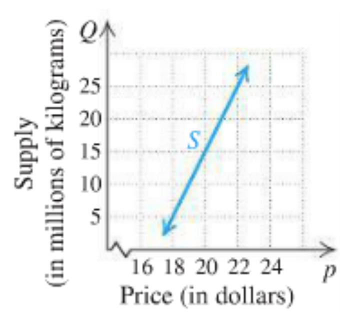
Demand Function, *D*

Price, *p*, per Kilogram	Quantity, *D*(*p*) (in millions of kilograms)
\$16.00	25
18.00	20
20.00	15
22.00	10
24.00	5

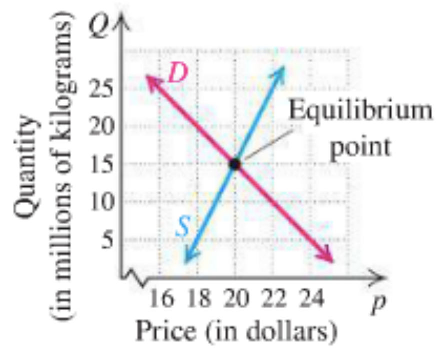


Supply Function, *S*

Price, *p*, per Kilogram	Quantity, *S*(*p*) (in millions of kilograms)
\$18.00	5
19.00	10
20.00	15
21.00	20
22.00	25



$D(p) = S(p).$



Example 3: if $D(p) = 9400 - 40p$ and $S(p) = 400 + 50p$ are demand and supply functions, respectively, find the equilibrium point.

$D(P) = S(P) \rightarrow$ Solve for P.

$9400 - 40P = 400 + 50P \Rightarrow -40P - 50P + 9400 = 400$

$\Rightarrow -90P + 9400 = 400 \Rightarrow -90P = 400 - 9400$

$\Rightarrow -90P = -9000 \Rightarrow P = \frac{-9000}{-90} \Rightarrow P = 100$

Eqm. Point.

$D(100) = 9400 - 40 \times 100 = 9400 - 4000 = 5400 \Rightarrow \boxed{(100, 5400)}$

Example 4: An electronics company is planning to introduce a new line of computers. For the first year, the fixed costs for setting up the production line are \$200,000. The variable costs for producing each computer are \$40 The revenue from each computer is \$6565. Find the total profit P(x) from the production and sale of x computers and the break-even point.

$C(x) = 40x + 200000$

$R(x) = 6565x$

$\rightarrow P(x) = R(x) - C(x)$

$P(x) = 0$

$\hookrightarrow$ Find x

$P(x) = 6565x - (40x + 200000)$

$= 6565x - 40x - 200000$

$\boxed{P(x) = 6525x - 200000}$

$\Rightarrow 6525x - 200000 = 0$

$\Rightarrow 6525x = 200000$

$$\Rightarrow x = \frac{200\,000}{6525}$$

$$\Rightarrow \boxed{x = 30.65} \leftarrow \underline{\text{Breakeven Point}}$$

Test Revision

(1) Find the equation of a line passing through the point (1, 1) and perpendicular to the line $2x + y = 1$.

Let the slope of the required line be m.

$y = -2x + 1 \Rightarrow$ slope $= -2$.

Because the lines are $\perp$, $m \times (-2) = -1$

$$\Rightarrow -2m = -1$$

$$\Rightarrow m = \frac{-1}{-2} \Rightarrow m = \frac{1}{2}$$

Using Point-slope form :-

$$\boxed{y - 1 = \frac{1}{2}(x - 1)} \Rightarrow 2(y-1) = 1(x-1)$$

$$\Rightarrow 2y - 2 = x - 1$$

$$\Rightarrow 2y = x - 1 + 2 \Rightarrow \boxed{2y = x + 1}$$

$$\Rightarrow \boxed{x - 2y + 1 = 0}$$

Ch2 Test #19

Determine without graphing whether the graphs of the equations are parallel, perpendicular, or neither.

$$y = -2x + 5$$
$$2y - x = 6$$

Ch2 Test #20

Find a linear function that has slope -5 and y-intercept $(0, -1)$ $\rightarrow (0, b)$

$y = mx + b \Rightarrow f(x) = -5x - 1$

$f(x)$

Ch2 Test #21

Incorrect

Find an equation in point-slope form of the line with slope 4 and containing (-2, -4)

$y + 4 = 4(x + 2)$ $\boxed{y + 4 = 4x + 2.}$

Ch2 Test #22

Using function notation, write a slope-intercept equation for the line containing (3, -1) and (4, -2)

$(y - (-1)) = m(x - 3)$, $m = \dfrac{-2 - (-1)}{4 - 3} = \dfrac{-2 + 1}{1} = -1$

$y + 1 = -1(x - 3) \Rightarrow y + 1 = -x + 3 \Rightarrow y = -x + 3 - 1 \Rightarrow y = -x + 2$

$\boxed{f(x) = -x + 2}$

Ch2 Test #23

Find an equation of the line containing (-3, 2) and parallel to the line $2x - 5y = 8$

$y - 2 = \dfrac{2}{5}(x + 3)$

Ch2 Test #24

Find an equation of the line containing (-3, 2) and perpendicular to the line $2x - 5y = 8$

Let slope be m. $m \times \dfrac{2}{5} = -1$ $\quad -5y = -2x + 8$

$\Rightarrow y - 2 = \dfrac{-5}{2}(x + 3)$ $\quad m = -\dfrac{5}{2}$ $\quad y = \dfrac{2}{5}x - \dfrac{8}{5}$ $\quad y = \dfrac{-2}{-5}x + \dfrac{8}{-5}$

Ch2 Test #25

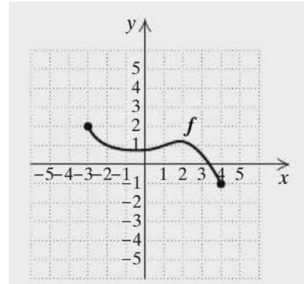
If you rent a truck for one day and drive it 250 mi, the cost is \$100. If you rent it for one day and drive it 300 mi the cost is \$115. Let *C(m)* represent the cost in dollars, of driving *m* miles.

a) Find a linear function that fits the data.

b) Use the function to determine how much it will cost to rent the truck for one day and drive it 500 mi.

Ch2 Test #26

For the following graph of *f* determine
(a) f(-2)
(b) the domain off
(c) any x-value for which f(x) = 1
(d) the range off



Ch2 Test #27

Given $g(x) = \frac{1}{x}$ and $h(x) = 2x + 1$, find $h(-5)$

Ch2 Test #28

Given $g(x) = \frac{1}{x}$ and $h(x) = 2x + 1$, find $(g + h)(x)$

$(g+h)(x) = g(x) + h(x) = \frac{1}{x} + 2x + 1$

$(g \cdot h)(2) = g(2) \cdot h(2) = \frac{1}{2} \cdot (2 * 2 + 1) = \frac{1}{2} \cdot 5 = \frac{5}{2}$

$(g/h)(1) = \frac{g(1)}{h(1)} = \frac{1}{2 \times 1 + 1} = \frac{1}{2+1} = \frac{1}{3}$

Find the domain of g/h.

$D_g = \{x \mid x$ is a real number and $x \neq 0\}$

$D_h = \{x \mid x$ is a real number$\}$

$D(g/h) = \{x \mid x$ is a real number and $x \neq 0$ and $x \neq \frac{-1}{2}\}$

$h(x) \neq 0 \Rightarrow 2x+1 \neq 0 \Rightarrow 2x \neq -1 \Rightarrow x \neq \frac{-1}{2}$

Math11000 Section 3962 Quiz 6
Summer 2023, May 18

Name: [1 pt]

Problem 1: Let $f(x) = \frac{1}{x}$ and $g(x) = x - 2$.

1. Find $(f.g)(4)$ [2 pts]
2. Find the domain of f/g. [3 pts]

① $(f\cdot g)(4) = f(4)\,g(4) = \frac{1}{4} \times (4-2) = \frac{1}{4} \times 2 = \frac{1}{2}$

② $Df = \{x \mid x$ is real number and $x \neq 0\}$

$Dg = \{x \mid x$ is real number$\}$

$D(f/g) = \{x \mid x$ is real number and $x \neq 0$ and $x \neq 2\}$

$g(x) = x-2 \neq 0 \Rightarrow x \neq 2$

Problem 2: Solve the system of linear equations $x + y = 3$ and $x - y = 1$. [4 pts]

$$\left.\begin{aligned} x \cancel{+y} &= 3 \\ x \cancel{-y} &= 1 \end{aligned}\right\} \text{Add}$$

$x + x = 3 + 1 \Rightarrow 2x = 4 \Rightarrow x = 2$

$x + y = 3 \Rightarrow 2 + y = 3 \Rightarrow y = 3 - 2 \Rightarrow y = 1$

$\boxed{x = 2,\ y = 1}$