Marwadi University	Marwadi University Faculty of Engineering & Technology Department of Information and Communication Technology	
Subject: Programming With Python (01CT1309)	**Aim:** Analysis of LTI System Responses to Standard Inputs Using Python	
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Aim: Analysis of LTI System Responses to Standard Inputs Using Python

IDE:

Analyzing Discrete-Time Systems Using Z-Transform

The Z-transform is used for analyzing discrete-time signals and systems. The Z-transform of a discrete-time signal $x[n]$ is given by:

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

where z is a complex variable, $X(z)$ represents the Z-transform of the signal.

Z-Transform Function

For an LTI system, the Z-transform function $H(z)$ is defined as:

$$H(z) = \frac{B(z)}{A(z)} = \frac{b_0 + b_1 z^{-1} + \cdots + b_m z^{-m}}{a_0 + a_1 z^{-1} + \cdots + a_n z^{-n}}$$

where $B(z)$ is the numerator polynomial, $A(z)$ is the denominator polynomial.

Stability

A discrete-time system is stable if all poles of its Z-transfer function lie inside the unit circle in the Z-plane. To check stability:

Calculate the poles of $H(z)$

Check if the magnitude of each pole is less than 1.

Causality

A system is causal if its impulse response $h[n]$ is zero $n < 0$. This generally means that the numerator polynomial should not have terms that depend on future values.

Time Invariance

A system is time-invariant if a time shift in the input results in an equivalent time shift in the output. For LTI systems, if the system is defined properly, it is generally assumed to be time-invariant.

Example

$$H(z) = \frac{(z^2 + 0.5)}{(z^2 - 1.5z + 0.5)}$$

Bode Plot Analysis

Stability:

- Check the gain and phase margins.
- Ensure that both margins are positive for stability.

Causality:

- Examine the magnitude and phase at low frequencies.
- Confirm that the system behaves as a causal system (magnitude starts lower, phase starts near 0 and decreases).

Time Invariance:

- If the system is LTI, it is inherently time-invariant.
- Analyse the impulse response (if available) to verify consistent responses to delayed inputs.

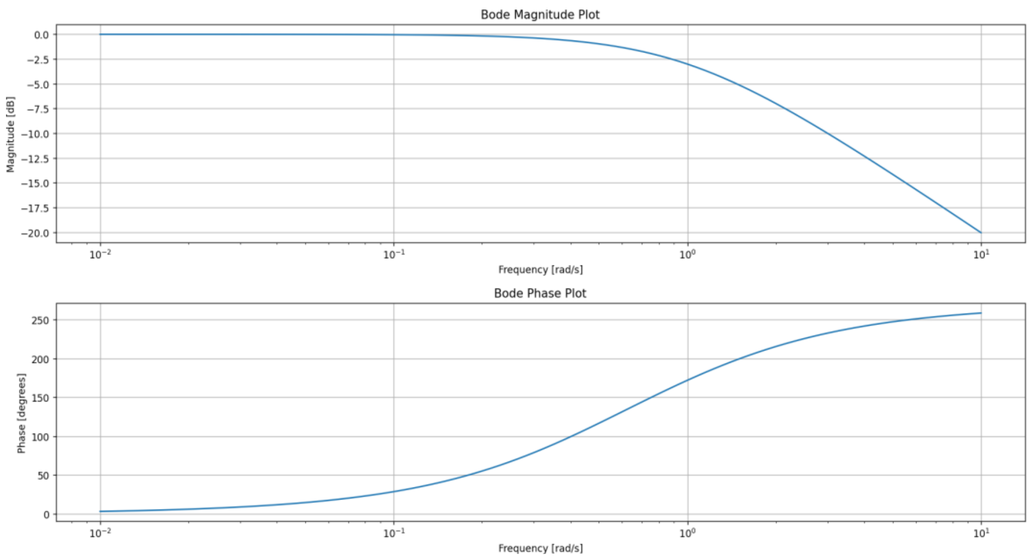
Python Implementation

```
import numpy as np

import matplotlib.pyplot as plt

from scipy.signal import TransferFunction, lti


def analyze_z_transfer_function(num, den):

    # Create a Transfer Function object

    system = TransferFunction(num, den)
```

```
# Get the poles and zeros
zeros = system.zeros
poles = system.poles
print("Zeros:", zeros)
print("Poles:", poles)
# Stability Analysis
stable = all(np.abs(pole) < 1 for pole in poles)
print("Stability:", "Stable" if stable else "Unstable")

# Causality Analysis
causal = all(num[i] == 0 for i in range(len(num) - 1) if num[i + 1] == 0)
print("Causality:", "Causal" if causal else "Non-Causal")
# Time Invariance Analysis
time_invariant = True  # For Z-transforms, generally time-invariant if system defined properly
print("Time Invariance:", "Time Invariant" if time_invariant else "Time Variant")
# Bode plot (magnitude and phase)
w, mag, phase = bode(system)
# Plot Bode plot
plt.figure(figsize=(12, 8))
plt.subplot(2, 1, 1)
plt.semilogx(w, mag)    # Bode magnitude plot
```

```
    plt.title('Bode Magnitude Plot')

    plt.xlabel('Frequency [rad/s]')

    plt.ylabel('Magnitude [dB]')

    plt.grid()

    plt.subplot(2, 1, 2)

    plt.semilogx(w, phase)  # Bode phase plot

    plt.title('Bode Phase Plot')

    plt.xlabel('Frequency [rad/s]')

    plt.ylabel('Phase [degrees]')

    plt.grid()

    plt.tight_layout()

    plt.show()

# Example: Analyzing a specific system H(z) = (z^2 + 0.5)/(z^2 - 1.5z + 0.5)

num = [1, 0.5]  # Numerator coefficients

den = [1, -1.5, 0.5]  # Denominator coefficients

analyze_z_transfer_function(num, den)
```

```
Zeros: [-0.5]
Poles: [1.  0.5]
Stability: Unstable
Casuality: Casual
Time invariance: Time Invariant
PS E:\python codes>
```



Transfer Function:

$$H(z) = \frac{0.5}{1 - 0.8z^{-1}}$$

Causality: This system is causal because the denominator has a non-negative exponent (i.e., all powers of z^{-1} are non-negative).

Stability: The system is stable if the poles (the roots of the denominator) lie inside the unit circle. Here, the pole is $z = 0.8$, which is inside the unit circle, so the system is stable.

Time Invariance: The system is time-invariant because the coefficients do not depend on time.

Transfer function:

$$H(z) = \frac{1 - z^{-1}}{1 - 0.5z^{-1}}$$

Causality: This system is causal.

Stability: The pole at $z = 0.5$ is inside the unit circle, making the system stable.

Time Invariance: The system is time-invariant

Post Lab Exercise:
https://github.com/keshvi1234/PWP_experiment

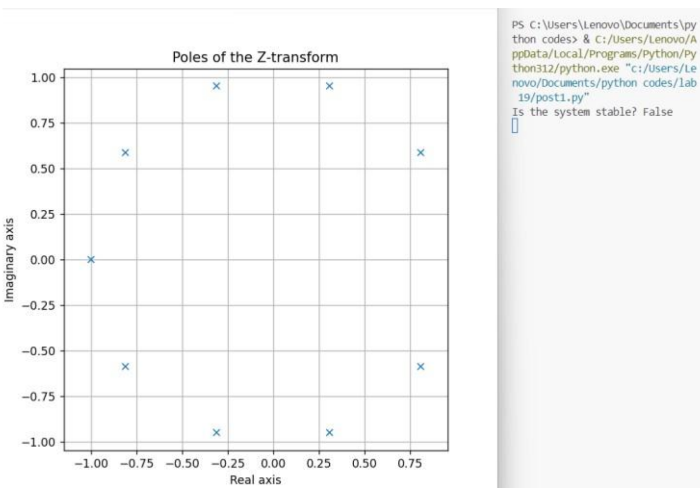
- Write a Python function to compute the Z-transform of an unit step function. verify whether the system is stable or unstable.

  Code:

```
import sympy as sp

import numpy as np

import matplotlib.pyplot as plt

n = 10

z_transform = [1] * n

poles = np.roots(z_transform)

is_stable = True

for pole in poles:

    if abs(pole) >= 1:

        is_stable = False

        break

print("Is the system stable?", is_stable)

plt.figure(figsize=(6, 6))

plt.plot(np.real(poles), np.imag(poles), 'x')

plt.xlabel('Real axis')

plt.ylabel('Imaginary axis')

plt.title('Poles of the Z-transform')
```

```
plt.grid(True)

plt.axis('equal')

plt.show()
```

Output:



- Implement this for the system $H(z) = \frac{0.5(z-0.7)(z-0.9)}{(z-0.6)(z-0.4)}$ and verify whether the system is stable or unstable.

Code:

```
import numpy as np

import matplotlib.pyplot as plt


num_coeffs = [0.5, -0.65, 0.315]

den_coeffs = [1, -1, 0.24]

def H_z(z):

    num = np.polyval(num_coeffs[::-1], z)
```

```
    den = np.polyval(den_coeffs[::-1], z)
    return num / den
def find_roots(coeffs):
    return np.roots(coeffs)
poles = find_roots(den_coeffs)
print("Poles:", poles)
stable = all(np.abs(pole) < 1 for pole in poles)
print("System is", "stable" if stable else "unstable")
w = np.linspace(0, np.pi, 1000)
z = np.exp(1j * w)
H = H_z(z)
plt.figure(figsize=(12, 6))
plt.subplot(121)
plt.plot(w, np.abs(H))
plt.xlabel('Frequency (rad/sample)')
plt.ylabel('Magnitude')
plt.title('Magnitude Response')
plt.grid()
plt.subplot(122)
plt.plot(w, np.angle(H))
plt.xlabel('Frequency (rad/sample)')
```

```
plt.ylabel('Phase (rad)')

plt.title('Phase Response')

plt.grid()

plt.tight_layout()

plt.show()
```

<u>Output:</u>