Q2:

i) $A^2 = A^3$, pre-multiply by A

$\Rightarrow A^3 = A^4$

$\Rightarrow A^4 = A^2$

$\Rightarrow A^4 - A^2 = 0$

$\Rightarrow (A^2)(A^2 - 1) = 0$

We have ~~expressed~~ found a polynomial p such that $p(A^2) = 0$ and this polynomial has only linear factors in A^2. Thus the minimal polynomial must have the same property.

As the minimal polynomial of A^2 has distinct linear factors, A^2 is diagonalisable.

ii) nilpotency of $A^2 - A$ implies $\exists n \in \mathbb{N}$:

$(A^2 - A)^n = 0$

try $n = 2$

$$(A^2 - A)^2 = A^4 - 2A^2 \cdot A + A^2$$

$$= A^4 - 2A^3 \cdot A + A^4 \quad , \quad \text{as } A^2 = A^3 = A^4$$

$$= A^4 - 2A^4 + A^4$$

$$= 0$$

$\therefore A^2 - A$ is nilpotent.