Aok 1 φυλ. 2

$X_1, \dots, X_{36}$ $\quad \mu = 78 \quad \sigma = 15$ $\quad P\left\{\sum_{n=1}^{36} X_n > 3000\right\}$

$X_1 + X_2$ $\quad \mu_{X_1+X_2} = \mu_{X_1} + \mu_{X_2}$

$\sigma^2_{X_1+X_2} = \sigma^2_{X_1} + \sigma^2_{X_2}$

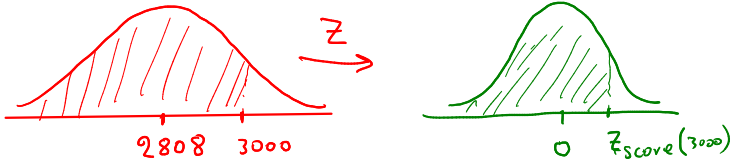
$Y \sim N(36 \cdot 78, 36 \cdot 15^2)$

$Y \sim N(2808, 8100)$

$P\{Y > 3000\} = 1 - P\{Y \le 3000\}$

$Y \to Z \sim N(0,1)$

$$Z = \frac{Y - 2808}{\sqrt{8100}} = \frac{Y - 2808}{90}$$



$$Z_{score(3000)} = \frac{3000 - 2808}{90} = 2.133$$

$P(Z \le Z_{score}(3000)) = 0.9834$

$P(Y > 3000) = 1 - 0.9834 = 0.016$ $\quad (1.6\%)$

Άσκηση

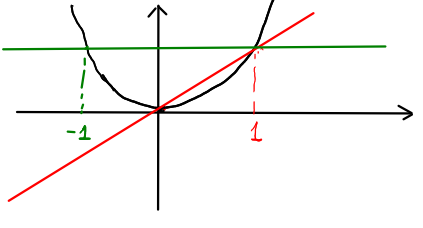
$\{-3x, 0, -2x, x, 8x\}$ $\quad x \neq 0$

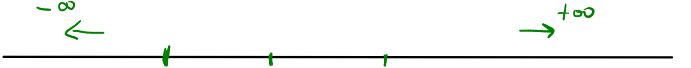
$x > 0 \longrightarrow \{-3x, -2x, 0, x, 8x\}$

$x < 0 \longrightarrow \{8x, x, 0, -2x, -3x\}$

Άσκηση

$\{x, x^2, 1\}$ $\quad x \in \mathbb{R}$





$x < -1$

$x < 1 < x^2$

$x \in (-1, 0)$

$x < x^2 < 1$

$x \in (0, 1)$

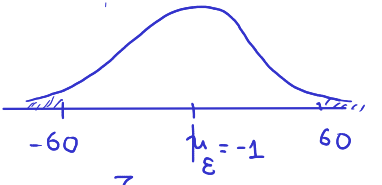
$x^2 < x < 1$

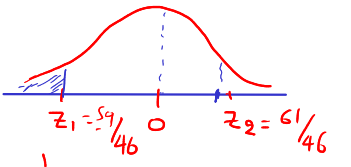
$x > 1$

$1 < x < x^2$

$\varepsilon \sim N(-1, 46^2)$

$P\{\varepsilon \notin [-60, 60]\}$





$P\{\varepsilon > 60\} + P\{\varepsilon < -60\}$

$1 - P\{\varepsilon \leq 60\} + P\{\varepsilon < -60\} =$

$= 1 - P\{Z \leq z_2\} + P\{Z < z_1\}$

$P\{Z < -\frac{59}{46}\} = 1 - P\{Z < \frac{59}{46}\}$

$$Z = \frac{\varepsilon - \mu_\varepsilon}{\sigma_\varepsilon} = \frac{\varepsilon + 1}{46}$$

$$z_1 = \frac{-60+1}{46} = \frac{-59}{46}$$

$$z_2 = \frac{60+1}{46} = \frac{61}{46}$$

$$P\{\varepsilon \notin [-60, 60]\} = 2 - P\{Z \le z_2\} - P\{Z \le |z_1|\} = 0.19$$

19 %

$\hat{y} = 1 + 3x$, $S_e = 2$, $SS_{xx} = 4$, $\bar{x} = 1$ Διαστήμα εμπ. για την μέση τιμή

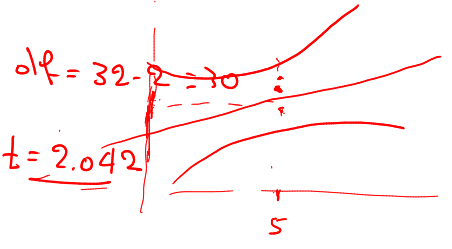
$N = 32$. $y = A + Bx + \varepsilon$ $x^* = 5$

$$\left[\mu_{\hat{y}|x^*} - t\, S_{\hat{\mu}|x^*},\ \mu_{\hat{y}|x^*} + t\, S_{\hat{\mu}|x^*}\right]$$

$\hat{y} = 1 + 3.5$

$$S_{\hat{\mu}|x^*} = S_e \sqrt{\frac{1}{N} + \frac{(x^* - \bar{x})^2}{SS_{xx}}} = 4.015$$

$\alpha = 0.05$



$\mu_{y|5} \in [7.8,\ 24.8]$ με πιθανότητα 95%

(b) $(1,2)$ $N' = N+1 = 33$

$df' = 31$

$$SS'_{xx} = \sum_{n=1}^{33} (x_n - \bar{x}')^2 = SS_{xx} + (x_{33} - \bar{x}')^2$$

$$\bar{x}' = \frac{1}{N'} \sum_{n=1}^{33} x_n = \frac{1}{33} \sum_{n=1}^{32} x_n + \frac{1}{33} = \frac{32}{33} \frac{1}{32} \sum_{n=1}^{32} x_n + \frac{1}{33} = \frac{32}{33} \bar{x} + \frac{1}{33} = 1$$

$\bar{y} = 1 + 3 \cdot 1 = 4$

$$\bar{y}' = \frac{1}{N'} \sum_{n=1}^{33} y_n \Rightarrow \bar{y}' = \frac{32}{33} \underbrace{\frac{1}{32} \sum y_n}_{4} + \frac{1}{33} \cdot 2 = \frac{32}{33} \cdot 4 + \frac{2}{33}$$

$$b' = \frac{SS'_{xy}}{SS'_{xx}} = \frac{SS'_{xy}}{SS_{xx}} < b$$

$$SS_{xy} = \sum_{n=1}^{33} (x_n - \bar{x})(y_n - \bar{y}')$$

$$= \sum x_n y_n - \frac{\sum x_n \sum y_n}{33} \approx SS_{xy}$$

$$s'_e = \sqrt{\frac{SS'_{yy} - b' SS'_{xy}}{N' - 2}}$$

Παράδειγμα (1,4)

$SS'_{xx} = SS_{xx}$ $\quad SS'_{yy} = SS_{yy}$

$\bar{y}' = \bar{y}$ $\quad \bar{x}' = \bar{x}$ $\quad SS'_{xy} = SS_{xy}$

$b' = b \quad a' = a$

$$S_e' = \sqrt{\frac{SS'_{yy} - b' SS_{xy}}{N' - 2}} = \sqrt{\frac{N-2}{N'-2}} \sqrt{\frac{SS_{yy} - b\, SS_{xy}}{N-2}} = \sqrt{\frac{30}{31}}\, S_e = 2\sqrt{\frac{30}{31}}$$