Homework #4

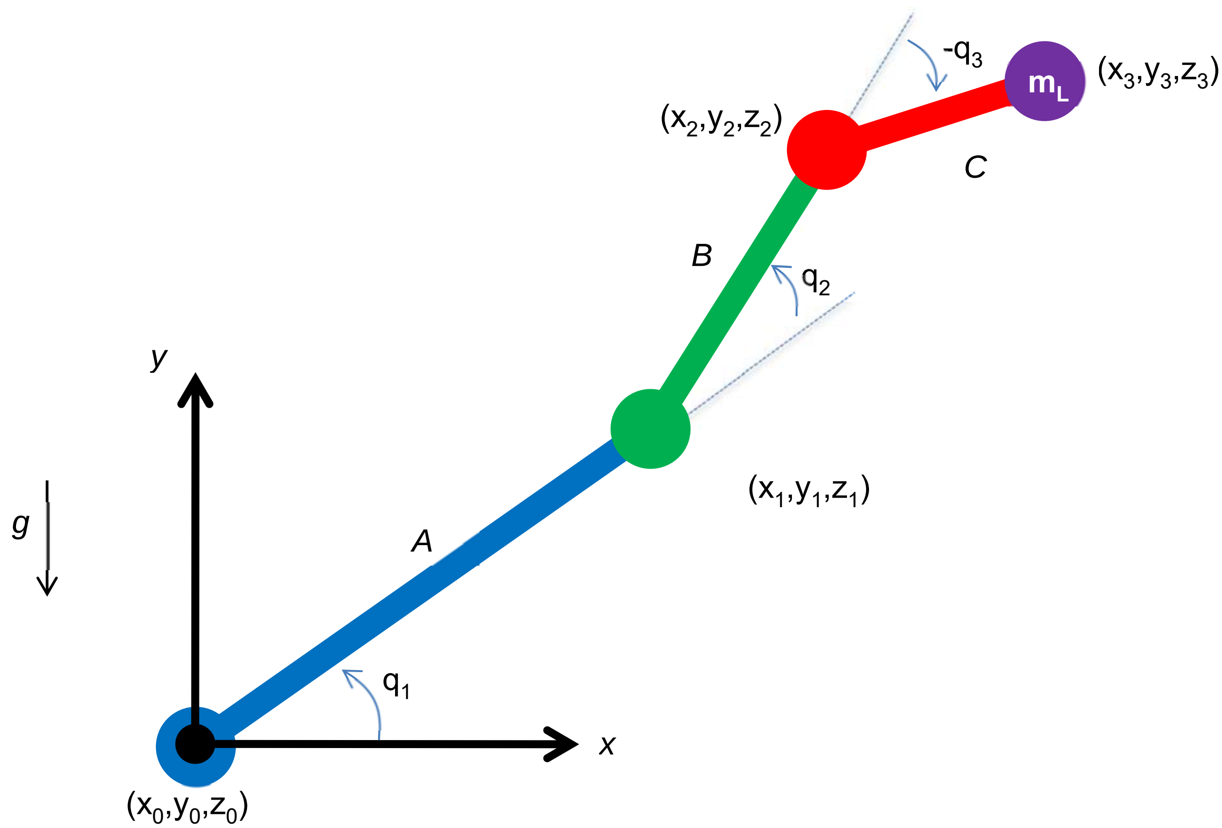
Robot Dynamics, Spring 2017

Due Tuesday, April 4 by 11:59 EST

Submit your work electronically through the Canvas submission link

The following questions are related to the figure of the 3-DOF robot arm below. The robot is a 3-link planar mechanism operating in the x-y plane (z=0). It can be assumed to have links of mass m_A, m_B, m_C with the masses located at the center of each link, and link lengths A, *B, C*. The joints are considered massless and have positive rotation relative to the previous link defined about the z-axis as shown, identified as q_1, q_2, and q_3, respectively. There is a load supported at the tip of Link 3 with a weight of m_L. Gravity is acting in the negative y_o direction at $g=9.8m/s^2$.

You are expected to perform the assignment using Matlab with the Symbolic Toolbox. You should define the terms as vector quantities (rather than individual scalars in each direction). Be sure to clearly define all steps and publish a clean output file for submission.



** Note that the intermediate frames and the tip frame pose are not specified, these intermediate frames may be defined as you choose and clearly identified.*

1. Position Kinematics:

   Symbolically calculate the forward position kinematics of the tip position (x_3, y_3, z_3) with respect to the base F_0. That is, define the 4x4 homogeneous transformation matrix for the base to the tip, T_0^3. *You are not required to use D-H parameters, but you must clearly show all intermediate steps.*

2. Velocity Kinematics:

   Symbolically calculate the forward velocity kinematics and write out in matrix form, clearly showing the full 6-DOF (6x3) Jacobian equation describing the 3 translational and 3 angular velocities of the tip at the end of the last link as a function the 3 joint velocities. *You must clearly show the full equation and identify what each of the elements correspond to, not just provide the Jacobian matrix.*

3. Numeric Kinematics Calculation:

   For this question, assume the configuration configured similarly to that shown in figure, where: A = 80cm, B = 40cm, C = 20cm and the current state of the configuration is $\theta_1 = 45^\circ$, $\theta_2 = 15^\circ$, $\theta_3 = -30^\circ$.

   a) Solve numerically for the tip position and orientation for the given configuration, present the result as a homogeneous transformation matrix, being sure to note units.

   b) Solve numerically for the full 6-DOF Jacobian for the given configuration. Then solve for the instantaneous tip velocity vector in Cartesian space with respect to the base frame F_0 if all three of the joint velocities are all set to 30°/sec (in positive direction about z-axis), being sure to note the units.

4. Joint Torque Calculation:

   a) For the same joint and link configuration in Question 3 *(assume massless links and joints for the purpose of this question)*, symbolically solve for the joint torques to support the weight of load m_L. That is, determine how you would solve for the necessary torques for gravity compensation.

   b) Numerically solve for the corresponding joint torques to perform gravity compensation (i.e. maintain its position) if m_L=1.5kg?

For the following questions, please use the following robot parameters:
***A* = *80*cm, *B* = 40cm, *C* = 20cm**
m_A = 2kg, m_B = 1kg, m_C = 0.5kg, m_L = 1.5kg

5. Lagrangian Dynamics:

   a) For the configuration described above, symbolically derive the kinetic and potential energy of the arm by modeling the links as point masses at the center of each of the links, with an additional load mass at the end of the last link (4 masses total).

   b) Solve for the Lagrangian for the 3-link arm shown. This should utilize the numbers provided above for the robot mass and link length parameters, leaving the joint parameters as symbolic values.

   c) Derive the dynamics of the arm using the Euler-Lagrange approach. Arrange the dynamics into the matrix-vector standard form for representing multi-link arm dynamics, labeling key components of the equation. Be sure to clearly identify the equation and do your best to factor into the standard form discussed in class.

6. Newtonian Dynamics:

   For the same configuration in Question 5, repeat the analysis to derive the system dynamics using the Newton-Euler approach. Again, utilize the numbers provided above for the robot mass and link length parameters, leaving the joint parameters as symbolic values.

   Be sure to clearly show the annotated free body diagrams of the links and organize and document the steps and results.

   Put in the same standard form and compare the results with that of Question 5.