Introductory Astronomy

Week 2: Newton's Universe

Clip 4: Gravity

Attractive Logic

- If an object of mass m moves in a circle of radius R with uniform speed v there must be a force acting, directed to center, of magnitude $F = mv^2/R$
- Moon orbits Earth so force directed towards Earth
- We notice Earth applies such a force to apples. Could this be the same force?
- Planets orbit Sun in (almost) circular orbits at (almost) uniform speed. Does Sun apply the force this implies on all planets?
- If so, Earth must apply a force to Sun directed towards Earth
- It all hangs together!!

Kepler and Newton

- Planet of mass m_P orbits Sun at radius R with speed v
- Force Sun applies to planet is thus $F = m_P v^2/R$
- Kepler says $v^2 = \dfrac{4\pi^2}{KR}$
- Find

It's Universal

- Sun applies a force given by $F = \frac{4\pi^2}{K}\frac{m_P}{R^2}$ to each planet, with same K
- So each planet applies force of same magnitude to Sun
- Law does not single out planet from Sun, so must have $F = \frac{GM_\odot m_P}{R^2}$ so $K = \frac{4\pi^2}{GM_\odot}$

Really Universal

- Earth also attracts the Moon, and all objects near it.
- So Moon and all other objects attract Earth.
- Everything attracts everything else!

$$F = \frac{G m_1 m_2}{R^2}$$

- Measured Newton's constant:

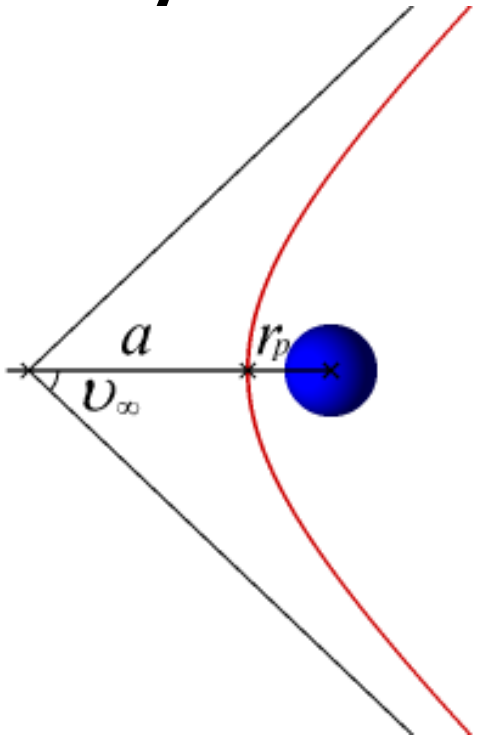
$$G = 6.67 \times 10^{-11} \, \frac{\mathrm{N\,m^2}}{\mathrm{kg^2}}$$

Thinking about Orbits

- Why doesn't the Moon fall on Earth?
- It does! Moon is constantly accelerating towards Earth. Orbiting is falling without ever hitting the ground

More Generally

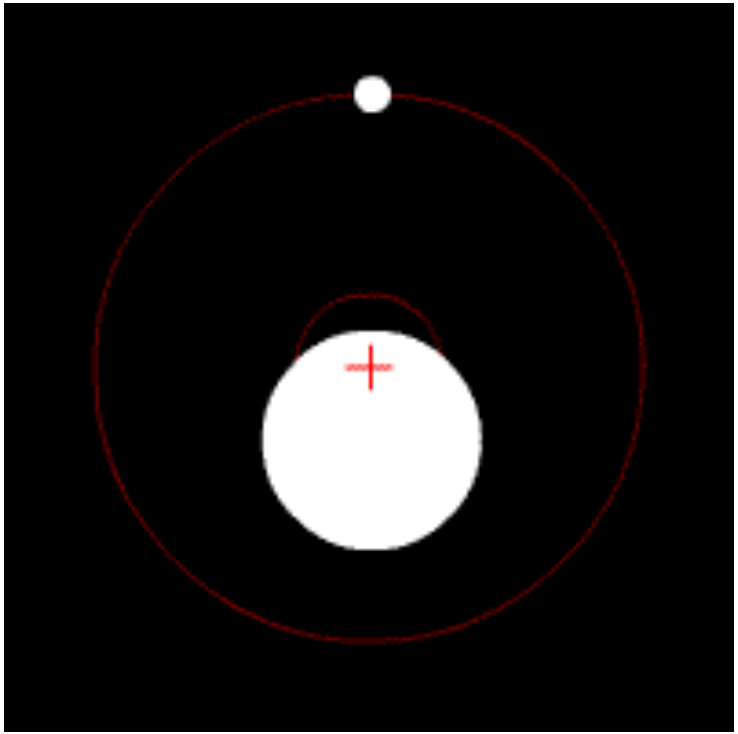
- Circular orbits at uniform speed are solutions to Newton's equations.
- So are elliptical orbits with Sun at focus satisfying Kepler II, as well as open hyperbolic trajectories



Even More Generally

- Newton's law is Universal. Apply to any two objects orbiting under mutual gravity. Find elliptical orbit about center of mass with $P^2 = Ka^3$

$$K = \frac{4\pi^2}{GM_{\text{total}}}$$



Example: Low-Earth Orbit

- ISS orbits at an altitude $h = 370\,\mathrm{km}$ so has orbital radius $R = 6371 + 370 = 6741\,\mathrm{km}$
- Period given by

$$\begin{aligned} P &= 2\pi \left(\frac{R^3}{GM_\oplus} \right)^{1/2} \\ &= 2\pi \left(\frac{\left(6.741 \times 10^6\right)^3}{6.67 \times 10^{-11} \times 5.972 \times 10^{24}} \right)^{1/2} \\ &= 5510\,\mathrm{s} = 91.8\,\mathrm{m} \end{aligned}$$

Wow

- Now we have the underlying, fundamental, universal laws
- It works: Halley predicts reappearance of comet to within days

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Credits

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- Demonstration videos: Duke Media Services
- Comet Halley: David Malin, Australian Astronomical Observatory