Quantitative Decision Making in Business

Summer University 2018

Topic 8: Simulation

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Technische Universität Berlin

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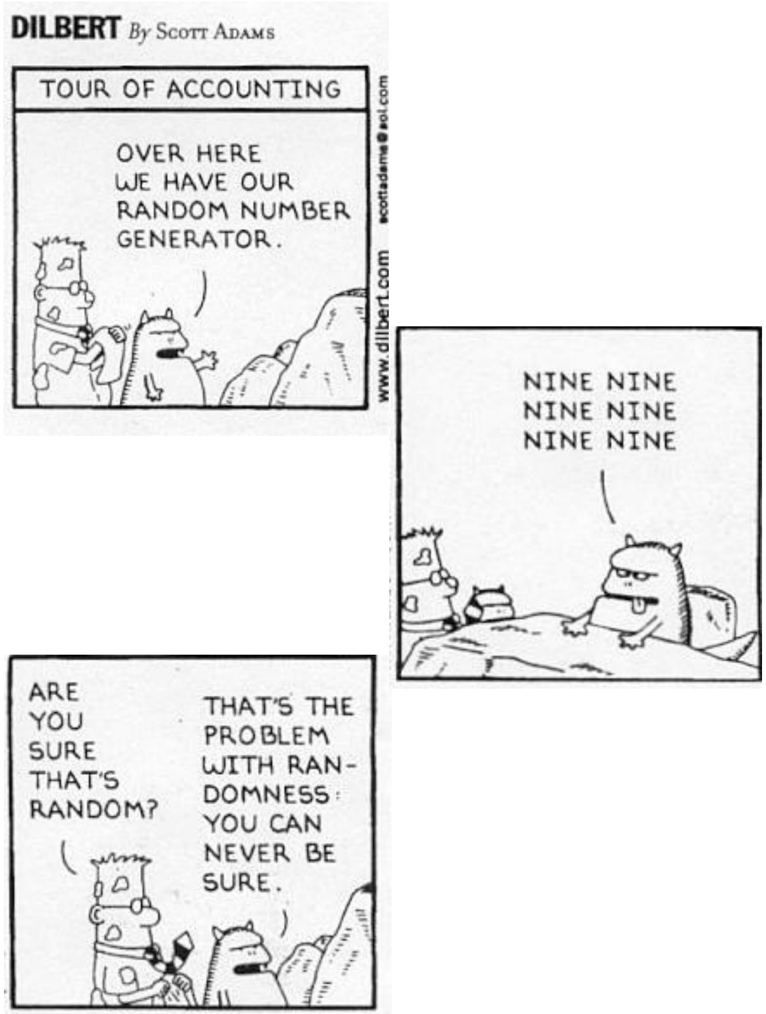
Simulation



Pictures : www.dilbert.com,
CT Zeitschrift, Januar 2008

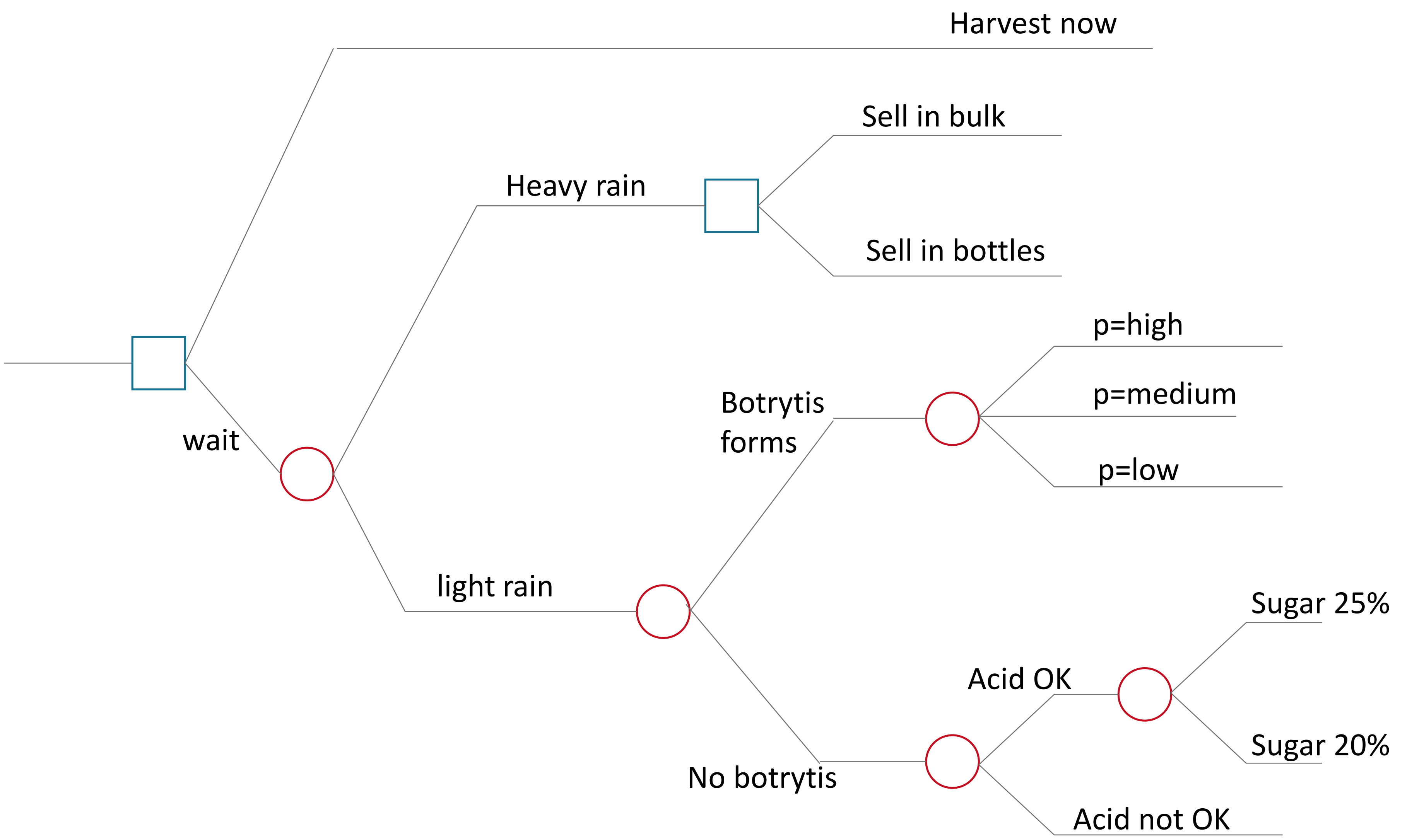
Monte Carlo Simulation

- Generating Random Numbers
- Transforming Random Numbers to Random Variates
- Monte Carlo Simulation

Uploaded files:

Case Study 2: Conley Fisheries, Inc.

What is simulation supposed to do?



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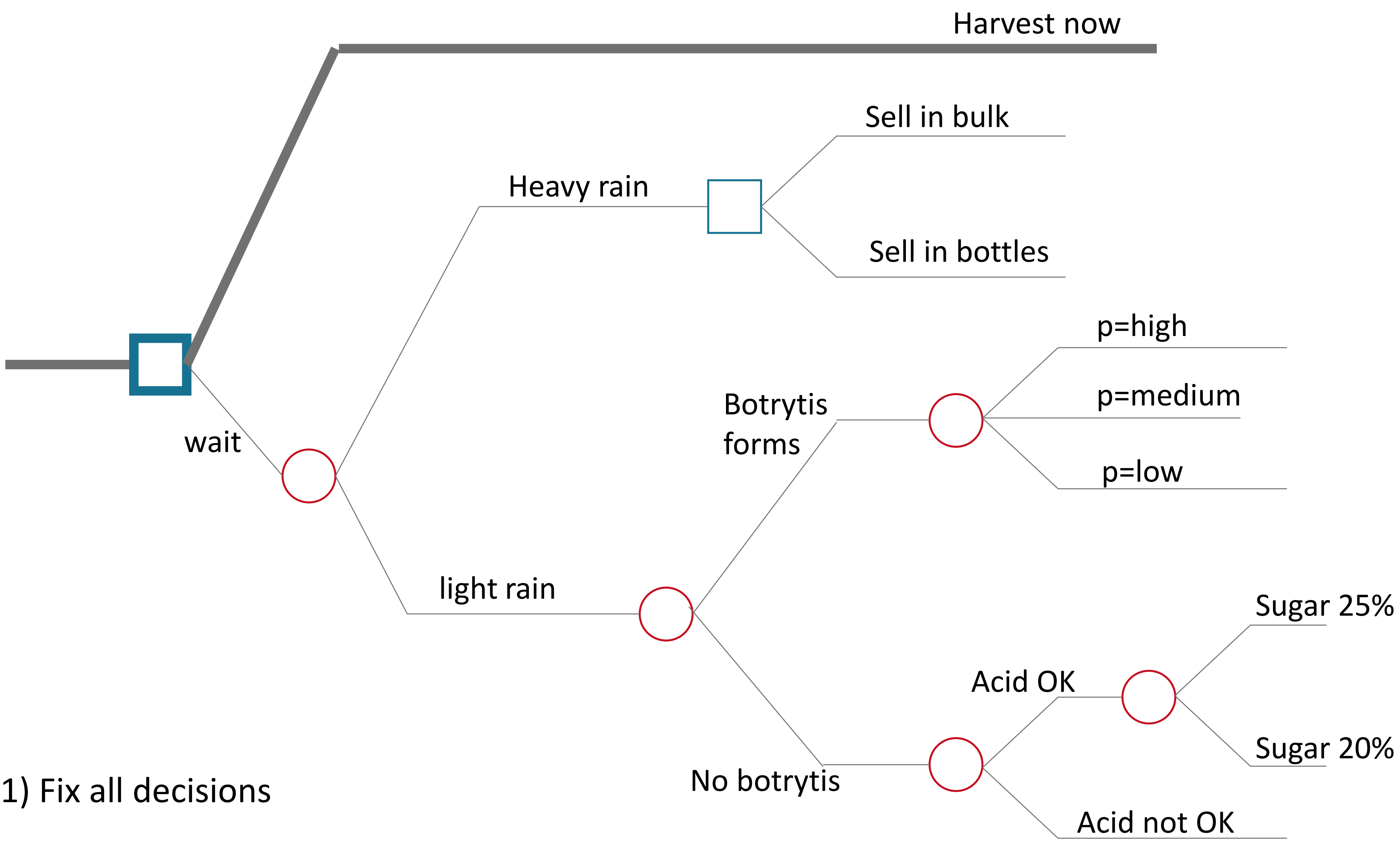
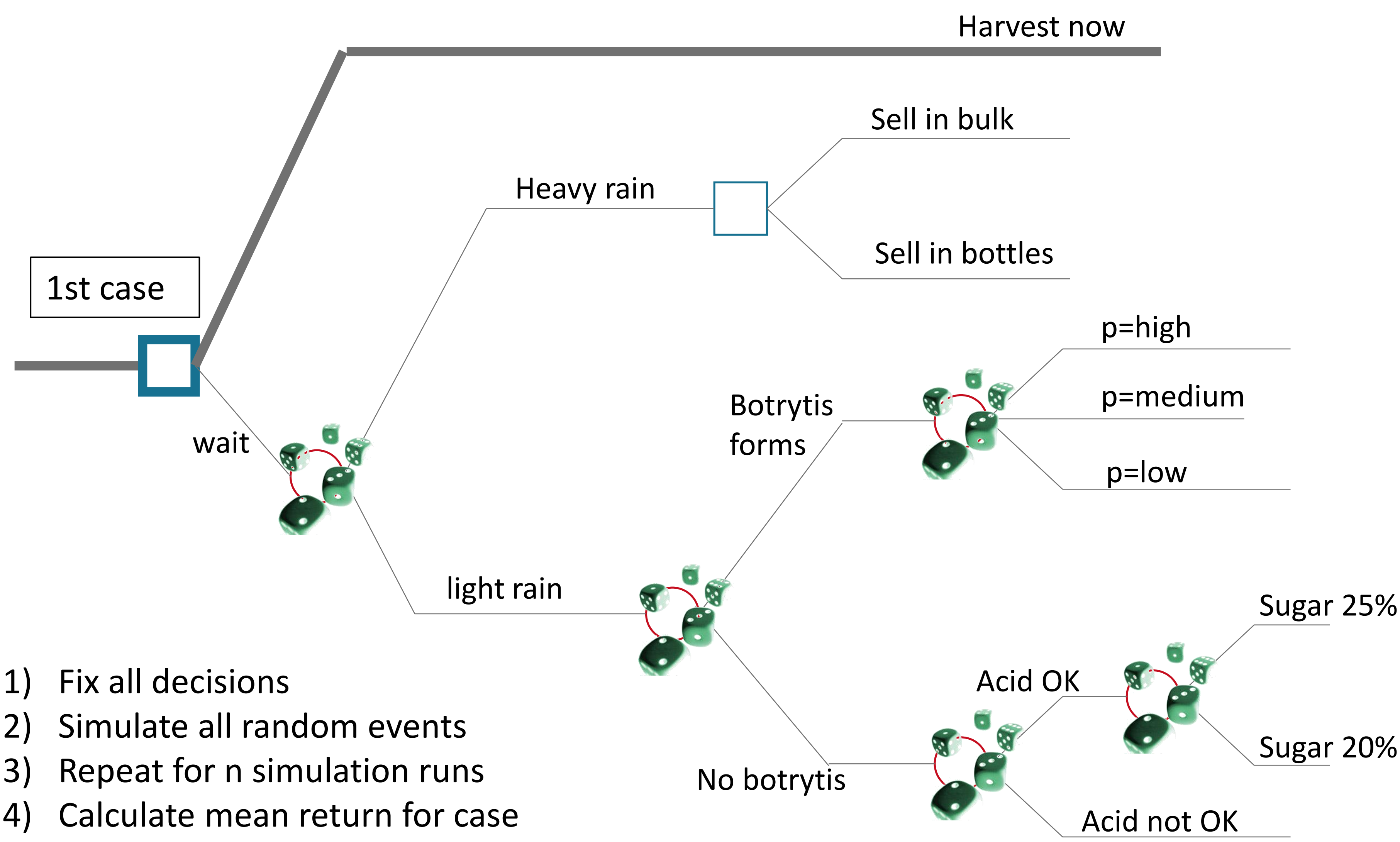
What is simulation supposed to do?



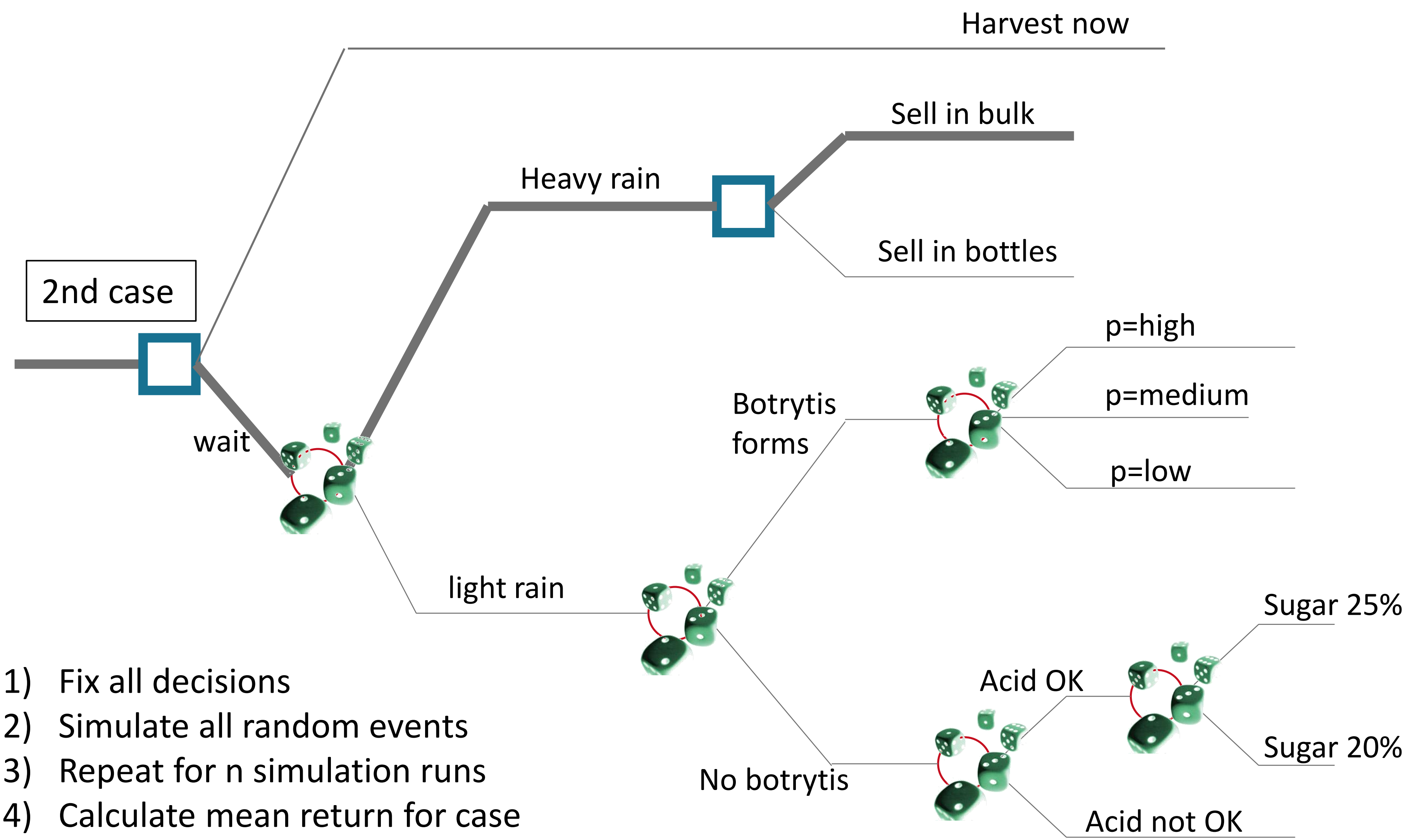
1) Fix all decisions

What is simulation supposed to do?



1) Fix all decisions
2) Simulate all random events
3) Repeat for n simulation runs
4) Calculate mean return for case

What is simulation supposed to do?



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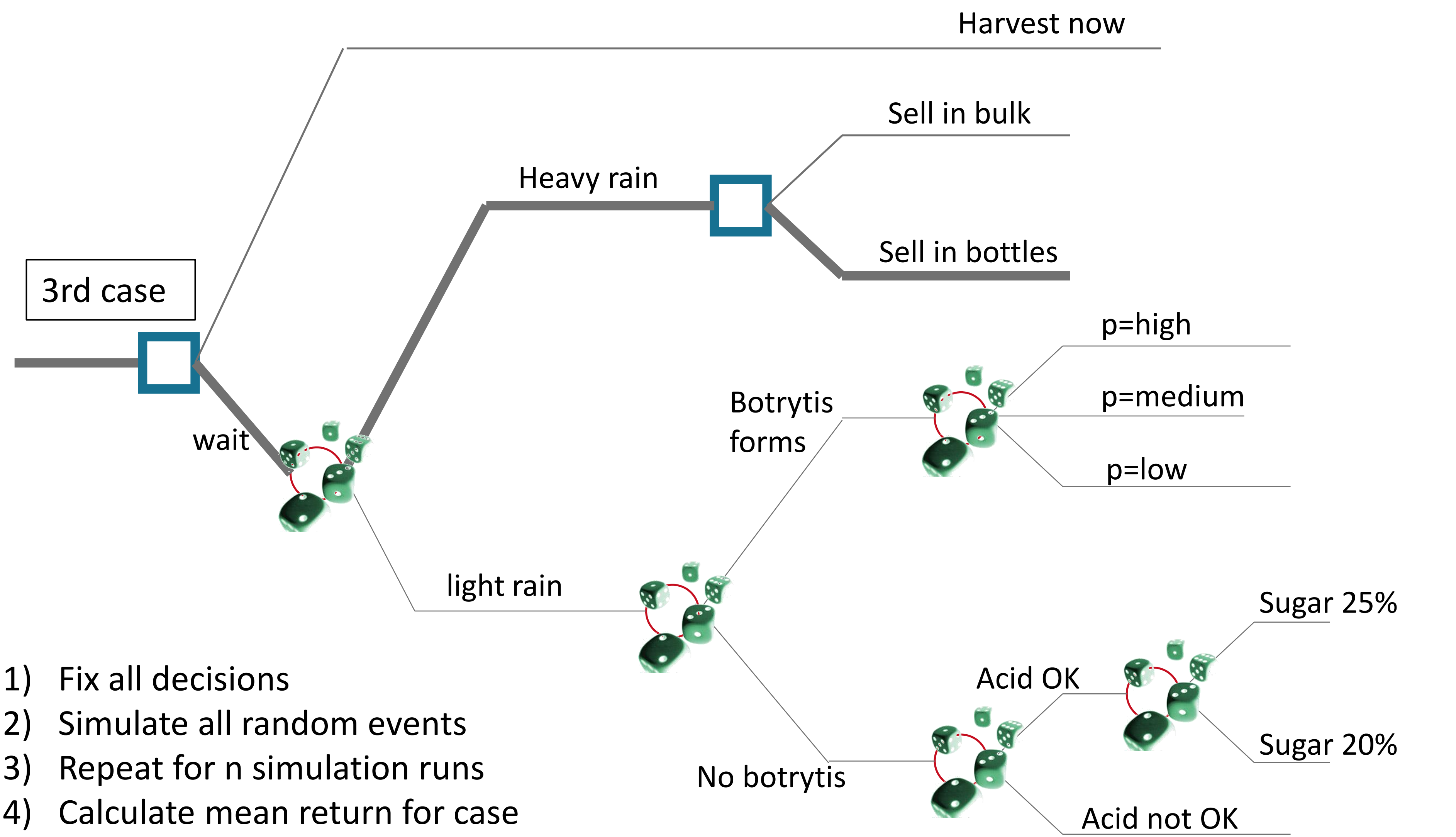
What is simulation supposed to do?



1) Fix all decisions
2) Simulate all random events
3) Repeat for n simulation runs
4) Calculate mean return for case

We could also simulate the process…

Advantages of Simulation

+ Most complex, real-world systems with stochastic elements cannot be accurately described by a mathematical model that can be evaluated analytically
+ Simulation allows one to estimate the performance of an existing system under some projected set of operating conditions
+ Alternative proposed system designs can be compared via simulation to see which best meets a specified requirement
+ In a simulation we can maintain much better control over experimental conditions than would generally be the possible when experimenting with the system itself
+ Simulation allows us to study a system with a long time frame in compressed time, or alternatively to study the detailed workings of a system in expanded time

Disadvantages

– Each run of a stochastic simulation model produces only estimates of a model‘s true characteristics; an analytical model can often easily produce the exact true characteristics.

– Simulations often are expensive and time-consuming to develop

– The large volume of numbers produced by a simulation study and the persuasive impact of a realistic animation often creates a tendency to place greater confidence in a study‘s results than is justified.

Simulation Languages vs. General Purpose Languages

A word on Excel: Although this is our tool in class, I would ***not recommend*** it for high stake Monte Carlo Simulations.

Advantages of general purpose languages:

- Most modelers already know a general-purpose language
- General-purpose languages are available on most computers
- General-purpose languages may provide more programming flexibility
- An efficiently written program in a general purpose language may require less execution time
- Software costs may be lower

Advantages of simulation languages:

- Simulation languages automatically provide most of the features needed in programming a simulation model → less programming time
- Basic building blocks are more closely akin to simulation
- Simulation models are easier to change when written in a simulation language
- Better error detection

Monte Carlo Simulation

Monte Carlo simulation is

„a scheme employing random numbers, [...] which is used for solving certain stochastic or deterministic problems.“

(Law, 2008)

Pictures : http://www.pharmaventures.com/pharmadeals/review/cartoon/2007
http://www.10best.com/destinations/monaco/monte-carlo/attractions/casinos/

Simulation: A Definition

This definition is very wide

- includes many forms of simulation,
- also includes our generation of random variates.

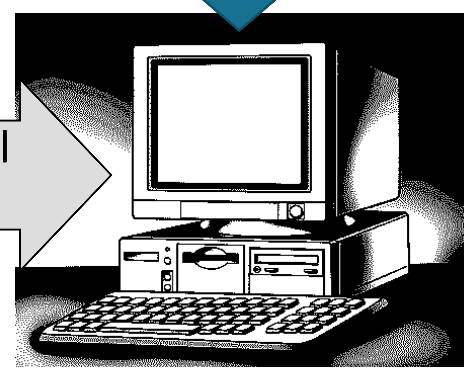
First major application: Manhattan project

Name: reference to the Monte Carlo Casino in Monaco

Simulation: From Input to Output

Random Elements:
generation and transformation of
random numbers u

Simplified model of reality



(Random) Output

Monte Carlo Simulation

Our view on simulation:

Estimate E(X) when one simulation run gives $X = h(U_1,\dots,U_k)$

Simulation = Estimating E(X)

- X_i : output from a single simulation (i^{th}) run
  e.g. X_i = earnings on day i, rainfall in week i,….
- x_i : realization of the random variable X_i resulting from the ith simulation run using the random numbers u_{i1}, u_{i2},…

We know from the strong law of large numbers that if μ is finite, then:

$$\lim_{n\to\infty} \frac{X_1 + .. + X_n}{n} = \mu$$

➔ For large n, the average output from n runs should equal the expected value of the random output.

How can we generate „Randomness“?

- Manually throw a die multiple times, shuffle cards, throw a coin, etc: Write down the numbers and save in a file
- Physical random number generator
- Pseudo random numbers

Arithmetical steps:

Step 1: Pseudo-Random or Physical random number generator

Step 2: Transforming Random Numbers to Random Variates

First Goal:

i.i.d. realizations of a random variable X following some given probability distribution.

Steps:

1) Generate i.i.d. realizations of a random variable uniformly distributed on (0,1)
2) Adapt those (random numbers) to the desired distribution

Step 1: Pseudo-Random Number Generators

Von Neumann (1951): “Any one who considers arithmetical methods of producing random digits is, of course, in a state of sin. For [...] there is no such thing as a random number – there are only methods to produce random numbers, and a strict arithmetic procedure of course is not such a method. [...] We are here dealing with mere ‘cooking recipes’ for making digits.”

When are these recipes good?

- Outcome should appear to be uniformly distributed and should not exhibit correlation with each other
- Generator should be fast and avoid the need for a lot of storage
- Possibility to reproduce a given stream of random numbers exactly (easier for debugging and verification and can be used for variance reduction techniques)
- One should be able to generate several separate “streams“ of random variables

Monte Carlo Simulation

First Goal:

i.i.d. realizations of a random variable X following some given probability distribution.

Steps:

1) Generate i.i.d. realizations of a random variable uniformly distributed on (0,1)
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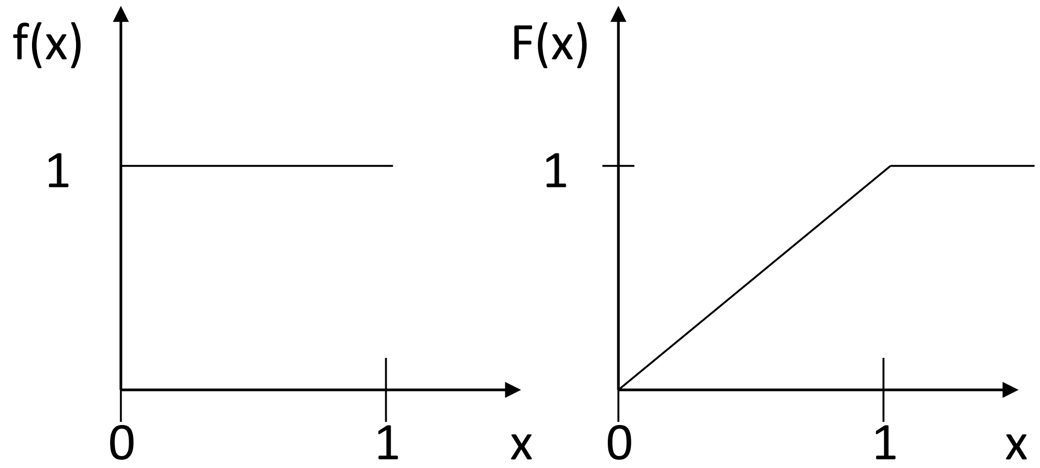
How can we generate Realizations of Random Variables?

Step 1: Generate realizations of a Uniform distribution on (0,1)



$$F(x) = \begin{cases} 0 & x < 0 \\ x & x \in [0,1] \\ 1 & x > 1 \end{cases}$$

Expectation: 1/2

Variance: 1/12

Step 1: Linear Congruential Generators (for example)

Initial value/seed: x_0

Recursively:

$x_n=(a\, x_{n-1}+c) \bmod m$

$a,m \in N$, $c \in N_0$

Random number given by

$u_n = x_n / m$

Example:

$X_n = (4 * X_{n-1} + 2) \bmod 9$, starting value 1

n	0	1	2	3	4	5	6	7	8
x_n	1								
u_n									

n	9	10	11	12	...
x_n					
u_n					

Of course, larger values are used in practice, e.g. m= 2147483647, a= 48271, c=0.

Monte Carlo Simulation

Initial value/seed: x_0

Recursively:

$x_n=(a\ x_{n-1}+c) \bmod m$

$a,m \in N,\ c \in N_0$

Random number given by

$u_n = x_n / m$

Step 1: Linear Congruential Generators

Properties:

Monte Carlo Simulation

Initial value/seed: x_0

Recursively:

$x_n=(a\, x_{n-1}+c) \bmod m$

$a,m \in N$, $c \in N_0$

Random number given by

$u_n = x_n / m$

The choice of x_0, a, c, and m is important!

Step 1: Linear Congruential Generators

More Examples:

$x_0=2$, $a=5$, $c=2$, $m=9$:

n	0	1	2	3	4	5	6	7	8	9	10	11	…
x_n	2	3	8	6	5	0	2	3	8	6	5	0	…

$x_0=4$, $a=5$, $c=2$, $m=9$:

n	0	1	2	3	4	5	6	7	8	9	10	11	…
x_n	4	4	4	4	4	4	4	4	4	4	4	4	…

$x_0=4$, $a=6$, $c=2$, $m=9$:

n	0	1	2	3	4	5	6	7	8	9	10	11	…
x_n	4	8	5	5	5	5	5	5	5	5	5	5	…

$x_0=0$, $a=1$, $c=1$, $m=111$:

n	0	1	2	3	4	5	6	7	8	9	10	11	…
x_n	0	1	2	3	4	5	6	7	8	9	10	11	…

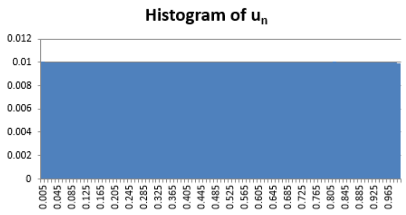
Monte Carlo Simulation

Even if linear congruential generators have full period, they might not be "good" random number generators.

Step 1: Linear Congruential Generators

How well do the generated random numbers resemble values of true i.i.d. U(0,1) random variates?

- Check whether the numbers appear to be **uniformly distributed** between 0 and 1 (by a chi-square-test etc. or graphically)
- Do a **Serial Test** (Generalization of the chi-square-test): If the random numbers were really i.i.d. U(0,1) random variates, the nonoverlapping d-tupels should be i.i.d. random vectors distributed uniformly on the d-dimensional unit hypercube. (This can be checked by a chi-square-test etc. or graphically.)
- (**Runs test**: Examine the sequence U_i for unbroken subsequences of maximal length within which the U_i increase monotonically (run up))



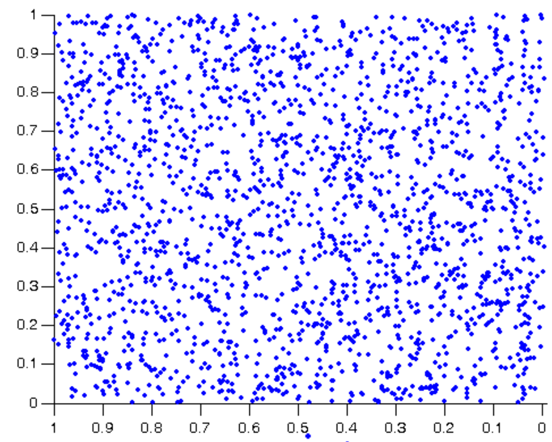
Judging the quality of a random number generator is difficult.

Step 1: Linear Congruential Generators

RANDU: random number generator in IBM's scientific subroutine package for System/360 mainframe computer systems (in the 1960s); LCG with $m=2^{31}$ $a=65539$ and $c=0$.

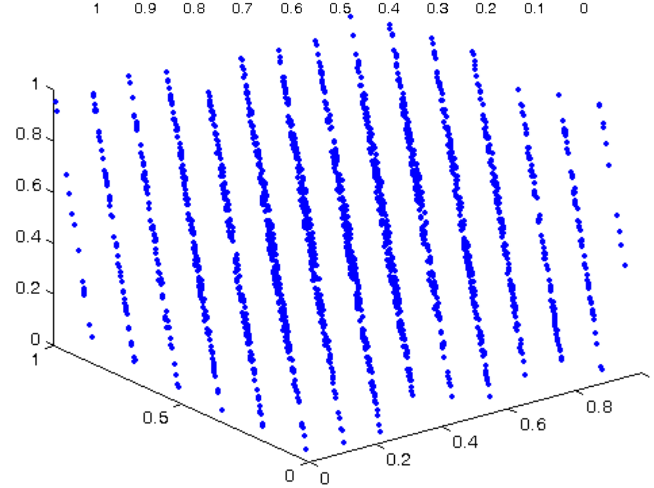
"...its very name RANDU is enough to bring dismay into the eyes and stomachs of many computer scientists!" (Donald Knuth)

Random numbers generated by RANDU for two-tupels:



Random numbers generated by RANDU for three-tupels:

All random numbers fall exactly on one of 15 hyperplanes!



First Goal:

i.i.d. realizations of a random variable X following some given probability distribution.

Steps:

1) Generate i.i.d. realizations of a random variable uniformly distributed on (0,1)
2) Adapt those (random numbers) to the desired distribution

Step 2: Transforming Random Numbers to Random Variates

Algorithms for obtaining realizations of a random variable (=random variates) of a given distribution should be

- Exact
- Efficient in storage space and execution time
- Non-complex (including conceptual as well as implementation related factors)
- Robust (i.e. it is efficient for all parameter values)

Monte Carlo Simulation

First Goal:

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Steps:

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Step 2: Transforming Random Numbers to Random Variates

How can we "build a die"?

Algorithm:

1) Generate random number u between 0 and 1

2) If u

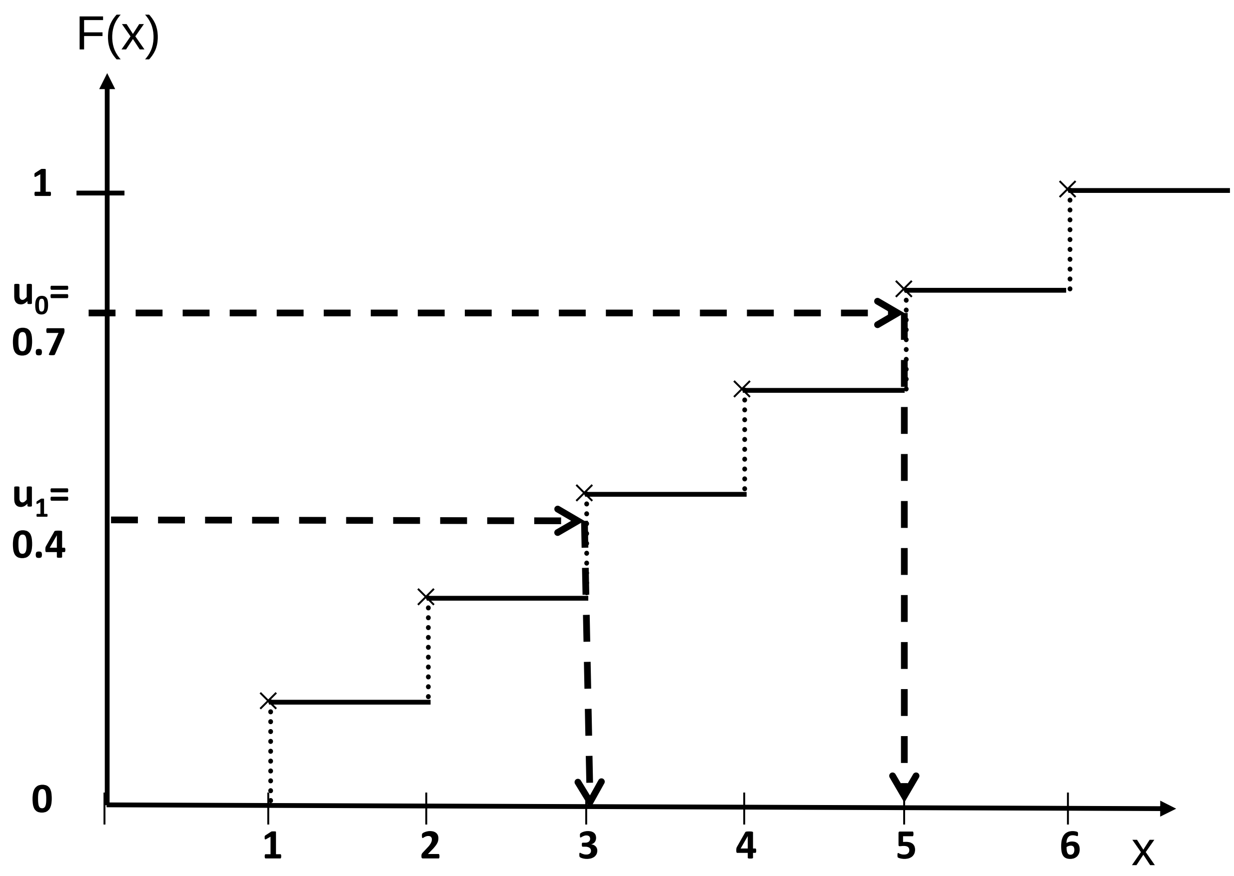
Example

u	0.9167	0.0167	0.4444	0.7878	0.3000

Step 2: Transforming Random Numbers to Random Variates

The Inverse Method is based on the use of the inverse cumulative distribution function F of the desired distribution.

How can we "build a die"?



Monte Carlo Simulation

Step 2: The Inverse Method (General Discrete Case)

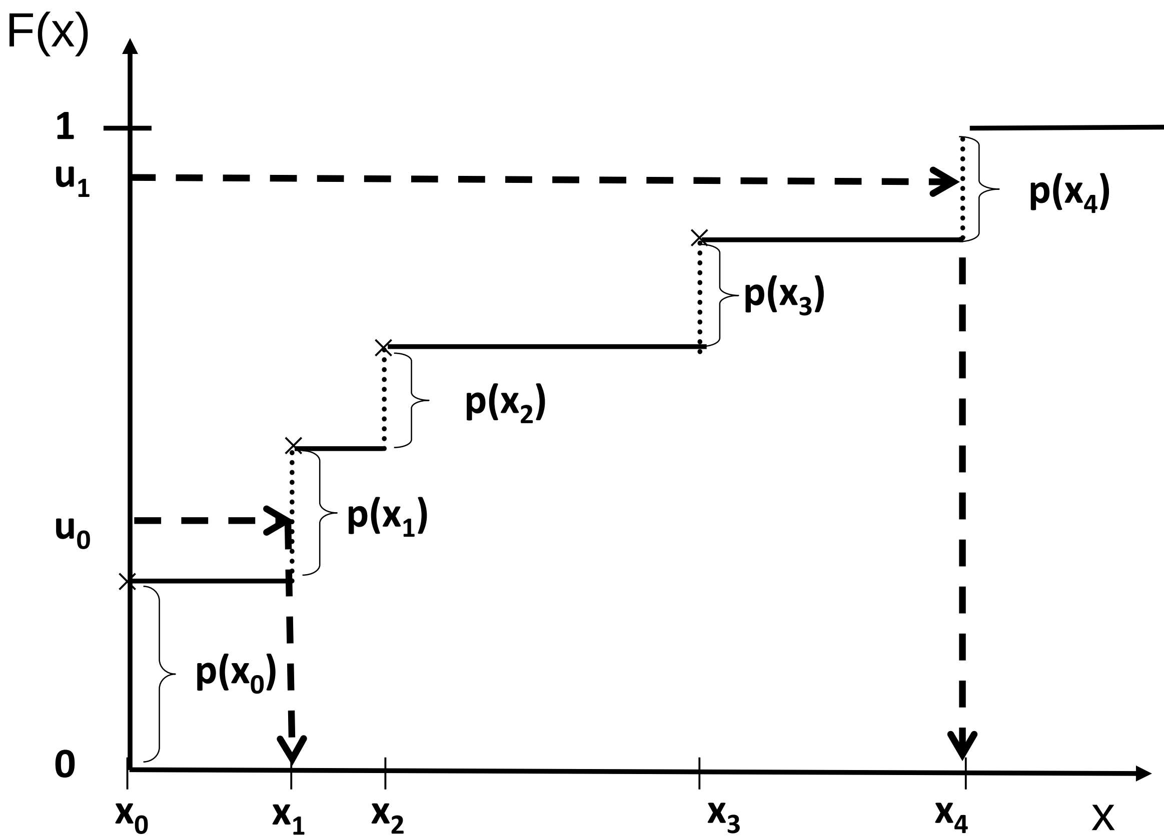
The Inverse Method is based on the use of the inverse cumulative distribution function F of the desired distribution.

If X is a discrete random variable with $P(X=x_i)=p_i$:

1) Generate random number u.
2) Set

$$x=\begin{cases} x_0 \text{ if } u<p_0 \\ x_1 \text{ if } p_0 \le u<p_0+p_1 \\ \dots \\ x_i \text{ if } \sum_{j=0}^{i-1} p_j \le u < \sum_{j=0}^{i} p_j \\ \dots \end{cases}$$

Illustrated:



Task 4

Random Variates (discrete case)

x	P(x)	F(x)
0	0.3	
1	0.2	
5	0.1	
10	0.4	

Consider a random variable that takes the value 0 with probability 0.3, the value 1 with probability 0.2, the value 5 with probability 0.1 and the value 10 otherwise.

1. Plot the cumulative distribution function.

Task 4

Random Variates (discrete case)

x	P(x)	F(x)
0	0.3	
1	0.2	
5	0.1	
10	0.4	

2. How would you transform them to obtain realizations of random numbers with the given distribution?

i	u_i
1	0.20951
2	0.79238
3	0.44533
4	0.21307
5	0.82943

Monte Carlo Simulation

Photo by Nguyễn Linh on Unsplash

Use:

171020_ConelyFishery_Template

Case 2) Conely Fisheries Inc.

Read the case study to answer the following questions:

1. What are the daily earnings if Clint chooses to sell his daily catch of codfish in Gloucester?
2. Simulate the earnings when selling in Rockport for a sample of 200 days in an Excel Spreadsheet with the following columns: Random number 1, Demand in Rockport, Random number 2, price in Rockport, Quantity Sold, Daily Earnings. (Using ConleyFishery_Template.xlsx will make part 3 easier.)
   a. How can you generate realizations of demand at Rockport?
   b. How can you generate realizations of price at Rockport?
   c. How can you obtain daily earnings from your answers to a. and b.?
3. What is the shape of the probability distribution of daily earnings from using Rockport?
4. On any given day, what is the probability that Conley Fisheries would earn more money from using Rockport instead of Gloucester?
5. On any given day, what is the probability that Conley Fisheries will lose money if they use Rockport?
6. What are the expected daily earnings from using Rockport?
7. What would you advise Clint to do?

Monte Carlo Simulation

The Inverse Method is based on the use of the inverse cumulative distribution function F of the desired distribution.

If X is a discrete random variable with $P(X=x_i)=p_i$:

1) Generate random number u.
2) Set

$$x=\begin{cases} x_0 & \text{if } u<p_0 \\ x_1 & \text{if } p_0 \le u<p_0+p_1 \\ \cdots & \\ x_i & \text{if } \sum_{j=0}^{i-1} p_j \le u < \sum_{j=0}^{i} p_j \\ \cdots & \end{cases}$$

Conely Fisheries Inc.

Question 2a

Discuss with a partner: How would you generate a realization of demand at Rockport?

	A	B	C	D
1	Day	Random Number for Demand	Demand in Rockport (kg)	Quantity Sold (kg)
2	1			
3	2			
4	3			
5	4			

General idea:

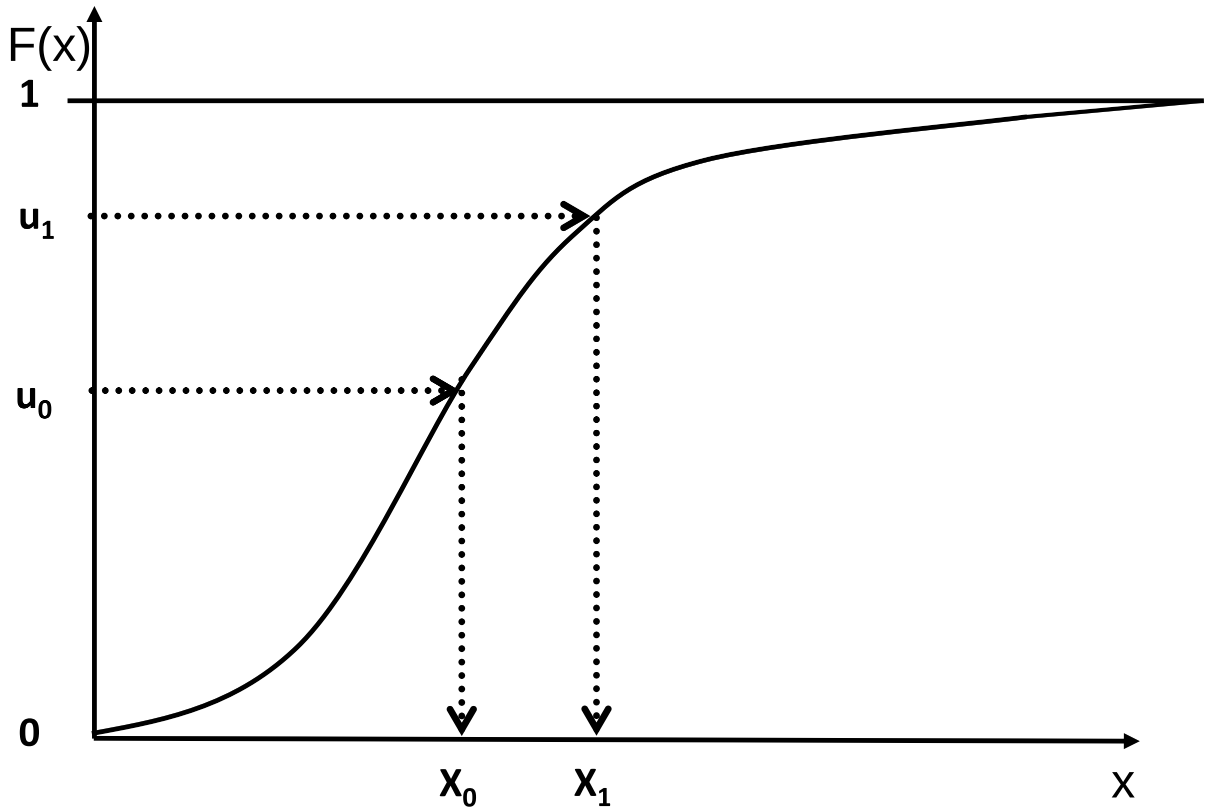
1. Generate realization u of U(0,1)
2. Return min{x: F(x) $\geq$ u}

If X is a random variable with a cumulative distribution function F that is strictly increasing and continuous on D={x|0<F(x)<1}, then $F^{-1}(u)$, the inverse, returns the value of x with F(x)=u. So:

1. Generate realization u of U(0,1)
2. Return $F^{-1}(u)$

Step 2: The Inverse Method (Continuous Case)

Illustrated:



Step 2: The Inverse Method (Continuous Case)

General idea:

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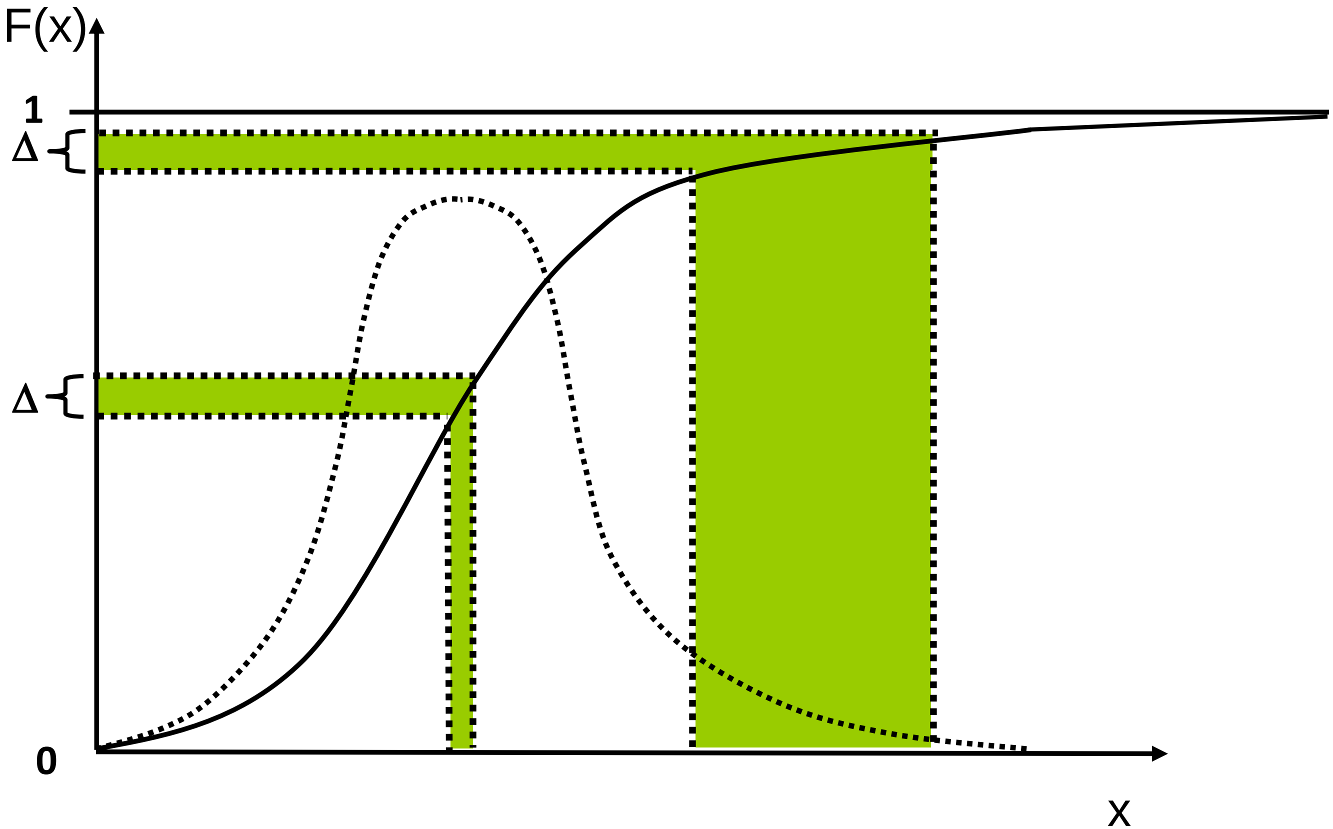
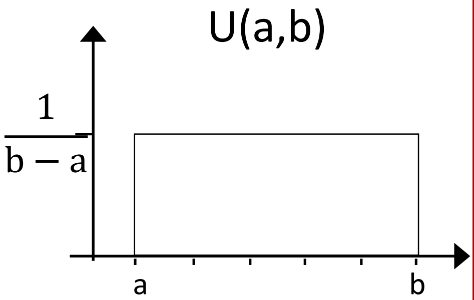
Illustrated:



Generating Random Variates of the Uniform Distribution



$$f(x)=\begin{cases} \frac{1}{b-a} & \text{for } a<x<b \\ 0 & \text{else} \end{cases}$$

$$F(x)=\begin{cases} 0 & \text{for } x \le a \\ \frac{x-a}{b-a} & \text{for } a<x<b \\ 1 & \text{for } b \le x \end{cases}$$

→ Steps:

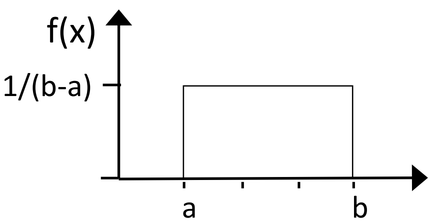
1. Generate random number u
2. Return a+(b-a) u

Task 5

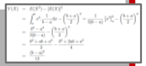
Random Variates (continous case)


Photo by Joseph Gonzalez on Unsplash

Uniform distribution between a and b:



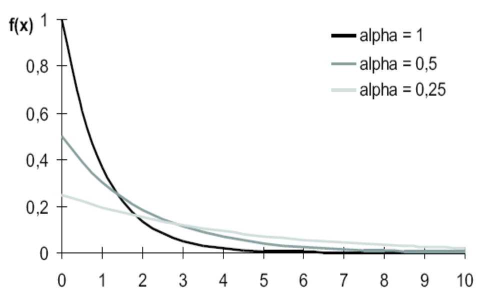
$$\mathrm{E}(X) = \frac{a+b}{2}, Var(X) = \frac{(b-a)^2}{12}$$



Let us assume that the number of popsicles sold from a vending machine on any given day is uniformly distributed between 160 and 400, and that sales on one day are independent of the sales on other days. We simulate popsicle sales.

1. Assume you run your simulation once, i.e. you generate one day of popsicle sales.
   a. What is the distribution of your output?
   b. What is the probability that the output of your simulation is smaller than 275?

Generating Random Variates of the Exponential Distribution



$$f(x) = \begin{cases} 0 & \text{for } x \leq 0 \\ \alpha e^{-\alpha x} & \text{for } x > 0 \end{cases}$$

$$F(x) = \begin{cases} 0 & \text{for } x \leq 0 \\ 1 - e^{-\alpha x} & \text{for } x > 0 \end{cases}$$

→ Steps:

1. Generate random number u
2. Return $-\ln(1-u)/\alpha$

General idea:

1. Generate realization u of U(0,1)

2. Return $\min\{x: F(x) \geq u\}$

Step 2: The Inverse Method – General Discussion

- The inverse method can also be used to generate realizations of a mixed random variable
- Only 1 random number is needed to generate 1 realization
- Very easy to combine with variance reduction techniques
- For some distributions no closed form of F^{-1} is available
- Not always the fastest method

Monte Carlo Simulation

General idea:

1. Generate realization u of U(0,1)
2. Return $\min\{x: F(x) \geq u\}$

If X is a random variable with a cumulative distribution function F that is strictly increasing and continuous on $D=\{x|0<F(x)<1\}$, then $F^{-1}(u)$, the inverse, returns the value of x with $F(x)=u$. So:

1. Generate realization u of U(0,1)
2. Return $F^{-1}(u)$

Conely Fisheries Inc.

Question 2b

How would you generate a realization of price at Rockport?

	A	B	C	D	E	F
1	Day	Random Number for Demand	Demand in Rockport (kg)	Quantity Sold (kg)	Random Number for Price	Price in Rockport ($/kg)
2	1					
3	2					
4	3					
5	4					
6	5					
7	6					

Monte Carlo Simulation

General idea:

1. Generate realization u of U(0,1)
2. Return min{x: F(x) $\geq$ u}

If X is a random variable with a cumulative distribution function F that is strictly increasing and continuous on D={x|0<F(x)<1}, then $F^{-1}(u)$, the inverse, returns the value of x with F(x)=u. So:

1. Generate realization u of U(0,1)
2. Return $F^{-1}(u)$

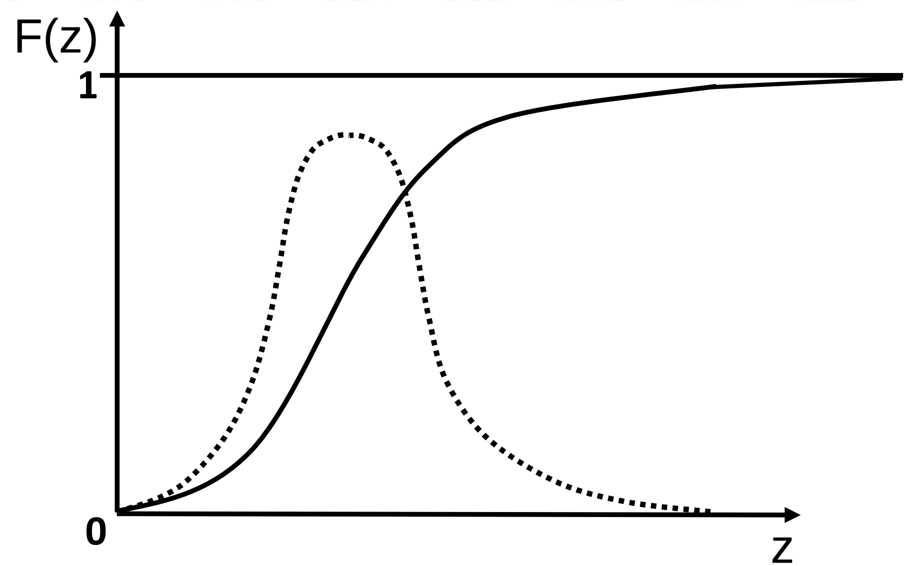
Conely Fisheries Inc.

Question 2b

How would you generate a realization of price at Rockport?

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82							
1.0	.84134	.84375	.84							
1.1	.86433	.86650	.86							
1.2	.88493	.88686	.88							
1.3	.90320	.90490	.90							
1.4	.91924	.92073	.92							
1.5	.93319	.93448	.93							
1.6	.94520	.94630	.94							
1.7	.95543	.95637	.95							
1.8	.96407	.96485	.96							
1.9	.97128	.97193	.97							
2.0	.97725	.97778	.97							
2.1	.98214	.98257	.98							
2.2	.98610	.98645	.98							
2.3	.98928	.98956	.98							
2.4	.99180	.99202	.99							
2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520



Conely Fisheries Inc.

General idea:

1. Generate realization u of U(0,1)
2. Return min{x: F(x) ≥ u}

If X is a random variable with a cumulative distribution function F that is strictly increasing and continuous on D={x|0<F(x)<1}, then $F^{-1}(u)$, the inverse, returns the value of x with F(x)=u. So:

1. Generate realization u of U(0,1)
2. Return $F^{-1}(u)$

Question 2c and 1

How would you simulate daily earnings at Rockport?

What are daily earnings at Gloucester?

	A	B	C	D	E	F	G
1	Day	Random Number for Demand	Demand in Rockport (kg)	Quantity Sold (kg)	Random Number for Price	Price in Rockport ($/kg)	**Daily Earnings ($)**
2	1						
3	2						
4	3						
5	4						
6	5						

Daily Earnings Gloucester:

Daily Earnings Rockport:

Monte Carlo Simulation

Output Analysis: Estimating E(X)

The Sample Mean

$$\overline{X} = \frac{X_1 + \ldots + X_n}{n}$$

The Central Limit Theorem (for the sample mean)

If n is large (say n≥30), the sample mean $\overline{X}$ is approximately normally distributed with mean μ and standard deviation $\sigma/\sqrt{n}$.

- X_1, X_2 … X_n are independent random variables having the same distribution with expectation μ and variance σ^2

- The **sample mean** has an expected value of μ

$$E\left[\overline{X}\right] = E\left[\frac{X_1 + \ldots + X_n}{n}\right] = \frac{1}{n}\left[E[X_1] + \ldots + E[X_n]\right] = \mu$$

- The **sample mean** has a variance of σ^2/n

$$Var\left[\overline{X}\right] = Var\left[\frac{X_1 + \ldots + X_n}{n}\right] = \frac{1}{n^2}\left[Var[X_1] + \ldots + Var[X_n]\right] = \frac{\sigma^2}{n}$$

→ When n is large the standard deviation of the sample mean tends to zero!

Task 5

Random Variates (continous case)

2. Assume you run your simulation 5 times, i.e. you simulate a total of 5 days, X_1 being the sales on day 1, X_2 being the sales on day 2, and so on. You generated the following sales:

172	161	386	250	201

a. Calculate the sample mean.

b. Calculate the sample standard deviation.

c. What is the expected value of the sample mean?

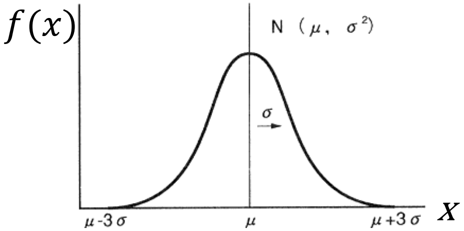
d. Calculate the standard deviation of the sample mean.

Task 5

Random Variates (continous case)


Photo by Joseph Gonzalez on Unsplash

Normal distribution with
$\mathrm{E}(X) = \mu, Var(X) = \sigma^2$



Standardize: $Z = \frac{X-\mu}{\sigma}$

2. Assume you run your simulation 144 times, i.e. you simulate a total of 144 days, X_1 being the sales on day 1, X_2 being the sales on day 2, and so on.
   a. What is the distribution of the mean number of popsicles sold?
   b. What is the probability that the mean number of popsicles sold is smaller than 275?

Monte Carlo Simulation

The Sample Mean

$$\overline{X} = \frac{X_1 + .. + X_n}{n}$$

The Central Limit Theorem

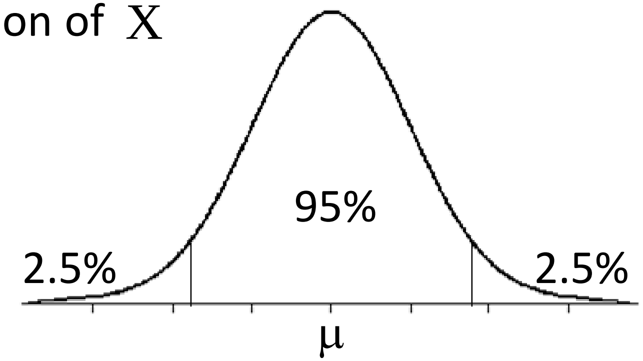
If n is large, the sample mean $\overline{X}$ approximately follows $N(\mu,\sigma^2/n)$.

The 95% Confidence Interval for the mean μ

$$\overline{X} \pm \underbrace{1.96 \frac{\sigma}{\sqrt{n}}}_{\text{Margin of Error (ME)}}$$

Output Analysis: A Confidence Interval for E(X)

Distribution of $\overline{X}$



There is a 95% probability that

$$\leq \overline{X} \leq$$

If we observe a sample mean $\overline{X}$, we can conclude:

- As n increases, the width of the confidence interval ....
- As the level of confidence increases, the width of the confidence interval ....
- As σ increases, the width of the confidence interval ....

Output Analysis: A Confidence Interval for E(X) when the variance is unknown

The 95% **Confidence Interval** for the mean μ (when n is sufficiently large):

$$\overline{X} \pm \underbrace{1.96\frac{S}{\sqrt{n}}}_{\text{Margin of Error (ME)}}$$

The Sample Variance

$$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2$$

Determining Sample Size (approximately)

$$n \geq \left(\frac{1.96\,S}{ME}\right)^2$$

- $X_1, X_2 \ldots X_n$ are independent random variables having the same distribution with expectation μ and variance σ^2
- The sample variance has an expected value of σ^2

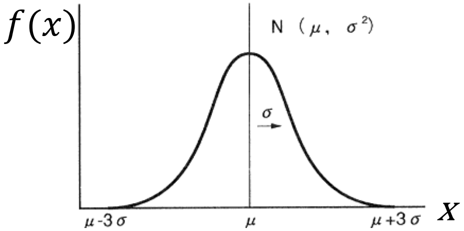
$$E\left[S^2\right] = E\left[\frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2\right] = \sigma^2$$

- If X follows a Normal distribution, then $\frac{\overline{X} - \mu}{S}$ approximately follows Student's t-distribution with k "degrees of freedom"; for moderately large n (around n>30), the Student's t-distribution can be approximated by a Normal distribution.

Task 5

Random Variates (continous case)

Normal distribution with
$\mathrm{E}(X) = \mu, Var(X) = \sigma^2$



Standardize: $Z = \frac{X-\mu}{\sigma}$

2. Assume you run your simulation 144 times, i.e. you simulate a total of 144 days, X_1 being the sales on day 1, X_2 being the sales on day 2, and so on.
   c. For a $\overline{X}$= 176 and s = 12, calculate the 95% confidence interval.
   d. Is the 99% confidence interval wider or more narrow?

Task 5

Standard normal distribution

Photo by Joseph Gonzalez on Unsplash

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

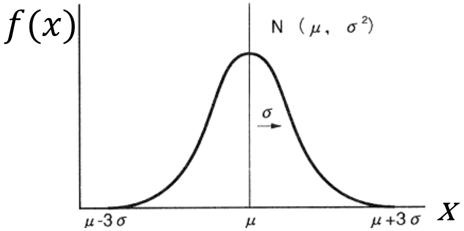
Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807

Task 5

Random Variates (continous case)

3. You decide to run the simulation for a total of 1,000 days this time.
   a. What is the distribution of your mean number of popsicles sold now?
   b. What is the probability that the mean number of popsicles sold is smaller than 275?

Normal distribution with
$\mathrm{E}(X) = \mu, Var(X) = \sigma^2$



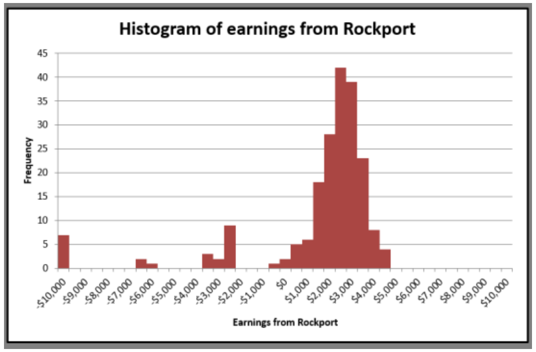
Standardize: $Z = \frac{X-\mu}{\sigma}$

Conely Fisheries Inc.

Questions 3-7

3. What is the shape of the probability distribution of daily earnings from using Rockport?
4. On any given day, what is the probability that Conley Fisheries would earn more money from using Rockport instead of Gloucester?
5. On any given day, what is the probability that Conley Fisheries will lose money if they use Rockport?
6. What are the expected daily earnings from using Rockport?
7. What would you advise Clint to do?

	A	B	C
1	Observed sample mean	$1,585	
2	sample standard deviation	$2,917	
3	P(Earnings>Gloucester)	0.7900	
4	P(Earnings<0)	0.1250	
5	95% confidence interval	$1,181	$1,990
6			



Take-Aways

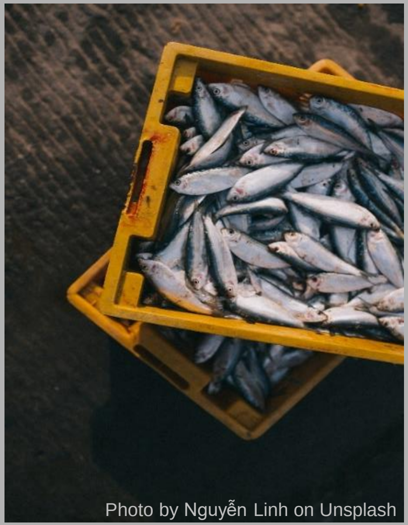


Photo by Nguyễn Linh on Unsplash

Simulation

- Monte Carlo Simulation can be used to capture uncertainties of any given distribution,
- we can to generate random numbers using LCG,
- we can generate realizations of a random variable using the inverse method,
- we can evaluate expected rewards of actions and their probability distributions,
- analyze risk via confidence intervals.