$\boxed{Q.1}$ Let X be a discrete random variable taking values $\{a_1, a_2, \ldots\}$ s.t. $P(X = a_n) = p_n$. Also, let $Z = g(X, Y)$. Then show that:

$$E[Z] = \sum_n E[g(a_n, Y) \mid X = a_n]\, p_n .$$

$\boxed{Q.2}$ X and Y are independent random variables s.t. $X \sim \text{Uniform}((0,1))$ and also $Y \sim \text{Uniform}((0,1))$. Let

$$W = \max(X, Y) \quad \text{and} \quad Z = \min(X, Y).$$

Find the density for (a) $W - Z$ and (b) $W + Z$.

$\boxed{Q.3}$ Let X and Y be independent and identically distributed (i.i.d.) $G(0,1)$ random variables. Let $Z = a_1 X + a_2 Y$ and $W = b_1 X + b_2 Y$, where a_1, a_2, b_1 and b_2 are reals. Find $f_{ZW}(\cdot, \cdot)$.

$\boxed{Q.4}$ Let X & Y be independent random variables s.t. $X \sim \exp(\lambda_1)$ and $Y \sim \exp(\lambda_2)$ for $\lambda_1, \lambda_2 > 0$. Find $F_Z(\cdot)$ for (1) $Z = aX + Y$ (2) $Z = aX - Y$ (3) $Z = X/Y$ (4) $Z = \max\{X, Y\}$ and (5) $Z = \min\{X, Y\}$.

$\boxed{Q.5}$ Let $f_{XY}(x, y) = \begin{cases} 2e^{-(x+y)} & 0 < x < y < \infty \\ 0 & \text{otherwise.} \end{cases}$

Let $Z = X + Y$ and $W = Y/X$. Prove or disprove: Z and W and independent.

Q.6 X & Y are identically distributed random variables. Prove or disprove:

$$\mathrm{Cov}(X+Y, X-Y) = 0.$$

Q.7 $f_{XY}(x,y) = e^{-y}/y$ for $x \in (0,y)$ & $y \in (0,\infty)$

$= 0$ o.w.

(i) Are r.v.'s X & Y independent?

(ii) Compute $E[X^3 | Y=y]$.

Q.8 Let $Z \sim G(0,1)$. Find $\mathrm{Cov}(Z, Z^2)$.

Q.9 $X \sim \mathrm{Poisson}(\lambda_1)$ and $Y \sim \mathrm{Poisson}(\lambda_2)$, and X and Y are independent. Find F_Z when $Z = X + Y$

Q.10 X & Y are independent and identically distributed $G(0,\sigma^2)$. Define,

$$Z = aX + bY \text{ and } W = bX - aY,$$ where

a, b are non-zero reals.

By looking at the definitions of Z & W, state your opion about whether Z and W are independent. Now, verify your opion with hand analysis.