Quantitative Decision Making: Mixed integer linear Programming for Supply Chain Network Design 2

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Agenda

1. Scenarion Planning
2. SCDN with Spreadsheets II

Baseline Model

> *A baseline is a model of the existing network.*

The baseline model is used as…

Validation of the model and measurement of fit to reality

"Actual Baseline"

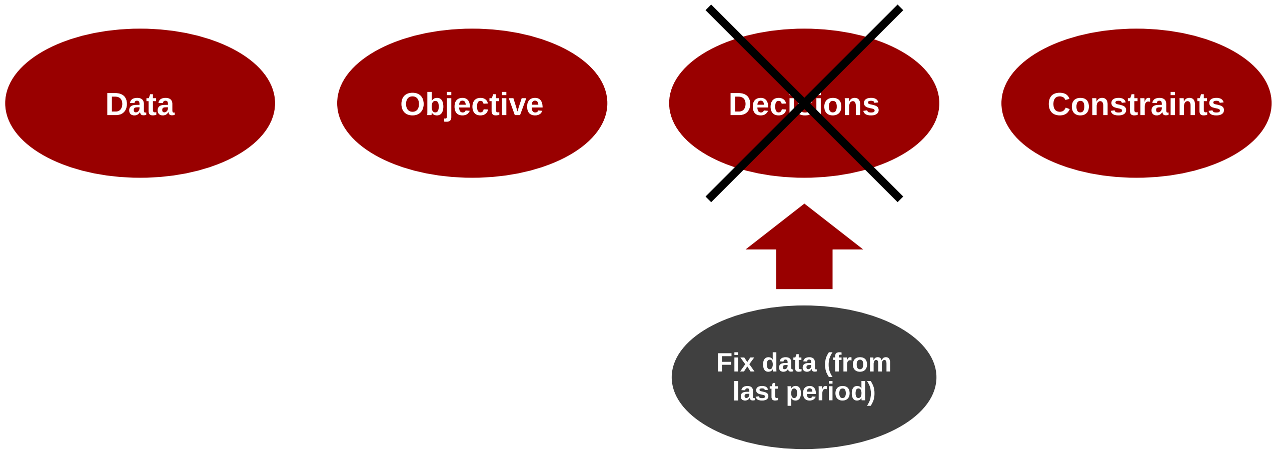
a comparison for a newly designed network

"Optimized Baseline"

Source: Watson et al. (2012)

Actual Baseline I/II

- The actual baseline is a representation of the supply chain exactly as it was run in the past
- It is supposed to reflect the business that is modeled
  - It is exactly the same model as the optimization model, but without „choices“ for the model

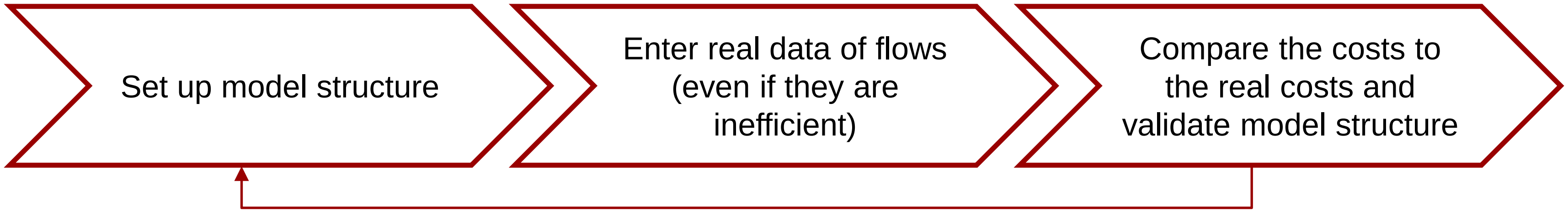


The purpose of the actual baseline model is to validate the optimization model. Thus, it is important to set the model for optimizations run up exactly as it is crafted for the actual baseline.

Source: Watson et al. (2012)

Actual Baseline II/II

Set up:



The model should come within 1 to 10% to the actual costs. The threshold depends on case and company.

- While the deviation between actual cost and model costs are a indicator for the models quality, a close gap does not necessarily mean, that the model is perfect.
- Having a 10% gap may result from
  - data issues and assumptions filling the gap
  - The ever changing character of the supply chain

Source: Watson et al. (2012)

Optimized Baseline I/II

- The optimized baseline replicates what should happen in the existing network based on the rules that are in place in the modeled business
- The definition of the optimized baseline is more flexible -> use several versions
- Optimize the model under the consideration of
  - Use all existing facilities
  - Use existing assignments of facilities <- This can be relaxed to create versions

First signs of needed improvement

- Large deviations of costs between the actual and the optimized baseline signal the need for improvement.
- To achieve the improvement, understanding about the behavior of the actual chain is needed
- Reorganizing the supply chain may already satisfy the need for improvement and revoke the need for redesign of the supply chain

Source: Watson et al. (2012)

Optimized Baseline II/II

Purpose of the model

Validation of model after optimization:

- Do results make sense?
- Do products flow as planned?
- Are costs reasonable?
- Are capacities respected?

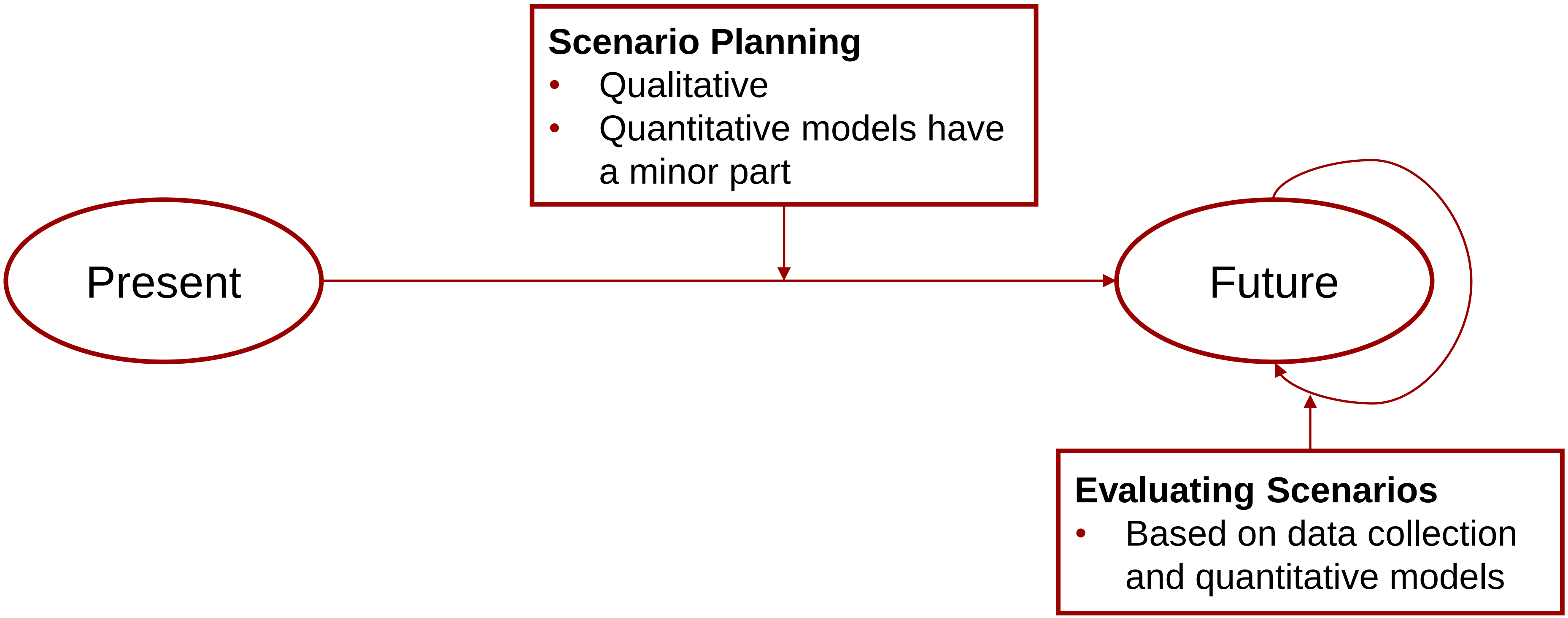
Benchmark for the optimization run with redesign elements of the network.

- The actual baseline may have high costs based on poor execution of the plan but not a poor plan.
- The optimized baseline should represent the plan and not the execution.
- Comparing optimization scenarios to the optimized version of the baseline represents a fair comparison

Source: Watson et al. (2012)

Scenario Planning

- Scenario planning is a process to define scenarios of their firm's long-term future that are consistent, plausible, and comprehensive.
- Goal: organize, enhance, and demystify individual and collective views of the future.



Source: Shapiro (2007)

Methodology of Scenario Planning I

> *Scenarios are focused descriptions of fundamentally different futures presented in coherent narratives.*

aims at overcoming human and organizational barriers to consistent and realistic assessment of the long-term future

- Focused: the scope is limited
- Fundamentally different: its not about predicting the future but bounding it and encourage consensus building of different expectations
- Coherent: all scenarios should cover the same issues

Source: Shapiro (2007)

Methodology of Scenario Planning II

1

Define the scope and strategic issues to be analyzed
Scope refers to the time frame for the strategic analysis, the products, suppliers, markets, geographic areas, technologies.

2

Identify the major stakeholders
Stakeholders are individuals or organizations with an interest in the strategic issues being addressed, including those who may be affected by strategic decisions and those who can influence them.

3

Identify current trends
This step involves the merging of perceptions of industry experts, managers, and knowledgeable outsiders. It is important to evaluate whether trends are mutually compatible during the time frame of analysis.

4

Identify key uncertainties
Uncertainties associated with the outcomes of key events must be identified, but at this point measurement of probabilities are not yet needed. Attention must also be paid to correlations among the uncertainties

5

Construct extreme scenarios
The initial step is to construct two, or a small number, of extreme scenarios. An example is one scenario of positive elements and the other of negative elements

Source: Shapiro (2007)

Methodology of Scenario Planning III

6

Assess internal consistency and plausibility of extreme scenarios
The extreme scenarios will very likely be internally inconsistent. (1) trends defining extreme scenarios may be incompatible. (2) outcomes of Uncertain events may be inconsistent. (3) stakeholder inconsistency will result if the major actors are placed in positions they dislike and can change.

7

Create representative scenarios
Using insights from step 6, create a variety of consistent scenarios that bracket a wide range of outcomes. Adjust scenarios to realistically reflect stakeholder behavior.

8

Identify research needs
Considering the scenarios identified in step 7, identify topics requiring further study to sharpen their definition and analysis

9

Develop and apply quantitative models
These are descriptive and normative models for forecasting and optimizing decisions associated with the uncertain future. This step may lead to further scenario definition and refinement.

10

Develop decision scenarios
Use the results of step 9 combined with managerial judgment to describe decisions for different scenarios, keeping in mind that they may suggest alternate future equilibria for the company and its industry

Source: Shapiro (2007)

Contingency Planning

> *Contingency planning can evaluate the impact of scenarios describing low-probability events that could have a significant impact on a company's supply chain performance*

The companies agility in responding to such events should be considered in the network design

- A network design model can be used to test the reaction of the network to such events
- It may also help to identify the most effective recovery plan

!

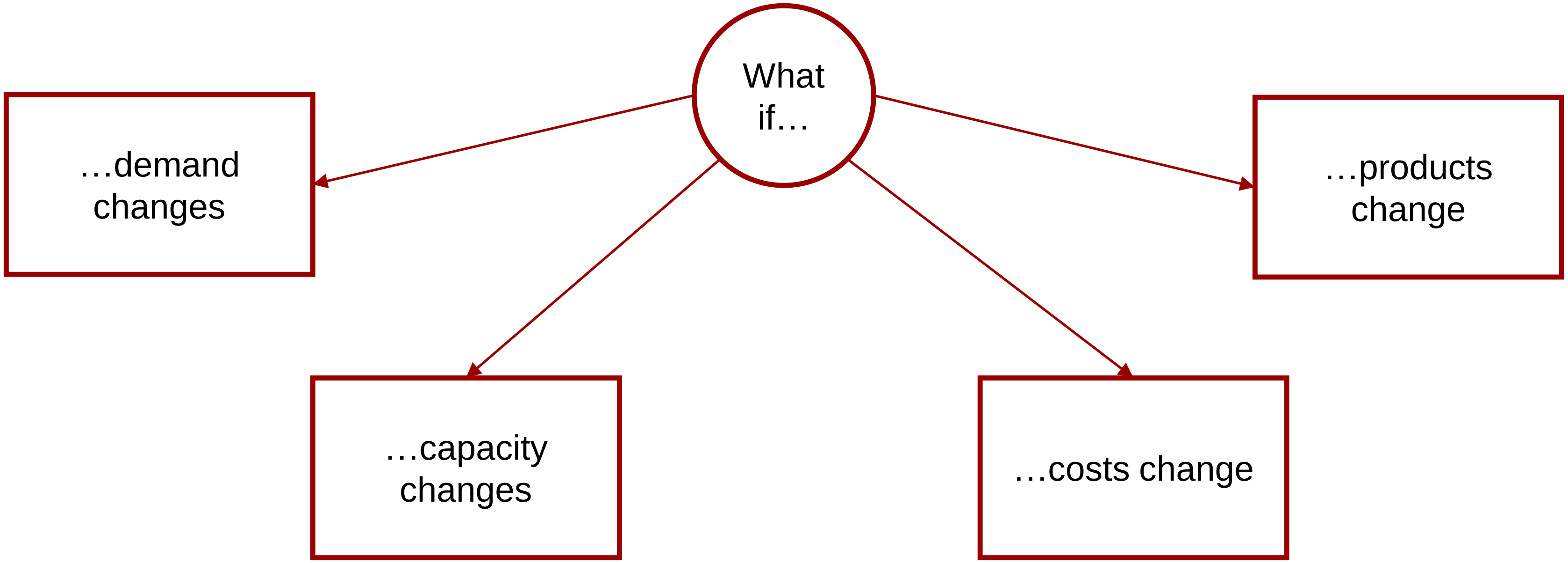
In case of a network design study:
If the network is designing according to one trend but another trend is occurring, the occurring trend could be considered as contingency.

Source: Shapiro (2007)

Lots of Scenarios…

- The impact of the decision to be made with the help of the model might be financially significant
- The main effort is setting up the model. Changing data for potential other scenarios is minor effort

…so run lots of scenarios!



Source: Watson et al. (2012)

Final Thoughts

> *Finally, although there is a desire for the optimization run to give us a single correct answer, the world is more complicated than that. In practice, when you're making an important strategic decision, it is more important to run multiple scenarios to understand the trade-offs, and understand the marginal value of adding facilities. This information can then be coupled with strategic business objectives to make a final decision.*

- The Models are based on assumptions for several different uncertain aspects (transportation costs, labor costs, demand). Thus, it should be tested whether the solution of an optimization is robust to deviations from the assumptions.
- Models fulfill the task to assess the implications of business decisions
- They do not necessarily predetermine decisions

Source: Watson et al. (2012)

Summary: Sceanrio Planning

- Scenarios are focused descriptions of fundamentally different futures presented in coherent narratives
  - But they don’t have to be the perfect representation of the future
- It‘s not about evaluating the most anticipated or „hoped for“ future. It‘s about challenging that anticipation and evaluate what could happed otherwise

Mixed-Integer (Linear) Programming

> Mixed-Integrer programming (MIP) / Mixed-Integer linear programming (MILP) are generalizations of linear programming, in which some variables are constrained to take nonnegative integer values

- **Integer variables**: any nonnegative integer value
- **Binary variables**: variables are constrained to take 0 or 1
- The remaining Variables can take any value and are called **continuous variables**

- Used to form:
  - Cost relationships
  - Logical conditions
  - Dependent constraints
- Optimization of Supply Chains related tasks:
  - Scheduling, routing or fixed costs

Source: Shapiro (2007)

Examples of Integer Variables in SCND

- Open or closing a facility / Operating or not operating a facility
  - Binary decision:
    - If facility is open/operating, it will create costs but will increase capacity or provide further options of routes
    - If closed, no costs should be generated. But capacity and routes are not available

- Piecewise linear cost function
  - Several binary decisions for the same facility or transportation lane
    - If the amount of flow exceeds or deceeds a threshold a tariff, a new tarif is entered with different costs and different thresholds

Source: Watson et al. (2012); Shapiro (2007), Ghiani (2005)

Opening / Operating a facility

- The opening of a facility is in itself a binary decision – a facility cannot be half open (Also used to assign operating costs for servicing a product in a facility)
- It will affect changes in the network
  - Routes with individual costs are created or closed
  - Capacity will be added to the network or removed
  - Costs for opening or closing occur, costs for running the facility are added or removed

Opening / Operating or not

$$y_i \in \{0,1\}$$

Decision variable indicating open or closed

objective

$$\min z = \sum_i \sum_j x_{ij} * c_{ij} + \sum_i y_i * F_i$$

May F be the fix costs of opening

Dependent capacity

$$x_{ij} \leq y_i * M$$

M indicates a “big number” in unconstrained optimization (for computational reasons, a value slightly higher than maximum demand should be used). In constrained optimization, the value is usually predetermined.

If y=0 -> x must be 0 / If y=1 -> x can be as high as M

Nomenclature

Values

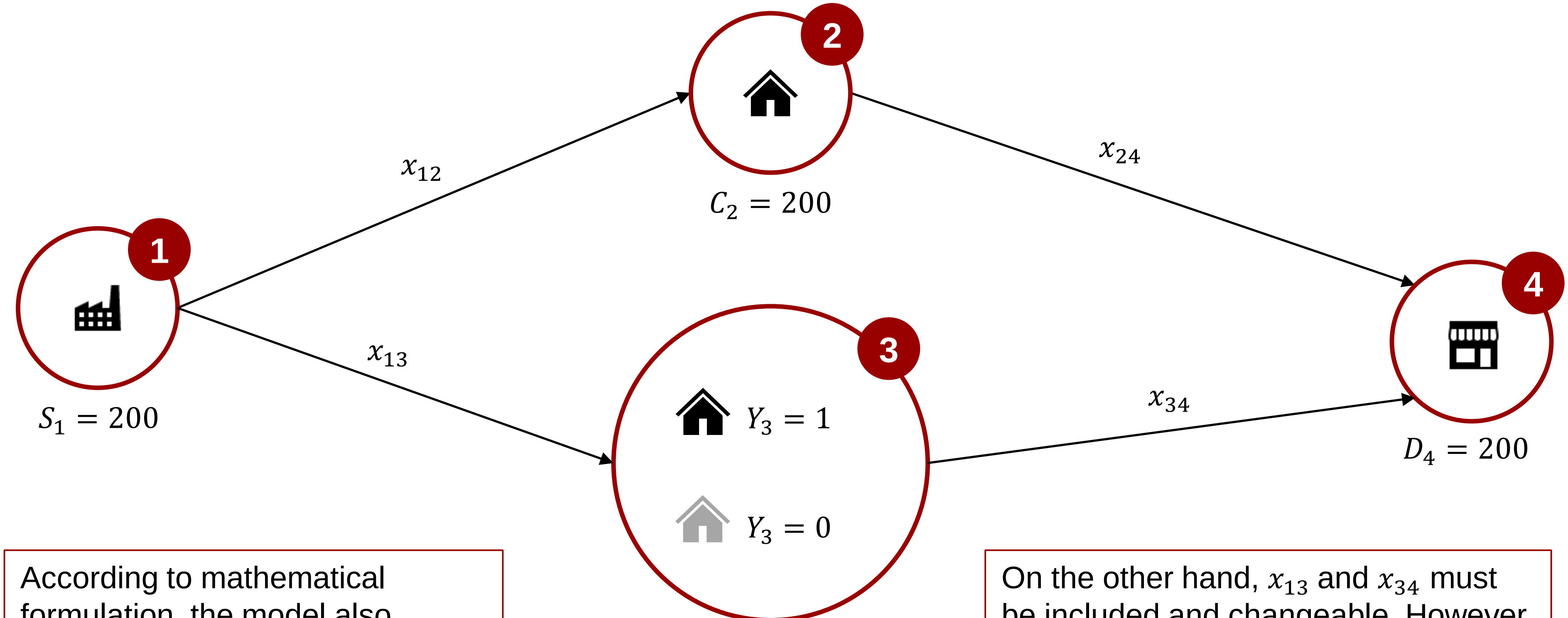
D – Demand
S – Supply / Capacity
c – cost
x – flow of goods
Y – open/closed
H – Amount of goods held
F – Fix costs of opening / operating
C – Capacity of a facility

Indices

j – Receiver (e.g. Customer)
i – Sender (e.g. plants)
k – hub (e.g. distribution center)
t – time period

Source: Watson et al. (2012); Shapiro (2007)

Opening / Operating a facility - Visualized



According to mathematical formulation, the model also contains S_4, D_1 or x_{41}. If the model is specifically listed, however, these can either be omitted or set to 0.

$$C_3 = \begin{cases} 0 & if\ Y_3 = 0 \\ 200 & if\ Y_3 = 1 \end{cases}$$

On the other hand, x_{13} and x_{34} must be included and changeable. However, since these edges cannot be used if location 3 does not exist (i.e. $Y_3 = 0$), x_{13} and x_{34} must be connected to Y_3 via an condition.

Opening / Operating a facility - Example Model

$$\min z = \sum_i \sum_j x_{ij} * c_{ij} + \sum_i y_i * F_i$$

Minimize costs including transportation (c) times the flow on a lane (x) and fix costs (F), if the facility is open (y)

s.t.

$$\sum_i x_{ij} \geq D_j \qquad \forall j \in J$$

All demand (D) has to be satisfied

$$\sum_j x_{ij} \leq S_i \qquad \forall i \in I$$

The supply (S) of a flow (x) of a facility i to any j is limited

$$\sum_i x_{ij} = \sum_i x_{ji} \qquad \forall j$$

For all facilities j holds: everything that goes in facility i, has to go out again.

$$\sum_i x_{ij} \leq y_j * C_j \qquad \forall j$$

If plant i is open, flow can be less or equal than M, less or equal zero otherwise. Here, M would be as set as high as S_i for the plants i with an opening/operating decision (y_i).

$$x_{ij} \geq 0 \qquad \forall i, j$$

All flows have to be greather or equal to zero

$$y_i \in \{0,1\}$$

Openining/operating decisions are binary

Nomenclature

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D – Demand
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j – Receiver (e.g. Customer)
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k – hub (e.g. distribution center)
t – time period

Problem 7-1

BEAR Home electronics manufactures and sells washing machines to customers in Barronvale (B1), Black Valley (B2) and Bridgeport (B3). The production facilities in East City (1) and Carlton (2) currently supply customers (see capacity in Table 1, current supply connections in Table 2 and costs in Table 3). ("**Network 1**")

To plan for the future, BEAR is considering opening in the following 3 locations: Sunset Beach (S1), Scottsville (S2), and/or Valleyside (S3). (30,000 capacity and 25,000 additional operating costs each)

a) Calculate the cost of the actual baseline.

b) Calculate the minimum cost of the Optimized Baseline.

c) Create a scenario 1 that has the new decision options described above (S1, S2, S3) ("**Network 2**") and make an appropriate cost comparison. Calculate the costs in the actual baseline.

Problem 7-1

	id	C
East City	1	45.000
Carlton	2	28.500

Table 1: Capacity

From \ To	id	B1	B2	B3
East City	1	6930	16940	15440
Carlton	2	7810	10890	4550

Table 2: Current flow

From \ To	id	B1	B2	B3
East City	1	1,61	2,12	3,58
Carlton	2	3,77	2,89	2,65

Table 3: Transportation Costs

From \ To	id	B1	B2	B3
Sunset Beach	S1	1,52	1,22	0,96
Scottsville	S2	1,76	3,39	1,73
Valleyside	S3	1,96	1,37	0,90

Table 4: Transportation costs of new Plants

Problem 7-1 –the actual baseline

Objective																					
min z	175.318,60																				
Data																					
transportation Costs																					
	1,61	2,12	3,58	3,77	2,89	2,65	1,52	1,22	0,96	1,76	3,39	1,73	1,96	1,37	0,9						
Operating Costs																					
																25.000	25.000	25.000			
Decision Variables																					
	x1_b1	x1_b2	x1_b3	x2_b1	x2_b2	x2_b3	xs1_b1	xs1_b2	xs1_b3	xs2_b1	xs2_b2	xs2_b3	xs3_b1	xs3_b2	xs3_b3	Ys1	Ys2	Ys3			
	6930	16940	15440	7810	10890	4550	0	0	0	0	0	0	0	0	0	0	0	0			
Constraints																					
Capacity (Plants)																					
	1	1	1																39310	<=	45.000
				1	1	1													23250	<=	28.500
							1	1	1							-30000			0	<=	0
										1	1	1					-30000		0	<=	0
													1	1	1			-30000	0	<=	0
Demand																					
	1			1			1			1			1						14740	>=	14.740
		1			1			1			1			1					27830	>=	27.830
			1			1			1			1			1				19990	>=	19.990

The resulting costs of entering the existing data into the model

Decision variables exist, but are not used

The actual demand provides demand constraints for the model on the current situation

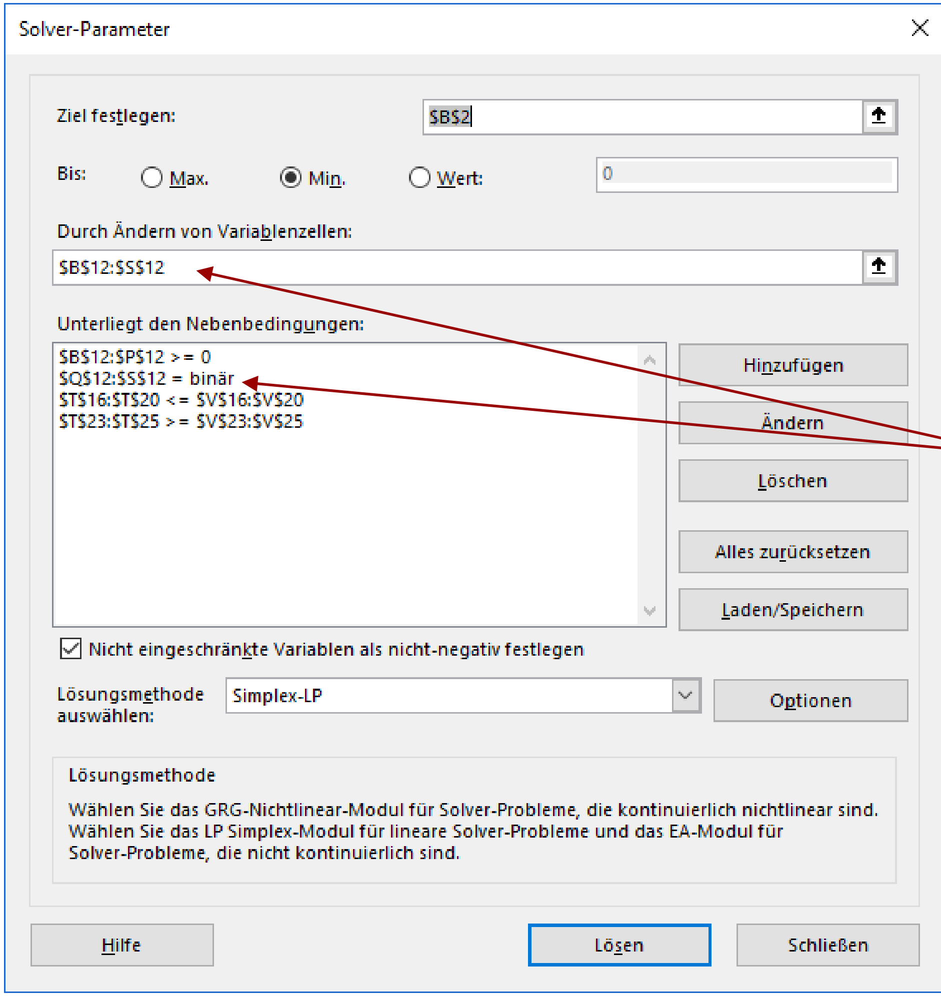
Problem 7-1 –the optimized baseline

Objective																					
min z	135.704,50																				
Data																					
transportation Costs																					
	1,61	2,12	3,58	3,77	2,89	2,65	1,52	1,22	0,96	1,76	3,39	1,7									
Operating Costs																					
Decision Variables																					
	x1_b1	x1_b2	x1_b3	x2_b1	x2_b2	x2_b3	xs1_b1	xs1_b2	xs1_b3	xs2_b1	xs2_b2	xs2_b3	xs3_b1	xs3_b2	xs3_b3	Ys1	Ys2	Ys3			
	14740	27830	0	0	0	19990	0	0	0	0	0	0	0	0	0	0	0	0			
Constraints																					
Capacity (Plants)																					
	1	1	1																42570	<=	45.000
				1	1	1													19990	<=	28.500
							1	1	1							-30000			0	<=	0
										1	1	1					-30000		0	<=	0
													1	1	1			-30000	0	<=	0
Demand																					
	1			1			1			1			1						14740	>=	14.740
		1			1			1			1			1					27830	>=	27.830
			1			1			1			1			1				19990	>=	19.990
Model Restriction																					
																1			0	=	0
																	1		0	=	0
																		1	0	=	0

The costs in an optimized state:
135.704,5 / 175.318,6 = 0,774

-> 22,6% Savings

Problem 7-1 – setting up the solver



The decision variables have to be extended and the operating variables have to be set on binary

Problem Y-1 – scenario 1 (no demand change)

=SUMMENPRODUKT(B6:P6;B12:P12)+
SUMMENPRODUKT(Q8:S8;Q12:S12)

Objective																					
min z	117.912,40																				
Data																					
transportation Costs																					
	1,61	2,12	3,58	3,77	2,89	2,65	1,52	1,22	0,96	1,76	3,39	1,73	1,96	1,37	0,9						
Operating Costs																					
																25.000	25.000	25.000			
Decision Variables																					
	x1_b1	x1_b2	x1_b3	x2_b1	x2_b2	x2_b3	xs1_b1	xs1_b2	xs1_b3	xs2_b1	xs2_b2	xs2_b3	xs3_b1	xs3_b2	xs3_b3	Ys1	Ys2	Ys3			
	14740	17820	0	0	0	0	0	10010	19990	0	0	0	0	0	0	1	0	0			
Constraints																					
Capacity (Plants)																					
	1	1	1																32560	<=	45.000
				1	1	1													0	<=	28.500
							1	1	1							-30000			-3,638E-12	<=	0
										1	1	1					-30000		0	<=	0
													1	1	1			-30000	0	<=	0
Demand																					
	1			1			1			1			1						14740	>=	14.740
		1			1			1			1			1					27830	>=	27.830
			1						1						1				19990	>=	19.990

New flow variables

Operating decision variables

It would be cheaper to produce in Valleyside already.

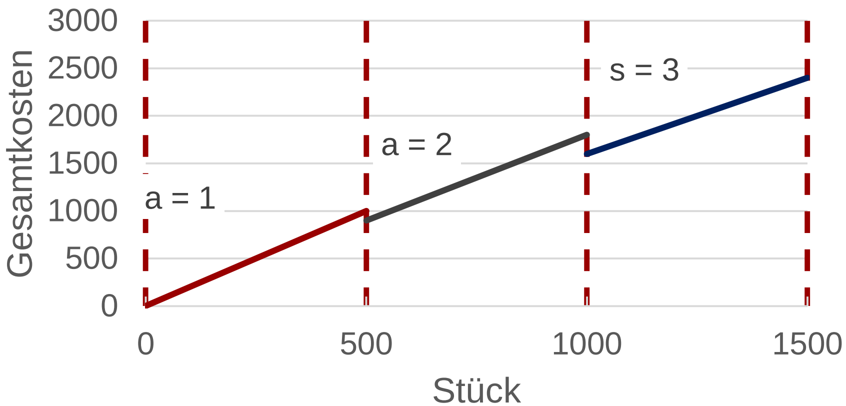
117.912,4 / 135.704,5 = 0,845

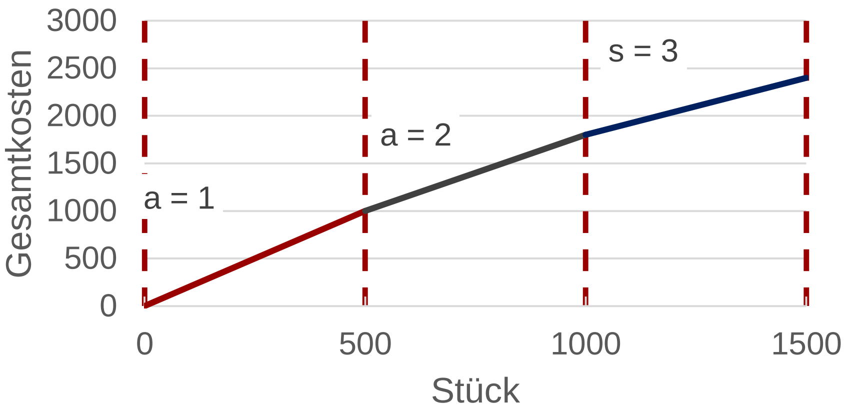
-> 13,1% Savings

Piecewise linear cost functions – Example model

Due to economy of scales several costs in logistics behave concave (e.g. operating costs of facilities, transportation costs), and cannot be handled by linear programming.

- To contribute that, stepwise cost functions with integer programming are used.
- In a model, the potential facility is replaced by artificial facilities (a) each representing a different cost level.





Variant 1:

At a threshold, cost per piece change:

- 500 Pieces → 2€/Pc * 500 Pc = 1000 €
- 501 Pieces → 1,8 €/Pc * 501 Pc = 901,8 €

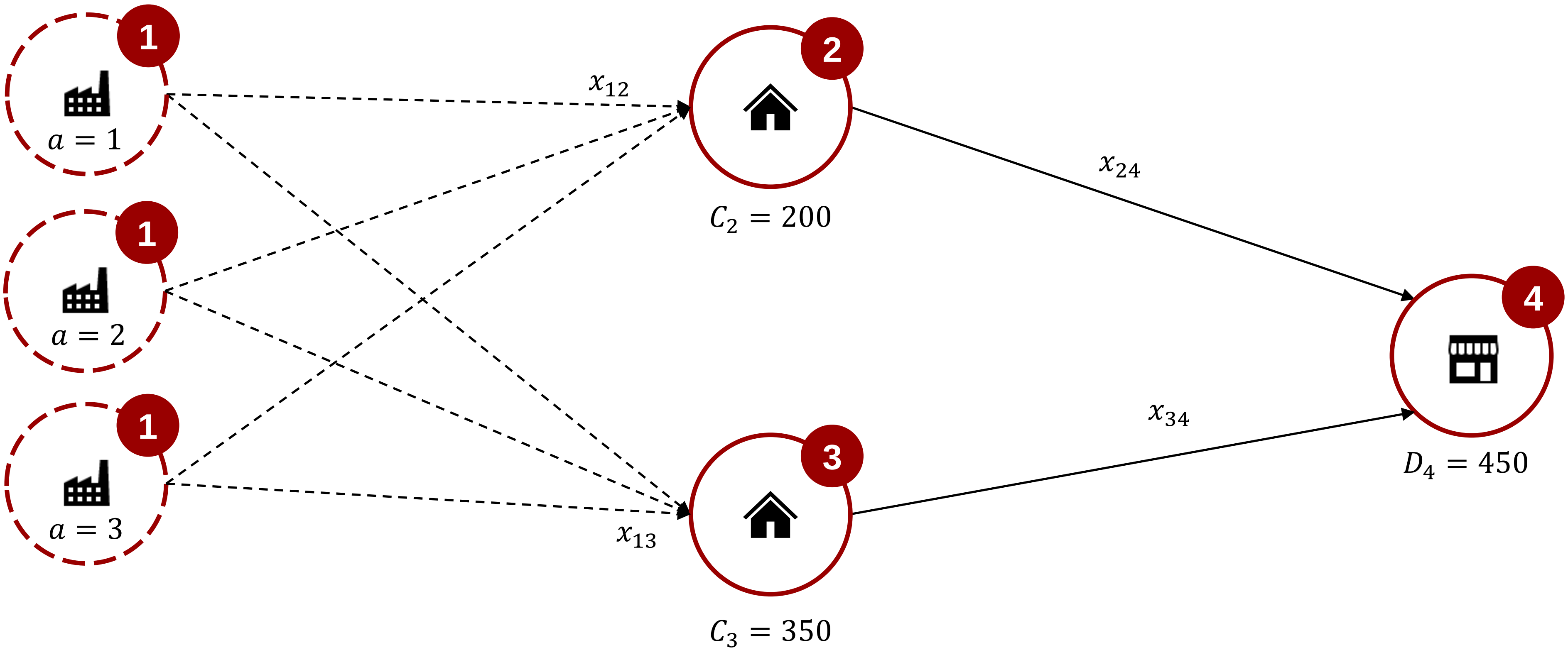
Variant 2:

At a threshold, cost of every additional piece is different:

- 500 Pieces → 2€/Pc * 500 Pc = 1000 €
- 501 Pieces → 1000 € + 1,8 €/Pc * 1 St = 1001,8 €

Actually 2 €/Pc * 500 Pc, but this would be pre-calculated for efficieny!

Piecewise linear cost functions - Visualized



$$S_1 = \begin{cases} \geq 0 \; and \; \leq 150 & if \; Y_{11} = 1 \\ > 150 \; and \leq 300 & if \; Y_{12} = 2 \\ > 300 \; and \leq 600 & if \; Y_{13} = 3 \end{cases}$$

Piecewise linear cost functions – Example model

Opening / Operating or not

$$y_{ia} \in \{0,1\}$$

Index a is an artificial facility.

Limit of parallel artificial facilities

$$\sum_a y_{ia} \leq 1 \quad \forall i$$

For all (potential) facilities i, only one cost level can exist. If zero exist, the facility is not open.

Dependent minimum Supply

$$\sum_a \sum_j x_{ija} \geq \sum_a S_{i(a-1)} * y_{ia} \quad \forall i$$

Dependend maximum supply

$$\sum_a \sum_j x_{ija} \leq \sum_a S_{ia} * y_{ia} \quad \forall i$$

Due to economies of scale, there is a minimum supply requirement (with $S_i^0 = 0$). If the pieces of the cost functions are connected, the supply minimum of a is the supply maximum of the previous (a-1).

Nomenclature

Values
- D – Demand
- S – Supply / Capacity
- c – cost
- x – flow of goods
- Y – open/closed
- H – Amount of goods held
- F – Fix costs of opening / operating
- C – Capacity of a facility

Indices
- j – Receiver (e.g. Customer)
- i – Sender (e.g. plants)
- k – hub (e.g. distribution center)
- t – time period
- a – artificial version of a facility

Piecewise linear cost functions – Example model

$$\min z = \sum_i \sum_j x_{ij} * c_{ij} + \sum_i \sum_a \sum_j x_{ija} * F_{ia}$$

Minimize costs including transportation (c) times the flow on a lane (x) and operatings costs (f_a), if the artificial facility is open (y_a) times all flow (x) leaving i to any j

s.t.

$$\sum_j x_{ij} \geq D_j \qquad \forall j \in J$$

All demand (D) has to be satisfied

$$\sum_a \sum_j x_{ija} \geq \sum_a S_{i(a-1)} * y_{ia} \qquad \forall i$$

The supply (S) of a flow (x) of a facility i to any j has to reach a minimum level, depenend on which artificial facility (y_a) of i is used.

$$\sum_a \sum_j x_{ija} \leq \sum_a S_{ia} * y_{ia} \qquad \forall i$$

The supply (S) of a flow (x) of a facility i to any j is limited, depenend on which artificial facility (y_a) of i is used.

$$\sum_a y_{ia} \leq 1 \qquad \forall i$$

For a potential facility i, which can be open or closed (y_a), of all artificial entities only one can exist.

$$x_{ij} \geq 0 \qquad \forall i, j$$

All flows (x) have to be greather or equal to zero

$$y_{ia} \in \{0,1\}$$

Openining/operating decisions are binary

Nomenclature

Values

D – Demand
S – Supply / Capacity
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x – flow of goods
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H – Amount of goods held
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C – Capacity of a facility

Indices

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i – Sender (e.g. plants)
k – hub (e.g. distribution center)
t – time period
a – artificial version of a facility

Problem 7-2

Example – No Exercise

- BAER Home electronics manufactures microwaves and sells them to customers in Barronvale and Black Valley. The Distribution center in East City is currently supplying the customers, but cannot fulfill the demand. BEAR has identified an external Distribution center closer to the market in Carlton leading to better transportation costs. However, the external DC generates operating costs per unit processed.
- What is the optimal flow?

	id	S
East City	1	35.000
Carlton	2	25.000

Table 1: Supply

	id	D
Barronvale	1	18.000
Black Valley	2	22.000

Table 2: Demand

From \ to	id	3	4
East City	1	1,72	1,96
Carlton	2	1,55	1,65

Table 3: Transportation costs

	Operating Cost	From … Units
East City	0	0
Carlton	0,32	0
	0,26	15.000
	0,22	20.000

Table 4: Operating costs

Service Levels

Service Level has different meanings in logistics (e.g. distance to customer or time to customer)

- It influences metrics like fill rate or percentage of late orders
- As objective the average service level (e.g. minimize average distance) or the proportion a service level (maximize percentage of customers within a distance) can be addressed

In a model, it is suggested to precompute necessary data (A) describing the service level of potential facilities.

- Average Distance (in objective function or in constraints) require the Distances of facilities to each other (A^{Dist})
- Minimum or maximum proportion within a Distance (in objective function or in constraints) require the (binary) data of the facilities being in that distance to each other or not ($A^{<MaxDist}$)

Example constraint for average Distance

$$\frac{\sum_i \sum_j x_{ij} * A_{ij}^{Dist}}{\sum_j D_j} \leq MaxAvgDist$$

Constraint assumes all flow (x) to to into Demand areas and all Demand will be fulfilled :

The Sum of all Flow multiplied with the distance of the flow divided by the total flow (which is the Demand) should be smaller than some specified Distance.

Problem 7-3

Example – No Exercise

- BAER Home electronics manufactures ambient lights and sells them to customers in Swissdale, Southgate and Starlight. The plants in Delleker, Dravosburg, and Delhi Hills currently ship cost optimally, but larger distances have proven unreliable for fast deliveries even though they are mostly operated by the cheaper Vender. BEAR now wants to enforce a maximum of 40% of the lanes being >50km.

- How much does this new policy cost BEAR?

	id	D
Swissdale	1	23.000
Southgate	2	36.000
Starlight	3	15.000

Table 1: Demand

	id	S
Delleker	4	20.000
Dravosburg	5	33.000
Delhi Hills	6	26.500

Table 2: Supply

From \ to	4	5	6
1	1,1	1,22	0,83
2	1,92	1,74	1,46
3	0,76	0,93	2,11

Table 3: Transportation costs

From \ to	4	5	6
1	0	1	1
2	0	0	1
3	1	0	0

Table 4: Distance >50km

Multi-Product flows

- In the previous exercises, the models have been limited on one product. However, companies usually have thousands of products or SKUs.
- Treating these products the same is not appropriate, but treating all different may create unnecessary effort, if products are similar in their relevant characteristics of the level of detail of the model.
- Products should be treated different,
  - When our overall pool of products includes large variations in storage and logistics characteristics (dry vs. frozen food, different sizes, different weight, hazardous vs. normal goods, …)
  - When some product types require specific customer service levels and therefore require specific transport modes
  - When products come from different source locations (it is important to source the right product from the right location)

Summary: Conditional Constraints

- Actual and optimized baseline set different points of cost reference to measure the improvement. Using the right point of reference is important to have a rightful impression of cost improvement potential.
- Mixed-integer (linear) programming in SCND is used for example for conditional decisions (adding/closing facilities) or cost relationships (piecewise linear cost functions).
- Multiple products should be treated differently, if the difference has an actual impact on the models level of detail.

References

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