Closed-loop Subspace Identification with Innovation Estimation

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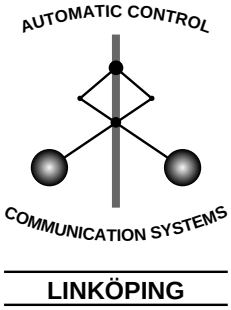

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CLOSED-LOOP SUBSPACE IDENTIFICATION WITH INNOVATION ESTIMATION

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Abstract: Most subspace identification algorithms are not applicable to closed-loop identification because they require future input to be uncorrelated with past innovation. In this paper, we propose a new subspace identification method that remove this requirement by using a parsimonious model formulation with innovation estimation. A simulation example is included to show the effectiveness of the proposed method.

Keywords: subspace identification; closed-loop identification; parsimonious models; innovation estimation

1. INTRODUCTION

Subspace identification methods (SIM) have gone through tremendous development over the last decade (Chou and Verhaegen, 1997; Moor *et al.*, 1988; Larimore, 1990; Moonen *et al.*, 1989; Overschee and Moor, 1993; Overschee and Moor, 1994; Verhaegen, 1991; Verhaegen, 1994; Viberg, 1994). Among these algorithms canonical variate analysis (CVA)(Larimore, 1983; Larimore, 1990), N4SID (Overschee and Moor, 1994), MOESP (Verhaegen and Dewilde, 1992), and IV-4SID (Viberg, 1995) are some of the representing algorithms. These SIM algorithms are now well explained in many papers and several books (Overschee and Moor, 1996), (Ljung, 1999). Typically, an SIM estimates the extended observability matrix with or without estimating the state sequence. Then the system matrices and the disturbance characteristics are estimated. The unifying theorem (Overschee and de Moor, 1995; Jansson and Wahlberg, 1998) formulates many of the SIM algorithms in a singular value decomposition framework, with differences in weighting matrices. The CVA and MOESP weighting matrices have been proven to be approximately optimal weighting (Gustafsson, 2002). In addition, statistical properties such as consistency have recently been explored (Bauer *et al.*, 1999; Deistler *et al.*, 1995; Heij and Scherrer, 1999; Jansson and Wahlberg, 1998).

Although the SIM algorithms are attractive because the state space form is very convenient for estimation, filtering, prediction of multivariable systems, severe drawbacks have been experienced. In general, the SIM estimates are not as accurate as the prediction error methods (PEM). Further, very few, if any, SIM methods are applicable to closed-loop identification, even though the data satisfy identifiability conditions for traditional methods such as PEMs.

In a companion paper (Qin and Ljung, 2003), we give the reasons why subspace identification approaches exhibit these drawbacks and propose parallel parsimonious SIM (PARSIM-P) for open-

loop applications. With the analysis of existing subspace formulation using the linear regression formulation (Jansson and Wahlberg, 1996; Jansson and Wahlberg, 1998; Knudsen, 2001), we reveal that the typical SIM algorithms actually use non-parsimonious model formulation, with extra terms in the model that appear to be non-causal. These terms, although conveniently included for performing subspace projection, are the causes for inflated variance in the estimates and partially responsible for the loss of closed-loop identifiability.

Removing the non-causal terms in the SIM formulation makes the model parsimonious, but it does not make the SIM methods automatically applicable to closed-loop identification. As pointed out in (Qin and Ljung, 2003), the PARSIM-P algorithm still requires that there is no correlation between future input u_k and past innovation e_k, which is not the case for closed-loop data.

In this paper, we propose a new parsimonious method that removes this requirement, thus making it applicable to closed-loop identification. We propose to estimate the innovation process e_k using the intermediate results in SIMs. Then the estimated innovation sequence is used in the subsequent projections in the SIM procedure. This method will be referred to as PARSIM-E, which means that the innovation process e_k is estimated first. A simulation example is used to demonstrate the effectiveness of the proposed PARSIM-E method for closed-loop identification with comparison to the PARSIM-P and MOESP algorithms.

2. SUBSPACE MODEL

2.1 *Conventional Subspace Models*

We begin with an innovation model formulation,

$$x_{k+1} = Ax_k + Bu_k + Ke_k \tag{1a}$$
$$y_k = Cx_k + Du_k + e_k \tag{1b}$$

where $y_k \in R^{n_y}$, $x_k \in R^n$, $u_k \in R^{n_u}$, and $e_k \in R^{n_y}$ are the system output, state, input, and innovation, respectively. A,B,C,D and K are system matrices with appropriate dimensions.

An extended state space model can be formulated as

$$Y_f = \Gamma_f X_k + H_f U_f + G_f E_f \tag{2a}$$
$$Y_p = \Gamma_p X_{k-p} + H_p U_p + G_p E_p \tag{2b}$$

where the extended observability matrix

$$\Gamma_f = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{f-1} \end{bmatrix} \tag{3}$$

and the Toeplitz matrices are

$$H_f = \begin{bmatrix} D & 0 & \cdots & 0 \\ CB & D & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{f-2}B & CA^{f-3}B & \cdots & D \end{bmatrix} \tag{4a}$$

$$G_f = \begin{bmatrix} I & 0 & \cdots & 0 \\ CK & I & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{f-2}K & CA^{f-3}K & \cdots & I \end{bmatrix} \tag{4b}$$

The input and output data are arranged in the following Hankel form:

$$U_f = \begin{bmatrix} u_k & u_{k+1} & \cdots & u_{k+N-1} \\ u_{k+1} & u_{k+2} & \cdots & u_{k+N} \\ \vdots & \vdots & \ddots & \vdots \\ u_{k+f-1} & u_{k+f} & \cdots & u_{k+f+N-2} \end{bmatrix} \tag{5a}$$
$$\triangleq \begin{bmatrix} u_f(k) & u_f(k+1) & \cdots & u_f(k+N-1) \end{bmatrix} \tag{5b}$$
$$U_p = \begin{bmatrix} u_{k-p} & u_{k-p+1} & \cdots & u_{k-p+N-1} \\ u_{k-p+1} & u_{k-p+2} & \cdots & u_{k-p+N} \\ \vdots & \vdots & \ddots & \vdots \\ u_{k-1} & u_k & \cdots & u_{k+N-2} \end{bmatrix} \tag{5c}$$
$$\triangleq \begin{bmatrix} u_p(k-p) & u_p(k-p+1) & \cdots & u_p(k-p+N-1) \end{bmatrix} \tag{5d}$$

Denoting

$$L^1 = \begin{bmatrix} L^1_{11} & L^1_{12} & \cdots & L^1_{1p} \\ L^1_{21} & L^1_{22} & \cdots & L^1_{2p} \\ \vdots & & \ddots & \\ L^1_{f1} & L^1_{f1} & & L^1_{fp} \end{bmatrix} \triangleq \begin{bmatrix} L^1_1 \\ L^1_2 \\ \vdots \\ L^1_f \end{bmatrix} \tag{6a}$$

$$L^2 = \begin{bmatrix} L^2_{11} & L^2_{12} & \cdots & L^2_{1p} \\ L^2_{21} & L^2_{22} & \cdots & L^2_{2p} \\ \vdots & & \ddots & \\ L^2_{f1} & L^2_{f1} & & L^2_{fp} \end{bmatrix} \triangleq \begin{bmatrix} L^2_1 \\ L^2_2 \\ \vdots \\ L^2_f \end{bmatrix} \tag{6b}$$

$$L^3 = \begin{bmatrix} L^3_{11} & L^3_{12} & \cdots & L^3_{1f} \\ L^3_{21} & L^3_{22} & \cdots & L^3_{2f} \\ \vdots & & \ddots & \\ L^3_{f1} & L^3_{f1} & & L^3_{ff} \end{bmatrix} \triangleq \begin{bmatrix} L^3_1 \\ L^3_2 \\ \vdots \\ L^3_f \end{bmatrix} \tag{6c}$$

the above problem is equivalent to f separate subproblems:

$$\begin{bmatrix} \hat{L}^1_i & \hat{L}^2_i & \hat{L}^3_i \end{bmatrix} = \arg\min\{J\} \tag{7}$$

where

$$J = \sum_{j=0}^{N-1} \left\| y(k+j+i-1) - \begin{bmatrix} L_i^1 & L_i^2 & L_i^3 \end{bmatrix} \begin{bmatrix} y_p(k-p+j) \\ u_p(k-p+j) \\ u_f(k+j) \end{bmatrix} \right\|^2 \quad \text{for } i = 1, 2, \ldots, f \tag{8}$$

For the case of $i = 1$, for example, the problem implies that the following model is specified:

$$\begin{aligned} y(k) &= \begin{bmatrix} L_1^1 & L_1^2 & L_1^3 \end{bmatrix} \begin{bmatrix} y_p(k-p) \\ u_p(k-p) \\ u_f(k) \end{bmatrix} + v(k) \\ &= \begin{bmatrix} L_1^1 & L_1^2 \end{bmatrix} \begin{bmatrix} y_p(k-p) \\ u_p(k-p) \end{bmatrix} + L_{11}^3 u(k) \\ &\quad + \sum_{j=2}^{f} L_{1j}^3 u(k+j-1) + v(k) \end{aligned} \tag{9}$$

Note that the third term on the RHS of the above equation is non-causal and unnecessary. Therefore, the model format used in SIM during the projection step is non-causal. This would result in non-causal models in the projection step. Although the non-causal terms are ignored at the step to estimate B, D, all the model parameters estimate have inflated variance due to the fact that extra and unnecessary terms are included in the model, making the model non-parsimonious. For $i > 1$ the number of non-causal terms will reduce, but they are unnecessary as long as $i < f$.

To avoid these problems the SIM model must not include these non-causal terms. The PARSIM-P algorithms remove: these terms by enforcing triangular structure of the Toeplitz matrix H_f at every step of the SIM procedure. The approach are referred to as parsimonious subspace identification methods (PARSIM) as it uses parsimonious model formulation.

2.2 *Parsimonious Subspace Models*

The key idea in the proposed method is to exclude those non-causal terms of U_f. To accomplish this we partition the extended state space model rowwise as follows:

$$Y_f = \begin{bmatrix} Y_{f1} \\ Y_{f2} \\ \vdots \\ Y_{ff} \end{bmatrix} ; \; Y_i \triangleq \begin{bmatrix} Y_{f1} \\ Y_{f2} \\ \vdots \\ Y_{fi} \end{bmatrix} ; i = 1, 2, \ldots, f \tag{10}$$

Partition U_f and E_f in a similar way to define U_{fi}, U_i, E_{fi}, and E_i, respectively, for $i = 1, 2, \ldots, f$. Denote further

$$\Gamma_f = \begin{bmatrix} \Gamma_{f1} \\ \Gamma_{f2} \\ \vdots \\ \Gamma_{ff} \end{bmatrix} \tag{11a}$$

$$H_{fi} \triangleq \begin{bmatrix} CA^{i-2}B & \cdots & CB & D \end{bmatrix} \tag{11b}$$

$$\triangleq \begin{bmatrix} H_{i-1} & \cdots & H_1 & H_0 \end{bmatrix} \tag{11c}$$

$$G_{fi} \triangleq \begin{bmatrix} CA^{i-2}K & \cdots & CK & I \end{bmatrix} \tag{11d}$$

$$\triangleq \begin{bmatrix} G_{i-1} & \cdots & G_1 & G_0 \end{bmatrix} \tag{11e}$$

$$\forall i = 1, 2, \cdots, f$$

where H_i and G_i are the Markov parameters for the deterministic input and innovation sequence, respectively. We have the following partitioned equations:

$$\begin{aligned} Y_{fi} &= \Gamma_{fi} X_k + H_{fi} U_i + G_{fi} E_i \\ &\forall i = 1, 2, \cdots, f \end{aligned} \tag{12}$$

Note that each of the above equation is guaranteed causal.

2.3 *Parallel Estimation of* Γ_{fi} *and* H_{fi}

By eliminating $e(k)$ in the innovation model through iteration, it is straightforward to derive the following relation (Knudsen, 2001),

$$X_k = L_z Z_p + A_K^p X_{k-p} \tag{13}$$

where

$$L_z \triangleq \begin{bmatrix} \Delta_p(A_K, K) & \Delta_p(A_K, B_K) \end{bmatrix} \tag{14a}$$

$$\Delta_p(A, B) \triangleq \begin{bmatrix} A^{p-1}B & \cdots & AB & B \end{bmatrix} \tag{14b}$$

$$A_K \triangleq A - KC \tag{14c}$$

$$B_K \triangleq B - KD \tag{14d}$$

Substituting this equation into Eq. 12, we obtain

$$\begin{aligned} Y_{fi} &= \Gamma_{fi} L_z Z_p + \Gamma_{fi} A_K^p X_{k-p} + H_{fi} U_i + G_{fi} E_i \\ &\forall i = 1, 2, \cdots, f \end{aligned} \tag{15}$$

Since the second term in the RHS of Eq. 15 tends to zero as p tends to infinity, we have the following least squares estimates:

$$\begin{aligned} \begin{bmatrix} \hat{\Gamma}_{fi} L_z & \hat{H}_{fi} \end{bmatrix} &= Y_{fi} \begin{bmatrix} Z_p \\ U_i \end{bmatrix}^+ \\ &\forall i = 1, 2, \cdots, f \end{aligned} \tag{16}$$

Qin and Ljung (Qin and Ljung, 2003) point out that the PARSIM-P algorithm requires that the input $u(k)$ and innovation sequence $e(k)$ are uncorrelated, i.e.,

$\frac{1}{N}E_iU_i^T \to 0$ as $N \to \infty$, to be unbiased. Because of this requirement, the PARSIM-P algorithm is biased for closed-loop identification. In the next section we propose a new PARSIM algorithm, PARSIM-E, that estimates the past innovation process first. The estimated innovation is treated as known data and the subsequent projections do not require future input to be uncorrelated with past innovation, hence the PARSIM-E method is applicable to closed-loop identification.

3. PARSIM WITH INNOVATION ESTIMATION

By ignoring the second term on the RHS of Eq. 15 and setting $i = 1$, we have

$$Y_{f1} = \begin{bmatrix} \Gamma_{f1}L_z & H_{f1} \end{bmatrix} \begin{bmatrix} Z_p \\ U_1 \end{bmatrix} + E_1 \qquad (17)$$

Therefore, a least squares estimate of the innovation process is:

$$\hat{E}_1 = Y_{f1} - \begin{bmatrix} \hat{\Gamma}_{f1}L_z & \hat{H}_{f1} \end{bmatrix} \begin{bmatrix} Z_p \\ U_1 \end{bmatrix} \qquad (18)$$

Now return to Eq. 15 for a general $i = 2, 3, \ldots, f$. Noticing that

$$E_i = \begin{bmatrix} E_{f1} \\ E_{f2} \\ \vdots \\ E_{fi} \end{bmatrix} = \begin{bmatrix} E_{i-1} \\ E_{fi} \end{bmatrix} \qquad (19)$$

and replacing E_{i-1} with $\hat{E}_{i-1}$, Eq. 15 becomes,

$$Y_{f1} = \begin{bmatrix} \Gamma_{fi}L_z & H_{fi} & G_{fi}^- \end{bmatrix} \begin{bmatrix} Z_p \\ U_i \\ \hat{E}_{i-1} \end{bmatrix} + E_{fi} \qquad (20)$$

where

$$G_{fi}^- = \begin{bmatrix} CA^{i-2}K & CA^{i-3}K & \ldots & CK \end{bmatrix}. \qquad (21)$$

The least squares estimate

$$\begin{bmatrix} \hat{\Gamma}_{fi}L_z & \hat{H}_{fi} & \hat{G}_{fi}^- \end{bmatrix} = Y_{f1} \begin{bmatrix} Z_p \\ U_i \\ \hat{E}_{i-1} \end{bmatrix}^+ \qquad (22)$$

now does not require future input u_k to be uncorrelated with past innovation e_k. It only requires that future innovation to be independent of past input, which is always true for both open-loop and closed-loop data. The innovation data are calculated recursively using

$$\hat{E}_i = \begin{bmatrix} \hat{E}_{i-1} \\ \hat{E}_{fi} \end{bmatrix} \qquad (23)$$

With the least squares estimates from Eq. 22, the system matrices A, B, C, D, K can be estimated similarly to the procedures given in (Qin and Ljung, 2003).

4. SIMULATION RESULTS

We simulate the following process

$$y_k + ay_{k-1} = bu_{k-1} + e_k + ce_{k-1} \qquad (24)$$

with a feedback controller

$$u_k = -Ky_k + r_k \qquad (25)$$

where $a = -0.9$, $b = 1$, and $c = 0.9$. The standard deviation for e_k is one and that for r_k is two; both of the signals are Gaussian white noise. Open-loop experiments are simulated with $K = 0$ and closed-loop experiments with $K = 0.6$. In both cases 2000 data points are collected and 20 Monte-Carlo simulations are performed. Figure 1 shows the pole estimates from PARSIM-E, PARSIM-P and MOESP for open-loop and closed-loop data. There is no observed difference for open-loop identification, while the closed-loop identification results are very different. The PARSIM-E gives the best estimate without bias.

Figure 2 shows the box plots of the parameter estimates from 20 simulations. In the open-loop case, all methods estimate a equally well. PARSIM-E and PARSIM-P give much better estimates for b than MOESP, showing the benefit of parsimonious formulation. PARSIM-E and PARSIM-P give equally good estimates for c, while MOESP does not estimate the stochastic parameters. In the closed-loop case, the PARSIM-E algorithm gives unbiased estimates for a and b. Both PARSIM-P and MOESP fail on closed-loop identification, with MOESP giving the worst results.

To examine the frequency responses of the identified models, Figure 3 gives the Bode plots by averaging the 20 closed-loop experiments. It is clearly shown that MOESP and PARSIM-P method fail to identify the steady state gain, while the PARSIM-E method is unbiased in all frequencies.

5. CONCLUSIONS

The proposed new subspace identification method with parsimonious models and innovation estimation gives unbiased results for closed-loop identification. For open-loop data both PARSIM-E and PARSIM-P algorithms give superior results than the contentional subspace model formulation.

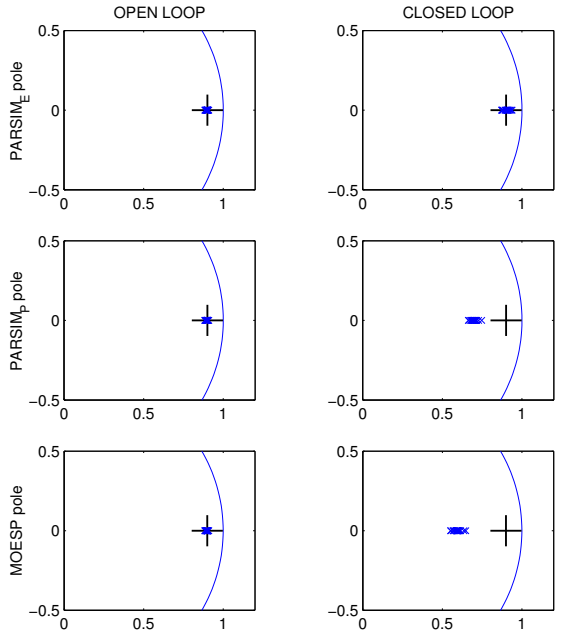

Fig. 1. Pole estimates for the simulation example.

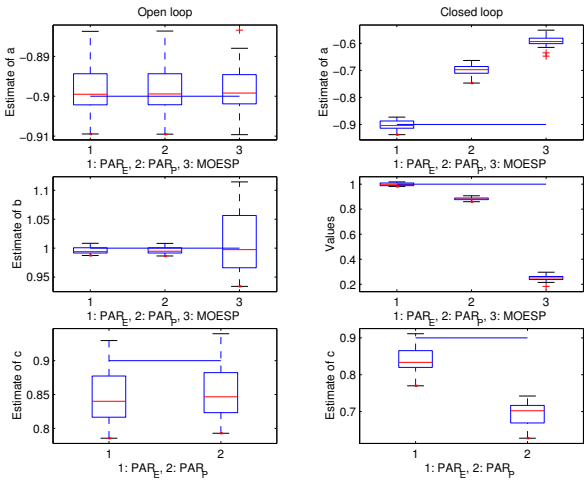

Fig. 2. Parameter estimates for the simulation example.

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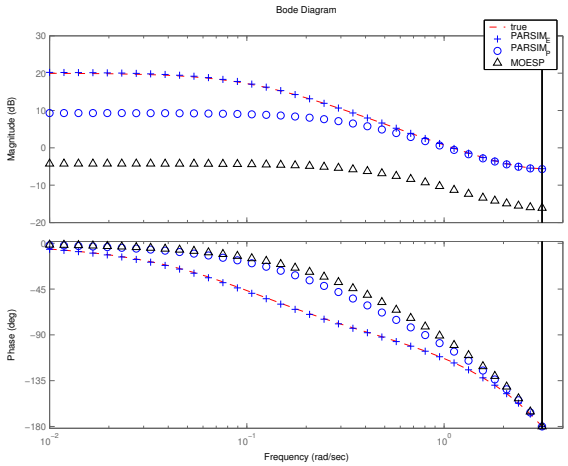

Fig. 3. Bode diagram for the closed-loop identification results

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