Keshuai Xu

1.1 $P_{ga} = P_{gs}$

$P_{ga} = \omega_{ga} \cdot \tau_{ga}$

$P_{gs} = \omega_{gs} \cdot \tau_{gs}$

$\theta_a = n\theta_s$ →

↓ take derivative

$\dot{\theta}_a = n\dot{\theta}_s$ (103)

$\omega_{ga} = n\omega_{gs}$

$$\frac{P_{ga}}{\tau_{ga}} = n\frac{P_{gs}}{\tau_{gs}}$$

$$\frac{1}{\tau_{ga}} = n\frac{1}{\tau_{gs}}$$

$\frac{1}{n}\tau_{gs} - \tau_{gs}$

$$\boxed{\tau_{gs} = n\tau_{ga}} \quad (101)$$

1.2

(3)+(4) $J_a\ddot{\theta}_a + B_a\dot{\theta}_a - k_i i + \tau_{ga} = 0$ (102)

(102)+(5) $J_a\ddot{\theta}_a + J_s\ddot{\theta}_s + B_a\dot{\theta}_a + B_s\dot{\theta}_s + \tau_L \boxed{-\tau_{gs} + \tau_{ga}} - k_i i = 0$ (103)

(103)+(7) $J_a\ddot{\theta}_a + J_s\ddot{\theta}_s + B_a\dot{\theta}_a + B_s\dot{\theta}_s + \boxed{\tau_{ga} - \tau_{gs}} - k_i i$

$+ml^2\ddot{\theta}_s + b\dot{\theta}_s + mgl\sin(\theta_s) = 0$ (104)

τ_{gs}

1.2

(105) $\ddot{\theta}_a = n\ddot{\theta}_s$ (108)

(3)(4) (108)(105) $J_a = (-B_a n\dot{\theta}_s + k_i i - \tau_{gs} \cdot n^{-1}) \cdot (n\ddot{\theta}_s)^{-1}$ (106)

(5) $J_s = (-B_s\dot{\theta}_s - T_L + T_{gs}) \cdot \ddot{\theta}_s^{-1}$ (107)

(8)(106)(107)

$$J = \begin{matrix} -B_a n^2\dot{\theta}_s\ddot{\theta}_s^{-1} & + k_i i n\ddot{\theta}_s^{-1} & \boxed{\cancel{-\tau_{gs}\ddot{\theta}_s^{-1}}} \\ -B_s\dot{\theta}_s\ddot{\theta}_s^{-1} & - T_L\ddot{\theta}_s^{-1} & \boxed{\cancel{+T_{gs}\ddot{\theta}_s^{-1}}} \end{matrix} + ml^2 \quad (108)$$

(7) $T_L = ml^2\ddot{\theta}_s + b\dot{\theta}_s + mgl\sin(\theta_s)$ (109)

(108)(7)

$$J = \begin{matrix} -B_a n^2\dot{\theta}_s\ddot{\theta}_s^{-1} + k_i i n\ddot{\theta}_s^{-1} \\ -B_s\dot{\theta}_s\ddot{\theta}_s^{-1} \\ -b\dot{\theta}_s\ddot{\theta}_s^{-1} - mgl\sin(\theta_s)\ddot{\theta}_s^{-1} \end{matrix} \quad +ml^2 - ml^2$$

(simplify) (w/(9)) $\ddot{\theta}_s = \begin{pmatrix} (-B_a n^2 - B_s - b)\dot{\theta}_s \\ + k_i \cdot i \cdot n - mgl\sin(\theta_s) \end{pmatrix} \cdot J^{-1}$

(ans) $\ddot{\theta}_s = (-B\dot{\theta}_s + k_i \cdot i \cdot n - mgl\sin(\theta_s)) \cdot J^{-1}$ (110)

1.3 decouple the model of the gearbox from the motor. make life easier.

1.4 $\begin{cases} \frac{di}{dt} = \dots & 1 \\ \ddot{\theta}_s = \dots & 2 \end{cases}$ $2+1 = \boxed{3}$

1.5 (1,2) $V_{in} = iR + L\frac{di}{dt} + k_\omega n \dot{\theta}_s$

$$\frac{di}{dt} = (V_{in} - k_\omega n \dot{\theta}_s - iR) \cdot L^{-1}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} i \\ \theta_s \\ \dot{\theta}_s \end{bmatrix}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} (V_{in} - k_\omega \cdot n \cdot x_3 - x_1 \cdot R) \cdot L^{-1} \\ x_3 \\ (-B\, x_2 + ki \cdot x_1 \cdot n - mg\, l \sin(x_2)) \cdot J^{-1} \end{bmatrix}$$

state variables have been changed.

4.1

$$u = kp \cdot (-x_1) + kd \cdot (-x_2)$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \theta_s \\ \dot{\theta}_s \end{bmatrix}$$

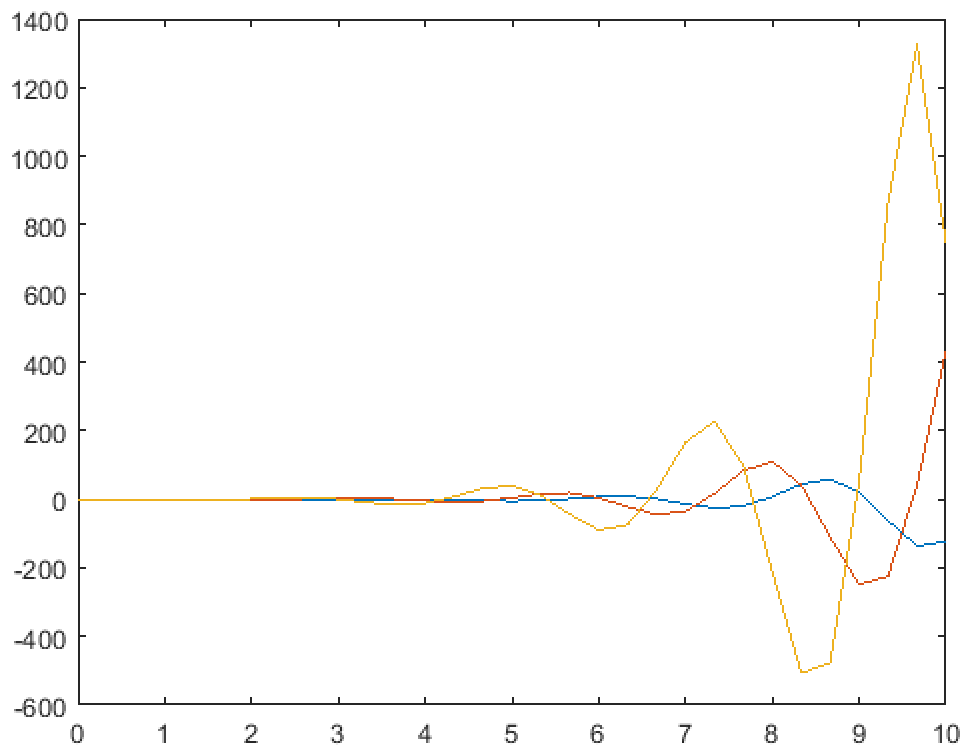
Table of Contents

Question 3.1

```
[XEuler, teu] = forSim(@sdof,[-pi/2 0
 0]',zeros(1,30),10,30,@eulerMet);
[XRK4, trk] = forSim(@sdof,[-pi/2 0
 0]',zeros(1,30),10,30,@rungeKutta4);
```

Euler

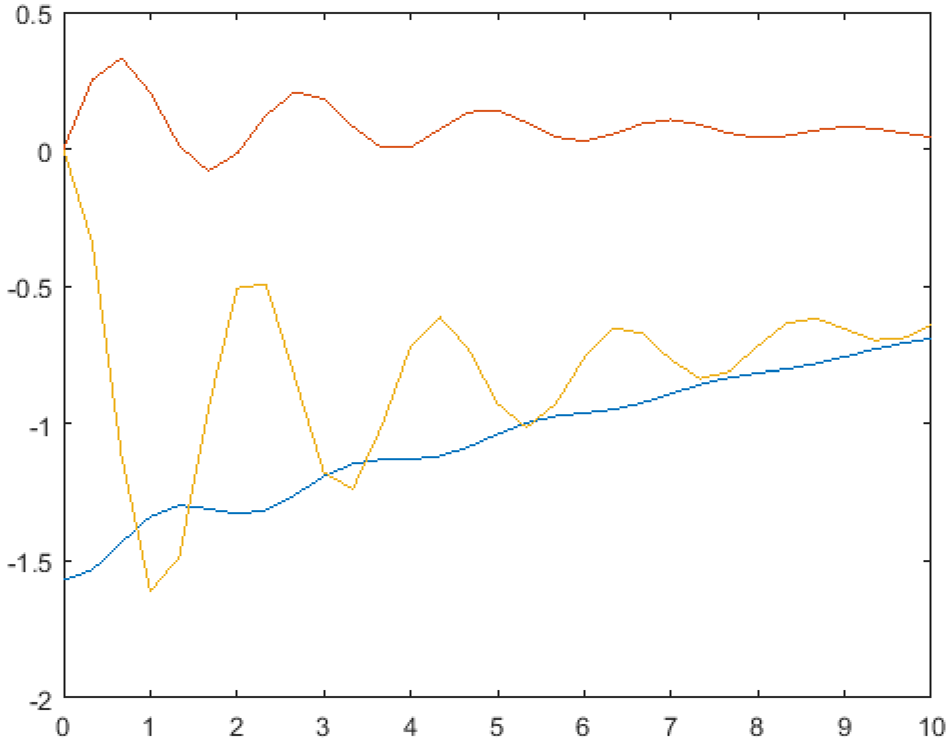
```
figure(1);
plot(teu, XEuler);
```



RK4

```
figure(2);
```

```
plot(trk, XRK4);
```



Pros and cons of RK4 compared to Euler

Pros - Better approximation when used with same step size. More robust. In figure 1, bad linear approximation of Euler method caused the values to explode.

Cons - More computational expensive.

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