MAST20022 Group Theory and Linear Algebra

Assignment 2

Due: 4pm Friday September 13

- ▷ Submission is by file upload on the LMS. Scans or photos must be of good quality. You are responsible for checking that your file(s) has uploaded correctly.
- ▷ Include your name, student number and tutorial time at the top of every page.
- ▷ **All answers should be fully justified.**
- ▷ Soliciting answers to assignment questions from internet forums (or elsewhere) is strictly forbidden.

1. Consider the matrix $M = \begin{bmatrix} i & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & i \end{bmatrix} \in M_3(\mathbb{C})$.

   (a) Find the minimal polynomial of M.

   (b) Use your answer for part (a) to determine whether M is diagonalizable.

   (c) Find the Jordan normal form of M.

2. Let $A \in M_n(\mathbb{C})$ and suppose that $A^3 = A^2$. Show that A^2 is diagonalisable and $A^2 - A$ is nilpotent.

3. Let $A \in M_3(\mathbb{R})$ be given by

$$A = \begin{bmatrix} 5 & 5 & -3 \\ -2 & -3 & 2 \\ 4 & 2 & -1 \end{bmatrix}$$

   (a) Show that $X - 1$ and $X^2 + 1$ are relatively prime in $\mathbb{R}[X]$.

   (b) Given that the characteristic polynomial of A is $(X - 1)(X^2 + 1)$, find the minimal polynomial $m(X) \in \mathbb{R}[X]$ of A.

   (c) Find matrices $B \in M_1(\mathbb{R})$, $C \in M_2(\mathbb{R})$, and $P \in M_3(\mathbb{R})$ such that P is invertible and

$$P^{-1}AP = B \oplus C$$

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4. Suppose $f\colon \mathbb{C}^5 \to \mathbb{C}^5$ is a linear transformation such that $f^5 = f^4$ and $\dim(\ker(f)) = 3$.

   (a) Describe all the possibilities for the Jordan normal form of f that are compatible with these conditions.

   (b) Suppose that, in addition to the above conditions, f satisfies $\dim(\ker(f^2)) = 5$. What now are the possibilities for the Jordan normal form of f?