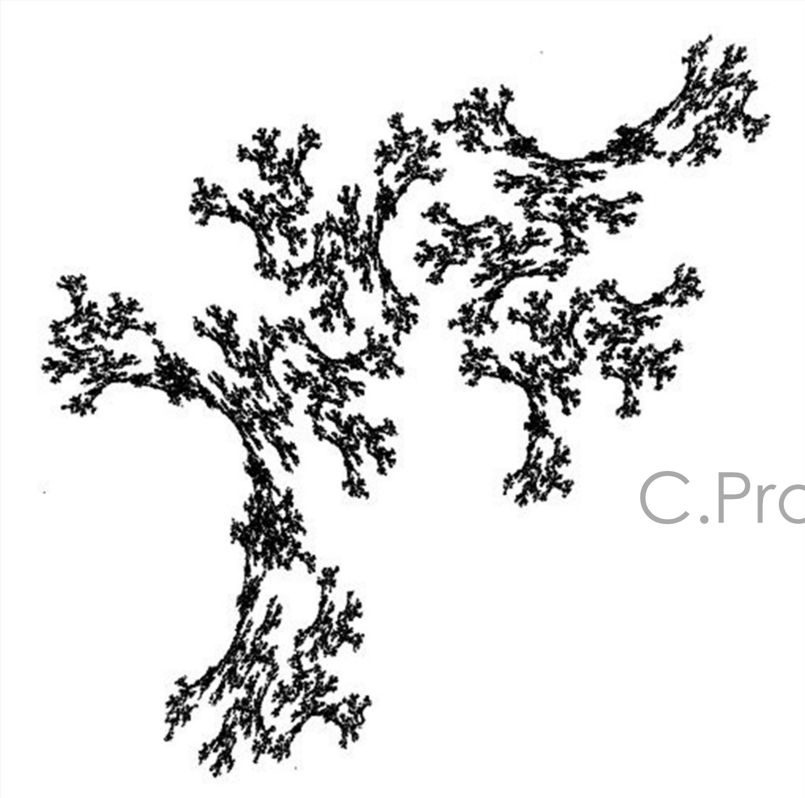
INTRODUCTION TO
DISCRETE STRUCTURES

C. Propositional Equivalence

1.1–1.2



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C.Propositional Equivalence

- Tautologies and contradictions
- Logical equivalences
- De Morgan's Law
- Satisfiability
- Direct proofs

Tautologies and contradictions

Tautology. A compound proposition that is always true despite the values of variables.

Contradiction. A compound proposition that is always false.

contingency. A compound proposition that neither a tautology or contradiction.

Precedence rules of logical operators

Operator precedence is an ordering of logical operators designed to allow dropping of the parenthesis in logical expressions.

$\neg$	← higher precedence
$\wedge$	
$\vee$	
$\Rightarrow \; \Leftrightarrow$	← lower precedence

un-parenthesized statements. associate the expression with the one with higher precedence.

$\neg p \wedge q$	$((\neg p) \wedge q)$
$p \wedge \neg q$	$(p \wedge (\neg q))$
$p \wedge q \vee r$	$((p \wedge q) \vee r)$
$p \vee q \wedge r$	$(p \vee (q \wedge r)$
$p \Rightarrow q \Rightarrow r$	$(p \Rightarrow (q \Rightarrow r))$
$p \Rightarrow q \Leftrightarrow r$	$(p \Rightarrow (q \Leftrightarrow r))$

Workshop

Identify each of the following as tautology, contradiction or contingency

1. $p \wedge q \wedge \neg p$
2. $(p \wedge q) \vee \neg p$
3. $p \rightarrow \neg p$
4. $p \vee q \rightarrow q$
5. $(p \rightarrow q) \vee (q \rightarrow p)$

Classwork

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2.2 Propositional Equivalence

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Logical Equivalence

Two compound propositions p and q are logically equivalent if p $\leftrightarrow$ q is a tautology.

Example: p $\rightarrow$ q and $\neg$p v q are logically equivalent (to be proved)

Notation. If p and q are logically equivalent, then we say p $\equiv$ q

workshop

Example. Show that p ➔ q and ¬p v q are logically equivalent

Example. Find a logically equivalent implication to p v ¬q

workshop

Show that $p \vee \neg p \Rightarrow q$ and q are logically equivalent

Useful Logical Equivalences

TABLE 6 **Logical Equivalences.**

Equivalence	*Name*
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws

TABLE 7 **Logical Equivalences Involving Conditional Statements.**

$p \rightarrow q \equiv \neg p \vee q$

$p \rightarrow q \equiv \neg q \rightarrow \neg p$

$p \vee q \equiv \neg p \rightarrow q$

$p \wedge q \equiv \neg(p \rightarrow \neg q)$

$\neg(p \rightarrow q) \equiv p \wedge \neg q$

$(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$

$(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r$

$(p \rightarrow q) \vee (p \rightarrow r) \equiv p \rightarrow (q \vee r)$

$(p \rightarrow r) \vee (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$

TABLE 8 **Logical Equivalences Involving Biconditional Statements.**

$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$

$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$

$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

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De Morgan's Law

TABLE 2 **De Morgan's Laws.**
$\neg(p \wedge q) \equiv \neg p \vee \neg q$
$\neg(p \vee q) \equiv \neg p \wedge \neg q$

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Satisfiability

A compound proposition is **satisfiable** if and only if **there exists at least one assignment** to the propositional variables that makes the entire proposition true.

Example. p v ¬p is satisfiable

Example. (p ➔ q) v (q ➔ p) is satisfiable

Example. p ➔ ¬p

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Direct Proofs

A method of proof where result is obtained by applying known equivalences

Example 1. Show that $\neg(p \rightarrow q)$ and $p \wedge \neg q$ are logically equivalent.

Example 2. Show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology.

workshop

Use De Morgan's laws to find the negation of each of the following statements.

a) Kwame will take a job in industry or go to graduate school.
b) Yoshiko knows Java and calculus.
c) James is young and strong.
d) Rita will move to Oregon or Washington.

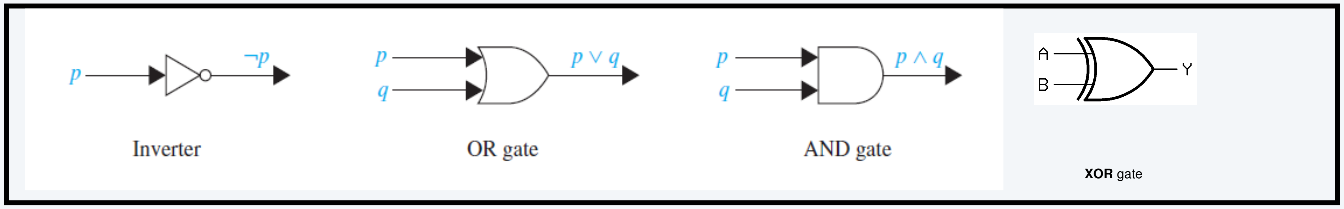
Problem #1

Find a compound proposition involving the propositional variables p, q, and r that is true when p and q are true and r is false, but is false otherwise. [*Hint:* Use a conjunction of each propositional variable or its negation.]

Problem #2

Logic gates

Logic gates. Is a big part of computer hardware design



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