N4SID and MOESP Subspace Identification Methods

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Abstract— **Multivariable Output Error State Space (MOESP) and Numerical algorithms for Subspace State Space System Identification (N4SID) algorithms are two well known subspace identification techniques discussed in this paper. Due to the use of robust numerical tools such as QR decomposition and singular value decomposition (SVD), these identification techniques are often implemented for multivariable systems. Subspace identification algorithms are attractive since the state space form is highly suitable to estimate, predict, filters as well as for control design. In literature, there are several simulation studies for MOESP and N4SID algorithms performed in offline and online mode. In this paper, order selection, validity and the stability for both algorithms for model identification of a glass tube manufacturing process system is considered. The weighting factor α, used in online identification is obtained from trial and error and particle swarm optimization (PSO). Utilizing PSO, the value of α is determined in the online identification and a more accurate result with lower computation time is obtained.**

***Keywords-** subspace identification; singular value decomposition; Hankel matrices; QR decomposition; MOESP; N4SID*

I. INTRODUCTION

Subspace identification methods (SIMs) have enjoyed a great development in both theory and practice since they can identify system matrices of the state space model directly from the input and output data. Subspace methods are based on robust numerical tools such as QR factorization and singular value decomposition (SVD) which makes them attractive from the numerical point of view. The most influential methods are Canonical Variate Analysis (CVA) proposed by [1], Multivariable Output Error State Space (MOESP) by [2] and Numerical Subspace State Space System Identification (N4SID) by [3]. Past research has revealed that the three algorithms use exactly the same subspace to determine the order and the extended observability matrix, but that the weighting matrix, used to calculate a basis for the column space of the observability matrix is different for those cases.

Subspace identification methods for linear time invariant systems can be classified into two groups. The first consists of methods that aim to recover the column space of the extended observability matrix and use the shift invariant structure of this matrix to estimate *A* and *C* matrices, subsequently the *B* and *D* matrices. The so-called MOESP methods [2], [4–6] are considered in the first group. The second group consists of methods that aim at approximating the state sequence of the system and use the approximated state in a second step to estimate the system matrices. The methods that constitute the second group are the N4SID methods [3], [7] as well as CVA methods [1]. In both groups, before identifying system matrices, the calculation of state vector sequences make use of measured input-output data only. In the identification procedure, the main step is to compute the singular value decomposition (SVD) of a block Hankel matrix, *H* constructed with input-output data. The computation of SVD can be done offline or online. The offline can be easily converted to an adaptive online version for a slow time varying system such as process control system.

Subspace methods have been developed for certain classes of nonlinear system such as Hammerstein [8–10], Wiener [11–14] and bilinear [15–17]. In this paper, model identification is implemented to show the effectiveness of SVD computation. The N4SID and MOESP subspace algorithms are used to identify state space models of a glass tube manufacturing process system.

II. PROBLEM FORMULATION

Consider the general linear discrete time invariant state space model:

$$x_{k+1} = A_k x_k + B_k u_k + w_k \tag{1}$$
$$y_k = C_k x_k + D_k u_k + v_k \tag{2}$$

where $x_k \in \Re^n$, $y_k \in \Re^{n_y}$ and $u_k \in \Re^{n_u}$ are the system state, output and input respectively. While $w_k \in \Re^n$ and $v_k \in \Re^{n_y}$ are additional unknown noise sequences. They will be recognized as the residuals of the set of equations for determining the system matrices and will be omitted for a while. Thus, consider the following linear discrete time invariant system;

$$x_{k+1} = A_k x_k + B_k u_k \tag{3}$$
$$y_k = C_k x_k + D_k u_k \tag{4}$$

The goal is to estimate systems matrices A, B, C and D with appropriate dimensions. One important equation in the derivation of subspace state space system identification algorithms is the data equation relating (block) Hankel matrices constructed from the input-output data samples. The output block Hankel matrices are defined as:

$$Y=\left[\begin{array}{ccccc} y_1 & y_2 & y_3 & \cdots & y_j \\ y_2 & y_3 & y_4 & \cdots & y_{j+1} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ y_i & y_{i+1} & y_{i+2} & \cdots & y_{i+j-1} \\ \hline y_{i+1} & y_{i+2} & y_{i+3} & \cdots & y_{i+j} \\ y_{i+2} & y_{i+3} & y_{i+4} & \cdots & y_{i+j+1} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ y_{2i} & y_{2i+1} & y_{2i+2} & \cdots & y_{2i+j-1} \end{array}\right]=\left[\begin{array}{c} Y_p \\ \hline Y_f \end{array}\right]. \tag{5}$$

where the subscript 'p' and 'f' stand for 'past' and 'future' and i block rows and j columns. The input block Hankel matrices U_p and U_f can also be defined in the same way. The matrix input-output plays an important role in the problem treated in linear subspace identification and it can be obtained by recursive substitution of (6) and (7).

$$Y_p = \Gamma_i X + H_t U_p \tag{6}$$
$$Y_f = \Gamma_i X + H_t U_f \tag{7}$$

The extended observability matrix is defined as

$$\Gamma = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{i-1} \end{bmatrix}. \tag{8}$$

The sequences of state vector, X as follows:

$$X = \begin{pmatrix} x_k & x_{k+1} & x_{k+2} & \ldots & x_{k+j-1} \end{pmatrix}. \tag{9}$$

The lower block Toeplitz matrix, H, is defined as:

$$H = \begin{bmatrix} D & 0 & \cdots & 0 \\ CB & D & \cdots & 0 \\ CAB & CB & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ CA^{i-2}B & CA^{i-3}B & \cdots & D \end{bmatrix}. \tag{10}$$

The block Toeplitz matrix, H is the concatenation of H_p and H_f, defined by 11 and 12.

$$H_p = \begin{bmatrix} u_k & u_{k+1} & \cdots & u_{k+j-1} \\ y_k & y_{k+1} & \cdots & y_{k+j-1} \\ u_{k+1} & u_{k+2} & \cdots & u_{k+j} \\ y_{k+1} & y_{k+2} & \cdots & y_{k+j} \\ \vdots & \vdots & \ddots & \vdots \\ u_{k+i-1} & u_{k+i} & \cdots & u_{k+j+i-2} \\ y_{k+i-1} & y_{k+i} & \cdots & y_{k+j+i-2} \end{bmatrix}. \tag{11}$$

$$H_f = \begin{bmatrix} u_{k+i} & u_{k+i+1} & \cdots & u_{k+j-1} \\ y_{k+i} & y_{k+i+1} & \cdots & y_{k+j-1} \\ u_{k+i+1} & u_{k+i+2} & \cdots & u_{k+j} \\ y_{k+i+1} & y_{k+i+2} & \cdots & y_{k+j} \\ \vdots & \vdots & \ddots & \vdots \\ u_{k+2i-1} & u_{k+2i} & \cdots & u_{k+j+i-2} \\ y_{k+2i-1} & y_{k+2i} & \cdots & y_{k+j+i-2} \end{bmatrix}. \tag{12}$$

A typical state space subspace system identification algorithm involves two steps:

1) Identification of the extended observability matrix and a block triangular Toeplitz matrix.
2) Estimation of the system matrices A, B, C and D from the identified observability matrix and Toeplitz matrix.

N4SID projects Y_f onto $[Y_p;U_p;U_f]$ and does a singular value decomposition (SVD) on the part corresponding to the past data, the right singular vectors are estimated as state variables and fit to the state space model. Meanwhile, the original MOESP does a QR decomposition on $[U_f : Y_f]$ and then a SVD on part of the R matrix. Part of the singular matrix is taken as Γ_f, based on which A and C matrices are estimated, while B and D are estimated based on least square fitting. As mentioned before, subspace methods are based on robust numerical tools such as QR factorization and singular value decomposition (SVD). However, they are inappropriate for online identification because of the computational complexity and storage costs. Thus, an online subspace identification which consist of recursive algorithm is developed. A significant contribution is the ability of the algorithm to update the estimation for the extended observability matrix online when new data arrives.

A. Offline subspace identification method [18]

Step 1: Calculate U and S in the SVD of H as shown in equations (8) and (9). Singular value decomposition produces a diagonal matrix S, of the same dimension as H and with nonnegative diagonal elements in decreasing order.

$$H = U \cdot S \cdot V^T = \begin{bmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{bmatrix} \cdot \begin{bmatrix} S_{11} & 0 \\ 0 & 0 \end{bmatrix} \cdot V^T$$

Step 2: Calculate the SVD of $U_{12}^T \cdot U_{11} \cdot S_{11}$

$$U_{12}^T \cdot U_{11} \cdot S_{11} = \begin{bmatrix} U_q & U_q^{\perp} \end{bmatrix} \begin{bmatrix} S_q & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_q^T \\ (V_q^{\perp})^T \end{bmatrix}$$

Step 3: Solve the following set of linear equation to get A, B, C and D matrices

$$\begin{bmatrix} U_q^t \cdot U_{12}^t \cdot U(m+l+1:(i+1)(m+l),:) \cdot S \\ U(mi+li+m+1:(m+l)(i+1),:) \cdot S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} U_q^t \cdot U_{12}^t \cdot U(1:mi+li,:) \cdot S \\ U(mi+li+1:mi+li+m,:) \cdot S \end{bmatrix}$$

B. Online Subspace Identification

Step 1: Construct new column *column* to be added to H, using the $2i$ latest I/O-measurements.

Step 2: Calculate SVD

$$U_k \cdot S_k \cdot V_k^T = \begin{bmatrix} \alpha \cdot U_{k-1} \cdot S_{k-1} & column \end{bmatrix}$$

where α is a weighting factor and partition

$$U_k \cdot S_k = \begin{bmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{bmatrix} \cdot \begin{bmatrix} S_{11} & 0 \\ 0 & 0 \end{bmatrix}$$

Step 3 and Step 4: Refer to an off-line algorithm step 2 and 3.

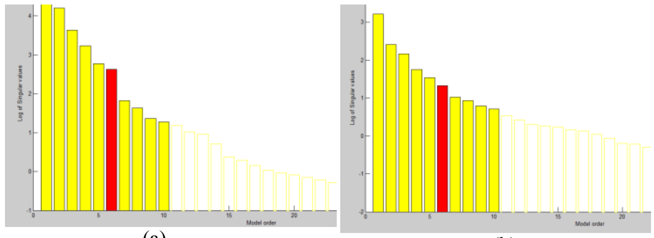
C. Particle Swarm Optimization

Particle swarm optimization (PSO) is a population-based heuristic optimization method inspired by animal behaviour such as bird flocking. The population is made up of potential solutions to the optimization problem referred as particles and the collection of particles is called swarm. The particles are initialized randomly which searches for best position in N-dimensional problem space. Each particle updates its velocity and position during searching based on its best experience and that of the whole population. This updating principle makes the swarm move closer to the location of best fitness value and eventually all the particles will converge to that point. In this paper, PSO is used in order to obtain the value of α.

III. Simulation Results

A. Offline subspace identification

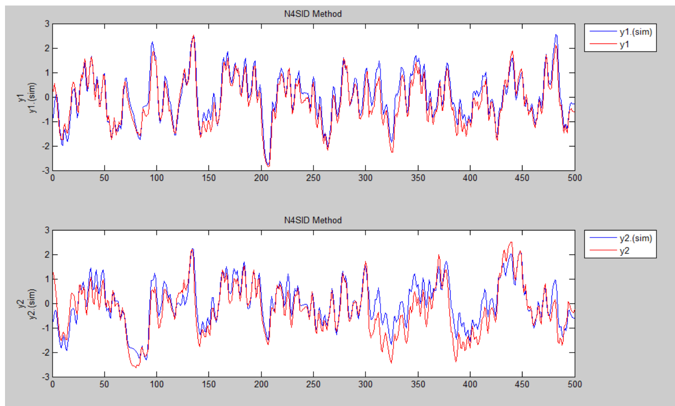
There are several ways to estimate the order of the system. In this project, singular value decomposition (SVD) is applied. Figure 1(a) and (b) show the SVD based on N4SID and MOESP method respectively. The best order for both N4SID and MOESP are six.



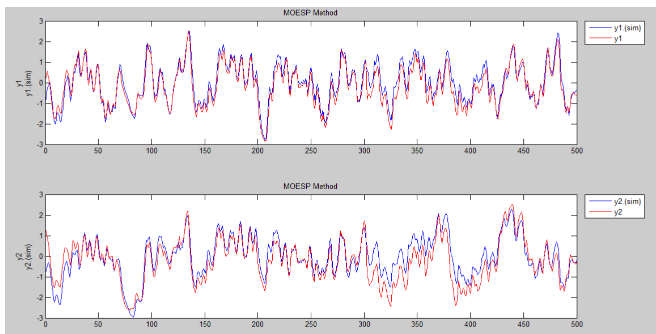
(a) (b)

Figure 1. (a) Singular Value Decomposition (SVD) based on N4SID (b) Singular Value Decomposition (SVD) based on MOESP

Figure 2 (a) and (b) show the validation based on N4SID and MOESP. In this paper, 3000 data are used for identification and 500 data are used for validation.



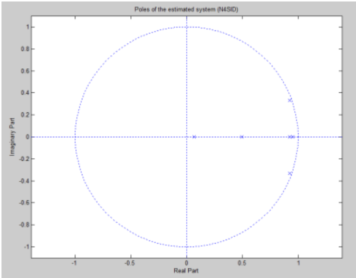
(a)



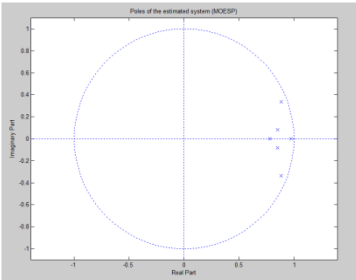
(b)

Figure 2. (a) Validation based on N4SID (b) Validation based on MOESP

The best fitness for output 1 and 2 are 96.71% and 84.79% respectively based on N4SID. Meanwhile, for MOESP method, the best fitness for output 1 and output 2 are 93.07% and 80.23% respectively. Figure 3 (a) and (b) show poles of the system based on N4SID and MOESP respectively. The stability is determined by checking the poles of the system or eigenvalues of the A matrices. The eigenvalues must be inside a unit circle. All the eigenvalues are in the unit circle, hence the systems for both methods are stable.



(a)

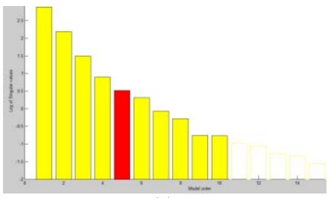


(b)

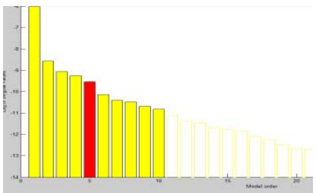
Figure 3. (a) Poles of the estimated system based on N4SID (b) Poles of the estimated system based on MOESP

B. *Online Subspace Identification Using Weighting Factor, α=1.*

Figure 4 (a) and (b) show the SVD based on N4SID and MOESP respectively. The result is taken when data sequence (k) is equal to 60. The best order for both methods is five.

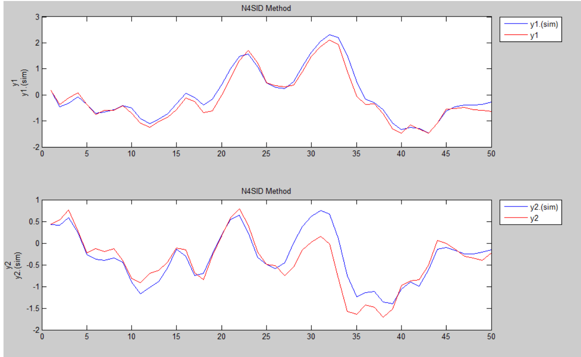


(a)

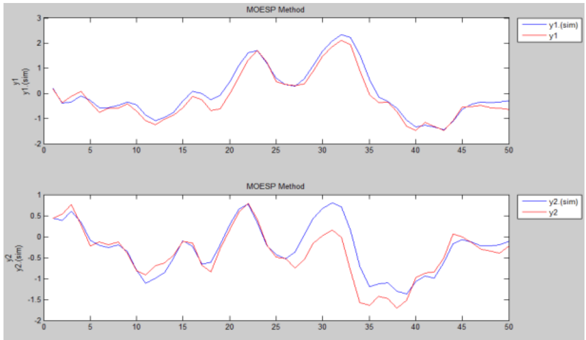


(b)

Figure 4. (a) Singular Value Decomposition (SVD) based on N4SID (b) SVD based on MOESP



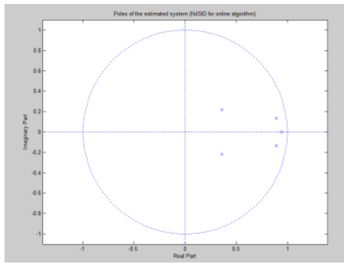
(a)



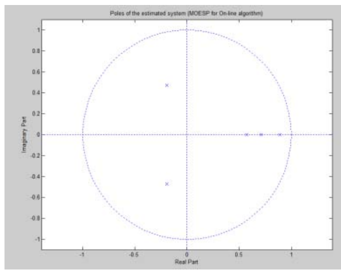
(b)

Figure 5. (a) Validation based on N4SID (b) Validation based on MOESP

From Figure 5(a), the best fitness for output 1 and output 2 are 82.79% and 72.66% respectively. Meanwhile, Figure 5 (b) shows that the best fitness for output 1 is 79.02% and for output 2 is 67.04%. The stability of the system is tested by determine the poles or eigenvalues. Since all the eigenvalues are inside the unit circle, the systems are stable for both methods.



(a)

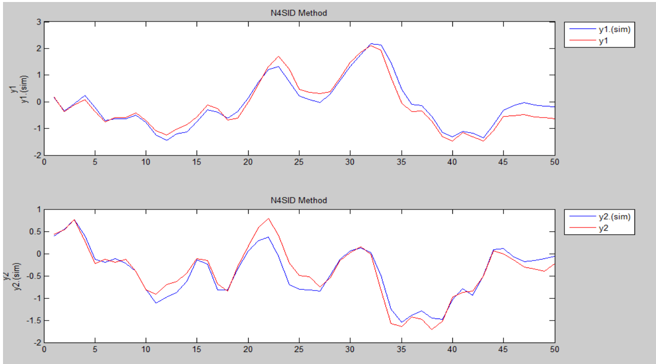


(b)

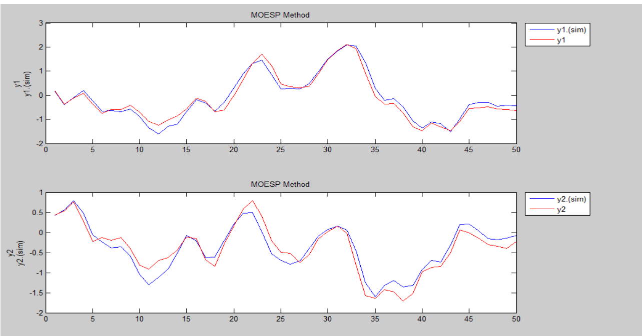
Figure 6. (a) Poles of the estimated system based on N4SID (b) Poles of the estimated system based on MOESP

C. Online Subspace Identification Using PSO

The value of weighting factor, α is chosen based on the value of global best or also known as *gbest.* The final value of α is chosen with the highest value of *gbest* PSO. 500 particles and 150 iterations are used for PSO part. The highest *gbest* value for N4SID method is 134.8900 which gives α=0.4859 and for MOESP method the highest *gbest* is 141.5678 which gives α=0.3123.



(a)



(b)

Figure 7. (a) Validation based on N4SID (b) Validation based on MOESP

Figure 7 (a) and (b) show the validation based on N4SID and MOESP. The best fitness for output 1 and output 2 with N4SID method are 74.42% and 89.29% respectively. Meanwhile, by MOESP method, the best fitness for output 1 and 2 are 79.26% and 87.63%.

IV. Conclusions

In this paper, an offline and online subspace identification models of a glass tube manufacturing process system are obtained based on two subspace algorithms which are N4SID and MOESP. For offline identification, the order of the system is fixed and for the online identification, it can track every change of order and system matrices in the system. The weighting factor, α, in online identification can be determined by using particle swarm optimization. In term of stability, since both results are within a unit circle, yields a stable system.

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