# Gravitational Collapse and the Information Loss Problem

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*in the*

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# Declaration of Authorship

I, Alton Vanie Kesselly, declare that this thesis titled, 'Gravitational Collapse and the Information Loss Problem' and the work presented in it are my own. I confirm that:

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- Where any part of this thesis has previously been submitted for a degree or any other qualification at the University of Cape Town or any other institution, this has been clearly stated.
- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.
- I have acknowledged all main sources of help.
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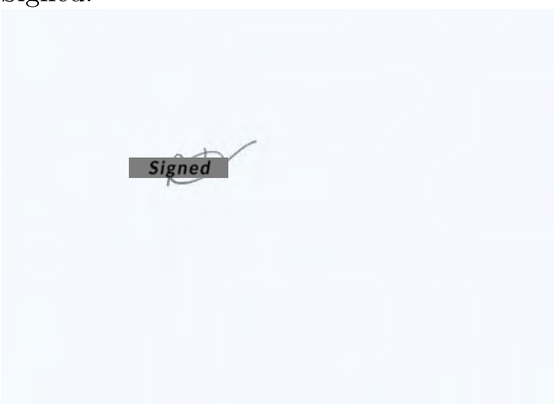


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*"I know that most men, including those at ease with problems of the greatest complexity, can seldom accept even the simplest and most obvious truth if it be such as would oblige them to admit the falsity of conclusions which they have delighted in explaining to colleagues, which they have proudly taught to others, and which they have woven, thread by thread, into the fabric of their lives."*

Leo Tolstoy

UNIVERSITY OF CAPE TOWN

# *Abstract*

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**Gravitational Collapse and the Information Loss Problem**

by Alton Vanie Kesselly

This thesis is intended to critically review the standard black holes. In this thesis, we used the intractability of the black hole Information loss problem and the current crisis stirred up by the black hole Firewall paradox to support the argument that nature is better off without black holes.

Keywords: Hawking radiation, trapped regions, horizons, semiclassical theory

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[1]was downloaded from http://www.LaTeXTemplates.com and modified for this paper.

# Contents

# Abbreviations

| | |
|---|---|
| **CC** | **Cosmological Constant** |
| **EC** | **E**nergy **C**onditions |
| **WEC** | **W**eak **E**nergy **C**ondition |
| **NEC** | **N**ull **E**nergy **C**ondition |
| **SEC** | **S**trong **E**nergy **C**ondition |
| **DEC** | **D**ominant **E**nergy **C**ondition |
| **BH** | **B**ekenstein **H**awking |
| **CCC** | **C**osmic **C**ensorship **C**onjecture |
| **AGN** | **A**ctive **G**alatic **N**uclei |
| **EH** | **E**vent **H**orizon |
| **AH** | **A**pparent **H**o1rizon |
| **CCC** | **C**osmic **C**ensorship **C**onjecture |

# Physical Constants

| Constant Name | Symbol | = | Constant Value (with units) |
|---|---|---|---|
| Speed of Light | $c$ | = | $2.9\ 792\ 4 \times 10^8 ms^{-1}$ |
| Newtonian gravitational constant | $G$ | = | $6.6\ 738\ 4 \times 10^{-11} m^3 kg^{-1} s^{-2}$ |
| Reduced Planck constant | $\hbar$ | = | $1.0\ 545\ 7 \times 10^{-34} Js$ |
| Boltzmann constant | $k_B$ | = | $1.3\ 806\ 4 \times 10^{-23} JK^{-1}$ |
| Solar mass | $M_\odot$ | = | $1.9\ 89\ 1 \times 10^{30} m^3 kg$ |
| Solar radius | $R_\odot$ | = | $6.9\ 550\ 0 \times 10^8 m$ |

# Symbols

| symbol | name | unit |
|---|---|---|
| $e_\mu$ | basis vector | $m^{-1}$ |
| $L$ | Lagrangian density | $m^{-4}$ |
| $\omega^\mu$ | one-form basis | $m$ |
| $J$ | angular momentum | $kgm^2s^{-1}$ |
| $Q$ | electric charge | $Coulomb$ |
| $T$ | Temperature | $Kelvin$ |
| $S$ | entropy | $JK^{-1}$ |

*Decidated to anyone who dares to think outside the box.*

# Chapter 1

# Introduction

## 1.1 Overview

Black holes are commonplace and yet are very strange objects. Everyone seems to have heard of them yet nobody seems to truly understand them. They were once thought of as the simplest objects in the universe but were found out to be among the most complex. They were expected to provide a connection among quantum theory, general relativity and thermodynamics but they are giving us reasons to question some basic tenets of these fundamental theories. There are very good reasons to think that black holes are physical objects to be observed and analysed and yet there are also good reasons to question their very existence.

The evidences for the existence of black holes are strong[1–3] and getting better all the time[4]. Observations such as the speeds of stars near centres of many galaxies[5], x-ray emissions from some binary systems[6], powerful jets of radiation from some compact objects [7], prodigious radiation thousand times those of an ordinary galaxy (i.e., the Milky way) coming from a region of one light year radius[8] and multiple images of the same object[9] are best explained by the effects of black holes. Amazingly, these observations dovetailed neatly in with predictions of general relativity made long before any one of these observations were even thought of.

Moreover, the singularity theorems[10–12], the black hole's Uniqueness theorem[13, 14], Hawking's area theorem and the other laws of black hole mechanics[15] and other detailed formalisms of black holes have taken black holes from being thought of as mere mathematical constructs[16–18] to being an indispensable part of theoretical physics.

Furthermore, the sheer beauty of the results obtained from the analyses of black hole and the striking similarities between the laws of black hole mechanics and the laws of thermodynamics give us reasons to take black holes as physical objects. For instance, the expressions for the temperature of black hole

$$T_H = \frac{\hbar c^3 \kappa}{2\pi G k_B} \tag{1.1}$$

and entropy of black hole

$$S_{BH} = \frac{k_B c^3 A}{4G\hbar} \tag{1.2}$$

are very amazing[1]. They are among the most beautiful relations in theoretical physics since they contain all of the fundamental constants. It is also of great beauty that these expressions are of the same forms for black holes of different types regardless of the masses, spins or charges of the black hole and even despite the number of spatial dimensions considered. Besides aestheticism, these relations connect thermodynamics, quantum theory and general relativity that were once thought to be disparate branches of Physics. Also, the astonishing fact that the expression for the black hole's temperature contains $\hbar$, $c$ and $G$ means that it must be of quantum gravity origin. Similarly, Because $S_{BH}$ as given by Equation 1.2 is non-zero, it must be of quantum gravitational origin since the classical black hole has zero entropy. These facts have made black holes very important in the quest for a viable theory of quantum gravity. It is the hope of many who are working on quantum gravity that black holes provide the means of testing the viability of all purported theories of quantum gravity. That is, clarifying and resolving outstanding issues associated with black holes should serve as the touchstone for any theory claiming to be a theory of quantum gravity.

From the above discussed, it is understandable why questioning the standard black holes or conventional views of them has never been very popular. There is no gainsaying that, to many experts on black holes, thought of the possibility that black hole might not exist amounts to a sacrilege.

Nonetheless, there are very good reasons to be wary of the conventional views on black holes. First, the observational evidence for black holes are circumstantial; there is no direct observational evidence for black holes[19]. What are observed as black holes are actually black hole candidates. That is, they could be compact objects which are simply mimicking the behaviours of black holes. Second, the prediction of singularities by Einstein's theory of general relativity raises questions about the reliability of the predictions of general relativity. The existence of singularity means that spacetime has a boundary beyond which the theory of general relativity is not applicable[12]. This contradicts the popular consensus that general relativity describes all of spacetime. In fact, the prediction of singularities by any theory ought to be seen as a genuine

[1] $\kappa$ is the surface tension of the black hole, $k_B$ is the Boltzmann's constant, $A$ is the area of the event horizon, $\hbar$ is the reduced Planck constant, $c$ is the speed of light and $G$ is the Newtonian gravitational constant

breakdown of that theory; for, anything could appear from the singularities since the physical laws governing the singularities are unknown. Thus, a complete theory is not expected to predict singularity. This observation is succinctly captured by Hawking:

> " once one allows that singular histories could occur anywhere and predictability would disappear completely. If the laws of Physics break down at singularities, they could break down anywhere. The only way to have a scientific theory is if the laws of Physics hold everywhere, including at the beginning of the universe" [20].

Furthermore, the existence of singularities creates puzzles and paradoxes such as the information loss problem[21–23]. As shall be discussed in chapter 2, black holes radiate and might evaporate completely. The evaporation of a black hole results in an irretrievable loss of information but an irretrievable loss of information is forbidden by quantum theory. Third, black holes are not directly defined by local physical quantities. They are defined in terms of the global property of spacetime. In short, the definition of black hole depends on observations that are, in practice, impossible to make. Fourth, the entire concept of black hole physics is built on an unproven conjecture.

So, there seems to be an ongoing crisis in our understanding of nature. On one hand general relativity and quantum theory individually agree perfectly well with all tests they have been subjected to. On the other hand, when both are taken into account for phenomena in which gravitational and quantum effects are both relevant, we encountered puzzles and paradoxes that defy solutions. This situation is brought to the fore by the radiation of black holes. As shall be discussed in this thesis, it has been shown that black holes radiate with exact thermal radiation[22, 24, 25]. Since black holes have no hair[2], the radiation observed outside the black hole is a mixed (or thermal) state. The radiation of black hole may lead to the complete evaporation of the black hole. If that was to happen then states which were initially pure quantum states could evolve into mixed states. This is a serious problem because quantum theory does not permit the evolution of pure states into mixed states. This is the famous Information Loss Problem.

Forty years of attempts at solving the information loss problem has culminated into the discovery of the black hole Firewall paradox[26]. The Firewall paradox is the suggestion that the information loss problem can only be resolved by violating at least one of the tenets of either quantum theory or general relativity. The implication of this is that the information loss paradox may be the tell-tale signal that the fundamental theories of science are not as sacrosanct as we have come to believe them to be. The Fire

[2]mass, electric charge and angular momentum are the only properties of the black hole accessible to an external observer.

wall paradox might be a signal that we must re-examine the very foundations of these fundamental theories. In light of this, the widely accepted belief that our understanding of everyday physics[3] is not impeded by the lack of a quantum theory of gravity is now more questionable than ever. With the Firewall paradox, it has become obvious that we need a viable theory of quantum gravity to ascertain the validity of some basis tenets of quantum theory and general relativity.

In this thesis, we look at these issues and use the intractability of the information loss problem to support the arguments against some of the conventional views on gravitational collapse. In the rest of this chapter, we give the background information needed for the arguments put forth in subsequent chapters of this thesis. This include a brief review of general relativity, black holes and quantum theory. In chapter 2, we derive the equation for the temperature of black holes in three independent ways in order to show the robustness of the Hawking's process. There, we will also discuss how the entropy of the black hole is derived. The information problem and some of the suggestions at solving it are also reviewed in chapter 2. In chapter 3, we look at alternatives to black holes. There we see that all assumptions of the singularity theorems are violable and we use the violations of the assumptions of the singularity theorems along with the need for the modification of general relativity to construct compact objects that mimic the behaviours of black holes. Our aim is neither to justify those solutions nor give reasons for their existence. Our sole intention is to show that black holes are not the only possible solutions for the end state of the gravitational collapse of very massive astronomical bodies. Also in chapter 3, we show that there are number of results that indicate that the Cosmic censorship conjecture is questionable. In chapter 4, we revisit the conventional views of black holes and try to identify pitfalls in them. We, particularly highlight the ongoing heated debate on whether black holes should be defined in terms of local horizons or global horizons and show that is a very good reason for questioning the very existence of black holes. In chapter 5, we summarized the key points of this thesis and conclude with an attempt an identifying sources of the seemingly contradictory views on gravitational collapse in particular and theoretical physics in general.

## 1.2 General Relativity

### 1.2.1 Einstein's Field Equations

In General relativity, spacetime is modelled as a four dimensional differential manifold$(M, g_{\alpha\beta})$. According to general relativity, spacetime is curved by the presence of energy or matter.

[3] Physics of low energy regimes

This means that far away from the concentration of matter or locally(i.e., in a sufficiently small region) spacetime is the Minkwoski spacetime. In other words, general relativity is the generalization of special relativity and Newton's law of universal gravitation. In addition, general relativity is built on the Equivalence Principle[27], of which there are two types:

1. The Strong Principle of Equivalence which states that a uniform static gravitational field and an accelerated reference frame are equivalent;
2. The Weak Principle of Equivalence which states that the inertial mass is equal to the gravitational mass or that the trajectory of a freely falling body is independent of the composition and structure of that body.

General relativity is also built on the principle that there is no fixed background spacetime.

To discuss general relativity and its implications unambiguously and make clearer the arguments of this thesis, we must first defined some terms including:

- Since far away from a concentration of mass spacetime is considered to be flat, we can construct a two-sheeted light cone at each point $p \in M$ in the tangent space $T_pM$

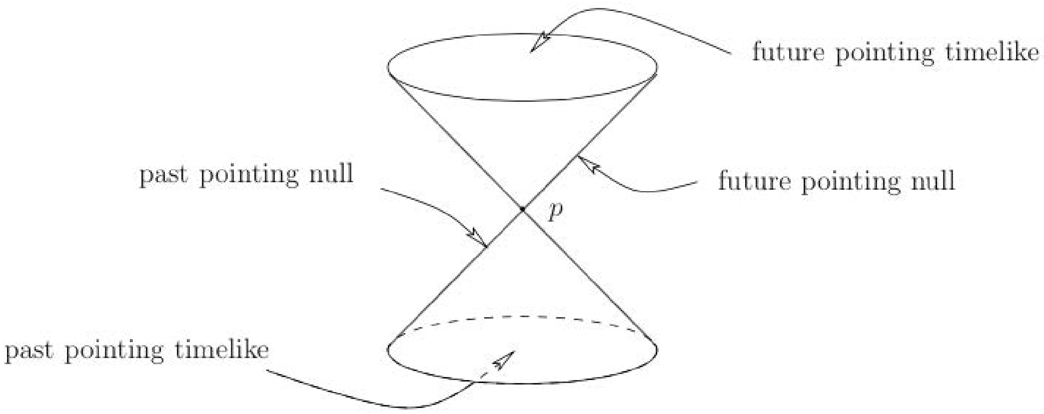


FIGURE 1.1

FIGURE 1.2: The light cone at $p$[28]

and use it to classify all non-zero vectors $v \in T_pM$ as either timelike, null or spacelike:

$$\left(g_{\mu\nu}v^{\mu}v^{\nu}\right)|_p \begin{cases} < 0 & \text{is timelike} \\ = 0 & \text{is null} \\ > 0 & \text{is spacelike.} \end{cases} \tag{1.3}$$

From the light cone, we can see that timelike vectors lie inside the light cone, null vectors on the light cone and spacelike vectors outside the light cone.

- One can use the light cone to arbitrarily assign the label future to half of the cone and the label past to the other half.
- If there is no ambiguity as to which half of the light cone is in the future of a point and which half is in the past then the spacetime is said to be time orientable.
- A timelike vector lying in the future half of the light cone is said to be a future directed timelike vector and a null vector lying on the future half of the light cone is said to be a future directed null vector.
- A vector that is either timelike or null is called a causal (or a non spacelike) vector.

The aforementioned definitions also apply to curves. For instance,

- for some interval $I \subseteq \Re$, a smooth curve $\gamma : I \mapsto M$ is timelike if its tangent vector $\varepsilon^\alpha = \frac{dx^\alpha}{d\lambda}$ at each point in $\gamma[I]$ is timelike and spacelike if its tangent vector is spacelike. Similarly, a curve is null if its tangent vector at each point is null.
- A curve is causal if its tangent vector at each point is either null or timelike.
- Particles with masses have timelike curves as their worldlines whereas massless particles (for instance, photons) have null curves as their worldlines. In other words, matter and information move along causal curves.
- The length of a causal curve $\gamma : [a, b] \mapsto M$ is defined by

$$\text{Length of } \gamma \equiv L(\gamma) = \int_a^b |\gamma'(t)| dt = \int_a^b \sqrt{g_{\alpha\beta} \frac{dx^\alpha}{d\lambda} \frac{dx^\beta}{d\lambda}} d\lambda, \tag{1.4}$$

  where $\lambda$ is an affine parameter[4].
- A curve $\gamma : I \to M$ is said to be maximal if there is another curve $\gamma' : I' \to M$ such that $\gamma(s) = \gamma'(s)$ for all $s \in I$.

Other terms that are needed for better understanding of causal relations between events in spacetime include:

- If $\lambda(t)$ is a future -directed causal curve then $p \in M$ is the future endpoint of $\lambda$ if for every neighbourhood $O$ of $p$ there exists a $t_0$ such that $\lambda(t) \in O$ for all $t > t_0$.

[4] an affine parameter $\lambda \geq 0$ has the relation $\lambda = a\sigma + b$ where $a$ and $b$ are constant and $\sigma$ is either the proper time $\tau$ or the proper distance $s$

- The curve $\lambda$ is said to be future inextendible if it has no future end points and it is said to be Past inextendible if it has no past end points.

- The chronological future of $p \in M$, denoted by $I^+(p)$, is defined by

$$I^+(p) \equiv \{q \in M | \text{there is a future-directed timelike curve from } p \text{ to } q\}. \quad (1.5)$$

- The chronological past of $p \in M$, denoted by $I^-(p)$, is defined by

$$I^-(p) \equiv \{q \in M | \text{there is a future-directed timelike curve from } q \text{ to } p\}. \quad (1.6)$$

- The causal future of $p \in M$, denoted by $J^+(p)$, is defined by

$$J^+(p) \equiv \{q \in M | \text{there is a future-directed causal curve from } p \text{ to } q\}. \quad (1.7)$$

  Physically, the causal future of $p$ represents events which , in principle, can be influenced by a signal emitted from $p$.

- The causal past of $p \in M$, denoted by $J^-(p)$, is defined by

$$J^-(p) \equiv \{x \in M | \text{there is a future-directed causal curve from } q \text{ to } p\}. \quad (1.8)$$

  Physically, the causal past of $p$ represents events which , in principle, influenced signals at $p$.

- The future horizon of $p \in M$, denoted $E^+(p)$, is defined by

$$E^+(p) = J^+(p) - I^+(p). \quad (1.9)$$

- If $S \subset M$ then

$$I^+(S) \equiv \bigcup_{p \in S} I^+(p), \quad J^+(S) \equiv \bigcup_{p \in S} J^+(p), \quad I^-(S) \equiv \bigcup_{p \in S} I^-(p) \quad \text{and} \quad J^-(S) \equiv \bigcup_{p \in S} J^-(p). \quad (1.10)$$

- Given $p \in M$ and any tangent vector $\frac{dx^\sigma}{d\lambda} \in T_pM$ then there exists a unique geodesic through $p$ given by

$$\frac{d^2x^\mu}{d\lambda^2} + \Gamma^\mu_{\sigma\nu} \frac{dx^\sigma}{d\lambda} \frac{dx^\nu}{d\lambda} = 0. \quad (1.11)$$

  In general relativity, a freely falling test particle with mass follows a geodesic with timelike tangent vector while a freely falling massless particle follows a null geodesic.

- The curve $x^\mu$ is considered geodesically complete if the solutions of Equation 1.11 are defined for all $\lambda \in \Re$
- The manifold $(M, g_{\alpha\beta})$ is called geodesically complete if the solutions of Equation 1.11 are defined for all $s \in R$.
- A set $S \subset M$ for which none of its elements are connected by causal curve is said to be achronal.
- If $S$ is a closed achronal set then the future domain of dependence of $S$, denoted $D^+(S)$, is defined by

$$D^+(S) = \{p \in M | \text{Every past inextendible causal curve through } p \text{ intersect } S\}. \tag{1.12}$$

  Since no information or signal can travel faster than light, any information or signal from $p \in D^+(S)$ must first register on $S$. Thus, initial conditions on $S$ can be used to predict what happens at $p \in D^+(S)$.
- Similarly, the past domain of dependence of $S$, denoted $D^-(S)$, is defined by

$$D^-(S) = \{p \in M | \text{Every future inextendible causal curve through } p \text{ intersect } S\}. \tag{1.13}$$

  From initial conditions on $S$, conditions on all $q \in D^-(S)$ can be retrodicted.
- We define the Domain of dependence, denoted $D(S)$ by

$$D(S) = D^+(S) \cup D^-(S). \tag{1.14}$$

- A closed achronal set $\Sigma$ for which

$$D(\Sigma) = M \tag{1.15}$$

  is called a Cauchy surface.
  A Cauchy surface can be used in general relativity to establish an appropriate notion of space on which initial value problems can be formulated.
- A manifold with at least one Cauchy surface is said to be globally hyperbolic.
  In a globally hyperbolic spacetime the future and past history of the entire spacetime (i.e., the universe) can be predicted or retrodicted from the initial conditions on a Cauchy surface. Conversely, there is breakdown of predictability for a spacetime that is not globally hyperbolic since a complete knowledge of condition on a Cauchy surface can never suffice to determine the entire history of the spacetime.

Centre to general relativity are the Einstein's field equations:

$$G_{\alpha\beta} \equiv R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R + \wedge g_{\alpha\beta} = 8\pi T_{\alpha\beta}. \tag{1.16}$$

The terms in Equation 1.16 are:

- $R_{\alpha\beta} \equiv R^{\mu}_{\alpha\mu\beta}$ is the Ricci tensor;
- $R^{\mu}_{\alpha\gamma\beta}$, defined by

  $$R^{\mu}_{\alpha\gamma\beta} = \Gamma^{\mu}_{\alpha\beta,\gamma} - \Gamma^{\mu}_{\alpha\gamma,\beta} + \Gamma^{\mu}_{\rho\gamma}\Gamma^{\rho}_{\alpha\beta} - \Gamma^{\mu}_{\rho\beta}\Gamma^{\rho}_{\alpha\gamma}, \tag{1.17}$$

  is the Riemann tensor;
- $\wedge$, which can be defined in terms of the energy density $\rho_{\wedge}$

  $$\rho_{\wedge} = \frac{\wedge}{8\pi G} \tag{1.18}$$

  satisfying an exotic equation of state $p = -\rho$, is called the Cosmological constant;
- $g_{\alpha\beta}$, which can be defined by

  $$ds^2 = g_{\alpha\beta} dx^{\alpha} dx^{\beta}, \tag{1.19}$$

  is the metric;
- $g$ is the determinant of the metric;
- $T_{\mu\nu}$, which represents the energy, momentum, pressure and stresses of all fields except the gravitational field can alternatively be called stress-energy-momentum tensor, the stress-energy tensor, the energy-momentum tensor or simply the stress tensor;
- $\Gamma^{\mu}_{\alpha\beta}$, which can be defined by

  $$\Gamma^{\mu}_{\alpha\beta} = \frac{1}{2} g^{\mu\rho}(g_{\rho\alpha,\beta} + g_{\rho\beta,\alpha} - g_{\alpha\beta,\rho}), \tag{1.20}$$

  are the Christoffel symbols.

Equation 1.16 can be obtained from

$$\delta S = 0, \tag{1.21}$$

where

$$S = \frac{1}{16\pi}\int_M d^4x\sqrt{-g}(R+2\wedge) + \int_M d^4x\sqrt{-g}\mathscr{L}_m + \frac{1}{8\pi}\int_{\partial M} d^3x\sqrt{h}K \tag{1.22}$$

is Einstein-Hilbert action (coupled to matter), $h$ is the determinant of the 3-dimensional metric of the boundary of the manifold ($\partial M$), $K$ is the trace of the extrinsic curvature and

$$\frac{2}{\sqrt{-g}}\frac{\delta}{\delta g^{\alpha\beta}}\Big[\int_M d^4x\sqrt{-g}\mathscr{L}_m\Big] \equiv T_{\alpha\beta}. \tag{1.23}$$

Contracting the Second Bianchi identity

$$R_{\alpha\beta[\gamma\delta;\mu]} \equiv R_{\alpha\beta\gamma\delta;\mu} + R_{\alpha\beta\delta\mu;\gamma} + R_{\alpha\beta\mu\gamma;\delta} = 0 \tag{1.24}$$

twice leads to

$$G^{\mu\nu}_{;\nu} = 0, \tag{1.25}$$

which in turn, implies the so-called conservation law

$$T^{\mu\nu}_{;\nu} = 0. \tag{1.26}$$

Equation 1.16 can give solutions with pathologies such as closed timelike curves and situations in which the speed of signals can exceed the speed of light. A way of avoiding getting pathological or non-physical solutions of Equation 1.16 is by imposing reasonable conditions on Einstein's field equations. Examples of reasonable conditions imposed on Equation 1.16 are the Energy conditions[10, 29–31] which include: the the Weak Energy Condition

$$T_{\alpha\beta}u^\alpha u^\beta \geq 0 \qquad \forall \text{ future directed timelike vector } u^\alpha, \tag{1.27}$$

the Null Energy Condition

$$T_{\alpha\beta}l^\alpha l^\beta \geq 0 \qquad \forall \text{ future directed null vector } l^\alpha, \tag{1.28}$$

the Strong Energy Condition

$$((T_{\alpha\beta} - \frac{1}{2}Tg_{\alpha\beta})u^\alpha u^\beta \geq 0 \qquad \forall \text{ future directed timelike vector } u^\alpha \quad \text{and} \tag{1.29}$$

and the Dominant Energy Condition.
The Dominant Energy Condition is the statement that the matter momentum density, $-T^\alpha_\beta v^\beta$, as measured by an observer with 4-velocity $v^\alpha$ must be causal.

Other conditions needed for ensuring non-physical solutions of the Einstein's field equations are the causality conditions, of which the strongest is global hyperbolicity. Globally hyperbolicity allows a consistent and global formulation of causality.

### 1.2.2 Black Holes

Given that Einstein's field equations are notoriously complex, finding exact solutions of them requires making simplifying assumptions. A simplifying assumption that is usually made is the that spacetime is spherically symmetric with a zero stress-energy tensor. With such an assumption, Equation 1.16 can be solved to obtain the Schwarzschild solution:

$$ds^2 = -\Big(1 - \frac{2M}{r}\Big)dt^2 + \frac{dr^2}{\Big(1 - \frac{2M}{r}\Big)} + r^2(d\theta^2 + \sin^2\theta d\phi^2). \tag{1.30}$$

Equation 1.30 represents spacetime outside of a spherically symmetric body.

Another popular assumption is that spacetime is spherically symmetric with the only non-zero contribution to the stress-energy tensor being that of an electric field with charged $Q$. In this case, the solution to Equation 1.16 is the Reissner-Nordström solution:

$$ds^2 = -\Big(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\Big)dt^2 + \frac{dr^2}{\Big(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\Big)} + r^2(d\theta^2 + \sin^2\theta d\phi^2). \tag{1.31}$$

Equation 1.31 represents the spacetime outside of a spherically symmetrical charged body.

If one assumes that spacetime is axial symmetric with zero stress-energy tensor then one will obtain the Kerr solution

$$\begin{aligned} ds^2 = &-\Big(\frac{\Delta - a^2\sin^2\theta}{\Sigma}\Big)dt^2 - 2a\sin^2\theta\Big(\frac{r^2 + a^2 - \Delta}{\Sigma}\Big)dtd\phi + \Big[\frac{(r^2+a^2)^2 - \Delta a^2\sin^2\theta}{\Sigma}\Big]\sin^2\theta d\phi^2 \\ &+ \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2 \end{aligned} \tag{1.32}$$

where

$$\Sigma = r^2 + a^2\cos^2\theta, \quad \Delta = r^2 - 2Mr + a^2, \quad a = \frac{J}{M}{}^{5}, \tag{1.33}$$

[5]Since one can choose $a > 0$ without a loss of generality, throughout this thesis we will assume that $a > 0$.

$J$ is the total angular momentum.
Equation 1.32 represents the spacetime outside an uncharged rotating body.

However, if the non-zero contribution to the stress-energy tensor is that of an electric field with charge $Q$ and that the spacetime is axial symmetric then one will obtain the Kerr-Newman solution:

$$ds^2 = -\Big(\frac{\Delta - a^2\sin^2\theta}{\Sigma}\Big)dt^2 - 2a\sin^2\theta\Big(\frac{r^2+a^2-\Delta}{\Sigma}\Big)dtd\phi + \Big[\frac{(r^2+a^2)^2 - \Delta a^2\sin^2\theta}{\Sigma}\Big]\sin^2\theta d\phi^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2 \tag{1.34}$$

where

$$\Sigma = r^2 + a^2\cos^2\theta, \quad \Delta = r^2 - 2Mr + a^2 + Q^2, \quad a = \frac{J}{M}. \tag{1.35}$$

It is easy to see that Equation 1.34 is the generalization of Equation 1.30,Equation 1.31 and Equation 1.32.

| | $Q = 0$ | $Q \neq 0$ |
|---|---|---|
| $a = 0$ | Schwarzschild | Reissner-Nordström |
| $a \neq 0$ | Kerr | Kerr-Newman |

It has been proven that, at late times, the solutions to Einstein-Maxwell equations[6] is one of the Kerr-Newman family of solutions (i.e., Equation 1.30,Equation 1.31, Equation 1.32, etc[7]). This is the so-called No-hair theorem or the Black hole uniqueness theorem[13, 14].

Although there seems to be singularities at $\Delta = 0$ and $\Sigma = 0$, by an appropriate transformation, it can be shown that the only true physical singularities are at $r = 0$. For example, by computing

$$R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} = \frac{48M^2}{r^6} \tag{1.36}$$

for the Schwarzschild solution or

$$R^{\alpha\beta\gamma\delta}R_{\alpha\beta\gamma\delta} = \frac{48M^2(r^2 - a^2\cos^2\theta)(\Sigma^4 - 16a^2r^2\cos^2\theta)}{\sqrt{\Sigma}} \tag{1.37}$$

for the Kerr-Newman solution[24], we see that $r = 2M$ and $\Delta = 0$ are mere coordinate singularities. That is, they appear as singularities due to the poor choice of coordinates.

[6] the Einstein-Hilbert action coupled to the Maxwell action
[7] Though the Schwarzschild solution suffices for most of the arguments in this thesis, we will occasionally use any member of the Kerr-Newman family of solutions to make a point.

Making the transformation

$$T = t + 2\sqrt{2Mr} + 2M\ln\left|\frac{\sqrt{\frac{r}{2M}}-1}{\sqrt{\frac{r}{2M}}+1}\right|, \tag{1.38}$$

Equation 1.30 becomes

$$ds^2 = -f(r)dT^2 + 2\sqrt{\frac{2M}{r}}dTdr + dr^2 + r^2d\Omega^2 = -dT^2 + \Big(dr + \sqrt{\frac{2M}{r}}dT\Big)^2 + r^2d\Omega^2. \tag{1.39}$$

In fact, if we make the general transformation[32] $t \to T = t + \psi(r)$ then Equation 1.30 becomes

$$ds^2 = -f(r)dT^2 + 2C(r)dTdr + f(r)^{-1}\Big[1 - C(r)^2\Big]dr^2 + r^2d\Omega^2, \tag{1.40}$$

where $f(r) = 1 - \frac{2M}{r}$, $C(r) \equiv f(r)\dfrac{d\psi(r)}{dr}$ and $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$.
Making the identification

$$f(r)^{-1}\Big[1 - C(r)^2\Big] = 1 \to C(r) = \sqrt{\frac{2M}{r}} \tag{1.41}$$

and we arrive at Equation 1.39.
If we had instead made the identification

$$f(r)^{-1}\Big[1 - C(r)^2\Big] = 0 \to T = t \pm \left(r + 2M\ln\left|\frac{r}{2M} - 1\right|\right) \equiv t \pm r^* \tag{1.42}$$

and had defined

$$v \equiv t + r^* \quad \text{and} \quad u \equiv t - r^*. \tag{1.43}$$

then Equation 1.30 could have been written as

$$ds^2 = -f(r)dv^2 + 2dvdr + r^2d\Omega^2 = -\Big(1 - \frac{2M}{r}\Big)dv^2 + 2dvdr + r^2d\Omega^2 \tag{1.44}$$

and

$$ds^2 = -f(r)du^2 - 2dudr + r^2d\Omega^2 = -\Big(1 - \frac{2M}{r}\Big)du^2 - 2dudr + r^2d\Omega^2. \tag{1.45}$$

In the analyses of the dynamics of a gravitationally collapsing spherically symmetric body, it is instructive to rewrite Equation 1.44 as

$$ds^2 = -\Big(1 - \frac{2M(v)}{r}\Big)dv^2 + 2dvdr + r^2d\Omega^2 \tag{1.46}$$

and Equation 1.45 as

$$ds^2 = -\Big(1 - \frac{2M(u)}{r}\Big)dv^2 - 2dudr + r^2 d\Omega^2. \tag{1.47}$$

Equation 1.39, Equation 1.44 and Equation 1.45 are the Schwarzschild solution written in terms of Painleve-Gullstrand coordinates $(T, r, \theta, \phi)$, ingoing Eddington-Finkelstein coordinates $(v, r, \theta, \phi)$ and outgoing Eddington-Finkelstein coordinates $(u, r, \theta, \phi)$ respectively. Equation 1.46 is the ingoing Vaidya solution and Equation 1.47 is outgoing Vaidya solution.
We can also make the transformation

$$U = -e^{-\frac{u}{4M}} \quad \text{and} \quad V = e^{\frac{v}{4M}} \tag{1.48}$$

to get the maximal analytic extension of the Schwarzschild solution

$$ds^2 = -\frac{32M^3}{r} e^{-\frac{r}{2M}} dUdV + r(U, V)d\Omega^2. \tag{1.49}$$

The coordinates $(U, V, \theta, \phi)$ are called the Kruskal-Szekeres coordinates.

Even though $r = 2M$ and $\Delta = 0$ are mere coordinate singularities, they are peculiar. For instance, for causal curves in the geometry given by Equation 1.30, we have[33]

$$2drdu = -\Big(\frac{2M}{r} - 1\Big)du^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \leq 0 \tag{1.50}$$

for the surface $r \leq 2M$. Since $du \geq 0$ by definition[8], Equation 1.50 is satisfied only if $dr < 0$. This means that a particle entering the region $r < 2M$ will never escape to the region $r > 2M$ and will unavoidably approach, in finite proper time, the singularity at $r = 0$. Thus, $r = 2M$ serves as a boundary which separates those events which are visible from those which invisible to an external observer. Such a boundary is called an event horizon. This idea can be used to define any object with a singularity within an event horizon as a black hole.

Discussions of event horizons, singularities and black holes are made easier and clearer by using Penrose (or Conformal) diagrams. A Penrose diagram, a conformal diagram or Penrose-Carter diagram is a diagram used for understanding the causal structure of spacetime. It provides an intuitively clear way to visualize the global geometry of black holes The main objective of the Penrose diagram is to project the entire spacetime on a finite diagram while maintaining the causal properties of the original geometry of the

[8]signals and material bodies can only move along future directed causal curves.

spacetime. This is possible by means of the Weyl (or conformal) transformation

$$ds^2 \to d\bar{s}^2 = \Omega^2(x^\mu)ds^2 \tag{1.51}$$

where $\Omega^2(x^\mu)$ depends on the spacetime point and is, in general, non vanishing and positive. On the Penrose diagram, infinities are classified either as

- future timelike infinity and denoted by $i^+$,
- past timelike infinity and denoted by $i^-$,
- spatial infinity and denoted by $i^0$,
- future null infinity and denoted by $\mathscr{I}^+$ or
- past null infinity and denoted by $\mathscr{I}^-$.

On the Penrose diagram, all timelike geodesics begin at $(i^-)$ and end at either $(i^+)$ or at the singularity $(r = 0)$, all null geodesics start $\mathscr{I}^-$ and end at $\mathscr{I}^+$ and all spacelike geodesics begin and end at $(i^0)$.

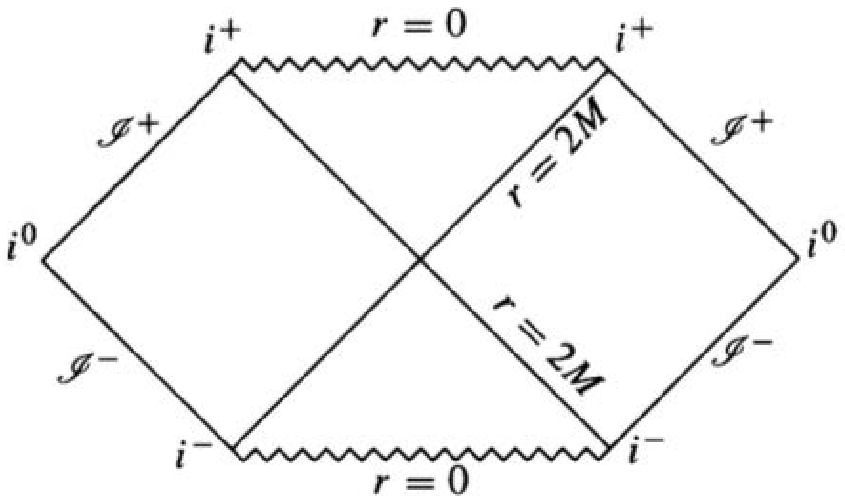


Figure 1.3: Penrose diagram for the Schwarzschild metric[29]

With the terms defined above and the Penrose diagram, We can now define a black hole, denoted $\mathscr{B}$, unambiguously as [29]

$$\mathscr{B} = M - J^-(\mathscr{I}^+) \tag{1.52}$$

and a future event horizon, denoted $\mathscr{H}$[9], as the boundary of the black hole formed from gravitational collapse[29]

$$\mathscr{H} = \partial\mathscr{B}. \tag{1.53}$$

It is not hard to see that $\mathscr{H}^+$ must be achronal.
Proof[10]:
If $p, q \in \mathscr{H}^+$ and if there is a timelike curve from $p$ to $q$, then displacing $q$ slightly to a point $q' = M - \mathscr{B}$ to the past of $q$, we could get a causal curve from $p$ to $q'$ and then escaping to $\mathscr{I}^+$, which is a contradiction of the definition of a black hole. q.e.d.
Also, it has been shown[35] that the event horizon is a null hypersurface generated by null geodesics that have no future end points.

The Raychaudhuri's equation for the null tangent vector $\ell^\mu$ to the congruence $O$

$$\frac{d\vartheta_\ell}{d\lambda} = -\frac{1}{2}\vartheta_\ell^2 - \sigma_{\ell\alpha\beta}\sigma_\ell^{\alpha\beta} + \omega_{\ell\alpha\beta}\omega_\ell^{\alpha\beta} - 8\pi T_{\alpha\beta}l^\alpha l^\beta, \tag{1.54}$$

can be used to investigate other peculiar properties of the event horizon. In Equation 1.54, the expansion, $\vartheta$, is defined by

$$\vartheta = \frac{1}{\mathscr{A}}\frac{d\mathscr{A}}{d\lambda}, \tag{1.55}$$

$\mathscr{A}$ is the cross-sectional area of the congruence and the spacelike quantities $\sigma_\ell^{\alpha\beta}$ and $\omega_\ell^{\alpha\beta}$ are the shear tensor and the twist tensor respectively.

For instance, if the spacetime is hypersurface orthogonal, which is the case for the Kerr-Newman family, then the twist tensor vanishes. If the null energy condition holds then Equation 1.54 can be written as

$$\vartheta^{-2}d\vartheta \leq \frac{1}{2}d\lambda \tag{1.56}$$

and with solution

$$\vartheta^{-1}(\lambda) \geq \vartheta(0) + \frac{\lambda}{2}. \tag{1.57}$$

Thus, for any $\vartheta(0) < 0$, $\vartheta(\lambda)$ must approach $-\infty$ within finite time. Therefore, event horizons that asymptote to become stationary($\frac{d\vartheta}{d\lambda} = 0$) must always have $\theta \geq 0$. This

[9]actually $\mathscr{H}^+$ for future event horizon in contrast to $\mathscr{H}^-$ for past event horizon but we will always ignore the plus sign for black hole since a physical black hole (i.e., black hole resulting from gravitational collapse) has no past event horizon.
[10]Proof is copied directly from [34]

implies that the event horizon never decreases with time

$$\frac{d\mathscr{A}}{d\lambda} \leq 0. \tag{1.58}$$

Equation 1.58 is the celebrated Hawking's area increase theorem for event horizon.

Differentiating Equation 1.54 and using Equation 1.55, we obtain

$$\frac{d^2\mathscr{A}}{d\lambda^2} = \Big(\frac{1}{2}\sigma_{\alpha\beta}\sigma^{\alpha\beta} - 8\pi T_{\alpha\beta}l^{\alpha}l^{\beta}\Big)\mathscr{A}. \tag{1.59}$$

Thus, though $\theta$ never increase, the rate of the expansion of the area of the horizon can increase unless $\theta = 0$.

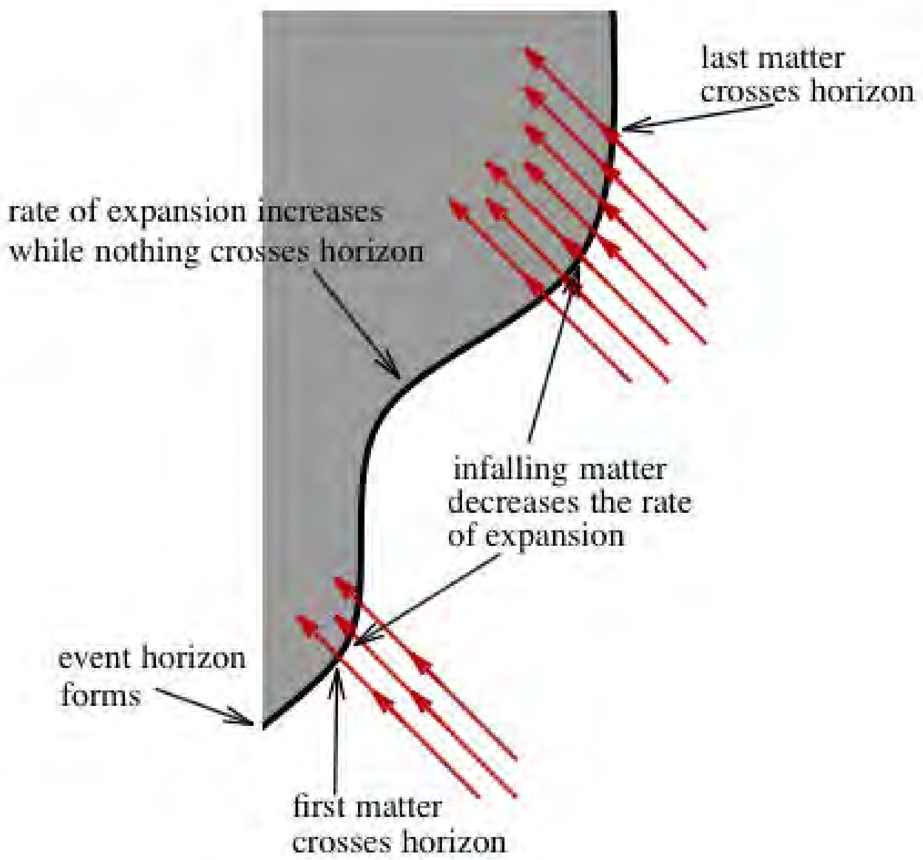


FIGURE 1.4: Event Horizon growing in Anticipation of matter that will fall across it in the future[36]

From Equation 1.59, we see that in falling matter or energy, rather than cause the expansion of the area of a black hole, reduces its rate of growth. The event horizon will expand at just the right rate so as to become stationary as soon as the last matter or energy that will ever pass through it does so. This means that the event horizon is not of local character. Its location depends on the entire spacetime. In other words, a black hole defined in term of an event horizon is teleological, non-local and of limited practical value.

If black holes are physical entities out in the universe to be observed and studies then there has to be definitions for them that make sense. For this reason, there have been attempts at defining black holes in terms of local (or apparent) horizons. Local horizons, for which there are many different types including slowly evolved horizon [37, 38], isolated horizon [39, 40], trapping horizon [41, 42] and dynamical horizon [40, 43][11], are defined in terms of trapped surfaces. By trapped surface, we mean a closed two dimensional spacelike surface having both expansion of its future null normals ($\vartheta^+$ and $\vartheta^-$) as negative. It is generally accepted that trapped surfaces signal strong gravitational effects.

Though it was initially thought that singularities were artefacts of the simplifying assumptions, such as spherically symmetric spacetime, made in solving the Einstein's field equations, it has been proven via the famous singularity theorems[10, 12, 30] that singularities are generic outcomes of general relativity. For instance, it has been proven[10, 30] that if

1. the weak energy condition holds or equivalently $R_{\alpha\beta}k^\alpha k^\beta \geq 0$ for all timelike vectors $k^\alpha$;
2. $(M, g_{\alpha\beta})$ is gobally hyperbolic;
3. there is trapped surfaces in $(M, g_{\alpha\beta})$

then $(M, g_{\alpha\beta})$ can not be geodesically complete.
Validity of the singularity theorems mean that spacetime has an edge beyond which predictability is lost. Since general relativity is supposed to describe all of spacetime, geodesic incompleteness is a genuine breakdown of general relativity even at the classical level. Given that the conditions of the singularity theorems are fairly easy to meet, singularities are inevitable in physically reasonable situations. This portends serious problem for general relativity in particular and all physical theory in general since spacetime is ubiquitous . Matter and information can suddenly appear at a singularity from nowhere and disappear to nowhere and therefore making all physical laws to cease to exist at a singularity. In other words, because the physical behaviours of singularities are unknown, if singularities can be observed from the rest of spacetime, all physical laws may lose their predictive powers. So, the fact that general relativity and quantum theory are known to pass, perfectly well, all tests they have been subjected to in our

[11]You can see [36, 44, 45] and references therein for general revision of apparent or local horizons

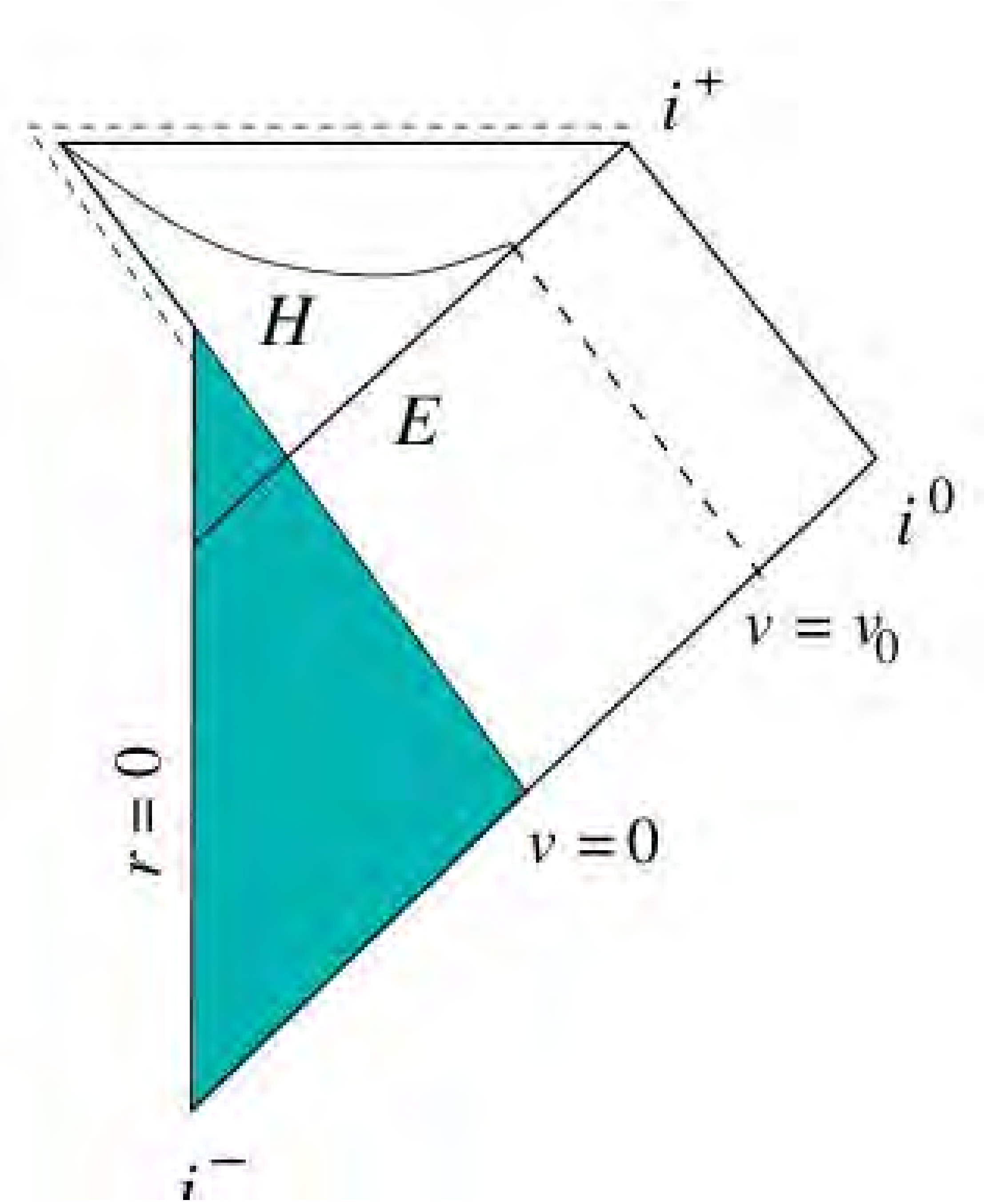


FIGURE 1.5: Event Horizon (E) vs Apparent horizon (H)[40]

low energy regime means that the unpredictability of the singularity is highly restricted. This motivates the so-called Cosmic censorship Conjecture[20, 35]. That is, all singularities, except those of the Big Bang, must exist inside black holes. Making use of initial value formulation, we can state the Weak Cosmic Censorship conjecture as [46]:

Let $\Sigma$ be a three manifold which, topologically, is the connected form of $\Re^3$ and a compact manifold. Let $(h_{ab}, K_{ab}, \psi)$ be non singular, asymptotically flat initial data on $\Sigma$ for a solution to the Einstein equation with suitable matter (where $\psi$ denotes the appropriate initial data for the matter). Then, generically, the maximal Cauchy evolution of the data is a spacetime $(M, g)$ which is asymptotically flat at future null infinity, with complete $\mathscr{I}^+$.

For the Kerr-Newman black hole, the Weak Cosmic Censorship Conjecture means that

$$M^2 - Q^2 - a^2 \geq 0. \tag{1.60}$$

### 1.2.3 Black Hole Mechanics

It was established in the early 1970's that black holes obey four laws that are analogous to the laws of thermodynamics[15]. These laws, called the four laws of black hole mechanics[12], are as follow:

1. The Zeroth law of black hole mechanics states that if Einstein's equation holds with matter stress-energy satisfying the dominant energy condition , then the surface gravity $\kappa$ is constant on the event horizon.
2. The First law states that if a stationary black hole of mass $M$, charge $Q$ and angular momentum $J$ with future event horizon with surface gravity $\kappa$, electric surface potential $\Phi_H$ and angular velocity $\mathit{\Omega}_H$ is perturbed such that it settles down to another black hole with mass $M + \delta M$, charge $Q + \delta Q$ and angular momentum $J + \delta J$ then

$$dM = \frac{1}{8\pi}\kappa dA + \Omega_H dJ + \Phi_H dQ, \tag{1.61}$$

3. The Second law states that the area of the black hole never decreases:

$$\delta A \geq 0^{13} \tag{1.62}$$

   and

4. The third law states that it is impossible by any procedure, no matter how idealized, to reduce the Black hole temperature to zero by a finite sequence of operations.

If one makes the identification that the mass of the black hole can be interpreted as its internal energy, the surface gravity as its temperature and the area of the horizon as the entropy then the four laws of black hole mechanics are just the ordinary laws of thermodynamics when applied to black holes.

[12] see [29, 33] for proofs and details

[13] See Equation 1.58

Table 1.1: thermodynamic laws [27]

| Law | Thermodynamic Systems | Black Holes |
|---|---|---|
| Zero | $T$ = constant in thermal equilibrium | $\kappa =$ on the horizon of stationary black holes |
| First | $dE = TdS + dW$ | $dM = \frac{\kappa}{8\pi}dA + \Omega dJ + \Phi dQ$ |
| Second | $\partial S \geq 0$ for all processes | $\partial A \geq 0$ for all processes involving black holes |
| Third | $T = 0$ can never be reached | $\kappa = 0$ can never be reached |

It is not hard to see that the existence of black hole is not compatible with the original second law of thermodynamics. If matter can fall into a black hole and disappear then the entropy of matter for the external observer can be decreased and to the external observer the entropy of the universe has decreased. In order for the existence of a black hole to be compatible with the second law of thermodynamics, it was postulated[47] that there has to exist a generalized second law:

$$\delta(S + S_{BH}) \geq 0, \tag{1.63}$$

where $S_{BH}$ is the entropy of a black hole and $S$ is the entropy of the matter outside the black hole.

## 1.3 Quantum Theory

Quantum field theory in Minkwoski spacetime is well worked out. Its computational techniques are elegant and often straightforward albeit mathematically involved. It is a very beautiful, powerful and successful theory. In fact, it is considered the most accurate of all physical theories. Quantum Field theory clarifies the meanings of several concepts including particle. It tells us that it is fields that are fundamental and that particle is a derived concept.

The concept of particle enters quantum field theory via the quantization of fields. That is, for instance, given the massless scalar field[14] $\varphi$ which obeys the Klein-Gordon equation

$$\partial_\mu \partial^\mu \varphi = 0, \tag{1.64}$$

one can quantize the scalar field $\varphi$ by promoting it to a quantum field operator with the operator satisfying the equal time commutation relations

$$[\phi(x,t), \Pi(x',t)] = \delta(x,x') \quad \text{and} \quad [\varphi(t,x), \varphi(t,x')] = 0 = [\Pi(t,x), \Pi(t,x')]. \tag{1.65}$$

[14] Though the argument remains the same for any field, the massless scalar field is chosen for the sake of simplicity

where $\Pi = \frac{\delta\mathscr{L}}{\delta(\nabla_0\varphi)}$ and $\mathscr{L} = -\frac{1}{2}\nabla_\alpha\phi\nabla^\alpha\phi$.

After quantization, the solution of Equation 1.64 can be expressed in terms of the plane wave basis (or positive frequency mode) $f \equiv e^{-ik_\mu x^\mu}$

$$\varphi = \sum_i \left[a_i e^{-ik_\mu x^\mu} + a_i^+ e^{ik_\mu x^\mu}\right] = \sum_i \left[a_i f_i + a_i^+ f_i^*\right], \tag{1.66}$$

where $k^\mu = (-\omega, k^i)$ is the wave vector, $x^\mu = (-t, x^i)$ is the spacetime coordinates and the time independent operators $a_i$ and $a_i^+$ are called the annihilation operator and creation operator respectively.

Using Equation 1.65, it is easy to see that

$$[a_i, a_j^+] = \delta_{ij} \quad \text{and} \quad [a_i, a_j] = 0 = [a_i^+, a_j^+]. \tag{1.67}$$

If we postulate a vacuum state $|0>$

$$a_i|0>= 0 \qquad \forall i \tag{1.68}$$

then we can construct the Fock space[15] by acting the creation operators $a_i^+$ on the vacuum state. For instance, $a_i^+|0>$ define the one-particle state $|1_i>$:

$$|1_i>= a_i^+|0> . \tag{1.69}$$

Using

$$a|n>= \sqrt{n}|n-1> \quad \text{and} \quad a^+|n>= \sqrt{n+1}|n+1> \quad \to |n>= \left[\frac{(a^+)^n}{\sqrt{n!}}\right]|n>, \tag{1.70}$$

we can construct the many-particle state by repeated application of $a^+$ on $|0>$. For example, the k-particle state is given by

$$|n_{i_1}^{(1)}, n_{i_2}^{(2)}, \cdots, n_{i_k}^{(k)}>= (n!^{(1)}n!^{(2)}\cdots n^{(k)}!)^{-\frac{1}{2}}(a_{i_1}^+)^{n^1}(a_{i_2}^+)^{n^2}\cdots(a_{i_k}^+)^{n^k}|0>, \tag{1.71}$$

with

$$N_i \equiv a_i^+ a_i \tag{1.72}$$

as count of the number of particles in the mode i.

[15]Most of the equations and comments in the rest of this section come from [48–52]

Since Minkwoski spacetime has the Poincare group of symmetry

$$x^{\mu'} = \wedge^{\mu}_{\nu} x^{\nu} + a^{\mu}, \quad {}^{16}, \tag{1.73}$$

the sign of the frequency modes is invariant[17] and therefore the concept of particle is defined unambiguously whenever there exists a Poincare-invariant ground state.

If, however, the spacetime lacks the Poincare symmetry then the definition of a particle becomes ambiguous. To see this, let consider a quantized massless Hermitian scalar field satisfying the general covariant wave equation

$$\Box\phi = \frac{1}{\sqrt{-g}}\partial_{\mu}\left[\sqrt{-g}g^{\mu\nu}\partial_{\nu}\phi\right] = 0. \tag{1.74}$$

If we define the Klein-Gordon inner product for two solutions, $f$ and $g$, of Equation 1.74 by

$$(f, g) = -i\int_{\Sigma} d\Sigma\sqrt{h}n^{\mu}\Big(f\nabla_{\mu}g^{*} - g^{*}\nabla_{\mu}f\Big), \tag{1.75}$$

where $\Sigma$ is a Cauchy surface with normal vector $n^{\mu}$ and induced metric $h$, then $\{f_i, f_i^*\}$ and $\{g_j, g_j^*\}$ form are orthonormal sets of bases in terms of which we may expand an arbitrary solution of Equation 1.74. For example, we can write

$$\psi = \int d\omega'(a_{\omega'}f_{\omega'} + a^{+}_{\omega'}f^{*}_{\omega'}) = \int d\omega(b_{\omega}g_{\omega} + b^{+}_{\omega}g^{*}_{\omega}). \tag{1.76}$$

Since the Klein-Gordon inner product is independent of choice of Cauchy surfaces (cf. Equation A.1), specifying a condition with respect to one Cauchy surface ensures that the condition will hold for all Cauchy surfaces and it will hold consistently throughout the spacetime.

It can be shown that $f_{\omega'}$ and $g_{\omega}$ satisfies

$$(f_{\omega}, f_{\omega'}) = \delta(\omega - \omega')) = -(f^{*}_{\omega}, f^{*}_{\omega'}) \quad \text{and} \qquad (f_{\omega}, f^{*}_{\omega'}) = 0, \tag{1.77}$$

for momenta $\omega > 0$ and $\Omega > 0$.

$$(g_{\Omega}, g_{\Omega'} = \delta(\Omega - \Omega') = -(g^{*}_{\Omega}, g^{*}_{\Omega'}) \quad \text{and} \quad = (g_{\Omega}, g^{*}_{\Omega'}) = 0, \tag{1.78}$$

It can also be shown that the creation and annihilation operators are time independent and satisfy the usual equal time commutation relations

[16] where $a^{\mu}$ is some constant.

[17] That is, the annihilation operator remains the coefficient of the positive frequency modes and the creation operator remains the coefficient of the negative frequency modes

$$[a_\omega, a^+_{\omega'}] = \delta(\omega - \omega'), \quad [b_\Omega, b^+_{\Omega'}] = \delta(\Omega - \Omega') \tag{1.79}$$

and

$$[a_\omega, a_{\omega'}] = [a^+_\omega, a^+_{\omega'}] = 0, \quad [b_\Omega, b_{\Omega'}] = [b^+_\Omega, b^+_{\Omega'}]. \tag{1.80}$$

The vacuum states $|0_a>$ or $|0_b>$ are defined as

$$a_\omega|0_a>=0 \quad \text{and} \quad b_\Omega|0_b>=0 \quad \forall\omega, \Omega. \tag{1.81}$$

Since $f_{\omega'}$ and $g_\omega$ are complete, we can expand one in terms of the other. For instance, we can write

$$g_\omega = \int_{-\infty}^{\infty} d\omega' \Big[\alpha_{\omega\omega'} f_{\omega'} + \beta_{\omega\omega'} f^*_{\omega,}\Big], \tag{1.82}$$

where $\alpha$ and $\beta$ are the Bogolubov coefficients and Equation 1.82 is the Bogolubov transformation.

Taking into account the relations Equation 1.77, we find that

$$\alpha_{\omega\omega'} \equiv (g_\omega, f_{\omega'}) \qquad \text{and} \quad \beta_{\omega\omega'} = -(g_\omega, f^*_{\omega'}). \tag{1.83}$$

From Equation 1.77 and Equation 1.78, we can find that

$$\int d\omega \Big[\alpha_{\Omega\omega}\alpha_{\Omega'\omega} - \beta_{\Omega\omega}\beta_{\Omega'\omega}\Big] = \delta(\Omega - \Omega') \tag{1.84}$$

Using Equation 1.83, Equation 1.76 and Equation 1.75 , we see that

$$(g_\omega, \psi) = \int d\omega' \Big[(a_{\omega'}(g_\omega, f_{\omega'}) + a^*_{\omega'}(g_\omega, f^*_{\omega'})\Big] = \int d\omega \Big[(b_\omega(g_\omega, g_\omega) + b^*_\omega(g_\omega, g^*_\omega)\Big] \tag{1.85}$$

or

$$b_\omega = \int d\omega' \Big[\alpha_{\omega\omega'} a_{\omega'} + \beta_{\omega\omega'} a^+_{\omega'}]. \tag{1.86}$$

We can use Equation 1.86 and its conjugate to calculate the number of particles in the $|0_f>$ as viewed with respect to the $g$ basis:

$$\begin{aligned} < N_\Omega > &=< 0_f|b^+_\Omega b_\Omega|0_f > \\ &=< 0_f|(\int d\omega[\alpha^*_{\omega\Omega} a^+_\omega - \beta^*_{\Omega\omega} a_\omega])(\int d\omega'[\alpha_{\omega'\Omega} a_{\omega'} + \beta_{\omega'\Omega} a^+_{\omega'}])|0_f >= \int d\omega|\beta_{\Omega\omega}|^2. \end{aligned} \tag{1.87}$$

Since $\beta_{ij} \neq 0$ in general , $|0_f>$ and $|0_g>$ do not necessarily coincide. This means that the concepts of vacuum and particle are observer dependent; what is regarded as a vacuum to one observer might be swarming with particles according to another observer.

One important consequence of this observer dependent of particle content is that there will be regimes in which there is no good definition for particle. For instance, there is ambiguity in the definitions of vacuum or particle in general spacetime.

One of the reasons for the ambiguity in the definitions of vacuum and particle for a general spacetime is that they are defined globally, in terms of field modes and so are sensitive to the large scale structure of spacetime. If, however, the spacetime is stationary or asymptotically flat at both early and late times then we can make use of the facts that killing vector operator $\partial_t$ and the Klein-Gordon operator commute(cf. Equation A.2)[18]

$$[\partial_t, \Box - m^2] = 0, \tag{1.88}$$

and the eigenvalues of the timelike Killing vector are purely imaginary since timelike Killing operator is anti-hermitian under the Klein-Gordon inner product(cf. Equation A.5) [20]

$$(f, Kg) = (-Kf, g) \tag{1.89}$$

to define positive frequency modes and negative frequency modes

$$\partial_t f_j = -i\omega f_j \quad \text{and} \quad \partial_t = i\omega f_j^* \qquad \omega > 0 \tag{1.90}$$

respectively for spacetimes that are stationary. This definition of positive frequency modes and negative frequency modes can be used to define unambiguous notion of vacuum and hence particles for stationary spacetimes. In other words, if we find that in a stationary spacetime the vacuum in two different basis does not coincide, say $|0>_g \neq |0>_f$, it implies particles creation. In short, the passage of the field through a time dependent potential or region of time dependent curvature has resulted in the creation or destruction of particles. Equivalently, a static gravitational field can create particle only if there exists a region where the Killing vector (which is future-directed and timelike at asymptotic infinity) becomes spacelike. This region lies inside the Killing horizon[21]. This implies that one can expect particle creation in a static spacetime only if it contains a black hole.

Contrary to the concept of vacuum or particle, quantities defined directly through the field $\hat{\phi}$, for example, the expectation value $<\psi|\hat{\phi}(x)|\psi>$ in some state $|\psi>$ are unambiguous. One of such unambiguous quantities is the expectation value of the stress-energy tensor,$<T_{\mu\nu}>(x)$ at the point $x$ which also serves as the source term in the

[18][19]

[20]see[51] for the proof.

[21]inside the event horizon of a Schwarzschild black hole.

Semi-classical Einstein equation

$$G_{\mu\nu} = 8\pi < \psi|\hat{T}_{\mu\nu}(x)|\psi > . \tag{1.91}$$

The motivation for the semiclassical theory is the fact that the left hand side and right hand side of Equation 1.16 are mathematically inconsistent. This is due to the fact that the stress-energy tensor on the right hand of Equation 1.16 is an operator whereas the left hand side of Equation 1.16 is a complex number. The widely accepted (and probably most popular) mean of harmonizing both sides of Equation 1.16 is by taking the expectation value of the stress-energy tensor. Equivalently, another motivation for the semiclassical theory is that the non-gravitational fields are known to be described by quantized fields and so it is assumed that all fields of a complete theory are quantized. Since, it is yet not possible to quantize the gravitational field, the accepted view is that for cases in which the curvature $R$ of the metric is far smaller than the Planck curvature

$$R_P \approx \frac{c^3}{\hbar G} \approx 3.829 \times 10^{65} cm^{-2}, \tag{1.92}$$

as is expected for a black hole of mass much greater than the Planck mass $M_P = \sqrt{\frac{\hbar c}{G}} \approx 2.177 \times 10^{-5} g$ then it is sufficient to quantize all fields except the gravitational field. The problem with taking the expectation value of the stress-energy tensor or quantizing the non-gravitational fields is that the result can be infinite, even in Minkwoski vacuum and therefore must be regularized and renormalized if the theory is to make sense.

The expectation value of the stress-energy is infinite because it contains the product of field operators at the same point. Naively, one would think that the usual procedure of normal ordering-finding the difference between the expectation values in a particular state and the expectation value in the vacuum state or putting the creation operator before the annihilation operator whenever they appear as products in an expression-as is done in quantum field theory would give finite results. This does not work because in curved spacetime absolute energy is of physical significant and also because the vacuum state is not unique in a general spacetime. Therefore, finding finite values from the infinite of the expectation values of quantities involving the product of field operators require elaborate and sophisticated schemes called regularization. Regularization means the introduction of an extra parameter ( or a so-called regulator) into the theory. The regulator is expected to make a divergent quantity convergent while at the same time recovering the original divergent quantity when the regulator is set to zero. After regularization, the theory can now be renormalized by taking limits which remove the extra parameter. For example, to renormalized a theory that gives the divergent quantity $F$

we first regularize the theory by introducing the regulator $\xi$ in to the theory

$$\Big|F\Big| = \infty \tag{1.93}$$

to obtain $F(\xi)$ where

$$\lim_{\xi \to 0} F(\xi) = F, \tag{1.94}$$

for finite but small $\xi$

$$\Big|F(\xi)\Big| < \infty. \tag{1.95}$$

After obtaining Equation 1.95 from Equation 1.93 by the introduction of the extra parameter $\xi$ then we can say that the theory has been regularized. Once the theory has been regularized, we can then calculate any quantity we want in terms of the quantities appearing in theory including the extra parameter we have introduced into the theory. One can then invoke certain conditions justified by either empirical evidences or physical considerations that enable one to get physical quantities that do not depend on the regulator. The standard view is that only quantities that are independent of the regulator are considered as physical quantities.

If, however, the difference of the expectation value of the stress-energy tensor in two different states that share the same singular short distance characteristics is well defined and finite then and this can be used as the criterion for the choice of states for which regularization procedures can be applied. This criterion, called the Hadamard property[31, 53], can be stated mathematically in terms of the two-point function $G(x, x')$:

$$G(x, x') = \frac{U(x, x')}{4\pi^2 \sigma_\epsilon(x, x')} + V(x, x') \ln \sigma_\epsilon(x, x') + W(x, x'), \tag{1.96}$$

where

$$\sigma_\epsilon(x, x') = g_{\mu\nu}(x^\mu - x'^\mu)(x^\nu - x'^\nu) + 2i\epsilon(t - t') + \epsilon^2, \tag{1.97}$$

$\epsilon \to 0^+$ is assumed and the functions $U(x, x')$, $V(x, x')$, $W(x, x')$ are all regular as $x \to x'$ and $U(x, x) = 1$. The function $U(x, x')$ and $V(x, x')$ are geometrical quantities independent of the quantum state and only $W(x, x')$ carries information about the state. Thus, the Hadamard condition is that only states whose two-point functions are of the same form as Equation 1.96 are renormalizable. If the two-point function of a state does not have the Hadamard form then it is considered to be unphysical[53]. It has been shown[54] that if the state is initially Hadamard in a suitable neighbourhood of some Cauchy surface, it will be Hadamard throughout the spacetime.

In practice, this regularization and renormalization scheme, also called the point-splitting procedure, is done by constructing the expectation value of the stress tensor $< T_{\mu\nu} >$ by replacing the same spacetime position for field operators being multiplied by slightly different spacetime positions. The limit of zero separation is then taken at the end of the calculation [54]. For example[22], the divergent values that results from taking the product of $\bar{\psi}(x)$ and $\psi(x)$ at the coincident point $x$ when electromagnetic current density $J_\mu^{em}(x)$

$$J_\mu^{em} x = e\bar{\psi}(x)\gamma_\mu \psi(x) \tag{1.98}$$

is calculated can be regularized by replacing the local product with slightly different separation with the introduction of the regulator $\xi$

$$J_\mu^{em} x = e\bar{\psi}(x+\xi)\gamma_\mu \psi(x-\xi). \tag{1.99}$$

or

For example, consider the massless minimally coupled scalar field for which

$$T_{\mu\nu} = \phi_{,\mu}\phi_{,\nu} - \frac{1}{2} g_{\mu\nu}\phi_{,\alpha}\phi^{,\alpha}. \tag{1.100}$$

The expectation value of $T_{\mu\nu}$ is

$$< T_{\mu\nu} >= \frac{1}{2} \lim_{x \to x'} \left[ \partial_\mu \partial_{\nu'} - \frac{1}{2} g_{\mu\nu} \partial_\alpha \partial^{\alpha'} \right] G(x, x'), \tag{1.101}$$

where $\partial_\mu$ denotes the derivative with respect to $x^\mu$ and $\partial_{\nu'}$ denotes the derivative with respect to $x'^\nu$.

Other procedures for regularization and renormalization include:

1. Dimensional regularization
   In this regularization scheme, divergent integrals are regularized in such a way that the dimension of the physical spacetime becomes a parameter that can be varied [56, 57]. For example [23] , the integral

$$\int \frac{d^4 p}{\left(p^2 + m^2\right)} \tag{1.102}$$

   which diverges can be made to give finite results by regularizing it as

$$\int \frac{d^{4-2\xi} p}{\left(p^2 + m^2\right)}. \tag{1.103}$$

[22] this example is copied directly from [55].
[23] this example is taken from http://www.phys.vt.edu/ ersharpe/6455/janhand1.pdf

2. Zeta-function regularization
   This regularization scheme uses the Riemann's zeta function to regularize divergent quantities. It most popular use is in the regularizing the vacuum energy density due to the Casimir effect. For example [24], the vacuum energy density $\varepsilon_0$ between two plates of separation $L$

$$\varepsilon_0 = \frac{1}{2L^2} \sum_{n=1}^{\infty} n = \infty \tag{1.104}$$

   can be regularised by introducing a regulator $\xi$

$$\varepsilon_0(L;\xi) = \frac{\pi}{2L^2} \sum_{n=1}^{\infty} n e^{-\frac{n\xi}{L}} \tag{1.105}$$

3. Adiabatic regularization
   In this regularization scheme, the quantity to be regularized (most often the stress-energy tensor) is first expressed in terms of well defined adiabatic basis. One then identify the terms giving rise to the divergence and subtract them mode by mode [49, 58, 59].

4. Lattice regularization
   In this regularization scheme the divergence is eliminated by discretizing spacetime and using the lattice spacing as the regulator.[60, 61]

[24]this example is lifted directly from [50]

# Chapter 2

# *Hawking Radiation*

## 2.1 Particle Production by Black Holes

Since the definition of positive frequency solutions is unambiguous on $\mathscr{I}^+$ and $\mathscr{I}^-$ for a star (which that is stationary at early time) that collapse to form a Schwarzschild black hole at late time, we can let $f_{\omega'}$ form a complete set of incoming positive frequency solutions of Equation A.11 on $\mathscr{I}^-$ and let $g_\omega$ form a complete set of outgoing positive frequency solutions of Equation A.11 on $\mathscr{I}^+$. The asymptotic form of $f_{\omega'}$ on $\mathscr{I}^-$ is

$$f_{\omega'} \backsim \frac{F_{\omega'}}{r\sqrt{4\pi}} e^{-i\omega' v} \tag{2.1}$$

and the asymptotic form of $g_\omega$ on $\mathscr{I}^+$ is

$$g_\omega \backsim \frac{G_\omega}{\sqrt{4\pi} r} e^{-i\omega u(v)} = \begin{cases} 0 & v > v_0 \\ \frac{G_\omega(r)}{r\sqrt{4\pi\omega}} e^{-i\omega \frac{1}{\kappa} \ln\left(\frac{\lambda}{C_1}\right)} & v < v_0. \end{cases} \tag{2.2}$$

In Equation 2.2, we have used Equation 2.17 since we are only interested in modes detected at $\mathscr{I}^+$ at late time when the geometry outside the black hole is expected to be the Schwarzschild geometry.

To derived the entropy of a black hole, we will follow Hawking's analysis [21, 62, 63] where the black hole formation and radiation is treated as a scattering process in which null rays with constant incoming null coordinate $v$ originate from $\mathscr{I}^-$, pass through the collapsing body to arrive at $\mathscr{I}^+$ as null rays with constant outgoing null coordinate $u(v)$ just before $\mathscr{H}^+$ is formed at $v = v_0$[1].

[1]the methods used in solving these equations follow those of [49, 51, 64]

If we choose an affine parameter $\lambda$ such that the affine separation between any two incoming null rays or outgoing null rays is constant along the entire length of the geodesic from $\mathscr{I}^-$ to $\mathscr{I}^+$ then the affine separation between $v$ and $v_0$ is equal to the affine separation between $u(v)$ and $u(v_0)$. Taking $\lambda = 0$ on the event horizon, $u(v)$ to be near the horizon and $v$ as an affine parameter then we can write

$$v_0 - v = b\lambda, \tag{2.3}$$

where $b$ is a negative constant[49].

Since the Schwarzschild metric is independent of the coordinates $t$ and $\phi$, we can use the Killing vector equations[2]

$$(1 - \frac{2M}{r})\frac{dt}{d\lambda} = E \tag{2.4}$$

and

$$r^2\frac{d\phi}{d\lambda} = L, \tag{2.5}$$

where $\lambda$ is an arbitrary affine parameter and $E$ and $L$ are constants along the geodesic $x^\alpha(\lambda)$. We can set $\theta = \frac{\pi}{2}$, without any loss in generality, and use Equation 2.4 and Equation 2.5 in

$$u_\alpha u^\alpha \equiv g_{\alpha\beta}\frac{dx^\alpha}{d\lambda}\frac{dx^\beta}{d\lambda} = -1 \tag{2.6}$$

to get

$$\left(\frac{dr}{d\lambda}\right)^2 + \frac{L^2}{r^2}\left(1 - \frac{2M}{r}\right) = E^2 \tag{2.7}$$

For radial geodesics (i.e., $L = 0$) Equation 2.7 becomes

$$\frac{dr}{d\lambda} = E \tag{2.8}$$

for outgoing radial null geodesics and

$$\frac{dr}{d\lambda} = -E \tag{2.9}$$

for ingoing radial null geodesics.
Setting $\lambda = 0$ at the horizon, we can solve Equation 2.8 and Equation 2.9 to get

$$r - 2M = E\lambda \tag{2.10}$$

and

$$r - 2M = -E\lambda \tag{2.11}$$

[2]The solutions here follow directly as done in[49].

respectively.

Equation 2.8 into Equation 2.4 gives

$$\frac{d}{d\lambda}\Big[t-\Big(1-\frac{2M}{r}\Big)^{-1}dr\Big] \equiv \frac{d}{d\lambda}(t-r^*) \equiv \frac{du}{d\lambda}=0. \tag{2.12}$$

We see from Equation 2.12 that the null coordinate $u$ is constant along any outgoing radial null geodesic.

Similarly, Equation 2.9 into Equation 2.4 gives

$$\frac{d}{d\lambda}\Big[t+\Big(1-\frac{2M}{r}\Big)^{-1}dr\Big] \equiv \frac{d}{d\lambda}(t+r^*) \equiv \frac{dv}{d\lambda}=0. \tag{2.13}$$

Differentiating $u$ with respect to $\lambda$ gives

$$\begin{aligned}\frac{du}{d\lambda} =&\frac{dt}{d\lambda}-\frac{dr^*}{d\lambda}=\frac{dt}{d\lambda}-\frac{dr^*}{dr}\frac{dr}{d\lambda}=2\Big(1-\frac{2M}{r}\Big)^{-1}E\\ &=2\Big(\frac{r}{r-2M}\Big)E=2\Big(\frac{-E\lambda+2M}{-E\lambda}\Big)E=2\Big(1-\frac{2M}{E\lambda}\Big)E\\ &=2E-\frac{4M}{\lambda},\end{aligned} \tag{2.14}$$

where we have used Equation 2.11.

A solution for Equation 2.14 is

$$u=2E\lambda-4M\ln\Big(\frac{\lambda}{C_1}\Big), \tag{2.15}$$

where $C_1$ is a negative constant.

Taking into account that $\lambda=0$ at the event horizon, we can write

$$u(v)\approx 2E\lambda \quad r\to\infty \tag{2.16}$$

$$u(v)\approx -4M\ln\Big(\frac{\lambda}{C_1}\Big)=-\frac{1}{\kappa}\ln\Big(\frac{\lambda}{C_1}\Big) \quad r\to 2M \tag{2.17}$$

Putting Equation 2.3 into  Equation 2.17 gives

$$u(v)=-4M\ln(\frac{\lambda}{a})=-4M\ln(\frac{v_0-v}{ab})=-\frac{1}{\kappa}\ln(\frac{v_0-v}{C}), \tag{2.18}$$

where $C\equiv ab$.

Equation 2.18 into Equation 2.2, we finally get

$$g_\omega \backsim \frac{G_\omega}{\sqrt{4\pi}r} e^{-i\omega u(v)} = \begin{cases} 0 & v > v_0 \\ \frac{G_\omega(r)}{r\sqrt{4\pi\omega}} e^{i\frac{\omega}{\kappa}} \ln(\frac{v_0 - v}{C}) & v < v_0. \end{cases} \tag{2.19}$$

as the basis modes on $\mathscr{I}^+$.

Actually,

$$\Sigma^+ = \mathscr{I}^+ \cup \mathscr{H}^+ \tag{2.20}$$

rather than $\mathscr{I}^+$ is a Cauchy surface since the most general solution of the wave equation will have a part that is incoming at $\mathscr{H}^+$ at late times. Therefore, we must introduce a set of solutions $q_\omega$ on $H^+$ such that a superposition of them at late times is localized near the $H^+$ and has zero Cauchy data on $I^+$. Equation 2.19 is only valid because we are interested in what distant observers perceive, rather than in what happens at the event horizon. If we need the part of the wave on $\Sigma^+$, which is the case for the calculation of probability densities, then we must specify a basis mode on $\mathscr{H}^+$. However, there is no natural choice for defining a positive frequency mode on $\mathscr{H}^+$ since there is no timelike Killing vector on $\mathscr{H}^+$. Thus, it is extremely difficult defining a meaningful basis on $\mathscr{H}^+$. However, following the argument made by Wald[65], we make the assumption that

$$h_\omega \backsim \begin{cases} 0 & v < v_0 \\ \frac{G_\omega(r)}{r\sqrt{4\pi\omega}} e^{i\frac{\omega}{\kappa}} \ln(\frac{v - v_0}{C}) & v > v_0 \end{cases} \tag{2.21}$$

since for $v > v_0$ all of the wave from $\mathscr{I}^-$ falls across $\mathscr{H}^+$ with none propagating to $\mathscr{I}^+$. In other words, we are making use of the fact that there is no correlation between the radiation before and after $v_0$ to construct $h_\omega$[3]. The motivation here is the desire to get the simplest and clearest result so that the expansion of $h_\omega$ in terms of $f_{\omega'}$ look very similar to the expansion of $g_\omega$ in terms of $f_{\omega'}$.

As discussed in section 1.3 , we can express any solution of the Klein-Gordon equation in terms of either basis:

$$\psi = \int d\omega' (a_{\omega'} f_{\omega'} + a^+_{\omega'} f^*_{\omega'}) = \int d\omega (b_\omega g_\omega + c_\omega h_\omega + b^+_\omega g^*_\omega + c^+_\omega h^*_\omega), \tag{2.22}$$

where $h_\omega$ and $h^*_\omega$ form a complete set on the horizon with normalization the same as given in and $a_{\omega'}$, $b_\omega$ and $c_\omega$ have their usual definitions:

$$a_{\omega'}|0_f>=0 \quad \forall \quad \omega', \quad b_\omega|0_g>=0 \quad \forall \quad \omega \quad \text{and} \quad c_{\omega'}|0_h>=0 \quad \forall \quad \omega. \tag{2.23}$$

[3]See [52] and references therein.

Since $g_\omega$ and $h_\omega$ are in disjointed regions at late times , their conserved product and commutation all vanish. Therefore, we can ignore $h_\omega$ when calculating particles density in $|0_f>$ as observed at $\mathscr{I}^+$.

Just as discussed in section 1.3, we can express the frequency modes in terms of each other:

$$g_\omega = \int d\omega'(\alpha_{\omega\omega'} f_{\omega'} + \beta_{\omega\omega'} f^*_{\omega'}), \tag{2.24}$$

and

$$h_\omega = \int d\omega'(\gamma_{\alpha\alpha'} f_{\omega'} + \eta_{\omega\omega'} f^*_{\omega'}), \tag{2.25}$$

where

$$b_\omega = \int d\omega'(\alpha^*_{\omega\omega'} a_{\omega'} - \beta^*_{\omega\omega'} a^+_{\omega'}) \tag{2.26}$$

and

$$c_\omega = \int d\omega'(\gamma^*_{\omega\omega'} a_{\omega'} - \eta^*_{\omega\omega'} a^+_{\omega'}). \tag{2.27}$$

Using Equation 2.1, Equation 2.2, Equation 2.24, multiplying the result by $e^{\pm\omega'' v}$ and then integrating over $v$, we get

$$\begin{aligned}\int_{-\infty}^{\infty} g_\omega e^{\pm i\omega''} dv &= \int_0^\infty d\omega'\left[\alpha_{\omega\omega'}\frac{F_{\omega'}(r)}{r\sqrt{4\pi\omega'}}\int_{-\infty}^{\infty} e^{i(-\omega'\pm\omega'')v} + \beta_{\omega\omega'}\frac{F_{\omega'}(r)}{r\sqrt{4\pi\omega'}}\int_{-\infty}^{\infty} e^{i(\omega'\pm\omega'')v}\right]\\ &= 2\pi\int_0^\infty d\omega'\left[\alpha_{\omega\omega'}\frac{F_{\omega'}(r)}{r\sqrt{4\pi\omega'}}\delta(-\omega'\pm\omega'') + \beta_{\omega\omega'}\frac{F_{\omega'}(r)}{r\sqrt{4\pi\omega'}}\delta(\omega'\pm\omega'')\right],\end{aligned} \tag{2.28}$$

where we have used

$$\int_{-\infty}^{\infty} e^{i(\omega-\omega')v} dv = 2\pi\delta(\omega-\omega'). \tag{2.29}$$

From Equation 2.28, we see that

$$\begin{aligned}\alpha_{\omega\omega'} =& \frac{r\sqrt{\omega'}}{\sqrt{\pi}F_{\omega'}}\int_{-\infty}^{\infty} dv e^{i\omega' v} g_\omega = \frac{1}{2\pi}\sqrt{\frac{\omega'}{\omega}}\frac{G_\omega(r)}{F_{\omega'}}\int_{-\infty}^{v_0} dv e^{\left[i\omega' v + i\frac{\omega}{\kappa}\ln(\frac{v_0-v}{K})\right]}\\ &\approx A\sqrt{\frac{\omega'}{\omega}}\int_{-\infty}^{v_0} dv e^{\left[i\omega' v + i\frac{\omega}{\kappa}\ln(\frac{v_0-v}{K})\right]}, \qquad r\to 2M\end{aligned} \tag{2.30}$$

and

$$\begin{aligned}\beta_{\omega\omega'} =& \frac{r\sqrt{\omega'}}{\sqrt{\pi}F_{\omega'}}\int_{-\infty}^{\infty} dv e^{-i\omega' v} g_\omega = \frac{1}{2\pi}\sqrt{\frac{\omega'}{\omega}}\frac{G_\omega(r)}{F_{\omega'}}\int_{-\infty}^{v_0} dv e^{\left[-i\omega' v + i\frac{\omega}{\kappa}\ln(\frac{v_0-v}{K})\right]}\\ &\approx A\sqrt{\frac{\omega'}{\omega}}\int_{-\infty}^{v_0} dv e^{\left[-i\omega' v + i\frac{\omega}{\kappa}\ln(\frac{v_0-v}{K})\right]}, \qquad r\to 2M,\end{aligned} \tag{2.31}$$

where $A \equiv \frac{1}{2\pi}\frac{G_\omega(r)}{F_{\omega'}}$ is a constant at the event horizon.

Given that the integrands of Equation 2.30 and Equation 2.31 have no singularities, we can make the transformation $v \to -v - \frac{i\pi}{\kappa}$ and then Equation 2.30 becomes

$$\alpha_{\omega\omega'} = e^{\frac{\pi\omega}{\kappa}} A\sqrt{\frac{\omega'}{\omega}} \int_{-\infty}^{v_0} dv e^{\left[-i\omega' v + i\frac{\omega}{\kappa}\ln(\frac{v_0 - v}{K})\right]} = e^{\frac{\pi\omega}{\kappa}} \beta_{\omega\omega'} \tag{2.32}$$

or

$$|\beta_{\omega\omega'}|^2 = e^{-2\frac{\pi\omega}{\kappa}} |\alpha_{\omega\omega'}|^2. \tag{2.33}$$

Similarly, using Equation 2.1, Equation 2.21 and Equation 2.24 and following the same procedure used to obtain Equation 2.33 , we can check that

$$\eta_{\omega\omega'} = -e^{-\frac{\pi\omega}{\kappa}} \alpha^*_{\omega\omega'} \quad \text{and} \qquad \beta_{\omega\omega'} = -e^{-\frac{\pi\omega}{\kappa}} \gamma_{\omega\omega'}. \tag{2.34}$$

Taking into account Equation 1.87, Equation 1.84, Equation 2.33 and setting $\Omega = \Omega'$, we see that

$$< n_\omega >= \frac{1}{e^{\frac{2\pi\omega}{\kappa}} - 1}, \tag{2.35}$$

where $< n_\omega >\equiv \frac{<N_\omega>}{\delta(0)}$ is the number of particles of mode $\omega$ in the volume of space $\delta(0)$. This expectation value of particle density coincides exactly with the Planck distribution of thermal radiation for boson. In other words, Equation 2.35 is just the Bose-Einstein distribution with temperature

$$T_H = \frac{\kappa}{2\pi} = \frac{1}{8\pi M}. \tag{2.36}$$

Equation 2.35 represents the average number of particles detected at $\mathscr{I}^+$ with a definite frequency $\omega$ with temperature given by Equation 2.36.

Since we are interested in the average number of particles observed at late time when in a realistic situation the black hole has settled down to a stationary configuration, we must replace the plane wave type modes, which are completely de-localized, by wave packets. One can introduce a complete orthonormal set of wave packet modes at $\mathscr{I}^+$, with discrete quantum numbers, as follows[63, 65]

$$g_{jn} = \frac{1}{\sqrt{\epsilon}} \int_{j\epsilon}^{(j+1)\epsilon} d\omega e^{-\frac{2\pi i\omega n}{\epsilon}} g_\omega. \tag{2.37}$$

Using Equation 2.37 we then find that

$$\alpha_{jn\omega'} = \frac{1}{\sqrt{\epsilon}}\int_{j\epsilon}^{(j+1)\epsilon} d\omega e^{-\frac{2\pi i\omega n}{\epsilon}}\alpha_{\omega\omega'} \quad \text{and} \quad \beta_{jn\omega'} = \frac{1}{\sqrt{\epsilon}}\int_{j\epsilon}^{(j+1)\epsilon} d\omega e^{-\frac{2\pi i\omega n}{\epsilon}}\beta_{\omega\omega'} \tag{2.38}$$

and it can be straightforwardly shown[52] that one will still arrive at Equation 2.36 and Equation 2.37. Thus, unless absolutely necessary, we will continue to use the plane wave modes but with the understanding that it is the wave packets that are needed in realistic situations.

To be certain that the black hole radiation is truly thermal, we need to check the probabilities of emitting different numbers of particles or particles in different modes. For instance, making use of Equation 2.30, Equation 2.31, Equation 2.38 and noting that[4]

$$\int_0^\infty d\omega'\beta_{\omega_1\omega'}\beta^*_{\omega_2\omega'} = \frac{1}{e^{8\pi M\omega_1}}\delta(\omega_2-\omega_1), \quad \int_0^\infty d\omega'\beta_{jn\omega'}\beta^*_{kn\omega'} = 0 \quad \text{for} \quad j\neq k, \tag{2.39}$$

$$\int_0^\infty d\omega'\alpha_{jn\omega'}\beta_{jn\omega'} = 0 \quad \text{since} \quad \delta(\omega_1+\omega_2) = 0, \tag{2.40}$$

and

$$\Big|\int_0^\infty d\omega'\beta_{jn\omega'}\beta^*_{kn\omega'}\Big|^2 = 0 \quad \text{and} \quad \Big|\int_0^\infty d\omega'\alpha_{jn\omega'}\beta^*_{kn\omega'}\Big|^2 = 0, \tag{2.41}$$

we can check that

$$\begin{aligned} < 0_f|N_{jn}N_{jn}|0_f > &=< 0_f|b^+_{jn}b_{jn}b^+_{jn}b_{jn}|0_f > \\ &= \int_0^\infty d\omega'|\beta_{jn\omega'}|^2 + 2(\int_0^\infty d\omega'|\beta_{jn\omega'}|)^2 + |\int_0^\infty d\omega'\alpha_{jn\omega'}\beta_{jn\omega'}|^2 \\ &= \frac{e^{-8\pi M\omega_j}(1+e^{-8\pi M\omega_j}}{(1-e^{-8\pi M\omega_j})^2} \equiv \sum_{N=0}^{\infty} N^2 P_{jn}, \end{aligned} \tag{2.42}$$

and

$$\begin{aligned} < 0_f|N_{jn}N_{kn}|0 > = (\int_0^\infty d\omega'|\beta_{jn\omega'}|^2)(\int_0^\infty d\omega'|\beta_{kn\omega'}|^2) + |\int_0^\infty d\omega'\beta_{jn\omega'}\beta^*_{kn\omega'}|^2 \\ + |\int_0^\infty d\omega'|\alpha_{jn\omega'}\beta^*_{kn\omega'}|^2 = \frac{1}{e^{8\pi M\omega_j}-1}\frac{1}{e^{8\pi M\omega_k}-1} \\ \equiv< 0_f|N_{jn}|0_f >< 0_f|N_{kn}|0_f >, \end{aligned} \tag{2.43}$$

[4] see [52] for details and proofs

where [52]

$$P_{jn} = (1 - e^{-8\pi M\omega_j})e^{-8\pi NM\omega_j}, \tag{2.44}$$

is the probability of emitting particle per volume in modes $jn$. Results similar to Equation 2.42 and Equation 2.43 are obtained for higher moments.

One can see from Equation 2.43 that there is a complete absence of correlation between different modes, as is the case for all thermal radiations. The profound implication of the absence of correlation between different modes is that the quantum state of the late time radiation at $\mathscr{I}^+$ is exactly described by a density matrix which is a tensor products of modes. That is, the state of quanta observed at $\mathscr{I}^+$ can be expressed as

$$\rho_{thermal} = \prod_{jn} \sum_{n=0}^{\infty} P(n_{jn}) |n_{jn} >< n_{jn}|, \tag{2.45}$$

where $|n_{jn}>$ is the state in the Fock space at $\mathscr{I}^+$ with $n$ particles of modes.

The backscattering can be taken into account by introducing the absorptivity factor $\Gamma_\omega$. This can be done by taking into the account the fact that part of the wave, say $g^{(1)}_\omega$, is scattered off the static exterior Schwarzschild geometry, reaching $\mathscr{I}^-$ with the original frequency while the remaining part of the wave, say $g^{(2)}_\omega$, enters the collapsing object and reaches $\mathscr{I}^-$ as a supposition of various positive and negative parts. Thus, on $\mathscr{I}^-$

$$g_\omega = g^{(1)}_\omega + g^{(2)}_\omega, \tag{2.46}$$

and

$$g^{(2)}_\omega = \int d\omega' (\alpha^{(2)}_{\alpha\alpha'} f_{\omega'} + \beta^{(2)}_{\omega\omega'} f^*_{\omega'}), \tag{2.47}$$

where $g^{(1)}_\omega = \alpha^{(1)}_{\omega\omega'} f_{\omega'}$. Since the part of the wave scattered off the static exterior geometry does not participate in particle product, $g^{(1)}_\omega$ will be ignored and we will drop the superscript from $g^{(2)}_\omega$. That is, throughout this thesis $g_\omega$ will refer to the part of the wave that passes through the collapsing body or the time dependent geometry to reach $\mathscr{I}^-$ as a supposition of various positive and negative parts.

Taking into account backscattering, we can write

$$< n_\omega >= \int_0^\infty d\omega' |\beta_{\omega\omega'}|^2 = \Gamma_\omega \Big[ e^{\frac{2\pi\omega}{\kappa}} - 1 \Big]^{-1} \tag{2.48}$$

where

$$\Gamma_\omega = \int_0^\infty d\omega' \Big( |\alpha_{\omega\omega'}|^2 - |\beta_{\omega\omega'}|^2 \Big). \tag{2.49}$$

The Hawking temperature can also be derived via quantum tunnelling of particles across the event horizon of black hole. This way of deriving the Hawking temperature is motivated by the fact that the Uncertainty Principle can not be reconciled with the idea of a vacuum or zero-particle state without insisting that virtual particles are constantly being created and annihilated in $\Delta t \leq \frac{1}{\Delta\omega}$ in what is supposed to be a vacuum. In the presence of an event horizon, some of these virtual particles can tunnel out of the horizon to be observed at infinity as the Hawking radiation and their anti-particle partners can fall into the singularity. To ensure the law of conservation of energy is not violated in this process, the mass of the black hole is reduced by an amount equal to the energy contained in the Hawking radiation.

As was done in [66], we consider a Schwarzschild black hole of mass $M-\omega$ after radiating quanta of energy $\omega$

$$ds^2 = -\Big[1 - \frac{2(M-\omega)}{r}\Big]dt^2 + 2\sqrt{\frac{2(M-\omega)}{r}}dtdr + dr^2 + r^2 d\Omega^2 \tag{2.50}$$

and with outgoing radial null geodesics satisfying

$$\frac{dr}{d\lambda} \equiv \dot{r} = 1 - \sqrt{\frac{2(M-\omega)}{r}}. \tag{2.51}$$

To compute the tunnelling amplitude $\gamma$, we use the WKB approximation given by

$$\gamma \approx e^{-2ImS}, \tag{2.52}$$

where[5]

$$\begin{aligned} \mathrm{Im}S = Im\int_{r_{in}}^{r_{out}} p_r dr = Im\int_{r_{in}}^{r_{out}}\int_0^{p_r} dp_r dr = Im\int_M^{M-\omega}\int_{r_{in}}^{r_{out}}\frac{dH_r}{\dot{r}}dr \\ = -Im\int_0^{\omega}\int_{r_{in}}^{r_{out}}\frac{d\omega'}{1-\sqrt{\frac{2(M-\omega')}{r}}}dr = 4\pi\omega\Big(M - \frac{\omega}{2}\Big). \end{aligned} \tag{2.53}$$

In Equation 2.53, we have used Equation 2.51, along with $\dot{r} = \dfrac{dH_r}{dp_r}$ and $H_r = M - \omega$. Using Equation 2.53, we therefore see that the probability for a pair created particle to tunnel across the event horizon of the Schwarzschild black hole is given by

$$\gamma \approx e^{-ImS} = e^{-8\pi\omega\left[M-\frac{\omega}{2}\right]}. \tag{2.54}$$

[5] see Equation A.14 for integration

This looks like the Bose-Einstein distribution with

$$T = \frac{\Gamma}{8\pi M}, \tag{2.55}$$

where we have defined $\Gamma \equiv e^{-\frac{8\pi\omega^2}{2}}$.

Alternatively, Hawking radiation can be viewed as pair creation occurring outside the black hole. Using the Feynmann-Stuekelberg interpretation[67, 68] that antiparticles can be interpreted as particles with positive energy moving backward in time then similar method used for deriving the Hawking radiation for the pair creation occurring inside the black hole can be used.

Since the particle is moving backward in time with positive energy, we must replaced $\sqrt{\frac{2(M-\omega)}{r}}$ by $-\sqrt{\frac{2(M+\omega)}{r}}$ and $M-\omega$ by $M+\omega$ in the equations used to derived the Hawking radiation for the case where the pair creation is occurring outside the event horizon. Thus, the Schwarzschild metric becomes

$$ds^2 = -\Big[1 - \frac{2(M+\omega)}{r}\Big]dt^2 - 2\sqrt{\frac{2(M+\omega)}{r}}dtdr + dr^2 + r^2 d\Omega^2, \tag{2.56}$$

and we use the ingoing radial null geodesics given by

$$ds^2 = -\Big[1 - \frac{2(M+\omega)}{r}\Big]dt^2 - 2\sqrt{\frac{2(M-\omega)}{r}}dtdr + dr^2 + r^2 d\Omega^2. \tag{2.57}$$

Following the same procedure used for the case of the pair creation outside the event horizon, we find that

$$ImS = Im\int_0^{-\omega}\int_{r_{out}}^{r_{in}} \frac{dr}{-1+\sqrt{\frac{2(M+\omega')}{r}}}d\omega' = 4\pi\omega(M - \frac{\omega}{2}) \tag{2.58}$$

from which we again get

$$\gamma = e^{-8\omega\pi(M-\frac{\omega}{2})} \rightarrow T = \frac{\Gamma}{8\pi M}. \tag{2.59}$$

An elegant way of deriving the temperature of a black hole is via Euclidean Path integral[20, 69]. One performs the usual wick rotation

$$t = -i\tau \tag{2.60}$$

on the metric to obtain

$$ds^2 = \Big(1 - \frac{2M}{r}\Big)d\tau^2 + \Big(1 - \frac{2M}{r}\Big)^{-1}dr^2 + r^2 d\Omega^2. \tag{2.61}$$

One can now define a new radial coordinate $\rho$ through

$$\rho = 4M\sqrt{1 - \frac{2M}{r}} \tag{2.62}$$

Using Equation 2.62 then Equation 2.61 becomes

$$ds^2 = \rho^2 \Big(\frac{d\tau}{4M}\Big)^2 + \Big(\frac{r^2}{4M^2}\Big)d\rho^2 + r^2 d\Omega^2. \tag{2.63}$$

Outside the black hole but very close to the event horizon (i.e., $r \backsim 2M$) and ignoring the angular part of the metric, Equation 2.63 becomes

$$ds^2 \approx \rho^2 \Big(\frac{d\tau}{4M}\Big)^2 + d\rho^2. \tag{2.64}$$

If we let

$$\frac{\tau}{4M} = \theta \tag{2.65}$$

then Equation 2.64 is just the metric of a two-dimensional flat space written in polar coordinates provided the coordinate $\theta$ is defined on the interval

$$0 < \theta < 2\pi. \tag{2.66}$$

Therefore, we can make the identification

$$\tau = 8\pi M. \tag{2.67}$$

If we let

$$t_2 - t_1 = -i\tau \qquad \text{and} \qquad \phi(x_1) = \phi(x_2) = \sum_n \phi_n \tag{2.68}$$

then the expectation for the quantum state $\phi(x_1, t_1)$ to evolve to quantum state $\phi(x_2, t_2)$

$$< \phi(x_2, t_2)|\phi(x_1, t_1) > = < \phi(x_2)|e^{-iE(t_2 - t_1)}|\phi(x_1) >, \tag{2.69}$$

can be written as

$$Z = \sum_n < \phi_n|e^{-\tau E}|\phi_n > . \tag{2.70}$$

Equation 2.70 is a partition function if we can make the identification that

$$\tau = \beta = \frac{1}{T}. \tag{2.71}$$

We would as well have used the Path integral to find the expectation for $\phi(x_1, t_1)$ to evolve to $\phi(x_2, t_2)$ :

$$< \phi(x_2, t_1)|\phi(x_1, t_1) >= \int d\phi \int D[\phi]e^{S[\phi]}, \tag{2.72}$$

where $D[\phi]$ is a measure on the space of all matter fields, $S$ is the action and the integral is taken over all field configurations with the given initial and final values. Since the action $S[\phi]$ is over a period field, it must satisfy the boundary condition

$$\phi(0) = \phi(\beta) = \phi \tag{2.73}$$

Comparing Equation 2.67, Equation 2.71 and Equation 2.73, we see that

$$T = \tau = \frac{1}{8\pi M}. \tag{2.74}$$

Although we have used a massless scalar field, a Schwarzschild black hole and a four dimensional spacetime, it has been shown [52] that even if we have used different types of black holes, massive scalar fields, Fermionic fields and even spacetimes with dimensions higher than four, we would have still found out that black holes radiate like black body with the Hawking temperature $T_H$. For instance, it can be shown that the temperature of the Kerr-Newman black hole is just Equation 2.36 but with $\frac{2\pi}{\kappa}\omega$ replaced by $\frac{2\pi}{\kappa}(\omega - m\Omega_H - q\Phi_H)$[6].

## 2.2 The Entropy and Evaporation of Black Holes

Using the the first law of thermodynamics or the First law of black mechanics(cf.subsection 1.2.3)

$$dM = TdS, \tag{2.75}$$

we compute the entropy of the Schwarzschild black hole to be

$$S = \frac{A}{4}. \tag{2.76}$$

If the Generalized Second law holds then we must interpret $S_{BH}$ as the physical entropy of a black hole and with the laws of black hole mechanics truly representing the ordinary laws of thermodynamics as applied to black holes. For, it is extremely strange to have one interpretation for the entropy of the matters outside the black, $S$, and a completely different interpretation for the entropy of the black hole, $S_{BH}$. The consequence

[6] where $m$ is the azimuthal quantum number of the spherical harmonics and $q$ is the charge

of interpreting $S_{BH}$ as the physical entropy of a black hole is that $S_{BH}$ would then somehow correspond to the number of quantum microstates that are consistent with the macrostate of the black hole. Explicitly writing out the $\hbar$, $G$, $c$ and $k_B$ in Equation 2.75, we see that

$$S_{BH} = \frac{k_B A}{4G\hbar} \approx 1.07 \times 10^{77} (\frac{M}{M_\odot})^2 k_B, \tag{2.77}$$

where $M_\odot \approx 10^{30}$kg is the mass of the Sun. We see from Equation 2.77 that a black hole has an entropy far larger than the entropy of the body that collapsed to form it. For example, while the entropy of a solar mass star is about $10^{58} k_B$, a solar mass black hole has an entropy of approximately $10^{77} k_B$. Moreover, since the entropy of a classical black hole is zero, the non-zero entropy of a black hole has to be solely as the result of quantum gravitational effects.

There are several outstanding questions about the entropy of the black hole including

- Does $S_{BH}$ correspond to states hidden behind the horizon?
- Does $S_{BH}$ correspond to the number of possible initial states from which the black hole might have formed?
- Can one understand the universality of these results (i.e., Equation 2.76)?
- What happens with $S_{BH}$ after the black hole has evaporated?

Since $T_{BH} \backsim 10^{-8} \frac{M_\odot}{M}$K[7], it is justifiable assuming that $\frac{dM}{dt} << M$ and it is therefore appropriate to use the the Stefan Boltzmann law to calculate the rate of change of the mass of the black hole:

$$\frac{dM}{dt} \approx AT^4 \approx M^2 \frac{1}{M^4} \approx \frac{1}{M^2}. \tag{2.78}$$

Equation 2.78 can be integrated to get

$$t \propto M_0^3 - M^3 \approx M_0^3, \tag{2.79}$$

where we have assumed $M|_{t=0} = M_0$.
This means that the lifetime of a black hole is proportional to the cube of its mass:

$$\tau_B \approx \frac{M_0^3}{M_\odot^3} 10^{65} \text{ years}. \tag{2.80}$$

[7]that is about six order of magnitude colder than the Cosmic Microwave background

This means that the radiation of black hole leads to the complete evaporation of the black hole in finite time. This evaporation of a black hole raises some serious yet unanswered questions. For example, what happens to the entropy of a black hole after the black hole has evaporated?

## 2.3 Information Loss Problem

Another very important question raised by the evaporation of the black hole is the Information Loss problem.
If we define [8] $|0>$ as the vacuum with respect to either $\mathscr{H}^+$ or $\mathscr{I}^+$

$$b_\omega|0>=0 \quad \text{and} \quad c_\omega|0>=0 \qquad \forall\omega, \tag{2.81}$$

then the probability distribution of $m$ particles outgoing at $\mathscr{I}^+$ and $n$ particles ingoing at $\mathscr{H}^+$ can be obtained.

From Equation 1.71, we see that

$$|n,m>=(n!)^{-\frac{1}{2}}(m!)^{-\frac{1}{2}}(c_\omega^+)^n(b_\omega^+)^m|0> . \tag{2.82}$$

Taking the inner product of Equation 2.82 with $|0>$ and making use of Equation 2.26, Equation 2.34 and Equation 1.70, we can check that the only non vanishing scalar product between $|n,m>$ and $|0>$ are the cases where $n=m$ and which is given by

$$(n,m|0>)=e^{-\frac{n\pi\omega}{\kappa}}(0|0>). \tag{2.83}$$

Considering Equation 2.43, we can write the most general form of Equation 2.83 as

$$\Big(\{n_\omega\}\{n_\omega\}|0>\Big)=\prod_\omega e^{-\frac{\pi\omega n}{\kappa}}(0|0>), \tag{2.84}$$

where $\{n_\omega\}$ is the basis state with $n_\omega$ particles outgoing at $\mathscr{I}^+$ and $n_\omega$ particles ingoing at $\mathscr{H}^+$ in mode $\omega$ for various different modes $\omega$.

Using Equation 2.84, we can find that[9]

$$|(0|0>)|^2=\prod_\omega\Big(1-e^{-\frac{2\pi\omega}{\kappa}}\Big)=exp\bigg(\sum_\omega \ln(1-e^{-\frac{2\pi\omega}{\kappa}})\bigg) \tag{2.85}$$

[8] these equations and most of the explanations are taken directly from [49, 52, 70]
[9] see [49, 70] for proofs

and therefore the probability of a state that is a vacuum at $\mathscr{I}^-$ to be observed with $n_\omega$ particles outgoing to $\mathscr{I}^+$ in various modes $\omega$ and $n_\omega$ particles in various modes $\omega$ ingoing at $\mathscr{H}^+$ is given by

$$|(\{n_\omega\},\{n_\omega\}|0>)|^2 = \prod_\omega \left[1-e^{-\frac{2\pi\omega}{\kappa}}\right]e^{-\frac{2\pi n_\omega}{\kappa}}. \tag{2.86}$$

Thus, the probabilities of occupation of each mode are independent and we can write

$$P_n(\omega) = \left[1-e^{-\frac{2\pi\omega}{\kappa}}\right]e^{-\frac{2\pi\omega n}{\kappa}} \tag{2.87}$$

for the probability of finding $n$ pairs in mode $\omega$.

Since only the the outgoing member of each pair is observed at $\mathscr{I}^+$, the outgoing radiation is described by a density matrix, with $P_n(\omega)$ being the probability of finding $n$ particles outgoing at $\mathscr{I}^+$:

$$\rho_{int} = tr_{int}|0>_M = \prod_\omega (1-e^{\frac{-2\pi\omega}{\kappa}})\sum_n e^{\frac{-2\pi n\omega}{\kappa}}|n_\omega^{ext}><n_\omega^{ext}|, \tag{2.88}$$

where *int* denotes the region inside the black hole and *ext* denotes the region outside the black hole.

As can be seen from Equation 2.78, the black hole loses mass with every radiation and will eventually evaporate with the part of the density matrix it contains. Thus, a state that starts as a pure state at $\mathscr{I}^-$ can evolve into a mix state at $\mathscr{I}^+$ but, in quantum theory, a pure state is not supposed to evolve to a mix state. This is the Information loss problem.

String theory, particularly the conjectured gauge-gravity duality [71], seems to imply that information can not be lost. This, along with the many problems associated with information loss (i.e., violation of unitarity), has motivated the quest to know where the semiclassical theory break down or identify a flaw in Hawking's analysis. This has lead to the proffering of several solutions for the information loss problem including:

1. The existence of Baby Universes. According to this view, black hole disappears completely but one or more separate universes branch off during the process and carry the information. The idea of the baby universe is that quantum gravitational effects, like the quantizing of space, result in the creation of baby universes that are causally disconnected from our universe. The information lost with the complete evaporation of a black hole is stored in these baby universes [72, 73]. The problem is that the idea of baby universes called for accepting the existence of multiple

universes. Also, the idea of baby universes shifts rather than solve the information loss problem.

2. The existence of Planck-sized remnant[74–76]. In this case, it is hypothesized that the evaporation of a black hole is halted in the late stage of the radiation when its mass has reduced to the order of the Planck mass[75, 76]. However, there are several problems with this suggestion. One problem is that since a collapsing body would be of arbitrarily large, the Planck-size remnant that result from the evaporating black hole formed from such an arbitrarily large body must be able to store an infinite amount of information. It has been argued [77] that since quantum states outside the black hole are entangled with states inside the black hole with a Von Neumann entropy $\sigma = \ln N$[10], the Planck-size remnant must have at least $N$ internal states. Given that there is no upper bound on $N$, there can be Planck-sized remnants of unbounded degeneracy which conflicts with the widely accepted notion that quantum theory limits the amount of information that can be stored in a quantum space. In the case of black hole remnants that slowly evaporate, they have huge amount of information to give up but small amount of energy because they are of Planck sized mass. This means that the final stage of the black holes evaporation takes extremely long time. In this sense, slowly evaporating black hole remnants are not that much different from permanent remnants and thus have the same problems as those of the permanent remnants. Also, the existence of black hole remnants is that they seem to spoil the nice analogies between thermodynamics and black hole mechanics. This is aptly stated in [22]:

> ”another displeasing feature of the remnant idea (in either of the two forms above) is that it leaves us without a reasonable interpretation for the Hawking-Bekenstein entropy. If information is really encoded in the Hawking radiation, then it seems to make sense to say that $e^{S(M)}$ counts the number of accessible black hole internal states for a black hole of mass M. But if the information stays inside the black hole, then the number of internal states has nothing to do with the mass of the black hole. Indeed (if the remnants are Planck size), we can prepare a black hole of mass M that holds an arbitrarily large amount of information, by initially making a much larger hole, and then letting it evaporate for a long time. Thus, the number of possible internal states for a black hole of mass M must really be infinite. The beautiful edifice of black hole thermodynamics then seems like an inexplicable accident.”

[10] where $N$ is the dimension of their joined Hilbert or their number of degrees of freedom

3. Another suggestion is that the outgoing radiation is not exactly thermal, but contains some subtle correlations that carries the information about the details of the matter which fell into the black hole[78]. Information about details of the black hole can either tunnelled out of evaporating black holes via some subtle correlations in the Hawking radiation [66, 79, 80] or it get out at late times when the semiclassical approximation is expect to break down and the radiation is expected to no longer be thermal[81, 82]. Some problems with the information leaving the black hole via subtle correlations include the facts that the black hole horizon is not some mere potential barrier through which information can just tunnel out [52] and it has not be shown how subtle correlations can get all the information out of the black hole before the complete evaporation of the black hole. Also, since entropy between the quanta in the Hawking radiation and those inside the black hole is given by $S_{entanglement} = N \ln 2$, with each succeeding emission the entanglement grows by $\ln 2$ [77]. Thus, unlike radiation from ordinary object[11], where the entanglement decreases with time, the entanglement increases with succeeding Hawking radiation.

   It has been argued [22, 52, 77, 81, 83] that Subtle correlations seem not to help significantly in resolving the information loss problem. Those arguments include the observation that near the event horizon of a black hole the pure state $\sum_i c_i|i>$ will evolve as

$$\sum_i c_i|i> \to \sum_i c_i|i>_{int} \otimes |i>_{ext} \tag{2.89}$$

   so that, the subsystems $|i>_{int}$ and $|i>_{ext}$ of the pure quantum state $\sum_i c_i|i>$ will generally be inseparable. For the subsystem $|i>_{ext}$ to be a pure quantum state then $|i>_{int}$ must to be in a unique state. That is, for the $|\alpha> = \sum_a c_i^\alpha |a> \in \Sigma_{ext}$ of

$$|\Psi> = \sum_{a,b} C_{ab}|a> \otimes |b> \tag{2.90}$$

   to be a pure state then $\beta> = \sum_b c_j^\beta |b> \in \Sigma_{int} = |b>_{int}$ so that $C_{a,b} = c_i^\alpha c_j^\beta$ . The only way this can happen is if there is some mechanism at the event horizon to strip matters off their information as they cross the event horizon but the presence of a "bleaching" mechanism at the event horizon violates the Equivalence principle. There just seems to be no way to retrieve all the information from an evaporating black hole without the violation of causality. Also, since the matters of which the star is made up or matters that falls into the black hole end at the singularity while the pair creation presumably occurs near the event horizon, the late times quanta are not created by the interaction with the matters or with the earlier

[11] like the burning of an encyclopaedia

created pairs, it is extremely difficult to see how correlations can exist between the matter, the earlier and late times radiation

4. A very popular suggestion is that the information is both emitted at the horizon and passes through it , so the observer outside would see it in the Hawking radiation and an observer who falls into the black hole would see it inside but no single observer would be able to confirm both pictures. This suggestion, called the Black Hole Complementarity principle[84], depends on three assumptions:

   (a) the unitarity of Hawking radiation

   (b) the validity of the effective field theory outside the stretched horizon

   (c) the equivalence Principle.

   If the assumptions of the Black hole complementary are true then there are two copies of information, one inside the horizon and the other outside the horizon. However, it has been shown[26, 85] that all of the postulates of the Black Hole Complementarity principle are not mutually consistent.
   By a thought experiment, it is shown [26] that a particle coming out of the black hole could fall back into the black leading to a single observer having access to the information and its copy. Another argument against the principle of black hole complementarity is that purity (i.e., unitarity) and no drama (i.e., Equivalence principle) can not existence simultaneously. That is, whereas the conservation of information requires that the earlier and late quanta of the Hawking radiation be maximally entangled, Hawking radiation is the result of pair creation in which the particles that are observed as the Hawking radiation are maximally entangled with their anti-particle partners that fall into the black hole. Thus, a late time quanta is maximally entangled simultaneously to its anti-partner that falls into the black hole and to an early time quanta but this is not to happen in quantum theory. According to quantum statistic, each of the $N$ maximally entangled subsystems of a system has the probability of $P_i = \frac{1}{N}$. Given that $\sum_i P_i = 1$, a subsystem can not being maximally entangled to the subsystems of more than one independent systems.

   In light of this, it has been suggested by what is called the Firewall Paradox [26] that the conservation of information demands that the entanglement between quanta in the Hawking radiation and quanta inside the black hole be broken. The process of breaking this entanglement generates an inferno which makes the event horizon a firewall, where anything that gets there is incinerated. Obviously, Firewall is a pronounced violation of the Equivalence Principle.

Responses to the Firewall paradox include:

1. the Einstein-Rosen bridge and the Einstein-Podolski-Rosen argument (EP/EPR) duality. In [86], it is argued that an Einstein-Rosen bridge can be created between two black holes by an EPR-like correlations between the microstates of the two black holes and that this prove the possibility for a subsystem to be maximally entangled to the subsystems of several independent systems. It is suggested that the quanta in the Hawking radiation are somewhat connected by wormholes to the quanta in the black hole. Thus, the quanta in the Hawking radiation and those in the black hole are not independent systems. It is further argued in [86] that a black holes, as well as elementary particles, are entangled to each other through wormhole. That is, there is a duality between the Einstein-Rosen bridge and Einstein-Podolski-Rosen argument . According to this suggestion, the Einstein-Podolski-Rosen argument ($EP = EPR$) can be used to eliminate the need for a Firewall. This purported EP/EPR duality is expected to eradicate the seemingly contradiction among the postulates of the black hole complementarity consistent with one another.

2. Hawking has proposed that no event horizon would form in the gravitational collapse of a massive astronomic body, only apparent horizons which persist for a period of time [87]. Since no event horizon is formed the entire discussion about information loss becomes irrelevant.

3. It was shown in [88] that once the backreaction of Hawking radiation is included in the interior dynamics of the star, then the collapse stops and the star bounces.

# Chapter 3

# Alternatives to Black Holes

## 3.1 Beyond the Singularity Theorems

The conventional views of the standard black holes are convincing so much so that the existence of the standard black hole is most often considered indisputable and therefore impervious to scrutiny. Yet, the elusiveness of solution to the information loss problem despite more than 40 years of intense investigations and researches by some of the brightest minds have motivated the advancement of alternate views on gravitational collapse. Also, the fact that singularities are undesirable in any theory that supposed to describe all of spacetime has motivated the need for alternative views on gravitational collapse. Yet another reason for exploring views other than the conventional views is based on the fact that the evidences for the existence of black holes are only circumstantial. That is, since there is no direct observational proof for the existence of a black hole, it is wise to investigate the possibility that natural phenomena attributed to black hole could be associated with physical entities with properties indistinguishable from those of black hole but without the pathological features of black hole such as singularities.

One way in which the formation of black hole would be avoided is by violations of the premises upon which the singularity theorems are built. For instance, by violating the energy conditions. Indeed, it has been shown[89–91] that all the energy conditions are violable.

One popular example of energy condition violation is the Casimir effect, in which plane parallel conducting plates in vacua experience an attractive force. In the case of infinite plane plates, separated through a distance $L$ along the $z-$ axis, one may compute the stress-energy tensor of the electromagnetic field to be [53]

$$T_{\mu\nu} = \frac{C(z)}{L^4}\mathrm{diag}(-1, 1, 1, -3), \tag{3.1}$$

where $C(z)$ is dimensionless.

Another example is the squeezed states of light[53, 89, 92, 93]. For instance, a quantum system that is a superposition of the vacuum state $|0>$ and a two-particle state $|2>= \frac{1}{\sqrt{2}}(a^+)^2|0>$ [53]:

$$|\psi>= \frac{1}{\sqrt{1+\epsilon^2}}\Big(|0>+\epsilon|2>\Big), \tag{3.2}$$

where the amplitude $\epsilon$ is a real number. If we assume that $<0|T_{00}|0>=0$ and $\epsilon$ sufficiently small then

$$<\psi|T_{00}|\psi>\approx \frac{1}{1+\epsilon^2}\Big[2\epsilon\Re(<0|T_{00}|2>)\Big]. \tag{3.3}$$

With an appropriate choice of $\epsilon$, it is possible for the energy density $<\psi|T_{00}|\psi>$ to be negative.

Yet another example of energy condition violation is the Hawking radiation. As we saw in chapter 2, energy conservation is ensured only if the Hawking radiation observed at infinite is accompanied by negative energy of the same magnitude that is crossing the horizon into the black hole[49, 52, 53, 59]. For instance, ingoing energy density near the event horizon of the Schwarzschild black hole is [49]

$$<0_B|T_{vv}|0_B>\approx -\frac{\hbar}{768\pi M^2}. \tag{3.4}$$

Violations of Energy conditions are also seen in most models of Cosmological inflation [94]. In fact, most models of the current acceleration of the universe[95] presuppose the violation of some of the energy conditions.

Another way to evade black hole formation is to avoid the formation of trapped surfaces. Indeed, it has been shown[96] that the very formation of a trapping surfaces is questionable. This has led to several suggestions on how the occurrence of trapped surface can be avoided. One such suggestions is that the avoidance of the formation of trapped surfaces could be achieved by the collapsing stars radiating away most of its matter before the advent of the formation of trapped surfaces[97]. Another suggestion is that the existence of scalar fields mediating a Fifth force[98, 99] makes it possible to realize the non-occurrence of trapped surfaces in the spacetime of a collapsing body. That is, a massive scalar fields interacting with gravity can create the possibility for the existence of black hole with non-trapping interior[100].

Since it is an assumption of the singularity theorems that causality holds (i.e., spacetime should be globally hyperbolic)[10, 12, 30], the violation of causality would be an alternative to the formation of black holes via gravitational collapse. One popular way causality can be violated is via the existence of closed trapped curves in spacetime. This can be seen from the fact that, on one hand, a closed timelike curve which intersects the Cauchy surface $\Sigma$ violates the achronality of $\Sigma$. While, On the other hand, a closed timelike curve which does not intersect $\Sigma$ violates global hyperbolicity since it could be used to define an inextendible causal curve which does not intersect $\Sigma$.

Closed trapped curves indeed appear in a number of solutions of the Einstein's equation. For example, the Kerr metric contains closed trapped surfaces[27, 33]. Whereas the closed timelike curves in the Kerr metric can be explained away by claiming that they are unphysical, there are ordinary systems in situations which may occur in the laboratory or in Astrophysics that contain closed timelike curves. One very good example[1] is a system comprising of two spinning masses $m_1$ and $m_2$ and constant angular momentum $h_1$ and $h_2$ with their spins both parallel to their line of separation. Assuming that the particles are fixed on the $z-$axis at $z = \pm b$, axial symmetric and time independent, we can write the metric for the system as

$$ds^2 = -f^{-1}[e^{\nu}(dz^2 + dr^2) + r^2 d\theta^2 + f(dt - \omega d\theta)^2], \tag{3.5}$$

where $f$, $\nu$ and $\omega$ are functions of only $z$ and $r$ and $-\infty < z < \infty$, $0 \leq r$, $0 \leq \theta \leq 2\pi$ and $-\infty < t < \infty$, with $\theta = 0$ and $\theta = 2\pi$ being identical. If $\theta$ is a spacelike coordinate then

$$g_{\phi\phi} = -f^{-1}[r^2 - f^2\omega^2] < 0 \tag{3.6}$$

but it turn out on the axis of symmetric $r = 0$ one can arrange this either between the particles or outside them but not both unless the parameter satisfy the relationship

$$m_1 h_2 + m_2 h_1 = 0 \tag{3.7}$$

Equation 3.7 can be written in the very simple form

$$a_1 + a_2 = 0. \tag{3.8}$$

where $a_1$ and $a_2$ are the angular momentum per unit mass of the two particles. There is no obvious reason why closed timelike curves should be absent in this case.

## 3.2 Modified or Extended Theories of Gravity

There are many good reasons for trying to modify or extend the theory of general relativity including the observations that the universe is accelerating, the existence of

[1]This example is copied directly from [101]

non-baryonic matters (i.e., Dark Matter)[102–104], the need for a theory of gravity void of singularities[10, 20, 30, 105, 106] and a theory of gravity that is renormalizable[107, 108]. In other words, it is the widely accepted view[108–110] that general relativity must be modified at both very low energies and very high energies in order to solve the outstanding shortcomings of general relativity.

Similarly, on the quantum side, it is the general consensus[111–113] that quantum theory in its present state (i.e., the Standard Model)[114, 115] is an effective theory in the sense that it fails to adequately explain physical phenomena such as neutrino oscillation[116–119], baryon asymmetry[120, 121], the hierarchy problem[122, 123],the nature of dark matter and dark energy[124, 125] and the gravitational interaction, the origin of the values of 19 parameters needed to make the predictions of the Standard Model fit well with experiments[126, 127], etc. In addition, the measurement problem in quantum theory[128] and the problem of interpreting many of the results of quantum calculations[129, 130] reinforce the belief that quantum theory like General relativity is incomplete.

One suggested way of modifying general relativity in order to take into account quantum effects is by introducing a minimal length $l_f$ of the order of the Planck distance[131, 132] $l_P = \sqrt{\frac{G\hbar}{c^3}} \backsim 10^{-33}cm$ and then assuming that lengths $l < l_f$ are meaningless. This means that classical notions such as causality or distance between events cannot be expected to be applicable at this scale and they must be replaced by some other, yet unknown, structures. As the consequence of the existence of a minimal length in spacetime, there can be no point-like physical quantities. All physical observables are smeared out on a scale $l_f$[133]. One of the manifestations of the inclusion of a minimal length in these theories is the Generalized Uncertainty principle (GUP)[134, 135]

$$\Delta x \geq \frac{\hbar}{2\Delta p} + \frac{\alpha}{c^3} G\Delta p \tag{3.9}$$

where $\alpha$ is a constant. One of the arguments for a generalized uncertainty principle is that since the Heisenberg Uncertainty Principle is based on the idea that spacetime is Minkwoskian, it ought to be modified when the spacetime is curved.

Similar to the idea of a minimal length is the notion that the consideration of both gravitational and quantum effects results in the noncommutativity of space:

$$[x^\mu, x^\nu] = i\ell^{\mu\nu} \tag{3.10}$$

where $l^{\mu\nu}$ is an antisymmetric matrix that can be diagonalized as

$$\ell^{\mu\nu} = \ell^2 \begin{pmatrix} \varepsilon_{ij} & 0 & 0 & \cdots & \\ 0 & \varepsilon_{ij} & 0 & \cdots & \\ \vdots & \vdots & \ddots & \cdots & \\ 0 & 0 & 0 & \cdots & \varepsilon_{ij} \end{pmatrix} \tag{3.11}$$

and $\ell$ is a constant with the dimension of length that determines the fundamental cell of spacetime in similar way as $\hbar$ discretizes the phase space.

Another idea is that there is a curvature limiting principle that ensures that there can be no spacetime point at which any invariant can become singular. The argument is, just as special relativity and quantum mechanics introduce a limiting speed $c$ and a limiting phase space $\hbar$, any viable theory of quantum gravity should introduce limiting bounds on invariant quantities, particular the spacetime curvature. That is, there is a limiting principle, dubbed the Limiting Curvature Hypothesis (LCH) that assumes

1. a finite number of invariants are bounded
2. When these invariants take on their limiting values, any solution of the field equations reduces to a definite nonsingular (e.g. de Sitter space), for which all invariants are automatically bounded.

Thus, based on the limiting curvature hypothesis (LHC), one looks for a theory in which a finite number of invariants are bounded by an explicit construction (e.g., $|R| \leq l_P^{-2}, \quad |R_{\mu\nu}R^{\mu\nu}| \leq l_P\nu^{-4}, \quad \text{and} \quad |C_{\alpha\beta\gamma\delta}C^{\alpha\beta\gamma\delta}| \leq l_P^{-8}$ ).

Yet, another idea is that there are more than three spatial dimensions. The introduction of extra dimensions has led to the development of theories like M-theory and Braneworld.

Interestingly, most of these modified theories of gravity predict results of gravitational collapse as compact objects other than the standard black holes.

## 3.3 Examples of Non-Singular Collapse

Given that the action is not unique(i.e., many different actions can generate the same equations of state) and the fact that the Lagrangian has no physical meaning of its own [2], we can always construct an action that give us the desire result we seek. Besides, the fact that general relativity has not really been tested in strong field gravity, that all

[2] there is no there is no obstacle such constructions as long as we ensure that they reproduce Einstein field equations in the weak field limit

the assumptions of the singularity theorems are violable along with the aforementioned prescriptions for modifying or extending general relativity, one can always write down "phenomenological" metrics for which singularities formed from gravitational collapse can be avoided.

One simple way of gravitational collapse resulting in the avoidance of singularity formation can be achieved by coupling non-linear Maxwell fields to the gravitational field. For instance, the coupling of the Einstein-Hilbert action to an appropriate action of a linear electromagnetic field can give [136–139] spacetime for a rotating regular black hole[140]:

$$\begin{aligned} ds^2 = &- f(r)dt^2 + \frac{\Sigma}{\Sigma f(r) + a^2 \sin^2\theta} dr^2 \\ &- 2a\sin^2\theta(1 - f(r))d\phi dt + \Sigma d\theta^2 + \sin^2\theta[\Sigma - a^2(f(r) - 2)\sin^2\theta]d\phi^2, \end{aligned} \tag{3.12}$$

where

$$f(r) \equiv \frac{1 - 2Mr\sqrt{\Sigma}}{(\Sigma + Q^2)^{\frac{3}{2}}} + \frac{Q^2\Sigma}{(\Sigma + Q^2)^2}, \tag{3.13}$$

$M$ and $Q$ are respectively the mass and charge parameters and $\Sigma$ and $\Delta$ are given by Equation 1.35. It can be shown that Equation 3.12 has no physical singularity. For instance, it is defined at $r = 0$

$$f(r) \to \frac{1}{(a^2\cos^2\theta + Q^2)^{\frac{3}{2}}} + \frac{Q^2 a^2 \cos^2\theta}{(a^2\cos^2\theta + Q^2)^2} \qquad \text{as} \quad r \to 0. \tag{3.14}$$

Also, for large $r$, Equation 3.12 reduces to the Kerr-Newman metric (cf.Equation 1.34). Since, deriving Equation 3.14 and Equation 1.34 from Equation 3.12 is quite involved, we will check that Equation 3.12 reduces to the Reissner-Nordström metric and the Schwarzschild metric when the appropriate conditions are considered. For instance, by setting $a = 0$, it is straightforward to see that Equation 3.12 becomes

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{3.15}$$

and Equation 3.13 becomes

$$f(r) = 1 - \frac{2Mr^2}{\left(r^2 + Q^2\right)^{\frac{3}{2}}} + \frac{Q^2 r^2}{\left(r^2 + Q^2\right)^2}. \tag{3.16}$$

For large $r$ (i.e., $r >> Q$),

$$f(r) \to 1 - \frac{2Mr^2}{r^3} + \frac{Q^2 r^2}{r^4} = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} \qquad r \to \infty \tag{3.17}$$

and Equation 3.15 is just the and Reissner-Nordström metric which obviously becomes the Schwarzschild metric when $Q = 0$. We can check that Equation 3.14 has no physical singularity. For $r = 0$, Equation 3.16 reduces to the de sitter metric[32]

$$f(r) \to 1 - \frac{2Mr^2}{Q^3} \equiv 1 - \frac{\wedge r^2}{3}, \tag{3.18}$$

where $\wedge \equiv \frac{2M}{Q^3}$.

Another singular free Schwarzschild like solution is given by [141, 142]:

$$ds^2 = -e^{4\beta(r)}\chi(r)du^2 + 2e^{2\beta(r)}dudr + r^2d\theta^2 + r^2\sin^2\theta d\phi^2, \tag{3.19}$$

where

$$\chi = 1 - \frac{2m(r)}{r}, \quad m(r) = \begin{cases} M & r \geq 2M \\ \frac{r^3}{8M^2}(10 - \frac{15r}{2M} + \frac{3r^2}{2M^2}) & r \leq 2M \end{cases} \tag{3.20}$$

$$\beta'(r) = \begin{cases} 0 & r \geq 2M \\ \frac{5r}{2M}(1 - \frac{r}{2M})^2 & r \leq 2M. \end{cases} \tag{3.21}$$

Many more regular black hole solutions have been obtained by coupling different types of fields to the metric[142, 143, 143–146]. In fact it has been proven there exists an infinitely many globally defined singularity-free solutions to the Einstein-Yang Mills Equations [147]. In fact, it turned out that there can be a very generic mechanism for singularity avoidance in classical general relativity, which requires what has been called a topology change[148].

That is, singularity formation is avoided by the evolution of spacetime slices from open become closed. In other words, spacelike slices evolve from a region in which they are non compact (i.e., $S^2 \times R$) into a region where they are compact($S^3$). For instance, the geometry of the spacetime switches from $S^2 \times R$ in the asymptotically flat part of the space to $S^3$ [3]

Another very popular nonsingular gravitational collapse model is the Boson star[149–152]. The idea is to have a (classical or semiclassical) scalar field configuration corresponding to a compact object without a singularity. For example, consider the action

$$S = \int \Big(\frac{1}{16\pi}R + L_\phi\Big)\sqrt{-g}d^4x, \tag{3.22}$$

[3] see theorem and proof in [141]

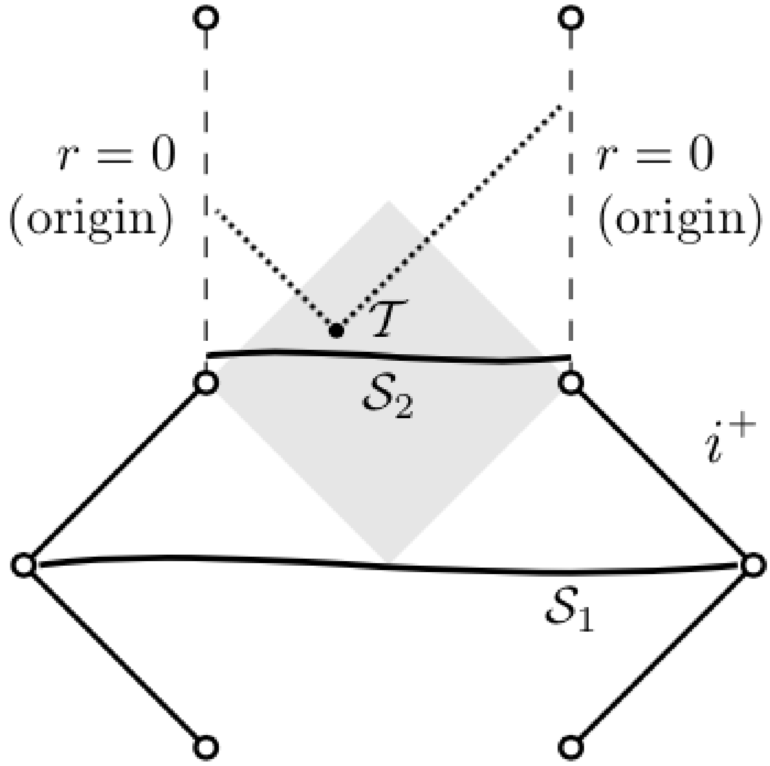


FIGURE 3.1: Singularities avoidance due to Topoloy Change[141]

where $R$ is the Ricci scalar and

$$L_\phi = -\frac{1}{2}\Big[\nabla_\mu \phi^* \nabla^\mu \phi + V(|\phi|^2)\Big] \tag{3.23}$$

is the Lagrangian density for the scalar field $\phi$. With an appropriate choice of the potential $V(|\phi|^2)$, one can obtain solutions for the Klein-Gordon equation

$$\nabla_\mu \nabla^\mu \phi = \frac{dV}{d|\phi|^2} \tag{3.24}$$

which contain no singularity. Some possible choices of $L_\phi$ are

$$L_\phi = -\frac{1}{2}\nabla_\mu \phi^* \nabla^\mu \phi - \frac{1}{2}m^2|\phi^2| - \frac{1}{4}\lambda|\phi^4| \tag{3.25}$$

and

$$L_\phi = -\frac{1}{2}\nabla_\mu \phi^* \nabla^\mu \phi + \frac{m^2}{\hbar^2}|\phi|^2 + \lambda(|\phi|^2 + \lambda(|\phi|^6 - a|\phi|^4)) \tag{3.26}$$

where $\lambda$ and $a$ are constants.

Other nonsingular collapse models include Gravitational Vacuum Condensate Stars model or the so-called gravastar[143, 153, 153, 154, 154–156, 156–159], noncommutative inspired models [160–162], Bardeen solution[139], Hayward regular black hole[163], so-called black stars[96, 164–167], Dark energy star [168, 169],Q-ball[170–173],Strange Star[174, 175],superstars[176],Preons[177],axionic fields[178],exotic [179], phantom [157, 180],chaplygin gas[181],quintessence[154, 182],Phantom Energy[183, 184], Palatini gravity[185]. There is also loop quantum gravity inspired models [186, 187, 187–191], BraneWorld inspired models [188], new exotic matter at extremely high densities such as quarks [173, 192], superstars[176] Q-balls[171], preons[177], chaplygin gas[181] that could drastically push up the Oppenheimer–Volkoff limit for unstoppable collapse.

## 3.4 Naked Singularity

An underpinning of black hole physics is that singularities that formed from gravitational collapse must lie within an event horizon. That is, the entire black hole physics is based on the premise that the Cosmic censorship is true. For example, on the basis of the Cosmic Censorship Conjecture, it is assumed that Equation 1.60 must always hold. In fact, the proofs of many of the important theorems of black hole physics, including the black hole uniqueness theorem[13, 193][4], the laws of black hole mechanics[15, 194] and the positive mass theorem[195–197] are all based on the assumption that the Cosmic Censorship Conjecture is true. Thus, black hole as a physical entity is questionable without the validity of the Cosmic Censorship Conjecture. This could mean that to even begin discussing the characteristics, properties and effects of black holes, one will have to first prove the Cosmic censorship Conjecture to be true. For, it is strange carrying out elaborate formalism of black hole physics or coming up with schemes for observing black holes when black holes may not even exist. However, there is yet no general proof of the Cosmic Censorship Conjecture and the prospective of obtaining a conclusive proof of the cosmic censorship in the near future is not promising. This is mainly because the Cosmic Censorship Conjecture is not rigorously formulated. For instance, without some physically reasonable generic set of initial conditions on the Einstein's equations, it is possible to find spacetime which satisfies the Einstein's equation that violates the Cosmic Censorship Conjecture. However, defining what constitutes physically reasonable generic set of initial conditions is still elusive[46, 198]. It seems like any proof for the Cosmic Censorship Conjecture will require much more knowledge on general global properties of Einstein equations than is currently known.

[4]and references therein

As we discussed in chapter 1, the condition of the cosmic censorship, particularly that the matter must be generic, makes it very hard to test the validity of cosmic censorship via the study of exact solutions. However, While no single solution could provide a negative answer to the question of cosmic censorship since it would be explained away as not being generic enough, an exact solution of the Einstein field equations with physically reasonable matter could be strong circumstantial evidence for the failure of cosmic censorship. After all, these are the same forms of matter that are widely used in discussing various astrophysical processes.
One good example of a physically reasonable matter that seems to violate Cosmic censorship is the radiation dust. For instance, consider the Vaidya solution representing a collapse of a body made of radiation dust

$$ds^2 = -(1 - \frac{2m(v)}{r})dv^2 + 2dvdr + r^2 d\Omega^2. \tag{3.27}$$

For $m(v) = \lambda v$ for $v \geq 0$, there is a curvature singularity at $r = 0$. It is possible to show that for $\lambda \leq \frac{1}{16}$ the singularity at $r = 0$ is a naked singularity[198]. That is, it can be viewed by observers at infinite[200]. By the way, by radiation dust we mean a pure radiation (dust-like null fluid) with the energy-momentum tensor

$$T_{\mu\nu} = \frac{1}{4\pi r^2}\frac{dm(v)}{dv}k_\mu k_\nu, \tag{3.28}$$

$m(v)$ is an arbitrary function of advanced time $v$ and $k_\mu$ is a null vector[32, 199]

The radiation dust is just one example of numerous physical reasonable matters[198, 199, 201, 201–203, 203–211][5] that seem to violate cosmic censorship.

Whereas exact solutions of Einstein equations with specific matters that violate cosmic censorship can be ruled out by saying that they are not generic enough, there are other naked singularities,like the ones due to the destruction of the event horizon that must be seen as true counterexamples to the Cosmic Censorship conjecture. For example, though it was initially thought that a naked singularity could not be created by destroying the event horizon of a black hole[212], It has been shown[213–216]that it is possible to violate the Cosmic Censorship Conjecture by overcharging or over spinning a black hole. These results show that the destruction of the event horizons of black holes is not forbidden as it should be for the Weak Cosmic Censorship Conjecture to be valid.

Cosmic censorship could also be violated if a black hole was big enough so that an observer within the horizon could live for many years without falling into the singularity

[5]and the many references in each of these papers and books

and would thus be able to observe the naked singularity. Since there is no theory governing what is happening at the singularity, an observer living inside a black hole would not be able to account for the physical phenomena being observed. It is the desire to avoid such unpredictabilities that motivates the introduction of a stronger version of the Cosmic Censorship Conjecture, called the Strong Cosmic Censorship[46].[6] For a given spacetime $(M, g_{\mu\nu})$, the Strong Cosmic Censorship Conjecture turns out to be equivalent to global hyperbolicity of $(M, g_{\mu\nu})$[217].

If the criterion for black hole formation from generic gravitational collapse is the Hoop Conjecture[46, 218]:

Black holes with horizons form when and only when a mass $M$ gets compacted into a region whose circumference in every direction is $C \leq 4\pi M$,

then not all naked singularities are ruled out from the spacetimes of collapsing bodies.

Also, in the Hawking process of black hole evaporation, the black hole produces a naked singularity just before it completely evaporate. If the area of the black hole goes to zero in finite time then the event that marks this vanishing of the area of a black hole is a naked singularity.

In short, the existence of naked singularities resulting from the gravitational collapse of astrophysical bodies seems not too far-fetched.

[6]What we have been calling the Cosmic Censorship up to this point is actually called the Weak Cosmic Censorship.

# Chapter 4

# Black Holes Reexamined

## 4.1 Some Questionable Conclusions

Several of the conclusions drawn about black holes, particularly the semiclassical black holes, are questionable. One of such questionable conclusions is that the ultimate fate of a collapsing astrophysical object whose mass exceed certain threshold[1] is a black hole. Without an actual understanding of the phenomena of gravitational collapse and the conditions which lead to the black hole formation, there should be scepticism of the claim that the end state of the gravitational collapse of a massive astrophysical body is a black hole. Until there is a good understanding of the equations of state of matter at extremely high densities, the claim that the final outcome of the gravitational collapse of a very massive astrophysical body is a black hole remains questionable. For example, given that there are mounting evidence for the existence of fundamental scalar fields[2], it is possible that there could exist a chameleon like fifth force which could result in a gravitational collapse deviating significantly from free fall. As it has been shown[96], if the gravitational collapse deviates significantly from free fall then the appearance of trapped surfaces can be delay indefinitely and the collapse can be halted.

Another questionable conclusion is that a radiating black hole will eventually evaporate completely because its mass has to compensate for its radiation since the conservation of energy must be ensured. Not only is there no consensus on the meaning of energy-momentum tensor in general relativity, the loss of mass by an evaporating black hole is implicitly quantum gravitational. That is, since the matter that made up the collapsing object and anything that crosses the black hole's event horizon end at the singularity, which is supposed to be outside of the spacetime, the very assumption that the mass

[1] i.e., Tolman–Oppenheimer–Volkoff limit[27]

[2] see [91] for details and references therein

of the black hole is decreasing with the Hawking radiation implicitly involves quantum gravity effect. Therefore a full understanding and consistent description of the phenomenon of black hole evaporation require some element of quantum gravity.

Yet another questionable conclusion is that the semiclassical theory is valid throughout the black hole except near the singularity. The argument is that since all components of the curvature tensor[3]

$$R_{\alpha\beta\gamma\delta} \backsim \frac{1}{r^3} \quad \text{and} \quad K = R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} \backsim \frac{1}{R^6}, \tag{4.1}$$

are very small for a large Schwarzschild black hole[4], the use of the semiclassical theory in all regions of an astrophysical black hole, except near the singularity is valid. Yet, gravity is supposed to be very strong inside the black hole, even very close to the event horizon, so as to trap anything including even light. Similarly, the fact that the entropy of the black hole is proportional to the area of the horizon suggests that the event horizon is special. That is, physical effects are occurring at the event horizon. This motivates the idea of a thermal atmosphere about a Planck distance outside of the black hole where real observable effects take place[219]. The existence of the thermal atmosphere is supported by calculations. For example, by excluding the thermal atmosphere, one can reduce the total entropy of the black hole exactly by the amount ascribed to the thermal atmosphere.[5] Also, the global and teleological nature of the event horizon make it possible for there to be a connection between the horizon and whatever is happening at the singularity. For instance, the Cosmic Censorship Conjecture implies that the singularity is influenced by the singularity. In fact, it has been showed[220] that the event horizon can be made to disappear by simply making alteration in the a Planck scale neighbourhood of the singularity. Similarly, it has been argued[221–223] that quantum gravitational effects might not be restricted to the Planck regime. According to [224], String theory has shown that the quantum gravity scale is dependent on $N_\alpha l_P^\alpha$ rather than just $l_P$ , where $N$ is the number of quantum state, $\alpha$ is some constant and $l_P$ is the Planck length. Accordingly, $N_\alpha l_P^\alpha$ can be of order of the event horizon for black holes.

## 4.2 Breakdown of the Semiclassical Theory

In addition to questionable conclusions drawn about black holes, there are questions about the validity of the semiclassical theory, including the transplanckian problem and

[3]and any invariant created from the curvature tensor

[4]While we using the Schwarzschild black hole for simplicity sake, these arguments are valid for any other type of black holes.

[5]see [24] and references therein

the backreaction problem. Although Hawking's derivation is supposed to involve the use of the Semiclassical theory, we can see from section 2.1 that the radiation involves modes with arbitrarily high energies. It is easy to see that the modes which appear at $\mathscr{I}^+$ at late times have to start with ultra high frequencies at $\mathscr{I}^-$. For instance, quanta are blue shifted to the order of $e^{\kappa t}$, where $t$ and $\kappa$ are the age and surface gravity of the black hole respectively . That is, from $v = G(u) = v_0 - Ce^{-\frac{u}{4M}}$, the frequency is of order of

$$\omega' = M^{-1} e^{\frac{t}{4M}} \tag{4.2}$$

If we take $t$ to be of the order of the expected black hole lifetime, $M^3$, then

$$\omega' \approx M^{-1} e^{(\frac{M}{M_P})^2} \tag{4.3}$$

where $M_P$ is the Planck mass. For example, for a black hole of mass one gram, this leads to $\omega' \approx 10^{10^{10}}$g which is far larger than the mass of the observable universe.
For a solar mass Schwarzschild black hole, the quanta is blue shifted by about $e^{105}$ in two seconds after the formation of the black hole. This is the so-called trans-Planckian problem [49, 50, 53, 54, 59, 225–228]. Of course, one can simply assume that there is a natural Planck-scale frequency cut-off for effective field theory in curved spacetimes. However, this way of solving the transPlanckian problem is worse than the problem itself, as it would also imply a shut-down of the Hawking radiation as soon as it has begun.

Closely related to the transplanckian problem is the backreaction problem. We saw in section 2.1 that the Hawking radiation leads to the reduction in the mass of the black hole and therefore a change in the geometry of the spacetime. Whether or not this change alters the predictions of the semiclassical theory can only be known by properly writing and solving the full semiclassical Einstein's field equations:

$$G_{\mu\nu}(g_{\mu\nu}) = 8\pi < T_{\mu\nu}(g_{\mu\nu}) > . \tag{4.4}$$

However, the calculation of $< \psi|T_{\mu\nu}(g_{\mu\nu})|\psi >$ is, in general, a rather elusive problem[54]. The complete four-dimensional computation of $< T_{\mu\nu} >$ has not yet been carried out[229]. What has been achieved is a good approximation to $< T_{\mu\nu} >$ in the static spherical symmetric case. Up to now, the only quantitative treatment available is in the context of the semiclassical theory. One of the reasons for this elusiveness is the two-parameter ambiguity in the definition of $< T_{\mu\nu} >$ corresponding to the addition of the two conserved local curvature terms which are quadratic in the curvature[53, 54]. Another reason is that Equation 4.4 is of higher derivatives than Equation 1.16. Namely, Equation 1.16 is of second order in derivatives of the metric but $< T_{\mu\nu} >$ contains terms of fourth-order in derivatives of the metric[54, 230]. Also, in generally $< T_{\mu\nu}$ is a highly nonlocal functional of the metric. Thus, arguments restricted to the context of quantum field theory in curved spacetime can not possibly uniquely determine $< T_{ab} >$.

In addition, the semiclassical theory is not really renormalizable. Arguments that are used in renormalizing the semiclassical theory either do not arise from any physical theory or there is no empirical basis for the use of these arguments. Also, most of the results of the regularized theories disregard basic properties of the theory being renormalized. For instance, invariances that are known to hold for the theories being regularized is do not hold for the renormalized theories. Also, the renormalized theories might introduce particles with negative metric or with wrong particle statistics.

## 4.3 Apparent Horizon or Event Horizon

If a black hole is to be defined as the region of spacetime from which causal signals can never escape then an event horizon defines its boundary. In realistic situations such as black holes being embedded inside the Cosmic Microwave background radiation and the presence of dark energy, it is impossible to determine the location of an event horizon by any local physical process. One will have to wait forever before determining the location of an event horizon. In order word, black holes as defined by event horizons depend on final conditions. That is, it depends on what is going to happen in the infinite future. Such non-local and teleological definition of a black hole do not fix well into the current framework of Physics, where theories are considered to be local and equations of state are considered to be dependent upon initial conditions. Results such as black hole mechanics and the Hawking radiation suggest that black hole are physical objects to be study and therefore must fix within the framework of current physical theories. The absurdity of waiting until the end of the time or the infinite future before knowing what is happening locally has motivated some experts on black holes to argue that black holes should be defined in terms of local or apparent horizons rather than global or event horizons.

While local horizons are depend on local measurements, they are, however, also problematic. First, local horizons are not invariantly defined; they are space-time foliations dependent [231]. Different foliations of the spacetime give infinite numbers of apparent horizons with many of them intersecting but with distinct features. Also, while a particular foliation might indicate a trapping surface, another might not. In other words, a black hole defined in terms of local horizons is non-unique and so is observer dependent. Also, it is not yet possible to find a single definition for the many different local horizons or to relate them to one another such that they can unambiguously define the surface of a black hole. Second, unlike event horizons that grow smoothly, local horizons are known to jump discontinuously as black holes grow. Such jumpiness makes it extremely difficult assigning a continuous entropy function to black holes. Third, local horizons are not necessarily acausal and this could lead to violation of Cosmic censorship Even though to ensure that cosmic censorship holds, most experts insist that local horizons should never be timelike[40, 43], it is easy to see that such strict restriction is problematic. Local horizons being only null or spacelike is difficult to reconcile with an evaporating black hole. The area of the local horizon of an evaporating black hole will shrink and therefore become timelike in contradiction to the claim that a local horizon can not be timelike. Fourth, local horizons are inside the black hole (within event horizon) [29, 32, 40] unless an energy condition (i.e., null energy condition) is violated[6]. If local horizons are inside black holes then their evolution can not be conclusively determined without an understanding of the singularities of black holes and the laws governing them. If local horizons are outside the black hole then either there is no event horizon or an energy condition is violated. For the former, the local horizons have to serve as the acasual boundaries of the black hole, which is not possible for local horizons that are timelike. For the latter, the very energy condition violation is one of the premises upon which gravitational collapse could lead to the formation of a black hole and therefore could have been used to avoid the formation of black holes from gravitational collapse in the first place.

Overall, whether defined in terms of local horizons or global horizons, the definition of black holes are problematic. Defining black holes in terms of event horizons and yet considering them as physical objects out there to be observed require moving beyond the realm of the current framework of Physics and experimentations. Defining them in terms of apparent or local horizons make the definition of black hole ambiguous, make it difficult to directly relate black hole mechanics with the laws of thermodynamics and could lead to violation of cosmic censorship. The very fact that black holes can not be defined unambiguously without conflicting with highly cherished fundamental tenets of current physical theories is good reason to question the very existence of black holes.

[6] see proof in either [232] and references therein

# Chapter 5

# Conclusions

## 5.1 Summary

In chapter 1, we discussed Einstein's field equations and some of the pathologies associated with their solutions. We emphasized that constraints, not impose by general relativity itself but motivated by experience and expediency have to be imposed on solutions of the Einstein's equations if they are to be physically meaningful. We also discussed the laws of black hole mechanics and their resemblance to the laws of thermodynamics. In particular, in order for the existence of black hole to be compatible with the second law of thermodynamics the generalized second law must hold and this in turn means that a black hole indeed has a physical entropy.

In chapter 2, we used the semiclassical theory of gravity, tunnelling and Euclidean quantum gravity to derive the expression for the temperature of a black hole and we then use the first law of thermodynamics to derive the expression of the entropy of a black hole. These different methods of derivation of the Hawking's temperature were meant to emphasize the robustness of the derivation of the expressions for black hole radiation. That is, the many different approaches to deriving the Hawking's temperature give the same result, regardless of any of the details of the black hole states. While this robustness of the derivation gives more supports for the existence of black hole, the huge number of entropy associated with the black rises a lot of not yet satisfactorily answered questions. We also saw that black hole radiates and eventually evaporate and this leads to the information loss problem. Several suggested solutions to the information loss problem were discussed and we saw that none of them provides a satisfactory solution.

In chapter 3, the needs for alternatives to black holes and the different possible ways of coming up with alternatives to black holes were discussed. One possibility was violations of some of the assumptions of the singularity theorems. Another possibility was a complete theory of gravity and quantum fields that could have general relativity and quantum field theory as effective theories that breaks down at some scales. We discussed that a spin-off of extending or modifying general relativity is gravitational collapsing avoiding the formation of singularities and we gave examples of some popular non-singular gravitational collapse models. In addition, we discussed that black holes may not be the only possible end state of the gravitational collapse of massive astronomic bodies. That, when pressure and the other dissipative processes associated with extreme densities are taken into account, a collapse might lead to a naked singularity. We discussed that it is possible to destroy the horizon of a black hole and therefore expose its singularity to external observers.

In chapter 4, we did a critical review of the current consensus views of black holes and questioned some of those views. We saw that the claim that a black hole will evaporate because of the Hawking radiation is implicitly quantum gravitational since the laws governing the singularities must first be known before we can be really sure that the mass of matter that falls into the singularity is related directly to the radiation observed at $\mathscr{I}^+$. We also discussed the transplankian problem and how rather than being tackle directly, is simply ignored. We look at how the scientific community is divided on whether black holes should be defined in terms of event horizons or local horizon and how this is a good reason for questioning the existence of the standard black holes.

## 5.2 Conclusion

The information loss problem brings to the fore a major crisis in physics that can no longer being ignored or glossed over. On one hand, there are ample evidences that quantum theory, general relativity and even the semiclassical theory are good descriptions of nature. For example, general relativity and quantum field theory have passed all the tests they have been subjected to; our low energy environment is well described by general relativity and the part of the microscopic world we have access to are completely described by quantum field theory. On the other hand, however, there are widespread violations of conditions needed to forbid general relativity, quantum mechanics and the semiclassical theory from making non-physical predictions. For instance, the energy conditions are violated by even reasonable physical systems. Yet, violations of the energy conditions are supposed to have dire consequences (such as violation of the second law of thermodynamics). Even the introduction of averaged energy conditions and quantum inequalities are insufficient to rid physics of the pathologies associated with negative energies since even classical systems are known to violate the energy conditions. Also, determinism of physical theories[1] which has long been an underpinning of physical theories becomes disputable if naked singularities truly exist. Then, there are causality violations such as the existence closed trapped surfaces even in normal classical systems.

With the advent of the Firewall paradox, the argument that everyday science or low energy regime Physics can be adequately done regardless of what is happening at the Planck level is no longer tenable. Until the information loss problem can be adequately solved[2], one can never be sure of the validity of some of the most cherished fundamental tenets of either quantum theory (i.e., unitarity) and general relativity (i.e., the Equivalence Principle).

From the researches made in writing this thesis, we can make the several tentative conclusions.

1. Hawking radiation is not really a test of quantum gravity[233]. In fact, derivations of Hawking radiation is based on the assumption that quantum gravitational effect are ignored.

2. Contradictory mechanisms or assumptions can lead to the same results or expressions for the Hawking radiation. For instance, though Hawking's original derivation is based on ignoring the backreaction, black hole radiation derived via particles tunnelling through the horizon relies on the backreaction and yet both derivations give the same expression for the black hole temperature. The fact that mutually inconsistent derivations of the Hawking radiation converge to the same result is a pointer to something deeper.

3. The prime source of the inconsistencies associated with any theory that naively combine[3] general relativity and quantum theory might be time. Since time has different meaning in general relativity and Quantum theory, it is reasonable to claim that time is ambiguous in any theory resulting from the naive combination of general relativity and quantum theory [234]. Also, canonical commutation is problematic in general relativity[10]. The idea of separating time and space goes against the very theme of general relativity.

The aforementioned tentative conclusions can be considered as supports for the argument that gravity and spacetime are emergent[235–237] in the sense they might just be an effective description emerging out at the macroscopic level. That is, the macroscopic description of gravity or spacetime might be completely different and independent of what

[1] determining the final state of a system from its initial state(i.e., the unitary evolution of quantum states)

[2] which by general consensus can only be adequately solved by a viable quantum theory of gravity

[3] by naively combined we mean direct combination without drastic alternations of the fundamental principles of each

is occurring at the microscopic level. Clues to gravity or spacetime being emergent can be seen from the gravity-gauge theory duality (or the AdS/CFT Correspondence)[71]. The fact that a duality can exist between theories that contain gravity and those that do not contain gravity means gravity can emerge from a microscopic description that got absolutely nothing to do with gravity. Also, the fact that the entropy of a black hole is proportional to the area of the black hole's horizon rather than its volume [238] reinforces the belief that the physics at the microscopic level does not necessarily conform to the naive intuition gained from familiarity with physics at the macroscopic level.

We surmise that the reason for the robustness and universality of the Hawking effect is because it is a coarse grained effect which is insensitive to the details of the "microstructure" of spacetime. We hypothesize that the pathologies, paradoxes, inconsistencies and problems of the solutions of general relativity or black holes are the result of trying to explain phenomena at the microscopic level by simply extrapolating familiar and intuitive concepts of the macroscopic world to the microscopic regime. We surmise that general relativity is a macroscopic approximation rather than a fundamental description and therefore singularities, the information loss problem and the other paradoxes, problems and contradictions associated with gravitational collapse represent a breakdown of our macroscopic approximation rather than the invalidation of general relativity and quantum theory.

In conclusion, though black holes seem to fill in the gaps of our knowledge about certain natural phenomena like active galactic nuclei and provide excellence opportunities for testing quantum gravity, the unresolved problems associated with them such as singularities and the information loss problem make the universe better off without black holes. It might be possible to envisage compact objects without the pathologies associated with black holes as the end products of gravitational collapse.

# Appendix A

## A.1

It is straightforward to show that the Klein-Gordon inner product is independent of the choice of Cauchy surface:

$$(f,g)_{\Sigma_2} - (f,g)_{\Sigma_1} = i \int_{\sqrt{-g}} n^\nu \Big( f^* \nabla_\nu g - g \nabla_\nu f^* \Big) = i \int_{\sqrt{-g}} n^\nu \nabla^\mu (f^* \partial_\nu g - g \nabla_\nu f) = 0 \tag{A.1}$$

where we have chosen $\Sigma_1$ and $\Sigma_2$ as two different Cauchy surfaces which form the boundary of the volume $V$ with $\Sigma = \Sigma_2 - \Sigma_1$ and we used the Gauss divergence theorem and considered that $f$ and $g$ are solutions of the Klein-Gordon equation.

## A.2

$$\begin{aligned} \partial_t \Box \psi =& \partial_t \Big( \nabla_\mu g^{\mu\nu} \partial_\nu \psi \Big) = g^{\mu\nu} \partial_t \Big( \partial_\mu \partial_\nu \psi - \Gamma^\sigma_{\mu\nu} \partial_\sigma \psi \Big) \\ =& g^{\mu\nu} \Big( \partial_t \partial_\mu \partial_\nu \psi - \Gamma^\sigma_{\mu\nu} \partial_t \partial_\sigma \psi ) \end{aligned} \tag{A.2}$$

and

$$\Box(\partial_t \psi) = \nabla_\mu g^{\mu\nu} \partial_\nu (\partial_t \psi) = g^{\mu\nu} (\partial_\mu \partial_\nu \psi) - \Gamma^\sigma_{\mu\nu} \partial_\sigma (\partial_t \psi) \tag{A.3}$$

Thus

$$[\partial_t, \Box - m^2] = 0. \tag{A.4}$$

## A.3

$$
\begin{aligned}
(f, Kg) = &\int_{\Sigma} n^{\mu}\sqrt{-h}\Big[(Kg^*)\nabla_{\mu} f - f\nabla_{\mu}(Kg^*)\Big] \\
&= \int_{\Sigma} n^{\mu}\sqrt{-h}\Big[g^*\nabla_{\mu}(\partial_t f) - (-\partial_t f)\nabla_{\mu} g^*\Big] \\
&= \int_{\Sigma} n^{\mu}\sqrt{-h}\Big[g^*\nabla_{\mu}(-Kf) - (-Kf)\nabla_{\mu} g^*\Big] \\
&= (-Kf, g)
\end{aligned}
\tag{A.5}
$$

## A.4

$$
\begin{aligned}
\Box\psi =& g^{\mu\nu}\nabla_{\mu}\nabla_{\nu}\psi = \frac{1}{\sqrt{-g}}\partial_{\mu}\Big[\sqrt{-g}g^{|\nu}\partial_{\nu}\psi\Big] \\
=& \Bigg[-\frac{r}{r-2M}\partial_t^2 + \frac{1}{r^2}\partial_r(r^2\frac{r-2M}{r}\partial_r) + \frac{1}{r^2}\hat{L}^2\Bigg]\psi = 0
\end{aligned}
\tag{A.6}
$$

where we have defined the operator

$$L^2 = \frac{1}{\sin^2\theta}\partial_{\theta}(\sin\theta\partial_{\theta}) + \frac{1}{\sin^2\theta}\partial_{\phi}^2 \tag{A.7}$$

If we make the transformation

$$dr^* = \frac{r}{r-2M}dr \tag{A.8}$$

and take the ansatz

$$\psi = \frac{R(r)}{r}e^{-i\omega t}Y_l^m(\theta,\phi), \tag{A.9}$$

where $Y_l^m(\theta,\phi)$ is the Spherical harmonics satisfying

$$L^2 Y_l^m(\theta,\phi) = -l(l+1)Y_l^m(\theta,\phi), \tag{A.10}$$

then Equation A.6 becomes

$$\Big[\frac{d^2}{dr^2} + \omega^2 - V(r)\Big]R(r) = 0, \tag{A.11}$$

where

$$V(r) \equiv (1-\frac{2M}{r})\Big[\frac{l(l+1)}{r^2} + \frac{2M}{r^3}\Big]. \tag{A.12}$$

## A.5

To show that the Klein-Gordon inner product is independent of the choice of Cauchy surface, we choose $\Sigma_1$ and $\Sigma_2$ are two different Cauchy surfaces which form the boundary of the volume $V$ and we use the Gauss divergence theorem to obtain

$$\begin{aligned}(f,g)_{\Sigma_1} - (f,g)_{\Sigma_2} &= -i \int_{=\Sigma_1 - \Sigma_2} d^3x \sqrt{\gamma} n^\mu \Big( f \nabla_\mu g^* - g^* \nabla_\mu f \Big) \\ &= - \int_{\partial\Sigma} d^4x \sqrt{-g} \nabla^\mu \Big( f \nabla_\mu g^* - g^* \nabla_\mu f \Big) \\ &= - \int_{\partial\Sigma} d^4x \sqrt{-g} \Big( f m^2 g^* - g^* m^2 f \Big) = 0\end{aligned} \tag{A.13}$$

## A.6

$$\begin{aligned}ImS &= -Im \int_0^\omega \int_{r_{in}}^{r_{out}} \frac{d\omega'}{1 - \sqrt{\frac{2(M-\omega')}{r}}} dr \\ &= -Im \int_0^\omega d\omega' \int_{z_{in}}^{z_{out}} \frac{2z^2 dz}{z - \sqrt{2(M - \omega' + i\epsilon)}} \\ &= Im 2\pi i . \frac{1}{2} \int_0^\omega d\omega' 4(M - \omega') = 4\pi\omega\Big(M - \frac{\omega}{2}\Big),\end{aligned} \tag{A.14}$$

where we have let $z = \sqrt{r}$ and add $i\epsilon$ to the energy which slightly shift the pole to the upper half plane.

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