Keshav Dandeva #11

Numerical Methods (ENUME) – Project
Assignment A: Analysis of accuracy of computer computation

1. Determine the functions characterising the propagation of the relative errors corrupting the data – $T_x(x,y)$, $T_y(x,y)$ – and the functions characterising the propagation of the relative errors caused by rounding the intermediate results of computing – $K_1(x,y)$, $K_2(x,y)$,… – for the following function:

$$z \equiv \frac{x^3 + \frac{\cos(y)}{3}}{y - \frac{\sin(y)}{2}} \quad \text{for} \quad (x,y) \in \mathbb{D} \equiv \{x \in [1,10],\ y \in [1,10]\}$$

Compare the results obtained by means of analytical differentiation and so-called epsilon calculus. Plot the graphs $T_x(x,y)$, $T_y(x,y)$, $K_1(x,y)$, $K_2(x,y)$,…

2. Assuming that the indicator of the accuracy of the floating-point representation is $eps = 10^{-12}$, assess the total error of computing the value of z by maximising the following indicator:

$$\delta z_{\text{sup}}^{(1)} = \sup\left\{|T_x(x,y)| + |T_y(x,y)| + |K_1(x,y)| + |K_2(x,y)| + \ldots \;\middle|\; (x,y) \in \mathbb{D}\right\} * eps$$

3. Compare the result obtained in Section 2 with the estimate:

$$\delta z_{\text{sup}}^{(2)} = \sup\left\{|\delta z(x,y)| \;\middle|\; (x,y) \in \mathbb{D}\right\}$$

obtained by means of the simulation method. In the above formula: $|\delta z(x,y)|$ is the largest magnitude of the error of the calculated value of z, which can appear when both the relative data errors and the relative rounding errors can assume only two values: $-eps$ and $+eps$.