Q2:

a) gcd(12378, -3054)

= gcd(12378, 3052) , Lemma 1.8:1)

12378 = 4 × 3054 + 162

3054 = 18 × 162 + 138

162 = 1 × 138 + 24

138 = 5 × 24 + 18

24 = 1 × 18 + 6

18 = 3 × 6 + 0

ie. gcd(18, 24) = 6

Therefore gcd(12378, -3054) = 6

b)

6 = (24) − 1 × (18)

= (162 − 1 × 138) − 1 × (138 − 5 × 24)

= (162 − 1 × 138) − 1 × (138 − 5 × (162 − 1 × 138))

= 6 × (162) − 7 × (138)

= 6 × (162) − 7 × (3054 − 18 × 162)

= 132 × (162) − 7 × (3054)

= 132 × (12378 − 4 × 3054) − 7 × (3054)

∴ 6 = 132 × (12378) − 535 × (3054) = 132 × (12378) + 535(−3054)

Therefore $x = 132$, $y = 535$