Keshav Dandeva #11

Numerical Methods (ENUME) – Project
Assignment C: Solving ordinary differential equations

1. Program the functions implementing the system of differential-algebraic equations of the form:

$$\left.\begin{aligned} \frac{d\mathbf{v}(t)}{dt} &= \mathbf{A}\cdot\mathbf{v}(t)+\mathbf{b}\cdot x(t) \\ y(t) &= \mathbf{c}^T\cdot\mathbf{v}(t) \end{aligned}\right\} \text{ where: } \mathbf{A}=\begin{bmatrix} 0 & 1 & -1 \\ 0 & 1 & -2 \\ 1200 & -282 & -62 \end{bmatrix},\quad \mathbf{b}=\begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \quad\text{and}\quad \mathbf{c}=\begin{bmatrix} 820 \\ -296.5 \\ -8 \end{bmatrix}$$

2. Solve this system of equations with zero initial conditions for:

$$x(t)=\begin{cases} 0 & \text{dla } t\le 0 \\ 1 & \text{dla } t>0 \end{cases} \qquad \text{for } t\in(0,5),$$

and for:

$$x(t)=\begin{cases} 0 & \text{dla } t\le 0 \\ \exp(-t) & \text{dla } t>0 \end{cases} \quad \text{for } t\in(0,5)$$

using the ***ode45*** procedure, setting the parameters ***'AbsTol'*** and ***'RelTol'*** to the values guaranteeing the maximum accuracy of the solution.

3. Solve the system of equations for both functions $x(t)$, defined in Section 2, using the following methods:

– the explicit Adams-Bashwithth method: $\mathbf{v}_n=\mathbf{v}_{n-1}+h\sum_{k=1}^{K}\beta_k f\left(t_{n-k},\mathbf{v}_{n-k}\right)$ with $K=2$;

– the implicit Gear method: $\mathbf{v}_n=\sum_{k=1}^{K}\alpha_k\mathbf{v}_{n-k}+h\cdot\beta_0 f\left(t_n,\mathbf{v}_n\right)$ with $K=5$;

for various values of the integration step $h\in\left[h_{\min},h_{\max}\right]$. For the multistep methods, determine the first K values of the vector $\mathbf{v}$ by means of the lower-order methods belonging to the same family. Choose the range $\left[h_{\min},h_{\max}\right]$ in such a way as to illustrate and explain the phenomenon of numerical instability for excessive values of h. Compute the estimate $\hat{y}(t)$ of $y(t)$ and determine the following indicators of its uncertainty:

$$\delta_2(h)=\frac{\left\|\hat{y}(t;h)-\dot{y}(t)\right\|_2}{\left\|\dot{y}(t)\right\|_2} \quad\text{and}\quad \delta_\infty(h)=\frac{\left\|\hat{y}(t;h)-\dot{y}(t)\right\|_\infty}{\left\|\dot{y}(t)_2\right\|_\infty}$$

$\dot{y}(t)$ is the solution obtained in Section 2. Plot the graphs of $\delta_2(h)$ and $\delta_\infty(h)$ for both methods and both functions $x(t)$.

	Adams-Bashforth Methods						Adams-Moulton Methods							Implicit Gear Methods						
K	β_1	β_2	β_3	β_4	β_5	β_6	β_0	β_1	β_2	β_3	β_4	β_5	β_6	β_0	α_1	α_2	α_3	α_4	α_5	α_6
1	1						$\frac{1}{2}$	$\frac{1}{2}$						1	1					
2	$\frac{3}{2}$	$-\frac{1}{2}$					$\frac{5}{12}$	$\frac{8}{12}$	$-\frac{1}{12}$					$\frac{2}{3}$	$\frac{4}{3}$	$-\frac{1}{3}$				
3	$\frac{23}{12}$	$-\frac{16}{12}$	$\frac{5}{12}$				$\frac{9}{24}$	$\frac{19}{24}$	$-\frac{5}{24}$	$\frac{1}{24}$				$\frac{6}{11}$	$\frac{18}{11}$	$-\frac{9}{11}$	$\frac{2}{11}$			
4	$\frac{55}{24}$	$-\frac{59}{24}$	$\frac{37}{24}$	$-\frac{9}{24}$			$\frac{251}{720}$	$\frac{646}{720}$	$-\frac{264}{720}$	$\frac{106}{720}$	$-\frac{19}{720}$			$\frac{12}{25}$	$\frac{48}{25}$	$-\frac{36}{25}$	$\frac{16}{25}$	$-\frac{3}{25}$		
5	$\frac{1901}{720}$	$-\frac{2774}{720}$	$\frac{2616}{720}$	$-\frac{1274}{720}$	$\frac{251}{720}$		$\frac{475}{1440}$	$\frac{1427}{1440}$	$-\frac{798}{1440}$	$\frac{482}{1440}$	$-\frac{173}{1440}$	$\frac{27}{1440}$		$\frac{60}{137}$	$\frac{300}{137}$	$-\frac{300}{137}$	$\frac{200}{137}$	$-\frac{75}{137}$	$\frac{12}{137}$	
6	$\frac{4277}{1440}$	$-\frac{7923}{1440}$	$\frac{9982}{1440}$	$-\frac{7298}{1440}$	$\frac{2877}{1440}$	$-\frac{475}{1440}$	$\frac{19087}{60480}$	$\frac{65112}{60480}$	$-\frac{46461}{60480}$	$\frac{37504}{60480}$	$-\frac{20211}{60480}$	$\frac{6312}{60480}$	$-\frac{863}{60480}$	$\frac{60}{147}$	$\frac{360}{147}$	$-\frac{450}{147}$	$\frac{400}{147}$	$-\frac{225}{147}$	$\frac{72}{147}$	$-\frac{10}{147}$