

UNIVERSITY OF CALIFORNIA  
Los Angeles

# **Closed-Loop Subspace Identification of a Quadrotor**

A thesis submitted in partial satisfaction  
of the requirements for the degree  
Master of Science in Engineering

by

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ABSTRACT OF THE THESIS

# Closed-Loop Subspace Identification of a Quadrotor

by

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Master of Science in Engineering

University of California, Los Angeles, 2013

Professor Steve Gibson, Chair

Ne quo feugiat tractatos temporibus, eam te malorum sensibus. Impetus voluptua  
senserit id mel, lucilius adipiscing at duo. Suas noster sanctus cu pro, movet  
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omnesque ex, essent percipitur intellegam an per. Iusto expetenda sed et.

The thesis of Andrew G. Kee is approved.

Steve Gibson, Committee Chair

University of California, Los Angeles

2013

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## NOMENCLATURE

$N$	Number of block rows
$s$	Number of block columns
$\mathbb{R}$	Set of all real numbers
$u$	System input sequence
$y$	System output sequence
$x$	System state sequence

# CHAPTER 1

## Introduction

Unmanned Aerial Vehicles (UAVs) have seen explosive growth in the past thirty years, performing a multitude of military and civilian tasks including surveillance, reconnaissance, armed combat operations, search and rescue, forest fire management, and domestic policing [9, 10]. A class of modern UAVs which have recently grown in popularity are quadrotors - Vertical Take Off and Landing (VTOL) vehicles powered by four rotors arranged in a cross configuration. The main advantage of the quadrotor lies in its mechanical simplicity. Adjusting the speed of one or more of the vehicle's fixed-pitch rotors provides full attitude control, eliminating the need for the swash plate mechanism found on single rotor helicopters [1, 2]. In spite of its mechanical simplicity, the quadrotor exhibits somewhat complex dynamics that are best modeled as a Multi-Input Multi-Output (MIMO) system.

Advances in MEMS sensors and light-weight high-powered lithium polymer batteries have contributed to the recent popularity of quadrotors, making them an attractive choice for research applications in flight dynamics and control, as in [3, 5, 6, 7]. One problem of particular interest is the development of mathematical models representing system dynamics based on experimentally gathered data. System identification provides a mechanism to relate this input-output data to the underlying system dynamics. Traditionally, system identification techniques have focused on developing a system model which minimizes prediction error. Identification methods of this form are commonly known as Prediction Error Methods (PEMs). PEMs have seen widespread use in both theoretical and real-world ap-

plications, but experience difficulties with MIMO systems as noted in [8, 11]. Subspace identification methods have recently grown in popularity and offer an alternative approach to the identification problem. These methods have a foundation in linear algebra and overcome the issues found in PEMs when identifying MIMO systems [4]. It is the goal of this research project to apply subspace identification techniques to a quadrotor using experimentally gathered closed-loop input and output data.

## 1.1 Related Work

For text, let's use the first words out of the ispell dictionary.

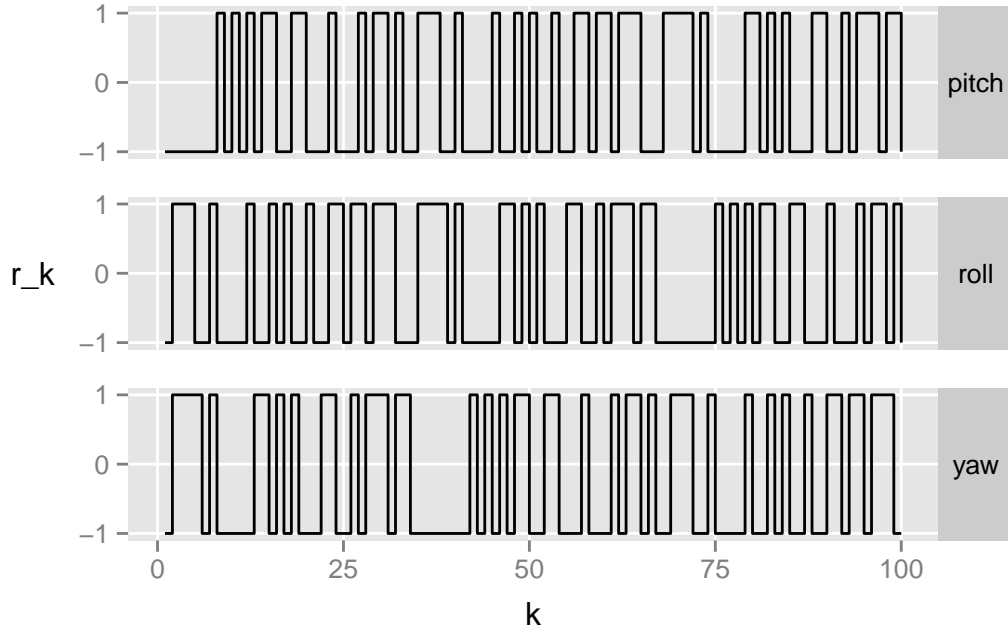


Figure 1.1: A figure caption

## 1.2 Motivation and Contributions

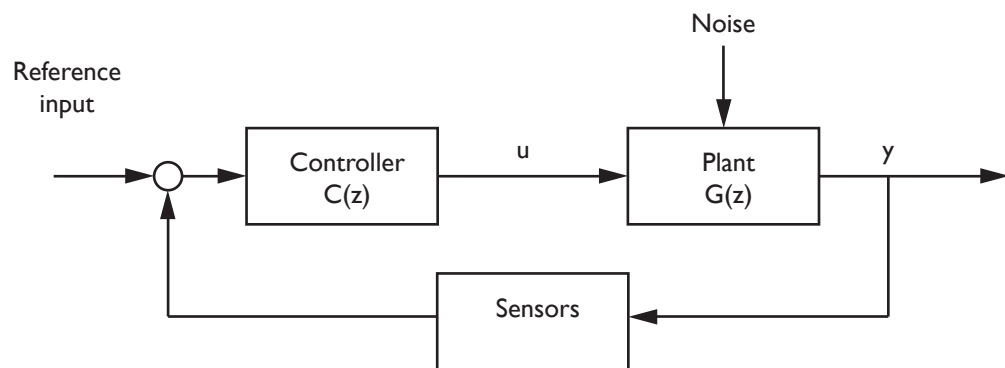


Figure 1.2: A figure caption

# CHAPTER 2

## Model and Assumptions

Introduce concept of LTI system here

### 2.1 State Space Model

We will consider a combined deterministic-stochastic LTI system written in innovation form as

$$x_{k+1} = Ax_k + Bu_k + Ke_k \quad (2.1a)$$

$$y_k = Cx_k + Du_k + e_k \quad (2.1b)$$

where  $x_k \in \mathbb{R}^n$  is the system state,  $u_k \in \mathbb{R}^m$  is the system input,  $y_k \in \mathbb{R}^l$  is the system output, and  $e_k \in \mathbb{R}^l$  is the innovation.  $A$ ,  $B$ ,  $C$ , and  $D$  are the system matrices with appropriate dimensions and  $K$  is the Kalman filter gain. The system represented in (2.1) can also be represented in predictor form as

$$x_{k+1} = A_K x_k + B_K u_k + K y_k \quad (2.2a)$$

$$y_k = Cx_k + Du_k + e_k \quad (2.2b)$$

where  $A_K = A - KC$  and  $B_K = B - KD$ .

The systems represented by (2.1) and (2.2) are equivalent from an input/output point of view, but because  $A_K$  is guaranteed stable even if the original process matrix  $A$  is unstable, the predictor form proves advantageous when considering unstable open-loop systems. We will use the state space model in innovation form

to derive the general subspace algorithm for identifying combined deterministic-stochastic LTI systems but will rely on the prediction form of the model when considering identification of closed-loop systems.

## 2.2 Assumptions

**[Assumption 1]:**  $A_K = A - KC$  is stable (i.e. its eigenvalues lie within the unit circle)

**[Assumption 2]:** The system is represented in its minimal form

## CHAPTER 3

### Subspace Identification Methods

Subspace identification methods (SIM) provide an approach to identifying an unknown LTI system in its state space form using input/output data. Subspace methods rely on linear algebra techniques to extract the system matrices from the column space of the system's extended observability matrix. Considering the combined deterministic-stochastic LTI system introduced in the previous chapter, the subspace identification problem is: given a set of input and output data, estimate the system matrices  $A$ ,  $B$ ,  $C$ , and  $D$  and the Kalman filter gain  $K$  up to within a similarity transform.

#### 3.1 Linear Algebra Tools

##### Hankel Matrices

A Hankel matrix is a matrix  $H \in \mathbb{R}^{m \times n}$  with constant skew-diagonals. In other words, the value of the  $(i, j)^{\text{th}}$  entry of  $H$  depends only on the sum  $i + j$ .

$$H_{m,n} = \begin{bmatrix} h_1 & h_2 & \cdots & h_n \\ h_2 & h_3 & \cdots & h_{n+1} \\ \vdots & \vdots & \ddots & \vdots \\ h_m & h_{m+1} & \cdots & h_{m+n-1} \end{bmatrix}$$

If each entry in the matrix is also a matrix, it is called a block Hankel matrix.

## Fundamental Matrix Subspaces

We require two of the fundamental matrix subspaces: the column space and the row space. The column space of a matrix  $A \in \mathbb{R}^{m \times n}$  is the set of all linear combinations of the column vectors of  $A$ . The dimension of the column space is called the rank. The row space of a matrix  $A \in \mathbb{R}^{m \times n}$  is the set of all linear combinations of the row vectors of  $A$ .

## Projections

### Singular Value Decomposition

Any matrix  $A \in \mathbb{R}^{m \times n}$  can be decomposed by a singular value decomposition (SVD) given by

$$A = U\Sigma V^T$$

where  $U \in \mathbb{R}^{m \times m}$  and  $V \in \mathbb{R}^{n \times n}$  are orthogonal matrices and  $\Sigma \in \mathbb{R}^{m \times n}$  is diagonal matrix of the singular values of  $A$  ordered such that

$$\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_k > 0$$

## 3.2 Subspace Identification of Combined Deterministic-Stochastic Systems

Considering the combined deterministic-stochastic LTI system in innovation form

$$x_{k+1} = Ax_k + Bu_k + Ke_k \tag{3.1a}$$

$$y_k = Cx_k + Du_k + e_k \tag{3.1b}$$

A detailed derivation of this procedure follows



### **3.3 Closed-Loop Subspace Identification**

#### **3.3.1 Identifying Systems Operating Under Feedback Control**

#### **3.3.2 Innovation Estimation Method**

#### **3.3.3 Whitening Filter**

## CHAPTER 4

### Experiments

#### 4.1 Experiment Design

## CHAPTER 5

### Results

## CHAPTER 6

### Conclusion

#### 6.1 Future Work

## REFERENCES

- [1] Anthony RS Bramwell, David Balmford, and George Done. *Bramwell's helicopter dynamics*. Butterworth-Heinemann, 2001.
- [2] Shweta Gupte, Paul Infant Teenu Mohandas, and James M Conrad. A survey of quadrotor unmanned aerial vehicles. In *Southeastcon, 2012 Proceedings of IEEE*, pages 1–6. IEEE, 2012.
- [3] Gabriel M Hoffmann, Haomiao Huang, Steven L Waslander, and Claire J Tomlin. Quadrotor helicopter flight dynamics and control: Theory and experiment. In *Proc. of the AIAA Guidance, Navigation, and Control Conference*, pages 1–20, 2007.
- [4] Tohru Katayama. *Subspace methods for system identification*. Springer, 2005.
- [5] Arda Özgür Kivrak. Design of control systems for a quadrotor flight vehicle equipped with inertial sensors. *Atilim University, December*, 2006.
- [6] Daniel Mellinger, Michael Shomin, and Vijay Kumar. Control of quadrotors for robust perching and landing. In *Proc. Int. Powered Lift Conf*, pages 119–126, 2010.
- [7] Nathan Michael, Daniel Mellinger, Quentin Lindsey, and Vijay Kumar. The grasp multiple micro-uav testbed. *Robotics & Automation Magazine, IEEE*, 17(3):56–65, 2010.
- [8] S Joe Qin. An overview of subspace identification. *Computers & chemical engineering*, 30(10):1502–1513, 2006.
- [9] Zak Sarris and STN ATLAS. Survey of uav applications in civil markets (june 2001). In *The 9 th IEEE Mediterranean Conference on Control and Automation (MED'01)*, 2001.
- [10] Kimon P Valavanis. *Advances in unmanned aerial vehicles: state of the art and the road to autonomy*, volume 33. Springer, 2007.
- [11] Mats Viberg. Subspace-based methods for the identification of linear time-invariant systems. *Automatica*, 31(12):1835–1851, 1995.