Q4 → $f^5 = f^4 \Rightarrow f^5 - f^4 = 0 \Rightarrow x^5 - x^4$ is a "multiple" of the minimal polynomial; $m(x)$ (i.e. $m(x) \mid x^5 - x^4$) (proposition 2.22).

⇒ the only possible candidates for eigenvalues are 0, 1 ~~dim~~ as $x^5 - x^4 = x^4(x-1)$, Lemma 2.24.

→ $\dim(\ker(f)) = 3 \Rightarrow$ precisely 3 Jordan blocks with ~~0~~ zero on the diagonal in the JNF of $[f]$ in any basis.

~~→ in the case where one is an eigenvector we have~~

~~→ from $f^5 - f^4 = 0$ we know the sum of the sizes of 0-Jordan blocks is at most 4.~~ NO!

→ as we have a 5x5 matrix, all Jordan block sizes must add to 5.

→ in the case where 1 is an eigenvalue we know that the size of the largest ~~sum of the sizes of~~ 1-Jordan blocks is at most 1.

i.e. JNF of $[f]_B = J(1,1) \oplus J(0,1) \oplus J(0,1) \oplus J(0,2)$ up to reordering of blocks. or $J(1,1) \oplus J(1,1) \oplus J(0,1) \oplus J(0,1) \oplus J(0,1)$

→ in the case where 1 is not an eigenvalue we use the facts that ~~① the sum of sizes of 0-Jordan blocks is at most 4~~ there are three 0-Jordan blocks, and there are two ways of partitioning 5: 1+1+3, 2+2+1.

$\therefore \quad [f]_B = J(0,1) \oplus J(0,1) \oplus J(0,3)$
or $= J(0,1) \oplus J(0,2) \oplus J(0,2)$

ie our options are (up to re-ordering of blocks)
$\begin{bmatrix} 1&0&0&0&0 \\ 0&0&0&0&0 \\ 0&0&0&0&0 \\ 0&0&0&0&1 \\ 0&0&0&0&0 \end{bmatrix}, \begin{bmatrix} 0&0&0&0&0 \\ 0&0&0&0&0 \\ 0&0&0&1&0 \\ 0&0&0&0&1 \\ 0&0&0&0&0 \end{bmatrix}, \begin{bmatrix} 0&0&0&0&0 \\ 0&0&1&0&0 \\ 0&0&0&0&0 \\ 0&0&0&0&1 \\ 0&0&0&0&0 \end{bmatrix} \begin{bmatrix} 1&0&0&0&0 \\ 0&1&0&0&0 \\ 0&0&0&0&0 \\ 0&0&0&0&0 \\ 0&0&0&0&0 \end{bmatrix}$ →