Aerodynamic Parameter Estimation of an Unmanned Aerial Vehicle Based on Extended Kalman Filter and Its Higher Order Approach

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***Abstract*—Aerodynamic parameter estimation provides an effective way for aerospace system modelling using measured data from flight test, especially for the purpose of developing elaborate simulation environments and control systems design of Unmanned Aerial Vehicle (UAV) with short design cycles and reduced cost. However, parameter identification of airplane dynamics is complicated because of its nonlinear identification models and the combination of noisy and biased sensor measurements. The combined difficulties mentioned above make the problem of state and parameter estimation a nonlinear filtering problem. Extended Kalman Filter (EKF) is an excellent tool for this matter with the property of recursive parameter identification and excellent filtering. The standard EKF algorithm is based on a first order approximation of system dynamics. More refined linearization techniques such as iterated EKF can be used to reduce the linearization error in the EKF for highly nonlinear systems, which leads to a theoretically better result. In this paper we concentrate on the application and comparison of EKF and iterated EKF for aerodynamic parameter estimation of a fixed wing UAV. The result shows that the two methods have been able to provide accurate estimations.**

Keywords-Unmannd Aerial Vehicle (UAV); Aerodynamic Parameter Estimation; Extended Kalman Filter (EKF)

I. INTRODUCTION

Aircraft system identification makes it possible to develop flight dynamic models by using measured data taken from flight test. Aerodynamic parameter estimation is just one example of how system identification techniques provide the capability to obtain essential information of an aircraft. It is obviously to see that aerodynamic parameter calculation from analysis of real flight data is normally much simpler and even more accurate than some conventional methodology such as CFD (computational fluid dynamics), which is more suitable for Unmanned Aerial Vehicle (UAV) with short design cycles and reduced cost.

We can roughly classify system identification in two groups: offline techniques and online or recursive techniques. The former employs iterated methods which need a complete time history of data, whereas the latter uses just last sample data as it becomes available and does not require availability of the whole time history. For aerodynamic parameter identification, recursive techniques handle flight data as it is measured through onboard sensors and estimate the required aerodynamic derivatives in real time. The measured flight data can contain considerable amount of noise. Furthermore there might be bias and unobserved states in the system model. Kalman filtering first developed in the 1960s is one of these methods [1]. In the following decades, extensive research and application has been done on this topic.

In the aerospace industry, planes are always described by nonlinear models in addition to noisy and biased sensor measurements. Hence, nonlinear filtering technique can be used. As a technique to handle this kind of problem, Extended Kalman Filter is the most popular method in the aerospace industry. So far, the EKF has been employed successfully in various aircraft aerodynamic parameter estimation problems [2]-[4].

EKF employs instantaneous first order linearization at each time step to approximate the nonlinearities of the system, which may lead to linearization error. Several alternative algorithms have been developed to overcome the theoretical disadvantage and have excellent applications such as Unscented Kalman Filter, iterated EKF and second order EKF [5]-[7].

In this paper, we compare two different recursive nonlinear filtering algorithms for aerodynamic parameter estimation from simulated flight data of a real UAV model. They are EKF and iterated EKF which reduces the linearization error in the EKF for highly nonlinear systems. The test aircraft model is a fixed wing UAV produced in University of Southampton, UK.

II. THE UAV MODEL

The system studied is an unmanned aerial vehicle employed by the National Oceanography Centre (NOC), Southampton, UK, for oceanographic parameters measurement. The vehicle has been equipped with onboard flight control system which contains a series of embedded sensors such as gyroscopes, accelerometers and dynamic pressure to provide measurements of angular rates, acceleration, airspeed and other states. The engine used to power the UAV is a 36 cc four-stroke engine. The NOC UAV is shown in Fig. 1.

The Aerosim block set by Unmanned Dynamics [8] associated with Simulink has been used to build the NOC UAV dynamic model and generate simulated flight data for

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EKF processing. The working practice with the model has been running a set of flights and data acquisition from a trim condition, adding elevator commands to study longitudinal dynamics. The output data has been treated using EKF and iterated EKF in Matlab as presented in this paper. Then, the longitudinal aerodynamic parameters and states were possible to estimate.

Figure 1. NOC UAV launched from research ship

III. KALMAN FILTERING

A. *Extended Kalman Filter*

Real engineering systems are governed by continuous time dynamics including aerospace systems whereas the measurements are obtained at discrete instants of time. In this subsection, we will derive the hybrid EKF, which considers system with continuous time dynamics and discrete time measurements. This is the most common situation encountered in practice.

The system dynamics are represented in generic continuous state space form along with the discrete measurement equation [7],

$$\begin{aligned} \dot{x} &= f(x,u,w,t) \\ y_k &= h_k(x_k, v_k) \\ w(t) &\sim (0,Q) \\ v_k &\sim (0,R_k) \end{aligned} \tag{1}$$

where x is the state vector having initial value x_0 at time t_0, u is the input vector, y_k is the measurement vector sampled at time k, f and h_k are the general nonlinear real valued functions. $w(t)$ is the continuous time process noise which is assumed to be zero mean normal distribution with covariance Q, whereas measurement noise v_k is discrete time Gauss white noise with covariance R_k.

The filter has two distinct phases: Prediction and Correction. The prediction phase is also called time-update which uses the state estimate at previous time step to produce an estimate of state at current time step. In the correction phase, measurement at current time step is taken to correct the prediction so that it will be more accurate.

1) Prediction: Initialize the filter as follows,

$$\begin{aligned} \hat{x}_0^+ &= E[x_0] \\ P_0^+ &= E[(x_0 - \hat{x}_0^+)(x_0 - \hat{x}_0^+)^T] \end{aligned} \tag{2}$$

for $k = 1,2,\ldots$, Integrate the state estimate and its covariance from time $(k-1)^+$ to k^- as follows,

$$\begin{aligned} \dot{\hat{x}} &= f(\hat{x},u,0,t) \\ \dot{P} &= FP + PF^T + LQL^T \end{aligned} \tag{3}$$

where F is the partial derivative of f with respect to x, H is the partial derivative of f with respect to w. The "-" superscript denotes that the estimate is a *priori* which doesn't include the current measurement. The "+" superscript means that the estimate is *a posteriori* including the current measurement. We begin this integration process with $\hat{x} = \hat{x}_{k-1}^+$ and $P = P_{k-1}^+$. At the end of this integration we have $\hat{x} = \hat{x}_k^-$ and $P = P_k^-$.

2) Correction: At time k, incorporate the measurement y_k into the state estimate and covariance estimate as follows,

$$\begin{aligned} K_k &= P_k^- H_k^T (H_k P_k^- H_k^T + M_k R_k M_k^T)^{-1} \\ \hat{x}_k^+ &= \hat{x}_k^- + K_k (y_k - h_k(\hat{x}_k^-, 0, t_k)) \\ P_k^+ &= (I - K_k H_k) P_k^- (I - K_k H_k)^T + K_k M_k R_k M_k^T K_k^T \end{aligned} \tag{4}$$

H_k and M_k are the partial derivatives of $h_k(x_k, v_k)$ with respect to x_k and v_k, and are both evaluated at $\hat{x}_k^-$. P_0^+ represents the confidence of the state estimate and should be known a *priori*. It is required to set a large value for P_0^+ if the knowledge of initial value for estimate states is absent. The process noise covariance Q and measurement covariance R should also be specified *a priori.* The tuning of R is easy as it only depends on the sensors used to acquire measures and the information is provided in the component manufacture. However, Q is difficult to determine which needs experimental work. The Runge-Kutta algorithm can be used for the integration in (3).

B. *Iterated Extended Kalman Filter*

Note that the EKF is a first order approximation of nonlinear dynamical system, which ignores higher order effects on the performance of the filter. When there are strong system nonlinearities, EKF may lead to biased estimates.

Various techniques are proposed to address the inaccuracies arising from the first order EKF implementation. Iterated EKF is one of the methods and has shown good estimation error reduction in specific application areas. It can be considered as higher order approach of the standard EKF.

Recall (4), the reason for expanding $h_k(x_k)$ around $\hat{x}_k^-$ is to take into account our best estimate of x_k before the measurement at time k. When we obtain the *a posteriori* estimate $\hat{x}_k^+$, we have a better estimate. So we can use Taylor expansion of $h_k(x_k)$ around our new estimate and use it to recalculate the measurement update equations to get a better a *posteriori* estimate of $\hat{x}_k^+$. This process can be repeated as many times as required.

We use the notation $\hat{x}_{k,i}^+$ to refer to the *a posteriori* estimate of x_k after re-linearizations have been performed. So $\hat{x}_{k,i}^+$ is the a *posteriori* estimate resulting from the standard EKF. Likewise, $P_{k,i}^+$, $K_{k,i}^+$, $H_{k,i}^+$ all have the same meanings. With this notation, we can describe the algorithm for the iterated EKF [7] as follows. It is only different from the standard EKF at the correction step.

1) Perform the measurement update by initializing the iterated EKF estimate to the standard EKF estimate,

$$\begin{aligned} \hat{x}_{k,0}^+ &= \hat{x}_k^- \\ P_{k,0}^+ &= P_k^- \end{aligned} \tag{5}$$

2) For $i = 0,1,...N$, evaluate the following equations (where N is the desired number of measurement-update iterations),

$$\begin{aligned} H_{k,i} &= \frac{\partial h}{\partial x}|_{\hat{x}_{k,i}^+} \\ M_{k,i} &= \frac{\partial h}{\partial v}|_{\hat{x}_{k,i}^+} \\ K_{k,i} &= P_k^- H_{k,i}^T (H_{k,i} P_k^- H_{k,i}^T + M_{k,i} R_k M_{k,i}^T)^{-1} \\ P_{k,i+1}^+ &= (I - K_{k,i} H_{k,i}) P_k^- \\ \hat{x}_{k,i+1}^+ &= \hat{x}_k^- + K_{k,i}\left[y_k - h(\hat{x}_{k,i}^+) - H_{k,i}(\hat{x}_k^- - \hat{x}_{k,i}^+) \right] \end{aligned} \tag{6}$$

3) The final state and covariance estimate are given as follows,

$$\begin{aligned} \hat{x}_k^+ &= \hat{x}_{k,N+1}^+ \\ P_k^+ &= P_{k,N+1}^+ \end{aligned} \tag{7}$$

IV. PARAMETER ESTIMATION

Estimation of parameters through a filtering approach is an indirect procedure, consisting of transforming the parameter estimation problem into a state estimation problem. This is done by augmenting the system state vector by artificially defining the unknown parameters as additional state variables. The system dynamics will have this generic form:

$$\begin{aligned} \dot{x} &= f(x, \beta, u, w, t) \\ y_k &= h_k(x_k, \beta_k, v_k) \end{aligned} \tag{8}$$

where β is the vector of unknown system parameters. Other notations are similar with those mentioned in the above section. Assume that β is a constant parameter vector and its derivative is zero.

In order to obtain the parameter vector β, we first form an augmented state vector x_a:

$$x_a = \begin{bmatrix} x \\ \beta \end{bmatrix} \tag{9}$$

Now the extended system can be written as:

$$\begin{aligned} \dot{x}_a &= f(x_a, u, w, t) = \begin{bmatrix} f(x, \beta, u, w, t) \\ w_\beta \end{bmatrix} \\ y_k &= h_{ak}(x_a, v_k) \end{aligned} \tag{10}$$

where w_β is an artificial small noise which allows the Kalman filter to more readily adjust its estimate of parameters.

Note that $f(x_a, u, w, t)$ is a nonlinear function of the augmented state x_a, we can therefore use EKF or other nonlinear to estimate x_a.

V. STATE SPACE MODEL OF THE UAV

In order to employ EKF and iterated EKF, a state space model of the UAV should be provided for the algorithm so that it can compare the measurements with predicted data to filter them and identify the parameters. With the objective that the lift, drag and pitching moment coefficients be estimated, the following model is used in the stability axes.

$$\begin{aligned} \dot{V} &= -\frac{\overline{q}S}{m} C_D + g\sin(\alpha - \theta) + \frac{Fe}{m}\cos(\alpha + \sigma_T) \\ \dot{\alpha} &= -\frac{\overline{q}S}{mV} C_L + q + \frac{g}{V}\cos(\alpha - V) - \frac{Fe}{mV}\sin(\alpha + \sigma_T) \\ \dot{\theta} &= q \\ \dot{q} &= \frac{\overline{q}Sc}{I_y} C_m + \frac{Fe}{I_y}(l_{tx}\sin\sigma_T + l_{tz}\cos\sigma_T) \end{aligned} \tag{11}$$

The aerodynamic coefficients are described as:

$$C_D = C_{D0} + C_{D\alpha}\alpha + C_{D\delta_e}\delta_e$$

$$C_L = C_{L0} + C_{L\alpha}\alpha + C_{Lq}\frac{q\bar{c}}{2V_0} + C_{L\delta_e}\delta_e \tag{12}$$

$$C_m = C_{m0} + C_{m\alpha}\alpha + C_{mq}\frac{q\bar{c}}{2V_0} + C_{m\delta_e}\delta_e$$

The observation equations are the following:

$$V_m = V; \alpha_m = \alpha; \theta_m = \theta; q_m = q$$

$$\dot{q}_m = \frac{\bar{q}S\bar{c}}{I_y}C_m + \frac{Fe}{I_y}(l_{tx}\sin\sigma_T + l_{tz}\cos\sigma_T)$$

$$a_{xm} = \frac{\bar{q}S}{m}C_X + \frac{Fe}{m}\cos\sigma_T \tag{13}$$

$$a_{zm} = \frac{\bar{q}S}{m}C_Z - \frac{Fe}{m}\sin\sigma_T$$

where the longitudinal and vertical force coefficients C_X and C_Z are given by:

$$C_X = C_L\sin\alpha - C_D\cos\alpha$$
$$C_Z = -C_L\sin\alpha - C_D\cos\alpha \tag{14}$$

In (10), V is airspeed, α the angle of attack, θ the pitch angle, q the pitch rate, δ_e the elevator deflection, F_e the thrust, σ_T the inclination angle of the engine, $\bar{q}$ the dynamic pressure, m the mass, S the wing area, $\bar{c}$ the wing chord, I_y the moment of inertia.

VI. APPLICATION OF THE FILTER

For this parameter identification problem, the unknown parameter vector β becomes:

$$\beta = [C_{D0} \quad C_{D\alpha} \quad C_{D\delta e} \quad C_{L0} \quad C_{L\alpha} \quad C_{Lq} \quad C_{L\delta e} \quad C_{m0} \quad C_{m\alpha} \quad C_{mq} \quad C_{m\delta e}] \tag{15}$$

It is important to note that augmenting system states with defining the unknown parameters as additional state variables will render the filtering problem effectively nonlinear.

The longitudinal motion is excited through two 3-2-1-1 multi-step elevator inputs leading to short period motion and a slight "pull up" and a "push down" pulse inputs resulting phugoid motion shown in Fig. 2. The rest of control commands have been left constant.

Measured data and filtered measurements are shown in Fig. 3, from which we can see that the filter can organize the noisy data flow coming from the sensors efficiently. The filtered results by iterated EKF and EKF are nearly coincident.

Next, the result of aerodynamic parameter identification convergence is displayed. All the initial values of the aerodynamic parameters are assumed to be unknown a *priori* and set to zero. The iteration number N is chosen to be 7 in iterated EKF. It is clearly shown in Fig. 4 that iterated EKF has a better performance than standard EKF. The iterated EKF method is faster than EKF in terms of time to convergence with less overshoot. However, all of the estimated parameters achieve to stable values in close vicinity of one another within 10 seconds for both of the two different algorithms. The estimated values of the parameters are show in TABLE 1.

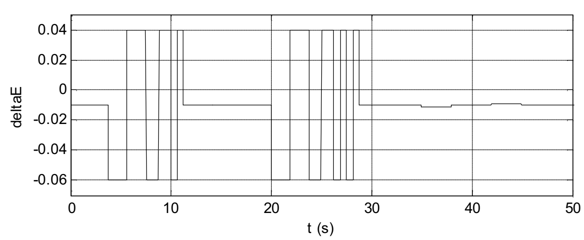

Figure 2. Elevator deflection

VII. CONCLUSION

In this paper, two recursive parameter identification algorithms are used and compared for estimating the aerodynamic parameters of an NOC UAV from the point of view of accuracy, time-consumption and algorithm complexity. They are iterated EKF and standard EKF. The result indicates that both of the two methods provide excellent performances during the identification process. As it can be seen, filtering measurements makes it possible to reconstruct the real maneuver correctly. Furthermore, all the parameters reach their stable values in less than 10 seconds and are available for elaborate simulation and control system design. These two approaches prove themselves to be powerful tools in real-time data analysis.

However, as a higher order modification of EKF which involves complex calculations, iterated EKF achieves faster convergence, higher accuracy and less oscillation than EKF. The price to be paid for the high performance of the iterated EKF is an increased level of computational effort. These trade-offs are problem dependent and must be investigated on an individual basis.

Future work will concern on the estimation of whole aerodynamic parameters of the NOC UAV, which involves a 6 degree of freedom dynamics. Moreover, other approach such as unscented Kalman filter is considered to be applied and compared alternatively.

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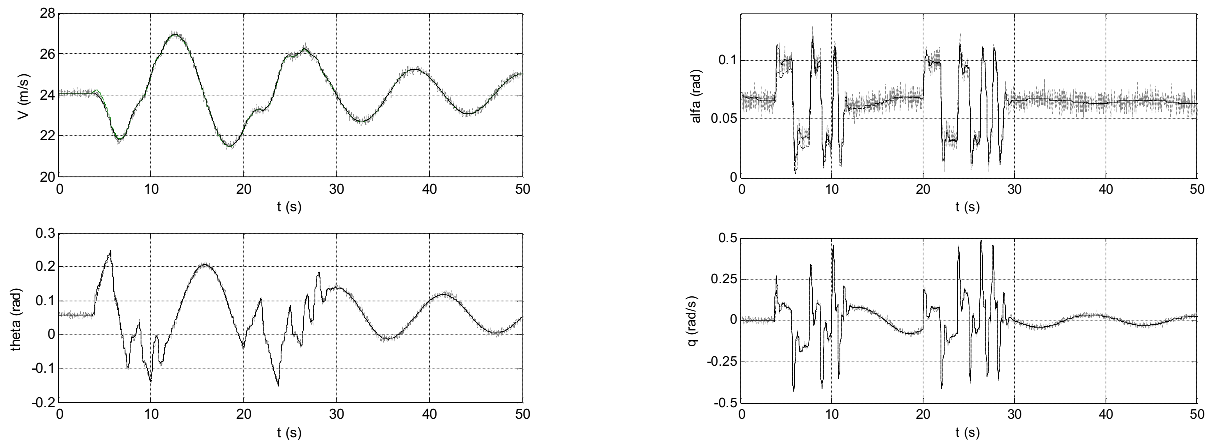
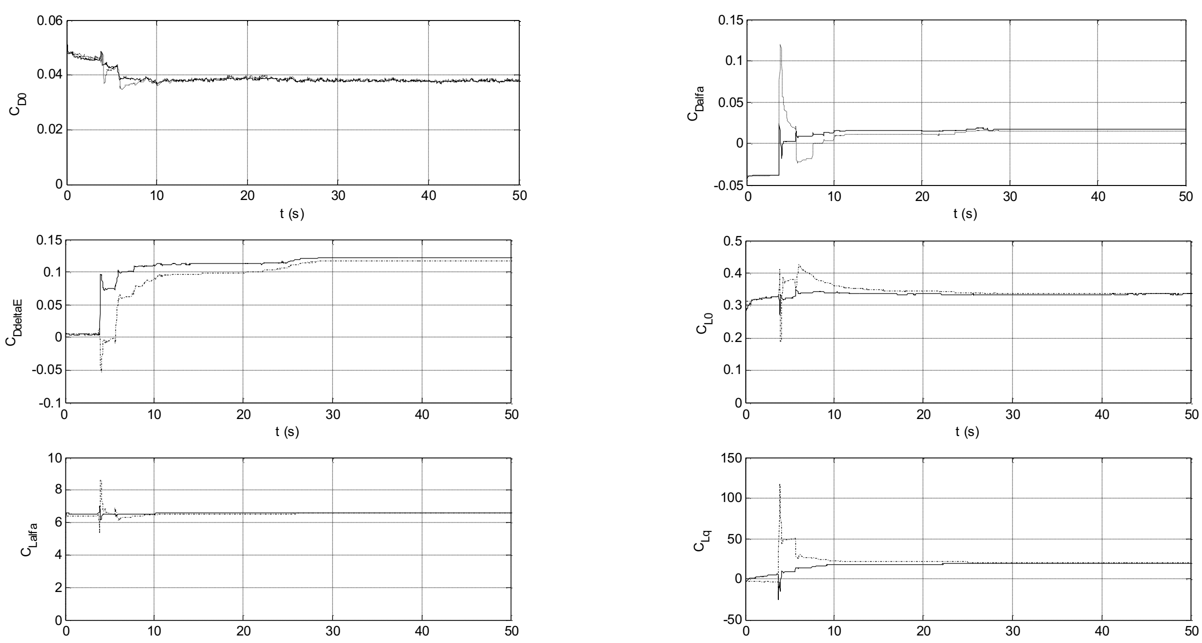


Figure 3. Noisy and filtered measurements (Noisy measurement is plotted in grey, filtered result by iterated EKF is in black line and EKF result is in dashed)



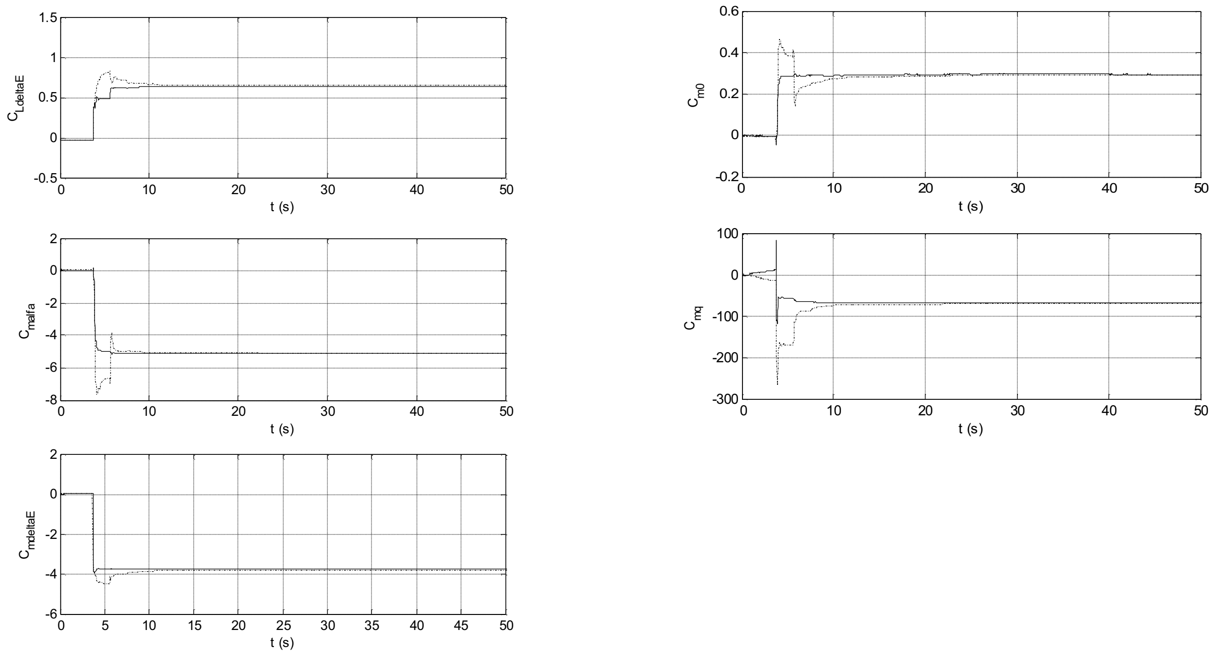


Figure 4. Parameter convergence (iterated EFK with line and EKF with dashed)

TABLE I. COMPARISON OF PARAMETER ESTIMATES WITH STANDARD DEVIATION VALUES IN PARENTHESIS

Parameter	**Recursive Algorithm**	
	Iterated EKF	***EKF***
C_{D0}	0.0377 (2.88E-4)	0.0380 (2.91E-4)
$C_{D\alpha}$	0.0168 (1.07E-5)	0.0153 (1.09E-5)
$C_{D\delta e}$	0.1224 (3.41E-5)	0.1181 (3.00E-5)
C_{L0}	0.3345 (5.48E-4)	0.3364 (5.56E-4)
$C_{L\alpha}$	6.5858 (5.40E-5)	6.5597 (3.28E-5)
C_{Lq}	19.0550 (4.63E-4)	20.4105 (4.13E-4)
$C_{L\delta e}$	0.6352 (1.03E-5)	0.6419 (8.38E-6)
C_{m0}	0.2936 (4.17E-4)	0.2921 (4.28E-4)
$C_{m\alpha}$	-5.1362 (7.62E-5)	-5.1164 (2.96E-5)
$C_{m\alpha}$	-68.4364 (2.07E-4)	-70.0106 (1.59E-4)
$C_{m\delta e}$	-3.7891 (1.41E-5)	-3.8041 (1.42E-5)