Compelling Propelling

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1 Hill Peterson, Problem 5.3

3. The idling engines of a landing turbojet produce forward thrust when operating in a normal manner, but they can produce reverse thrust if the jet is properly deflected. Suppose that while the aircraft rolls down the runway at 150 km/h the idling engine consumes air at 50 kg/s and produces an exhaust velocity of 150 m/s.

a. What is the forward thrust of this engine?
b. What are the magnitude and direction (i.e., forward or reverse) if the exhaust is deflected 90° without affecting the mass flow?
c. What are the magnitude and direction of the thrust (forward or reverse) after the plane has come to a stop, with 90° exhaust deflection and an airflow of 40 kg/s?

First, we easily see that **Part A** is asking for thrust; then we must ask ourselves: do any of the given variables, when combined, provide thrust? If this is not clear for you then let me help show whether or not these variables provide thrust.

I will start with Newton's Second Law:

$$F = ma \tag{1}$$

Backtrack Newton's equation into a differential and we can see a few things, don't forget about product rule,

$$F = \frac{d}{dt}(m \cdot \vec{v}) \tag{2}$$
$$= \frac{dm}{dt} \cdot \vec{v} + m \cdot \frac{d\vec{v}}{dt} \tag{3}$$

This problem can be a bit difficult, as we notice an acceleration term. This could be confusing to understand, but there is no acceleration in this instant. Conversely, the acceleration term is only produced by the mass flow rate, but we are not solving for acceleration (or deceleration). Instead, we are focused on thrust. Keep in mind that we should assume that the mass flow rates at the inlet and exhaust are equal due to continuity.

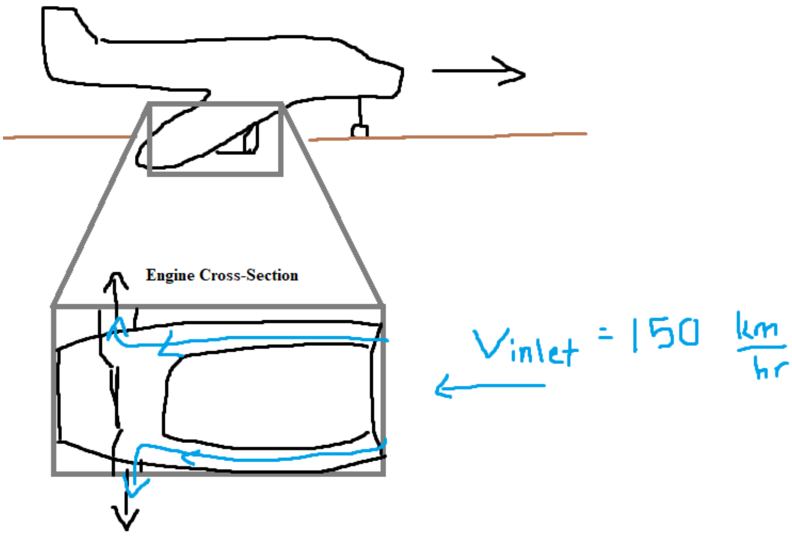
$$\left(\frac{dm}{dt}\right)_{inlet} = \left(\frac{dm}{dt}\right)_{exhaust} \tag{4}$$

With this in mind, we need to include both exhaust and inlet thrusts. Exhaust and inlet have opposite signs — something I can explain at a different time. Utilize equation 3 and equation 4,

$$T = \left(\frac{dm}{dt} \cdot \vec{v}\right)_{exhaust} - \left(\frac{dm}{dt} \cdot \vec{v}\right)_{inlet} \tag{5}$$
$$= \frac{dm}{dt} \cdot (v_{exhaust} - v_{inlet}) \tag{6}$$

$$T = \frac{dm}{dt} \cdot (v_{exhaust} - v_{inlet}) = 50(150 - 150 \cdot \frac{1000}{3600}) = \boxed{5416.7N}$$

Second, **Part B** wants to see what happens when thrust is deflected 90 degrees. Do not forget, thrust is defined as a horizontal movement relative to the ground.



Upon viewing my semi-impromptu diagram, we can see that the vertical mass flow rate (and consequently velocity) are now pointing upwards/downwards at 90 degrees. **Note: mass is a scalar quantity, so velocity is the vector in our thrust equation.**

Isolating the horizontal forces, we can adjust equation 5 to obtain the following:

$$T = \left(\frac{dm}{dt} \cdot v \cdot cos(\theta)\right)_{exhaust} - \left(\frac{dm}{dt} \cdot v \cdot cos(\theta)\right)_{inlet} \tag{7}$$

$$\tag{8}$$

After applying our known theta in degrees, our equation simplifies into

$$T = \left(\frac{dm}{dt} \cdot v \cdot cos(90)\right)_{exhaust} - \left(\frac{dm}{dt} \cdot v \cdot cos(0)\right)_{inlet} \tag{9}$$

$$= -\left(\frac{dm}{dt} \cdot v \cdot 1\right)_{inlet} \tag{10}$$

$$= -\frac{dm}{dt} \cdot v_{inlet} \tag{11}$$

$$T = -\frac{dm}{dt} \cdot v_{inlet} = -50 \cdot 150 \cdot \frac{1000}{3600} = \boxed{-2083.3N}$$

Lastly, **Part C** is the easiest of all. Since the vehicle is no longer moving, our $V_{\text{inlet}} = 0$. While thrust is a function of mass flow rate and velocity, $\text{T}(\dot{\text{m}}, \tilde{\text{v}})$, and there is mass flow rate, that should consequently produce air movement (velocity). However, this is not true relative to the engine. In other words, the engine is not experiencing air entering the system due to the aircraft moving.

Therefore, our velocity inlet term is zero and we can rewrite equation 9 as

$$T = \left(\frac{dm}{dt} \cdot v \cdot cos(90)\right)_{exhaust} - \left(\frac{dm}{dt} \cdot v \cdot cos(0)\right)_{inlet} \tag{12}$$

$$= \frac{dm}{dt} \cdot 0 \tag{13}$$

$$\boxed{T = 0N}$$