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Q3:

a) Consider the polynomials $\left(\frac{1}{2}\right) \cancel{(-x-1)}\ \frac{1}{2}(-x-1)\ \frac{1}{2} \in \mathbb{R}[X]$

notice $\frac{1}{2}(-x-1)(x-1) + \frac{1}{2}(x^2+1)$

$\cancel{= \frac{1}{2}(-x^2-1)}$

$\frac{1}{2}(-x-1)(x-1) + \frac{1}{2}(x^2+1)$

$= \frac{1}{2}[-x^2+1] + \frac{1}{2}[x^2+1]$

$= 1$, $\therefore$ x^2+1 and $x-1$

~~are~~ have gcd 1

i.e. are co-prime.

b)

The minimal polynomial $m(x)$ divides ~~th~~ $(x-1)(x^2+1)$ but must have $(x-1)$ and (x^2+1) as factors,

therefore $m(x) = (x-1)(x^2+1)$

c)

as $\mathbb{R}^3$ is a finite dimensional vector space with pairwise relatively prime factors in A. we can use proposition 2.27.

- find a basis for $\ker(A-I)$:

$$A-I = \begin{bmatrix} 4 & 5 & -3 \\ -2 & -4 & 2 \\ 4 & 2 & -2 \end{bmatrix} \sim \begin{bmatrix} 4 & 5 & -3 \\ 0 & -3 & 1 \\ 0 & -3 & 1 \end{bmatrix} \sim \begin{bmatrix} 4 & -4 & 0 \\ 0 & 3 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$\Rightarrow \{(1,1,3)\}$ forms a basis

- find a basis for $\ker(A^2+I)$ (B_1)

$$A^2+I = \begin{bmatrix} 5 & 5 & -3 \\ -2 & -3 & 2 \\ 4 & 2 & -1 \end{bmatrix}^2 + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 4 & -2 \\ 4 & 4 & -2 \\ 12 & 12 & -6 \end{bmatrix}$$

$$\sim \begin{bmatrix} 2 & 2 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

, equivalently find basis vectors that span the plane $2x+2y-z=0$ in $\mathbb{R}^3$

$\rightarrow$