Assignment 2, Group Theory and Linear Algebra, Sam Lloyd 994940

Q1:

a) Characteristic polymial:

$c(x) = (x-i)^2(x+)$

$\Rightarrow$ Candidates for minimal polymial:

$(x-i)(x+1)$, $(x-i)^2(x+1)$

as eigenvalues are $i, -1$ ~~and~~ $\Rightarrow (x-i), (x+1)$ are linear factors of $m(x)$ and $m(x) \mid c(x)$ (Lemma 2.22, 2.24)

try $(x-i)(x+i)$:

$$\begin{bmatrix} 0 & 0 & 0 \\ 1 & -1-i & 0 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} i+1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & i+1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \neq 0$$

Therefore $(x-i)^2(x+1)$ is the minimal polynomial.

b) ~~M is a 3×3 matrix, yet there are only two distinct eigenvalues. Therefore the matrix is not diagonalisable.~~

$m(x)$ is not the product of distinct linear factors Therefore is <u>not</u> diagonalisable. (2.35)

c)

~~the~~ $m(x) = (x-i)^2(x+1)$, and $c(x) = (x-i)^2(x+1)$

$\Rightarrow$ ① Size of largest i-Jordan block is two,

② Sum of sizes of i-Jordan blocks is 2

③ Size of largest -1-Jordan block is 1

$\therefore\ J(i,2) \oplus J(-1,1)$