

for  $f(n) = o(g(n))$

$$\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = 0$$

i.  $(1/3)^n + 17$

term gets smaller as  $n \rightarrow \infty$

but term 17 remains constant

$$17 > (1/3)^n$$

ii)  $\lim_{n \rightarrow \infty} \frac{17}{\log \log n} = 0$

iii)  $\lim_{n \rightarrow \infty} \frac{\log \log n}{\log n}$  let  $\log n = y$

l'Hopital's rule

$$\lim_{y \rightarrow \infty} \frac{\log y}{y} = \lim_{y \rightarrow \infty} \frac{1}{y} = 0$$

ii)  $\lim_{n \rightarrow \infty} \frac{\log n}{2^{\sqrt{\log n}}}$  let  $\log n = y$

$$\lim_{y \rightarrow \infty} \frac{y}{2^{\sqrt{y}}}$$
 l'Hopital's rule

$$\lim_{y \rightarrow \infty} \frac{1}{2^{\sqrt{y}} \cdot \frac{1}{2\sqrt{y}}} = 0$$

v)  $\lim_{n \rightarrow \infty} \frac{2^{\sqrt{\log n}}}{\sqrt{n}}$  let  $\log n = y$   
 $n = e^y$

$$= \lim_{y \rightarrow \infty} \frac{2^y}{\sqrt{e^{2y}}} = \frac{2^y}{e^y} = 0$$

$$e^y > 2^y$$

v<sub>i</sub>)

$$\lim_{y \rightarrow \infty} \frac{\sqrt{n}}{\sqrt{n} \log n} = \lim_{y \rightarrow \infty} \frac{1}{\log n} = 0$$

$$\sqrt{n} \log n > \sqrt{n}$$

$$\text{vii)} \lim_{n \rightarrow \infty} \frac{\sqrt{n} \log n}{\frac{n}{\log n}} = \lim_{n \rightarrow \infty} \frac{\sqrt{n} \cdot (\log n)^2}{n}$$

$$\text{let } \log n = y$$

$$= \lim_{y \rightarrow \infty} \frac{y^2}{e^{y/2}}$$

$$n = e^y$$

l'Hopital's rule

$$= \lim_{y \rightarrow \infty} \frac{2y}{\frac{1}{2} e^{y/2}} = 0$$

$$\therefore \frac{n}{\log n} > \sqrt{n} \log n$$

$$\text{viii)} \lim_{n \rightarrow \infty} \frac{n/\log n}{2n} = \lim_{n \rightarrow \infty} \frac{n}{\log n \times 2n} = \frac{1}{\log n} = 0$$

$$\therefore 2n > \frac{n}{\log n}$$

$$\text{ix)} \lim_{n \rightarrow \infty} \frac{2n}{(1.5)^n}, \text{ l'Hopital's rule}$$

$$\lim_{n \rightarrow \infty} \frac{2}{n(1.5)^{n-1}} = 0 \quad \therefore (1.5)^n > 2n$$

$$\text{x)} \lim_{n \rightarrow \infty} \frac{(1.5)^n}{\left(\frac{n}{2}\right)^{\log n}} \quad \text{let } \log n = y$$

$$n = e^y$$

$$\lim_{y \rightarrow \infty} \frac{(1.5)^{e^y}}{\left(\frac{e^y}{2}\right)^y} = \lim_{y \rightarrow \infty} \frac{(1.5)^{e^y}}{e^{y^2}} \times 2^y = 0$$

$$\left(\frac{n}{2}\right)^{\log n} > (1.5)^n$$