University of California
Los Angeles

# Closed-Loop Subspace Identification of a Quadrotor

A thesis submitted in partial satisfaction
of the requirements for the degree
Master of Science in Engineering

by

**Andrew G. Kee**

2013

Abstract of the Thesis

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Professor Steve Gibson, Chair

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The thesis of Andrew G. Kee is approved.

Steve Gibson, Committee Chair

University of California, Los Angeles

2013

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# Nomenclature

| | |
|---|---|
| $N$ | Number of block rows |
| $s$ | Number of block columns |
| $\mathbb{R}$ | Set of all real numbers |
| $u$ | System input |
| $y$ | System output |
| $x$ | System state |

# CHAPTER 1

# Introduction

Unmanned Aerial Vehicles (UAVs) have seen explosive growth in the past thirty years, performing a multitude of military and civilian tasks including surveillance, reconnaissance, armed combat operations, search and rescue, forest fire management, and domestic policing [9, 10]. A class of modern UAVs which have recently grown in popularity are quadrotors - Vertical Take Off and Landing (VTOL) vehicles powered by four rotors arranged in a cross configuration. The main advantage of the quadrotor lies in its mechanical simplicity. Adjusting the speed of one or more of the vehicle's fixed-pitch rotors provides full attitude control, eliminating the need for the swash plate mechanism found on single rotor helicopters [1, 2]. In spite of its mechanical simplicity, the quadrotor exhibits somewhat complex dynamics that are best modeled as a Multi-Input Multi-Output (MIMO) system.

Advances in MEMS sensors and light-weight high-powered lithium polymer batteries have contributed to the recent popularity of quadrotors, making them an attractive choice for research applications in flight dynamics and control, as in [3, 5, 6, 7]. One problem of particular interest is the development of mathematical models representing system dynamics based on experimentally gathered data. System identification provides a mechanism to relate this input-output data to the underlying system dynamics. Traditionally, system identification techniques have focused on developing a system model which minimizes prediction error. Identification methods of this form are commonly known as Prediction Error Methods (PEMs). PEMs have seen widespread use in both theoretical and real-world applications, but experience difficulties with MIMO systems as noted in [8, 11]. Subspace identification methods have recently grown in popularity and offer an alternative approach to the identification problem. These methods have a foundation in linear algebra and overcome the issues found in PEMs when identifying MIMO systems [4]. It is the goal of this research project to apply subspace identification techniques to a quadrotor using experimentally gathered closed-loop input and output data.

## 1.1 Related Work

For text, let's use the first words out of the ispell dictionary.

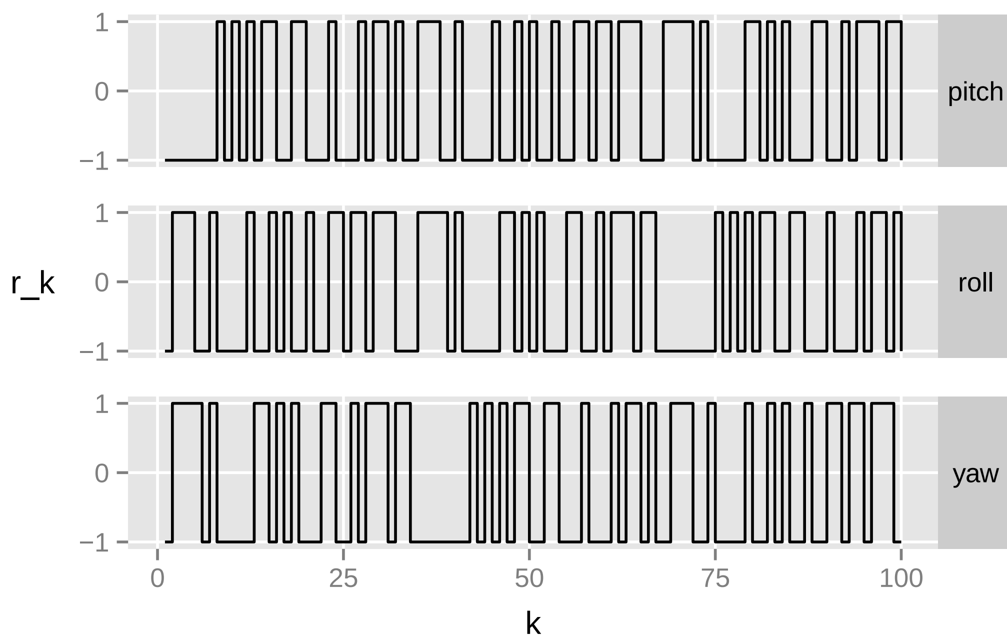


Figure 1.1: A figure caption

## 1.2 Motivation and Contributions

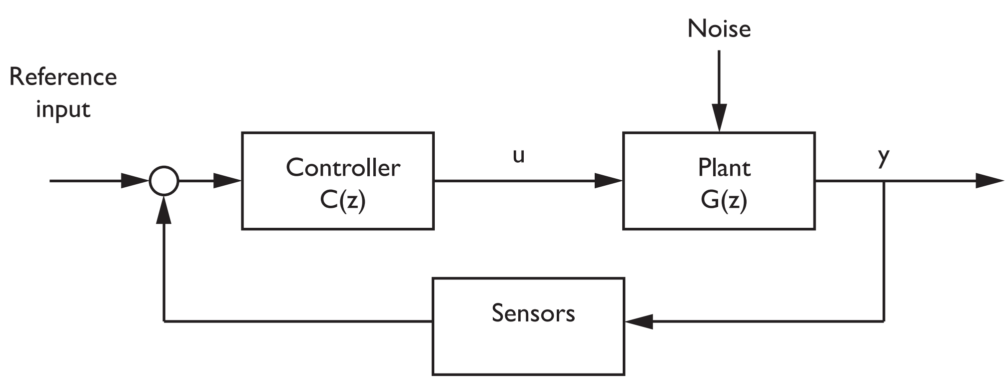


Figure 1.2: A figure caption

# CHAPTER 2

# Model and Assumptions

Introduce concept of LTI system here

## 2.1 State Space Model

We will consider a combined deterministic-stochastic LTI system written in innovation form as

$$x_{k+1} = Ax_k + Bu_k + Ke_k \tag{2.1a}$$

$$y_k = Cx_k + Du_k + e_k \tag{2.1b}$$

where $x_k \in \mathbb{R}^n$ is the system state, $u_k \in \mathbb{R}^m$ is the system input, $y_k \in \mathbb{R}^l$ is the system output, and $e_k \in \mathbb{R}^l$ is the innovation. $A$, $B$, $C$, and $D$ are the system matrices with appropriate dimensions and $K$ is the Kalman filter gain. The system represented in (2.1) can also be represented in predictor form as

$$x_{k+1} = A_K x_k + B_K u_k + Ky_k \tag{2.2a}$$

$$y_k = Cx_k + Du_k + e_k \tag{2.2b}$$

where $A_K = A - KC$ and $B_K = B - KD$.

The systems represented by (2.1) and (2.2) are equivalent from an input/output point of view, but because $A_K$ is guaranteed stable even if the original process matrix $A$ is unstable, the predictor form proves advantageous when considering unstable open-loop systems.

## 2.2 Assumptions

[**Assumption 1**]**:** $A_K = A - KC$ is stable (i.e. its eigenvalues lie within the unit circle)

[**Assumption 2**]**:** The system is represented in its minimal form

# CHAPTER 3

# Subspace Identification Methods

## 3.1 Subspace Identification

### 3.1.1 General Subspace Identification Problem

### 3.1.2 Geometric Tools

Matrices with Hankel structure Matrix Row/Column space Projections

### 3.1.3 Subspace Identification of Combined Deterministic-Stochastic Systems

## 3.2 Closed-Loop Subspace Identification

### 3.2.1 Identifying Systems Operating Under Feedback Control

### 3.2.2 Innovation Estimation Method

### 3.2.3 Whitening Filter

# CHAPTER 4

# Experiments

## 4.1 Experiment Design

# CHAPTER 5

# Results

# CHAPTER 6

# Conclusion

## 6.1 Future Work

# References


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