Sensor Data Fusion Using Kalman Filter

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Abstract - *Autonomous Robots and Vehicles need accurate positioning and localization for their guidance, navigation and control. Often, two or more different sensors are used to obtain reliable data useful for control system. This paper presents the data fusion system for mobile robot navigation. Odometry and sonar signals are fused using Extended Kalman Filter (EKF) and Adaptive Fuzzy Logic System (AFLS). The signals used during navigation cannot be always considered as white noise signals. On the other hand, colored signals will cause the EKF to diverge. The AFLS was used to adapt the gain and therefore prevent the Kalman filter divergence. The fused signal is more accurate than any of the original signals considered separately. The enhanced, more accurate signal is used to guide and navigate the robot.*

Keywords: Autonomous Robots, Guidance, Navigation and Control, Sensor fusion, Kalman Filter, Adaptive Fuzzy Logic System

Introduction

To follow the designed path, an autonomous vehicle has to be equipped with three systems: navigation, guidance, and control system [1], [2]. Navigation system is used to provide estimation of position and velocity of the vehicle. Guidance system is used to determine the optimal trajectory from a current position and velocity to a desired position and velocity. Control system is used to determine the commands for vehicle's actuators to drive the actual velocity and attitude of the vehicle to the value commanded by the guidance scheme.

There are two basic position-estimation methods applied in navigation system, i.e. absolute and relative positioning. [3], [4], [5], [6]. Absolute positioning is based on navigation beacons, active or passive landmark, map matching, or satellite-based navigation signal, where absolute positioning sensors interact with dynamic environment. Relative positioning is usually based on odometry sensors, or inertial sensors

Internal and external sensors are usually used in positioning problems. Internal sensors measure physical variables on the vehicle. The examples of these sensors are encoders, gyroscopes, accelerometers and compasses. External sensors measure relationships between the robot and its environment, which can be natural or artificial objects. The examples of external sensors are sonar sensor, radars and laser range finders.

Both of these sensors have advantages and disadvantages. For short period of time, measurements using internal sensors are quite accurate. However, for long-term estimation, the measurements usually produce a drift. On the contrary, external sensors do not produce the drift, however, the measurements from these sensors are usually not always available [7].

To get the optimal result, both sensors are usually combined. Because the results of both measurements contain errors, a special method has to be used to combine the results. The commonly used method is fusing those two measurements so it will produce the best desire estimation by using the Extended Kalman Filter (EKF) [6], [8], [9], and [10]. Internal sensors can be used to estimate the position of the vehicle during a particular period. External sensors are then implemented to correct the errors that come from internal sensors.

The type of internal sensors that are widely used in navigation is odometry sensor. It is mounted on the robot's driving wheels and register angular movements of the wheels. These angular movements are then translated into linear movements. This process has limited accuracy, for example, if slip has occurred on the wheel, the odometry would register the movement, but in fact, the vehicle may stay on its position. In the long period, the incremental motion of odometry will cause the accumulative error in positioning process. On the other hand, the advantage of using odometry is that the measurement signal is always available.

Beside the typical drift error in odometry, other errors can also occur in odometry sensors [11]. One important error is systematic error. This error will cause the bias in one direction of the movement of the vehicle, so the final position of the vehicle will deviate from the designed path. One method used to reduce this error is by conducting a benchmark experiment prior to regular operation of the vehicle [3]. This experiment can find the systematic error and, subsequently, this error is applied to correct the control system parameters. However, if the systematic errors occur frequently, this method may not be sufficient. For example, if the vehicle uses elastic tires, the benchmarking process has to be performed each time the unequal diameter occurs. It is beneficial if the error correction can be done in real time operation.

Sonar sensor is one type of external sensor. It measures absolute position of the vehicle based on pre-defined environment. If the sonar sensor is implemented on the vehicle with odometry sensor, it can be used to correct the systematic error by fusing both measurements using EKF.

When using EKF to fuse the signal, it is widely known that poorly designed mathematical model for the EKF will lead to the divergence. If the plant parameters are subject to perturbations and dynamics of the system are too complex to be characterized by an explicit mathematical model, an adaptive scheme is needed.

The adaptation scheme that usually used is based on fuzzy logic. Many methods have been implemented using this logic. Two recent paper that explain about this method are the paper presented by Jetto [6] and Sasiadek [10]. Jetto used Fuzzy Logic Adapted Kalman Filter (FLAKF) to prevent the filter from divergence when fusing measurement from odometry and sonar sensors. In this method, the ratio of innovation and covariance of innovation is used as input to the fuzzy logic, and the output is used to weight the process noise covariance in EKF. Sasiadek used exponential data weighting to prevent the divergence. Mean value and covariance of innovation are used as the input of the Fuzzy Logic Adaptive Controller (FLAC). The output is then used to weight process noise and measurement noise covariance in EKF. This FLAC is implemented on the flying vehicle navigating in three-dimensional space. Both those methods have shown improvement in the estimation of the vehicle position in comparison with the EKF only.

In this paper, the systematic error in odometry sensor is corrected during real-time operation of the vehicle by using measurements result from the sonar sensor. EKF is applied to fuse those two signals to find the best estimation of position. Adaptive Fuzzy Logic System (AFLS) is used to prevent the filter from divergence. The model of vehicle used in this experiment is based on a differential-drive. This type vehicle can be steered by differentiating the wheels angular velocity. The objective of this paper is to develop an efficient method for signal fusing to get accurate positioning.

Two-Wheeled Vehicle

The configuration of the two-wheeled vehicle is presented in Figure 1. This configuration has two opposed drive wheels mounted on the left and right sides of the vehicle. The motion of the midpoint of the axis represents the movement of the vehicle.

The incremental movement of the left and right drive wheel can be formulated as:

$$\Delta S_{L,R} = \frac{\pi D_n}{nC_e} N_{L,R} \tag{1}$$

where

D_n = nominal wheel diameter

C_e = encoder resolution

n = gear ratio between the motor and drive wheel

N = pulse increment shown by encoder.

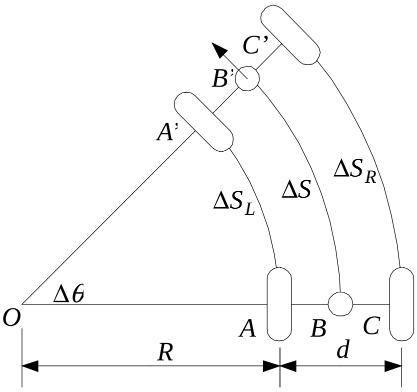

Figure 1. Two-wheeled vehicle model.

If the sampling interval of encoder is T, the linear and angular velocity of the incremental movement of the vehicle can be shown as:

$$\bar{v} = \frac{\Delta S_R + \Delta S_L}{2T} \tag{2}$$

$$\bar{\omega} = \frac{\Delta S_R - \Delta S_L}{dT} \tag{3}$$

where d is the distance between the odometry encoder.

If the sampling period is made to limit to zero, the kinematic model of this vehicle then can be described by the following equations:

$$\dot{x}(t) = v(t)\cos\theta(t) \tag{4}$$

$$\dot{y}(t) = v(t)\sin\theta(t) \tag{5}$$

$$\dot{\theta}(t) = \omega(t) \tag{6}$$

If we denote the state variable of the vehicle as $\mathbf{x}(t) = [x(t) \quad y(t) \quad \theta(t)]^T$, and the vehicle control input as $\mathbf{u}(t) = [v(t) \quad \omega(t)]^T$, the kinematic model in Eqs. (4 - 6) can be written in the form of stochastic differential equation as:

$$\dot{\mathbf{x}}(t) = f(\mathbf{x}(t), \mathbf{u}(t)) + \mathbf{w}(t) \tag{7}$$

where $\mathbf{w}(t)$ is a zero-mean Gaussian white noise with covariance matrix $\mathbf{Q}(t)$, which represents the model inaccuracies.

Exponentially Weighted EKF

Linearizing Eq. (7), and making the sampling period is small enough, the equation of EKF can be found. By assuming that during this time sampling, the linear and angular velocities are constant, and that the vehicle is following an arc path (see Wang [12]), then, the equations for Extended Kalman Filter can be expressed by:

$$\mathbf{x}_{k+1}^{-} = \mathbf{x}_k + \mathbf{B}_k \mathbf{u}_k \tag{8}$$

$$\mathbf{P}_{k+1}^{-} = \mathbf{A}_k \mathbf{P}_k \mathbf{A}_k^T + \mathbf{Q}_k \tag{9}$$

$$\mathbf{K}_{k+1} = \mathbf{P}_{k+1}^{-} \mathbf{C}_{k+1}^T [\mathbf{C}_{k+1} \mathbf{P}_{k+1}^{-} \mathbf{C}_{k+1}^T + \mathbf{R}_{k+1}]^{-1} \tag{10}$$

$$\mathbf{x}_{k+1} = \mathbf{x}_{k+1}^{-} + \mathbf{K}_{k+1} [\mathbf{y}_{k+1} - \mathbf{C}_{k+1} \mathbf{x}_{k+1}^{-}] \tag{11}$$

$$\mathbf{P}_{k+1} = [\mathbf{I} - \mathbf{K}_{k+1} \mathbf{C}_{k+1}] \mathbf{P}_{k+1}^{-} \tag{12}$$

where:

$$\mathbf{x}_k = [x_k \quad y_k \quad \theta_k]^T \tag{13}$$

$$\mathbf{B}_k = \begin{bmatrix} T\cos\left(\theta_k + \dfrac{\Delta\theta_k}{2}\right) & 0 \\ T\sin\left(\theta_k + \dfrac{\Delta\theta_k}{2}\right) & 0 \\ 0 & 1 \end{bmatrix} \tag{14}$$

$$\mathbf{A}_k = \begin{bmatrix} 1 & 0 & -v_k T \sin\theta_k \\ 0 & 1 & v_k T \cos\theta_k \\ 0 & 0 & 1 \end{bmatrix} \tag{15}$$

$$\mathbf{Q}_k = [\mathbf{Q}_1 \quad \mathbf{Q}_2 \quad \mathbf{Q}_3] \tag{16}$$

$$\mathbf{Q}_1 = \begin{bmatrix} Q_{11}T + Q_{33}\,(T^3/3) v_k^2 \sin^2\theta_k \\ -Q_{33}\,(T^3/3) v_k^2 \sin\theta_k \cos\theta_k \\ -Q_{33}\,(T^2/2) v_k \sin\theta_k \end{bmatrix} \tag{17}$$

$$\mathbf{Q}_2 = \begin{bmatrix} -Q_{33}\,(T^3/3) v_k^2 \sin\theta_k \cos\theta_k \\ Q_{22}T + Q_{33}\,(T^3/3) v_k^2 \cos^2\theta_k \\ Q_{33}\,(T^2/2) v_k \cos\theta_k \end{bmatrix} \tag{18}$$

$$\mathbf{Q}_3 = \begin{bmatrix} -Q_{33}\,(T^2/2) v_k \sin\theta_k \\ Q_{33}(T^2/2) v_k \cos\theta_k \\ Q_{33}T \end{bmatrix} \tag{19}$$

and, $Q_{11} = \sigma_x^2$, $Q_{22} = \sigma_y^2$, and $Q_{33} = \sigma_z^2$ are diagonal elements of covariance matrix $\mathbf{Q}(t)$ of $\mathbf{w}(t)$ in Eq. (7).

The measurement, in this case, will consist of the measurement from odometry sensor and sonar sensor. The size of the measurement vector depends on the number of active sonar sensor. In general, this vector can be expressed as (See Jetto et. al. [7]):

$$\mathbf{y}(\mathbf{x}_k, \Pi) = [x_k \quad y_k \quad \theta_k \quad d_{1k} \quad d_{2k} \quad \ldots \quad d_{nk}]^T \tag{20}$$

where d_{nk} is the measurement of sonar *n*th at time *k*.

There are many methods that can be implemented to fuse the signals using EKF. Four methods are described here. The first method is direct pre-filtering method, which was presented by Green and Sasiadek [13]. In this method, both measurement signals are filtered prior of comparison process. The error of those signals is then used to correct the measurement signal. The scheme of this method is shown in Figure 2.

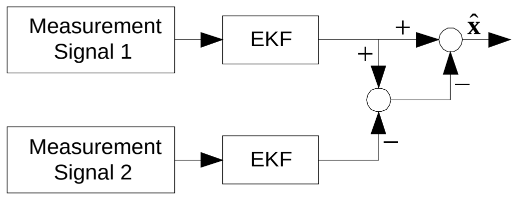

Figure 2. Direct Pre-filtering scheme

The second method is by applying the total state, such as position and velocity, into the filter. The measurement signals are combined together before fed to the EKF. The advantage of using this method is that although only one measurement signal is available, the correction still can be done. The more the measurement signals become available, the estimation will be more accurate. The disadvantage of this method is that one has to know the dynamic model of the sensor in order find the predicted position. This method is sometime called direct or total state space EKF [14]. The scheme of this method is presented in Figure 3.

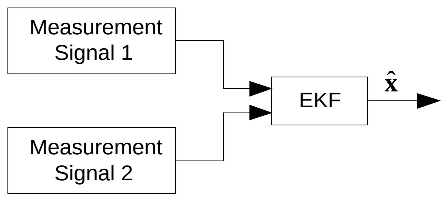

Figure 3. Direct EKF scheme

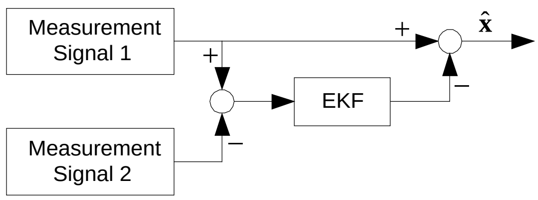

Figure 4. Indirect feed forward EKF scheme

The third method is indirect feed forward EKF. In this method, the signals are compared before fed into the EKF. The estimation error is then fed forward into one of the measured signal. The scheme of this method is presented in Figure 4.

The last method is almost the same as the third one. In spite of feed the estimation error forward as in the third method, in this method, the error is fed back into one of the measured signal. Sasiadek and Wang [10] presented this method when fusing the signals, which come from INS and GPS. Using the last two methods, the dynamic model of the sensor is less important, but, when one measurement signal is not available, the correction cannot be performed. The last method is sometime referred as indirect feedback EKF [14]. The scheme of this method is shown in Figure 5.

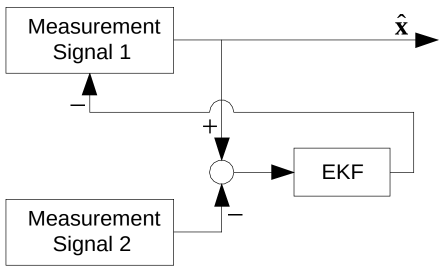

Figure 5. Indirect feed back EKF scheme

In this paper, the second method, that is the direct EKF method, is used. The measurement signals, which come from odometry and sonar sensors, are combined into one measurement vector. This measurement result is then fed into the EKF.

To prevent the filter from divergence, exponentially weighted EKF is used (See Lewis [15]). Using this method, the equations of the EKF as explained in the beginning of this section have to be adjusted. For exponentially weighted EKF, the weighted process and measurement noise covariance can be written as:

$$\mathbf{R}_k = \mathbf{R}\alpha^{-2(k+1)} \tag{21}$$

$$\mathbf{Q}_k = \mathbf{Q}\alpha^{-2(k+1)} \tag{22}$$

where $\alpha \geq 1$. $\mathbf{Q}$ and $\mathbf{R}$ are constant matrices of process and measurement noise covariance. For $\alpha > 1$, as time k increases, $\mathbf{Q}_k$ and $\mathbf{R}_k$ will decrease, which means that the most recent measurement is given higher weighting.

If the weighted estimation covariance is defined as:

$$\mathbf{P}_k^{\alpha-} = \mathbf{P}_k^{-}\alpha^{2k} \tag{23}$$

then the EKF equations become:

$$\mathbf{x}_{k+1}^{-} = \mathbf{x}_k + \mathbf{B}_k\mathbf{u}_k \tag{24}$$

$$\mathbf{P}_{k+1}^{\alpha-} = \alpha^2\mathbf{A}_k\mathbf{P}_k^{\alpha}\mathbf{A}_k^T + \mathbf{Q}_k \tag{25}$$

$$\mathbf{K}_{k+1} = \mathbf{P}_{k+1}^{\alpha-}\mathbf{C}_{k+1}^T[\mathbf{C}_{k+1}\mathbf{P}_{k+1}^{\alpha-}\mathbf{C}_{k+1}^T + \frac{\mathbf{R}_{k+1}}{\alpha^2}]^{-1} \tag{26}$$

$$\mathbf{x}_{k+1} = \mathbf{x}_{k+1}^{-} + \mathbf{K}_{k+1}[\mathbf{y}_{k+1} - \mathbf{C}_{k+1}\mathbf{x}_{k+1}^{-}] \tag{27}$$

$$\mathbf{P}_{k+1}^{\alpha} = [\mathbf{I} - \mathbf{K}_{k+1}\mathbf{C}_{k+1}]\mathbf{P}_{k+1}^{\alpha-} \tag{28}$$

Adaptive Fuzzy Logic System

In Kalman filter model, both process noise $\mathbf{w}_k$ and measurement noise $\mathbf{v}_k$ are assumed zero-mean white noise sequence with covariance $\mathbf{Q}_k$ and $\mathbf{R}_k$. If the model of EKF is tuned perfectly, the residual between actual and predicted measurement should be a zero-mean white noise process.

Often, we do not know all parameters of the model or we want to reduce the complexity of modeling. Therefore, in real application, the exact value of $\mathbf{Q}_k$ and $\mathbf{R}_k$ is not known. If the actual process and measurement noises are not a zero-mean white noise, the residual in Kalman filter will also not be a white noise. If this is happened, the Kalman filter would diverge or at best converge to a large bound.

Jetto et. al. [7] used fuzzy logic adapted Kalman filter to prevent the filter from divergence. The fuzzy logic controller uses one input and one output. The ratio between innovation and covariance of innovation process is used as an input. The output is a constant, which is used to weight the process noise covariance. The controller uses five fuzzy rules and is implemented in a wheeled mobile robot equipped with odometry and sonar sensors.

Sasiadek and Wang [10] used fuzzy logic adapted controller (FLAC) to prevent the filter from divergence when fusing signals coming from INS and GPS on flying vehicle. Nine rules were used. There were two inputs, which are the mean value and covariance of innovation, and the output is a constant that is used to weight exponentially the model and measurement noise covariance.

In the case of fusing signals that come from odometry and sonar sensors, sometime only odometry measurements are available. In this case, the innovation will be a white noise as long as the process and measurement noises are assumed as a white noise. But when the sonar measurements become available, and combined with the odometry measurement, the innovation might be not a white noise anymore. This will cause the filter to diverge.

When systematic error occurs in the vehicle, the process and measurement noise actually are not a gaussian white noise. This will deviate from the requirement of the EKF and the divergence in filter will occur. AFLS can be used to adapt the filter gain so that the divergence can be avoided. The adaptation process used in this paper is based on exponential data weighting (Lewis [15]). The adaptation process will change the $\mathbf{Q}$ and $\mathbf{R}$, and subsequently the Kalman gain $\mathbf{K}$ of the EKF. The scheme of the adaptation process is shown in Figure 6.

The membership function used in this AFLS is displayed in Figure (7 – 9). The AFLS uses nine rules , which are summarized in Table 1.

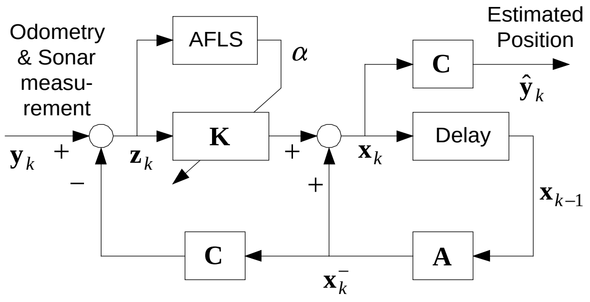

Figure 6. Adaptive Fuzzy Logic System (AFLS) scheme

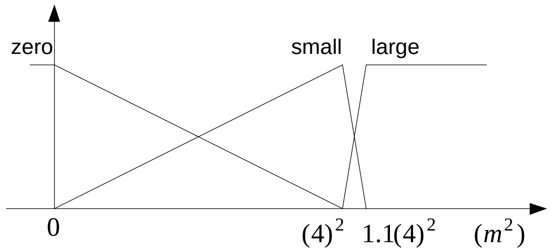

Figure 7. MF of innovation process covariance

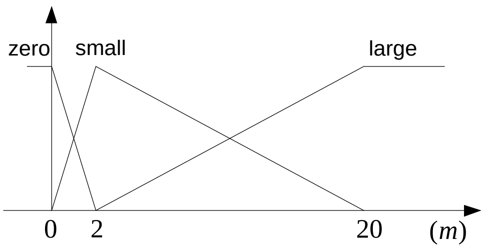

Figure 8. MF of innovation process mean value

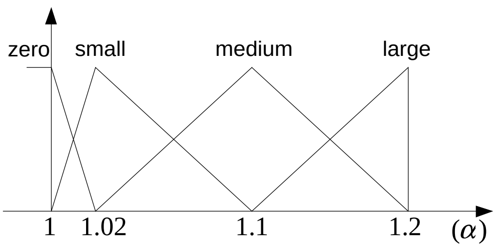

Figure 9. MF of α

Table 1. Rules table for AFLS

α		Innovation process mean value		
		Zero	Small	Large
Innovation process covariance	Zero	Small	Zero	Large
	Small	Zero	Large	Medium
	Large	Large	Medium	Zero

Experiments and Results

Simulation experiments have been conducted to show the implementation of AFLS when fusing the signals that come from odometry and sonar sensor. Systematic error in odometry measurement, which comes from unequal in wheel's diameter, is also considered. The vehicle is planned to follow sinus path in in-door environment. The map of the in-door environment along with the movement of the mobile vehicle that has systematic error is shown in Figure 10.

Three simulation experiments have been performed. The first experiment is to show the implementation of EKF in the mobile robot using odometry sensor, where the sensor has systematic error. The result of this experiment is shown in Figure 11. In this experiment, it shows that the implementation of EKF with only one measurement signal is available, cannot be used to correct the systematic error. The EKF in this case only filters the gaussian white noise of the odometry measurement error. The systematic error however, still present in the movement of the mobile vehicle.

The second experiment is to use the EKF to fuse measurement signals that come from odometry and sonar sensor without using AFLS. This experiment result is shown in Figure 12. The present of sonar sensor, which measures the relation of the mobile vehicle and its environment, reduces the systematic error, and the mobile vehicle can follow the designed path. However, the movement of the mobile vehicle in this case is not smooth. The result of sonar measurement in this experiment is not used efficiently to improve the position estimation.

The third experiment is to use AFLS to adapt the gain of EKF to prevent the filter from divergence. In this experiment, when the sonar measurement becomes available, the EKF uses this signal to improve its estimation. AFLS makes the position estimation smoother than without AFLS. The result of this experiment is shown in Figure 13.

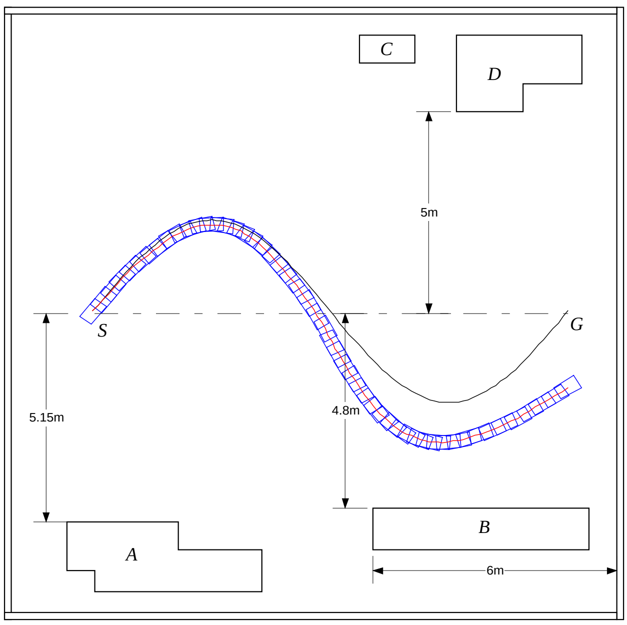

Figure 10. Map of in-door environment

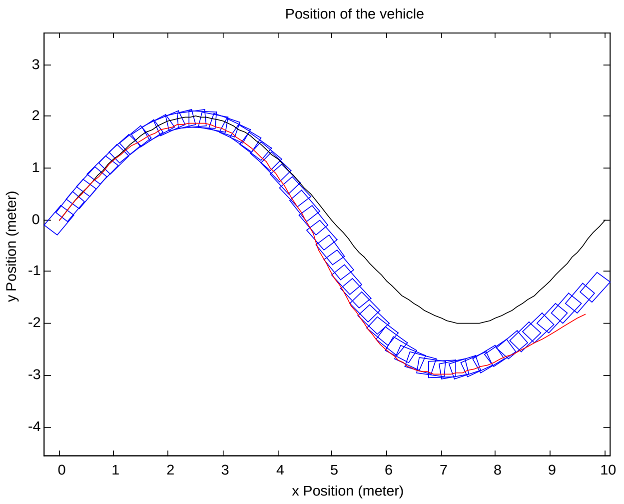

Figure 11. Simulation experiment result using EKF with only odometry measurement

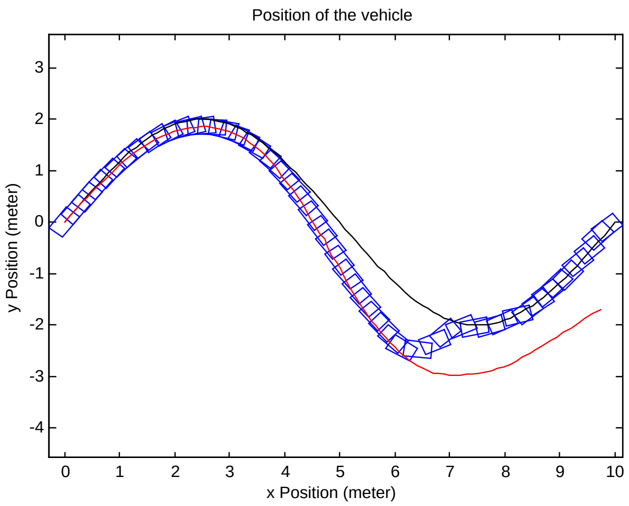

Figure 12. Simulation experiment result using EKF with odometry and sonar measurement

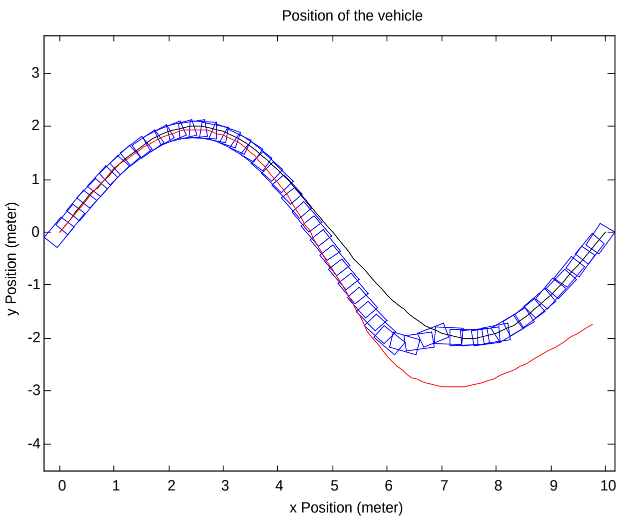

Figure 13. Simulation experiment result using EKF with odometry and sonar measurement, adapted by AFLS

Conclusions

In this paper, Extended Kalman Filter (EKF) has been used to estimate the position of the mobile vehicle. To prevent the filter from divergence, the innovation and covariance of innovation process are monitored by using Adaptive Fuzzy Logic System (AFLS). The result is an adaptation in the gain of EKF.

Odometry and sonar sensors have been used to simulate the method. From the simulation experiment, it shows that beside the improvement in the estimation of position, the method can also be used to correct the systematic error. Using this method, real-time operation of the vehicle can be reduced.

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