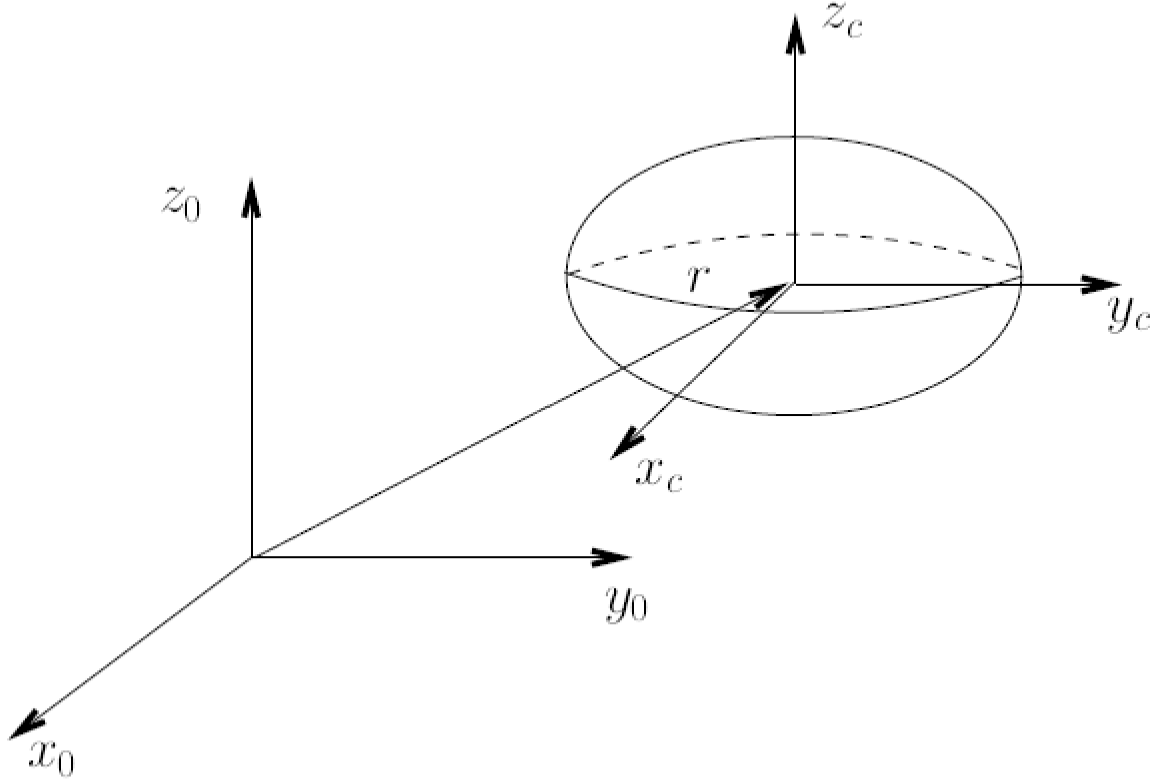
Manipulator Dynamics Including Link Inertia

But.....

real robots aren’t point masses?!?

Generic Representation of a Rigid Body



Kinetic Energy of Rigid Body

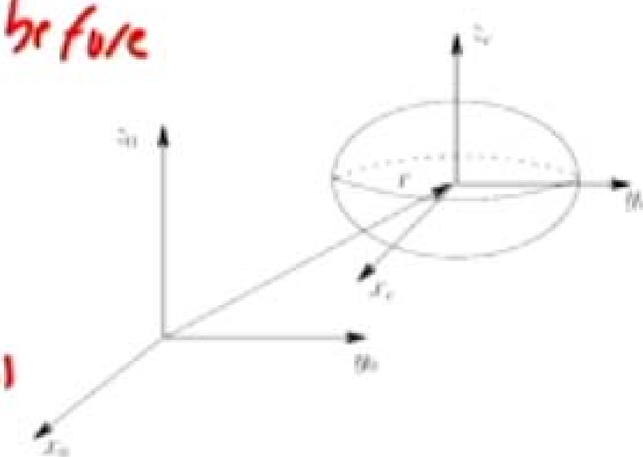
Total Kinetic Energy:

was 0 before

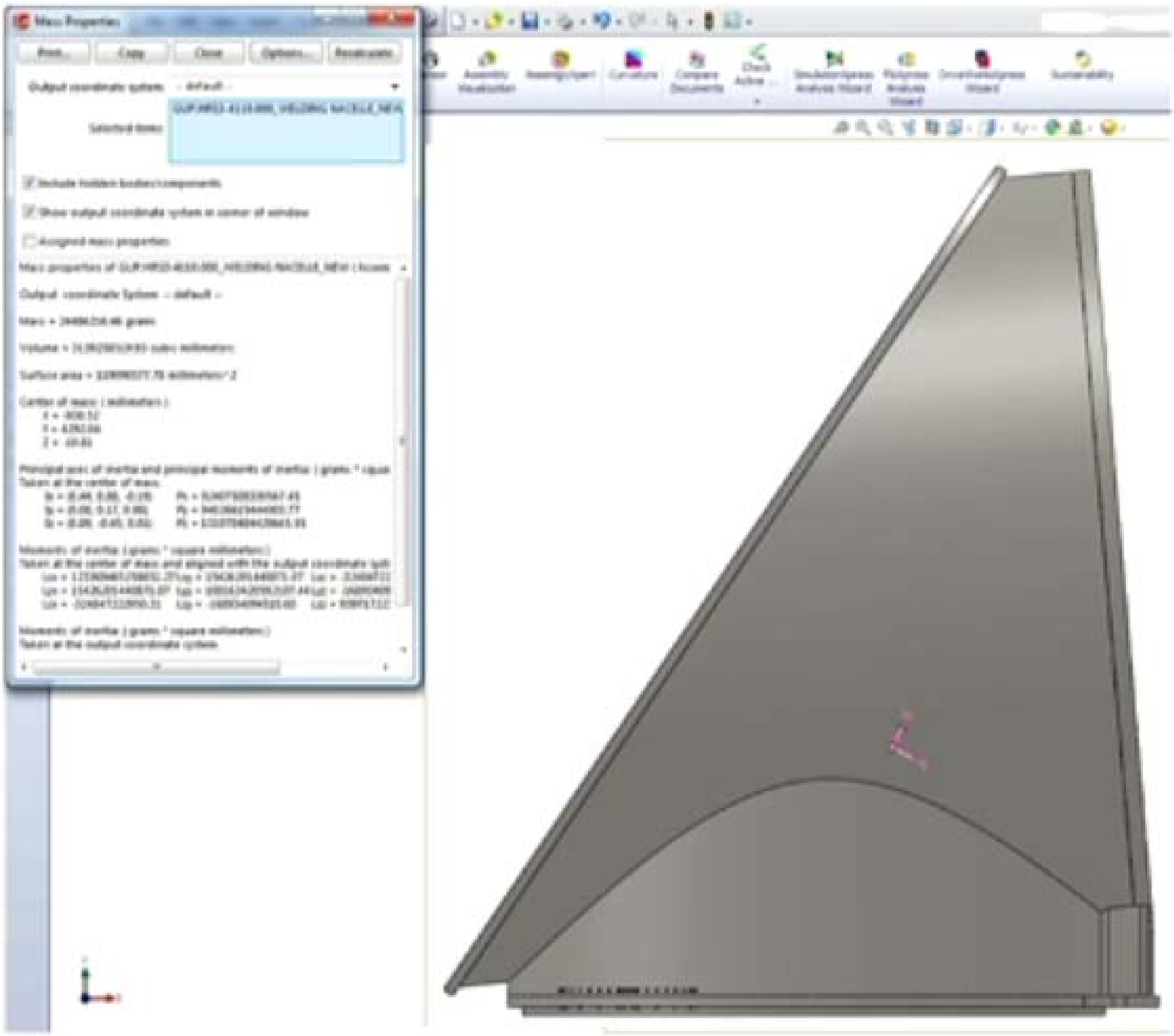
$$\mathcal{K} = \frac{1}{2} m v^T v + \frac{1}{2}\omega^T \mathcal{I}\omega$$

translational term

rotational term



Example Moments of Inertia



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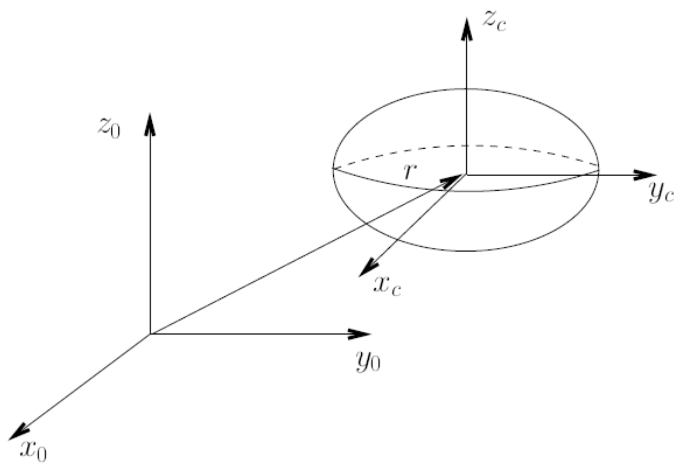

Transforming Inertia Reference Frame:

$$\mathcal{I} = R I R^T$$

Angular Velocities

Skew Sym Matrix

Angular Velocity of Rotating Frame:

3x3

$$\dot{R}R^T = S(\omega_0).$$

3x1 vec

$$\dot{R} = S(\omega_o)R.$$

Where:

$$\omega_0 = R\omega$$

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Where:

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Combining Angular Velocities

Series of Rotations:

$$R_n^0 = R_1^0 R_2^1 \cdots R_n^{n-1}$$

Determining the Derivative:

$$\dot{R}_n^0 = S(\omega_n^0) R_n^0$$

Solving for the Angular Velocity

$$\omega_n^0 = \omega_1^0 + R_1^0 \omega_2^1 + R_2^0 \omega_3^2 + R_3^0 \omega_4^3 + \cdots + R_{n-1}^0 \omega_n^{n-1}$$

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Solving for the Angular Velocity

$$\begin{bmatrix} c & -s & 0 \\ s & c & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \theta_1 \end{bmatrix}$$

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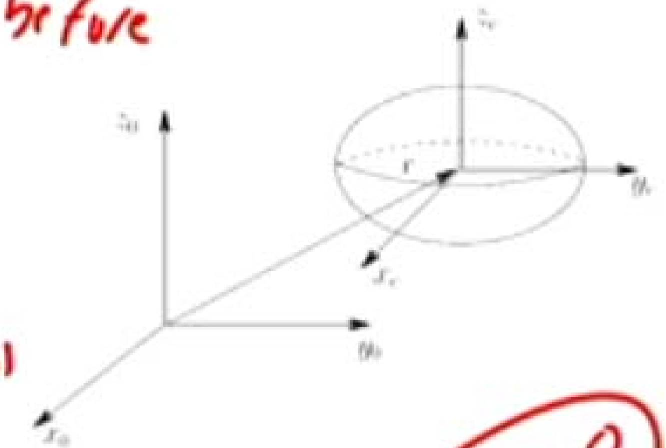

Transforming Inertia Reference Frame:

O

$R/\vec{a}$

E_i

$$\mathcal{I} = R I R^T$$

usually fixed to F_0

$T_i^0 = \begin{bmatrix} R_i^0 & p_i^0 \\ 0 & 1 \end{bmatrix}$