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Closed-Loop Subspace Identification of a Quadrotor

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by

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Abstract of the Thesis

Closed-Loop Subspace Identification of a Quadrotor

by

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Master of Science in Engineering

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Professor Steve Gibson, Chair

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The thesis of Andrew G. Kee is approved.

Steve Gibson, Committee Chair

University of California, Los Angeles

2013

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Nomenclature

N	Number of block rows
s	Number of block columns
$\mathbb{R}$	Set of all real numbers
u	System input sequence
y	System output sequence
x	System state sequence

CHAPTER 1

Introduction

Unmanned Aerial Vehicles (UAVs) have seen explosive growth in the past thirty years, performing a multitude of military and civilian tasks including surveillance, reconnaissance, armed combat operations, search and rescue, forest fire management, and domestic policing [9, 10]. A class of modern UAVs which have recently grown in popularity are quadrotors - Vertical Take Off and Landing (VTOL) vehicles powered by four rotors arranged in a cross configuration. The main advantage of the quadrotor lies in its mechanical simplicity. Adjusting the speed of one or more of the vehicle's fixed-pitch rotors provides full attitude control, eliminating the need for the swash plate mechanism found on single rotor helicopters [1, 2]. In spite of its mechanical simplicity, the quadrotor exhibits somewhat complex dynamics that are best modeled as a Multi-Input Multi-Output (MIMO) system.

Advances in MEMS sensors and light-weight high-powered lithium polymer batteries have contributed to the recent popularity of quadrotors, making them an attractive choice for research applications in flight dynamics and control, as in [3, 5, 6, 7]. One problem of particular interest is the development of mathematical models representing system dynamics based on experimentally gathered data. System identification provides a mechanism to relate this input-output data to the underlying system dynamics. Traditionally, system identification techniques have focused on developing a system model which minimizes prediction error. Identification methods of this form are commonly known as Prediction Error Methods (PEMs). PEMs have seen widespread use in both theoretical and real-world applications, but experience difficulties with MIMO systems as noted in [8, 11]. Subspace identification methods have recently grown in popularity and offer an alternative approach to the identification problem. These methods have a foundation in linear algebra and overcome the issues found in PEMs when identifying MIMO systems [4]. It is the goal of this research project to apply subspace identification techniques to a quadrotor using experimentally gathered closed-loop input and output data.

1.1 Related Work

For text, let's use the first words out of the ispell dictionary.

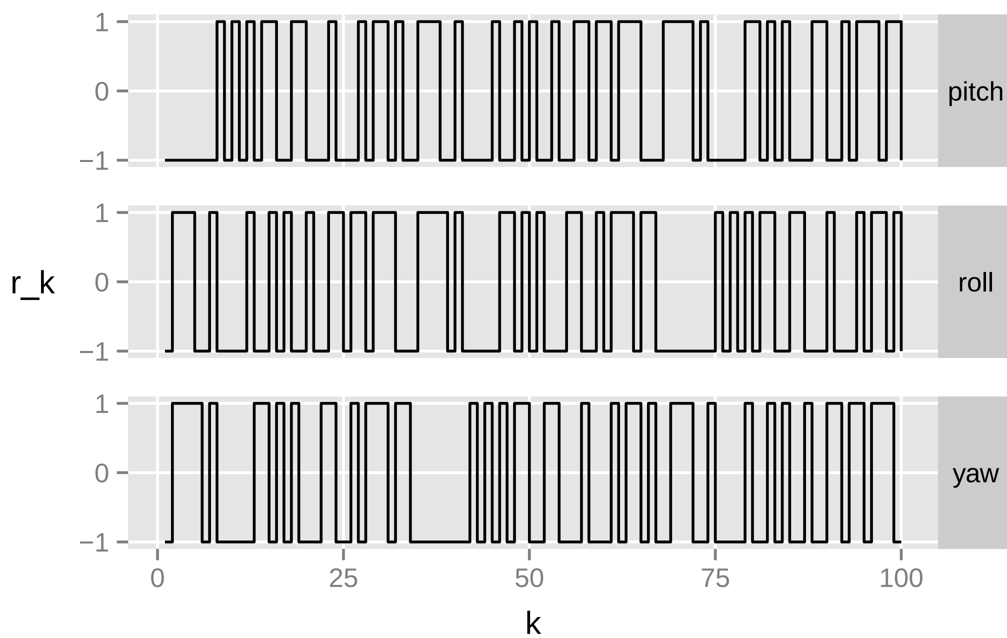

Figure 1.1: A figure caption

1.2 Motivation and Contributions

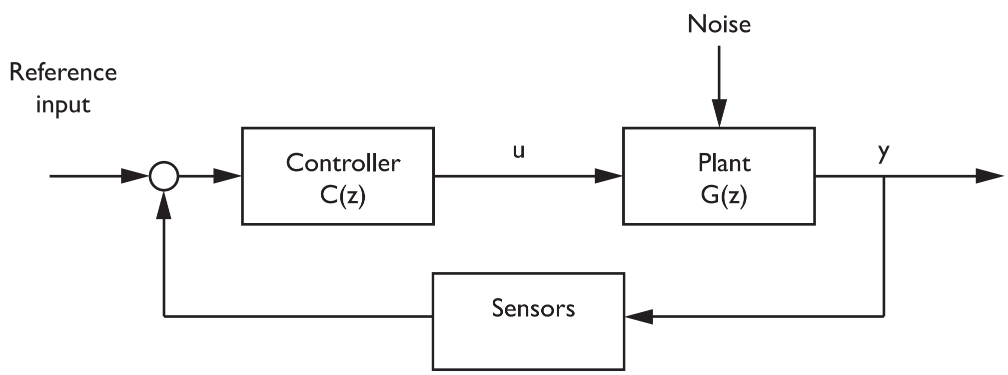

Figure 1.2: A figure caption

CHAPTER 2

Model and Assumptions

Introduce concept of LTI system here

2.1 State Space Model

We will consider a combined deterministic-stochastic LTI system written in innovation form as

$$x(k+1) = Ax(k) + Bu(k) + Ke(k) \tag{2.1a}$$

$$y(k) = Cx(k) + Du(k) + e(k) \tag{2.1b}$$

where $x_k \in \mathbb{R}^n$ is the system state, $u_k \in \mathbb{R}^m$ is the system input, $y_k \in \mathbb{R}^l$ is the system output, and $e_k \in \mathbb{R}^l$ is the innovation. A, B, C, and D are the system matrices with appropriate dimensions and K is the Kalman filter gain. The system represented in (2.1) can also be represented in predictor form as

$$x(k+1) = A_K x(k) + B_K u(k) + Ky(k) \tag{2.2a}$$

$$y(k) = Cx(k) + Du(k) + e(k) \tag{2.2b}$$

where $A_K = A - KC$ and $B_K = B - KD$.

The systems represented by (2.1) and (2.2) are equivalent from an input/output point of view, but because A_K is guaranteed stable even if the original process matrix A is unstable, the predictor form proves advantageous when considering unstable open-loop systems. We will use the state space model in innovation form to derive the general subspace algorithm for identifying combined deterministic-stochastic LTI systems but will rely on the prediction form of the model when considering identification of closed-loop systems.

2.2 Assumptions

[**Assumption 1**]: $A_K = A - KC$ is stable (i.e. its eigenvalues lie within the unit circle)

[**Assumption 2**]: The system is represented in its minimal form

CHAPTER 3

Subspace Identification Methods

Subspace identification methods (SIM) provide an approach to identifing an unknown LTI system in its state space form using input/output data. Subspace methods rely on linear algebra techniques to extract the system matrices from the column space of the system's extended observability matrix. Considering the combined deterministic-stochastic LTI system introduced in the previous chapter, the subspace identification problem is: given a set of input and output data, estimate the system matrices A, B, C, and D and the Kalman filter gain K up to within a similarity transform.

!!!!!Give basic overview of the two steps here!

3.1 Open-Loop Subspace Identification

Considering the combined deterministic-stochastic LTI system in innovation form

$$x(k+1) = Ax(k) + Bu(k) + Ke(k) \tag{3.1a}$$

$$y(k) = Cx(k) + Du(k) + e(k) \tag{3.1b}$$

we wish to estimate A, B, C, D, and K to within a similarity transform using system input/output data. Based on the state space model in (3.1), an extended state space model can be formulated as

$$Y_f = \Gamma_f X_k + H_f U_f + G_f E_f \tag{3.2}$$

where the subscript f denotes the future horizon. The extended observability matrix is

$$\Gamma_f = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{f-1} \end{bmatrix} \tag{3.3}$$

and H_f and G_f are Toeplitz matrices of the Markov parameters of the deterministic and stochastic subsystems, respectively

$$H_f = \begin{bmatrix} D & 0 & 0 & \cdots & 0 \\ CB & D & 0 & \cdots & 0 \\ CAB & CB & D & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ CA^{f-2}B & CA^{f-3}B & CA^{f-4}B & \cdots & D \end{bmatrix} \tag{3.4a}$$

$$G_f = \begin{bmatrix} I & 0 & 0 & \cdots & 0 \\ CK & I & 0 & \cdots & 0 \\ CAK & CK & I & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ CA^{f-2}K & CA^{f-3}K & CA^{f-4}K & \cdots & I \end{bmatrix} \tag{3.4b}$$

We arrange the input data in the following Hankel form:

$$U_f = \begin{bmatrix} u(k) & u(k+1) & \cdots & u(k+N-1) \\ u(k+1) & u(k+2) & \cdots & u(k+N) \\ \vdots & \vdots & \ddots & \vdots \\ u(k+f-1) & u(k+f) & \cdots & u(k+f+N-2) \end{bmatrix} \tag{3.5}$$

We similarly arrange the output data Y_f and noise E_f according to the same Hankel form.

!!!!! Need to discuss past and future splitting !!!!!

$$Z_p = \begin{bmatrix} U_p \\ Y_p \end{bmatrix}$$

3.1.1 Estimation of the Extended Observability Matrix

Estimate Γ_f from (3.2) via MOESP: linear regression followed by an SVD to estimate EOM.

First, eliminate U_f by post-multiplying (3.2) by $\Pi_{U_f}^{\perp}$ (eliminate the influence of the input) giving

$$Y_f \Pi_{U_f}^{\perp} = \Gamma_f X_k \Pi_{U_f}^{\perp} + G_f E_f \Pi_{U_f}^{\perp} \tag{3.6}$$

Recalling E_f is uncorrelated with U_f,

$$Y_f \Pi_{U_f}^{\perp} = \Gamma_f X_k \Pi_{U_f}^{\perp} + G_f E_f \tag{3.7}$$

Next we eliminate the influence of the noise term E_f. From Kalman filter theory, E_f is uncorrelated with Z_p (cite Qin overview paper on this one):

$$\lim_{N\to\infty} \frac{1}{N} E_f Z_p^T = 0$$

Thus multiplying (3.7) on the right by Z_p gives

$$Y_f \Pi_{U_f}^{\perp} Z_p = \Gamma_f X_k \Pi_{U_f}^{\perp} Z_p \tag{3.8}$$

3.2 Closed-Loop Subspace Identification

Under open-loop conditions, E_f is uncorrelated with U_f. That is,

$$\lim_{N\to\infty} \frac{1}{N} E_f U_f^T = 0$$

or

$$E_f \Pi_{U_f}^{\perp} = E_f (I - U_f^T (U_f U_f^T)^{-1} U_f) = E_f$$

3.2.1 Identifying Systems Operating Under Feedback Control

3.2.2 Innovation Estimation Method

3.2.3 Whitening Filter

CHAPTER 4

Experiments

4.1 Experiment Design

CHAPTER 5

Results

CHAPTER 6

Conclusion

6.1 Future Work

6.2 Linear Algebra Tools

Hankel Matrices

A Hankel matrix is a matrix $H \in \mathbb{R}^{m \times n}$ with constant skew-diagonals. In other words, the value of the $(i, j)^{\text{th}}$ entry of H depends only on the sum $i + j$.

$$H_{m,n} = \begin{bmatrix} h_1 & h_2 & \cdots & h_n \\ h_2 & h_3 & \cdots & h_{n+1} \\ \vdots & \vdots & \ddots & \vdots \\ h_m & h_{m+1} & \cdots & h_{m+n-1} \end{bmatrix}$$

If each entry in the matrix is also a matrix, it is called a block Hankel matrix.

Fundamental Matrix Subspaces

We require two of the fundamental matrix subspaces: the column space and the row space. The column space of a matrix $A \in \mathbb{R}^{m \times n}$ is the set of all linear combinations of the column vectors of A. The dimension of the column space is called the rank. The row space of a matrix $A \in \mathbb{R}^{m \times n}$ is the set of all linear combinations of the row vectors of A.

Projections

Singular Value Decomposition

Any matrix $A \in \mathbb{R}^{m \times n}$ can be decomposed by a singular value decomposition (SVD) given by

$$A = U \Sigma V^T$$

where $U \in \mathbb{R}^{m\times m}$ and $V \in \mathbb{R}^{n\times n}$ are orthogonal matrices and $\Sigma \in \mathbb{R}^{m\times n}$ is diagonal matrix of the singular values of A ordered such that

$$\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_k > 0$$

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