for $f(n) = o(g(n))$

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} = 0$$

i $e^{\log n^3} = o(n^{3.01})$

$$\lim_{n \to \infty} \frac{e^{\log n^3}}{n^{3.01}} = \lim_{n \to \infty} \frac{n^3}{n^{3.01}} = 0$$

ii

$$\lim_{n \to \infty} \frac{n^{3.01}}{2^{\sqrt{n}}},$$

since both numerator & denominator tend to ∞ apply L'Hopital's rule

$$\lim_{n \to \infty} \frac{3.01\, n^{2.01} \ldots}{\frac{2^{\sqrt{n}} \ln(2)}{2\sqrt{n}} \ldots} = 0$$

iii) $\lim_{n \to \infty} \frac{2^{\sqrt{n}}}{2^{n^3}} = 0$

also $2^{\sqrt{n}}$ grows slower than 2^{n^3} just by inspection.

for $f(n) = o(g(n))$

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} = 0$$

i) $\lim_{n \to \infty} \frac{1 + \log^3 n}{n^{1.6}} \propto \frac{\log^3 n}{n^{1.6}} = \frac{\log n \log n \log n}{n^{1.6}}$

let

$\log n = y$

$n = e^y$

2) $\lim_{y \to \infty} \frac{y^3}{e^{1.6y}}$

Apply L'Hopital's rule.

$$\lim_{y \to \infty} \frac{3y^2 \ldots}{1.6 e^{1.6y} \ldots} = 0$$

ii) $\lim_{n \to \infty} \frac{n^{1.6}}{n^{\log n}} = 0$

We can apply L'Hopital's rule or also by inspection we can say $n^{\log n}$ grows faster

iii) $\lim_{n \to \infty} \frac{n^{\log n}}{\sqrt{n}!} = 0$

for $f(n) = o(g(n))$

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} = 0$$

i) $\lim_{n \to \infty} \frac{(\log(\log n))^2}{n^3 \log n}$, consider $\log n = y$ $\therefore n = e^y$

as $n \to \infty$ both numerator & denominator $\to \infty$, apply l'Hopital's rule.

$$\lim_{y \to \infty} \frac{(\log y)^2}{e^{3y} \cdot y} \Rightarrow \lim_{y \to \infty} \frac{2 \cdot \log y \times 1/y \cdots}{3 \cdot e^y \cdots} = 0$$

ii)

$$\lim_{n \to \infty} \frac{n^3 \log n}{(n+4)^9}$$

$(n+4)^9 \approx n^9$, apply l'Hopital's rule

$$= \lim_{n \to \infty} \frac{n^3 \log n}{n^9} = \frac{n^3 \cdot \frac{1}{n} + \log n \times 3n^2}{9n^8} \cdots = 0$$

numerator gets exhausted $(=1)$ before the denominator.

iii) $\lim_{n \to \infty} \frac{(n+4)^9}{2^{n\sqrt{n}}} = 0$ (Same as above).

5.

$f(n) = 2^{\sqrt{n}}$

$$\lim_{n \to \infty} \frac{n^2}{2^{\sqrt{n}}}$$

l'Hospital's rule

numerator → constant

$$\Rightarrow \lim_{n \to \infty} \frac{2n}{2^{\sqrt{n}} \frac{1}{2\sqrt{n}}} \ldots = 0$$

$$\lim_{n \to \infty} \frac{2^{\sqrt{n}}}{\sqrt{2^n}}.$$

$$= \cancel{\lim_{n\to\infty}} \; 2^{n/2} > 2^{(n)^{1/2}}$$

$$\therefore \lim_{n \to \infty} \frac{2^{\sqrt{n}}}{\sqrt{2^n}} = 0.$$

6.

for $1 : \sqrt{n}$ → $\sqrt{n}$

: $K = 1$; $l = 1$

while $l < n$

$K = K + 2$

$l = l + 2$.

Lets consider $n = 16$

→ while loop

1st. $l = 1$

$K = K + 2 = 1 + 2 = 3$

$l = l + K = 1 + 3 = 4$

2nd. $l = 4$

$K = K + 2 = 3 + 2 = 5$

$l = l + K = 4 + 5 = 9$

3rd $l = 9$

$K = K + 2 = 5 + 2 = 7$

$l = l + K = 9 + 7 = 16$

1, 4, 9 ...

$= \sqrt{n} - 1$ terms

$$= O(\sqrt{n}) \cdot O(\sqrt{n} - 1) \quad (\approx \sqrt{n})$$

$$= \underline{O(n)}$$

10) for i = 1 : n − 1 $\rightarrow (n-1)$

if i is odd then

for j = 1 : j $\rightarrow (n/2)$

end for

for k = i+1 : n $\rightarrow (n/2)$

end for

$\Big\} \; \frac{n}{2} + \frac{n}{2} = n$

$\therefore O(n^2)$

11) for : i = n−1 : 1 $\rightarrow (n-1) \approx n$

if i is odd

for j = 1 : j $\rightarrow (n/2)$

~~end for~~

for k = k+1 : n $\rightarrow (n/2)$

end for

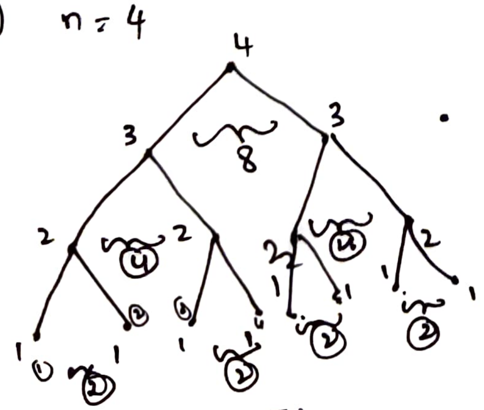
end for.

$\Big\} \; \frac{n}{2} \cdot \frac{n}{2} \approx n^2$

$\therefore O(n^3)$

(15)

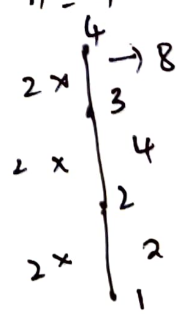
13: a) n = 4



total Vertices = 15

$\Rightarrow \Theta(2^n - 1)$ output = $2^{(n-1)}$

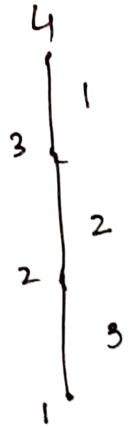
(b) n = 4



O(n) & output = 2^{n-1}

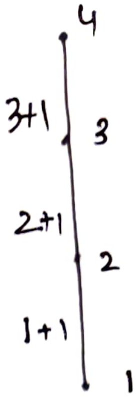
13. c)

$n = 4$



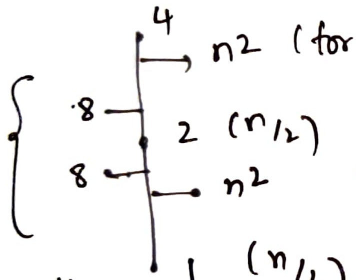
$O(n)$, & output = 1

d) $n = 4$



$O(n)$ & output = n

14) $n = 4$



$\Rightarrow$ $n^2 (n/2)$

$O(n^3)$

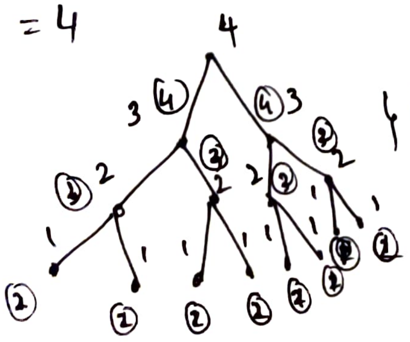
over all

recursion : $\frac{n}{2}$

$\therefore \frac{n}{2} \times (n^2)$ ← for loops

$\Rightarrow \approx O(n^3)$

(15) $n = 4$



& $(2^n - 1)$ recursion

returns (n)