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~~Bernoulli Trials~~

Geometric distribution

$$p(x) = \begin{cases} q^x p & , x = 0,1,2,\ldots \\ 0 & \text{o/w} \end{cases}$$

$$Mgf = E(e^{xt}) = \sum_{x=0}^{\infty} e^{xt} p(x)$$

$$= \sum e^{xt} q^x p$$

$$= \sum (qe^t)^x p$$

$$= p\left[1 + qe^t + (qe^t)^2 + \ldots\right]$$

$\downarrow (1-x)^{-1}$

$(1-e^t)$

-0

$$= \frac{p}{\cancel{q}[1-\cancel{q}e^t]}$$

$M_x t$

$M_x(t) \rightarrow$ Coeffic of $\frac{t^r}{r!}$

$$\frac{d^r}{dt^r} M_x(t)\Big|_{t=0}$$

$$\text{Mean} = \mu_1' = \frac{d\,M_x(t)}{dt}\Big|_{t=0}$$

$$\Rightarrow \frac{d\left(\frac{p}{\cancel{q}(1-\cancel{q}e^t)}\right)}{dt} = \boxed{\frac{+p}{q}\cancel{(1-e^t)^{-2}} \times e^t}$$

$t=0$

$$\frac{p}{q}(1-e$$

$$= \frac{d\, p}{\frac{d(1-qe^t)}{dt}}$$

$$\Rightarrow \frac{d\, p\,(1-qe^t)^{-1}}{dt}$$

$$= \left(-p\,(1-qe^t)^{-2} \times (-qe^t)\right)$$

$$= p\,q\,e^t\,(1-qe^t)^{-2}$$

$$= p\,q\,(1-q)^{-2}$$

$$= \frac{pq}{(1-q)^2} = \frac{pq}{p^2} = \frac{q}{p}$$

$$\mu_2' = \frac{d^2\,(M_x t)}{dt^2}\Bigg|_{t=0}$$

$$\mu_2' = \frac{d\,\frac{q}{p}}{dt^2}$$

$$\mu_2' = \frac{q}{p} + \frac{2q^2}{p^2}$$

Lack of Memory

↳ Suppose an event E can occur at one of the times $t_1 = 0, 1, 2, \ldots$ & the occurrence time X (waiting time) has its geometric distribution with parameter p then probabilit of

$$P(X = t) = q^t p \quad , t = 0, 1, 2, \ldots$$

Suppose we know that Event E has not occur before k. i.e

$X \geq k$

Let $Y = X - k$ so Y is the amount of additional time needed time to occur. then we can proof that probability of

$$\boxed{P[Y = t \mid X \geq k] = P[X = t]}$$

which implies that the additional time to wait has the same distribution as the initial time to wait.

Proof:-

RHS $P[Y = t / X \geq k] = \underline{P[Y \geq t / X \geq k]} - \underline{P[Y \geq t+1]}$ (A)

$$P[Y \geq t / X \geq k] = \frac{P(Y \geq t \cap X \geq k)}{P(X \geq k)}$$

$$= \frac{P[X-k \geq t \cap X \geq k]}{P[X \geq k]}$$

$$\frac{P[X \geq k+t \cap X \geq k]}{P[X \geq k]}$$

$$= \frac{P[X \geq k+t]}{P[X \geq k]} \quad (1)$$

Geometric dis $\rightarrow q^x p$

$$P[X \geq r] = \sum_{x=r}^{\infty} q^x p$$

$$= \sum q^r p + q^{r+1} p + q^{r+2} p \cdots$$

$$= \cancel{p}\left[\frac{q^r}{\cancel{1-q}}\right]$$

$$= q^r$$

from eq^n

$$= \frac{q^{k+t}}{q^k} = q^t$$

$$P[Y \geq t+1] = q^{t+1}$$

from eq A

$$= q^t - q^{t+1}$$

$$= q^t[1-q] = q^t[p]$$

$$= P[X = t]$$

RHS = RHS

Hyper geometric distribution :-

1) When population is finite & sampling is done without replacement so that events are dependent

although random
then we obtain. Hyper Geom. distr.

Eg Consider an urn with ~~capital~~ ~~N~~
'total $\underline{N}$ balls
with M white
& N − M are red. Suppose that
we draw a ~~small~~ sample of n balls
at random without replacement
from urn, then the prob of getting
small 'k' white balls where
$k \le n$ given by

$$= \frac{{}^{M}C_{k} \; {}^{N-M}C_{n-k}}{{}^{N}C_{n}}$$

P.M.F of
A discrete random variable with parameters
'N', 'M' & '

$$\text{pm.f} = \begin{cases} \dfrac{{}^{M}C_{k} \; {}^{N-M}C_{n-k}}{{}^{N}C_{n}} & k = 0,1,2,\ldots \min\{n, M\} \\ 0 & o/w \end{cases}$$

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N, M, n, x all are natural number.

Mean $\dfrac{nM}{N}$

Variance $\dfrac{NM(N-M)(N-n)}{N(N^2-1)}$

Imp.

Proof that Binomial distribution is the limiting case of Hyper geometric distribution.

H.G.D tends to B.D as

1) $N \to \infty$
2) $\dfrac{M}{N} \to p$

$$p(x) = \begin{cases} \dfrac{{}^{M}C_{k}\,{}^{N-M}C_{n-k}}{{}^{N}C_{n}} & ; k = 0, 1, 2, 3. \\ 0 & \text{o/w.} \end{cases}$$

$$= \frac{M!}{(M-k)!\,k!} \cdot \frac{(N-M)!}{(n-k)!\,(N-M-(n-k))!} \times \frac{n!\,(N-n)!}{N!}$$

$$= \frac{M(M-1)\ldots(M-(k-1))}{k!} \, \frac{(N-M)(N-M-1)\ldots(N-M-(n-k-1))}{(n-k)!}$$

$$= \frac{{}^{n}C_k \frac{M}{N}\left(\frac{M}{N}-\frac{1}{N}\right)\cdots\left(\frac{M}{N}-\frac{(k-1)}{N}\right)\cdot\left[1-\frac{M}{N}\right]\left[1-\frac{M-1}{N}\right]\cdots\left[1-\frac{M-(N-k-1)}{N}\right]}{1\left(1-\frac{1}{N}\right)\cdots\left(1-\frac{n-1}{N}\right)}$$

$$= \frac{{}^{n}C_k \underbrace{p\,(p)\cdots(p)}_{k \text{ times}}\ \underbrace{(1-p)\cdots(1-p)}_{(n-k) \text{ times}}}{1}$$

$$= {}^{n}C_k\, p^k (1-p)^{n-k}$$

$$= {}^{n}C_k\, p^k (q)^{n-k}$$

$$= \boxed{{}^{n}C_x\, p^x (q)^{n-x}}$$

$m-(n-k-1)$

$$\frac{n!}{N(N-1)\cdots(N-(n-1))}$$