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Parameter Identification of an Autonomous Quadrotor

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Abstract—**This paper describes one of possible parameter identification approach for a quadrotor. The unknown parameter of the quadrotor will be identified using state estimation method with the implementation of *Unscented Kalman Filter* (UKF). In the identification of state and parameter for nonlinear dynamic system, UKF has grown to be superior techniques. Two main processes highlighted in this paper are dynamic modeling of quadrotor and the implementation of UKF algorithm. The aim is to identify and estimate the needed parameters for an autonomous quadrotor. The obtained results demonstrate the performance of UKF based on the flight test applied to the quadrotor system.**

Keywords-component; Unscented Kalman Filter (UKF); State and Parameter Estimation; Autonomous Quadrotor.

I. INTRODUCTION

In general, for nonlinear sytems, the most extensively applied algorithm for parameter identification is *Extended Kalman Filter* (EKF). However, if the systems have severe nonlinearities, EKF can be hard to tune and often gives unreliable estimation. It is due to the linearization relies by the EKF in order to propagate the mean and covariance of the states. Thus, the UKF is introduced. [1][2]

The UKF is proposed by *Julier and Ulhman* and was first published in 1995. Since then, it has been expounded upon in many publications.[2][3] A deterministic sampling techniques known as the *Unscented Transform* (UT) is employs by UKF. The UT is used to pick a minimal set of sample points (called sigma points) around the mean. Then, these sigma points are propagated through the nonlinear function before the covariance of the estimate is then recovered. [1]

The system studied in this research is a four-rotor *Unmanned Air Vehicles* (UAV's) called as quadrotor. Quadrotor as illustrated in Figure 1, is referring to a small agile vehicle that has four rotors located at the front, rear, left, and right ends of a cross frame. It requires no cyclic or collective pitch. Quadrotor can be highly maneuverable, has the potential to hover, take off, fly and land in small areas and needs only simple control mechanisms. It is mechanically simple and is controlled by only changing the speed of rotation for the four rotors. [4][5]

Figure 1. Quadrotor Flying-robot

The quadrotor used as the testbed in this paper is a modified commercially available *Remote Controlled* (R/C) quadrotor. The electronics that come with the R/C quadrotor is replaced with a new on-board component. It consists of a flight stabilizer sensor which is a six degree of freedom *Inertial Measurement Unit* (IMU), an ultrasonic sensor, a transducer and a microcontroller. The IMU equipped in the quadrotor consists of three analog gyros and three analog accelerometers. The IMU is used to directly measure the Euler angles and the angular rates for x, y, z axes respectively. The test data obtained from the running flight test are then used in the UKF algorithm to provide accurate aerodynamic parameters.

The working method started by running the model with a set of flights; take off, hover and landing in an indoor structured environment, starting a data acquisition from trim condition, and ends with treating the output data using UKF implemented in MATLAB.

This paper begins with quadrotor dynamic modeling in Section II. We then discussed state estimation and parameter identification in Section III. This section also analyzed the results obtained, in form of values and graphs. Section IV will conclude the analysis.

II. QUADROTOR DYNAMIC MODELLING

In order to model the quadrotor dynamics, two frames have to be defined, which are the earth inertial frame and the body-fixed frame. The dynamics model equations are based on Newton-Euler formalism, where the nonlinear dynamics is obtained in both frame defined earlier. A concept of quadrotor

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is shown in Figure 2. Each rotor produces a lift force and moment.

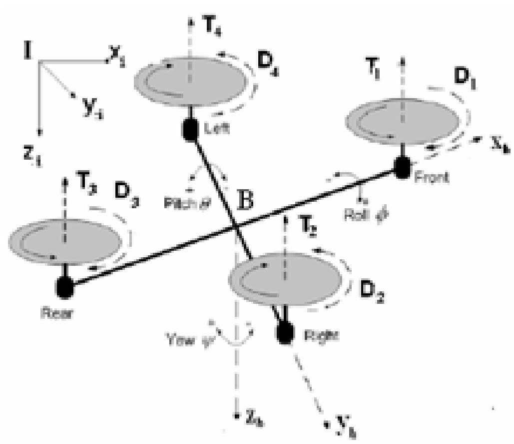

Figure 2. Quadrotor inertial and body frame.

Consider: I -- Inertial Frame and;
B -- Body Frame

The assumption for dynamic modelling as in [6]

1. The structure of quad-rotor is rigid and symmetrical.
2. The centre of mass of quad-rotor coincides with the origin of body frame, B.
3. The propellers are rigid in plane. Pitch is fixed.

A. Force Equations

In general, the orientation of quad-rotor is given by three Euler Angles:

$$\Omega^T = (\phi, \theta, \psi) \tag{1}$$

While the position of quad-rotor in the inertial frame is given by the vector:

$$\mathrm{r}^{\mathrm{T}} = (\mathrm{x}, \mathrm{y}, \mathrm{z}) \tag{2}$$

Along with that, the transformation vector from body to inertial frame is given by the rotational matrix:

$$R = \begin{pmatrix} c\psi c\theta & cos\psi s\theta s\phi - s\psi c\phi & c\psi s\theta c\phi + s\psi s\phi \\ s\psi c\theta & s\psi s\theta s\phi + c\psi c\phi & s\psi s\theta c\phi - c\psi s\phi \\ -s\theta & c\theta s\phi & c\theta c\phi \end{pmatrix} \tag{3}$$

where, $c\theta$ denotes $\cos\theta$ and $s\theta$ denotes $\sin\theta$.

Based from the momentum theory, the aerodynamic thrust force of a propeller is proportional to the square of its rotational speed. The trust force generated by rotor i =1,2,3,4 is defined as:

$$F_i = b.\omega_i \tag{4}$$

where, b is the thrust factor and ω i is the rotational speed of rotor i.

Based from that, the total thrust force applied to the four rotors will be given by:

$$T = \sum_{i=1}^{4} |F_i| = b \sum_{i=1}^{4} {\omega_i}^2 \tag{5}$$

$$T = \left|b{\omega_1}^2\right| + \left|b{\omega_2}^2\right| + \left|b{\omega_3}^2\right| + \left|b{\omega_4}^2\right| = u_i \tag{6}$$

Since linear momentum balance in inertial frame is defined as:

$$\ddot{r} = \begin{pmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{pmatrix} = g.\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} - R.\frac{T}{M}.\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{7}$$

Thus, the first set of differential equation that describes the acceleration of the quadrotor can be written as:

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} -(c\phi s\theta c\psi + s\phi s\psi).\frac{u_1}{m} \\ -(c\phi s\theta s\psi - s\phi c\psi).\frac{u_1}{m} \\ g - (c\phi c\theta).\frac{u_1}{m} \end{bmatrix} \tag{8}$$

B. Moment Equations

The second set of differential equation is obtained by deriving the angular momentum balance equation in body frame, which is:

$$I.\ddot{\Omega} = -(\dot{\Omega} \times I.\dot{\Omega}) - M_G + M \tag{9}$$

where, I is the inertia matrix, I_R is the rotor inertia, M is the torque applied to the quadrotor's body and M_G is the gyroscopic torque. With the assumption that the structure of quadrotor to be symmetrical with respect to the x and y axes, the center of gravity is then located at the center of the quadrotor. Therefore, inertial matrix is derived as diagonal matrix with the inertia *Ix, Iy, Iz.*

Torque applied to the quad-rotor's body is defined as:

$$M = \begin{pmatrix} Lb({\omega_3}^2 - {\omega_4}^2) \\ Lb({\omega_1}^2 - {\omega_2}^2) \\ d({\omega_1}^2 + {\omega_2}^2 - {\omega_3}^2 - {\omega_4}^2) \end{pmatrix} \tag{10}$$

with d as the drag factor and L as the length of the lever. While gyroscopic torque is defined as:

$$M_G = I_R.\left(\dot{\Omega}\, x \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\right).(\omega_1 + \omega_2 - \omega_3 - \omega_4) \tag{11}$$

The gyroscopic torques depends on the rotational velocities of the rotors. It is caused by the moment produced by each pair of rotors.

Substituting all of the equation in the angular momentum balance equation in body frame will give:

$$I.\ddot{\Omega} = -(\dot{\Omega}\ x\ I.\dot{\Omega}) - \left(I_R.\left(\dot{\Omega}\ x \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\right).(\omega_1 + \omega_2 - \omega_3 - \omega_4) \right) + \begin{pmatrix} Lb({\omega_3}^2 - {\omega_4}^2) \\ Lb({\omega_1}^2 - {\omega_2}^2) \\ d({\omega_1}^2 + {\omega_2}^2 - {\omega_3}^2 - {\omega_4}^2) \end{pmatrix} \quad (12)$$

Thus, the second set of differential equation is:

$$\begin{bmatrix} \ddot{\phi} \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} = -I^{-1}\left(\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} x\ I. \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \right) - I^{-1}\left(I_R.\left(\dot{\Omega}\ x \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\right).(\omega_1 + \omega_2 - \omega_3 - \omega_4) \right) + I^{-1}\begin{pmatrix} Lb({\omega_3}^2 - {\omega_4}^2) \\ Lb({\omega_1}^2 - {\omega_2}^2) \\ d({\omega_1}^2 + {\omega_2}^2 - {\omega_3}^2 - {\omega_4}^2) \end{pmatrix} \quad (13)$$

Considering four kinematics movements for quadrotor which are throttle movement (u_1), roll movement (u_2), pitch movement (u_3), and yaw movement (u_4) as the input variables in real application, the artificial input variables can be defined as follows: [6]

$$u_1 = b({\omega_1}^2 + {\omega_2}^2 + {\omega_3}^2 + {\omega_4}^2) \quad (14)$$

$$u_2 = b({\omega_3}^2 - {\omega_4}^2) \quad (15)$$

$$u_3 = b({\omega_1}^2 - {\omega_2}^2) \quad (16)$$

$$u_4 = d({\omega_1}^2 + {\omega_2}^2 - {\omega_3}^2 - {\omega_4}^2) \quad (17)$$

Full throttle movement is provided by increasing or decreasing the velocity of all rotors by the same amount. For roll movement, along the x-axes of the body frame, the angular velocity of rotor (*2*) is increased and the angular velocity of rotor (*4*) is decreased while keeping the whole thrust constant. Similarly, the angular velocity of rotor (*3*) is increased and the angular velocity of rotor (*1*) is decreased to produce a pitch movement along y-axes of the body frame and for yawing movement, along z-axes of the body frame, the velocity of rotors (*1,3*) are increased and decreased the velocity of rotors (*2,4*).

The differential equation will be finally derived as:

$$\begin{bmatrix} \ddot{\phi} \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} = \begin{pmatrix} \dot{\theta}\dot{\psi}\left(\frac{I_Y - I_Z}{I_X}\right) - \frac{I_R}{I_X}\dot{\theta}g(u) + \frac{L}{I_X}u_2 \\ \dot{\phi}\dot{\psi}\left(\frac{I_Z - I_X}{I_Y}\right) - \frac{I_R}{I_Y}\dot{\phi}g(u) + \frac{L}{I_Y}u_3 \\ \dot{\theta}\dot{\phi}\left(\frac{I_X - I_Y}{I_Z}\right) + \frac{L}{I_Z}u_4 \end{pmatrix} \quad (18)$$

Table I illustrates quadrotor parameters calculated and measured from the testbed used for this paper. The thrust factors and drag factor are defined based on the results generated through force lift test done prior to the flight test.

TABLE I. QUADROTOR PARAMETERS

Parameters	Value	Unit	Remark
g	9.81	m/s2	gravity
m	0.65	kg	Mass of quadrotor
L	0.165	m	Length from motor to motor
b	2.107 x 10-5	kg.m2	Lift (Thrust) factor
d	4.566 x 10-7	kg.m2	Drag factor
Ct1	0.17	N	Thrust factor for motor 1 (resulted from the force lift test performed)
Ct2	0.182	N	Thrust factor for motor 2
Ct3	0.1707	N	Thrust factor for motor 3
Ct4	0.185	N	Thrust factor for motor 4

III. PARAMETER IDENTIFICATION

Theoretically, identification of unknown parameter for quadrotor can be done in two different ways; one can be by direct computation of the quadrotor geometry and the other way is by analysis of flight data. The first method requires very deep mathematical and physical calculations with the excessive computational load while the second method needs only an algorithm which is normally simpler and provides even more accurate results. [7] One of possible algorithms is the UKF.

A. The Unscented Kalman Filter

The UKF has two distinct phases: Predict and Update. The predict phase uses the state estimate from the previous time step to produce an estimate of the state at the current time step. In the update phase, measurement information at the current step is used to correct this prediction so that it will get to a new and hopefully more accurate state estimate, again for the current time step. [1][2][9]

The standard UKF algorithm can be summarized as follows:[2][5]

Firstly, the time point is set to zero, $k = 0$, then built up the augmented state vector ${\hat{x}_k}^a$ and covariance matrix ${\hat{P}_k}^a$ as the initial values of augmented state vector and output vector.

Initialization with;

$$\hat{x}_o = E(x_o) \quad (19)$$

$$P_o = E[(x_o - \widehat{x_o})(x_o - \widehat{x_o})]^T \tag{20}$$

$$\hat{x}_o{}^a = E[x^a] = [\hat{x}_o{}^a \; 0 \; 0]^T \tag{21}$$

Subsequently, we calculate sigma point. The UKF algorithm requires the definition of *2na +1* sigma points, where na is the total number of states to be estimated. The sigma points are defined this way: [9]

$$S_i = (X_i, w_i) \quad i = 1, .., 2n + 1 \tag{22}$$

$$X_{k-1}^a = \left[\hat{x}_{k-1}^a \quad \hat{x}_{k-1}^a \pm \sqrt{(n+\lambda)P_{k-1}^a}\right] \tag{23}$$

Followed by, time update process. The sigma points are then propagated through the transition function f using numerical integration methods:

$$\hat{X}_{k-1} = \hat{X}_k(K) + \int_{t_{k-1}}^{t_k} f(k, X_k)dt \tag{24}$$

The weighted sigma points are recombined to produce the predicted state and covariance:

$$\widehat{x_k}^- = \sum_{i=0}^{2n} W_i^{(m)} \hat{X}_{i,\ k-1} \tag{25}$$

Predicted estimate covariance:

$$\widehat{P_k}^- = \sum_{i=0}^{2n} w_i{}^c \left[\hat{x}_{i,k+1} - \hat{x}_{k+1}\right]\left[\hat{x}_{i,k+1} - \hat{x}_{k+1}\right]^T$$

$$w_i{}^c = \frac{1}{2(n+k)} \tag{26}$$

Next, since the time update equations are done, we implement the measurement update equations. Innovation or measurement residual is defined as:

$$y_{k+1} = h(\hat{X}_{k+1}, u_{k+1}) \tag{27}$$

$$y_{k+1} = \sum_{i=0}^{2n} w_i \hat{y}_{i,\ k+1} \tag{28}$$

While, the updated estimate covariance:

$$P_{\hat{y}\hat{y}_k} = \sum_{i=0}^{2n} w_i{}^c \left[y_{i,k} - \hat{y}_{k-1}\right]\left[y_{i,k} - \hat{y}_{k-1}\right]^T + R$$

$$P_{\hat{x}\hat{y}_{k+1}} = \sum_{i=0}^{2n} w_i{}^c \left[\hat{x}_{i,k} - \hat{x}_{k-1}\right]\left[y_{i,k} - \hat{y}_{k-1}\right]^T \tag{29}$$

Finally, the measurement update of state estimate can be performed using the normal Kalman filter equations as shown in followings equations:

$$K_k = P^{-1}{}_{\hat{y}\hat{y}_k} P_{\hat{x}\hat{y}_k} \tag{30}$$

$$\widehat{x_k}^+ = \hat{x}_k{}^- + K_k(y_k - \hat{y}_k) \tag{31}$$

$$\widehat{P_k}^+ = P_k^- - K_k P_y K_k^T \tag{32}$$

B. Application to Quadrotor

Basically, identification of parameter through filtering approach transformed the parameter estimation problem into a state estimation problem. It employs a continuous estimation model for prediction and the measurements that are recorded at discrete time step for the correction. The system dynamic of quadrotor is represented in continuous state space equation along with the measurement model as follows:[8]

For the quad-rotor system with continuous-time system dynamics:

$$\dot{x} = f[x(t), u(t), w(t), t] \tag{33}$$

$$y(t) = h[x(t), v(t), t] \tag{34}$$

$$w \sim [0, Q] \tag{35}$$

$$v \sim [0, R] \tag{36}$$

where $x(t) \in R^n$ is the system state, $u(t) \in R^m$ is the control input, $y(t) \in R^m$ is the system input, while w and v are the measurement noise.

Expanding the system equations will give:

$$\dot{x} = Ax + Bu + Lw \tag{37}$$

$$y = Cx + Mv \tag{38}$$

Since the state vector of the quad-rotor:

$$x_k = (x, y, z, \dot{x}, \dot{y}, \dot{z}, \phi, \theta, \psi, \dot{\phi}, \dot{\theta}, \dot{\psi})^T \tag{39}$$

Thus, the quad-rotor system equations with state and input matrices are derived as follows:

$$\dot{x} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \ddot{x} \\ \ddot{y} \\ \ddot{z} \\ \dot{\emptyset} \\ \dot{\theta} \\ \dot{\psi} \\ \ddot{\emptyset} \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} = \begin{cases} \dot{x_1} = x_4 \\ \dot{x_2} = x_5 \\ \dot{x_3} = x_6 \\ \dot{x_4} = -(c\phi s\theta c\psi + s\phi s\psi)\frac{u_1}{m} \\ \dot{x_5} = -(c\phi s\theta s\psi - s\phi c\psi).\frac{u_1}{m} \\ \dot{x_6} = g - (c\phi c\theta).\frac{u_1}{m} \\ \dot{x_7} = x_{10} \\ \dot{x_8} = x_{11} \\ \dot{x_9} = x_{12} \\ \dot{x_{10}} = (\dot{\theta}\dot{\psi}\left(\frac{I_Y - I_Z}{I_X}\right) - \frac{I_R}{I_X}\dot{\theta}g(u) + \frac{L}{I_X}u_2) \\ \dot{x_{11}} = (\dot{\phi}\dot{\psi}\left(\frac{I_Z - I_X}{I_Y}\right) - \frac{I_R}{I_Y}\dot{\phi}g(u) + \frac{L}{I_Y}u_3) \\ \dot{x_{12}} = (\dot{\theta}\dot{\phi}\left(\frac{I_X - I_Y}{I_Z}\right) + \frac{L}{I_Z}u_4) \end{cases} \tag{40}$$

$$
\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \ddot{x} \\ \ddot{y} \\ \ddot{z} \\ \dot{\emptyset} \\ \dot{\theta} \\ \dot{\psi} \\ \ddot{\emptyset} \\ \ddot{\theta} \\ \ddot{\psi} \end{pmatrix} = \begin{bmatrix} 0_{3x3} & 1_{3x3} & 0_{3x3} & 0_{3x3} \\ 0_{3x3} & L_1 & 0_{3x3} & 0_{3x3} \\ 0_{3x3} & 0_{3x3} & 0_{3x3} & 1_{3x3} \\ 0_{3x3} & 0_{3x3} & 0_{3x3} & L_2 \end{bmatrix} \begin{pmatrix} x \\ y \\ z \\ \dot{x} \\ \dot{y} \\ \dot{z} \\ \emptyset \\ \theta \\ \psi \\ \dot{\emptyset} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} + \begin{bmatrix} 0_{3x1} & 0_{3x3} \\ L_3 & 0_{3x3} \\ 0_{3x1} & 0_{3x3} \\ 0_{3x1} & L_4 \end{bmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix}
$$

Where: $L_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & g \end{bmatrix}$, $L_2 = \begin{bmatrix} 0 & \dot{\psi}\left(\frac{I_Y - I_Z}{I_X}\right) - \frac{I_R}{I_X} g(u) & 0 \\ \dot{\psi}\left(\frac{I_Z - I_X}{I_Y}\right) - \frac{I_R}{I_Y} g(u) & 0 & 0 \\ \dot{\phi}\left(\frac{I_X - I_Y}{I_Z}\right) & 0 & 0 \end{bmatrix}$

$$
L_3 = \begin{bmatrix} \frac{-(C\phi S\theta C\psi + S\phi S\psi)}{m} \\ \frac{-(C\phi S\theta S\psi - S\phi C\psi)}{m} \\ \frac{-(C\phi C\theta)}{m} \end{bmatrix} \quad \text{and} \quad L_4 = \begin{bmatrix} \frac{L}{I_X} \\ \frac{L}{I_y} \\ \frac{1}{I_Z} \end{bmatrix}
$$

For the measurement matrix, the responds of quadrotor for a sequence of flight, starting from low speed motors rotation to high speed motors rotation, followed by the take off, hover and landing are analyzed. The time taken for full flight test is 6seconds. The measured data for velocities, $\dot{x}, \dot{y}, \dot{z}$ and angular rates, $\dot{\emptyset}, \dot{\theta}, \dot{\psi}$ are read IMU sensor and recorded.

Therefore the measurement matrix is derived as:

$$
\begin{pmatrix} x \\ y \\ z \\ \dot{x} \\ \dot{y} \\ \dot{z} \\ \emptyset \\ \theta \\ \psi \\ \dot{\emptyset} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&0&0&0&1 \end{bmatrix} \begin{pmatrix} x \\ y \\ z \\ \dot{x} \\ \dot{y} \\ \dot{z} \\ \emptyset \\ \theta \\ \psi \\ \dot{\emptyset} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} \tag{41}
$$

The unknown parameter vector Θ consists of system parameter β, the measurement Δy and the input estimates Δu.

$$
\Theta^T = \{ \beta^T, \Delta y^T, \Delta u^T \} \tag{42}
$$

Finally, based on all of the system equation the parameters to be estimated and identified are:

I_{xx} : moment of inertia about x-axis in body frame

I_{yy} : moment of inertia about y-axis in body frame

I_{zz} : moment of inertia about y-axis in body frame

I_R : moment of inertia for rotor

$\dot{x}$: quadrotor velocity along x-axes in body frame

$\dot{y}$: quadrotor velocity along y-axes in body frame

$\dot{z}$: quadrotor velocity along z-axes in body frame

$\dot{\emptyset}$: quadrotor angular rates along x-axes in body frame

$\dot{\theta}$: quadrotor angular rates along x-axes in body frame

$\dot{\psi}$: quadrotor angular rates along x-axes in body frame

The state filtering performance is shown in Figure 3(*a*) through Figure 3(*f*). Figure 3(*a*) to 3(*c*) shows the measure and the estimate velocities for x, y, z-axes respectively. Conversely Figure 3(*d*) to 3(*f*) shows the measure and the estimate angular velocities for *x*, *y*, and *z*-axes respectively. From the illustrations, the error of the estimation for velocity at *x*-axes is less than 0.001, while the errors at both *y*-axes and *z*-axes are less than 0.0015. For estimation of angular velocities at *x*, *y*, and *z*-axes, the errors are less than 0.0015. Error of estimation is referring to the difference between the measure and the estimate value for velocities and angular velocities at *x*, *y*, and *z* axes. From the errors computed, it can be concluded that the UKF output matches with the measured output and the measured noise is well filtered by the UKF.

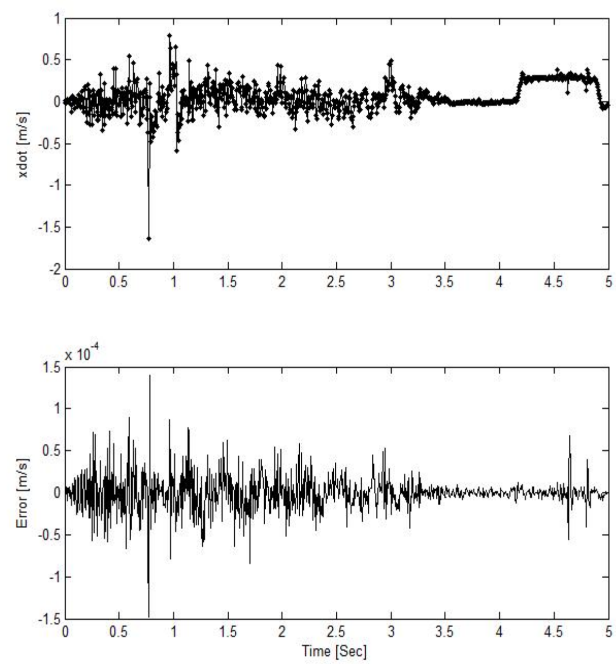

Figure 3(a). UKF state filtering; measured and estimated for velocity at x-axes.

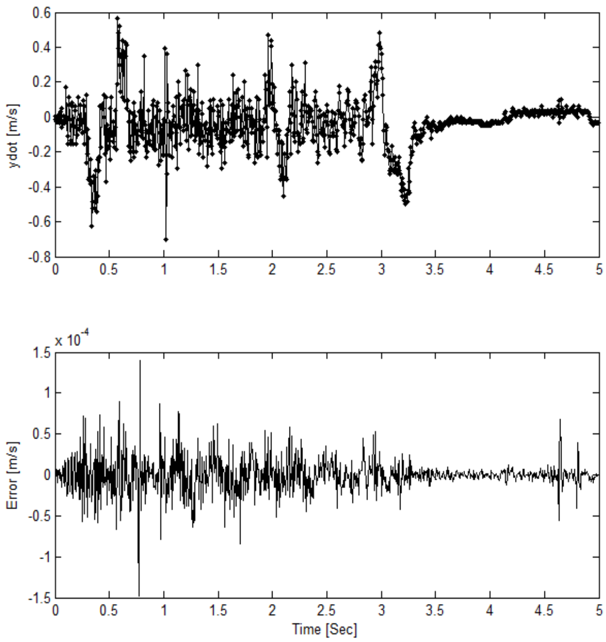

Figure 3(b). UKF state filtering; measured and estimated for velocity at y-axes.

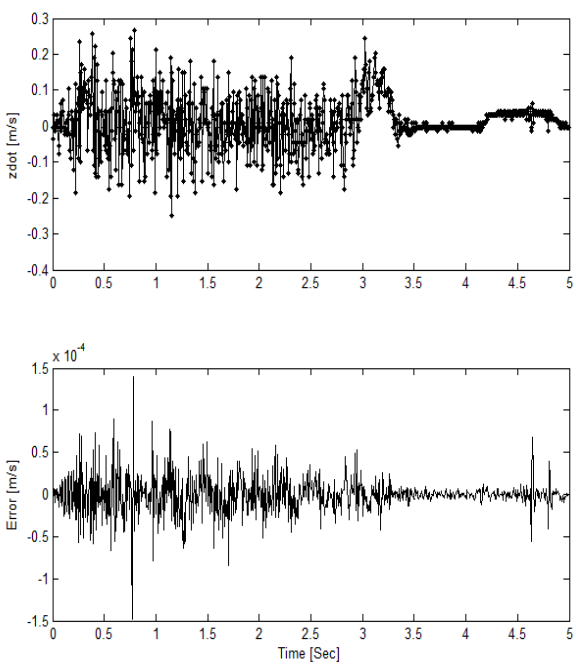

Figure 3(c). UKF state filtering; measured and estimated for velocity at z-axes.

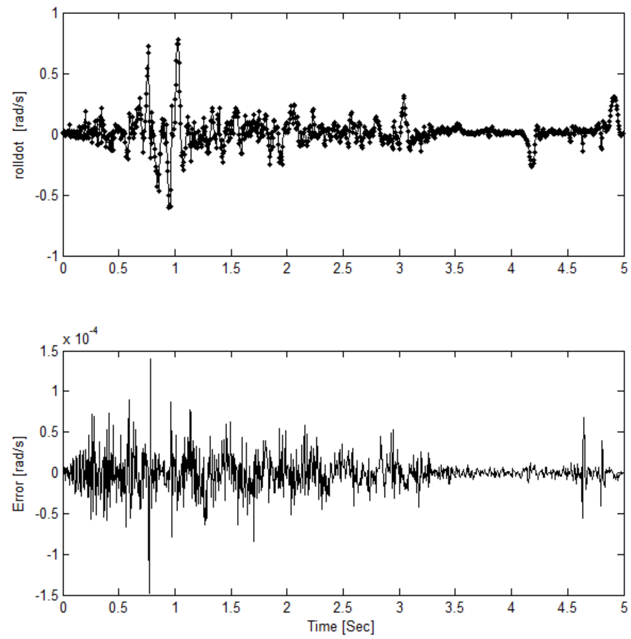

Figure 3(d). UKF state filtering; measured and estimated for angular velocity at x-axes.

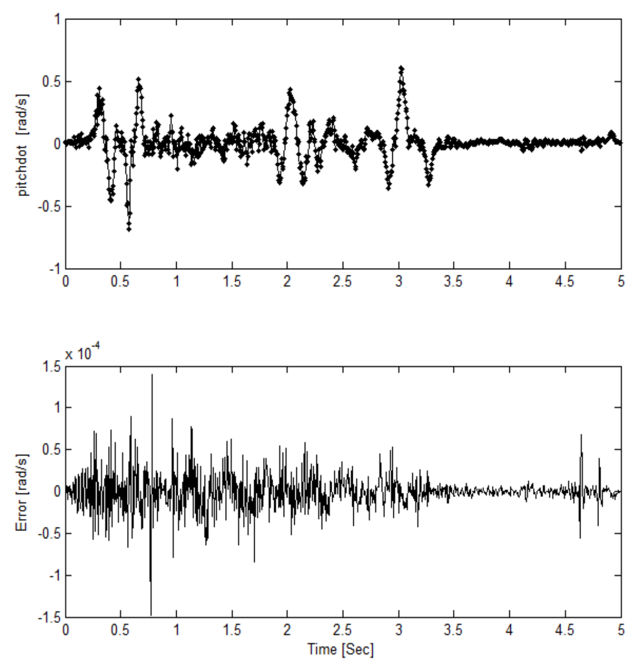

Figure 3(e). UKF state filtering; measured and estimated for angular velocity at y-axes.

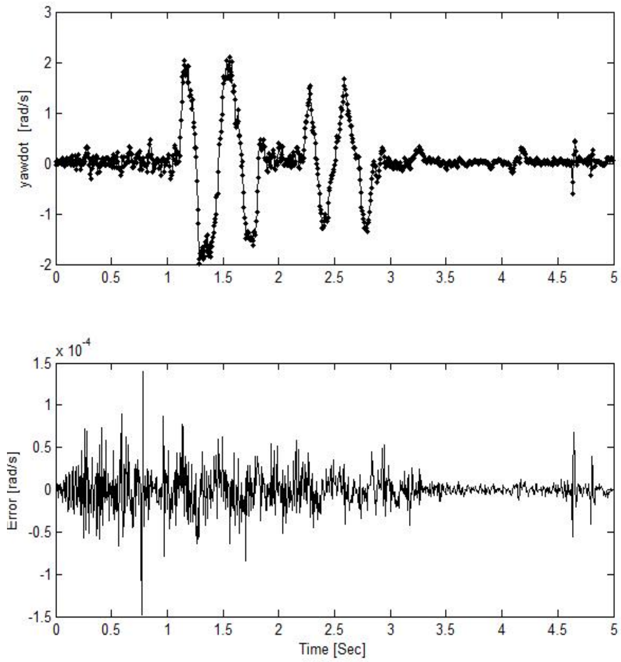

Figure 3(f). UKF state filtering; measured and estimated for angular velocity at z-axes

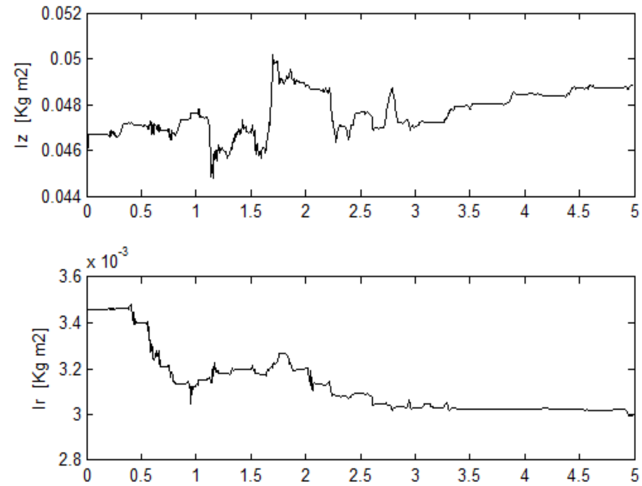

Figure 4(b). Reconstruct of moment of inertia by UKF algorithm for z axes as well as moment for rotor.

TABLE II. QUADROTOR PARAMETERS

Parameters	**Value**	**Unit**	**Mean Square Error**
Ixx	0.0224	kg.m2	0.0228
Iyy	0.0224	kg.m2	0.0060
Izz	0.0436	kg.m2	0.035
I_R	3.46E-3	kg.m2	0.0031

The parameters reconstructed by UKF algorithm for quadrotor hovering are illustrated in Figure 4(*a*) and 4(*b*). Figure 4(*a*) illustrate the reconstruction values for moment of inertia at *x* and *y*-axes. The calculated values for moment of inertia at *x*-axes and *y*-axes are 0.0224, while the reconstructed values are 0.0258 and 0.024 respectively. Reconstruction values for moment of inertia at *z* and rotor inertia are illustrated in Figure 4(*b*). The calculated value for moment of inertia at *z*-axes is 0.049 and rotor inertia is 0.00346, while the reconstructed values are 0.0258 and 0.0032 respectively. The numerical mean square errors for all of reconstructed values are presented in Table II.

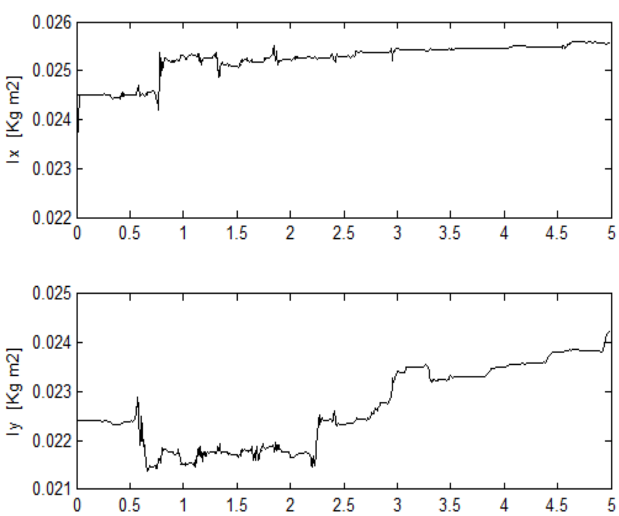

Figure 4(a). Reconstruct of moment of inertia by UKF algorithm for x and y axes respectively.

IV. CONCLUSION AND FUTURE WORK

In this paper, the performance of UKF state filtering for parameter identification purposes applying on an autonomous quadrotor has been discussed. A continuous estimation model with additive noise represented by state vector is filtered using a first order approximation for propagation of the covariance matrix.

In conclusion, the results indicate that the UKF is a practical tool for parameter identification. With appropriate identification model and identification data, UKF can be a better algorithm. It has good convergence time and relatively reliable for the estimations. It leads to better filtered result which more accurately captures the true mean and covariance. It has proved itself to be one of useful tool in the field of real-time data analysis.

As future work, it could be interesting comparing the performance of UKF with EKF, from the point of view of results accuracy, time and cost consumption and algorithm complexity.

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