⑥ $(a+bi)(x+yi)$

$= a(x+yi) + bi(x+yi)$

$= ax + ayi + bxi - by$

$= (ax - by) + (ay + bx)i$

$(x+yi)(a+bi)$

$= x(a+bi) + yi(a+bi)$

$= ax + bxi + ayi - by$

$= (ax - by) + (ay + bx)i$

$\therefore$ commutative multiplication

⑦ For any $a+bi$, 1 satisfies

$1 \times (a+bi) = a+bi$

$\therefore$ multiplicative identity

⑧ $(x+yi)[(a+bi) + (\alpha+\beta i)]$

$= x[(a+\alpha) + (b+\beta)i] + yi[(a+\alpha) + (b+\beta)i]$

$= ax + \alpha x + bxi + \beta xi + yai + y\alpha i - yb - y\beta$

$= (ax + \alpha x - by - \beta y) + (bx + \beta x + ay + \alpha y)i$

$(x+yi)(a+bi) + (x+yi)(\cancel{a+bi}\ \alpha+\beta i)$

$= ax - by + bxi + ayi + \alpha x - \beta y + \beta xi + \alpha yi$

$= (ax + \alpha x - by - \beta y) + (bx + \beta x + ay + \alpha y)i$

$\therefore$ distributative

Therefore, $\mathbb{Q}(i)$ is a commutative ring

$0, 1 \in \mathbb{Q}(i)$ and for any $a+bi$

$\frac{a-bi}{a^2+b^2} \in \mathbb{Q}(i)$ satisfies the multiplicative inverse property

$$\frac{a-bi}{a^2+b^2} \times a+bi = \frac{(a-bi)(a+bi)}{a^2+b^2} = \frac{a^2 + \cancel{abi} - \cancel{abi} + b^2}{a^2+b^2} = 1$$

assuming $a^2+b^2 \neq 0$ i.e. $a+bi$ is non-zero

* $\mathbb{Q}(i)$ is commutative ring with at least two elements and all non-zero elements have a multiplicative inverse

Therefore $\mathbb{Q}(i)$ is a field.