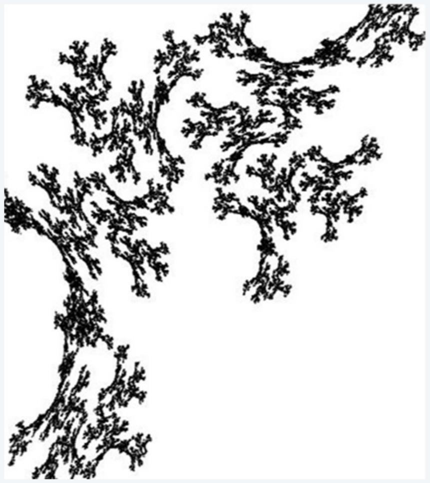
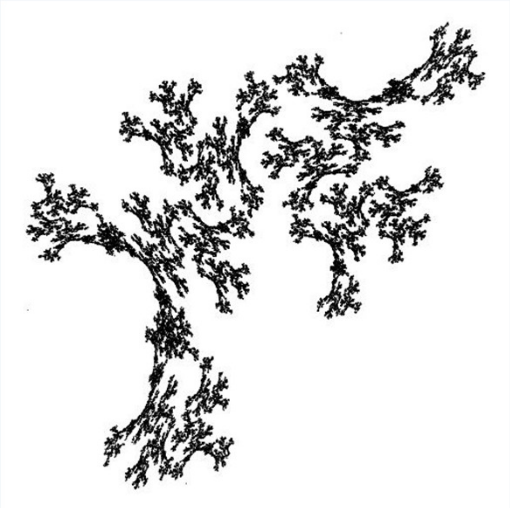


DISCRETE STRUCTURES
FOR COMPUTER SCIENCE

1.B. Applications of Propositions

1.1–1.2



DISCRETE STRUCTURES
FOR COMPUTER SCIENCE

1.B. Applications of Propositions

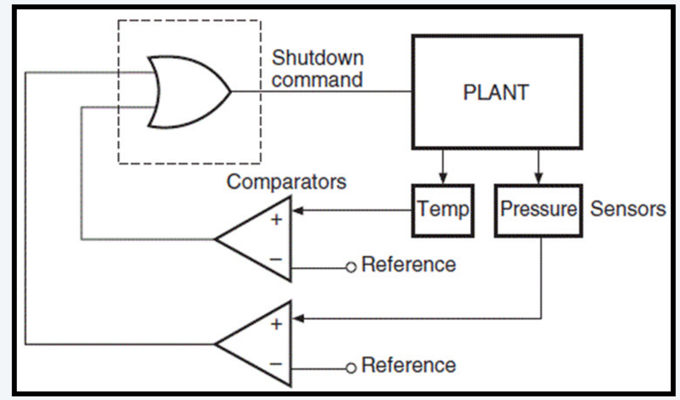
- introduction
- Natural language to propositions
- Consistent systems
- puzzles

Introduction

English. Sentences could be ambiguous.

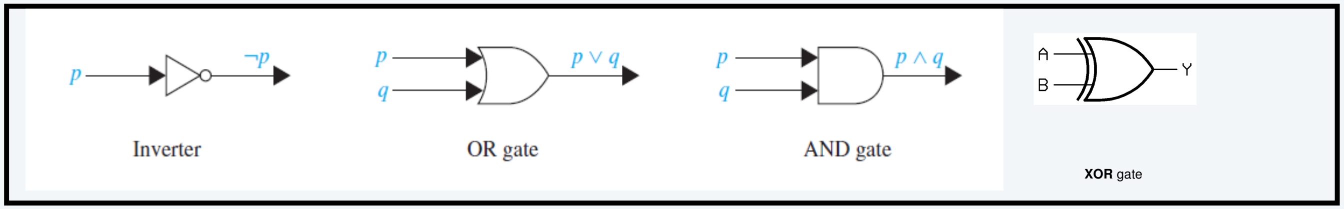
Logic. can be used to describe real world applications.

Propositional logic. Can be used to design systems with real world applications.



Logic gates

Logic gates. Is a big part of computer hardware design

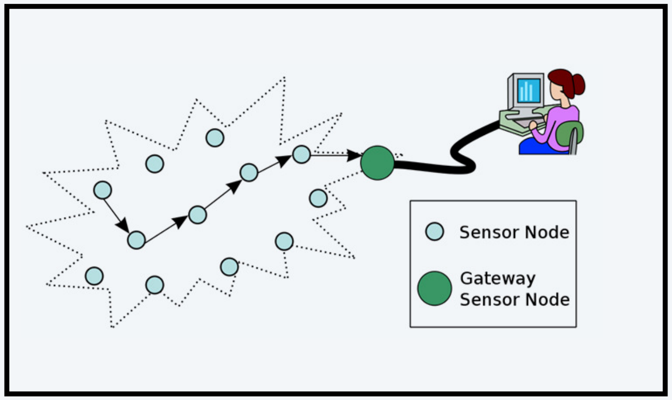


Sensor network

Requirements. Assume there is a 3-sensor network. If 2 or more sensors are true, then we must send TRUE to control station.

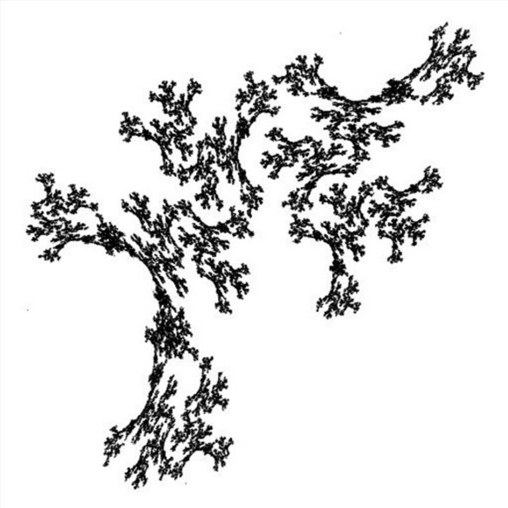
Design a logic circuit that meets these requirements

Assume p, q and r are the sensors



p q r	
0 0 0	➔ 0
0 0 1	➔ 0
0 1 0	➔ 0
0 1 1	➔ 1
1 0 0	➔ 0
1 0 1	➔ 1
1 1 0	➔ 1
1 1 1	➔ 0

~p ^ q ^ r



DISCRETE STRUCTURES
FOR COMPUTER SCIENCE

1.B Applications of Propositions

- introduction
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Many ways of saying p ➔ q

If p, then q

p is sufficient for q

q if p

q when p

a necessary condition for p is q

~q unless p

p implies q

p only if q

a sufficient condition for q is p

q whenever p

q is necessary for p

q follows from p

Examples

Convert the following statements to logical propositions

roads will be wet, if it rains

student will pass the exam if he studies tonight

you can only pass the exam if you study tonight

It is below freezing and snowing

It is either below freezing or snowing, but not both

I will go to class if I feel like it

workshop

Convert the following statements to logical propositions

I will pass the exam **if** I can study tonight

I will not pass the exam **unless** I study tonight

I will pass the exam **if** I can buy a book unless the book-store is closed

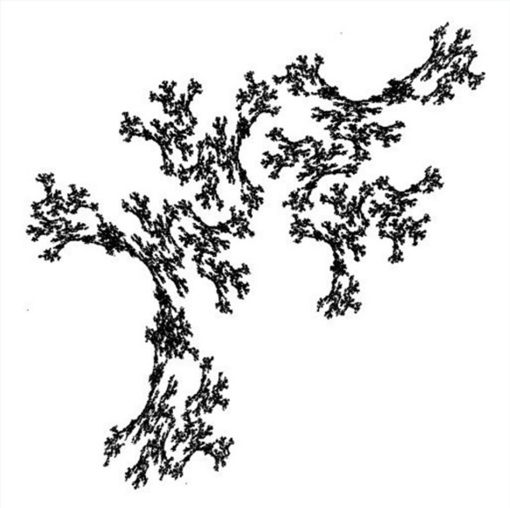
she is a poor little rich girl

My sister is jealous of me **because** I'm an only child

workshop

Convert the following statements to one logical propositions

You can access internet from campus only if you are a computer science major or you are not a freshman



DISCRETE STRUCTURES
FOR COMPUTER SCIENCE

2.1 Applications of Propositions

- introduction
- Natural language to propositions
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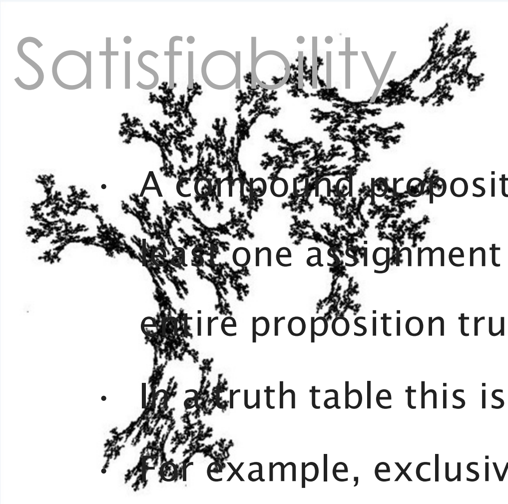
- puzzles

Definition – Consistent System

Definition: A propositional system is **consistent** if **there exists a set of values** for each proposition, such that the entire system is consistent

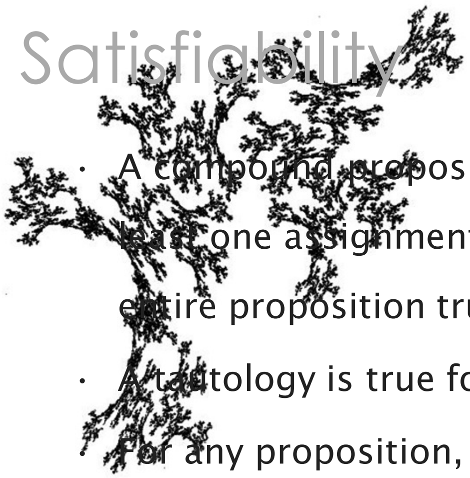
Satisfiability



- A compound proposition is satisfiable if and only if there exists at least one assignment to the propositional variables that makes the entire proposition true
- In a truth table this is a line that is true.
- For example, exclusive OR had two such assignments.

p	q	$p \otimes q$
T	T	F
T	F	T
F	T	T
F	F	F

Satisfiability

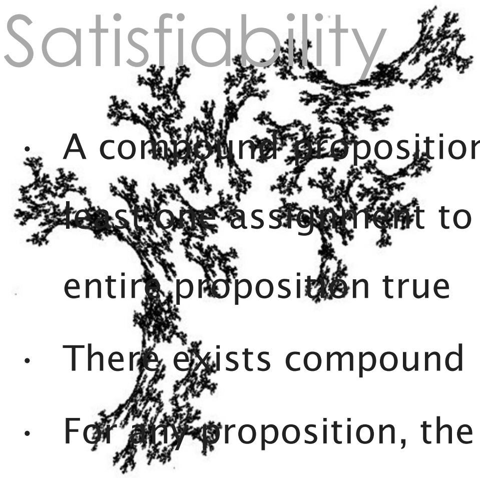


- A compound proposition is satisfiable if and only if there exists at least one assignment to the propositional variables that makes the entire proposition true
- A tautology is true for every assignment to the variables
- For any proposition, the following is a tautology:

Example: p = “There was a blizzard on April 1, 1997 in Cambridge, MA.”

p	$\neg p$	$p \vee \neg p$
T	F	T
F	T	T

Satisfiability



- A compound proposition is satisfiable if and only if there exists at least one assignment to the propositional variables that makes the entire proposition true
- There exists compound propositions that are unsatisfiable.
- For any proposition, the following is unsatisfiable:

Example: p = "There was a blizzard on April 1, 1997 in Cambridge, MA."

p	$\neg p$	$p \wedge \neg p$
T	F	F
F	T	F

Consistency

- A set of compound propositions are consistent if there exists truth assignment such that all propositions are satisfied at the same time.
- First, we will show how two compound propositions using the atomic propositions, p and q, are consistent.
- Then we will introduce a third proposition to show when they are not longer consistent.

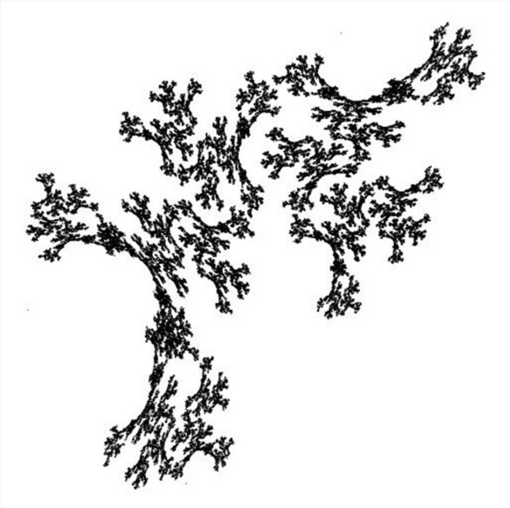
Example: p = "There was a heatwave in Los Angeles in July 2019. "
q = "There was a heatwave in London in July 2019."

p	q	$p \vee q$	$p \vee \neg q$	$\neg p$
T	T	T	T	F
T	F	T	T	F
F	T	T	F	T
F	F	F	T	T

workshop

Are these system specifications consistent?

- Whenever the **system software is being upgraded(p)**, users cannot **access the file system(~q)**. (p ➔ ~q)
- If users **can access the file system**(q), then they **can save new files (r)**. (q ➔ r)
- If users cannot save new files(~r), then the system software is not being upgraded(~p)." (~r ➔ ~p)



DISCRETE STRUCTURES
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1.B Applications of Propositions

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De Morgan's Law

TABLE 2 De Morgan's Laws.
$\neg(p \wedge q) \equiv \neg p \vee \neg q$
$\neg(p \vee q) \equiv \neg p \wedge \neg q$

Puzzle

There is an island with two kinds of inhabitants. Those who always tell the truth (knights) and those who always lie (knaves). You encounter two people A and B.

What are A and B, if A says, “B is a knight” and B says “two of us are opposite types”

Workshop

Brown, Jones and Smith are suspected of income tax evasions. They testify under oath as follows.

Brown : Jones is guilty(~J) and smith(S) is innocent

Jones : If Brown is guilty(~B), then so is smith

Smith : I am innocent but at least one of the others is guilty

Cases

- Assume everyone told the truth. Are the statements consistent?
- Assuming honest told the truth and others lied (we have to consider many cases). In this case, make sure you NEGATE whatever the statement that liars made

Solution – case all are telling the truth

Suppose everyone is telling the truth

1. B says: Jones is guilty and smith is innocent

2. J says: if S is innocent, then B is innocent ➔ Since Smith is innocent, therefore Brown is innocent

3. So now we have: S, B and ~J is true

4. This is also consistent with the statement : S ^ (~B v ~J) is TRUE

Case – not all telling the truth

1. Suppose we do not know who is telling the truth and who is lying
   1. There are 7 possibilities in this case for B J S and the statements they made must be consistent

      B J S

      1. *T T F -*
         1. **(~J ^S)** ^ (~B➔~S) ^ (~(S^(~B v ~J))) show this is not consistent (**bold** is false, therefore the whole thing)
         2. By plugging in B=T, J=T, S=F show that this whole statement is FALSE
      2. *T F T -*
         1. (~J ^S) ^ **~(S➔B)** ^ (S^(~B v ~J))) show this is not consistent (~(S➔B) means ~(~S V B) means S ^~B = F
         2. Plug in B=T, J=F, S=T
      3. *T F F -*
         1. **(~J ^S)** ^ ~(S➔B) ^ ~(S^(~B v ~J))) show this is not consistent
         2. Plug in B=T, J=F, S=F
      4. *F T T -*
         1. ~(~J ^S) ^ **(S➔B)** ^ (S^(~B v ~J))) show this is not consistent
         2. Plug in B=F, J=T, S=T

Solution continued…

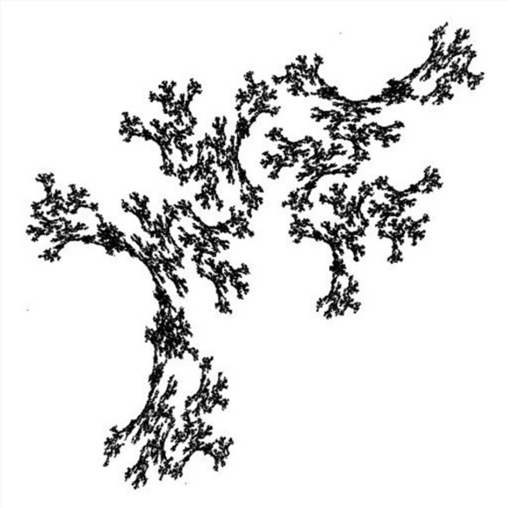
1. *F T F - Is this consistent with all statements? - **VERY VERY NICE ARGUMENT PROF Guna***
   1. *~(~J ^S) ^ (S➔B) ^ ~(S^(~B v ~J))) show this is not consistent*
   2. *Plug in B=F, J=T, S=F*

2. *F F T -*
   1. **~(~J ^S)** ^ ~(S➔B) ^ (S^(~B v ~J))) show this is not consistent
   2. Plug in B=F, J=F, S=T
3. *F F F -*
   1. ~(~J ^S) ^ ~(S➔B) ^ **(S^(~B v ~J)))** show this is not consistent
   2. Plug in B=F, J=F, S=F

Practice problem

A tourist come to a Y junction and the city may be to the left or to the right. There is a native person standing at the junction who knows the answer. But the person may be lying or telling the truth and they only answer with YES or NO.

What question can the tourist ask, so that if the answer is “yes’ he will go left and if the answer is no, then he will go right.



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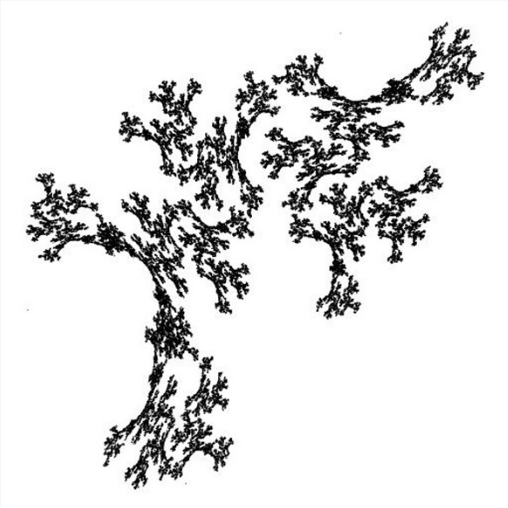

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