INTRODUCTION TO DISCRETE STRUCTURES

5.1 Introduction to sets

1.1–1.2

5.1 Introduction to sets

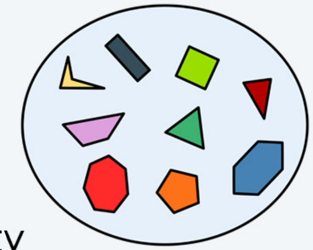
- Definitions
- Operations on sets
- Proofs involving sets

Definition

A *set* is an unordered collection of objects, called *elements* or *members* of the set. A set is said to *contain* its elements.

We write a ∈ A to denote that a is an element of the set A.

The notation a ∉ A denotes that a is not an element of the set A



Operations. on sets includes, intersection, union, difference, compliment, equality

Sets as mathematical objects

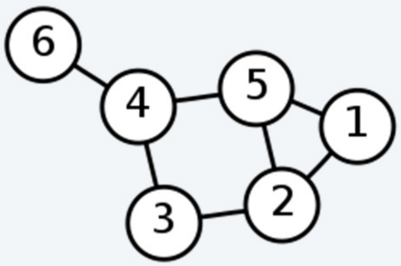
Much of discrete mathematics is devoted to the study of discrete structures, used to represent discrete objects.

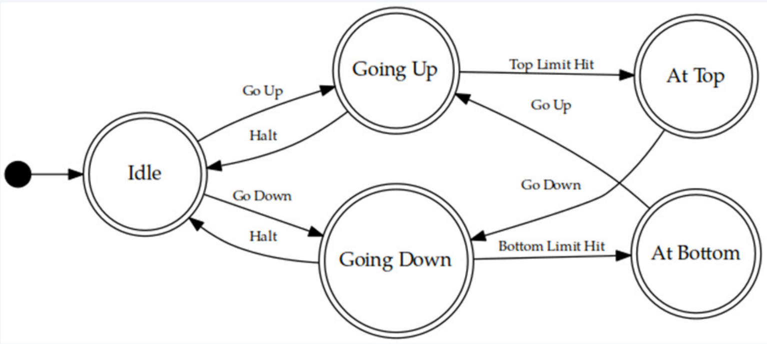
Many important discrete structures are built using sets, which are collections of objects.

Among the discrete structures built from sets are combinations, unordered collections of objects used extensively in counting;

Relations are sets of ordered pairs that represent relationships between objects; graphs, sets of vertices and edges that connect vertices;

Finite state machines, used to model computing machines are represented by a set of states.





Example of well-known sets

$\mathbf{N} = \{0, 1, 2, 3, \ldots\}$, the set of **natural numbers**
$\mathbf{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$, the set of **integers**
$\mathbf{Z}^+ = \{1, 2, 3, \ldots\}$, the set of **positive integers**
$\mathbf{Q} = \{p/q \mid p \in \mathbf{Z}, q \in \mathbf{Z}, \text{ and } q \neq 0\}$, the set of **rational numbers**
$\mathbf{R}$, the set of **real numbers**
$\mathbf{R}^+$, the set of **positive real numbers**
$\mathbf{C}$, the set of **complex numbers**.

Java set interface

```
public interface Set<E>
extends Collection<E>
```

A collection that contains no duplicate elements. More formally, sets contain no pair of elements `e1` and `e2` such that `e1.equals(e2)`, and at most one null element.

The `Set` interface places additional stipulations, beyond those inherited from the `Collection` interface, on the contracts of all constructors and on the contracts of th
accompanying these declarations have been tailored to the `Set` interface, but they do not contain any additional stipulations.)

The additional stipulation on constructors is, not surprisingly, that all constructors must create a set that contains no duplicate elements (as defined above).

Note: Great care must be exercised if mutable objects are used as set elements. The behavior of a set is not specified if the value of an object is changed in a mann
contain itself as an element.

Some set implementations have restrictions on the elements that they may contain. For example, some implementations prohibit null elements, and some have restr
`ClassCastException`. Attempting to query the presence of an ineligible element may throw an exception, or it may simply return false; some implementations will ex
result in the insertion of an ineligible element into the set may throw an exception or it may succeed, at the option of the implementation. Such exceptions are marke

This interface is a member of the Java Collections Framework.

Since:

1.2

See Also:

`Collection`, `List`, `SortedSet`, `HashSet`, `TreeSet`, `AbstractSet`, `Collections.singleton(java.lang.Object)`, `Collections.EMPTY_SET`

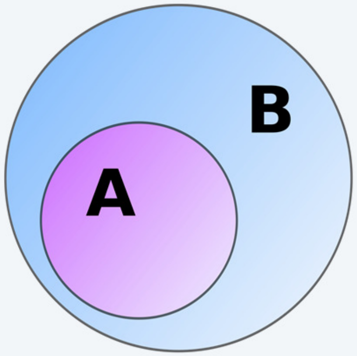
Source: Java API

data type in a programming language can be considered a set of values and operations on them

Sets can be represented in a computer using binary patterns

A subset of a set

A is a subset of B, if every element in A is also an element of B



Question. Given a set of n elements, how many subsets can be drawn from it?

Subset games

Subset take-away is a two-player game involving a fixed finite set, A. Players alternately choose nonempty subsets of A with the conditions that a player may not choose

- the whole set A, or
- any set containing a set that was named earlier.

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DISCRETE STRUCTURES

5.1 Introduction to sets

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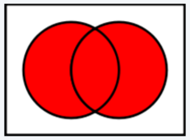
Set equality

Two sets A and B are equal, if they have the same elements. That is, for all x in A, implies x in B and vice versa.

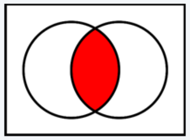
Question. Write an algorithm to test if two sets are equal. What is the big-O complexity of your algorithm?

Union, intersection, difference and compliment

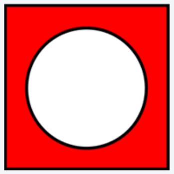
Let A and B be sets. The *union* of the sets A and B, denoted by $A \cup B$, is the set that contains those elements that are either in A or in B, or in both.



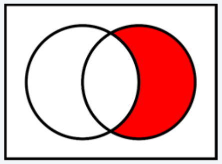
Let A and B be sets. The *intersection* of the sets A and B, denoted by $A \cap B$, is the set containing those elements in both A and B.



Let U be the universal set. The *complement* of the set A, denoted by $\overline{A}$, is the complement of A with respect to U. Therefore, the complement of the set A is $U - A$.



Let A and B be sets. The *difference* of A and B, denoted by $A - B$, is the set containing those elements that are in A but not in B. The difference of A and B is also called the *complement of B with respect to A*.



Question. How do we find union and intersection algorithmically?

The size or the cardinality of a set

Let S be a set. If there are exactly n distinct elements in S where n is a nonnegative integer, we say that S is a *finite set* and that n is the *cardinality* of S. The cardinality of S is denoted by $|S|$.

Examples.

|N| = infinite but countable
|Q| , |Z|, |Z+| ➔ infinite but countable

|R| = uncountable

Most sets we encounter in life are finite.

S = {set of all n-bit strings}

Power set of a set

Given a set S, the *power set* of S is the set of all subsets of the set S. The power set of S is denoted by $\mathcal{P}(S)$.

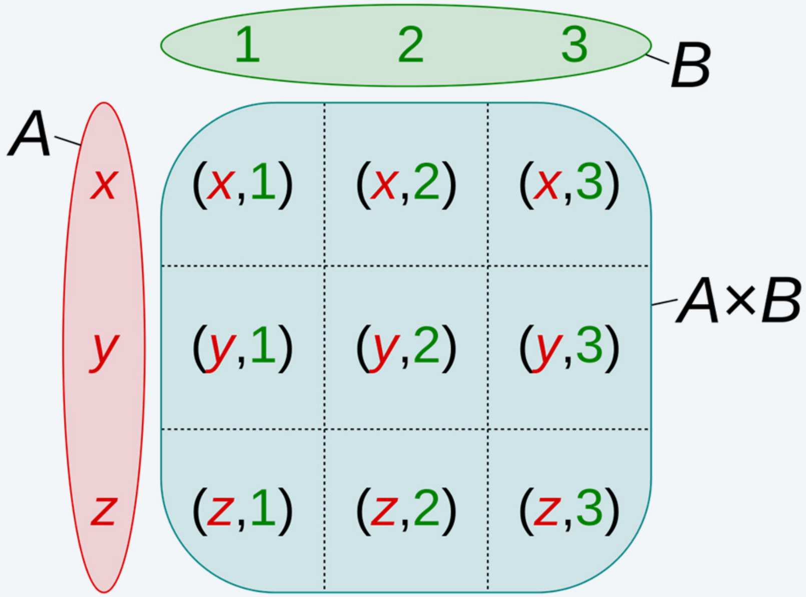
Question: What is |P(S)| in terms of |S| ?

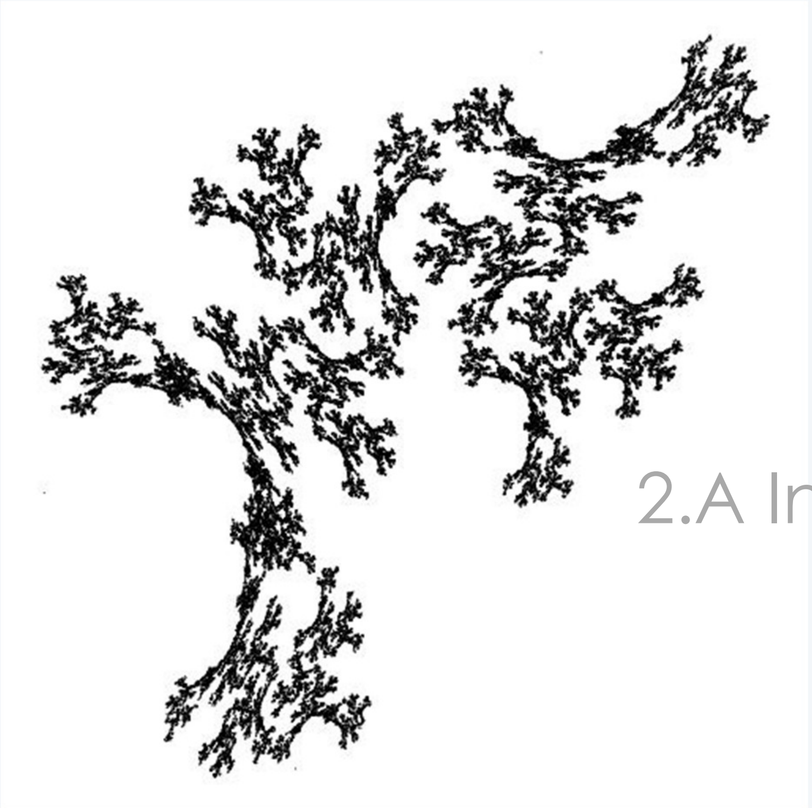
Question. How many n-bit strings are there?

Answer. |power(n)| = 2^n

Cartesian product of two sets

Cartesian product of two sets A and B, denoted by A X B is the set of all pairs of the form (a , b) where a is an element in A and b is an element in B.





INTRODUCTION TO
DISCRETE STRUCTURES

2.A Introduction to sets

- Definitions
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Set identities

Identity	*Name*
$A \cap U = A$ $A \cup \emptyset = A$	Identity laws
$A \cup U = U$ $A \cap \emptyset = \emptyset$	Domination laws
$A \cup A = A$ $A \cap A = A$	Idempotent laws
$\overline{(\overline{A})} = A$	Complementation law
$A \cup B = B \cup A$ $A \cap B = B \cap A$	Commutative laws
$A \cup (B \cup C) = (A \cup B) \cup C$ $A \cap (B \cap C) = (A \cap B) \cap C$	Associative laws
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	Distributive laws
$\overline{A \cap B} = \overline{A} \cup \overline{B}$ $\overline{A \cup B} = \overline{A} \cap \overline{B}$	De Morgan's laws
$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$	Absorption laws

workshop

Question 1. Show that $(A \cap B) \cup A = A$ (do not use Venn diagrams)

Hint: to show two sets are equal, you need to show set1 is a subset of set2 and vice versa.

Usually you accomplish this by taking an element x in set1 and then show by inference x is also in set2. You also need to show the opposite direction.

Question 2. Show that if A and B are distinct sets and A and B have common elements, then $A - (A \cap B) \neq A$

Truth set of a predicate

Given a predicate P(x) : $0 < x^2 < 100$, and domain Z (set of integers), find the truth set of P(x).
The truth set of a predicate is the set of all integers such that P(x) is true.

workshop

Given sets A, B and C, prove the following

1. $A - B = A \cap \overline{B}$
2. $(A - B) - C \subseteq A - C$
3. $(A - C) \cap (C - B) = \phi$

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DISCRETE STRUCTURES

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1.1–1.2