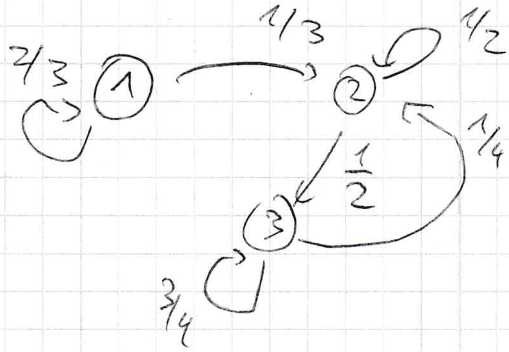
Aufgabe 5)



(a) $$P = \begin{pmatrix} 2/3 & 1/3 & 0 \\ 0 & 1/2 & 1/2 \\ 0 & 1/4 & 3/4 \end{pmatrix}$$

(b) $$\mathbb{P}(X_n = 1) = \mathbb{P}(X_n = 1 \mid X_{n-1} = 1) + \overset{\overset{0}{\shortparallel}}{\mathbb{P}(X_n = 1 \mid X_{n-1} = 2)} + \mathbb{P}(X_n = 1 \mid X_{n-2} = 3)^{=0}$$

$$= \mathbb{P}(X_{n-1} = 1) \cdot \frac{2}{3}$$

$$\overset{\text{Induktion}}{=} \mathbb{P}(X_{n-k} = 1) \cdot \left(\frac{2}{3}\right)^k$$

$$= \mathbb{P}(X_1 = 1) \cdot \left(\frac{2}{3}\right)^{n-1}$$

$$= \left(\frac{2}{3}\right)^{n-1}$$

(c) Sei $\mu = (\mu_1, \mu_2, \mu_3)$ stationär

Wegen $\mu P = \left(\frac{2}{3}\mu_1, -, -\right)$ gilt also

$\mu_1 = \frac{2}{3}\mu_1 \Rightarrow \mu_1 = 0$

Nun $\mu P = \left(0, \frac{1}{2}\mu_2 + \frac{1}{4}\mu_3, \frac{1}{2}\mu_2 + \frac{3}{4}\mu_3\right)$

Also $\mu_2 = \frac{1}{2}\mu_2 + \frac{1}{4}\mu_3 \Rightarrow \mu_2 = \frac{1}{2}\mu_3$

$\mu_3 = \frac{1}{2}\mu_2 + \frac{3}{4}\mu_3 \Rightarrow \mu_2 = \frac{1}{2}\mu_3$

Wegen $1 = \mu_1 + \mu_2 + \mu_3 = \mu_2 + \mu_3$

folgt also eindeutig $\mu_2 = \frac{1}{3}, \mu_3 = \frac{2}{3}$.
(einfaches LGS)

Also ist $\mu = \left(0, \frac{1}{3}, \frac{2}{3}\right)$ <u>die</u> stationäre Verteilung von P (X_n)