Q3:

a) i)

$$\begin{cases} [2]_5 x + [3]_5 y = [4]_5 & \text{—— ①} \\ [3]_5 x + [-2]_5 y = [1]_5 & \text{—— ②} \end{cases}$$

① ← ① + ② $\Rightarrow$ $[5]_5 x + [1]_5 y = [5]_5$

$\Rightarrow$ ~~$[-1]_5 y$~~ $[0]_5 x + [1]_5 y = [0]_5$

$\Rightarrow$ $[y]_5 = [0]_5$ —— ③

③ → ② $\Rightarrow$ $[3]_5 x + [-2]_5 [0]_5 = [1]_5$

$\Rightarrow$ $[3]_5 x = [1]_5$

$\Rightarrow$ $[2]_5 [3]_5 x = [2]_5 [1]_5$

$\Rightarrow$ $[1]_5 x = [2]_5$

$\Rightarrow$ $[x]_5 = [2]_5$

Therefore $x = [2]_5$, $y = [0]_5$

NB: Scalar multiplication is not defined for congruence classes, we should have $[5][x]_5$ etc.

~~b)~~ ii)

$$\begin{cases} [2]_{11} x + [3]_{11} y = [7]_{11} & \text{—— ①} \\ [3]_{11} x + [-2]_{11} y = [1]_{11} & \text{—— ②} \end{cases}$$

① ← ① × $[7]_{11}$ $\Rightarrow$ $[3]_{11} x + [-1]_{11} y = [6]_{11}$

② ← ② − 2 × ① $\Rightarrow$ $[-5]_{11} x + [0]_{11} y = [-11]_{11}$

$\Rightarrow$ $[-5]_{11} x = [0]_{11} + [0]_{11} y$

$\Rightarrow$ $[-4]_{11} [-5]_{11} x = [0]_{11}$

$\Rightarrow$ $[1]_x = [0]_{11}$

$\Rightarrow$ $[x]_{11} = [0]_{11}$

$\therefore x = [0]_{11}$ —— ③

③ → ① $\Rightarrow$ $[2]_{11} \cdot [0]_{11} + [3]_{11} y = [9]_{11}$

$\Rightarrow$ $[4][3]_{11} y = [9]_{11} [9 - 2 \cdot 0]_{11}$

$\Rightarrow$ $[1]_{11} y = [5]_{11}$

$\Rightarrow$ $[y]_{11} = [5]_{11}$

$\therefore y = [5]_{11}$

Therefore $x = [0]_{11}$, $y = [5]_{11}$