Q5:

a) $ed \equiv 1 \pmod{44}$

$\Rightarrow 44 \mid ed - 1$

$\Rightarrow \exists k \in \mathbb{Z}$ s.t.

$44k = ed - 1$

$= 15d - 1$

i.e. $44k + 15(-d) = 1$, linear Diophantine equation

Solve using Euclids gcd algorithm

$44 = 2 \times 15 + 14$ $\qquad 1 = 15 - 1 \times (14)$

$15 = 1 \times 14 + 1 \longrightarrow = 15 - 1 \times (44 - 2 \times 15)$ i.e. d

$14 = 14 \times 1 + 0$ $\qquad \Rightarrow 1 = 3 \times 15 - 1 \times 44$

$\therefore$ Therefore $\boxed{3}$ satisfies $15d \equiv 1 \pmod{44}$, i.e. $\boxed{d = 3}$

b)

to decrypt Y, take $Y^d \pmod{m}$

$= Y^3 \pmod{69}$

Y (encrypted)	$Y^3 \pmod{69}$	decrypted
11	20	S
28	10	I
11	20	S
30	21	T
54	6	E
43	19	R

a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27

$\Rightarrow$ code word is "SISTER"