Quantitative Decision Making in Business

Summer University 2018

Topic 5: Decision Trees and Probabilities

Technische Universität Berlin

Lecturer:
Lissy Langer, M.Sc.
Chair of Production and Operations Management

1. Decision Trees

Picture : http://www.savagechickens.com/images/chickendecisions.jpg

Decision Trees

- Components
- Structure
- Building a Tree
- Solving for the Expected Monetary Value
- The Risk Profile
- Excursus on Probabilities
- Value of Information
- Sensitivity Analysis

Uploaded files:

Case Study 1: Mr. Jaeger's Vineyard

Introduction: An offer

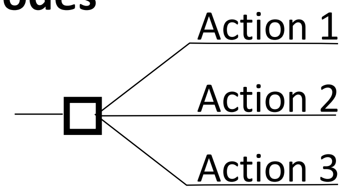
Asset N

Pays 5 when national
0 otherwise

Envelope tells you if the outcome of the coin flip was heads or tails

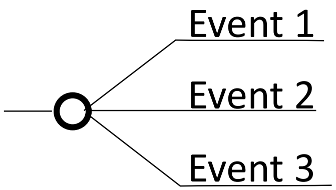
Components of a Decision Tree

Decision / Action Nodes



A **decision node** (*a square*) is a point in time when the decision maker makes a choice/ takes an **action**. Branches (*straight lines*) emanating from a decision node represent actions that the decision maker takes. The decision maker can choose which branch to follow.

Chance / Event Nodes



A **chance node** (a *circle*) is a point in time when we get to know the outcome of a random experiment. Branches (*straight lines*) emanating from a chance node represent events/ outcomes of the random experiment described by the node. The decision maker has no choice over which branch occurs. The probabilities listed on chance branches are conditional on the events that have already been observed on the sequence of branches leading to the chance node. The probabilities on branches leading out of a given node must sum to 1.

Structure of a Decision Tree

Flow of time / information

time →

The nodes and branches represent the random events and decisions of the problem proceeding from beginning (left) to end (right). The sequence of branches leading to a node from the left record the history of what has been observed up to that point in time. Branches to the right have yet to occur.

Payoffs

Payoff 1

Payoff 2

Payoff 3

Payoffs or costs are shown on the tips of the rightmost branches and take into account all of the decisions and events that preceded them.

“Solving” the Tree / Folding back:

← solve

We are looking for the maximal expected monetary value (EMV). Starting from the right of the tree and proceeding to the left (backwards in time):

- Take expected values at chance nodes.
- Take maximums (for payoffs) or minimums (for costs) at decision nodes.

Case Study 1: Mr. Jaeger‘s Vineyard

Read the case study to answer the following questions:

1. What are the main elements of Mr. Jaeger’s decision problem?
   a. What are the decisions Mr. Jaeger needs to make about the Riesling harvest? (Do not suggest which decision he should take, just list all of his options.)
   b. What are the uncertainties Mr. Jaeger is facing concerning the Riesling harvest?

2. How much revenue does Freemark Abbey obtain from its Riesling grapes if Mr. Jaeger decides to have them harvested before the storm?

3. How would you come to a decision whether to harvest the Riesling grapes right away or to wait for the storm (given risk-neutrality)?

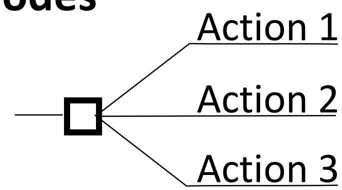
4. What is the maximum expected monetary value Mr. Jaeger can achieve?

Decision Trees

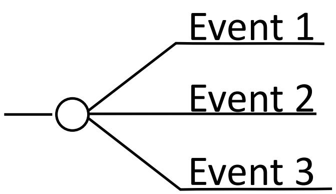
Case Discussion: Mr. Jaeger's Problem

Pictures: http://napavintners.blogspot.com/2009/10/ halloween-pics-from-freemark-abbey.html

Decision / Action Nodes



Chance / Event Nodes



Flow of Time:

time →

Payoffs

Payoff 1

Payoff 2

“Solving” the Tree:

← solve

Building a Decision Tree and Solving for Expected Monetary Value

1. List all decisions and uncertainties

2. Arrange them in a tree using decision and chance nodes

   time →

3. Label the tree with probabilities (for chance nodes) and payoffs (at least at the end of each branch).

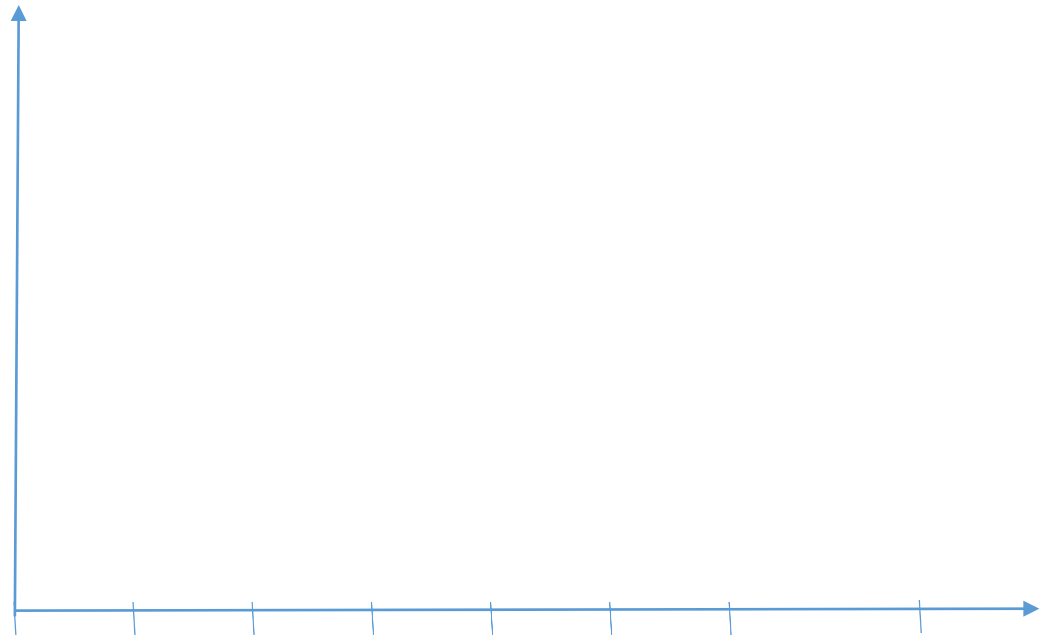
4. Solve the tree/ fold back by taking the maximum of expected payoffs/ minimum of expected costs for decision nodes and the expected value for chance nodes to get the EMV (expected monetary value)

   ← solve

Visualizing Risk

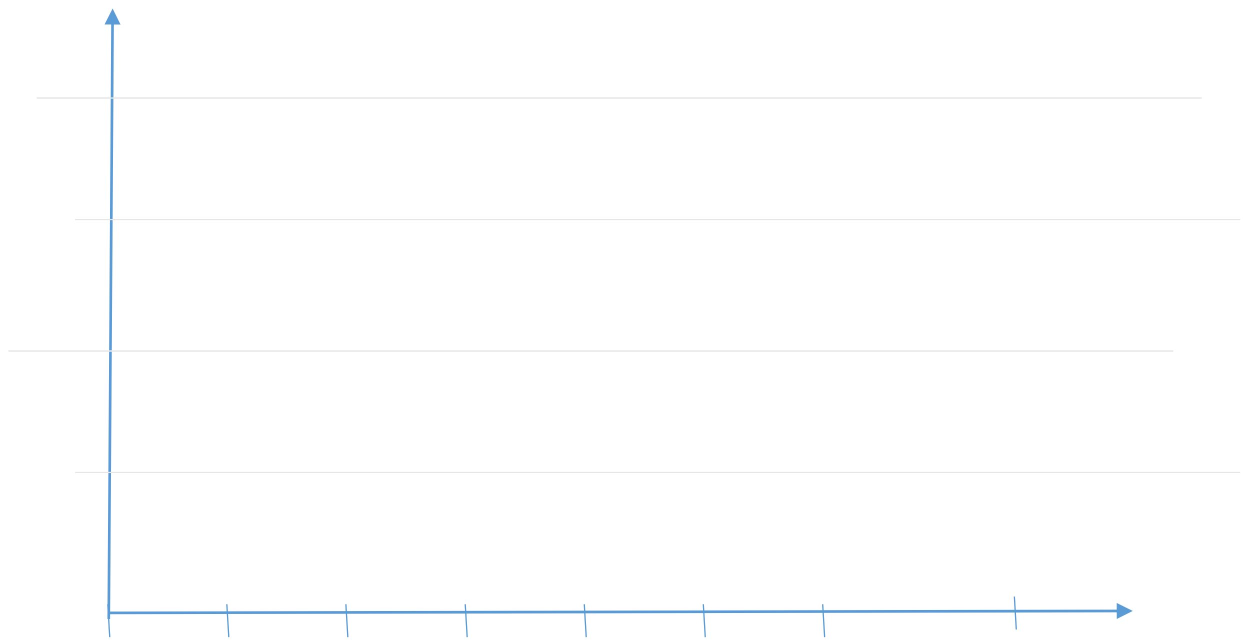
The Risk Profile
The probability mass function (PMF) of the net payoffs.

The risk profile can be viewed as a measure of riskiness (tighter means less risk).

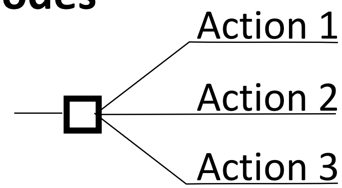


Visualizing Risk

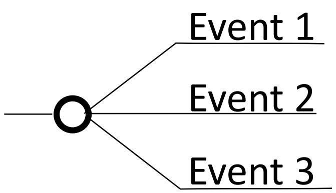
The Cumulative Risk Profile
The cumulative distribution function of the net payoffs.



Decision / Action Nodes



Chance / Event Nodes



Flow of Time:

time →

Payoffs

Payoff 1

Payoff 2

“Solving” the Tree:

← solve

Case Discussion (continued)

5. What is the maximum price Mr. Jaeger should be willing to pay for a perfect weather forecast that tells him with certainty if light or heavy rain with hail will develop?

Perfect vs. Imperfect Information

The Expected Value of (Perfect) Information
=EMV(with all information)-EMV(without added information)

The expected value of perfect information equals the EMV of the decision situation when all information is available minus the EMV of the decision situation when no additional information is available.
It is the most a risk-neutral decision maker is willing to pay for information.

The Expected Value of (Imperfect) Information
=EMV(with information)-EMV(without added information)

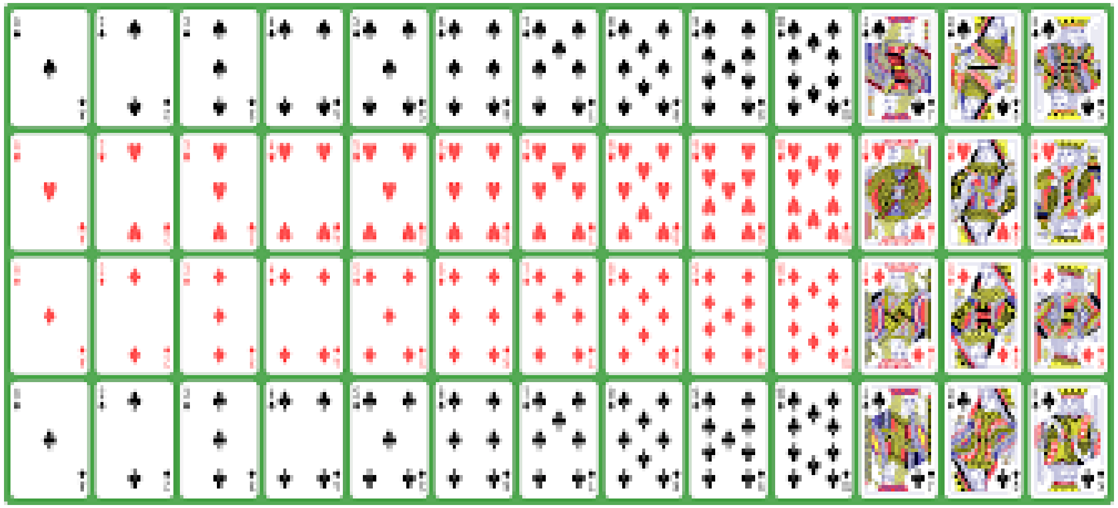
The expected value of imperfect information equals the EMV of the decision situation when a particular piece of information is available minus the EMV of the decision situation when no additional information is available.
It is the most a risk-neutral decision maker is willing to pay for this piece of information.

Probabilities

Photo by Jack Hamilton on Unsplash

Short excursus on probabilities

Say we have an ordinary deck of cards with 52 cards total:



Event: The card picked is…	Symbol	Probability
a spade.		
a seven.		
an odd numbered card.		
a jack, queen, or king.		

Short excursus on probabilities

Photo by Jack Hamilton on Unsplash

First law of Probability: The probability of any event is a number between zero and one. A larger probability corresponds to the intuitive notion of greater likelihood. An event whose associated probability is 1.0 is virtually certain to occur; an event whose associated probability is 0.0 is virtually certain not to occur.

Second law of Probability: If A and B are mutually exclusive events, then $P(A \text{ or } B) = P(A \cup B) = P(A) + P(B)$

Example
Event A = pick ten of diamonds
Event B = pick jack of any suit

Third law of Probability: If A and B are two events, then

$$\boldsymbol{P(A|B) = \frac{P(A \cap B)}{P(B)}} \quad \text{or} \quad P(A \cap B) = P(A|B) \times P(B)$$

Example
Event A = pick queen of any suits
Event B = pick a face card

Photo by Jack Hamilton on Unsplash

Short excursus on probabilities

distribution	EU-citizen	international	
Men			
Women			

percentage	EU-citizen	international	
Men			
Women			

When picking a random student from our class, what is the probability that we picked a male student?

When picking a random student from our class, what is the probability that we picked a male EU-citizen?

When picking a random student from our class, and we know we picked a male student, what is the probability that he is an EU-citizen?

Short excursus on probabilities

Photo by Jack Hamilton on Unsplash

Fourth law of probabilities: If A and B are independent events, then
P(A|B) = P(A)

Example
Event A = pick a five
Event B = pick a club

Third: $P(A|B) = \frac{P(A \cap B)}{P(B)}$

Implications of the fourth law of probabilities:

Third law:
P(A ∩ B) = P(A|B) x P(B)

Fourth law:
P(A|B) = P(A)

So:
P(A ∩ B) = P(A) x P(B)

Photo by Jack Hamilton on Unsplash

Short excursus on probabilities

Bayes' Theorem:

Third law:

$P(A \cap B) = P(A|B) \times P(B)$

$P(B \cap A) = P(B|A) \times P(A)$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

So: $$P(B|A) = \frac{P(A|B)\ x\ P(B)}{P(A)}$$

Example

Event A = pick queen of any suits

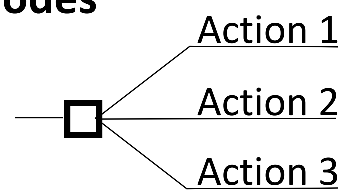
Event B = pick a face card

Case Discussion (continued)

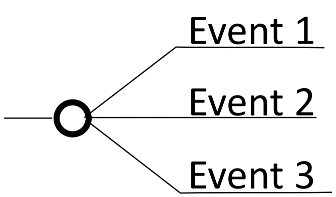
Mr. Jaeger also has the option to rent a Hightech weather detector for $4,000 per use. It provides a very accurate local forecast that might be extremely valuable, as the heavy rain is mostly only a local phenomenon. The machine, however, is not entirely reliable; 8 out of 10 times it predicted the heavy rain that occurred. For light rain, it has proven to be a bit more accurate, i.e. in 9 out of 10 times that there was an effective light rain, it actually predicted it. Mr. Jaeger is wondering if the rental might be worth a try.

What probabilities are given in the case extension? How does the decision problem change? Which probabilities do you need?

Decision / Action Nodes



Chance / Event Nodes



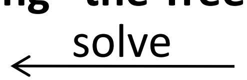
Flow of Time:

Payoffs

Payoff 1

Payoff 2

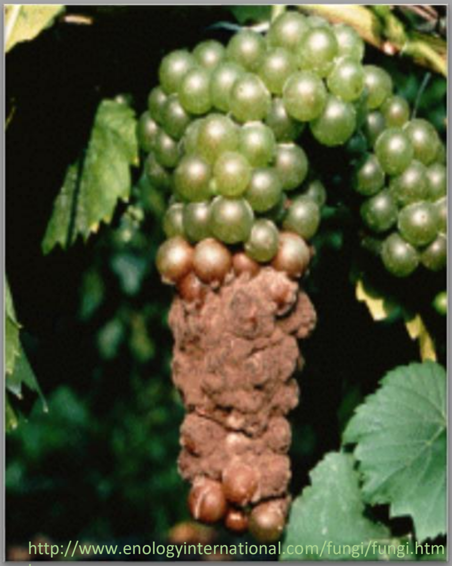
“Solving” the Tree:



Case Discussion (continued)

6. What is the maximum price Mr. Jaeger should be willing to rent the SuperDoppler? Would you advise him to rent it at the price given in the case?

Case Discussion (continued)



The trade magazine WineToday had also recently run an article about Mr. Harvey Borz of Borz's Direct Mail Mold Spores. Harvey sells botrytis spores and guarantees that if you use his spores and it rains lightly, botrytis mold will develop. One application would be enough to treat Freemark Abbey's Riesling grapes and would cost $40,000. Mr. Jaeger must pay the $40,000 upfront (before the storm).

7. Should Mr. Jaeger buy the spores for the suggested price?

Case Discussion (continued)

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7. Should Mr. Jaeger buy the spores for the suggested price?

Groupwork:

Task 1

Motion Pictures

Photo by Jakob Owens on Unsplash

The manager of a motion picture studio faces three marketing choices for a new film. He can either sell the film to a TV network for 10 million dollars or distribute the film as an "A" feature, or distribute the film as a "B" feature. The values associated with distributing the film as an "A" or "B" feature depends on whether the film is a box office success or not and are summarized in the table given below (all figures are in millions of dollars).

BOX OFFICE RESULT	DISTRIBUTE AS "A" FEATURE	DISTRIBUTE AS "B" FEATURE
Success	50	30
Failure	-20	-10

The probability of box office success has been assessed at 0.3.

a) Draw the associated decision tree.

b) Suppose that the motion picture company could somehow learn in advance if the movie will be a box office success or not. How much would that information be worth to the company?

c) A consultant suggested that an expenditure of 2 million dollars on a promotional campaign over the planning horizon will effectively increase the probability of a box office success to 0.4. Is it a good investment?

d) The consultant reconsiders its promotion strategy and comes up with a plan to target a more selected crowd and will sell this campaign to you at 2 million dollars. How much has the success probability to increase to make it an offer worth considering?

Groupwork: Task 1

Photo by Jakob Owens on Unsplash

Motion Pictures

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Groupwork: Task 1

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Motion Pictures

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Groupwork:

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Motion Pictures

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Motion Pictures

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Groupwork:

Task 2

The Rider Club

The Rider Club, has a small side business selling tractor lawn-mowers to their high-end members. Rider buys the mowers at a price of $4,800. Rider offers an installment plan at a net present value of $5,400 to their customers. The default rate of the customers on the installment plan is 10%. On average, Rider recovers $1,400 from defaulters. (Non-defaulters pay the full value of $5,400.)

Assume that customers who sign up for the installment plan can not buy tractor lawn-mowers from Rider if they were not offered the installment plan.

a) Would you recommend offering the proposed installment plan or should they stop offering it? What is the expected monetary value of the installment plan?

b) What is the expected value of information about a customer being a defaulter (D) or a non-defaulter (ND)?

c) The company can buy credit-rating information on customers before signing them up for an installment plan. This information will indicate whether a customer is a high-risk (HR) or a low-risk (LR). The credit-rating service claims the following conditional probabilities (obtained from historical frequencies): P(HR|D) = 0.65 and P(LR|ND) = 0.7. The rating service charges $20 per customer for this information. Should Rider buy this information?

Groupwork:

Task 2

The Rider Club

1. Would you recommend offering the proposed installment plan or should they stop offering it? What is the expected monetary value of the installment plan?

2. What is the expected value of information about a customer being a defaulter (D) or a non-defaulter (ND)?

Groupwork:

Task 2

The Rider Club

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The rating service charges $20 per customer for this information.

3. Should Rider buy this information?

Groupwork:

Task 3

Rusty Properties

Rusty Properties is planning to build a condominium development in El Segundo, California. The company is trying to decide between building a small, medium, or large development. The payoffs received for each size of development will depend on the market demand for condominiums in the area, which could be low or high. The payoff matrix (given in $1,000) is given. The owner of the company estimates a 25% chance that market demand will be low.

Size of Development	**Market Demand**	
	Low	High
Small	300	300
Medium	200	500
Large	-400	900

1. What decision should be made to maximize the expected monetary value?
2. Suppose the probability for low market demand was unknown. For what range should the company build which development?

Groupwork: Task 3


Photo by Jakob Owens on Unsplash

Rusty Properties

Size of Development	Market Demand	
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Groupwork: Task 3


Photo by Jakob Owens on Unsplash

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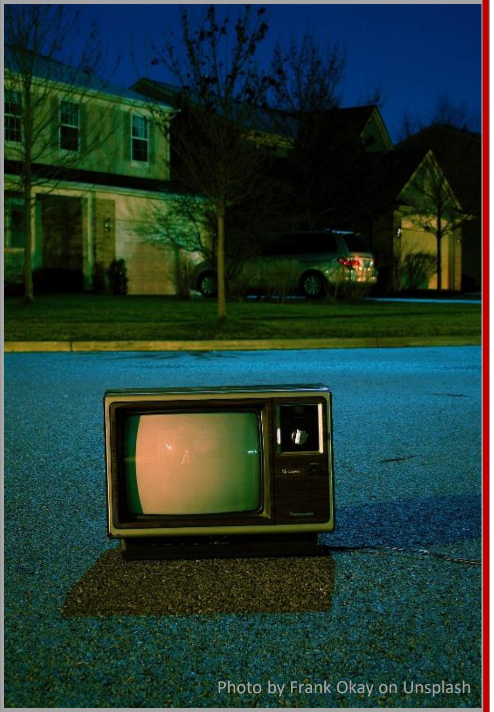


Photo by Frank Okay on Unsplash

Cable television

A cable television company is considering extending its services to a rural community. The company's managing director believes that there is a 0.60 chance that profits from the service will be high and amount to $760,000 in the first year (case H), and a 0.40 chance that profits will be low and amount to $400,000 for the year (case L). An alternative operation promises a sure profit of $500,000 for the period in question (case A).

a) Construct the decision tree and determine the decision to maximize the expected monetary value.

b) The company may test the potential of the rural market at a cost of $25,000. The test has a 90% probability of correctly detecting the state of nature, i.e. of forecasting high in case H and forecasting low in case L. Determine:

   i. the joint probability of the test yielding a high forecast and observing high profits;

   ii. the (marginal) probability of the test yielding a high forecast;

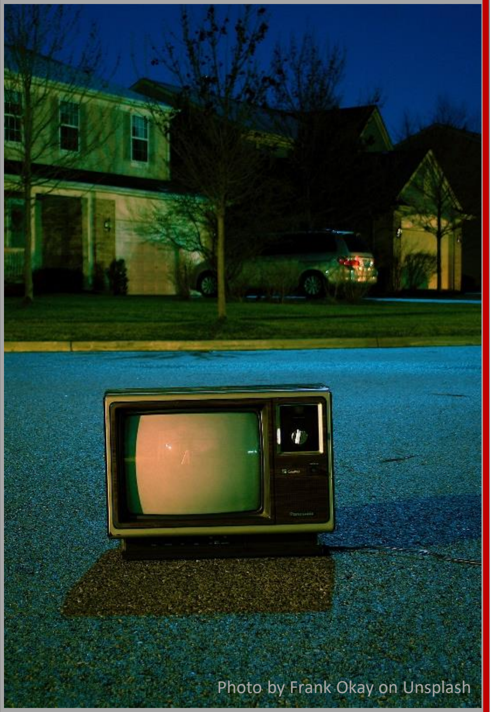
   iii. the probability of observing high profits given a high forecast;

   iv. and the probability of observing high profits given a low forecast.

c) Would you recommend buying the test described in part b?

Groupwork:

Task 4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Groupwork:

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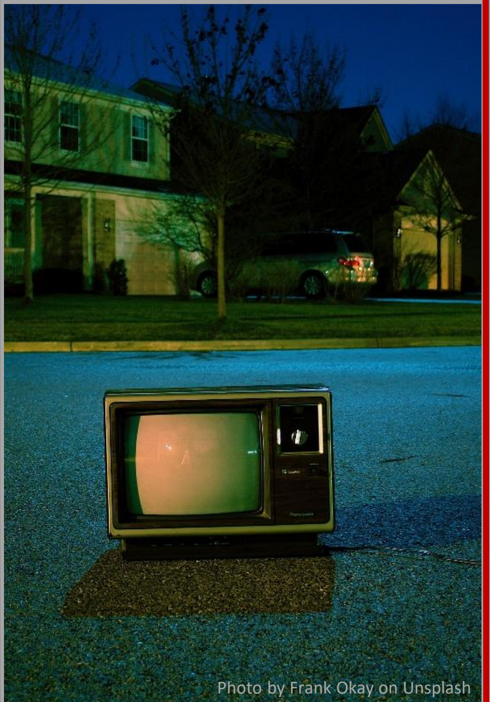

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c) Would you recommend buying the test described in part b?

	H	L	
TH	$P(TH \cap H) =$	$P(TH \cap L) =$	$P(TH)=$
TL	$P(TL \cap H) =$	$P(TL \cap L) =$	$P(TL)=$
	$P(H)=$	$P(L)=$	1

$P(L|TL))=$

$P(H|TH)=$

Groupwork:

Task 4

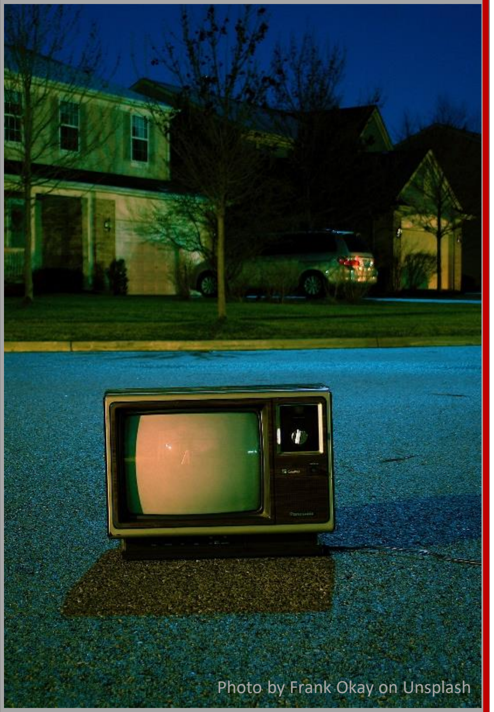

Photo by Frank Okay on Unsplash

Cable television

c) Would you recommend buying the test described in part b?

	H	L	
TH	P(TH ∩ H) =	P(TH ∩ L) =	P(TH)=
TL	P(TL ∩ H) =	P(TL ∩ L) =	P(TL)=
	P(H)=	P(L)=	1

P(L|TL))=

P(H|TH)=

Take-Aways

Decision Trees

- Decision Trees can be used to evaluate business situations where we face uncertainties and sequential decision making,
- we can determine the maximum EMV and perform sensitivity analysis on our decisions,
- we can analyze risks portfolios of different decisions,
- we can monetarize the value of additional information offered to us.

- We know how to apply probabilities in decision trees,
- we know how to derive joint and conditional probabilities,
- we know how to apply Bayes‘ theorem and the laws of probability.