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Closed-Loop Subspace Identification of a Quadrotor

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by

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Abstract of the Thesis

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Master of Science in Engineering

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Professor Steve Gibson, Chair

Ne quo feugiat tractatos temporibus, eam te malorum sensibus. Impetus voluptua senserit id mel, lucilius adipiscing at duo. Suas noster sanctus cu pro, movet dicam intellegebat pri ad, esse utamur vulputate ut per. Admodum facilisi sea an, omittam molestiae pertinacia vim eu, et eam illud graeco. Enim persius duo ei, mea te posse congue putent.

Has ut commodo inermis definiebas. Eu pro audire fabellas. Te qui zril omnes forensibus, ius an dico tempor maiorum. Ea mel illud adipisci.

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The thesis of Andrew G. Kee is approved.

Steve Gibson, Committee Chair

University of California, Los Angeles

2013

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Nomenclature

N	Number of block rows
s	Number of block columns
$\mathbb{R}$	Set of all real numbers
u_k	System input sequence
y_k	System output sequence
x_k	System state sequence
f	Future time horizon
p	Past time horizon

CHAPTER 1

Introduction

Unmanned Aerial Vehicles (UAVs) have seen explosive growth in the past thirty years, performing a multitude of military and civilian tasks including surveillance, reconnaissance, armed combat operations, search and rescue, forest fire management, and domestic policing [12, 13]. A class of modern UAVs which have recently grown in popularity are quadrotors - Vertical Take Off and Landing (VTOL) vehicles powered by four rotors arranged in a cross configuration. The main advantage of the quadrotor lies in its mechanical simplicity. Adjusting the speed of one or more of the vehicle's fixed-pitch rotors provides full attitude control, eliminating the need for the swash plate mechanism found on single rotor helicopters [1, 3]. In spite of its mechanical simplicity, the quadrotor exhibits somewhat complex dynamics that are best modeled as a Multi-Input Multi-Output (MIMO) system.

Advances in MEMS sensors and light-weight high-powered lithium polymer batteries have contributed to the recent popularity of quadrotors, making them an attractive choice for research applications in flight dynamics and control, as in [4, 6, 8, 9]. One problem of particular interest is the development of mathematical models representing system dynamics based on experimentally gathered data. System identification provides a mechanism to relate this input-output data to the underlying system dynamics. Traditionally, system identification techniques have focused on developing a system model which minimizes prediction error. Identification methods of this form are commonly known as Prediction Error Methods (PEMs). PEMs have seen widespread use in both theoretical and real-world applications, but experience difficulties with MIMO systems as noted in [10, 17]. Subspace identification methods have recently grown in popularity and offer an alternative approach to the identification problem. These methods have a foundation in linear algebra and overcome the issues found in PEMs when identifying MIMO systems [5]. It is the goal of this research project to apply subspace identification techniques to a quadrotor using experimentally gathered closed-loop input and output data.

1.1 Related Work

For text, let's use the first words out of the ispell dictionary.

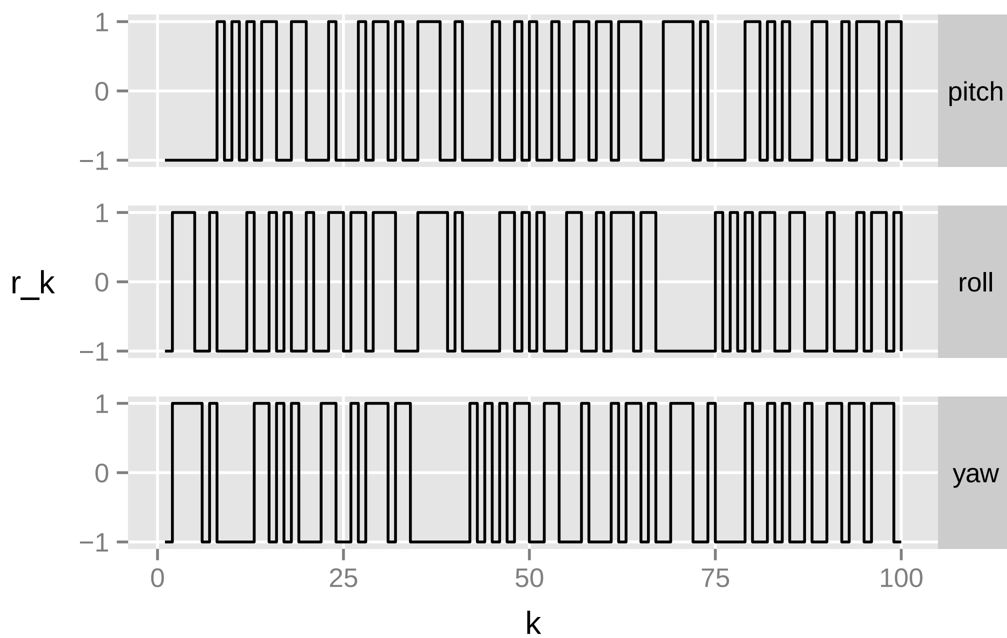

Figure 1.1: A figure caption

1.2 Motivation and Contributions

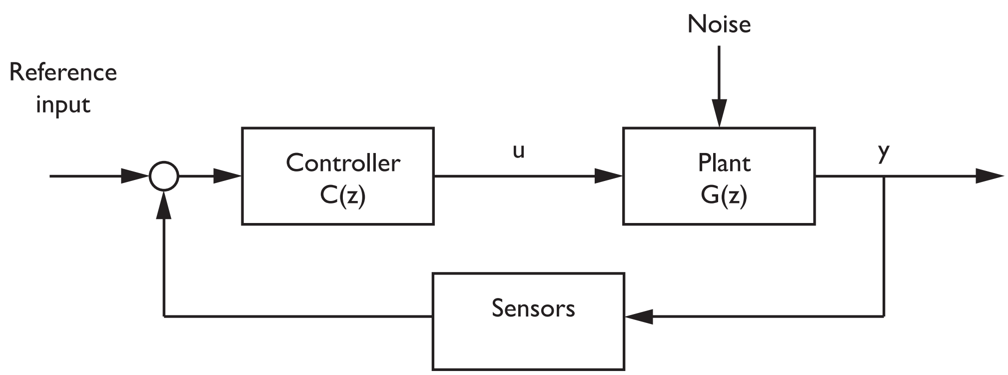

Figure 1.2: A figure caption

CHAPTER 2

Model and Assumptions

Introduce concept of LTI system here

2.1 State Space Model

We will consider a combined deterministic-stochastic LTI system written in innovation form as

$$x(k+1) = Ax(k) + Bu(k) + Ke(k) \tag{2.1a}$$

$$y(k) = Cx(k) + Du(k) + e(k) \tag{2.1b}$$

where $x_k \in \mathbb{R}^n$ is the system state, $u_k \in \mathbb{R}^m$ is the system input, $y_k \in \mathbb{R}^l$ is the system output, and $e_k \in \mathbb{R}^l$ is the innovation. A, B, C, and D are the system matrices with appropriate dimensions and K is the Kalman filter gain. The system represented in (2.1) can also be represented in predictor form as

$$x(k+1) = A_K x(k) + B_K u(k) + Ky(k) \tag{2.2a}$$

$$y(k) = Cx(k) + Du(k) + e(k) \tag{2.2b}$$

where $A_K = A - KC$ and $B_K = B - KD$.

The systems represented by (2.1) and (2.2) are equivalent from an input/output point of view, but because A_K is guaranteed stable even if the original process matrix A is unstable, the predictor form proves advantageous when considering unstable open-loop systems. We will use the state space model in innovation form to derive the general subspace algorithm for identifying combined deterministic-stochastic LTI systems but will rely on the prediction form of the model when considering identification of closed-loop systems.

2.2 Assumptions

[**Assumption 1**]: $A_K = A - KC$ is stable (i.e. its eigenvalues lie within the unit circle)

[**Assumption 2**]: The system is represented in its minimal form

CHAPTER 3

Subspace Identification Methods

Subspace identification methods (SIM) provide an approach to identifing LTI systems in their state space form using input-output data. SIMs provide an attractive alternative to Prediction Error Methods (PEM) because of their ability to identify MIMO systems and their non-iterative solution nature, making them suitable for working with large data sets. In general, the subspace identification problem is: given a set of input and output data, estimate the system matrices (A, B, C, D) up to within a similarity transform.

Extensive work in both the theory and application of SIMs in the last 20 years has resulted in the development of a number of popular algorithms, including the canonical variate analysis (CVA) method proposed by Larimore [7], the multi-variable output-error state space (MOESP) method proposed by Verhaegen [16], and the numerical algorithms for subspace state space system identification (N4SID) proposed by Van Overschee and De Moor [14]. A unifying theorem proposed by Van Overschee and De Moor [15] links these algorithms and provides a generalized approach to the subspace identification problem.

As described in Van Overschee and De Moor's unifying theorem, all SIMs follow the same general two step procedure. First, estimate the subspace spanned by the columns of the extended observability matrix (Γ_k) from input-output data u_k, y_k. The dimension of Γ_k determines the order n of the estimated system. The system order is determined and Γ_k reduced accordingly. Second, the system matrices are determined, either directly from the extended observability matrix or from the realized state sequence X_k.

Until recently, SIMs were unable to identify systems operating in the presence of feedback control(i.e. closed-loop). In the open-loop case, input data is uncorrelated with past noise. When a system is operating in closed-loop, the presence of feedback control causes the input to be correlated with past noise as the controller attempts to eliminate system disturbances [10]. The result is a system that is biased when identified using traditional SIMs. Several new approaches to identifying closed-loop systems by decoupling inputs from past noise have been proposed, most notably the innovation estimation method (IEM) proposed by Qin and Ljung [11] and the whitening filter approach (WFA) proposed by Chiuso and Picci [2].

3.1 Open-Loop Subspace Identification

When a system is operating in open-loop (i.e. no feedback), the input data is assumed to be independent of past noise. In this case, the traditional SIMs (MOESP, N4SID, CVA) can be used without modification.

3.1.1 Extended State Space Model

Recalling the combined deterministic-stochastic LTI system is given in its innovation form as

$$x(k+1) = Ax(k) + Bu(k) + Ke(k) \tag{3.1a}$$

$$y(k) = Cx(k) + Du(k) + e(k) \tag{3.1b}$$

Based on the state space model in (3.1), an extended state space model can be formulated as

$$Y_f = \Gamma_f X_k + H_f U_f + G_f E_f \tag{3.2}$$

where the subscript f denotes the future horizon. The extended observability matrix is

$$\Gamma_f = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{f-1} \end{bmatrix} \tag{3.3}$$

and H_f and G_f are Toeplitz matrices of the Markov parameters of the deterministic and stochastic subsystems, respectively

$$H_f = \begin{bmatrix} D & 0 & 0 & \cdots & 0 \\ CB & D & 0 & \cdots & 0 \\ CAB & CB & D & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ CA^{f-2}B & CA^{f-3}B & CA^{f-4}B & \cdots & D \end{bmatrix} \tag{3.4a}$$

$$G_f = \begin{bmatrix} I & 0 & 0 & \cdots & 0 \\ CK & I & 0 & \cdots & 0 \\ CAK & CK & I & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ CA^{f-2}K & CA^{f-3}K & CA^{f-4}K & \cdots & I \end{bmatrix} \tag{3.4b}$$

Equation (3.2) relates matrices of input-output data to matrices of the system matrices. We will leverage this structure to identify the unknown system matrices from known input-output data. In particular, we will estimate the column space of the extended observability matrix. Knowledge of this subspace is sufficient to then estimate the unknown system matrices.

3.1.2 Input-Output Data

The structure of (3.2) requires the input, output, and noise sequences be represented in block Hankel form. For the input sequence

$$U_f = \begin{bmatrix} u(k) & u(k+1) & \cdots & u(k+N-1) \\ u(k+1) & u(k+2) & \cdots & u(k+N) \\ \vdots & \vdots & \ddots & \vdots \\ u(k+f-1) & u(k+f) & \cdots & u(k+f+N-2) \end{bmatrix} \tag{3.5}$$

We similarly arrange the output data Y_f and noise E_f according to the same Hankel form.

!!!!! Need to discuss past and future splitting !!!!!

$$Z_p = \begin{bmatrix} U_p \\ Y_p \end{bmatrix}$$

3.1.3 Estimation of the Extended Observability Matrix

Estimate Γ_f from (3.2) via MOESP: linear regression followed by an SVD to estimate EOM.

First, eliminate U_f by post-multiplying (3.2) by $\Pi_{U_f}^{\perp}$ (eliminate the influence of the input) giving

$$Y_f \Pi_{U_f}^{\perp} = \Gamma_f X_k \Pi_{U_f}^{\perp} + G_f E_f \Pi_{U_f}^{\perp} \tag{3.6}$$

Recalling E_f is uncorrelated with U_f,

$$Y_f \Pi_{U_f}^{\perp} = \Gamma_f X_k \Pi_{U_f}^{\perp} + G_f E_f \tag{3.7}$$

Next we eliminate the influence of the noise term E_f. From Kalman filter theory, E_f is uncorrelated with Z_p (cite Qin overview paper on this one):

$$\lim_{N\to\infty} \frac{1}{N} E_f Z_p^T = 0$$

Thus multiplying (3.7) on the right by Z_p gives

$$Y_f \Pi_{U_f}^{\perp} Z_p = \Gamma_f X_k \Pi_{U_f}^{\perp} Z_p \tag{3.8}$$

3.1.4 Determination of the System Matrices

3.2 Closed-Loop Subspace Identification

Under open-loop conditions, E_f is uncorrelated with U_f. That is,

$$\lim_{N \to \infty} \frac{1}{N} E_f U_f^T = 0$$

or

$$E_f \Pi_{U_f}^{\perp} = E_f (I - U_f^T (U_f U_f^T)^{-1} U_f) = E_f$$

3.2.1 Identifying Systems Operating Under Feedback Control

3.2.2 Innovation Estimation Method

3.2.3 Whitening Filter

CHAPTER 4

Experiments

4.1 Experiment Design

CHAPTER 5

Results

CHAPTER 6

Conclusion

6.1 Future Work

APPENDIX A: LINEAR ALGEBRA TOOLS

6.2 Linear Algebra Tools

Hankel Matrices

A Hankel matrix is a matrix $H \in \mathbb{R}^{m \times n}$ with constant skew-diagonals. In other words, the value of the $(i,j)^{\text{th}}$ entry of H depends only on the sum $i+j$.

$$H_{m,n} = \begin{bmatrix} h_1 & h_2 & \cdots & h_n \\ h_2 & h_3 & \cdots & h_{n+1} \\ \vdots & \vdots & \ddots & \vdots \\ h_m & h_{m+1} & \cdots & h_{m+n-1} \end{bmatrix}$$

If each entry in the matrix is also a matrix, it is called a block Hankel matrix.

Fundamental Matrix Subspaces

We require two of the fundamental matrix subspaces: the column space and the row space. The column space of a matrix $A \in \mathbb{R}^{m \times n}$ is the set of all linear combinations of the column vectors of A. The dimension of the column space is called the rank. The row space of a matrix $A \in \mathbb{R}^{m \times n}$ is the set of all linear combinations of the row vectors of A.

Projections

Singular Value Decomposition

Any matrix $A \in \mathbb{R}^{m \times n}$ can be decomposed by a singular value decomposition (SVD) given by

$$A = U \Sigma V^T$$

where $U \in \mathbb{R}^{m \times m}$ and $V \in \mathbb{R}^{n \times n}$ are orthogonal matrices and $\Sigma \in \mathbb{R}^{m \times n}$ is diagonal matrix of the singular values of A ordered such that

$$\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_k > 0$$

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