$2x + 2y - z = 0$

fix $x=0$, : $(0, 1, \cancel{-}2)$

fix $y=0$: $(1, 0, \cancel{-}2)$

$\therefore \{(0,1,\cancel{-}2), (1,0,\cancel{-}2)\}$ forms our basis (B_2)

– now we find $[M|_{ker(q_i(M))}]_{B_i}$:
where q_i are our polynomials and B_i are our bases

$$M \times \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}, \quad M \times \begin{bmatrix} 0 \\ 1 \\ \cancel{-}2 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$$

$$M \times \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix} = 2 \times \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$$

$B =$
$\Rightarrow [1] \in M_1(\mathbb{R})$
is $[M|_{ker(M-I)}]_{B_1}$

$$\Rightarrow C = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix} \in M_2(\mathbb{R})$$
is $[M|_{ker(M-I)}]_{B_2}$

now we must find P, P^{-1}
we are ~~usin~~ changing ~~M into~~ basis $\{b_1, b_2\}$, then applying $A \oplus B$ and returning to the standard basis.

$\cancel{A \oplus B =}$

$$P(B \oplus C)P^{-1} = A, \quad P = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 3 & 2 & 2 \end{bmatrix}, \text{ to find } P^{-1}: \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 3 & 2 & 2 & 0 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 2 & -1 \\ 0 & 1 & 0 & -2 & -1 & 1 \\ 0 & 0 & 1 & -1 & -2 & 1 \end{array}\right]$$

$\Rightarrow B \oplus C = P^{-1} A P$, where

$$B = [1], \quad C = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}, \quad P = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 3 & 2 & 2 \end{bmatrix}$$

$$\therefore P^{-1} = \begin{bmatrix} 2 & 2 & -1 \\ -2 & -1 & 1 \\ -1 & -2 & 1 \end{bmatrix}$$

P is invertible.