Q4:

b)

Consider the polynomial $X^2 \mp 2 \in \mathbb{Q}(i)[X]$

the root of this polynomial (assuming it exists)

Satisfies $x^2 = 2$, let $a+bi = x$, $a, b \in \mathbb{Q}$

ie $\cancel{*}$ $(a+bi)^2 = 2$

$\Rightarrow a^2 - b^2 + 2abi = 2$

equating co-efficients, we get

$\underset{①}{a^2 - b^2 = 2}$, $\underset{②}{2ab\cancel{i} = 0}$

② $\Rightarrow a = 0$ or $b = 0$

if $a = 0$, $-b^2 = 2$, no solutions in $\mathbb{Q}$

if $b = 0$, $a^2 = 2$, no solutions in $\mathbb{Q}$

ie $a^2 = 2$, $a \in \mathbb{Q}$ and $-b^2 = 2$, $b \in \mathbb{Q}$

are both contradictory statements.

Therefore, the roots of $X^2 + \cancel{2}$ do not exist in $\mathbb{Q}(i)$

by contradiction

Therefore, $\mathbb{Q}(i)$ is not algebraically closed.