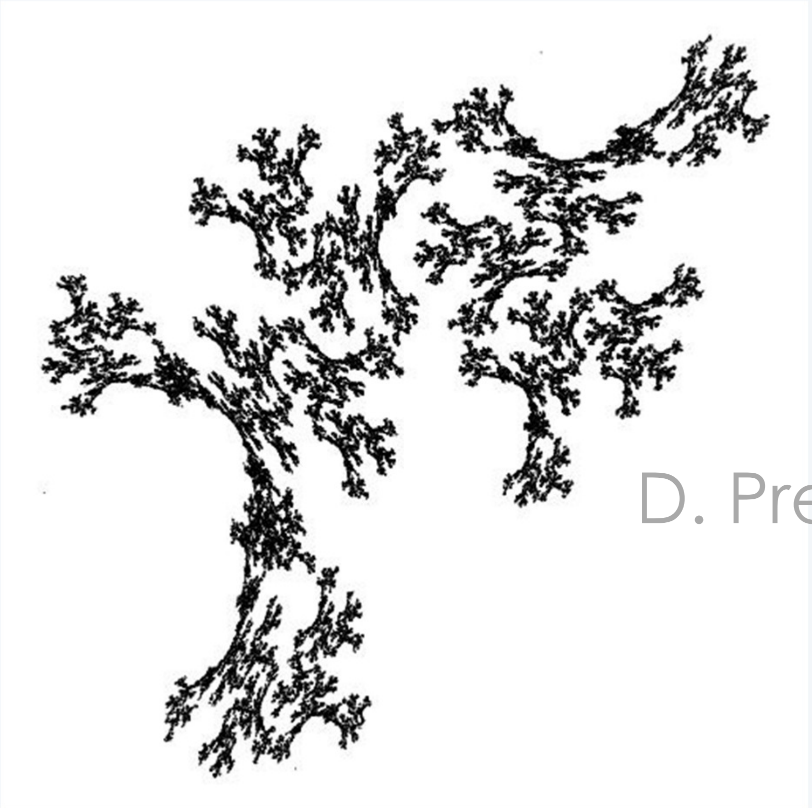
INTRODUCTION TO
DISCRETE STRUCTURES

D. Predicates and Quantifiers

1.1–1.2



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meaning of p ➔ q

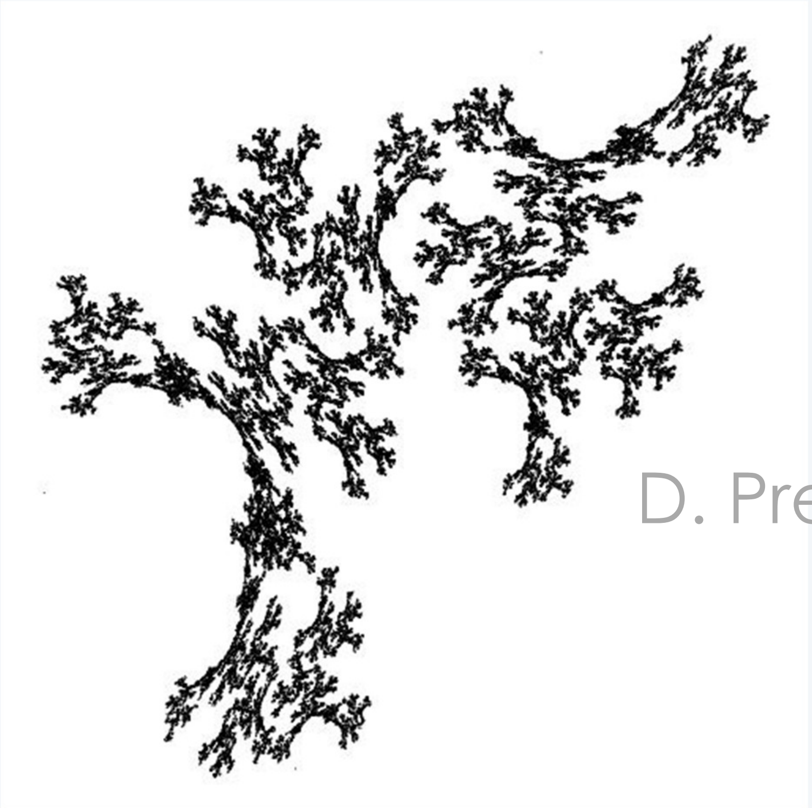
"if p, then q"	"p implies q"
"if p, q"	"p only if q"
"p is sufficient for q"	"a sufficient condition for q is p"
"q if p"	"q whenever p"
"q when p"	"q is necessary for p"
"a necessary condition for p is q"	"q follows from p"
"q unless $\neg p$"	

workshop

1. Show that $p \wedge (\neg p \vee p)$ is equivalent to p

2. Show that $p \Rightarrow \neg p$ is a contingency

3. Show that $p \Rightarrow \neg q$ is logically equivalent to $q \Rightarrow \neg p$



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Definition of a Predicate

A predicate is a proposition that depends on some state of its propositional variables.

Example.

Every computer connected to the university network is functioning properly

is expressed as

If n is a computer AND n is connected to university network, then n is functioning properly

Notation

P(x) is a predicate that involves the propositional variable x

P(x,y) is a predicate that involves the propositional variables x and y

In general, $P(x_1, x_2, ..., x_n)$ is a predicate that involves the propositional variables $x_1, x_2, ..., x_n$

workshop

1. Let $P(x)$ denote the statement "$x \leq 4$." What are these truth values?

a) $P(0)$ **b)** $P(4)$ **c)** $P(6)$

2. Let $P(x)$ be the statement "the word x contains the letter a." What are these truth values?

a) P(orange) **b)** P(lemon)
c) P(true) **d)** P(false)

3. Let $Q(x, y)$ denote the statement "x is the capital of y." What are these truth values?

a) Q(Denver, Colorado)
b) Q(Detroit, Michigan)
c) Q(Massachusetts, Boston)
d) Q(New York, New York)

4. State the value of x after the statement **if** $P(x)$ **then** $x := 1$ is executed, where $P(x)$ is the statement "$x > 1$," if the value of x when this statement is reached is

a) $x = 0$. **b)** $x = 1$.
c) $x = 2$.

Useful domains when using predicates

N – The set of natural numbers {0,1,…}

Z – The set of all integers {…, -3, -2, -1, 0, 1, 2, 3, …}

Q – The set of all rational numbers {m/n where m and n are integers}

R – The set of all real numbers

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Precondition and Postcondition

Precondition. A condition that is assumed to be true at the beginning of a program block

Postcondition. A condition that is expected to be true at the end of a program block.

Example.

***PRE:* *assert*($x = a$, $y = b$)**

$x = x + y$

$y = y - x$

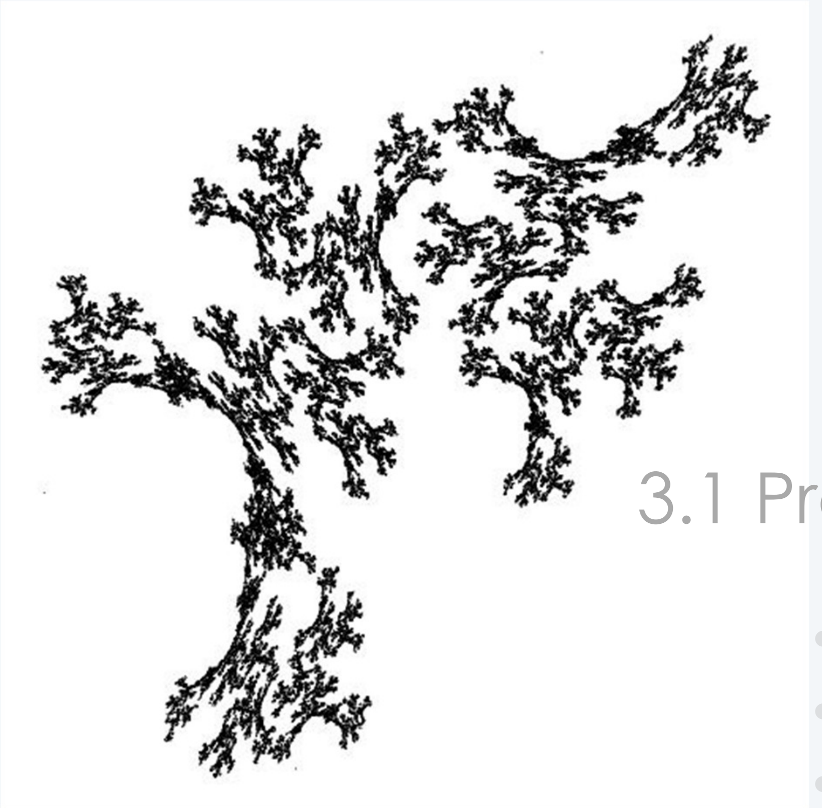
$x = x - y$

***POST:* *assert*($x = ?$, $y = ?$)**

workshop

What are pre and post conditions for the following code?

```
int f(int x, int y) {
  int r = 1;
  while (y > 1) {
    if (y % 2 == 1) {
      r = x * r;
    }
    x = x * x;
    y = y / 2;
  }
  return r * x;
}
```



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quantifiers

Quantifiers. Expresses the truth of a propositional statement over a domain.

There are two kinds of quantifiers.

Universal. expresses the truth of a predicate over an entire domain.

A proposition P(x) is true for all x in X

Existential. Expresses the truth of a predicate for at least one instance.

There exist some x, such that P(x) is true.

workshop

Determine the truth value of each of these statements if the domain consists of all integers.

a) $\forall n(n+1>n)$

b) $\exists n(2n=3n)$

c) $\exists n(n=-n)$

d) $\forall n(3n \leq 4n)$

workshop

For each of these statements find a domain for which the statement is true and a domain for which the statement is false.

a) Everyone speaks Spanish.

b) There is someone older than 21 years.

c) Every two people have the same first name.

d) Someone knows more than two other people.

a. DomainT = all people in US, domainF = all people in a Mexican restaurant
b. domain = all college students, domain = all elementary students
c. Domain T = father and son , domain – general population
d. Domain T = college students, domain F = hermit

workshop

[highlight here to select this problem] Let $P(x)$, $Q(x)$, $R(x)$, and $S(x)$ be the statements "x is a baby," "x is logical," "x is able to manage a crocodile," and "x is despised," respectively. Suppose that the domain consists of all people. Express each of these statements using quantifiers; logical connectives; and $P(x)$, $Q(x)$, $R(x)$, and $S(x)$.

a) Babies are illogical.
b) Nobody is despised who can manage a crocodile.
c) Illogical persons are despised.
d) Babies cannot manage crocodiles.

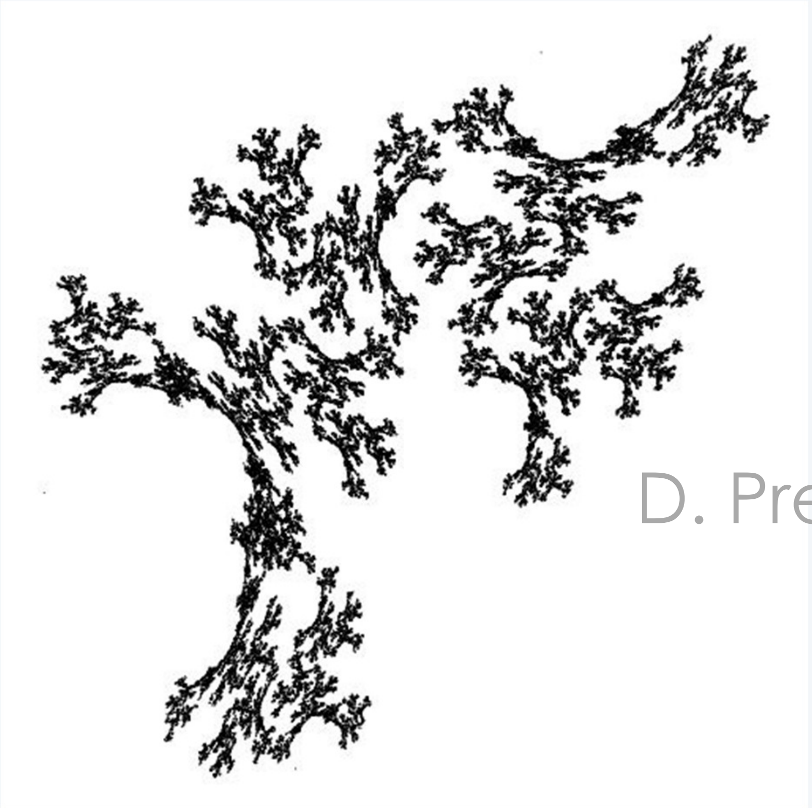
Precedence of quantifiers

The quantifiers $\forall$ and $\exists$ have higher precedence than all logical operators from propositional calculus. For example, $\forall x P(x) \vee Q(x)$ is the disjunction of $\forall x P(x)$ and $Q(x)$. In other words, it means $(\forall x P(x)) \vee Q(x)$ rather than $\forall x (P(x) \vee Q(x))$.

Logical equivalences of quantifiers

Statements involving predicates and quantifiers are *logically equivalent* if and only if they have the same truth value no matter which predicates are substituted into these statements and which domain of discourse is used for the variables in these propositional functions. We use the notation $S \equiv T$ to indicate that two statements S and T involving predicates and quantifiers are logically equivalent.

Example. P(x) : x is even for all x
Q(x) : x is divisible by 2 for all x
R(x) : x = 2y for some y



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Negating quantifiers

Negating universal. Leads to a statement of existential

not (all humans are mammals) is equivalent to there is at least one human who is not mammal

Negating existential. Leads to a statement of universal

it is not the case (at least one human is not mammal) is equivalent all humans are mammals

$$\neg \forall x\, P(x) \equiv \exists x\, \neg P(x).$$

De Morgan’s Law and quantifiers

TABLE 2 **De Morgan’s Laws for Quantifiers.**

Negation	*Equivalent Statement*	*When Is Negation True?*	*When False?*
$\neg \exists x P(x)$	$\forall x \neg P(x)$	For every x, $P(x)$ is false.	There is an x for which $P(x)$ is true.
$\neg \forall x P(x)$	$\exists x \neg P(x)$	There is an x for which $P(x)$ is false.	$P(x)$ is true for every x.

workshop

Express the negation of these propositions using quantifiers, and then express the negation in English.

a) Some drivers do not obey the speed limit.

b) All Swedish movies are serious.

c) No one can keep a secret.

d) There is someone in this class who does not have a good attitude.

Let $Q(x)$ be the statement "$x + 1 > 2x$." If the domain consists of all integers, what are these truth values?

a) $Q(0)$ **b)** $Q(-1)$ **c)** $Q(1)$

d) $\exists x Q(x)$ **e)** $\forall x Q(x)$ **f)** $\exists x \neg Q(x)$

g) $\forall x \neg Q(x)$

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