

# Attitude Control System Design for a Quadrotor Flying Robot

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**Abstract** - This study describes the development of an attitude control system for a quadrotor flying robot. In this paper, we introduces the linearized mathematical model from experimental data using system identification method and design of the optimal control. And then we verify the control system through simulation and performing the experimental tests.

**Keywords** - Quadrotor Flying Robot, Attitude Control, Optimal Control, Model Identification, Prediction Error Minimization method(PEM)

## 1. Introduction

Recently, much effort is being made to develop efficient Unmanned Aerial Vehicles (UAV). Small scale UAVs are very promising platform for monitoring, surveillance, and emergency aid in isolated area. Unmanned helicopter is most suitable for monitoring and surveillance because of its hovering capability [1-3]. Helicopter has been the favorable platform for UAV development. However, the manufacturing process and controller designing of a helicopter involves a lot of effort since it is complex and very unstable system. The quadrotor possesses some special characteristics that make it attractive comparing with the conventional helicopters [4, 5]. One is the superior payload capacity. The other is the simplicity of its control system. The Control of a quadrotor is accomplished by independently adjusting the speed of each rotor. This system is particularly suitable for small UAVs, because it reduces the mechanical complexity of the rotors and simplifies the control algorithms required for autonomous flight. Due to these advantages, the quadrotor has become a standard platform in the experiments and applications of UAVs. But utilization of a quadrotor is not so easy problem because it demands continuous and simultaneous adjustment of four motors' speeds. In this paper, we describes our work to develop an attitude control system for a quadrotor flying robot including extraction of the linear model, design the optimal controller and test of the autonomous hovering performance.

## 2. Quadrotor's Dynamics and Simplified Model

The quadrotor is only controlled by independently adjusting the speed of the four rotors. The quadrotor schematic is shown in Fig. 1. Let  $T_i$  and  $\tau_i$  be the thrust and torque for  $i$ th rotor respectively ( $i = 1, \dots, 4$ ). These values are normalized with the moment of inertia and mass, respectively.

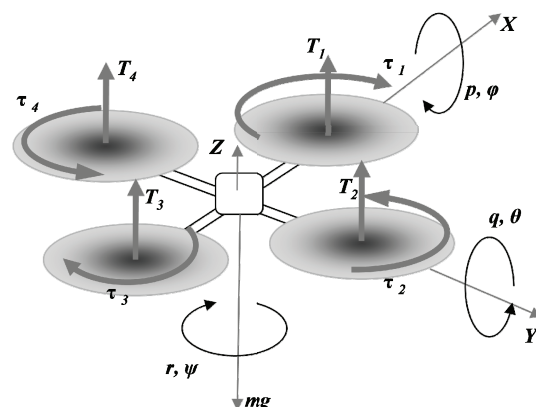


Fig. 1. Quadrotor schematic.

Denoting the distance of the rotor from the center of mass by  $l$ , we can introduce a set of four control input  $u_i$  as function of normalized individual thrusts and torques as follows.

The total thrust is given by

$$u_1 = T_1 + T_2 + T_3 + T_4 \quad (1)$$

The rolling moment is achieved by controlling the rotor#2's and rotor#4's speeds and it is given as

$$u_2 = l(T_4 - T_2) \quad (2)$$

The pitching moment is achieved by controlling the rotor#1's and rotor#3's speeds and it is given as

$$u_3 = l(T_1 - T_3) \quad (3)$$

The yawing moment is obtained from the torque resulting from the difference of the counterclockwise antitorques and the clockwise ones and it is given as

$$u_4 = \tau_1 - \tau_2 + \tau_3 - \tau_4 \quad (4)$$

The quadrotor dynamic model describing the roll, pitch and yaw rotations contains three terms which are the gyroscopic effect resulting from the rigid body rotation, the gyroscopic effect from the propeller rotation coupled with the body rotation and finally the actuators action [6].

$$\begin{aligned} I_{xx}\ddot{\phi} &= \dot{\theta}\dot{\phi}(I_{yy} - I_{zz}) - J\dot{\theta}\Omega_r + u_2 \\ I_{yy}\ddot{\theta} &= \dot{\phi}\dot{\theta}(I_{zz} - I_{xx}) - J\dot{\phi}\Omega_r + u_3 \\ I_{zz}\ddot{\psi} &= \dot{\phi}\dot{\theta}(I_{xx} - I_{yy}) + u_4 \end{aligned} \quad (5)$$

The variables  $I_{xx}$ ,  $I_{yy}$ ,  $I_{zz}$  denote inertial moment of the body. The variables  $J$  and  $\Omega_r$  are the inertial moment and angular rate of the propeller. And the variables  $\phi$ ,  $\theta$ , and  $\psi$  are the roll, pitch and yaw, respectively.

The control vector in an attitude control system,  $u$  is defined as

$$u = [u_2, u_3, u_4]^T \quad (6)$$

And state vector  $x$  is defined as

$$x = [\phi, p, \theta, q, \psi, r]^T \quad (7)$$

### 3. Model Identification

The first principle and system identification method are two branches of approach for determining a linear model of vehicle dynamics. The first principle approach analytically derives the equations of motion from the laws of flight physics while system identification develops model using experimental data. In this work, we estimated a linearized model for a quadrotor using the Prediction Error Minimization (PEM), system identification method [7]. An advantage of this method is that linear models can be extracted directly from flight data using a priori knowledge of a linearized model structure.

#### 3.1 Flight control system

The experimental prototype quadrotor and the Flight Control Computer are shown in Fig. 2. And the functional block diagram of Flight Control System(FCS) is given in Fig. 3.

It consists of four brushless DC motors with speed controllers, one AHRS(Attitude Heading Reference System), a microprocessor, and Bluetooth. It also has RC receiver to get remote commands from the user. The attitude data obtained by AHRS are transmitted to the ground host computer for model identification and monitoring.

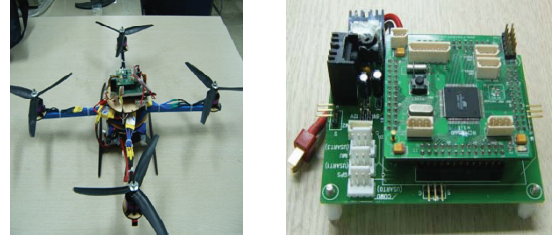


Fig. 2. Quadrotor prototype and Flight Control Computer

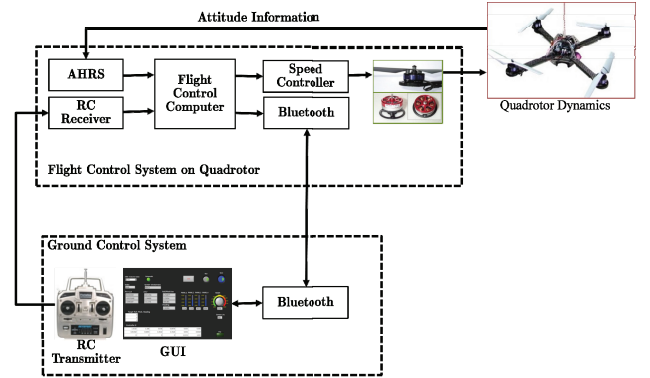


Fig. 3. Structure of the Flight Control System .

Fig. 4. describes the software for ground control system. This software can display the status of a quadrotor like attitude, auto/manual mode, reference angle, and motor outputs. It can record flight data to text file and t reference angle and control gain

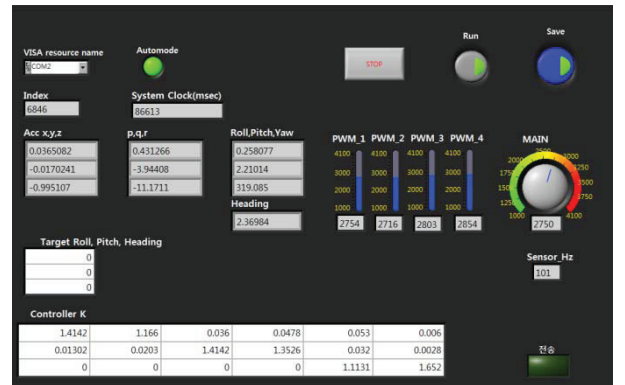


Fig. 4. Ground control system.

#### 3.2 Flight data acquisitions

The flight data acquisition method uses dynamic response time-history test data generated from pilot or computer generated inputs, such as sweeps or other inputs. These inputs excite the system dynamics of a quadrotor. The procedure of flight test in this work typically started by manually taking the Remote Controller to a hovering condition after taking-off. The control input for each channel and the responses are recorded in an onboard computer. The captured flight data used for the parameter estimation consists of four control inputs and six sensor

outputs. Real flight data of the quadrotor are shown in Fig.5 - Fig.7.

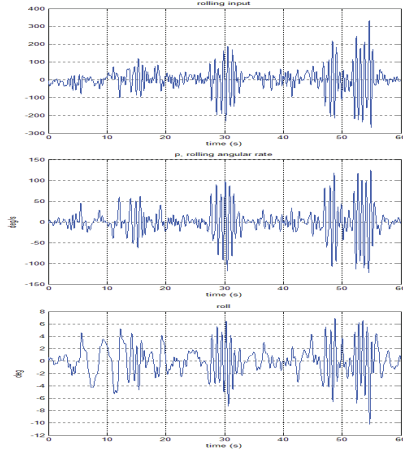


Fig. 5. Lateral input - output response.

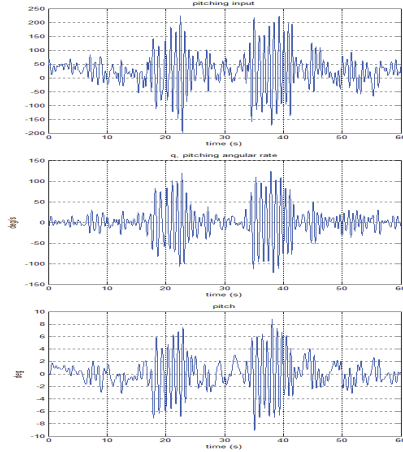


Fig. 6. Longitudinal input - output response.

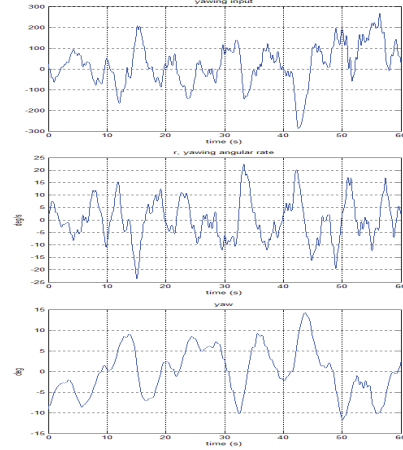


Fig. 7. Directional input - output response.

### 3.3 Model identification

System identification procedures are carried out by Prediction Error Minimization method (PEM) algorithm. PEM is a time domain method which can be used to obtain linear models. In the process of system identification, the flight data is divided into two parts: one part is used during optimization and the other part is used

for validation. The model is identified by matching predicted time histories against measured time histories. This method must be started by assuming or otherwise identifying the parametric model structure. The PEM algorithm gives the parameters set A, B, K, C and D in Eq (8) that minimize the quadratic error given as Eq (9).

We consider a discrete-time state space model

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + Ke(k) \\ y(k) &= Cx(k) + Du(k) + e(k) \end{aligned} \quad (8)$$

The PEM seeks to minimize the quadratic error function :

$$V_N(A, B, K, C, D) = \sum_{k=1}^N e^2(k) \quad (9)$$

The estimated model is simulated with the input and output of flight data. The response of estimated model is plotted together with the corresponding measured output. Fig. 8 shows the responses of flight data and the identified model. It can be observed that the roll and pitch rate show an excellent matching and the matching of yaw rate is also satisfactory. The obtained linear model with the exception of very small values are represented as follows Eq (10):

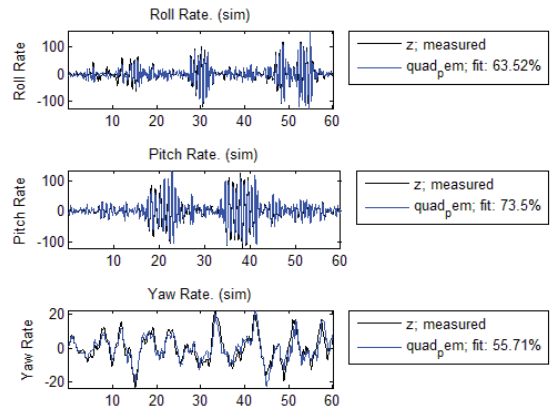


Fig. 8. Comparison with response of flight data and predicted model.

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -0.048 & -4.92 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -0.022 & -6.88 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -0.023 & -0.44 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 \\ 3.3047 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 5.6146 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0.057643 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 & 0 \\ 0.49942 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0.7169 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0.10045 \end{bmatrix} \quad (10)$$

## 4. Attitude Controller Design

### 4.1 Formulation of the LQR problem

We used optimal control scheme to design an attitude controller for a quadrotor. Linear Quadratic Regulator (LQR) is one of the preferable control designs for multi-input, multi-output air vehicle [1]. For a linear state-space model of the plant dynamics

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (11)$$

The control input is given as

$$u(t) = r - Kx(t) \quad (12)$$

$r$  is a reference command. And the control gain  $K$  is chosen so that it minimizes the performance index defined as

$$J = \int_0^\infty (x^T Q x + u^T R u) dt \quad (13)$$

where, the matrices  $Q$  and  $R$  are the weighting matrices.

Then the closed loop system is given as

$$\dot{x}(t) = (A - BK)x(t) + Br \quad (14)$$

### 4.2 Controller design and simulation

Fig. 9 is shown a feedback system which consists of plant, controller and extra gain  $\bar{N}$ . We seek  $K$  having good performance by adjusting weighting matrix  $Q$  and  $R$ . For good tracking performance, we want output  $y(t)$  track reference command  $r$  in short time. So, the closed loop transfer function should be approximately 1. Input gain  $\bar{N}$  is used to scale the closed loop transfer function. The simulation result is shown in Fig. 10. Simulation conditions are given as

initial euler angle : [10 10 10] degree  
reference euler angle : [0 0 0] degree

Fig. 10 shows that the responses follow the reference commands well. The responses reach the reference commands in 6 seconds. The yaw response was slower

than that of roll and pitch. It is similar with the experimental data for model identification. It was considered the characteristics of our flight.

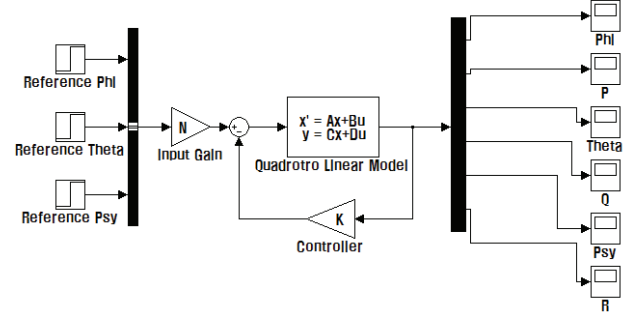


Fig. 9. Feedback system with controller

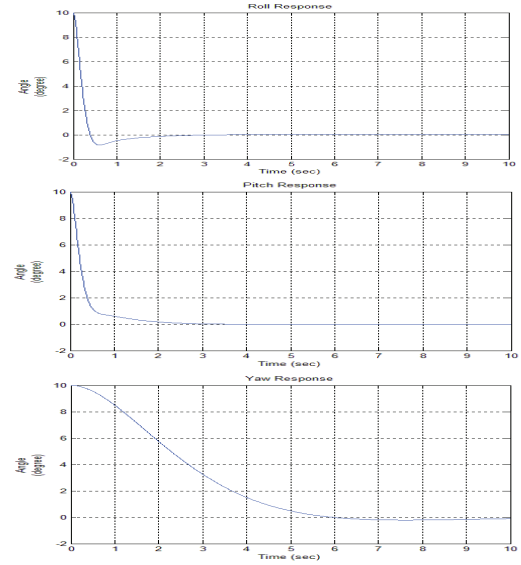


Fig. 10. Simulation result.

## 5. Experimental Results

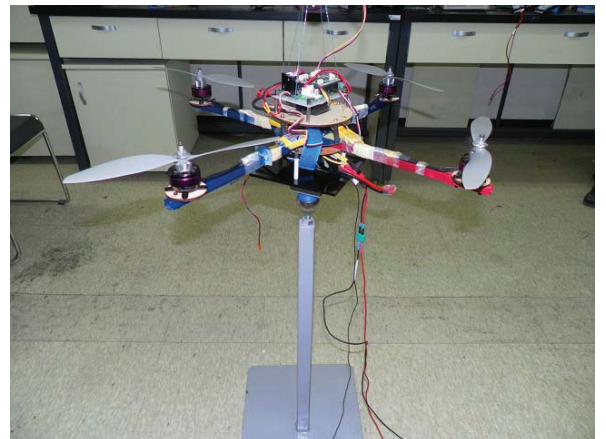


Fig. 11. Experimental test set up

Prior to test the designed controller under a real flight condition, we performed experiment on a test apparatus like Fig. 11. This test apparatus holds the flight with boll permitting 3 axis rotational motion. We adjusted the controller gain parameter considering control energy in

real system. The control is started after the speeds of rotors reach the hovering conditions. The reference attitude is the zero state, i.e., the hovering condition. The results are shown in Fig. 12. The quadrotor can stabilize itself in roll, pitch and remain within 4 degrees. But the response of yaw was slow and also have a little big drift range. This is considered a characteristic of the flight.

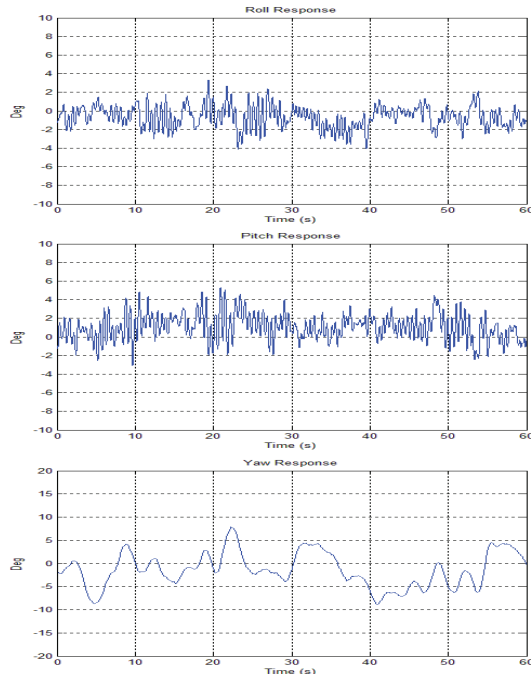


Fig. 12. Attitude Responses in test

## 6. Conclusion

An attitude control system was developed for a quadrotor helicopter. A linear model was extracted from raw flight data. The prediction error minimization method (PEM) algorithm was used to extract the linearized model from the experimental data. Then, an attitude controller with an optimal control scheme was designed and the performance of controller system was validated through simulation and experimental test. The controller was working in principle but need to be improved a little more.

## References

- [1] B. Mettler, M. Tischler, T. Kanade, "System Identification of Small Size Unmanned Helicopter Dynamics," American Helicopter Society 55<sup>th</sup> Forum, Montreal, Quebec, Canada, 1999.
- [2] B. Mettler, "Identification Modelling and Characteristics of Miniature Rotorcraft," Kluwer Academic Publishers, 2003.
- [3] D. H. Shim, H. J. Kim, Sastry, "Control System Design for Rotorcraft-based Unmanned Aerial Vehicles using Time-domain System Identification," IEEE International Conference on Control Application, 2000, pp. 808-813.
- [4] Castillo P., Dzul A., Lozano R., "Real-time stabilization and tracking of a four-rotor mini rotorcraft. Control Systems Technology," IEEE Trans Automat, vol.12, 2004, pp. 510-516.
- [5] V. M. Martinez, "Modelling of the flight dynamics of a quadrotor helicopter," A MSc Thesis in Cranfield University, 2007
- [6] Bouabdallah S., "Design and Control of Quadrotors with Application to Autonomous Flying," A doctoral thesis in EPFL, Swiss, 2007
- [7] A. Budiyo, and S.S. Wibowo, "Optimal Tracking Controller Design for A Small Scale Helicopter," J Bionic Eng., 2007, 04(04), pp. 271-280