UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS: Part III Physics
MASTER OF ADVANCED STUDY IN PHYSICS

Monday 20th January 2025   10:00 to 12:00

MAJOR TOPICS
Paper 1/PP (Particle Physics)

*Answer **two** questions only. The approximate number of marks allocated to each part of a question is indicated in the right-hand margin where appropriate. The paper has content on 6 sides, including this one, and is accompanied by a book giving values of constants and containing mathematical formulae which you may quote without proof.*

*You should use a **separate Answer Book** for each question.*

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
2x20-page answer books	Mathematical Formulae Handbook
Rough workpad	Approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

The information in this box may be used in any question.

The Pauli-matrices are:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

The Dirac representation of the gamma matrices is:

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \gamma^k = \begin{pmatrix} 0 & \sigma_k \\ -\sigma_k & 0 \end{pmatrix}, \gamma^5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} = i\gamma^0\gamma^1\gamma^2\gamma^3.$$

The Dirac representation of the gamma matrices has the following properties:

$$(\gamma^0)^* = \gamma^0,\ \ (\gamma^1)^* = \gamma^1,\ \ (\gamma^2)^* = -\gamma^2,\ \ (\gamma^3)^* = \gamma^3 \ \ \textit{and}\ \ \gamma^2(\gamma^\mu)^* = -\gamma^\mu\gamma^2.$$

Using the above representation, the Part III Particles lecture course defined the following particle and anti-particle spinors:

$$u_\uparrow = N\begin{pmatrix} c \\ e^{i\phi}s \\ \frac{|\vec{p}|}{E+m}c \\ \frac{|\vec{p}|}{E+m}e^{i\phi}s \end{pmatrix}, \qquad u_\downarrow = N\begin{pmatrix} -s \\ e^{i\phi}c \\ \frac{|\vec{p}|}{E+m}s \\ -\frac{|\vec{p}|}{E+m}e^{i\phi}c \end{pmatrix},$$

$$v_\uparrow = N\begin{pmatrix} \frac{|\vec{p}|}{E+m}s \\ -\frac{|\vec{p}|}{E+m}e^{i\phi}c \\ -s \\ e^{i\phi}c \end{pmatrix}, \qquad v_\downarrow = N\begin{pmatrix} \frac{|\vec{p}|}{E+m}c \\ \frac{|\vec{p}|}{E+m}e^{i\phi}s \\ c \\ e^{i\phi}s \end{pmatrix}$$

for objects whose three-momentum $\vec{p}$ is given by $|\vec{p}|(\cos\phi\sin\theta, \sin\phi\sin\theta, \cos\theta)$ where $c = \cos\frac{\theta}{2}$ and $s = \sin\frac{\theta}{2}$. The normalising constant is $N = \sqrt{E+m}$.

$\hbar \approx 1.05 \times 10^{-34}$ kg m^2/s, $\qquad c \approx 3.00 \times 10^8$ m/s, $\qquad e \approx 1.60 \times 10^{-19}$ C.

$m_e = 5.11 \times 10^{-4}$ GeV. $\qquad m_p = m_n = 1.67 \times 10^{-27}$ kg.

1 In electron-proton scattering, the Lorentz invariant quantities:

$$s = (p_1 + p_2)^2 , \quad Q^2 = -q^2 = -(p_1 - p_3)^2 , \quad x = \frac{Q^2}{2p_2 \cdot q} \text{ and } y = \frac{p_2 \cdot q}{p_1 \cdot p_2}$$

are defined in terms of p_1 and p_2, the four-momenta of the initial-state electron and proton respectively, and p_3, the four-momentum of the scattered electron. Neglecting the Q^2 dependence of the structure functions, F_1^{ep} and F_2^{ep}, the differential cross section for electron-proton deep inelastic scattering can be written as

$$\frac{\mathrm{d}^2\sigma^{ep}}{\mathrm{d}x\,\mathrm{d}Q^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y)\frac{F_2^{ep}(x)}{x} + y^2 F_1^{ep}(x)\right].$$

(a) For the case where the proton is at rest, express s, Q^2, x and y in terms of the proton mass, m_p, the electron scattering angle θ in the lab frame, and the energies of the incoming and scattered electron, E_1 and E_3. [4]

(b) In the parton model, show that x can be interpreted as the fraction of the proton's momentum carried by the struck quark in a frame where the proton has infinite momentum. Explain any assumptions made. [4]

(c) The differential cross section for electron-quark scattering can be written as

$$\frac{\mathrm{d}\sigma^{eq}}{\mathrm{d}Q^2} = \frac{4\pi\alpha^2 e_q^2}{Q^4}\left[(1-y) + \frac{y^2}{2}\right]$$

where e_q is the charge of the quark. Using the parton model, including contributions from the light quarks (u, d, s) only, show that

$$\frac{F_2^{ep}(x)}{x} = \frac{4}{9}[u(x) + \bar{u}(x)] + \frac{1}{9}[d(x) + \bar{d}(x) + s(x) + \bar{s}(x)]$$

where $u(x)$, $d(x)$ and $s(x)$ are the up-, down- and strange-quark parton distribution functions for the proton. Obtain a similar expression for the electron-neutron structure function $F_2^{en}(x)$. [6]

(d) Stating clearly any assumptions made, show that

$$\int_0^1 \frac{\left[F_2^{ep}(x) - F_2^{en}(x)\right]}{x}\,\mathrm{d}x = \frac{1}{3} + \frac{2}{3}\int_0^1 [\bar{u}(x) - \bar{d}(x)]\mathrm{d}x$$

and comment on the consequences of the observed value being 0.24 ± 0.03. [6]

(e) In the parton model for neutrino-nucleon scattering the structure functions are

$$F_2^{\nu p}(x) = 2x[d(x) + s(x) + \bar{u}(x)] \quad \text{and} \quad F_2^{\nu n}(x) = 2x[u(x) + \bar{s}(x) + \bar{d}(x)].$$

Assuming $s(x) = \bar{s}(x)$, obtain an expression for $xs(x)$ in terms of the structure functions for neutrino- and electron-nucleon scattering, $F_2^{\nu N}(x) = \frac{1}{2}\left(F_2^{\nu p}(x) + F_2^{\nu n}(x)\right)$ and $F_2^{eN}(x) = \frac{1}{2}\left(F_2^{ep}(x) + F_2^{en}(x)\right)$. [7]

(f) Provide possible physical explanations for why $\bar{d}(x) > \bar{u}(x) > \bar{s}(x)$. [3]

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2 This question concerns the colour wave functions of hadrons (baryons, mesons, tetraquarks, pentaquarks, *etc*.). Within this question the basis colour states r, g and b are sometimes written as c_1, c_2 and c_3, respectively, to allow them to be used in indexed sums. Similarly $\bar{c}_1$, $\bar{c}_2$, and $\bar{c}_3$ refer to $\bar{r}$, $\bar{g}$ and $\bar{b}$, respectively.

(a) Write down the action of the SU(3)-colour ladder operators $T^c_\pm$, $U^c_\pm$ and $V^c_\pm$:

(i) on each of the colour basis states r, g and b; and

(ii) on each of the anti-colour basis states $\bar{r}$, $\bar{g}$ and $\bar{b}$. [2]

(b) What is the defining property of a singlet state S under a general SU(n) symmetry? [Note that this question specifies general SU(n), not SU(3)-colour!] [1]

(c) What special role do colour-singlet states have within QCD? [1]

(d) Which of the following states A, B, C and D are colour singlet states? (Justify your answers!) [2]

$$
\begin{aligned}
A &= rgb. \\
B &= \frac{1}{\sqrt{6}}\left(rgb - rbg + brg - bgr + gbr - grb\right). \\
C &= \frac{1}{\sqrt{3}}\left(r\bar{r} + g\bar{g} + b\bar{b}\right). \\
D &= \frac{1}{\sqrt{3}}\left(r\bar{r}r\bar{r} + g\bar{g}g\bar{g} + b\bar{b}b\bar{b}\right).
\end{aligned}
$$

It may be shown that if S_1 and S_2 are both colour singlet states, then all of the following states X_1, X_2, X_3 and X_4 are also colour singlet states (when non-zero), albeit not-necessarily normalised.

$X_1 = S_1 S_2$.

$X_2 = \lambda_1 S_1 + \lambda_2 S_2$, for any constants λ_1 and λ_2.

X_3 = [S_1 symmetrized (or antisymmetrized) over any two colour states].

X_4 = [S_1 symmetrized (or antisymmetrized) over any two anti-colour states].

[*In the above, it is intended that the words 'symmetrization over first and third colour states' would describe the linear transformation which would take a term like $c_1c_2c_3$ (within some singlet wave function) into the new expression $c_1c_2c_3 + c_3c_2c_1$. Similarly 'antisymmetrization over anti-colour states' would take a term like $c_1\bar{c}_2c_3\bar{c}_4$ (within some singlet wave function) into the new expression $c_1\bar{c}_2c_3\bar{c}_4 - c_1\bar{c}_4c_3\bar{c}_2$, and so on.*]

(e) Prove **only one** of the last four results just given – i.e. prove either that X_1 is a singlet, or that X_2 is a singlet, or that X_3 is a singlet, or that X_4 is a singlet. [1]

The rest of this question will focus on colour singlet wave functions that could potentially belong to tetraquark (i.e. $q\bar{q}q\bar{q}$) states. You may assume that these will be linear combinations of terms of the form $c_i\bar{c}_jc_k\bar{c}_l$ for various values i, j, k and l. [We will call these 'four-colour' singlet wave functions even though they contain two colours and two anti-colours.]

(f) Using the results from earlier in this question (or otherwise) write down two linearly-independent four-colour singlet wave functions F_S and F_A. Choose F_S such that it is **symmetric** under exchange of the colour states (positions 1 and 3 in the wave function) while also being symmetric with respect to exchange of the anti-colour states (position 2 and 4) in the wave function. Similarly, choose F_A to be **antisymmetric** with respect to those same interchanges.

[*When answering, avoid of multiplying out long expressions unless it helps you. Of the* $3^4 = 81$ *colour combinations* $c_i\bar{c}_jc_k\bar{c}_l$ *there are fifteen which (like* $r\bar{g}g\bar{r}$*) have* $I_3^c = Y^c = 0$*, and so you will want to avoid manipulating explicit linear combinations of such terms wherever possible! If you define suitable notation you should mostly be able to avoid multiplying out or manipulating long expressions.*]

[10]

(g) If your states F_S and F_A were unit normalised (like states A, B, C and D of part (d) above) and were written in the forms:

$$F_S = \frac{1}{\sqrt{24}} \sum_{i,j,k,l=1}^{3} \lambda_{i,j,k,l} \cdot c_i\bar{c}_jc_k\bar{c}_l, \quad \text{and} \qquad F_A = \frac{1}{\sqrt{12}} \sum_{i,j,k,l=1}^{3} \mu_{i,j,k,l} \cdot c_i\bar{c}_jc_k\bar{c}_l$$

in terms of 81 indexed constant coefficients $\lambda_{i,j,k,l}$ and 81 indexed constant coefficients $\mu_{i,j,k,l}$, then state the values you would need for:

(i) $(\lambda_{1,1,1,1}, \lambda_{1,1,2,2}, \lambda_{1,2,2,1}, \lambda_{1,2,3,1})$ and

(ii) $(\mu_{1,1,1,1}, \mu_{1,1,2,2}, \mu_{1,2,2,1}, \mu_{1,2,3,1})$.

[4]

Recall that $3 \otimes \bar{3} = 8 \oplus 1$ when an SU(3) colour is combined with an anti-colour.

(h) By multiplying multiplets (or otherwise) determine how many singlets appear in the product $3 \otimes \bar{3} \otimes 3 \otimes \bar{3}$, and thereby state whether or not there exist other tetraquark colour wave functions linearly independent of those already found in part (f). [7]

(i) State two physical consequences you might predict for the spectrum of tetraquark states given your results in this question. You need not consider more than the two lightest quarks. [2]

3 Although some of the questions below are discursive rather than mathematical, long essays are not required. Indeed, succinct answers which include all the relevant points will be rewarded more highly than lengthy answers which digress into areas of tangential relevance. You are therefore strongly encouraged to keep your answers focused narrowly on what has been asked.

(a) What can electron deep inelastic scattering tell us about nucleon structure that neutrino deep inelastic scattering cannot, and why? [6]

(b) Why is one of the central members of the $L = 0$, $S = 0$, $J = 0$, $P = -1$ psuedoscalar meson nonet (the η') considerably heavier than all the other members of that nonet? [6]

(c) Outline the most important parts of the mechanism whereby charged leptons and quarks acquire mass in The Standard Model. [6]

(d) How did the Super-Kamiokande discriminate between electron and muon neutrinos and determine their direction? Why was detection of solar tau neutrinos historically difficult in many neutrino detectors? Describe the innovative features of the experiment that first measured the total solar flux of all three neutrino types. [12]

END OF PAPER