Q.1 Let $\{x_1, \dots, x_n\}$ be iid r.v. with marginal distr. function $F(\cdot)$. Let $X_{(k)}$ denote the k^{th} largest r.v.

(a) Find distribution of $X_{(k)}$.

(b) Find joint distribution of ($X_{(1)}$ and $X_{(n)}$)

(c) Find density of $X_{(1)}$ if $F = \exp(\lambda)$

Q.2 Let X_k's are iid $\exp(\lambda)$ random variables

(a) Find density of $\sum_{k=1}^{n} X_k$.

(b) Let $N \sim \text{Geometric}(p)$ r.v. independent of X_k's. Find density of $\sum_{k=1}^{N} X_k$.

$$P(N=n) = (1-p)^{n-1}p \quad \text{for } n = 1, 2, \dots$$
$$= 0 \quad \text{o.w.}$$

(c) Find $P\left(\sum_{k=1}^{n} X_k \le t, \sum_{k=1}^{n+1} X_k > t\right)$.

Q.3 Show that

$$E[g(X_1, X_2) \mid X_3]$$
$$= E\left[E[g(X_1, X_2) \mid X_2, X_3] \mid X_3 \right]$$

Q.4 Let X, Y and Z be iid Uniform$([0,1])$ random variables. Let

$$U = X + Y + Z$$
$$V = XY \quad \text{and} \quad W = YZ.$$

Find Joint density of U, V & W.