System Identification of UAV Based on EWC-LMS

Zijing Zhou, Jinchun Hu, Shiqian Liu, Badong Chen
Department of Computer Science and Technology, Tsinghua University,
100084, Beijing, P.R. China
Phone: 86-10-62796460, Email: Zhouzj03@mails.tsinghua.edu.cn

Abstract:

Mean-Squared Error (MES) can at best provide biased solution in the presence of additive disturbances (both correlated and white) on the input and the output signals. However, in the system identification of Unmanned Aerial Vehicle (UAV), typically, in the takeoff, the UAV suffers serious disturbance e.g. wind gusts, turbulence, ground influence and sampling noise. For EWC, we can get the unbiased parameter estimation in white noise, and the system identification in this paper prove the noise rejection ability of EWC.EWC (Error Whitening Criterion) was first put forward by Mr. Rao in 2002 and applied in filtering, and further developed theoretically. But EWC has not been applied in practical problems yet.

This paper applies EWC in the system identification of real self-developed UAV's takeoff motion, and compares the result of EWC-LMS with traditional LMS (Least Mean Square) and TLS (Total Least Square). The result shows that in comparison with traditional LMS and TLS, EWC-LMS improve the performance significantly.

Keyword: Error whitening criterion; LMS; UAV; System identification

I. INTRODUCTION

MSE is widely used in data process, control and system identification. The stochastic LMS and RLS are two popular algorithm based on MSE criterion in the system identification. However, MSE criterion based algorithm can at best achieve biased estimation at the existence of turbulence signals, either white noise or correlated. In the flying test, turbulence is unavoidable. Some methods was put forward intend to deal with noise signal, i.e. Subspace Wiener Filtering [HAY,96] based on Principal Subspace Analysis (PSA). However, these methods are quite costly, and become unaffordable when the system dimension increases. TLS can achieve unbiased estimation, only when the variances of input and output noise are equal [DOU,96]. For EWC, we can get the unbiased parameter estimation in white noise. The EWC [RAO,03] formulation enforces zero autocorrelation of the error signal beyond a certain lag, and hence the name Error Whitening Criterion, while MSE minimizes the mean-square error.

It is generally recognized now that the majority of UAV accidents take place during the launch and recovery cycles, and that most can be attributed to operator failure. This fact is pushing manufacturers and users alike in the direction of fully automated systems. And system identification is critical in the research of a fully autonomous flight control of the UAV.

In this paper, EWC-LMS is applied in the system identification of the UAV which suffers from serious turbulence. The Error Whitening Criterion and EWC-LMS algorithm is firstly introduced. In section 4 the UAV system is briefly described and some data pre-process methods are introduced and applied in the following identification. In section 5, with the measured flying test input and output data, we apply EWC-LMS in the identification of UAV's take-off. In section 6, a brief conclusion and discussion are given.

II. ERROR WHITENING CRITERION

Considering the UAV system identification problem in Fig1., the parameter vector is $\mathbf{w}_T = \Re^N$, where $(\mathbf{x}_k, d_k)$ are the real input and output of the system. Meanwhile, we assume u_k and $\mathbf{v}_k$ as the measurement error and system disturbance, without knowledge of their variance. Therefore, the problem can be described as: for certain noisy data pair $(\hat{\mathbf{x}}_k, \hat{d}_k)$, where $\hat{\mathbf{x}}_k = \mathbf{x}_k + \mathbf{v}_k$, $\hat{d}_k = d_k - u_k$, we are supposed to estimate the parameter vector $\mathbf{w} = \Re^M$. For the sake of generality, we assume the length of parameter vector w should be at least N, while the parameter vector of real system is $M \geq N$.

Since $d_k = \mathbf{x}_k^T \mathbf{w}_T$, the error should be

$$\hat{e}_k = \mathbf{x}_k^T(\mathbf{w}_T - \mathbf{w}) + u_k - \mathbf{v}_k^T \mathbf{w}$$

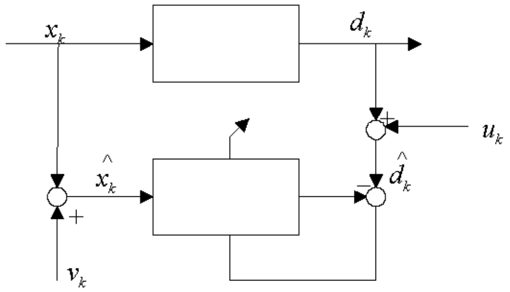

Fig.1 Scheme of system identification of UAV

Defining vector $\varepsilon = \mathbf{w}_T - \mathbf{w}$, for arbitrary lag L , the corresponding error autocorrelation can be determined as:

$$\rho_{\hat{e}}(L) = \varepsilon^T E[\mathbf{x}_k \mathbf{x}_{k-L}^T]\varepsilon + \mathbf{w}^T E[\mathbf{v}_k \mathbf{v}_{k-L}^T]\mathbf{w} \qquad (1)$$

Obviously, if $L \ge M$, then $E[\mathbf{v}_k \mathbf{v}_{k-L}^T] = 0$. And given matrix $E[\mathbf{x}_k \mathbf{x}_{k-L}^T]$ exists and is full rank, $\rho_{\hat{e}}(L) = 0$, only $\mathbf{w} = \mathbf{w}^T$. Consequently, for any $L \ge M$; when the error autocorrelation become zero, unbiased estimation of system parameter vector is achieved [RAO, 05]. In other words, EWC simply whitening error signals whose lag are larger than the length of the system, while the classical Wiener solution tries to minimize the zero-lag autocorrelation of the error.

Define: $\dot{\hat{e}} = (\hat{e}_k - \hat{e}_{k-L})$, Eq1. can be re-written as:

$$\mathrm{J}(\mathbf{w}) = E(\hat{e}_k^{\,2}) + \beta E(\dot{\hat{e}}_k^{\,2}) \qquad (2)$$

Where β is constant. It is obviously when $\beta = -0.5$, Eq2. equals to Eq1. And it is reduced to cost function of MSE when $\beta = 0$. Namely, MSE is just a specific situation of EWC.

Here we give the definition of following symmetric matrices:

$$\mathbf{R} = E[\mathbf{x}_k \mathbf{x}_k^T]$$

$$\hat{\mathbf{R}} = E[\hat{\mathbf{x}}_k \hat{\mathbf{x}}_k^T]$$

$$\mathbf{R}_L = E[\mathbf{x}_{k-L}\mathbf{x}_k^T + \mathbf{x}_k \mathbf{x}_{k-L}^T]$$

$$\hat{\mathbf{R}}_L = E[\hat{\mathbf{x}}_{k-L}\hat{\mathbf{x}}_k^T + \hat{\mathbf{x}}_k \hat{\mathbf{x}}_{k-L}^T]$$

The input noise autocorrelation matrices

$$\mathbf{V} = \mathbf{E}[\mathbf{v}_k \mathbf{v}_k^T]$$

$$\mathbf{V}_L = E[\mathbf{v}_{k-L}\mathbf{v}_k^T + \mathbf{v}_k \mathbf{v}_{k-L}^T]$$

The input derivative autocorrelation matrices

$$\mathbf{S} = E[\dot{\mathbf{x}}_k \dot{\mathbf{x}}_k^T]$$

$$\hat{\mathbf{S}} = E[\dot{\hat{\mathbf{x}}}_k \dot{\hat{\mathbf{x}}}_k^T]$$

For the cross-correlation matrices we can following the same structure, and figure out the correlation between the input vector and the desired signal by

$$\mathbf{P} = E[\mathbf{x}_k d_k]$$

$$\hat{\mathbf{P}} = E[\hat{\mathbf{x}}_k \hat{d}_k]$$

$$\mathbf{P}_L = E[\mathbf{x}_{k-L} d_k + \mathbf{x}_k d_{k-L}]$$

$$\hat{\mathbf{P}}_L = E[\hat{\mathbf{x}}_{k-L} \hat{d}_k + \hat{\mathbf{x}}_k \hat{d}_{k-L}]$$

The correlation between the input vector and desired signal derivatives

$$\mathbf{Q} = E[\dot{\mathbf{x}}_k \dot{d}_k]$$

$$\hat{\mathbf{Q}} = E[\dot{\hat{\mathbf{x}}}_k \dot{\hat{d}}_k]$$

We cite following theorem without proof. The detail proof is given by Rao in [RAO, 05].

Theorem 1: (Analytical EWC Solution) Supposed the training data are noisy, i.e., we have $(\hat{\mathbf{x}}_k, \hat{d}_k)$. The following are equivalent expressions for the stationary point of the EWC performance surface given in Eq.2.

$$\hat{\mathbf{w}}_* = (\hat{\mathbf{R}} + \beta \hat{\mathbf{S}})^{-1}(\hat{\mathbf{P}} + \beta \hat{\mathbf{Q}}) \qquad (3)$$

$$\hat{\mathbf{w}}_* = [(1+2\beta)\hat{\mathbf{R}} - \beta \hat{\mathbf{R}}_L]^{-1}[(1+2\beta)\hat{\mathbf{P}} - \beta \hat{\mathbf{P}}_L] \qquad (4)$$

Theorem 2: (Noise Rejection with EWC) If $\beta = -1/2$, $\mathbf{V}_L = 0$, and $\mathbf{R}_L$ is invertible, then the optimal solution of EWC obtained using noisy data is equal to the true weight vector of the reference model that generated the data.

In the system identification of UAV, since the output noise vector consists of delayed versions of the contaminating noise signal (we may only consider the white noise here), selecting $L \ge n$, where n is the order of the model, will guarantee the $\mathbf{V}_L = 0$.

III. EWC-LMS ALGORITHM

Substituted $\hat{e}_k = \hat{d}_k - \hat{\mathbf{x}}_k^T$, $\dot{\hat{e}}_k = \dot{\hat{d}}_k - \dot{\hat{\mathbf{x}}}_k^T$, we obtained following equations:

$$\mathrm{J}(\mathbf{w}) = E[\hat{e}_k^2] + \beta E[\dot{\hat{e}}_k^2]$$

$$= E[\hat{d}_k^2] + \beta E[\dot{\hat{d}}_k^2] + \mathbf{w}^T(\hat{\mathbf{R}} + \beta \hat{\mathbf{S}})\mathbf{w} - \mathbf{2}(\hat{\mathbf{P}} + \beta \hat{\mathbf{Q}})^T \mathbf{w}$$

(5)

Taking the gradient with respect to $\mathbf{w}$ and equating to zero:

$$\frac{\partial J(\mathbf{w})}{\partial \mathbf{w}} = 2(\hat{\mathbf{R}}+\beta\hat{\mathbf{S}})\mathbf{w} - \mathbf{2}(\hat{\mathbf{P}}+\beta\hat{\mathbf{Q}}) = 0$$

Namely:

$$\hat{\mathbf{w}}_* = (\hat{\mathbf{R}}+\beta\hat{\mathbf{S}})^{-1}(\hat{\mathbf{P}}+\beta\hat{\mathbf{Q}}) \qquad (6)$$

We can remove the expectation operators to obtain the stochastic instantaneous gradient Eq.7 for the approximation of the exact gradient.

$$\frac{\partial J(\mathbf{w})}{\partial \mathbf{w}} \approx -2(\hat{e}_k \hat{\mathbf{x}}_k + \beta \hat{\dot{e}}_k \hat{\dot{\mathbf{x}}}_k) \qquad (7)$$

Generally, optimization problems look for the maximum or minimum to obtain optimum value. However, for EWC, the stationary point might be the saddle point, consequently, the traditional fixed-sign gradient algorithm failed to arrive at the stationary point.

Therefore, we take the local curvature of the performance surface into consideration, and modify the gradient accordingly to converge to the desired saddle point.

For the offline training, the convexity of a local parabola passing through the current weight vector and extending along the gradient direction could be used to determine the sign. We take negative gradient if parabola is convex, vice versa. But we have to keep track of the input covariance matrices $\mathbf{R}$ and $\mathbf{S}$, which is costly. Here we utilize the stochastic estimation of the cost of EWC at every update to determine the sign. This yields the EWC-LMS algorithm in below.

$$\mathbf{w}_{k+1} = \mathbf{w}_k + \eta_k \operatorname{sign}(\hat{e}_k^2+\beta\hat{\dot{e}}_k^2)(\hat{e}_k\hat{\mathbf{x}}_k+\beta\hat{\dot{e}}_k\hat{\dot{\mathbf{x}}}_k)$$

(6)

The convergence analysis is given in [RAO, 05].

IV UAV MODEL

We've completed the development of the prototype of UAV, which have to takeoff and land under manual control and is autonomous during the flight which incurs the majority of UAV accidents. The purpose of our research is to realize the fully autonomous flight control

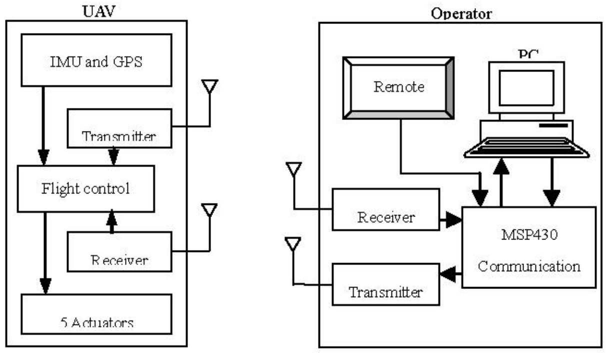

Fig.2 Configuration of the UAV

of UAV. Therefore, precise modeling is a critical process in the whole research especially the takeoff and land motion. However, UAV suffered from serious disturbance especially in the takeoff and land process, consequently, we need to choose a system identification algorithm to eliminate the bias incurred by these white and correlated noise. And here we adopt EWC-LMS which proved to be effective in the simulation result.

A. System description

The structure of the self-developed UAV system is illustrated in Fig2.

The autonomous flight of UAV is under the control of the flight control computer. Installing the communication module, UAV is able to takeoff and land under the remote control from the operator on ground.

Here we adopt Auto-Regressive Moving Average exogenous (ARMAX) model to describe the dynamics of UAV's take-off process.

TABLE I PERFORMANCE OF UAV

Gross Weight	50kg
Payload Weight	5kg
Endurance	4.5h
Radius	50km
Max Speed	120km/h
Stall Speed	100km/h
Takeoff Means	Runway
Landing Means	Runway
Navigation	Preprogrammed (points)/autonomous/direct control

B. Data pre-process

In 3 flying test, we obtained 3 sets of test data for the take-off motion under manual control.

All these data was collected by the anemometer, barometer, gyro sensor, GPS, underwent the processes of amplification, coding, modulation, emitting, receiving and demodulation incurring serious measured errors. Consequently, it is rewarding to revise the test data before system identification.

In this paper, following pre-processing methods are adopted:

Outlier detection and correction

In the flying test, environment disturbance and accidentally discontinuity of instrument may bring in unreasonable point in the flying test, here we call outlier. We set trust probability and trust interval. And we consider a point as outlier, if the corresponding error exceed the interval. All these points will be deleted from the data set.

For the steady state experiments, we can simply clarify those points whose deviation $v(i)$ exceed 3 times of the standard deviation σ as outlier.

However for the flying test, the data are time-varying, the expectation of observation can not be determined by average. Therefore, we adopt polynomial sliding fitting to detect outliers.

Avoiding back-propagation of the outlier, we use 7 points 2 orders difference algorithm.

$$\hat{y}_1 = \frac{32y_1 + 15y_2 + 3y_3 - 4y_4 - 6y_5 - 3y_6 + 5y_7}{42}$$

$$\hat{y}_2 = \frac{5y_1 + 4y_2 + 3y_3 + 2y_4 + y_5 - y_7}{14}$$

$$\hat{y}_3 = \frac{y_1 + 3y_2 + 4y_3 + 4y_4 + 3y_5 + y_6 - 2y_7}{14}$$

$$\hat{y}_4 = \frac{-y_1 + 3y_2 + 6y_3 + 7y_4 + 6y_5 + 3y_6 - 2y_7}{21}$$

$$\hat{y}_5 = \frac{-2y_1 + y_2 + 3y_3 + 4y_4 + 4y_5 + 3y_6 + y_7}{14}$$

$$\hat{y}_6 = \frac{-y_1 + y_2 + 2y_4 + 3y_5 + 4y_6 + 5y_7}{14}$$

$$\hat{y}_i = \frac{5y_{i-6} - 3y_{i-5} - 6y_{i-4} - 4y_{i-3} + 3y_{i-2} + 15y_{i-1} + 32y_i}{42}$$

$$(i = 7, 8, 9, \cdots, N)$$

Computing $\hat{y}_i$ and residual $v_i = y_i - \hat{y}_i$ consecutively, we detect outliers with the equation below

$$\left| y_k - \hat{y}_k \right| > 2.2\sqrt{\sum (y_i - \hat{y}_i)^2 / 6} \equiv E$$

However for the consecutive outliers, when $\left| y_{k+i} - y_K \right| < E \quad (i = 1, 2, \cdots, m)$, and $m > 3$, $y_k, y_{k+1}, \cdots, y_{k+m}$ are no longer considered as outliers.

Those determined outlier $y_k, y_{k+1}, \cdots, y_{k+m}$ are substitute with correction data computed by Lagrange interpolating polynomial

$$y_l = \sum_{j=k-3}^{k+m+3} \prod_{i=k-3}^{k+m+3} \frac{t_l - t_i}{t_j - t_i} y_i$$

$$(i, j \neq k, k+1, \cdots, k+m, i \neq j, l = k, k+1, \cdots, k+m)$$

Low pass data filter

The frequency of UAV is generally less than 10Hz. However the test data usually contain high frequency part, therefore low pass data filter is quite necessary.

2 orders low pass digit filter is adopted here, with pass-band cut-off frequency ω_p, stop-band cut-off frequency ω_q, sample period T

$$\bar{y}(n) = \frac{1}{A}\{\bar{\omega}_p^2[y(n) + 2y(n-1) + y(n-2)]$$
$$-[C\bar{y}(n-1) + B\bar{y}(n-2)]\}$$

Where, y is the real test data, $\bar{y}$ is the resulting data of the filter.

$$\bar{\omega}_p = \tan\frac{\omega_p T}{2}$$

$$\bar{\omega}_q = \tan\frac{\omega_q T}{2}$$

$$A = 1 + 1.414\bar{\omega}_p + \bar{\omega}_p^2$$

$$B = 1 - 1.414\bar{\omega}_p + \bar{\omega}_p^2$$

$$C = 2(\bar{\omega}_p^2 - 1)$$

V. SYSTEM IDENTIFICATION

We record one set of complete flying test data. In the first step, the data pre-process methods are applied to the test data to eliminate some of the common errors in the

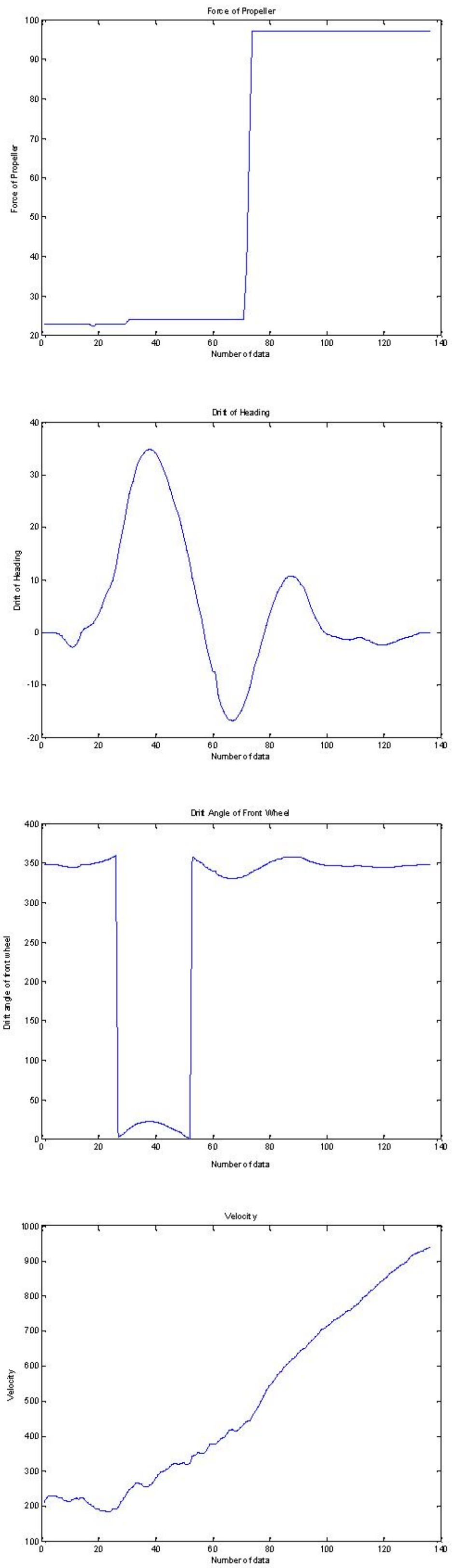

Fig.3 Pre-processed data for system identification

system identification of aerial vehicle. We obtain the processed data illustrated in the figures below.

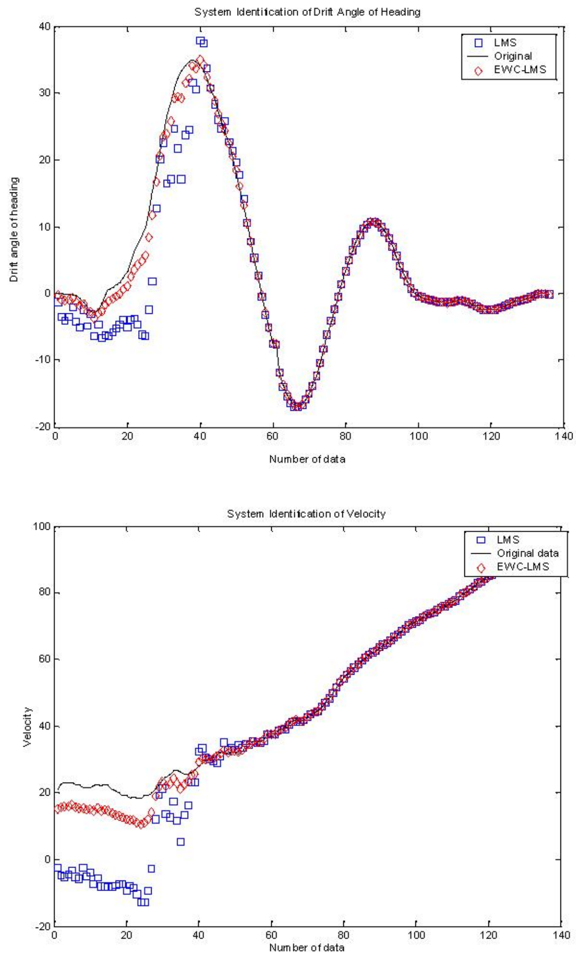

Fig.4 System identification with ARMAX model via LMS and EWC-LMS

We will now verify the performance improvement of EWC-LMS algorithm in the system identification problem. With the model of ARMAX mentioned before. Fig4 illustrates time responses of the 5 step predicted output of the ARMAX model and the measured output. Here, we apply both traditional LMS and EWC-LMS for the identification. The results showed the superiority of the EWC-LMS over the LMS equivalents in the application of UAV identification. As we have stated before, this is due to the noise rejection ability of EWC. EWC-LMS has outstanding performance at low SNR values, while the UAV is just a typical case for low SNR value.

VI. CONCLUSION

In this paper, we apply EWC to the system identification of UAV. Theoretically, the principal advantage for EWC criterion is its ability to estimate the parameters of a given noisy input and desired signal, especially at low SNR value. Here, we apply EWC-LMS to the pragmatic system identification of UAV. The result proves the effectiveness of EWC-LMS. Further more, the modeling result will benefit the future research of the autonomous take-off of UAV.

ACKNOWLEDGMENTS

This work is supported by National Aviation Cooperation Research Foundation of P.R. China Grant 10577012. The authors would like to acknowledge the research group of unmanned aerial vehicle in Tsinghua University.

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