

DSA-501
LECTURE # 8
TOPIC: DP-4

Edit Distance →

auto complete

str1 = abcde
str2 = abcg) convert

(I) abcde
2 delete
abc
+ 1 add ~ ⇒ 3
abcg

oper^{ns}

- ① insertⁿ a char
- ② deleteⁿ " "
- ③ replace a char

(II) abcd~~e~~
1 delete

abcd~~e~~
1 replace

abcg

Min edit dist = 2

Edit Distance

→ Min # operations on S1 to convert it to S2

S1 = "xyz"

S2 = "xypz"

S1 = ^xab~~f~~de

S2 = abxde

edit dis = ? 1

opⁿ = replace

S1 = appleⁱ
S2 = apple^j

[compare char by char]

Case 2:
No match

Case 1:
Match (i--, j--)

S1 = appleⁱ

S2 = apple^j

match / j

No match

S1: abcdef*f*i Min(-, -, -)

S2: abcdej

(i=j)

15

Insert

insert s2[j]

+1

S1: abcdef*f*e

S2: abcdej

(i=i
j=j-1)

+

Delete i

S1: abcdef*f*

S2: abcdej

1 + (i=i-1
j=j) abcde
 abcde

Match
i--
j--

12

Replace

20

S1: abcdef*f*e

S2: abcdej

1 + (i=i-1
j=j-1)

S1: abcdee

S2: abde

+

Recursive f^N

editDist (str1, str2, i, j)

{
 // 1. base condⁿ
 if (i == 0) ret j
 if (j == 0) ret i

// 2. recursive calls

 // Match
 if (str1[i-1] == str2[j-1])
 ret editD(str1, str2, i-1, j-1)

 // No Match

 else Min (
 1 + editD(str1, str2, i, j-1) // insertⁿ
 1 + editD(str1, str2, i-1, j) // deleteⁿ
 1 + editD(str1, str2, i-1, j-1) // replaceⁿ
)

}

s1: abc ⁱ ~~p~~ ^j ~~q~~ ^k ~~r~~ ^l ~~s~~ ^m ~~t~~ ⁿ ~~u~~ ^v ~~w~~ ^x ~~y~~ ^z ~~z~~

s2: ⁰ ~~p~~ ¹ ~~q~~ ² ~~r~~ ³ ~~s~~ ⁴ ~~t~~ ⁵ ~~u~~ ⁶ ~~v~~ ⁷ ~~w~~ ⁸ ~~x~~ ⁹ ~~y~~ ¹⁰ ~~z~~ ¹¹ ~~z~~

editD = i

⇒ editDist (s1, s2, len1, len2)

TC = $O(3^N)$

Memoization

I) Create Data Struc $\left[\begin{array}{l} \text{vars} = 2 \\ dp[N+1][M+1] = -1 \end{array} \right. // \text{all with } -1$

II) Fill dp array recursively

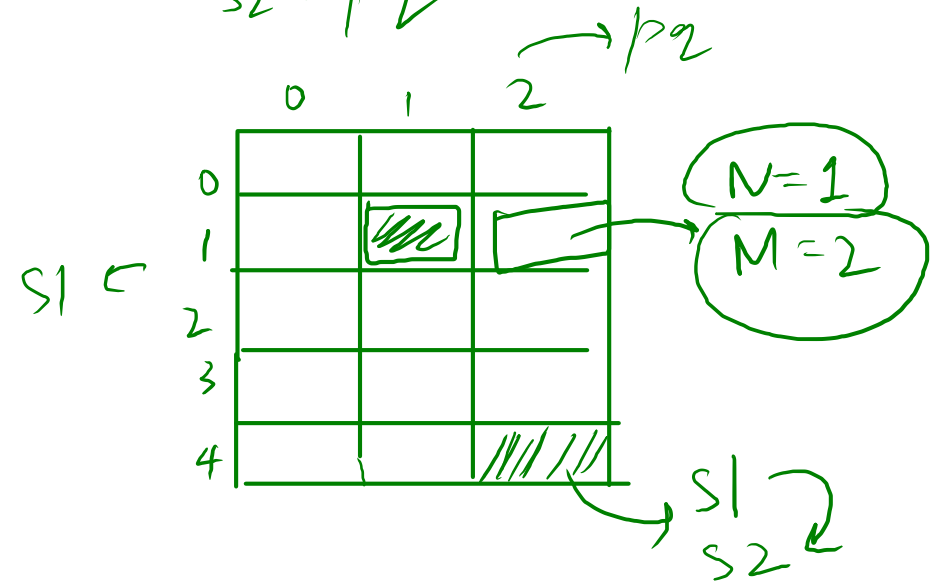
EditDist(str1, str2, N, M)

```
{  
    if(N==0) return M;  
    if(M==0) return N;  
  
    if(dp[N][M] != -1) return dp[N][M]  
  
    //recursive calls  
    //match  
    if( str1[N-1] == str2[M-1])  
        return dp[N][M] = EditDist(str1, str2, N-1, M-1)  
  
    //no match  
    else{  
        return dp[N][M] = min( 1+ EditDist(str1, str2, N, M-1) //ins  
                                , 1+ EditDist(str1, str2, N-1, M) //del  
                                , 1+ EditDist( str1, str2, N-1, M-1) )  
    }  
}
```

$TC = O(N * M)$
 $SC = O(N * M)$

$N \rightarrow \text{len1}$
 $M \rightarrow \text{len2}$

str1: abpq $dp[N+1][M+1]$
str2: |pq $dp[5][3]$



TABULATION

I) create data struct

$dp[N+1][M+1]$

✓ II) Initialize

✓ III) Filling iteratively

//init

for($i=0; i \leq M; i++$)

$dp[0][i] = i$

for($i=1; i \leq N; i++$)

$dp[i][0] = i$

$TC = O(N * M)$

$SC = O(N * M)$

//filling rest of the matrix

for($i=1; i \leq N; i++$) {

for($j=1; j \leq M; j++$) {

if($s1[i-1] == s2[j-1]$)

$dp[i][j] = dp[i-1][j-1]$

else

$dp[i][j] = 1 + \min(dp[i][j-1], dp[i-1][j], dp[i-1][j-1])$

}

}

return $dp[N][M]$

s1
s2

$s1 = \text{abc}$
 $s2 = \text{aec}$

$ed = 2$

$i = 1$

$j = 1$
 $s1[3] = s2[0]$

$j = 2$
 $s1[3] = s2[1]$

$j = 3$
 $s1[3] = s2[2]$

$N=4$
 $M=3$

$dp[5][4]$

	0	1	2	3	
0	0	1	2	3	a
1	1	0	1	2	aec
2	2	1	1	2	
3	3	2	2	1	
4	4	3	3	2	dp

s1: abc
s2: aec