Introductory Astronomy

Week 2: Newton’s Universe

Clip 5: More Gravity

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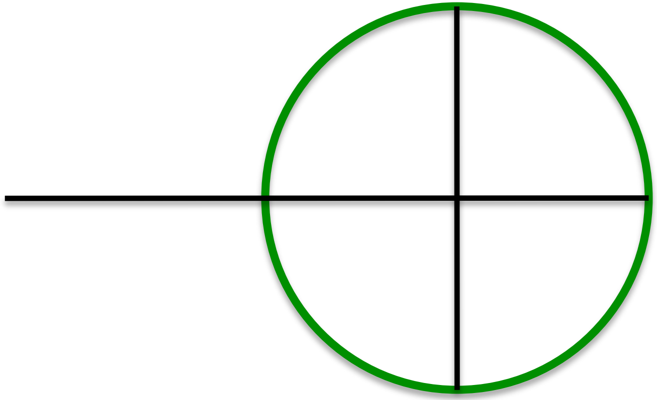
Working with Newton

- When I am near Earth, every bit of Earth exerts a bit of attractive force, directed towards it. To get total, add them up.
- Newton shows that for any round shell the total force it exerts is
- Adding it all up, outside Earth we can compute force by considering entire mass located at center



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Ronen's Gravitas

- Force on me $m_R = 59\,\mathrm{kg}$

$$
\begin{aligned}
F &= \frac{G M_\oplus m_R}{R_\oplus^2} \\
&= \frac{6.674 \times 10^{-11} \times 5.972 \times 10^{24}}{(6.371 \times 10^6)^2} m_R \\
&= 9.82 m_R = 579\,\mathrm{N}
\end{aligned}
$$

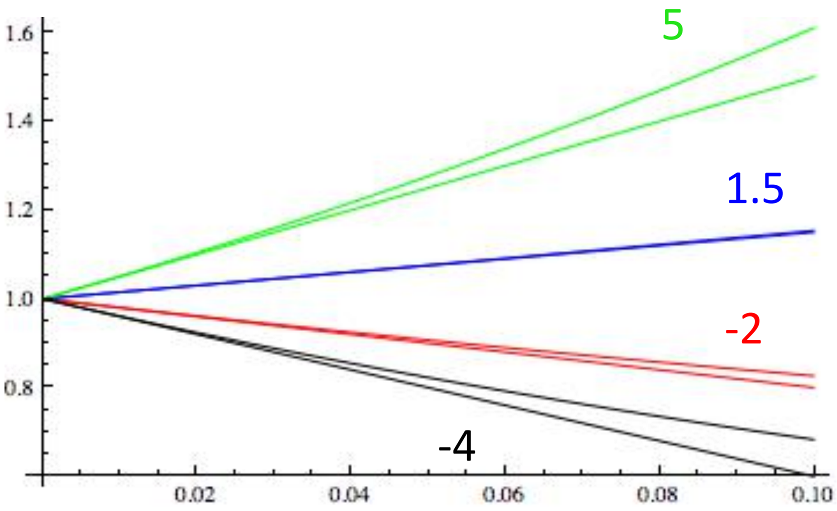
Gravity Here and There

- As I get further from Earth force decreases

$$F = \frac{GM_\oplus m_R}{(R_\oplus + h)^2}$$
$$= m_R g \left(\frac{R_\oplus}{R_\oplus + h}\right)^2$$
$$= m_R g \left(1 + (h/R_\oplus)\right)^{-2}$$
$$F \sim m_R g \left(1 - 2(h/R_\oplus)\right) \quad h << R_\oplus$$

- Newton:

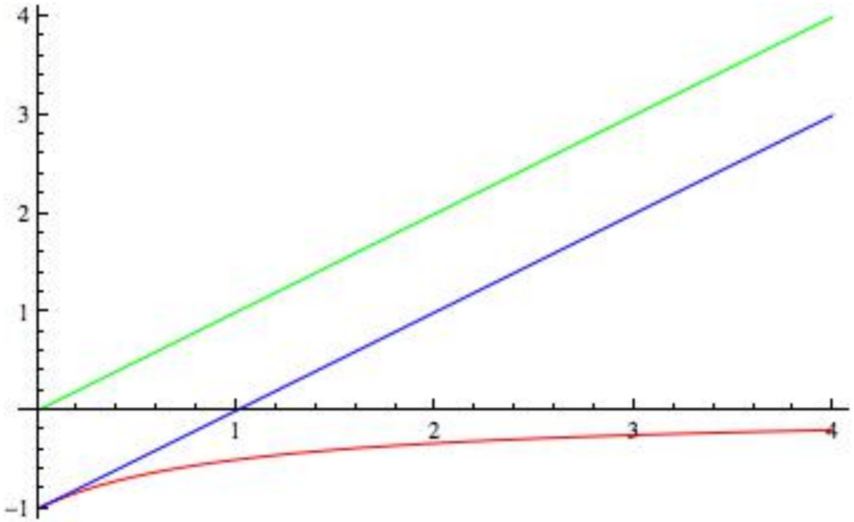
$$(1+x)^a \sim 1 + ax \quad x \ll 1$$



Potential Energy

- We said potential energy was mgh. This true if force constant so valid near surface.
- Since force decreases, height gain costs less energy at large distance. Find

$$U = -\frac{GM_{\oplus}m}{R}$$



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Energy in Orbit

- At radius R potential energy $U = -\frac{GMm}{R}$
- Speed $v^2 = \frac{4\pi^2}{KR} = \frac{GM}{R}$ so kinetic energy $\frac{mv^2}{2} = \frac{GMm}{2R}$
- Total energy $E = \frac{mv^2}{2} + U = -\frac{GMm}{2R}$
- Negative energy orbits are bound, closed
- Positive energy orbits unbound

The Principle of Equivalence

- S. Hawking is weightless because gravity is weaker in space?
- No! *h=400km* so

$$
\begin{aligned}
F &\sim mg\,(1-2(h/R_{\oplus})) \\
&= mg(1-2\frac{400}{6371}) = 0.87mg
\end{aligned}
$$

- Hawking is in free fall

In free fall there is no gravity

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Leftovers

- Earth is in free fall under gravity of Sun, so
- Sun's gravity has no effect on Earth!
- Almost none. There are remnants of gravity even in freefall: tidal forces
- These are due to the fact that gravitational acceleration is different at different points. So not all points of an extended object can possibly be simultaneously in free-fall
- Difference a_T in free-fall acceleration (from center of Earth) acts as a tidal "force" $F_T = ma_T$

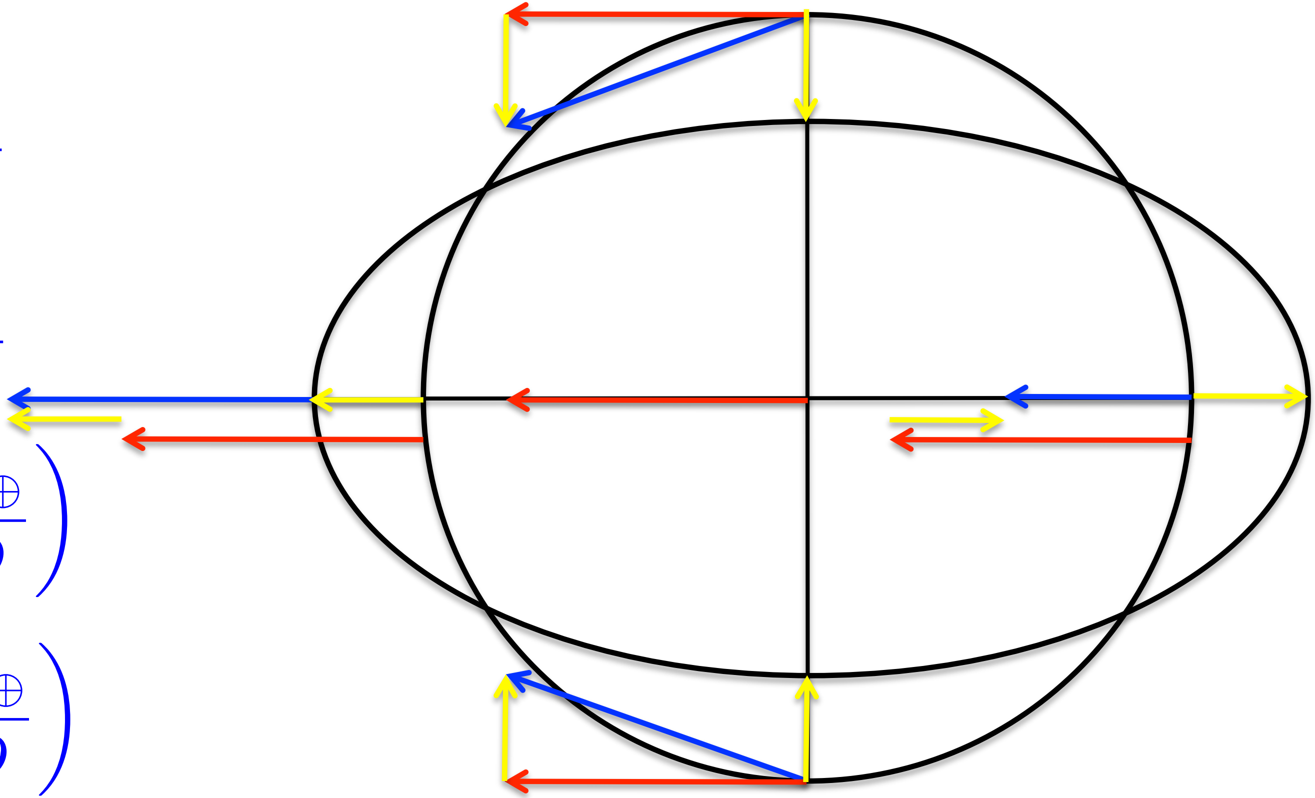
$$a_\oplus = \frac{GM_\odot}{D_\odot^2}$$

$$a_+ = \frac{GM_\odot}{(D_\odot - R_\oplus)^2}$$

$$a_- = \frac{GM_\odot}{(D_\odot + R_\oplus)^2}$$

$$a_+ \sim a_\oplus \left(1 + 2\frac{R_\oplus}{D}\right)$$

$$a_- \sim a_\oplus \left(1 - 2\frac{R_\oplus}{D}\right)$$



How Strong is this Force?

$$
\begin{aligned}
a_T^{\odot} &= \frac{2GM_{\odot}R_{\oplus}}{D_{\odot}^3} \\
&= 2\frac{GM_{\oplus}}{R_{\oplus}^2}\left(\frac{M_{\odot}}{M_{\oplus}}\right)\left(\frac{R_{\oplus}}{D_{\odot}}\right)^3 \\
&= 2g\left(\frac{1.989\times 10^{30}}{5.972\times 10^{24}}\right)\left(\frac{6371}{1.496\times 10^{8}}\right)^3 \\
&= 5.14\times 10^{-8}g
\end{aligned}
$$

What about the Moon?

$$
\begin{aligned}
a_T^{\mathrm{Moon}} &= a_T^{\odot}\left(\frac{M_{\mathrm{Moon}}}{M_{\odot}}\right)\left(\frac{D_{\mathrm{Moon}}}{D_{\odot}}\right)^{-3} \\
&= a_T^{\odot}\left(\frac{7.348\times 10^{22}}{1.989\times 10^{30}}\right)\left(\frac{3.844\times 10^{5}}{1.496\times 10^{8}}\right)^{-3} \\
&= 2.2a_T^{\odot}
\end{aligned}
$$

The Tides

- Moon deforms water so bulge faces Moon. As Earth rotates, bulge moves around Earth so tides repeat every 24h 44m
- Earth's rotation drags bulge East so lags Moon by about 12m
- Sun exerts tidal force towards Sun about ½ as strong. At full/new Moon act together creating intense spring tides. At quarter Moon counteract to create weak neap tides

Even More Tides

- When Moon formed – molten and closer - Earth's tidal forces deformed it so it froze with permanent bulge. Tidal forces keep this bulge aligned with direction to Earth: tidal locking is why we always see same side of the Moon
- Since tidal bulge on Earth is dragged East of Moon, tidal force of Moon tries to align it. This in fact slows Earth's rotation, transferring angular momentum to the Moon which thus recedes into higher orbit (G. Darwin, 1898)

What Now, Aristotle?

- Applying universal laws leads to unified understanding of many phenomena!
- In space, everything is in free-fall. Trajectories are Keplerian orbits. Internal structure controlled by tidal forces
- $\vec{F} = m\vec{a}$ is powerful. Learn more about matter and forces

Credits

- S. Hawking weightless: NASA
- Astronomy Animations: University of Nebraska-Lincoln Astronomy Education Group http://astro.unl.edu/
- Plots: Mathematica
- Demonstration videos: Duke Media Services
- M51: NOAO/AURA/NSF/T.A.Rector & M.Ramirez