Programming Assignment Report

EE5121: Convex Optimization – Logistic Regression

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Objective

The goal of this assignment is to implement logistic regression from scratch using gradient descent. We study the sigmoid function and evaluate performance on synthetic and real-world datasets.

Problem Statement

Given the logistic regression loss function:

$$L(\theta) = -\frac{1}{n}\sum_{i=1}^{n}\left[y_i \log(\hat{y}_i) + (1-y_i)\log(1-\hat{y}_i)\right], \quad \text{where } \hat{y}_i = \sigma(x_i^\top \theta) = \frac{1}{1+e^{-x_i^\top \theta}}$$

we are asked to:

1. Derive the gradient of $L(\theta)$
2. Plot the sigmoid function
3. Implement logistic regression using gradient descent
4. Analyze convergence
5. Test the model on synthetic and real-world data

Gradient Derivation

Let $\hat{y}_i = \sigma(x_i^\top \theta)$. The gradient is:

$$\nabla_\theta L(\theta) = \frac{1}{n}\sum_{i=1}^{n}(\hat{y}_i - y_i)x_i$$

or in vectorized form:

$$\boxed{\nabla_\theta L(\theta) = \frac{1}{n}X^\top(\hat{y} - y)}$$

Sigmoid Function

The sigmoid function is defined as:

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

It maps real-valued inputs to (0, 1). It is commonly used to represent probability in classification tasks. The function is S-shaped and centered at $\sigma(0) = 0.5$.

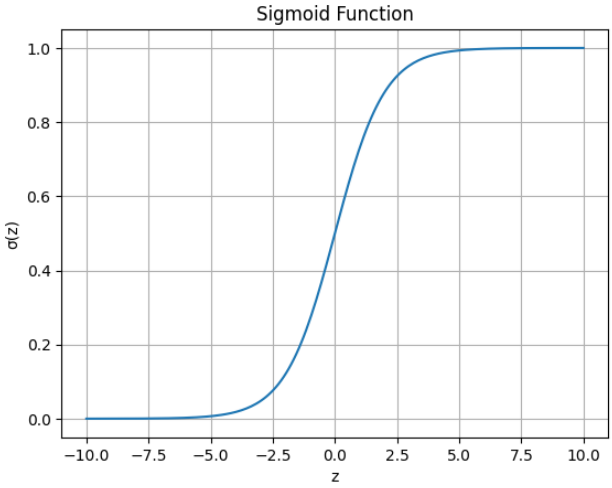

Figure 1: Sigmoid function $\sigma(z)$

Implementation Summary

The logistic regression model is trained using the following steps:

- Initialize θ
- For each iteration:
  a) Compute $\hat{y} = \sigma(X\theta)$
  b) Compute loss and gradient
  c) Update: $\theta \leftarrow \theta - \alpha \nabla_\theta L(\theta)$

Experiments and Results

Synthetic Dataset

- Generated using `make_classification()` from scikit-learn
- Input: 2 features + intercept
- Gradient descent shows smooth convergence

Real-World Dataset: Breast Cancer

- Used `sklearn.datasets.load_breast_cancer`
- Applied feature scaling and added bias term
- Achieved test accuracy: **97–99%**

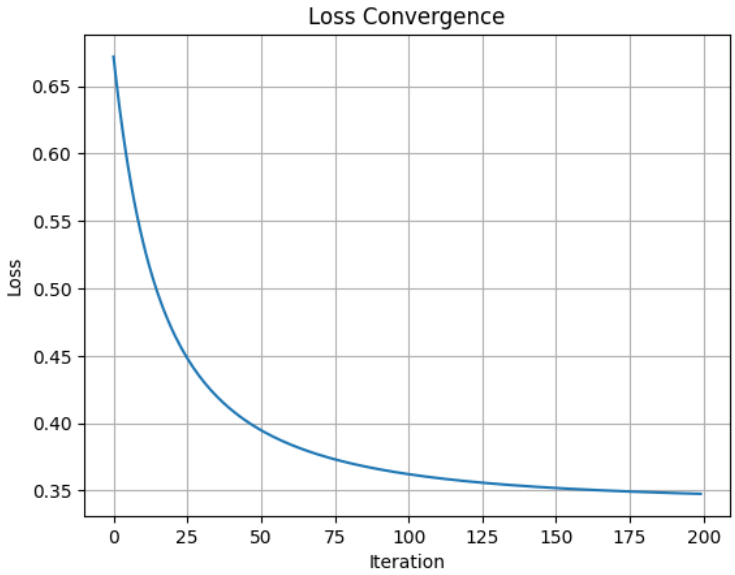

Figure 2: Loss vs. Iteration (Convergence Plot)

Conclusion

- Successfully derived and implemented gradient of logistic regression loss
- Built gradient descent from scratch
- Validated the model on synthetic and real-world datasets
- Observed strong convergence behavior and high accuracy on test data