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Generic Nonlinear Model of Reduced Scale UAVs

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Abstract— **This paper proposes, through a survey of models of several UAV-Structures, a generic nonlinear model for reduced scale aerial robotic vehicles (6 DOF)[1]. Dynamics of an aircraft and some VTOL UAV (quadricopter, ducted fan and classical helicopter) are illustrated. This generic model focuses only on the key physical efforts acting on the dynamics in order to be sufficiently simple to design a controller. The *Small Body Forces* expression which can introduce a zero dynamics is then discussed.**

I. INTRODUCTION

Usually, UAVs can be classified into close, short and long ranges according to their areas of mission. There exist numerous types of UAVs to fit for various demands about altitude, range and duration, in the same way than payload capability, volume capability and control capability.

With a range of applications in both civilian and military scenarios, the development of automated aerial robots are an increasingly important field of robotics research. Such vehicles have strong commercial potential in remote surveillance applications such as monitoring traffic congestion, regular inspection of infrastructure such as bridges, dam walls and power cables or investigation of hazardous environments, to name but a few of the possibilities. The development of such robotic vehicles states a number of problems in sensoring and control. A key challenge is to develop light aerial vehicles able of autonomous navigation.

To develop the flight control systems for maneuverable autonomous reduced scale aerial vehicles, dynamic models that are accurate for their flight envelope are needed. For example, in the case of a standard helicopter, dynamic models explicitly must take into account the effects such that the rotor/fuselage coupling. However, in order to design nonlinear control law, minimal complexity models are prefered.

The main difficulties (at a theoretical level) for design stable feedback controllers for such vehicles come from nonlinearities and couplings (for the solid mechanics part) and from the fact that inputs are not torques nor forces but displacements of some elements which enter the dynamics through aerodynamical forces/torques.

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[1]Terms 'light' and 'reduced scale' implie that the flight range of aerial vehicles under interest is lower than 30mn.

Design, modeling and control of autonomous flying systems have now become very challenging areas of research, as shown by a large literature since 90's decade [20], [6], [8], [18], [1], [2], [12], [16], [18] for small-size helicopter and [25][13][27] for nonlinear full-scale rotorcraft models. However, there is no work that has been made on the design of a general (generic) aerodynamic model valid for all autonomous flying system.

The main contribution of this paper is to present a generic nonlinear model of reduced scale UAVs. A panorama of dynamics of an aircraft and some VTOL UAV (quadricopter, ducted fan and classical helicopter) is then presented. A generic model focuses only on the key physical efforts acting on the dynamics in order to be simple sufficiently enough to design a controller. In addition, the *Small Body Forces* expression which can introduce a zero dynamics is then discussed in this general case.

The outline of the paper is as follows. Section II is dedicated to the Background of the study. In Section III different UAV structures are reviewed. The Generic 6DOF State Model and Control Strategy are the subjects of section IV. Finally, we present the conclusion of this work in section V.

II. PROBLEM STATEMENT

In this section, analytic expressions for the forces and moments on the rigid body are derived. The forces and moments are referred to a system of body-fixed axes, centered at the center of gravity.

There are in general two approaches in deriving equations of motion. One is to apply Newton's law and Euler's law which can give some physical insight through the derivation. The other one is more complicated, it provides the linkage between the classical framework and the Lagrangian or Hamiltonian framework.

In this paper, applying Newton's laws of motion relating the applied forces and moments to the resulting translational and rotational accelerations assembles the equations of motion for the 6 degrees of freedom.

We will make in the sequel some simplifying assumptions : the vehicle is rigid and the earth fixed reference frame is inertial, the gravitational field is constant, the aerial vehicle is supposed to be a rigid body, the density of air is supposed to be uniform.

A. Rigid body dynamics

We begin by define the reference frames are considered in the derivation of the kinematics and dynamical equations of

motion[2].

- Let $\mathcal{I} = \{e_1^i, e_2^i, e_3^i\}$ denote the frame attached to the earth can be assumed inertial. e_3^i denotes the downwards vertical direction and e_1^i points to the magnetic north.
- Let $\mathcal{B} = \{e_1^b, e_2^b, e_3^b\}$ be a body-fixed frame whose center coincides with the center of mass of the mobile.
- Let $\mathcal{A} = \{e_1^a, e_2^a, e_3^a\}$ denote the air-frame[3] attached to the body fixed frame, as the previous frame, its center coincides with the center of mass of the mobile.
- The attitude of the body-fixed frame is represented by a rotation matrix $R : \mathcal{B} \to \mathcal{I}$. Let $\bar{R} : \mathcal{A} \to \mathcal{B}$ denote the rotation matrix made up the angle of attack α and the sideslip angle β. The set of all rotation matrices is termed the Special Orthogonal group and denoted by $SO(3)$.

Let $\xi^i,\ v^i,\ R$ and Ω^b denote respectively the linear position and velocity of the center of mass, the attitude and the angular velocity of the aerial vehicle, *i.e*

$$\begin{array}{rcl} \xi^i & = & [x\ y\ z]^T \\ v^i & = & [v_x\ v_y\ v_z]^T \\ R & = & \begin{bmatrix} c_\theta c_\psi & s_\phi s_\theta c_\psi - c_\phi s_\psi & c_\phi s_\theta c_\psi + s_\phi s_\psi \\ c_\theta s_\psi & s_\phi s_\theta c_\psi + c_\phi c_\psi & c_\phi s_\theta s_\psi - s_\phi c_\psi \\ -s_\theta & s_\phi c_\theta & c_\phi c_\theta \end{bmatrix} \\ \Omega^b & = & [p\ q\ r]^T \end{array} \tag{1}$$

with $\eta = [\phi\ \theta\ \psi]^T$ is the Euler angles vector and $c_\chi,\ s_\chi$ denoting, respectively, $\cos\chi$ and $\sin\chi$.

Let m and J be the mass and the inertia matrix of the rigid object assumed to be constant. The motion of a rigid body subjected to forces and torques is then described by the Newton-Euler equations [7]

$$\begin{array}{rcl} \dot{\xi}^i & = & v^i \\ m\dot{v}^i & = & mge_3^i + RF^b \\ \dot{R} & = & Rsk(\Omega^b) \\ J\dot{\Omega}^b & = & -sk(\Omega^b)J\Omega + C^b \end{array} \tag{2}$$

where $g \approx 9.81\ [m.s^{-2}]$ the gravitational acceleration, F^b and C^b the resulting forces (excluding the gravity force) and torques acting on the rigid body.

The notation $sk(\Omega)$ denotes the skew-symmetric matrix such that $sk(\Omega)v = \Omega \times v$ for the vector cross-product and any vector $v \in \mathbb{R}$.

The expressions of F^b and C^b are detailed in the next section for some classical UAV's architecture.

III. Examples of UAV's dynamics

The complexity of a state model essentially depends on the expression of aerodynamic forces and torques.

[2]In the sequel of the paper, a vector denoted w^* means that w is expressed in $\{e_1^*, e_2^*, e_3^*\}$ frame with $^* \in \{i, a, b\}$

[3]Also called Aerodynamic Reference Frame.

A. A reduced scale aircraft (Fig. 1)

In this case, the force F^b is composed by F_A^b (produced by the airframe) and F_T^b (produced by the propeller thrust)

$$F^b = F_A^b + F_T^b$$

The propeller thrust $F_T^b = F_t e_1^b,\ F_t > 0$ generates a sufficient airspeed V_t. The displacement of a streamlined surface in the air generates lift and drag forces. Assuming small angle of attack α and sideslip angle β, linearized aerodynamic forces F_A^b can be expressed as (with coefficients dimensioless C_*)[19]

$$\begin{array}{rcl} F_A^b & = & \bar{q}S\bar{R} \begin{bmatrix} C_{X0} + C_{X\alpha}\alpha + C_{X\beta}\beta \\ C_{Y\beta}\beta \\ C_{Z0} + C_{Z\alpha}\alpha \end{bmatrix} + \Sigma\Gamma \\ \Sigma & = & \begin{bmatrix} C_{X\delta_a} & C_{X\delta_e} & C_{X\delta_r} \\ C_{Y\delta_a} & C_{Y\delta_e} & C_{Y\delta_r} \\ C_{Z\delta_a} & C_{Z\delta_e} & C_{Z\delta_r} \end{bmatrix} \end{array} \tag{3}$$

with C_Z lift coefficient, C_X and C_Y logitudinal and lateral drag coefficients, $\bar{q} = \frac{1}{2}\rho V_t^2$ the dynamic pressure (ρ is the air density), S the wing surface and $\Gamma = [\delta_a\ \delta_e\ \delta_r]^T$ the control-surface deflection vector (aileron, elevator and rudder) used to generate the torque which turns the airplane the desired way.

The torque C^b is composed by the gyroscopic torque $C_{P\ gyros}^b$ induced by the propeller and the aerodynamic torque C_A^b

$$C^b = C_{P\ gyros}^b + C_A^b$$

The gyroscopic torque induced by the propeller is expressed by

$$C_{P\ gyros}^b = -J_P\omega_P sk(\Omega^b)e_1^b \tag{4}$$

with J_P the inertia matrix of the propeller and ω_P its angular velocity. As the aerodynamical force F_A^b, the aerodynamical torque C_A^b can be linearized which gives [19]

$$\begin{array}{rcl} C_A^b & = & \bar{q}S \begin{bmatrix} C_{L\beta}\beta + C_{Lp}\hat{p} + C_{Lr}\hat{r} \\ C_{M0} + C_{M\alpha}\alpha + C_{Mq}\hat{q} \\ C_{N\beta}\beta + C_{Nr}\hat{r} \end{bmatrix} + K\Gamma \\ K & = & \begin{bmatrix} C_{L\delta_a} & 0 & 0 \\ 0 & C_{L\delta_e} & 0 \\ 0 & 0 & C_{L\delta_r} \end{bmatrix} \end{array} \tag{5}$$

where :

$$\hat{p} = \frac{bp}{2\bar{V}_t},\ \hat{q} = \frac{\bar{c}q}{2\bar{V}_t},\ \hat{r} = \frac{br}{2\bar{V}_t} \tag{6}$$

with : b the wingspan (the distance from the left wingtip to the right wingtip), c the mean chord of wing.

B. VTOL vehicles

Different rotorcraft structures exist to perform hover or vertical take-off and landing. All these structures have, at least, one rotor such that its thrust directly confronts gravity force. By supposing that the vehicle has k rotors, the thrust induced by a rotor j ($1 \leqslant j \leqslant k$) may be written [25]

$$F_{T_j} = \rho\left(C_{M_j}(\theta_{T_j} - \alpha_{T_j}) - C_{D_j}\right)\omega_j^2 = b_j\omega_j^2 \tag{7}$$

Fig. 1. **SIG Rascal 110.**

with ω_j the angular velocity of the rotor, θ_{T_j} the collective angle, α_{T_j} the angle of attack of blades and the coefficient C_{D_i} the drag of the rotor. The constant $C_{M_j} = \frac{1}{4}R_j^3 n_j \bar{c}_j$ depends on the radius, the number and the mean chord of blades. Besides, the rotor induces a torque $C^b_{air_j}$ due to the air resistance and proportional to ω_j^2.

To develop the flight control systems for maneuverable autonomous miniature helicopter, dynamic models that are accurate for their flight envelope are needed. However, in order to design nonlinear control law, minimal complexity models are preferred. This models, developped by using linear system identification for example, are only valid in the vicinity of the nominal operating point, for example at hover.

- **Classical helicopter** (Fig. 2): longitudinal and lateral cyclic angles control the main rotor flapping dynamics. When this dynamics tends to equilibrium, the total lift of the main rotor is tilted in comparison to the motor shaft [25]

$$\begin{array}{rcl} F^b_{T_m} & = & F_{T_m} e^b_{T_m} \\ e^b_{T_m} & = & [-a_{1s} \;\; b_{1s} \;\; -1]^T \end{array} \tag{8}$$

with a_{1s} and b_{1s} the longitudinal and lateral flapping angles assumed to be small. Under quasi-stationary flying conditions, these angles depend algebraically on cyclic angles (*i.e.*: mechanical control inputs) [25], [17]. The main rotor induces a torque C^b_{mot} acting on the fuselage which is compensated by the tail rotor. This smaller rotor controls the yaw motion which yields

$$F^b_{T_t} = F_{T_t} e^b_2 \tag{9}$$

From (8) and (9), one gets the expression of F^b (Equation. 2).
The resulting torque is composed by

  - the propulsion momentum

$$C^b_T = F_{T_m} sk(l^b_{T_m}) e^b_{T_m} + F_{T_t} sk(l^b_{T_t}) e^b_2$$

    induced by the lever arms $l^b_{T_{m(/t)}} = [l^1_{T_{m(/t)}} \;\; l^2_{T_{m(/t)}} \;\; l^3_{T_{m(/t)}}]^T$ of the two rotors,
  - the air resistance torque $C^b_{air} = C_{air_m} e^b_3 + C_{air_t} e^b_2$,
  - the damping torque $C^b_{damp} = -\kappa_{T_m} sk(e^b_{T_m}) e^b_3$ which resists the *rotor disk* distortion ($\kappa_{T_m} > 0$ denote the elasticity constant of the main rotor) [26],
  - the gyroscopic torque induced by the two rotors expressed by

$$C^b_{T\ gyros} = -J_{T_m} \omega_{T_m} sk(\Omega^b) e^b_3 - J_{T_t} \omega_{T_t} sk(\Omega^b) e^b_2 \tag{10}$$

with J_{T_m} and J_{T_t}, respectively, the inertia matrix of the main and the tail rotor, ω_{T_m} and ω_{T_t}, respectively, its angular velocity.

By introducing $\Gamma = [-F_{T_m} a_{1s} \; F_{T_m} b_{1s} \; F_{T_t}]^T$ as control input vector, F_b and C_b may be written [6]

$$\begin{array}{rcl} F_b & = & -F_{T_m} e^b_3 + \Sigma\Gamma \\ C_b & = & C^b_{air} + C^b_{damp} + C^b_{mot} + F_{T_m} K^b_0 + K\Gamma \\ \Sigma & = & LK^{-1} \end{array} \tag{11}$$

with $L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$, $K^b_0 = [l^2_{T_m} \;\; -l^1_{T_m} \;\; 0]^T$ and $K = \begin{bmatrix} 0 & -l^3_{T_m} & -l^3_{T_t} \\ l^3_{T_m} & 0 & 0 \\ -l^2_{T_m} & l^1_{T_m} & l^1_{T_t} \end{bmatrix}$,
$l_i = \sum_{k=1}^{3} l^k_i E^a_k$ (i stands for m, t) represents the distance between the center of gravity and the application point of the force.

Fig. 2. **VARIO Benzin Helicopter.**

- **Quadricopter** (Fig.3): This architecture composed by four identical rotors has some advantages with respect to conventional helicopters. In fact, gyroscopic effects and air resistance torques tend to cancel in trimmed flight because front and rear motors rotate counterclockwise while the other two rotate clockwise. Besides, the collective angle is constant because of the small radius of blades, the thrust of each rotor being then simply controlled by their angular velocities. The collective input is the sum of the thrusts of each rotor. Pitch motion is obtained by increasing (reducing) the speed of the rear motor while reducing (increasing) the speed of the front motor. Roll motion is obtained by a similar way by the lateral motors. The yaw movement is obtained by increasing (decreasing) the speed of the front and rear motors while decreasing (increasing) the speed of the lateral motors. This attitude motion is performed while keeping the total thrust constant. From this control

principle, control input vector reads as [3], [2], [9]

$$\begin{bmatrix} F_T \\ \Gamma^1 \\ \Gamma^2 \\ \Gamma^3 \end{bmatrix} = P \begin{bmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{bmatrix}$$

$$P = \begin{bmatrix} b_0 & b_0 & b_0 & b_0 \\ 0 & db_0 & 0 & -db_0 \\ db_0 & 0 & -db_0 & 0 \\ C_{air_0} & -C_{air_0} & C_{air_0} & -C_{air_0} \end{bmatrix} \quad (12)$$

with d the distance between the rotor shaft and the center of mass. Given the parameters d, b and C_{air_0}, P is a full rank matrix. The gyroscopic torque induced by the four rotors reads as

$$C^b_{T\ gyros} = -\sum_{i=1}^{4} J_T \omega_i sk(\Omega^b) e_3^b \quad (13)$$

with J_T the inertia matrix of a rotor and ω_i the angular velocity of the rotor i. Consequently, F_b and C_b of system (2) may be written as [3], [2], [9]:

$$\begin{array}{rcl} F^b & = & -F_T e_3^b \\ C^b & = & \Gamma + C^b_{T\ gyros} \end{array} \quad (14)$$

Theoretically, translational and rotational dynamics (*i.e.*: $\Sigma\Gamma = 0_{3\times 1},\ \Sigma = 0_{3\times 3}$) are decoupled.

Fig. 3. **Dragonflyer V TI-RC.**

- **Ducted fan** (Fig.4): This UAV is composed by two counter-rotary rotor in order to eliminate the tail rotor and the gyroscopic effect induced by the rotor. Four control surfaces located at a distance d of the center of mass induce a control torque Γ by deflecting the air flow in order to control the attitude of the vehicle (see: Hovereye (Bertin Technology), Kestrel (Honeywell) or ISTAR (Allied Aerospace)). Payload, on board electronics and batteries are located above this ducted fan in an axi-symmetrical fuselage. Consequently, F_b and C_b reads as [23]

$$\begin{array}{rcl} F^b & = & -F_T e_3^b - \frac{1}{d} sk(e_3^b)\Gamma \\ C^b & = & \Gamma \end{array} \quad (15)$$

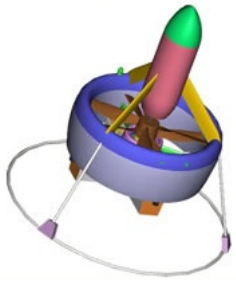

Fig. 4. **HoverEye (Berthin Technology ©).**

IV. Generic 6DOF state model

From the previous expressions of resulting forces and torques F_b et C_b, a generic nonlinear 6DOF state model is proposed covering a large class of reduced scale UAVs, *i.e* from aircraft to rotorcraft under quasi-stationary flight conditions.

A. Nonlinear model

Equation (2) describing the motion of a rigid body can be developped as follows

$$\begin{array}{rcl} \dot{\xi}^i & = & v^i \\ m\dot{v}^i & = & R\left(F_T e_T^b + mgR^T e_3^i + F^b_{air} + \Sigma\Gamma + \delta F^b\right) \\ \dot{R} & = & Rsk(\Omega^b) \\ J\dot{\Omega}^b & = & -sk(\Omega^b)J\Omega + C^b_{inter} + C^b_{air} + C^b_{T\ gyros} + \\ & & K\Gamma + \delta C^b \end{array} \quad (16)$$

with F_T the resulting thrust, e_T^b the thrust direction, F^b_{air} the aerodynamic force, $\Sigma\Gamma$ the *Small Body Force* induced by the torque control input Γ, δF^b the disturbance force, C^b_{inter} the interaction torque between the different part of the vehicle, C^b_{air} the aerodynamic torque, $C^b_{T\ gyros}$ the gyroscopic torque induced by the rotor, K the efficiency matrix of the torque control input Γ and δC^b the disturbance torque. In appendix, the expression of each term on previous platforms is described.

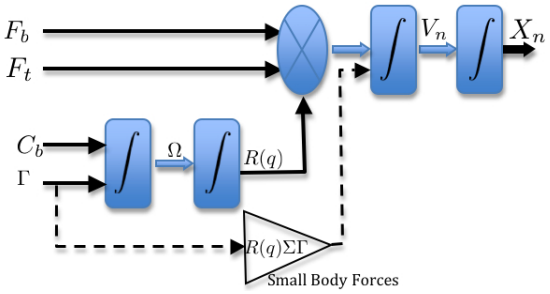

Fig. 5. **Scheme of UAV's dynamics.**

B. Zero dynamics

The study of PVTOL [12], [16] brings into relief that the coupling matrix Σ between translational and rotational dynamics

induce the *Small Body Force* $\Sigma\Gamma$ from the torque control input Γ, which generates a zero-dynamics if $\Sigma \neq 0_{3\times3}$ (Fig 5). In fact, assuming that the center of mass of the vehicle ξ^i perfectly tracks the desired trajectory $(\xi^i)^d$, one gets

$$\begin{aligned} \xi^i - (\xi^i)^d &= 0_{3\times1} \\ v^i - (v^i)^d &= 0_{3\times1} \\ \dot{v}^i - (\dot{v}^i)^d &= 0_{3\times1} \end{aligned} \tag{17}$$

From (17) and the two first equations of (16), one gets

$$\Sigma\Gamma = -F_T e_T^b - mgR^T e_3^i - F_{air}^b - R^T \delta F^b \tag{18}$$

By multiplying the last equation of (16) by $\bar{\Sigma} = \Sigma K^{-1}$ and substuting $\Sigma\Gamma$ by equation (18), yields

$$\begin{aligned} \bar{\Sigma} J\dot{\Omega}^b &= \bar{\Sigma}\left(-sk(\Omega^b)J\Omega^b + C_{inter}^b + C_{air}^b \right. \\ &\quad \left. + C_{T\ gyros}^b + \delta C^b\right) - F_T e_T^b \\ &\quad - mgR^T e_3^i - F_{air}^b - \delta F^b \end{aligned} \tag{19}$$

The attitude R^d corresponding to the equilibrium position $\Omega^b = 0_{3\times1}$, $\dot{\Omega}^b = 0_{3\times1}$ is given by

$$\begin{aligned} mg(R^d)^T e_3^i &= -\bar{\Sigma}\left(C_{air}^b + C_{inter}^b + \delta C^b\right) \\ &\quad + F_T e_T^b + F_{air}^b + \delta F^b \end{aligned} \tag{20}$$

$\tilde{R} = R^T R^d$ denotes the error rotation matrix and may be linearized by

$$\tilde{R} = Id_{3\times3} + \delta\alpha sk(\Delta e)$$

with $\Delta e \in I\!R^3$ the rotation axis and $\delta\alpha \in [0\ 2\pi]$ the rotation angle. The third equation of (16) may be also linearized by $\dot{\Delta} = \Omega^b$ and (19) by $\bar{\Sigma} J\dot{\Omega}^b = -mgsk((R^d)^T e_3^i)\Delta$ with $\Delta = \delta\alpha\Delta e$. Finally, one gets

$$\begin{aligned} S\ddot{\bar{\Delta}} &= -A\bar{\Delta} \\ \bar{\Delta} = V\Delta,\ A &= mgU^{-1}(\bar{\Sigma}J)^T sk((R^d)^T e_3^i)V^{-1} \end{aligned} \tag{21}$$

where the matrices U, S and V correspond to the singular value decomposition of the symmetrical matrix $(\bar{\Sigma}J)^T(\bar{\Sigma}J)$ (U and V are unity matrices and S is a diagonal matrix with nonnegative diagonal elements in decreasing order).

The matrix A depends exclusively of geometry and inertia of the UAV. If A is non Hurwitz, then the attitude diverges; else, the attitude periodically swings around the equilibrium attitude R^d as illustrated for the helicopter [6]. This oscillation is not damped, undesirable but with a small amplitude, it may also be assumed to neglect *Small Body Forces* in the control design and to *a posteriori* verify its robustness [12], [16].

C. Control strategy

$F_T \in I\!R$ and $\Gamma \in I\!R^3$ correspond respectively to the control inputs acting on the fuselage in order to control the position of the center of mass and the attitude of the rigid body. There is only one force to control the translational dynamics and three torques for the rotational dynamics. These vehicles may be also considered as underactuated.

- The *guidance* problem finally consists in the design of a control law for the translational dynamics

$$\begin{aligned} \dot{\xi}^i &= v^i \\ m\dot{v}^i &= R(F_T e_T^b + mgR^T e_3^i + F_{air}^b + \Sigma\Gamma) + \delta F^b \end{aligned} \tag{22}$$

  $F_T R e_T$ is considered as the control input so that ξ^i converges to the desired position $(\xi^i)^d$. A desired attitude R^d is then deduced.

- The *control* problem consists in the design of a control law for the translational dynamics:

$$\begin{aligned} \dot{R} &= Rsk(\Omega^b) \\ J\dot{\Omega}^b &= -sk(\Omega^b)J\Omega + C_{inter}^b + C_{air}^b + \\ &\quad C_{T\ gyros} + K\Gamma + \delta C^b \end{aligned} \tag{23}$$

  Γ is then the control input.

By neglecting *the Small Body Forces* (assumed stable), the state model (16) has a triangular structure adapted to the use of *Backstepping* techniques for instance. A first approach is to design a nonlinear controller with the full 4^{th} order state model by considering a dynamical extension of the thrust control input (*i.e.*: $\ddot{F}_T = u$) [17], [10], [6], [4]. An other way is to separate the full order state model into a slow and a fast timescale [14], [15]. Consequently, the translational dynamics represents the slow timescale and determines a desired attitude R^d to reach which is viewed constant by the fast-timescale rotational dynamics. There is also no need of a dynamical extension of the thrust control input and the control design is simplified [19], [24], [9], [5]

V. Conclusion

The main contribution of this paper was to present a generic nonlinear model of reduced scale UAVs in order to be simple sufficiently enough to design a controller. After a presentation of different architectures of some VTOL UAV (quadricopter, ducted fan and classical helicopter), a generic model focuses only on the key physical efforts acting on the dynamics is then proposed. In addition, the *Small Body Forces* expression, which can introduce a zero dynamics, was also studied.

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APPENDIX

	Aircraft	X4
F_{air}	$\bar{q}S \begin{bmatrix} C_A \\ C_Y \\ C_N \end{bmatrix}$	$0_{3\times1}$
C_{inter}	0_{3x1}	$0_{3\times1}$
C_{air}	$\bar{q}S \begin{bmatrix} C_l \\ C_m \\ C_n \end{bmatrix}$	$0_{3\times1}$
K	$\bar{q}S \begin{bmatrix} bC_{l\delta_a} & 0 & 0 \\ 0 & cC_{m\delta_e} & 0 \\ 0 & 0 & bC_{n\delta_r} \end{bmatrix}$	$Id_{3\times3}$
Σ	$\bar{q}S \begin{bmatrix} C_{A\delta_a} & C_{A\delta_e} & C_{A\delta_r} \\ C_{Y\delta_a} & C_{Y\delta_e} & C_{Y\delta_r} \\ C_{N\delta_a} & C_{N\delta_e} & C_{N\delta_r} \end{bmatrix}$	$0_{3\times3}$

	Helicopter	Ducted fan
F_{air}	$0_{3\times1}$	$0_{3\times1}$
C_{inter}	$F_{t_P} \begin{bmatrix} l_P^2 \\ -l_P^1 \\ 0 \end{bmatrix} - \Lambda_P$	$0_{3\times1}$
C_{air}	$C_P e_3 - C_Q e_2$	$0_{3\times1}$
K	$\begin{bmatrix} 0 & -l_P^3 & -l_Q^3 \\ l_P^3 & 0 & 0 \\ -l_P^2 & l_P^1 & l_Q^1 \end{bmatrix}$	$Id_{3\times3}$
Σ	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} K^{-1}$	$-\frac{1}{L} sk(e_3)$