Pg 5/8

Q4:

First, we show that $\mathbb{Q}(i)$ is a commutative ring

~~① / $\forall x,y,z \in \mathbb{Q}(i)$~~ let

let $x+yi$, $a+bi$, $\alpha+\beta i$ be elements of $\mathbb{Q}(i)$

ie. $x, y, a, b, \alpha, \beta \in \mathbb{Q}$

and 0 represent $0+0i$, 1 represent $1+0i$

① $[(x+yi)+(a+bi)] + (\alpha+\beta i)$

$= [(x+a)+(y+b)i] + (\alpha+\beta i)$

$= (x+a+\alpha) + (y+b+\beta)i$

Similarly

$(x+yi) + [(a+bi)+(\alpha+\beta i)]$

$= (x+a+\alpha) + (y+b+\beta)i$ $\square$

$\therefore$ associative addition

② $(x+yi)+(a+bi)$

$= (x+a)+(y+b)i$

$= (a+bi)+(x+yi)$

$\therefore$ commutative addition

③ 0 satisfies

$(a+bi)+0 = (a+bi)$ $\therefore$ additive identity

④ for $a+bi$, $-a-bi$ satisfies

$(a+bi)+(-a-bi) = 0$ $\therefore$ additive inverse

⑤ $[(a+bi)\times(\alpha+\beta i)] \times (x+yi)$

$= [(a\alpha - b\beta) + (a\beta + b\alpha)i] \times (x+yi)$

$= (xa\alpha - xb\beta) + (xa\beta + xb\alpha)i + (ya\alpha - yb\beta)i + (ya\beta + yb\alpha)$

$= (xa\alpha - xb\beta - ya\beta - yb\alpha) + (xa\beta + xb\alpha + ya\alpha - yb\beta)i$

$(a+bi) \times [(\alpha+\beta i)\times(x+yi)]$

$= (a+bi) \times [(\alpha x - \beta y) + (\alpha y + \beta x)i]$

$= a(\alpha x - \beta y) + a(\alpha y + \beta x)i + bi(\alpha x - \beta y) - b(\alpha y + \beta x)$

$= (xa\alpha - y\beta a - b\alpha y - b\beta x) + (a\alpha y + a\beta x + \alpha b x - \beta b y)i$

$= (xa\alpha - xb\beta - ya\beta - yb\alpha) + (xa\beta + xb\alpha + ya\alpha - yb\beta)i$

$\therefore$ ~~multip~~ Associative multiplication