Example 5

What is the potential difference between points x and y in the network?

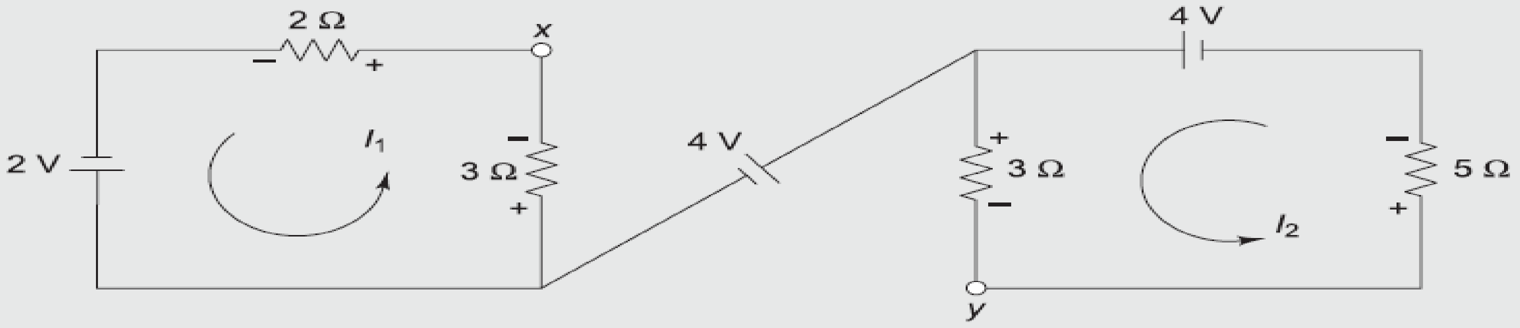


Fig. 2.11

Solution

$$I_1 = \frac{2}{2+3} = 0.4 \text{ A}$$

$$I_2 = \frac{4}{3+5} = 0.5 \text{ A}$$

Potential difference between points x and $y = V_{xy} = V_x - V_y$

Writing KVL equation for the path x to y,

$$V_x + 3I_1 + 4 - 3I_2 - V_y = 0$$

$$V_x + 3(0.4) + 4 - 3(0.5) - V_y = 0$$

$$V_x - V_y = -3.7$$

$$V_{xy} = -3.7 \text{ V}$$

Example 7

Determine the potential difference V_{AB} for the given network.

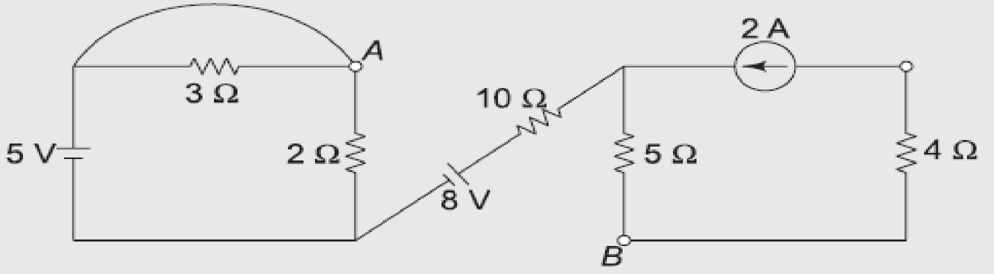


Fig. 2.14 [May 2014]

Solution The resistor of 3 Ω is connected across a short circuit. Hence, it gets shorted.

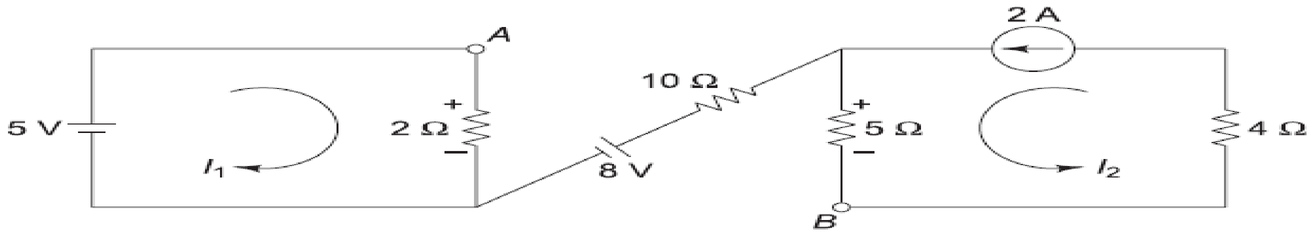


Fig. 2.15

$$I_1 = \frac{5}{2} = 2.5 \text{ A}$$

$$I_2 = 2 \text{ A}$$

Potential difference $V_{AB} = V_A - V_B$

Writing KVL equation for the path A to B,

$$V_A - 2I_1 + 8 - 5I_2 - V_B = 0$$

$$V_A - 2(2.5) + 8 - 5(2) - V_B = 0$$

$$V_A - V_B = 7$$

$$V_{AB} = 7 \text{ V}$$

Example 18

Find the value of current flowing through the 4 Ω resistor.

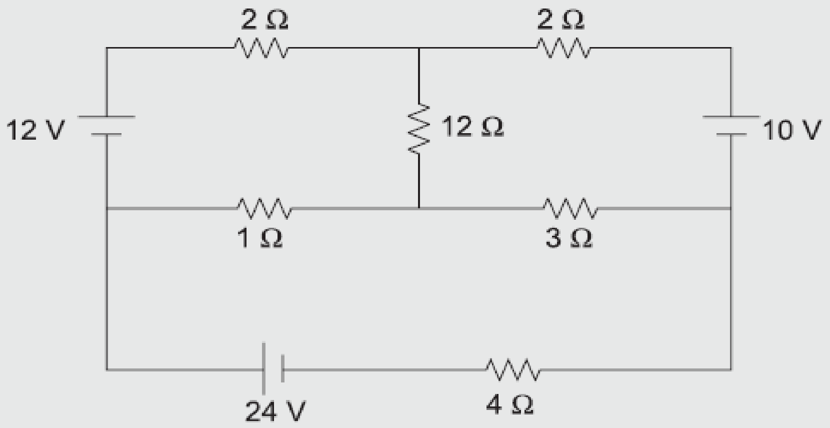


Fig. 2.33

Solution Assigning currents to all the branches,

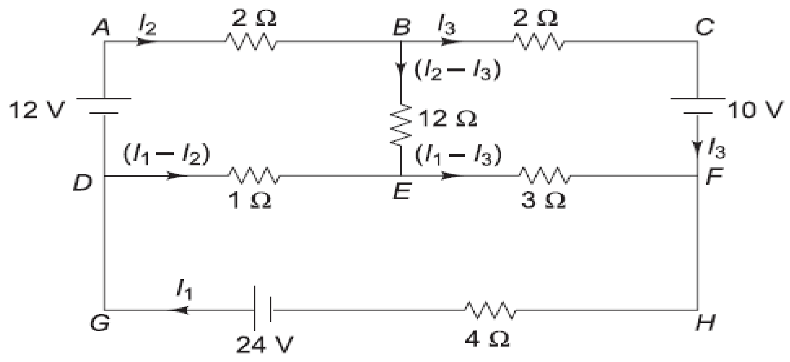


Fig. 2.34

Applying KVL to the closed path *ABEDA*,

$$-2I_2 - 12(I_2 - I_3) + 1(I_1 - I_2) + 12 = 0$$

$$I_1 - 15I_2 + 12I_3 = -12 \qquad (1)$$

Applying KVL to the closed path *BCFEB*,

$$-2I_3 - 10 + 3(I_1 - I_3) + 12(I_2 - I_3) = 0$$

$$3I_1 + 12I_2 - 17I_3 = 10 \qquad (2)$$

Applying KVL to the closed path *DEFHGD*,

$$-1(I_1 - I_2) - 3(I_1 - I_3) - 4I_1 + 24 = 0$$

$$-8I_1 + I_2 + 3I_3 = -24 \qquad (3)$$

Solving Eqs (1), (2) and (3),

$$I_1 = 4.11 \text{ A}$$

$$I_1 = 2.72 \text{ A}$$

$$I_3 = 2.06$$

Current through the 4 Ω resistor $= I_1 = 4.11$ A

2.7 STAR-DELTA TRANSFORMATION

When a circuit cannot be simplified by normal series–parallel reduction technique, the star-delta transformation can be used.

Figure 2.175(a) shows three resistors R_A, R_B and R_C connected in delta.

Figure 2.175(b) shows three resistors R_1, R_2 and R_3 connected in star.

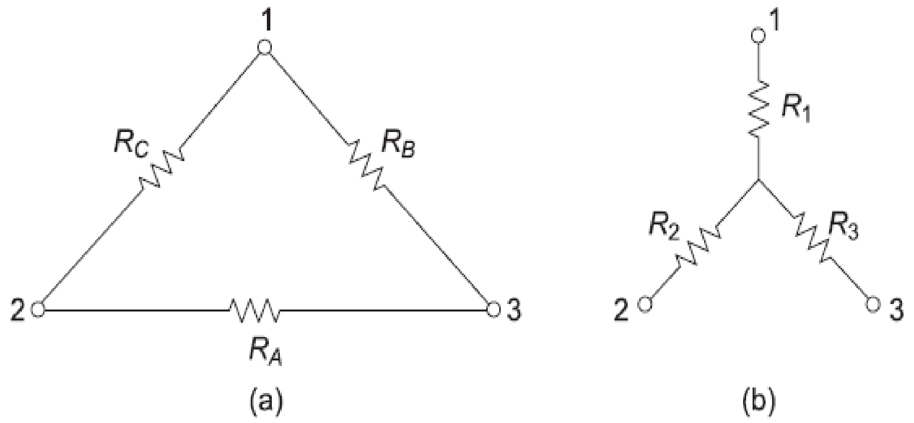
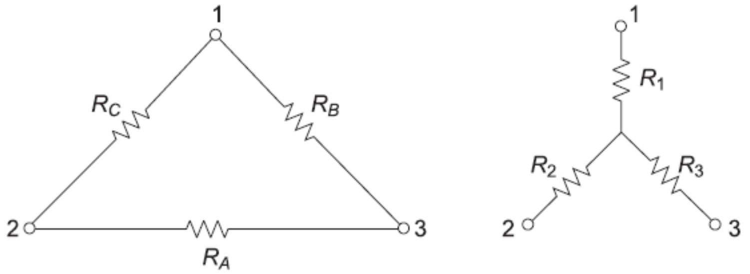


Fig. 2.175 *Delta and star networks*

These two networks will be electrically equivalent if the resistance as measured between any pair of terminals is the same in both the arrangements.

2.7.1 Delta to Star Transformation

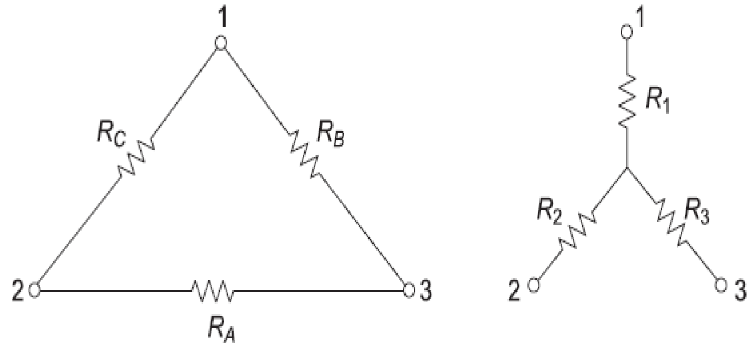


$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C} \quad (2.10)$$

$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C} \quad (2.11)$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C} \quad (2.12)$$

2.7.2 Star to Delta Transformation



$$R_A = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$= R_2 + R_3 + \frac{R_2R_3}{R_1}$$

$$R_B = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$= R_1 + R_3 + \frac{R_3R_1}{R_2}$$

$$R_C = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

$$= R_1 + R_2 + \frac{R_1R_2}{R_3}$$

Example 2

Find an equivalent resistance between terminals A and B.

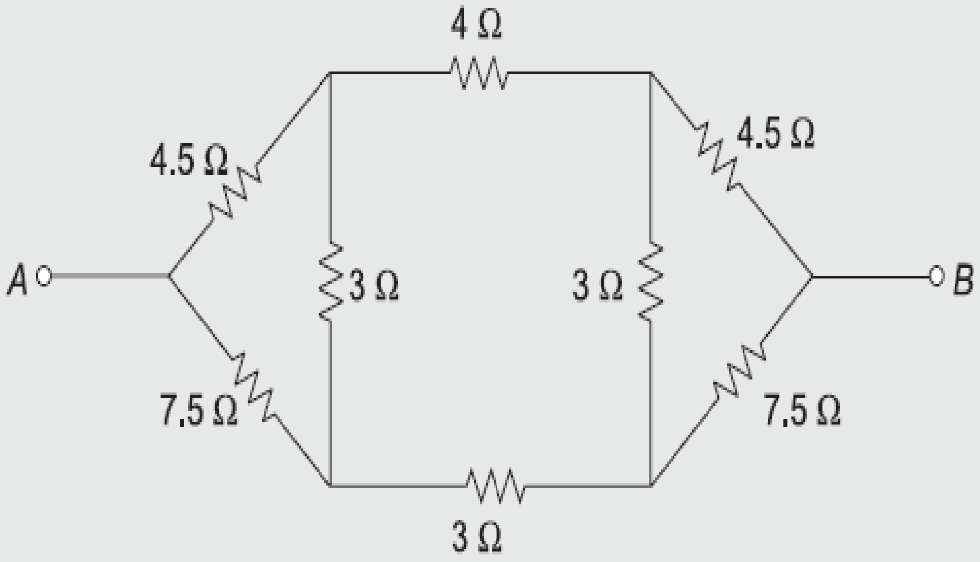


Fig. 2.179

Example 2

Find an equivalent resistance between terminals A and B.

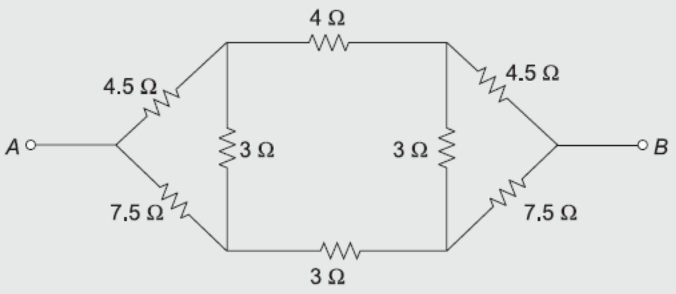


Fig. 2.179

Solution Converting the two delta networks formed by resistors of 4.5 Ω, 3 Ω and 7.5 Ω into equivalent star networks,

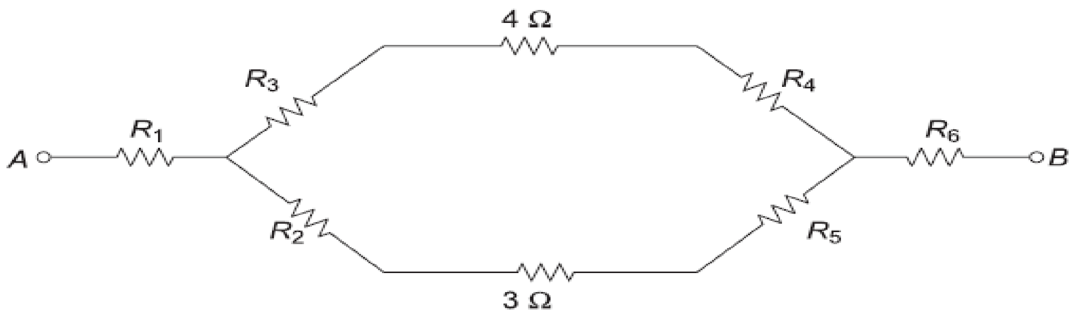


Fig. 2.180

$$R_1 = R_6 = \frac{4.5 \times 7.5}{4.5 + 7.5 + 3} = 2.25\ \Omega$$

$$R_2 = R_5 = \frac{7.5 \times 3}{4.5 + 7.5 + 3} = 1.5\ \Omega$$

$$R_3 = R_4 = \frac{4.5 \times 3}{4.5 + 7.5 + 3} = 0.9\ \Omega$$

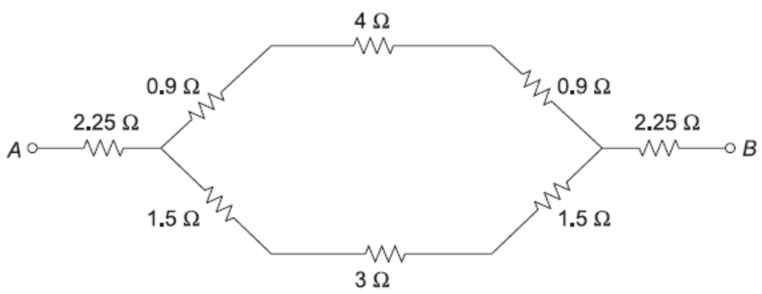


Fig. 2.181

Simplifying the network,

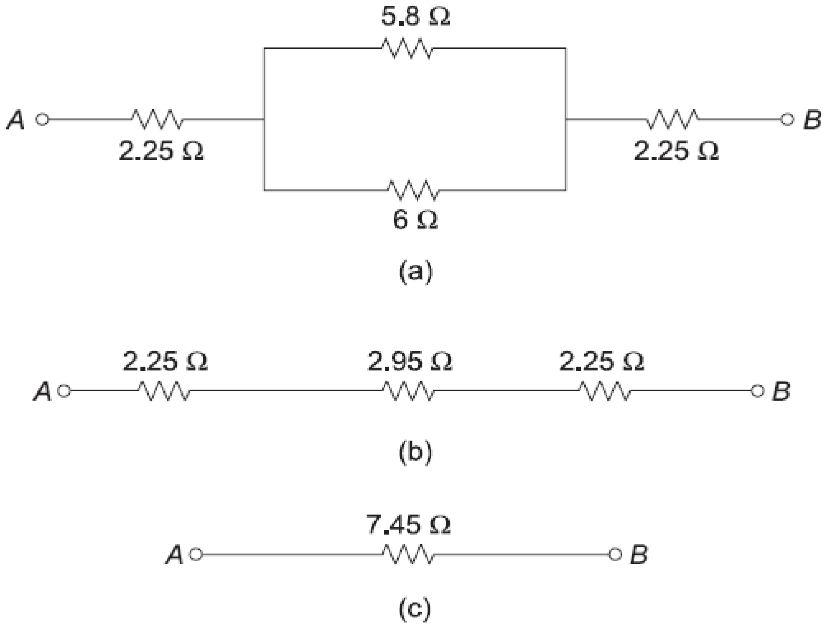


Fig. 2.182

$$R_{AB} = 7.45\ \Omega$$

Example 3

Find an equivalent resistance between terminals A and B.

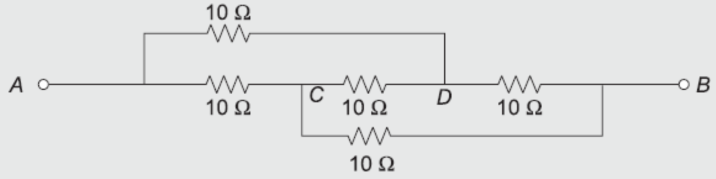


Fig. 2.183

Solution Redrawing the network,

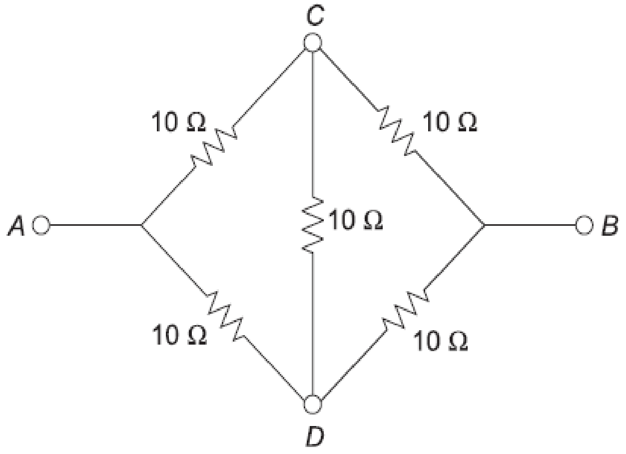


Fig. 2.184

Converting the delta network formed by three resistors of 10 Ω into an equivalent star network,

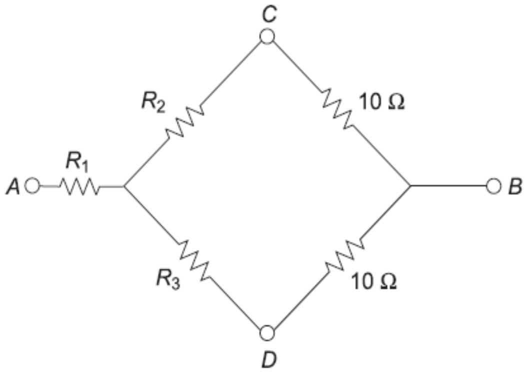


Fig. 2.185

$$R_1 = R_2 = R_3 = \frac{10 \times 10}{10 + 10 + 10} = \frac{10}{3}\ \Omega$$

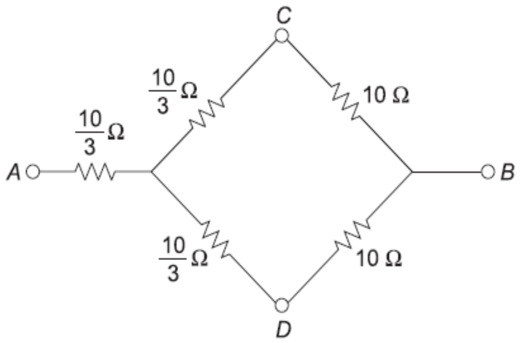


Fig. 2.186

Example 3

Find an equivalent resistance between terminals A and B.

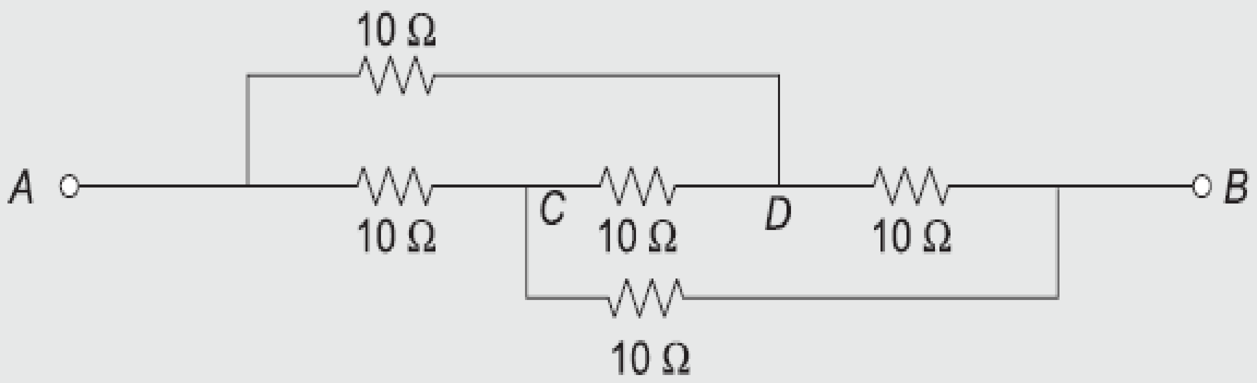


Fig. 2.183

Simplifying the network,

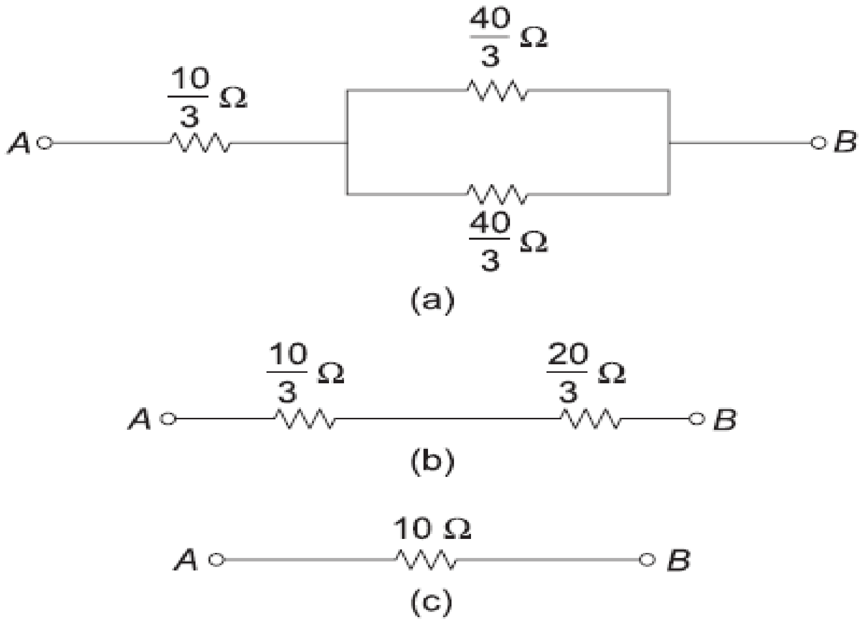


Fig. 2.187

$$R_{AB} = 10\ \Omega$$

Example 5

Find an equivalent resistance between terminals A and B.

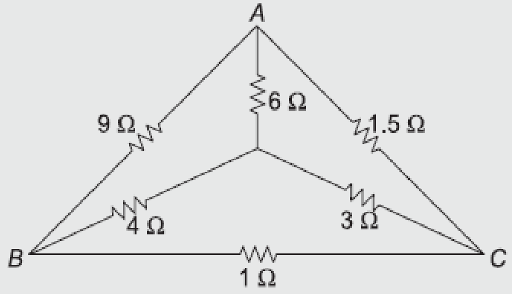


Fig. 2.192

Solution Converting the star network formed by resistors of 3 Ω, 4 Ω and 6 Ω into an equivalent delta network,

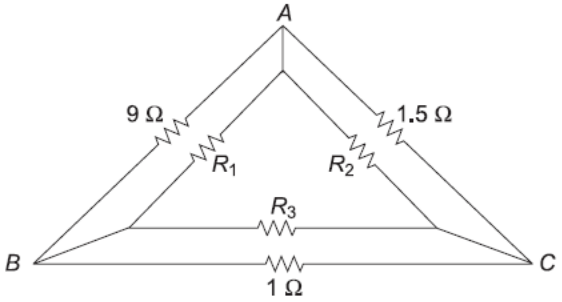


Fig. 2.193

$$R_1 = 6 + 4 + \frac{6 \times 4}{3} = 18\ \Omega$$

$$R_2 = 6 + 3 + \frac{6 \times 3}{4} = 13.5\ \Omega$$

$$R_3 = 4 + 3 + \frac{4 \times 3}{6} = 9\ \Omega$$

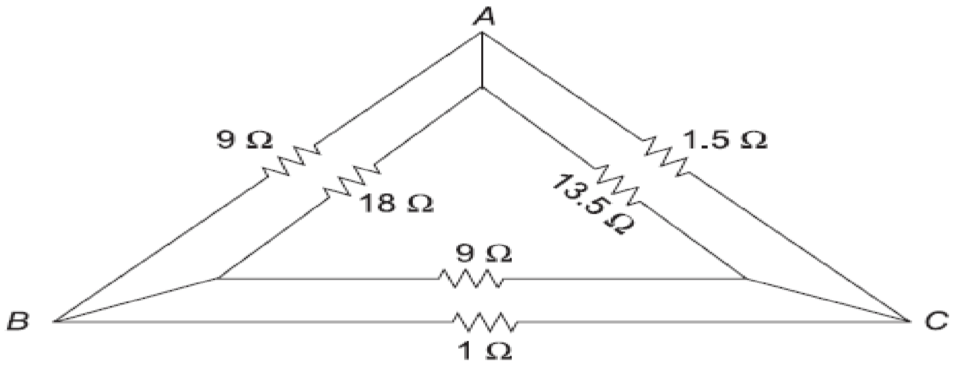


Fig. 2.194

Simplifying the network,

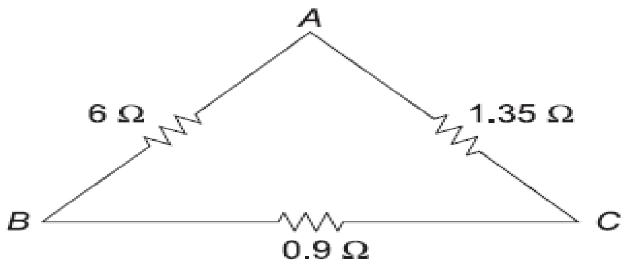


Fig. 2.195

$$\begin{aligned} R_{AB} &= 6 \parallel (1.35 + 0.9) \\ &= 6 \parallel 2.25 \\ &= 1.64\ \Omega \end{aligned}$$

MESH ANALYSIS

2.2.1 Steps to be Followed in Mesh Analysis

1. Identify the mesh, assign a direction to it and assign an unknown current in each mesh.
2. Assign the polarities for voltage across the branches.
3. Apply KVL around the mesh and use Ohm's law to express the branch voltages in terms of unknown mesh currents and the resistance.
4. Solve the simultaneous equations for unknown mesh currents.

Consider the network shown in Fig. 2.60 which has three meshes. Let the mesh currents for the three meshes be I_1, I_2 and I_3 and all the three mesh currents may be assumed to flow in the clockwise direction. The choice of direction for any mesh current is arbitrary.

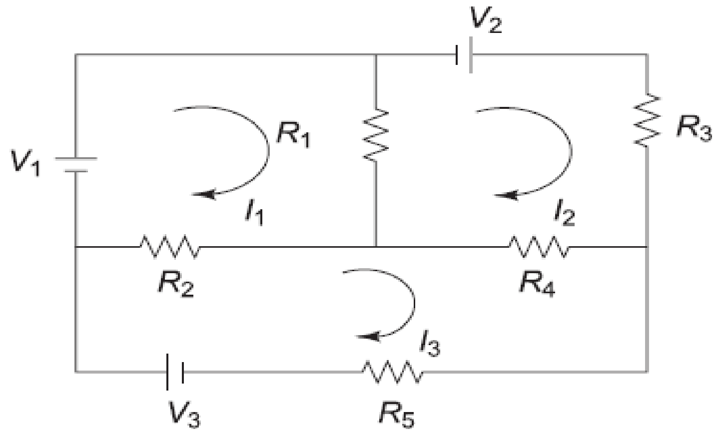


Fig. 2.60 *Mesh analysis*

Example 1

Find the value of current flowing through 1 Ω resistor.

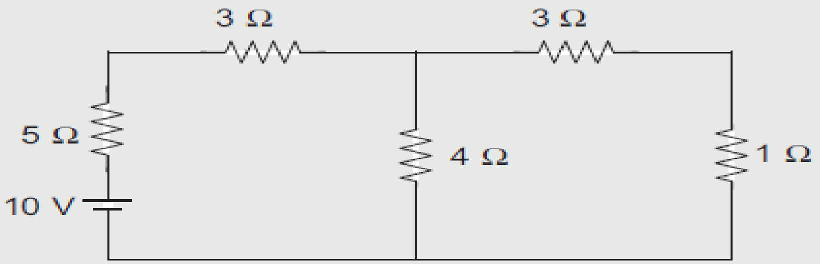


Fig. 2.61 [May 2015]

Solution Assigning clockwise currents in two meshes,

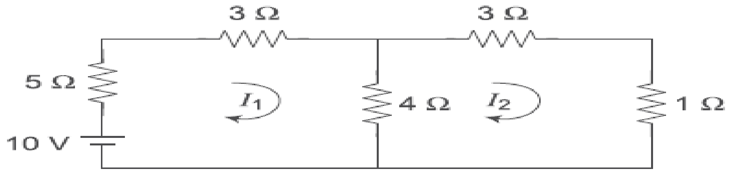


Fig. 2.62

Applying KVL to Mesh 1,

$$10 - 5I_1 - 3I_1 - 4(I_1 - I_2) = 0$$

$$12I_1 - 4I_2 = 10 \qquad (1)$$

Applying KVL to Mesh 2,

$$-4(I_2 - I_1) - 3I_2 - 1I_2 = 0$$

$$-4I_1 + 8I_2 = 0 \qquad (2)$$

Solving Eqs (1) and (2),

$$I_1 = 1 \text{ A}$$

$$I_2 = 0.5 \text{ A}$$

$$I_{1\,\Omega} = I_2 = 1.5 \text{ A}$$

Example 2

Find the value of current flowing through 5 Ω resistor.

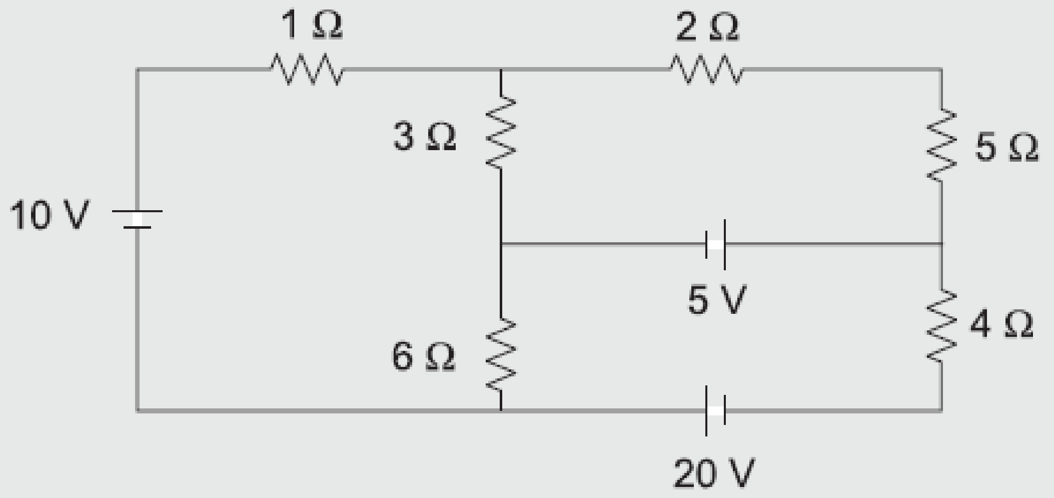


Fig. 2.63

Example 2

Find the value of current flowing through 5 Ω resistor.

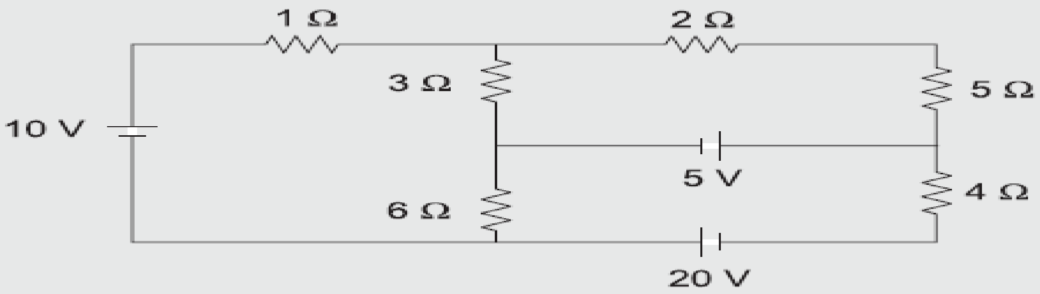


Fig. 2.63

Solution Assigning clockwise currents in three meshes,

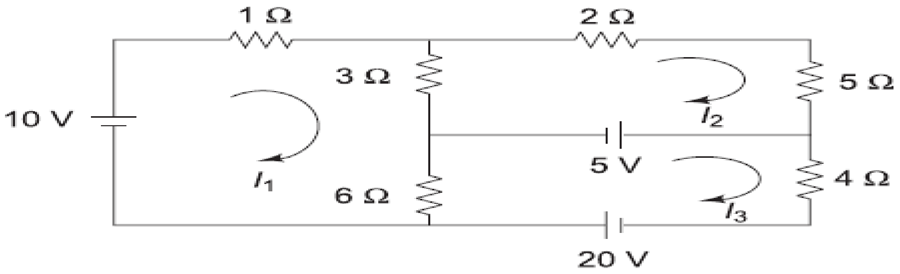


Fig. 2.64

Applying KVL to Mesh 1,

$$10 - 1(I_1) - 3(I_1 - I_2) - 6(I_1 - I_3) = 0$$

$$10I_1 - 3I_2 - 6I_3 = 10 \quad (1)$$

Applying KVL to Mesh 2,

$$-3(I_2 - I_1) - 2I_2 - 5I_2 - 5 = 0$$

$$-3I_1 + 10I_2 = -5 \quad (2)$$

Applying KVL to Mesh 3,

$$-6(I_3 - I_1) + 5 - 4I_3 + 20 = 0$$

$$-6I_1 + 10I_3 = 25 \quad (3)$$

Writing equations in matrix form,

$$\begin{bmatrix} 10 & -3 & -6 \\ -3 & 10 & 0 \\ -6 & 0 & 10 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10 \\ -5 \\ 25 \end{bmatrix}$$

Solving Eqs (1), (2) and (3),

$$I_1 = 4.27 \text{ A}$$

$$I_2 = 0.78 \text{ A}$$

$$I_3 = 5.06 \text{ A}$$

$$I_{5\Omega} = I_2 = 0.78 \text{ A}$$

Example 2

Example 4

Determine the value of current flowing through the 5 Ω resistor.

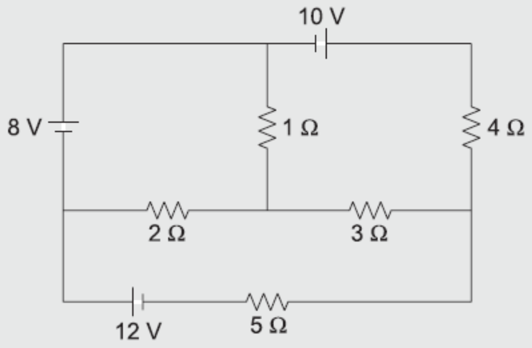


Fig. 2.67

Solution Assigning clockwise currents in the three meshes,

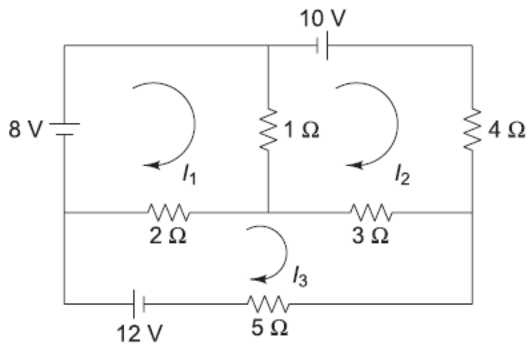


Fig. 2.68

Solution Assigning clockwise currents in the three meshes,

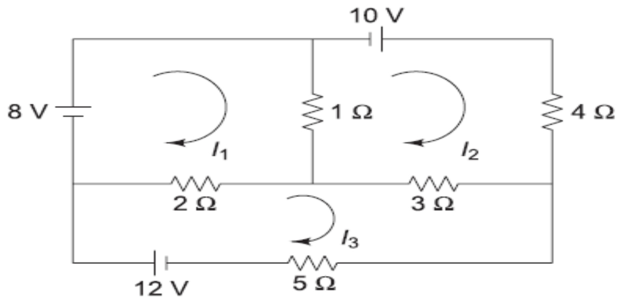


Fig. 2.68

Applying KVL to Mesh 1,

$$8 - 1(I_1 - I_2) - 2(I_1 - I_3) = 0$$

$$3I_1 - I_2 - 2I_3 = 8 \quad (1)$$

Applying KVL to Mesh 2,

$$10 - 4I_2 - 3(I_2 - I_3) - 1(I_2 - I_1) = 0$$

$$-I_1 + 8I_2 - 3I_3 = 10 \quad (2)$$

Applying KVL to Mesh 3,

$$-2(I_3 - I_1) - 3(I_3 - I_2) - 5I_3 + 12 = 0$$

$$-2I_1 - 3I_2 + 10I_3 = 12 \quad (3)$$

Writing equations in matrix form,

$$\begin{bmatrix} 3 & -1 & -2 \\ -1 & 8 & -3 \\ -2 & -3 & 10 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 10 \\ 12 \end{bmatrix}$$

Solving Eqs (1), (2) and (3),

$$I_1 = 6.01 \text{ A}$$

$$I_2 = 3.27 \text{ A}$$

$$I_3 = 3.38 \text{ A}$$

$$I_{5\Omega} = I_3 = 3.38 \text{ A}$$

Example 6

Find the value of current supplied by the battery.

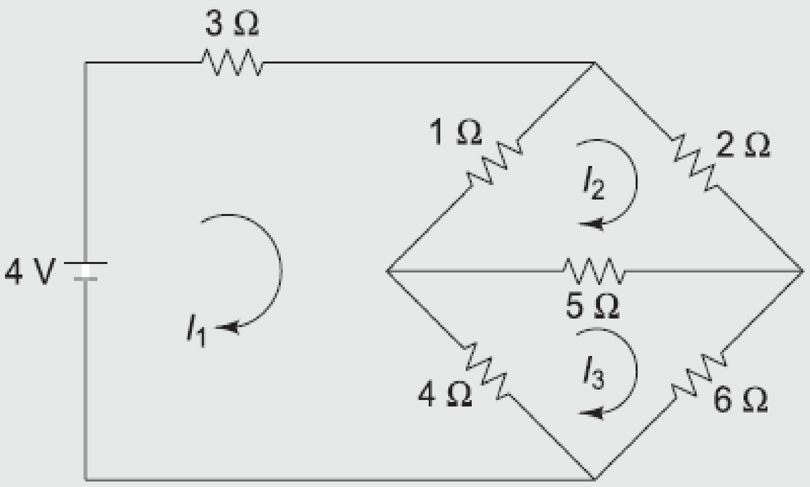


Fig. 2.71

Solution

Applying KVL to Mesh 1,

$$4 - 3I_1 - 1(I_1 - I_2) - 4(I_1 - I_3) = 0$$

$$8I_1 - I_2 - 4I_3 = 4 \qquad (1)$$

Example 6

Find the value of current supplied by the battery.

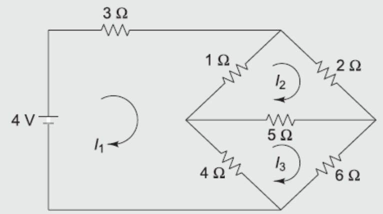


Fig. 2.71

Applying KVL to Mesh 2,

$$-2I_2 - 5(I_2 - I_3) - 1(I_2 - I_1) = 0$$

$$-I_1 + 8I_2 - 5I_3 = 0 \quad (2)$$

Applying KVL to Mesh 3,

$$-6I_3 - 4(I_3 - I_1) - 5(I_3 - I_2) = 0$$

$$-4I_1 - 5I_2 + 15I_3 = 0 \quad (3)$$

Writing equations in matrix form,

$$\begin{bmatrix} 8 & -1 & -4 \\ -1 & 8 & -5 \\ -4 & -5 & 15 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix}$$

Solving Eqs (1), (2) and (3),

$$I_1 = 0.66 \text{ A}$$

$$I_2 = 0.24 \text{ A}$$

$$I_3 = 0.26 \text{ A}$$

Current supplied by the battery $= I_1 = 0.66$ A.

2.3 SUPERMESH ANALYSIS

Meshes that share a current source with other meshes, none of which contains a current source in the outer loop, form a supermesh. A path around a supermesh doesn't pass through a current source. A path around each mesh contained within a supermesh passes through a current source. The total number of equations required for a supermesh is equal to the number of meshes contained in the supermesh. A supermesh requires one mesh current equation, that is, a KVL equation. The remaining mesh current equations are KCL equations.

Example 1

Find the current through the 10 Ω resistor of the network shown in Fig. 2.78.

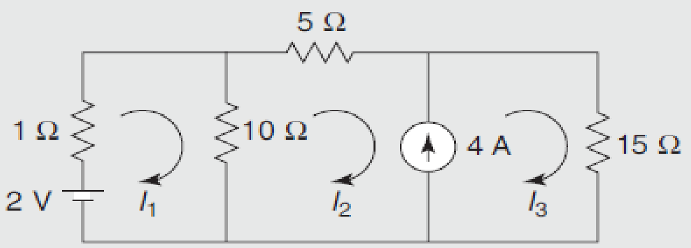


Fig. 2.78

Example 1

Find the current through the 10 Ω resistor of the network shown in Fig. 2.78.

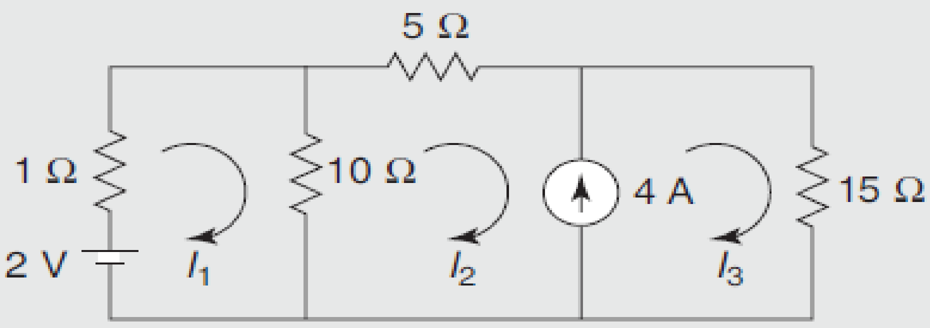


Fig. 2.78

Solution Applying KVL to Mesh 1,

$$2 - 1I_1 - 10(I_1 - I_2) = 0$$

$$11I_1 - 10I_2 = 2 \qquad (1)$$

Since meshes 2 and 3 contain a current source of 4 A, these two meshes will form a supermesh. A supermesh is formed by two adjacent meshes that have a common current source. The direction of the current source of 4 A and current $(I_3 - I_2)$ are same, i.e., in the upward direction.

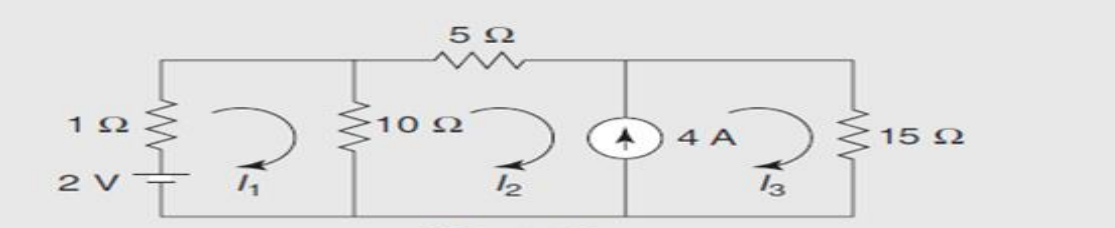


Fig. 2.78

Writing current equation to the supermesh,1

$$I_3 - I_2 = 4 \qquad (2)$$

Applying KVL to the outer path of the supermesh,

$$-10(I_2 - I_1) - 5I_2 - 15I_3 = 0$$

$$10I_1 - 15I_2 - 15I_3 = 0 \qquad (3)$$

Solving Eqs (1), (2) and (3),

$$I_1 = -2.35 \text{ A}$$

$$I_2 = -2.78 \text{ A}$$

$$I_3 = 1.22 \text{ A} \qquad (4)$$

Current through the 10 Ω resistor $= I_1 - I_2 = -(2.35) - (-2.78) = 0.43$ A

Example 2

Find the current in the 3 Ω resistor of the network shown in Fig. 2.79.

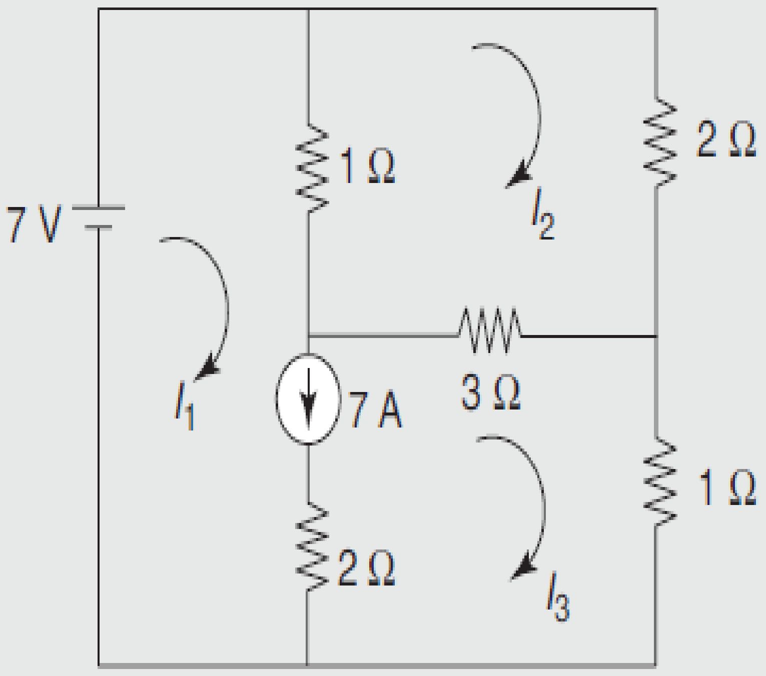


Fig. 2.79

Solution Meshes 1 and 3 will form a supermesh.

Writing current equation for the supermesh,

$$I_1 - I_3 = 7 \tag{1}$$

Applying KVL to the outer path of the supermesh,

$$7 - 1(I_1 - I_2) - 3(I_3 - I_2) - 1I_3 = 0$$

$$-I_1 + 4I_2 - 4I_3 = -7 \tag{2}$$

Applying KVL to Mesh 2,

$$-1(I_2 - I_1) - 2I_2 - 3(I_2 - I_3) = 0$$

$$I_1 - 6I_2 + 3I_3 = 0 \tag{3}$$

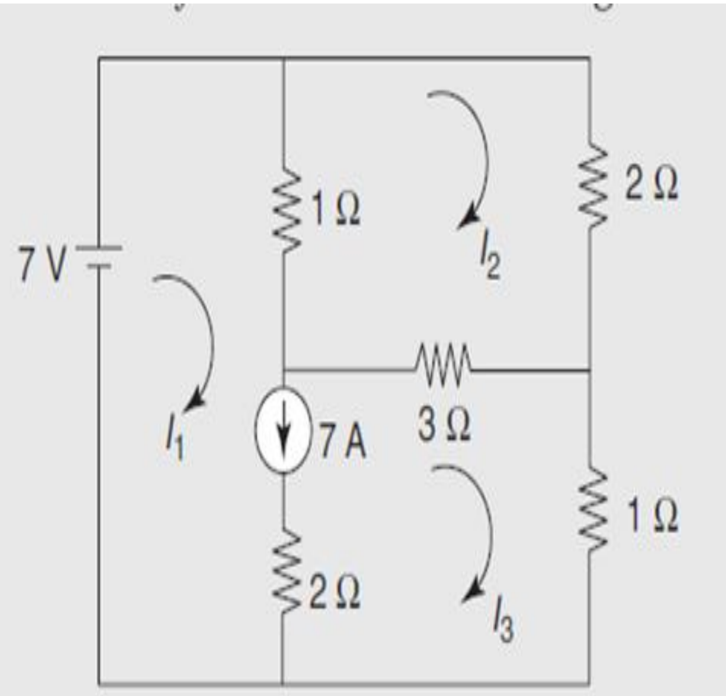


Fig. 2.79

Solving Eqs (1), (2) and (3),

$$I_1 = 9\text{ A}$$

$$I_2 = 2.5\text{ A}$$

$$I_3 = 2\text{ A}$$

Current through the 3 Ω resistor = $I_2 - I_3 = 2.5 - 2 = 0.5\text{ A}$

2.4 NODAL ANALYSIS

Nodal analysis is based on Kirchhoff's current law which states that the algebraic sum of currents meeting at a point is zero. Every junction where two or more branches meet is regarded as a node. One of the nodes in the network is taken as *reference node* or *datum node*. If there are n nodes in any network, the number of simultaneous equations to be solved will be $(n - 1)$.

2.4.1 Steps to be followed in Nodal Analysis

1. Assuming that a network has n nodes, assign a reference node and the reference directions, and assign a current and a voltage name for each branch and node respectively.
2. Apply KCL at each node except for the reference node and apply Ohm's law to the branch currents.
3. Solve the simultaneous equations for the unknown node voltages.
4. Using these voltages, find any branch currents required.

Example 1

Calculate the current through 2 Ω resistor for the network shown in Fig. 2.93.

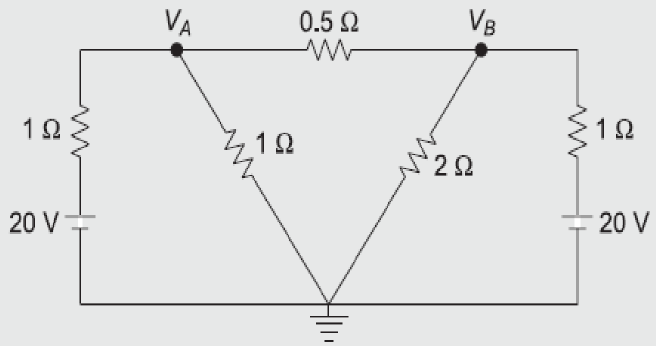


Fig. 2.93

Solution Assume that the currents are moving away from the nodes.

Applying KCL at node *A*,

$$\frac{V_A - 20}{1} + \frac{V_A}{1} + \frac{V_A - V_B}{0.5} = 0$$

$$\left(\frac{1}{1} + \frac{1}{1} + \frac{1}{0.5}\right) V_A - \frac{1}{0.5} V_B = \frac{20}{1}$$

$$4V_A - 2V_B = 20 \qquad (1)$$

Applying KCL at node *B*,

Applying KCL at node B,

$$\frac{V_B - V_A}{0.5} + \frac{V_B}{2} + \frac{V_B - 20}{1} = 0$$

$$-\frac{1}{0.5}V_A + \left(\frac{1}{0.5} + \frac{1}{2} + \frac{1}{1}\right)V_B = \frac{20}{1}$$

$$-2V_A + 3.5V_B = 20 \qquad (2)$$

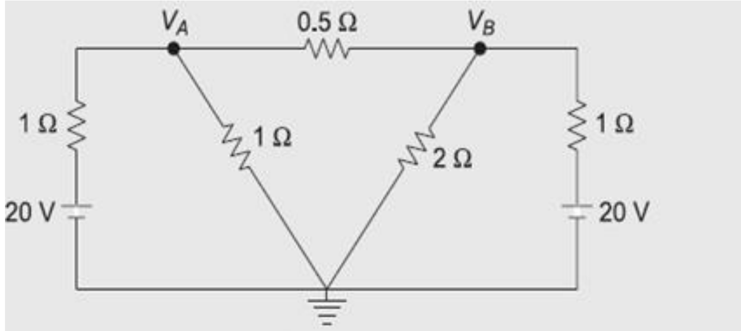


Fig. 2.93

$$4V_A - 2V_B = 20 \qquad (1)$$

Applying KCL at node B,

$$\frac{V_B - V_A}{0.5} + \frac{V_B}{2} + \frac{V_B - 20}{1} = 0$$

$$-\frac{1}{0.5}V_A + \left(\frac{1}{0.5} + \frac{1}{2} + \frac{1}{1}\right)V_B = \frac{20}{1}$$

$$-2V_A + 3.5V_B = 20 \qquad (2)$$

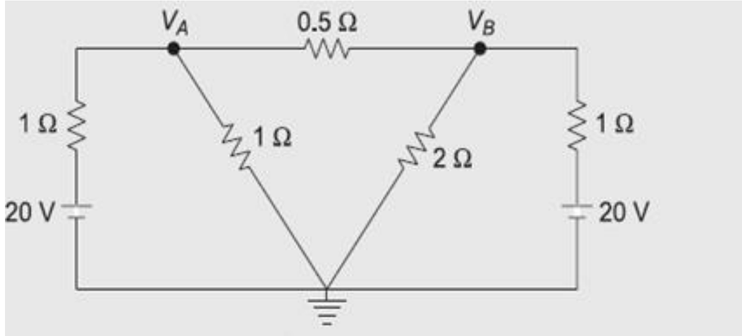


Fig. 2.93

Solving Eqs (1) and (2),

$$V_A = 11 \text{ V}$$
$$V_B = 12 \text{ V}$$

$$\text{Current through } 2\ \Omega \text{ resistor} = \frac{V_B}{2} = \frac{12}{2} = 6 \text{ A}$$

2.5 SUPERNODE ANALYSIS

Nodes that are connected to each other by voltage sources, but not to the reference node by a path of voltage sources, form a *supernode*. A supernode requires one node voltage equation, that is, a KCL equation. The remaining node voltage equations are KVL equations.

Example 1

Determine the current in the 5 Ω resistor for the network shown in Fig. 2.110.

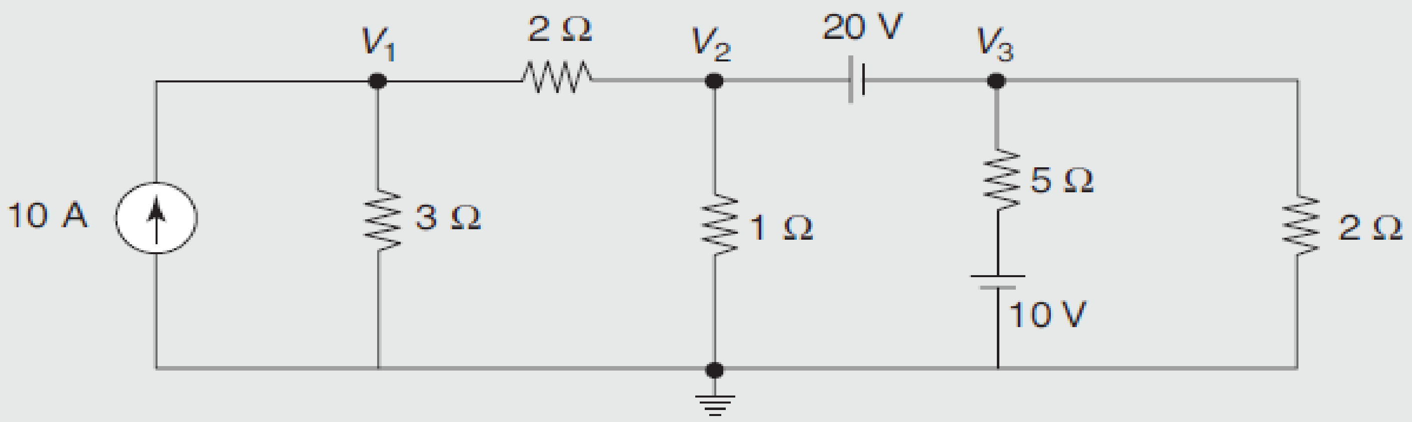


Fig. 2.110

Solution Assume that the currents are moving away from the nodes.

Applying KCL at Node 1,

$$10 = \frac{V_1}{3} + \frac{V_1 - V_2}{2}$$

$$\left(\frac{1}{3} + \frac{1}{2}\right)V_1 - \frac{1}{2}V_2 = 10$$

$$0.83\,V_1 - 0.5\,V_2 = 10 \qquad (1)$$

Nodes 2 and 3 will form a supernode.

Writing voltage equation for the supernode,

$$V_2 - V_3 = 20 \qquad (2)$$

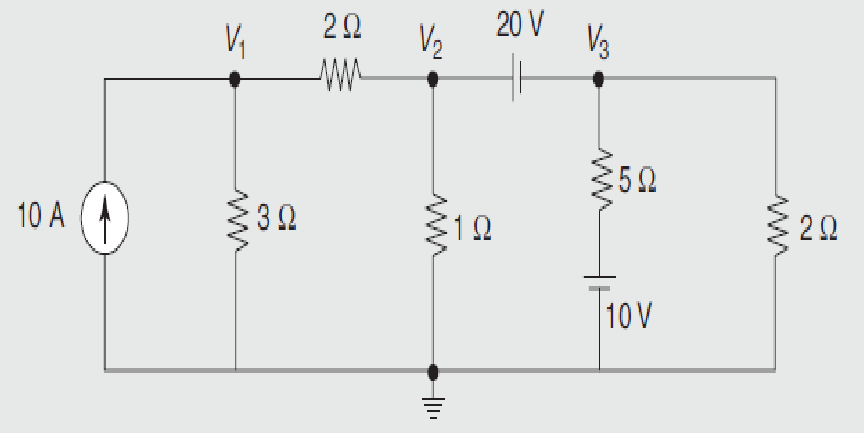


Fig. 2.110

Applying KCL at the supernode,

$$\frac{V_2 - V_1}{2} + \frac{V_2}{1} + \frac{V_3 - 10}{5} + \frac{V_3}{2} = 0$$

$$-\frac{1}{2}V_1 + \left(\frac{1}{2} + 1\right)V_2 + \left(\frac{1}{5} + \frac{1}{2}\right)V_3 = 2$$

$$-0.5\,V_1 + 1.5\,V_2 + 0.7\,V_3 = 2 \qquad (3)$$

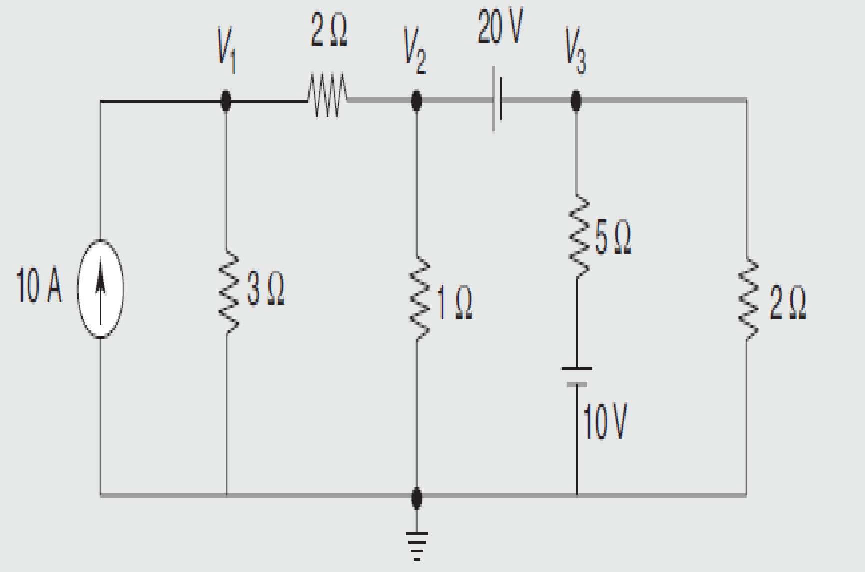


Fig. 2.110

Solving Eqs (1), (2) and (3),

$$V_1 = 19.04\text{ V}$$

$$V_2 = 11.6\text{ V}$$

$$V_3 = -8.4\text{ V}$$

$$I_{5\,\Omega} = \frac{V_3 - 10}{5} = \frac{-8.4 - 10}{5} = -3.68\text{ A}$$

Example 2

Find the power delivered by the 5 A current source in the network shown in Fig. 2.111.

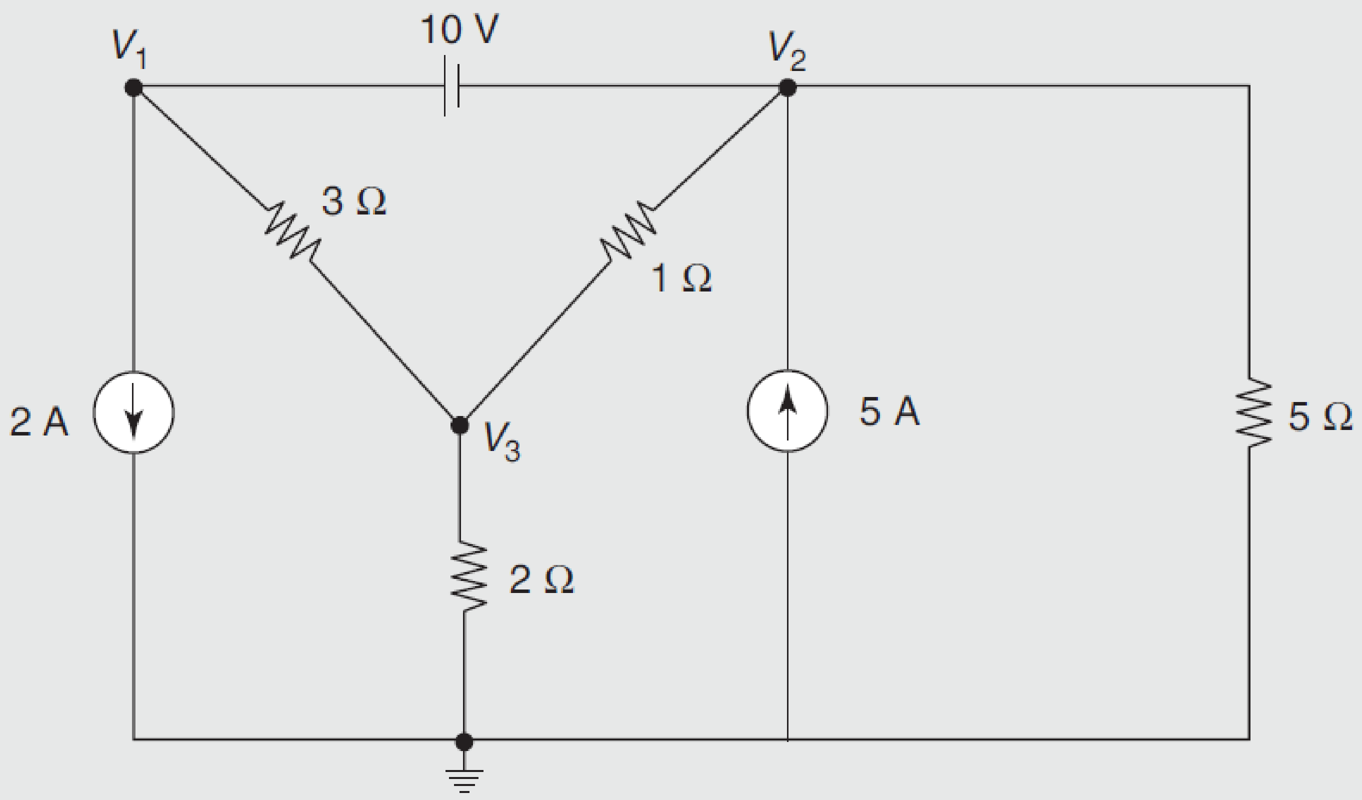


Fig. 2.111

Solution Assume that the currents are moving away from the nodes.

Nodes 1 and 2 will form a supernode.

Writing voltage equation for the supernode,

$$V_1 - V_2 = 10 \qquad (1)$$

Applying KCL at the supernode,

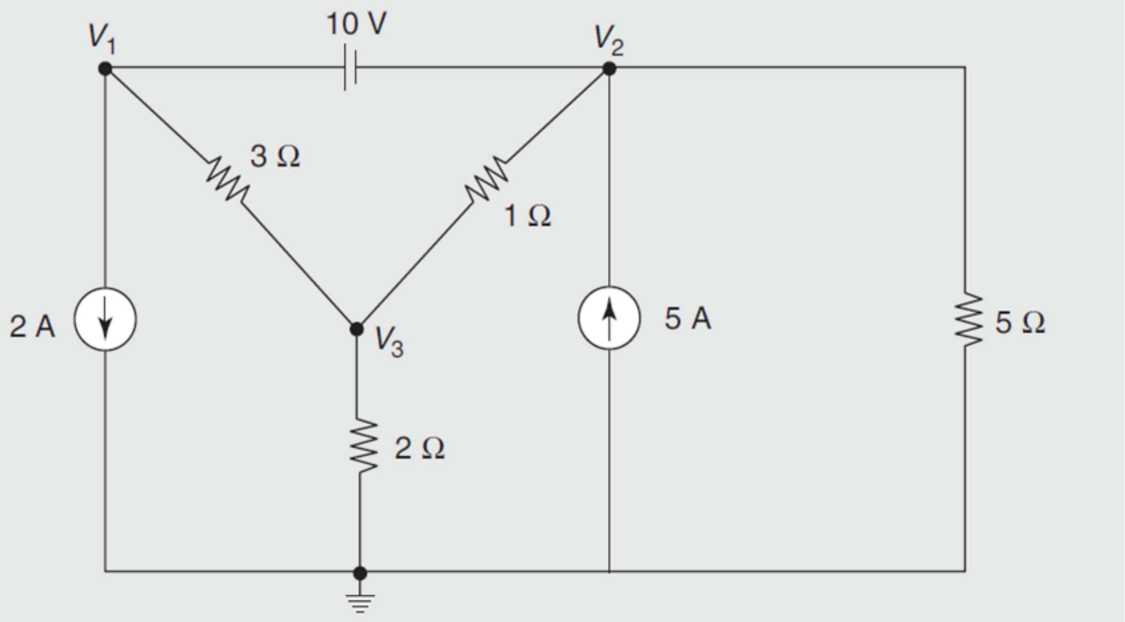


Fig. 2.111

Applying KCL at the supernode,

$$2+\frac{V_1-V_3}{3}+\frac{V_2}{5}+\frac{V_2-V_3}{1}=5$$

$$\frac{1}{3}V_1+\left(\frac{1}{5}+1\right)V_2-\left(\frac{1}{3}+1\right)V_3=3$$

$$0.33\,V_1+1.2\,V_2-1.33\,V_3=3 \qquad (2)$$

Applying KCL at Node 3,

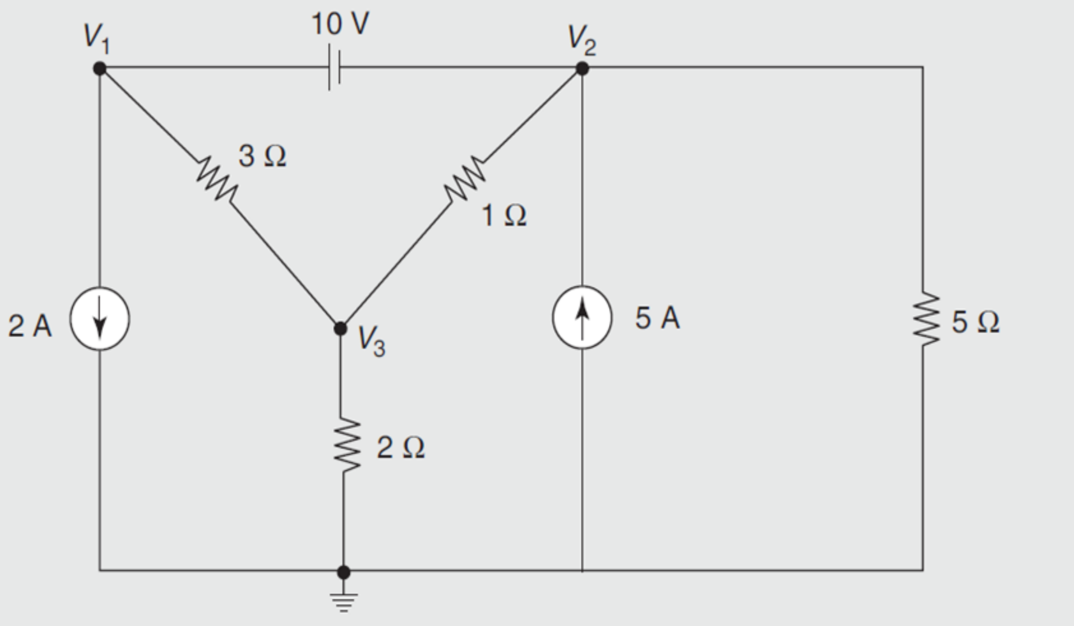


Fig. 2.111

Applying KCL at Node 3,

$$\frac{V_3 - V_1}{3} + \frac{V_3 - V_2}{1} + \frac{V_3}{2} = 0$$

$$-\frac{1}{3}V_1 - V_2 + \left(\frac{1}{3} + 1 + \frac{1}{2}\right)V_3 = 0$$

$$-0.33\,V_1 - V_2 + 1.83\,V_3 = 0 \qquad (3)$$

Solving Eqs (1), (2) and (3),

$$V_1 = 13.72 \text{ V}$$

$$V_2 = 3.72 \text{ V}$$

$$V_3 = 4.51 \text{ V}$$

Power delivered by the 5 A source = $5\ V_2 = 5 \times 3.72 = 18.6$ W

2.9 THEVENIN'S THEOREM

It states that *'Any two terminals of a network can be replaced by an equivalent voltage source and an equivalent series resistance. The voltage source is the voltage across the two terminals with load, if any, removed. The series resistance is the resistance of the network measured between two terminals with load removed and constant voltage source being replaced by its internal resistance (or if it is not given with zero resistance, i.e., short circuit) and constant current source replaced by infinite resistance, i.e., open circuit.'*

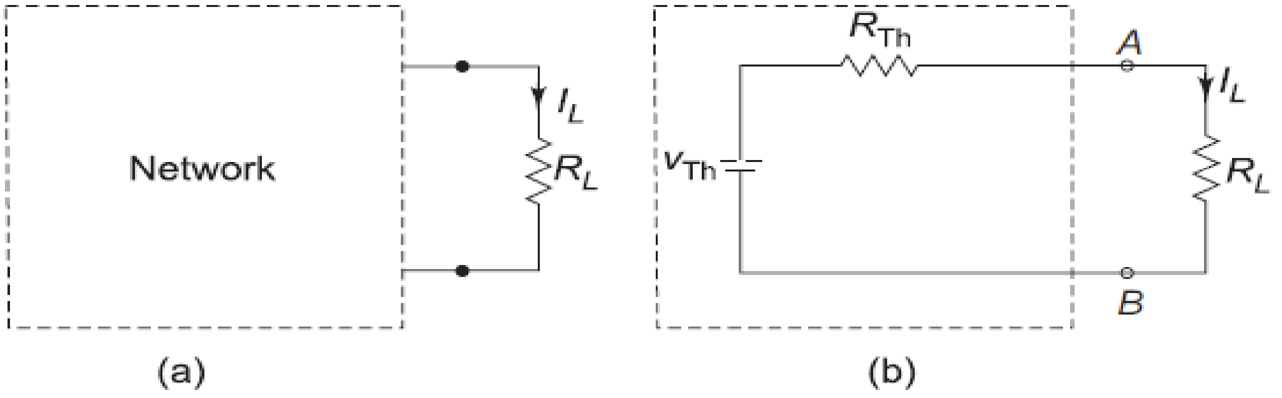


Fig. 2.367 *Thevenin's theorem*

2.9.1 Steps to be followed in Thevenin's Theorem

1. Remove the load resistance R_L.
2. Find the open circuit voltage V_{Th} across points A and B.
3. Find the resistance R_{Th} as seen from points A and B with the voltage sources and current sources replaced by internal resistances.
4. Replace the network by a voltage source V_{Th} in series with resistance R_{Th}.
5. Find the current through R_L using Ohm's law.

$$I_L = \frac{V_{\text{Th}}}{R_{\text{Th}} + R_L}$$

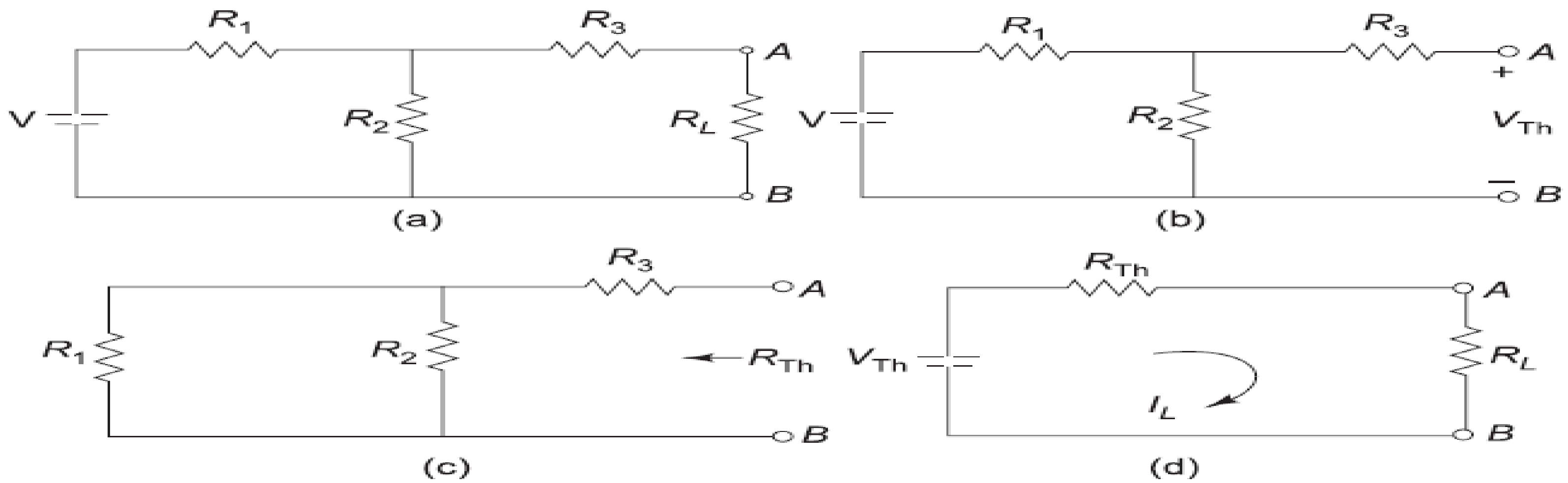


Fig. 2.368 Steps in Thevenin's theorem

Example 1

Find the value of current flowing through the 2 Ω resistor.

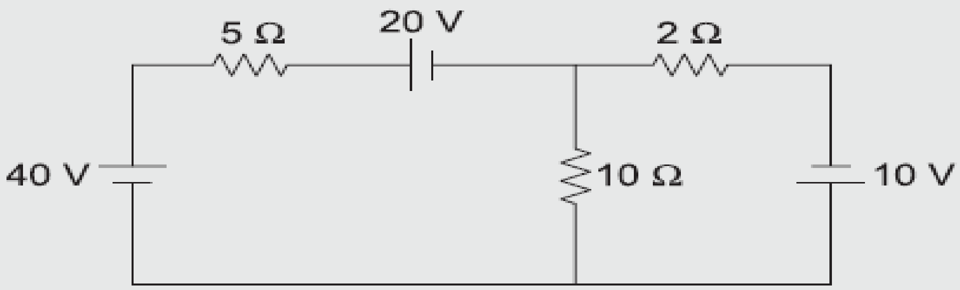


Fig. 2.369

Solution *Step I : Calculation of V_{Th}*

Removing the 2 Ω resistor from the network,

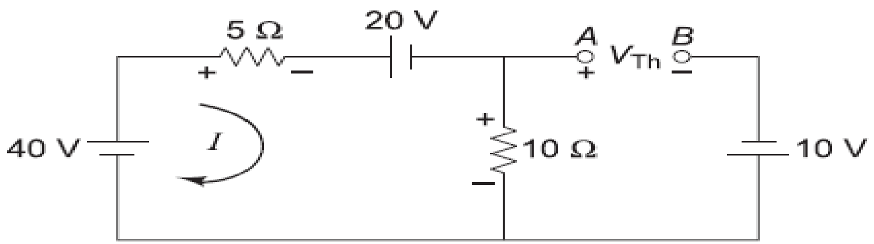


Fig. 2.370

Applying KVL to the mesh,

$$40 - 5I - 20 - 10I = 0$$
$$15I = 20$$
$$I = 1.33 \text{ A}$$

Writing V_{Th} equation,

$$10I - V_{Th} + 10 = 0$$
$$V_{Th} = 10I + 10$$
$$= 10\,(1.33) + 10$$
$$= 23.33 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing voltage sources by short circuits,

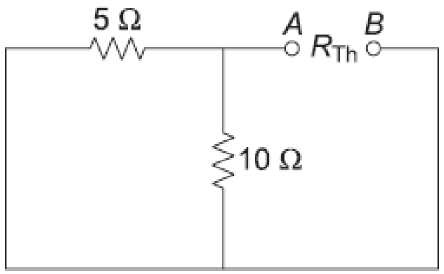


Fig. 2.371

$$R_{Th} = 5 \,||\, 10 = 3.33\ \Omega$$

Step III: Calculation of I_L

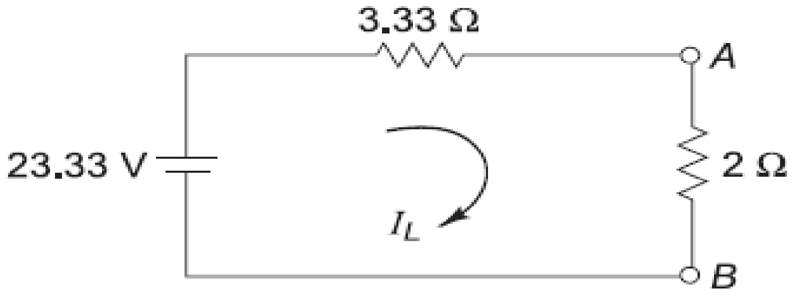


Fig. 2.372

$$I_L = \frac{23.33}{3.33 + 2} = 4.38 \text{ A}$$

Example 2

Find the value of current flowing through the 8 Ω resistor.

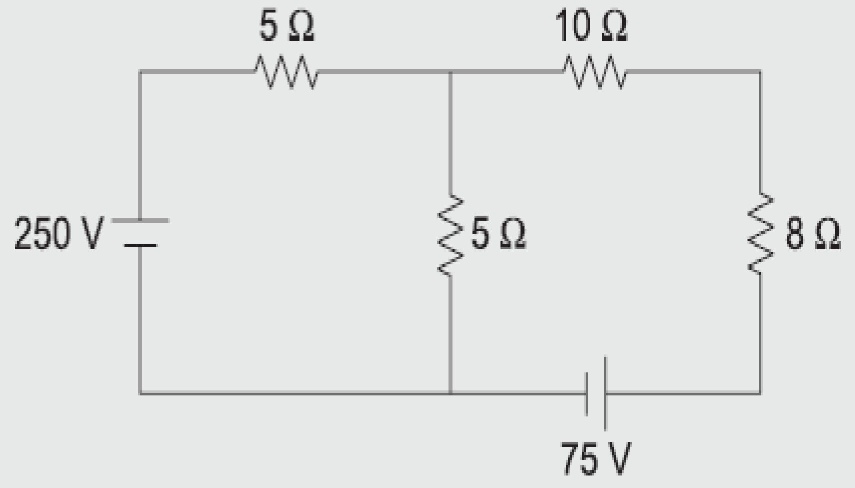


Fig. 2.373

Example 2

Find the value of current flowing through the 8 Ω resistor.

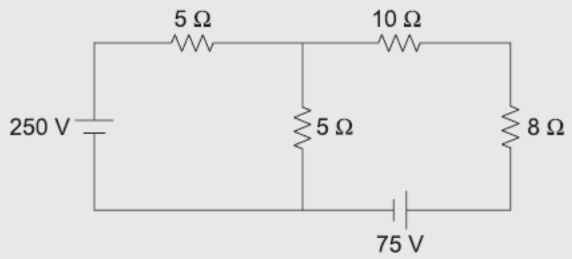


Fig. 2.373

Solution *Step I: Calculation of V_{Th}*

Removing the 8 Ω resistor from the network,

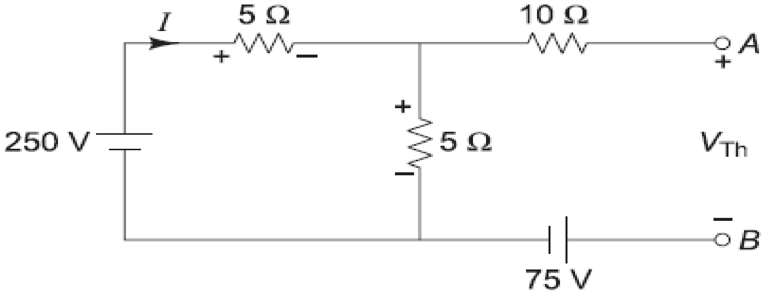


Fig. 2.374

$$I = \frac{250}{5+5} = 25 \text{ A}$$

Writing V_{Th} equation,

$$250 - 5I - V_{Th} - 75 = 0$$

$$V_{Th} = 175 - 5I$$

$$= 175 - 5\,(25)$$

$$= 50 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing voltage sources by short circuits,

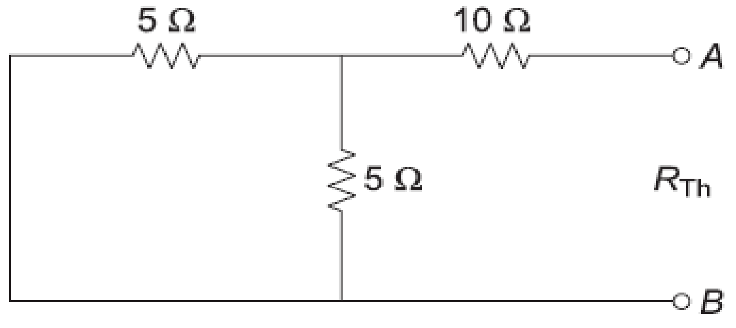


Fig. 2.375

$$R_{Th} = (5 \parallel 5) + 10 = 12.5\ \Omega$$

Step III: Calculation of I_L

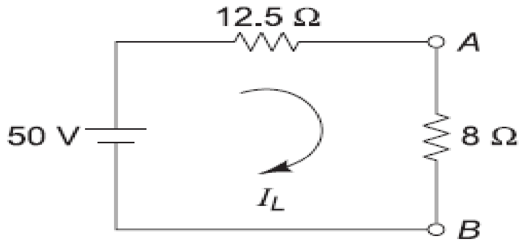


Fig. 2.376

$$I_L = \frac{50}{12.5 + 8} = 2.44\ \text{A}$$

Example 3

Find the value of current flowing through the 2 Ω resistor connected between terminals A and B.

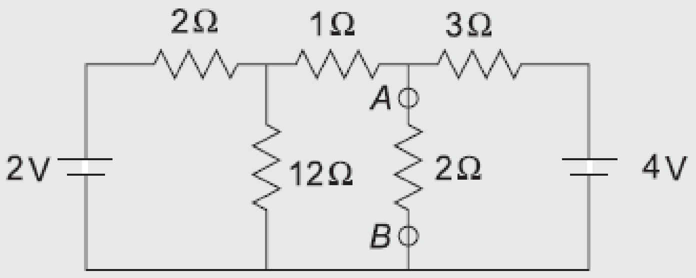


Fig. 2.377

Solution

Step I: Calculation of V_{Th}

Removing the 2 Ω resistor connected between terminals A and B,

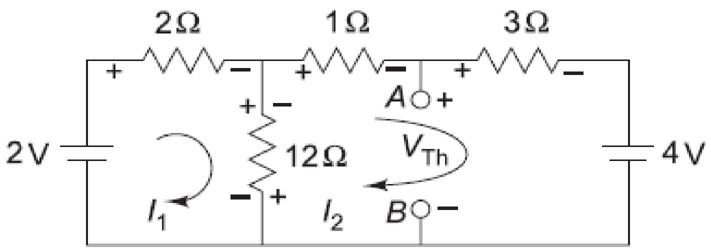


Fig. 2.378

Applying KVL to Mesh 1,

$$2 - 2I_1 - 12(I_1 - I_2) = 0$$

$$14I_1 - 12I_2 = 2 \qquad (1)$$

Applying KVL to Mesh 2,

$$-12(I_2 - I_1) - 1I_2 - 3I_2 - 4 = 0$$

$$-12I_1 + 16I_2 = -4 \qquad (2)$$

Solving Eqs (1) and (2),

$$I_2 = -0.4 \text{ A}$$

Writing V_{Th} equation,

$$V_{Th} - 3I_2 - 4 = 0$$

$$V_{Th} = 4 + 3I_2$$

$$= 4 + 3(-0.4)$$

$$= 2.8 \text{ V}$$

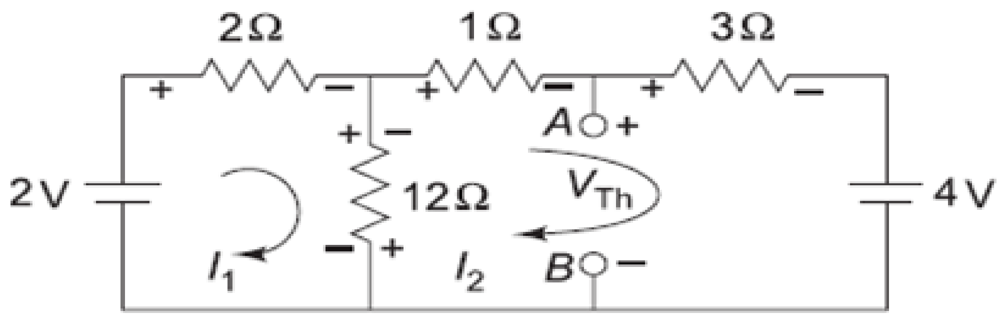


Fig. 2.378

– 2.8 V

Step II: Calculation of R_{Th}

Step II: Calculation of R_{Th}

Replacing all voltage sources by short circuits,

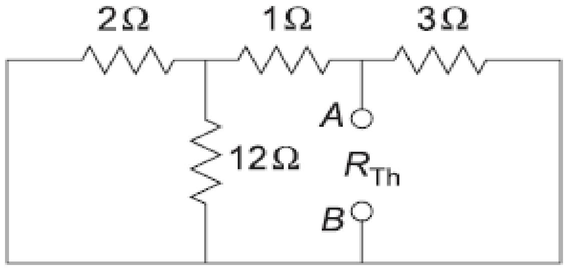


Fig. 2.379

$$R_{Th} = [|(2||\ 12) + 1]\ ||\ 3 = 1.43\ \Omega$$

Step III: Calculation of I_L

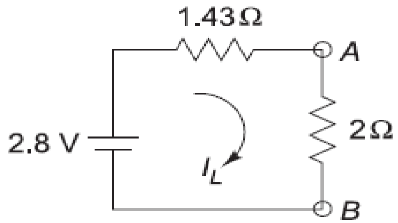


Fig. 2.380

$$I_L = \frac{40}{5 + 1.67} = 0.82\text{ A}$$

Example 9

Determine the value of current flowing through the 24 Ω resistor.

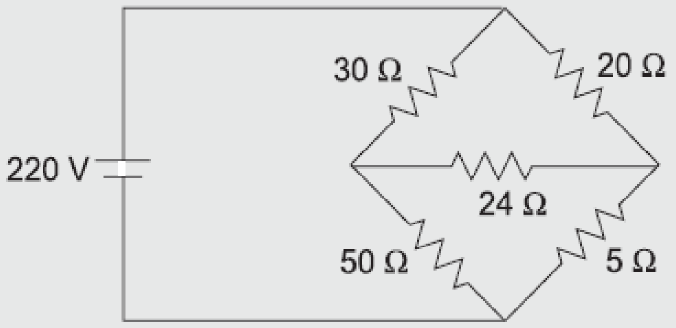


Fig. 2.401

Solution

Step I: Calculation of V_{Th}

Removing the 24 Ω resistor from the network,

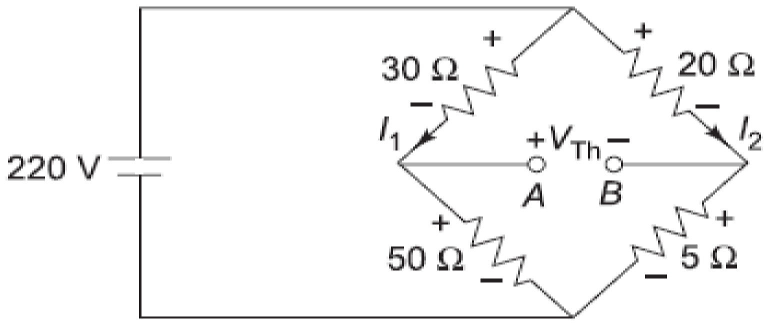


Fig. 2.402

$$I_1 = \frac{220}{30+50} = 2.75 \text{ A}$$

$$I_2 = \frac{220}{20+5} = 8.8 \text{ A}$$

Writing V_{Th} equation,

$$V_{Th} + 30I_1 - 20I_2 = 0$$

$$V_{Th} = 20I_2 - 30I_1$$

$$= 20\,(8.8) - 30\,(2.75)$$

$$= 93.5 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing the voltage source by short circuit,

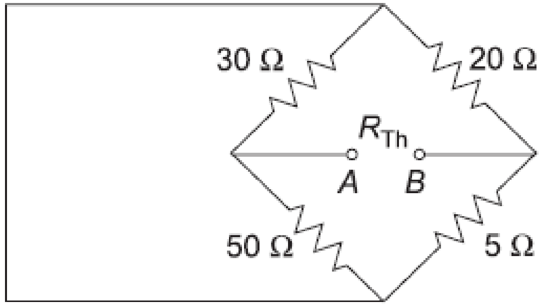


Fig. 2.403

Solution

Step I: Calculation of V_{Th}

Removing the 24 Ω resistor from the network,

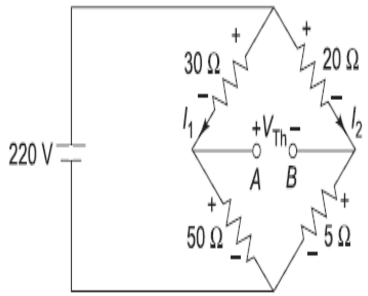


Fig. 2.402

$$I_1 = \frac{220}{30+50} = 2.75 \text{ A}$$

$$I_2 = \frac{220}{20+5} = 8.8 \text{ A}$$

Redrawing the circuit,

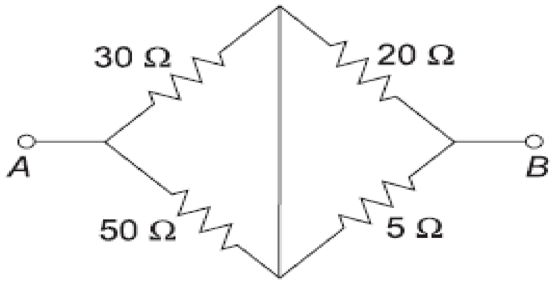


Fig. 2.404

$$R_{Th} = (30 \,||\, 50) + (20 \,||\, 5) = 22.75\ \Omega$$

Step III: Calculation of I_L

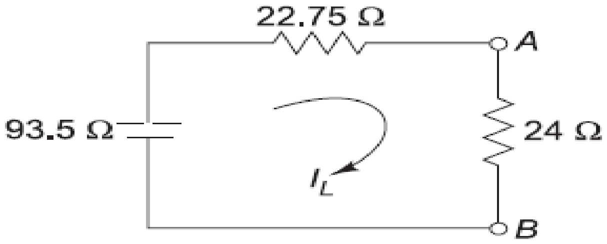


Fig. 2.405

$$I_L = \frac{93.5}{22.75 + 24} = 2\text{ A}$$

Example 10

Find the value of current flowing through the 3 Ω resistor.

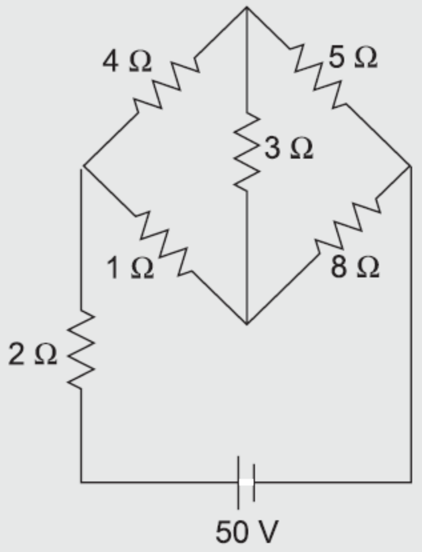


Fig. 2.406

Solution

Step I: Calculation of V_{Th}

Removing the 3 Ω resistor from the network,

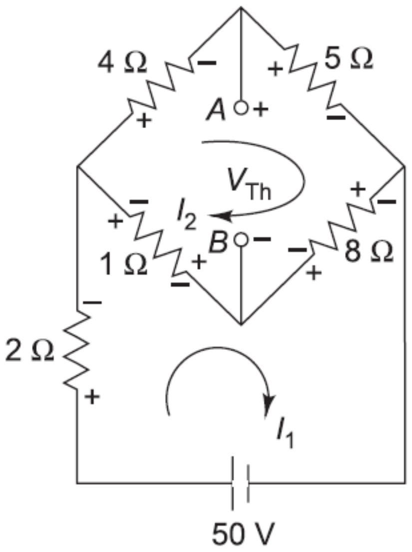


Fig. 2.407

Applying KVL to Mesh 1,

$$50 - 2I_1 - 1(I_1 - I_2) - 8(I_1 - I_2) = 0$$

$$11I_1 - 9I_2 = 50 \qquad (1)$$

Applying KVL to Mesh 2,

$$-4I_2 - 5I_2 - 8(I_2 - I_1) - 1(I_2 - I_1) = 0$$

$$-9I_1 + 18I_2 = 0 \qquad (2)$$

Solving Eqs (1) and (2),

$$I_1 = 7.69 \text{ A}$$

$$I_2 = 3.85 \text{ A}$$

Writing V_{Th} equation,

$$V_{Th} - 5I_2 - 8(I_2 - I_1) = 0$$

$$V_{Th} = 5I_2 + 8(I_2 - I_1)$$

$$= 5(3.85) + 8(3.85 - 7.69)$$

$$= -11.47 \text{ V}$$

$$= 11.47 \text{ V (the terminal } B \text{ is positive w.r.t. } A)$$

Step II: Calculation of R_{Th}

Replacing the voltage source by a short circuit,

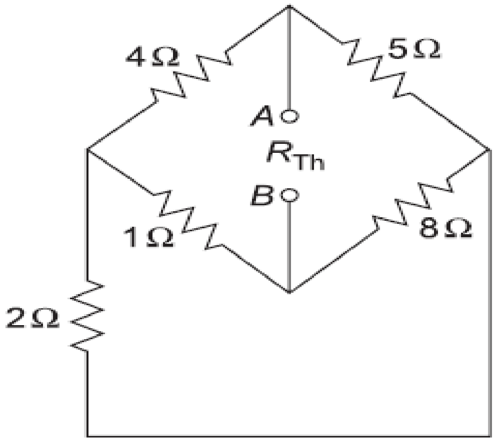
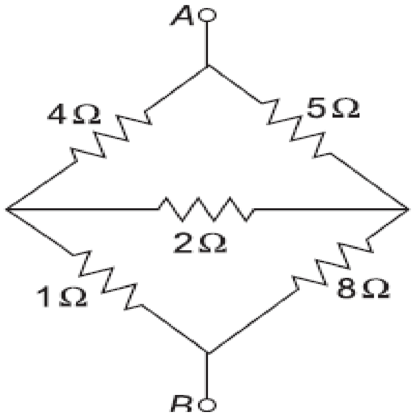


Fig. 2.408

Redrawing the network,



Converting the upper delta into equivalent star network,

$$R_1 = \frac{4 \times 2}{4+2+5} = 0.73\ \Omega$$

$$R_2 = \frac{4 \times 5}{4+2+5} = 1.82\ \Omega$$

$$R_3 = \frac{5 \times 2}{4+2+5} = 0.91\ \Omega$$

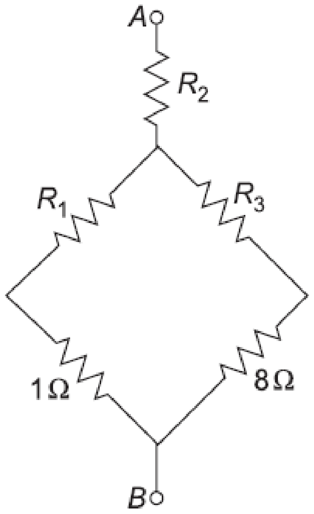


Fig. 2.410

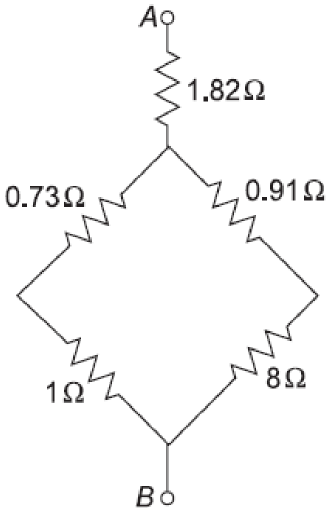


Fig. 2.411

Simplifying the network,

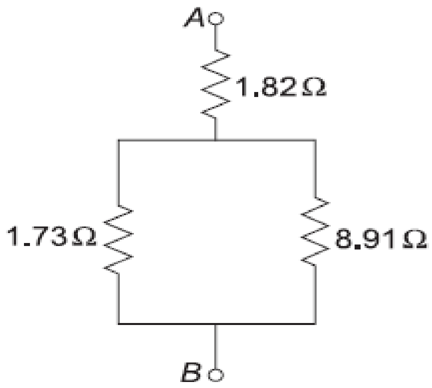


Fig. 2.412

$$R_{Th} = 1.82 + (1.73 \,||\, 8.91) = 3.27\ \Omega$$

Step III: Calculation of I_L

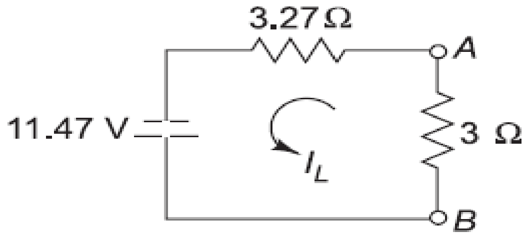


Fig. 2.413

$= 1.83\ \text{A}\ (\uparrow)$

2.10 NORTON'S THEOREM

[Dec 2013]

It states that *'Any two terminals of a network can be replaced by an equivalent current source and an equivalent parallel resistance.' The constant current is equal to the current which would flow in a short circuit placed across the terminals. The parallel resistance is the resistance of the network when viewed from these open-circuited terminals after all voltage and current sources have been removed and replaced by internal resistances.*

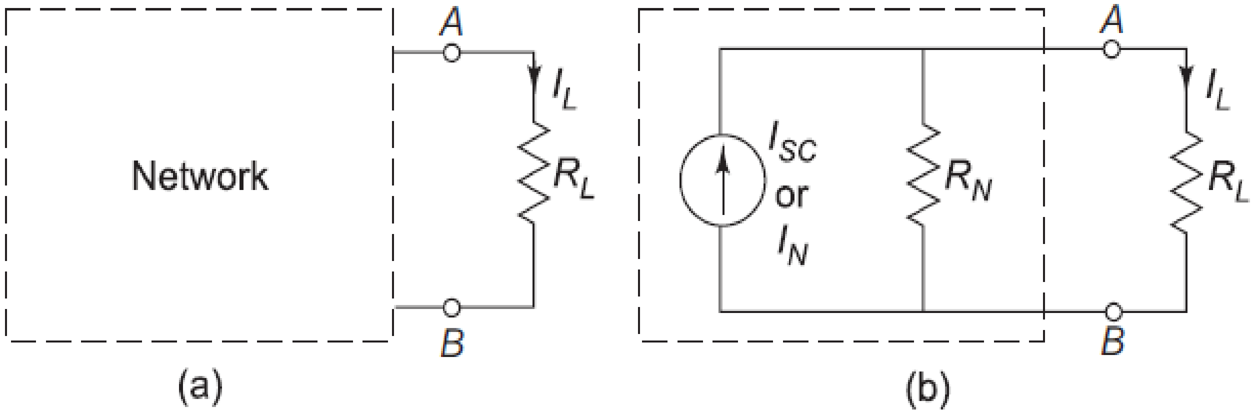


Fig. 2.467 Norton's theorem

2.10.1 Steps to be followed in Norton's Theorem

1. Remove the load resistance R_L and put a short circuit across the terminals.
2. Find the short-circuit current I_{sc} or I_N.
3. Find the resistance R_N as seen from points A and B by replacing the voltage sources and current sources by internal resistances.

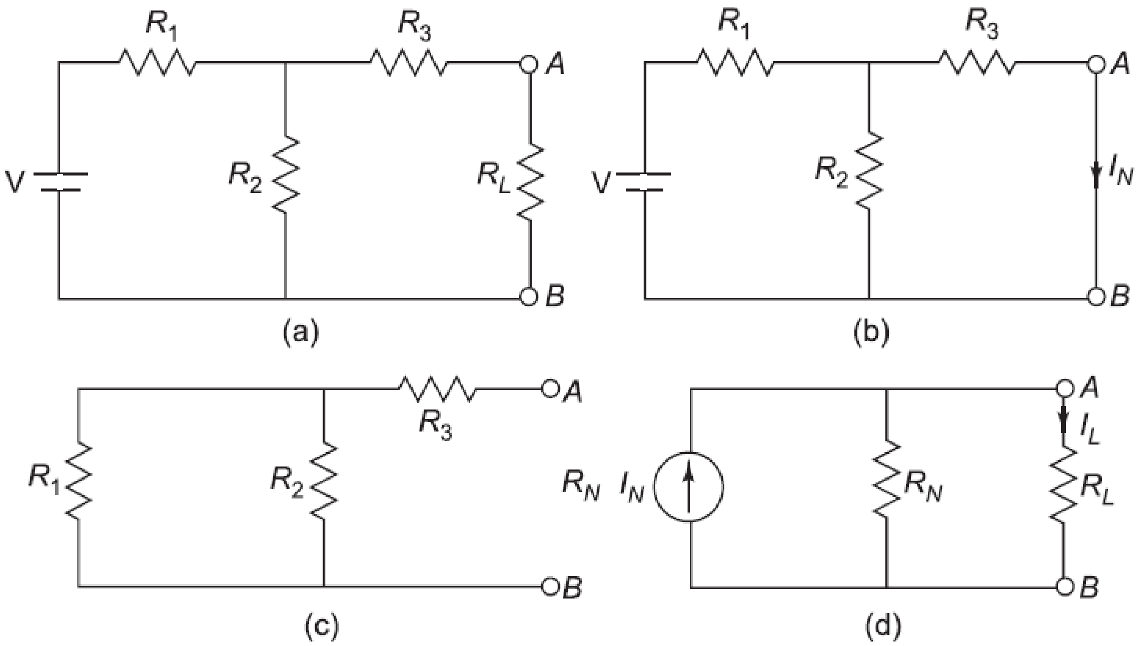


Fig. 2.468 *Steps in Norton's theorem*

4. Replace the network by a current source I_N in parallel with resistance R_N.
5. Find current through R_N by current–division rule,

$$I_L = \frac{I_N R_N}{R_N + R_L}$$

Example 1

For the given circuit in Fig. 2.539, find the Norton equivalent between points A and B.

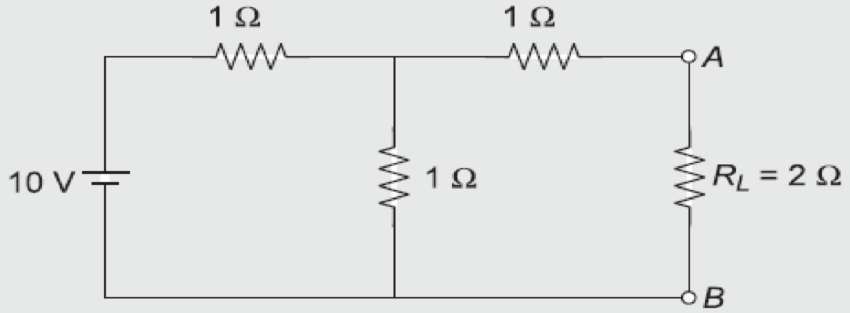


Fig. 2.469 **[May 2015]**

Solution

Step I: Calculation of I_N

Replacing 2 Ω resistor by short circuit,

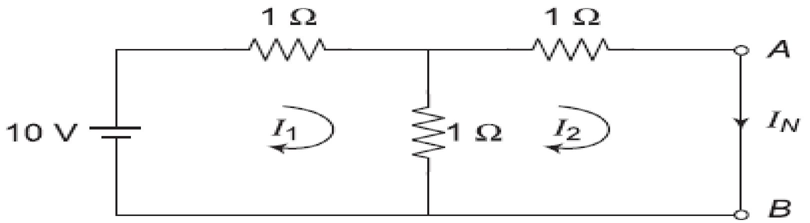


Fig. 2.470

Applying KVL to Mesh 1,

$$10 - 1I_1 - 1(I_1 - I_2) = 0$$

$$2I_1 = I_2 = 10 \qquad \ldots(1)$$

Applying KVL to Mesh 2,

$$-1(I_2 - I_1) - 1I_2 = 0$$

$$-I_1 + 2I_2 = 0 \qquad \text{...(2)}$$

Solving Eqs (1) and (2),

$$I_1 = 6.67 \text{ A}$$

$$I_2 = I_N = 3.33 \text{ A}$$

Step II: Calculation of R_N

Replacing voltage source by short circuit,

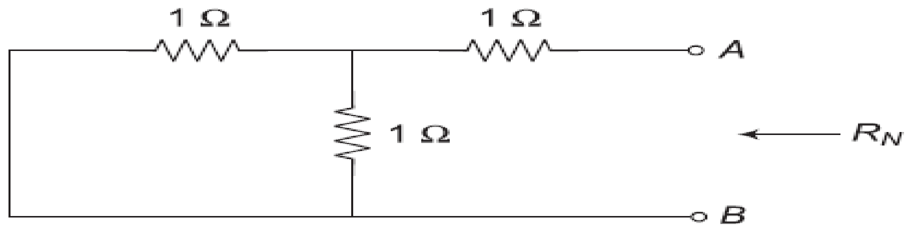
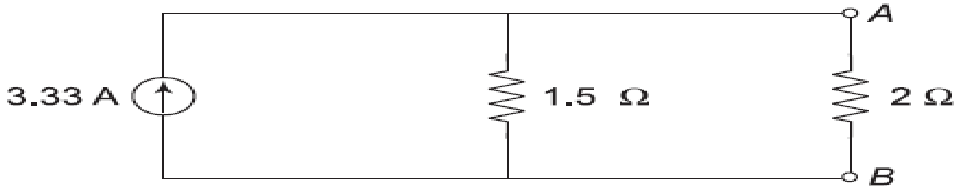


Fig. 2.471

$$R_N = 1.5\ \Omega$$

Step III: Norton's equivalent network



Example 2

Find the value of current through the 10 Ω resistor.

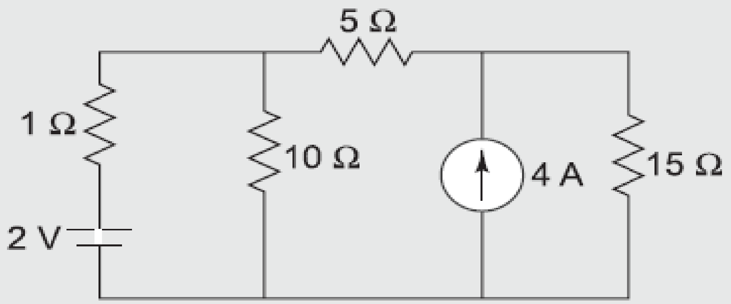
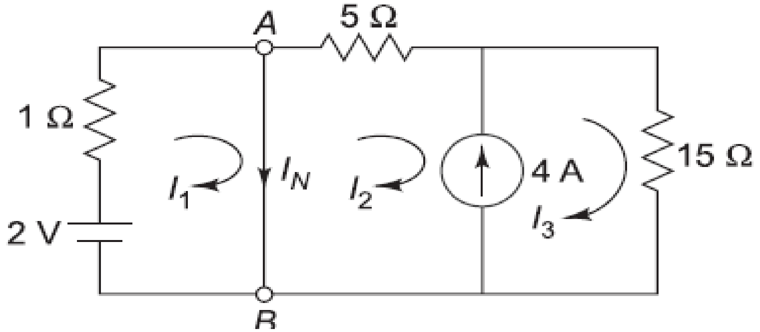


Fig. 2.473

Solution

Step I: Calculation of I_N

Replacing the 10 Ω resistor by a short circuit,



Applying KVL to Mesh 1,

$$2 - 1I_1 = 0 \quad (1)$$

$$I_1 = 2$$

Meshes 2 and 3 will form a supermesh.

Writing current equation for the supermesh,

$$I_3 - I_2 = 4 \quad (2)$$

Applying KVL to the supermesh,

$$-5I_2 - 15I_3 = 0 \quad (3)$$

Solving Eqs (1), (2) and (3),

$$I_1 = 2\ \text{A}$$

$$I_2 = -3\ \text{A}$$

$$I_3 = 1\ \text{A}$$

$$I_N = I_1 - I_2 = 2 - (-3) = 5\ \text{A}$$

Step II: Calculation of R_N

Replacing the voltage source by a short circuit and current source by an open circuit,

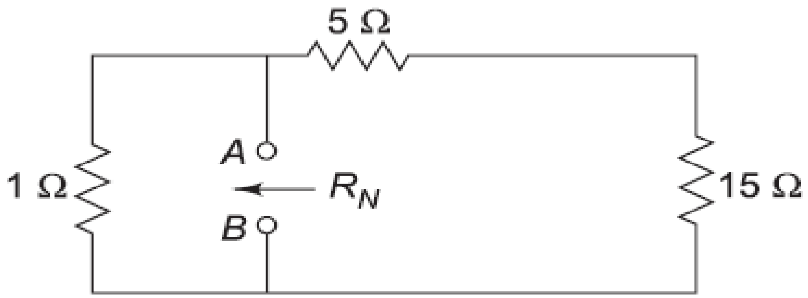
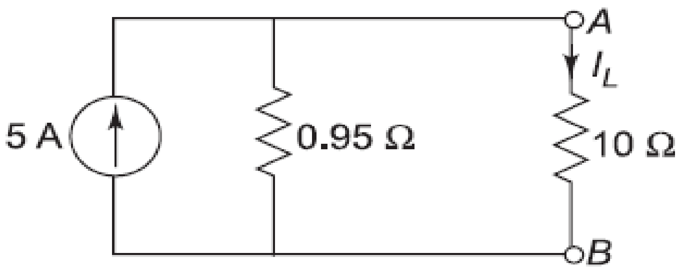
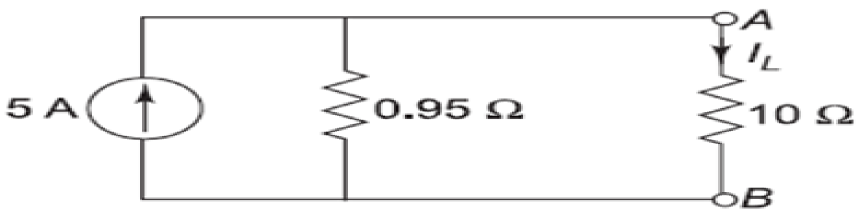


Fig. 2.475

$$R_N = 1 \,||\, (5 + 15) = 0.95\ \Omega$$

Step III: Calculation of I_L





$$I_L = 5 \times \frac{0.95}{10 + 0.95} = 0.43 \text{ A}$$

Example 3

Calculate the value of current flowing through the 15 Ω load resistor in the given circuit.

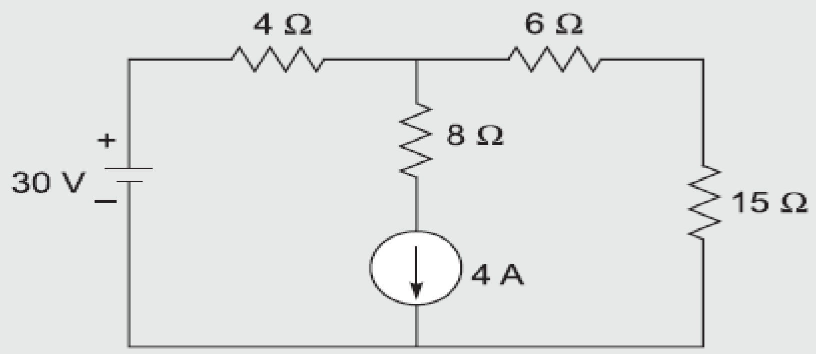


Fig. 2.477 [May 2013]

Solution

Step I: Calculation of I_N

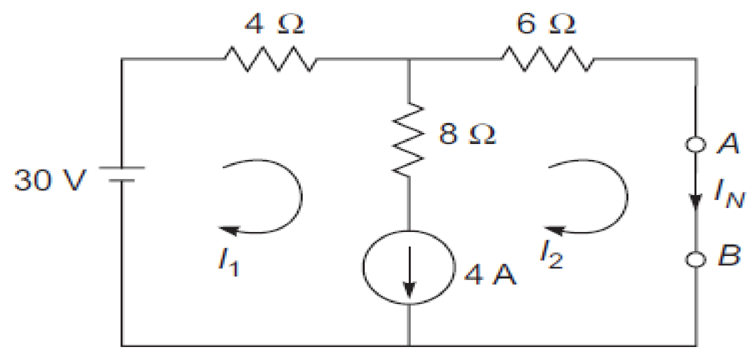


Fig. 2.478

Solution

Step I: Calculation of I_N

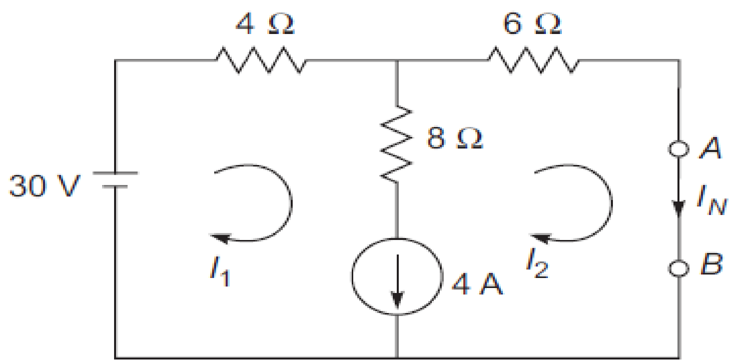


Fig. 2.478

Writing the current equation for the supermesh,

$$I_1 - I_2 = 4 \quad (1)$$

Writing the voltage equation for the supermesh,

$$30 - 4I_1 - 6I_2 = 0$$

$$4I_1 + 6I_2 = 30 \quad (2)$$

Solving Eqs (1) and (2),

$$I_1 = 5.4 \text{ A}$$

$$I_2 = 1.4 \text{ A}$$

$$I_N = I_2 = 1.4 \text{ A}$$

Step II: Calculation of R_N

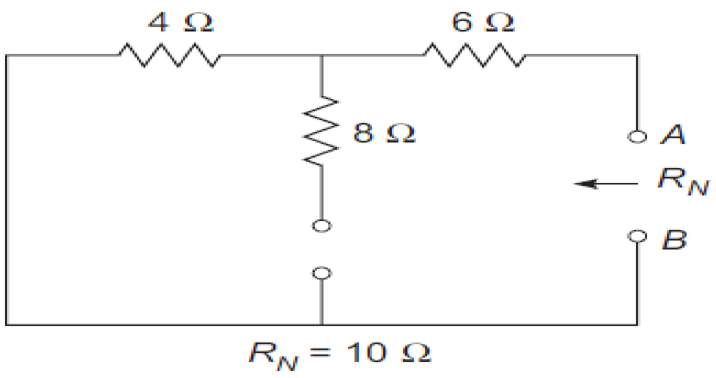


Fig. 2.479

Step III: Calculation of I_L

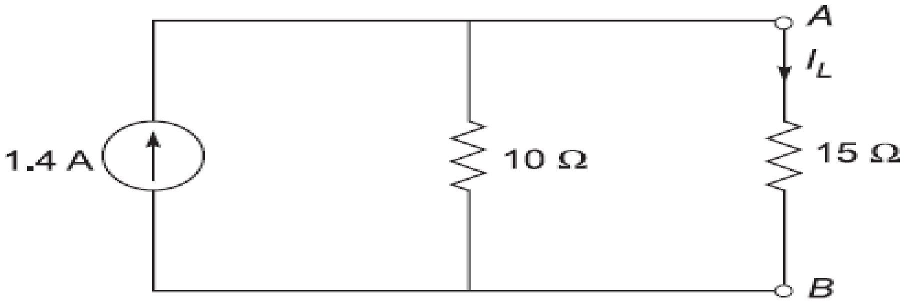


Fig. 2.480

$$I_L = 1.4 \times \frac{10}{10+15} = 0.56 \text{ A}$$

Example 10

Find value of current flowing through the 1 Ω resistor.

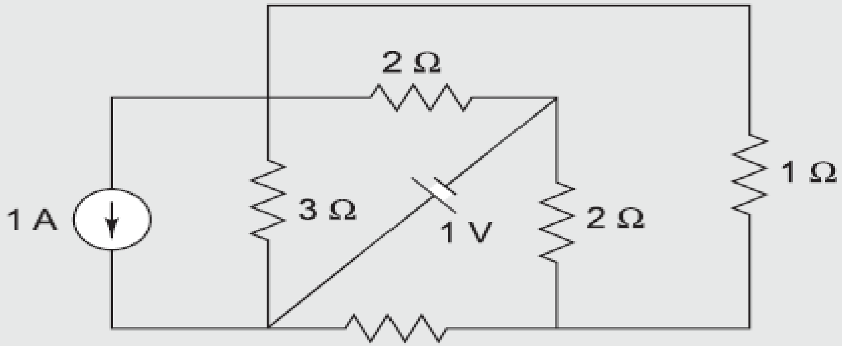


Fig. 2.507

Solution

Step I: Calculation of I_N

Replacing the 1 Ω resistor by a short circuit,

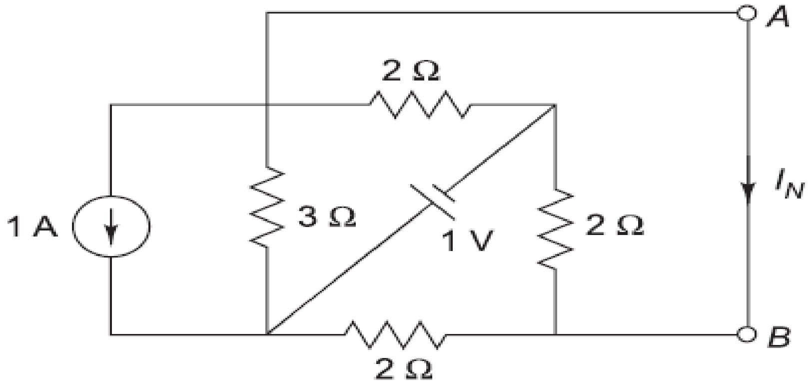


Fig. 2.508

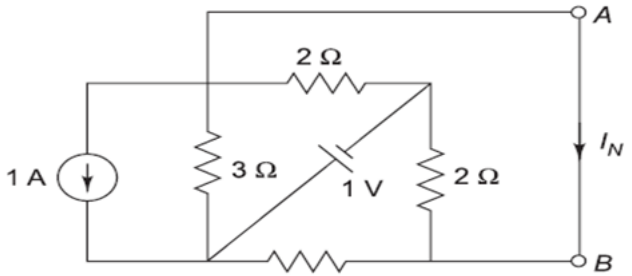


Fig. 2.508

By source transformation,

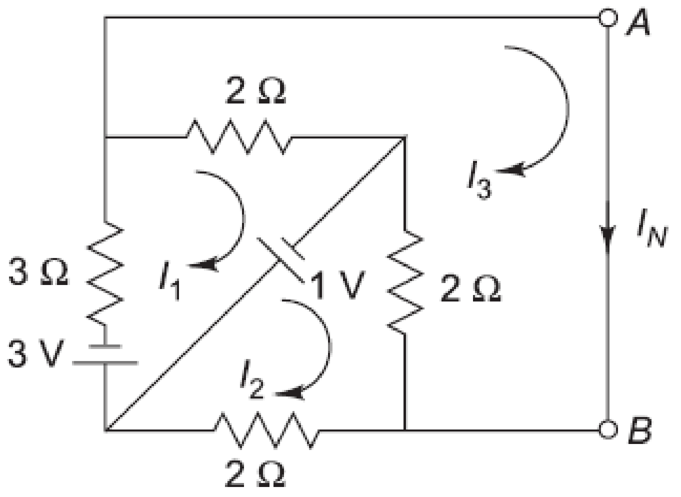


Fig. 2.509

Fig. 2.509

Applying KVL to Mesh 1,

$$-3 - 3I_1 - 2(I_1 - I_3) + 1 = 0$$

$$5I_1 - 2I_3 = -2 \qquad (1)$$

Applying KVL to Mesh 2,

$$-1 - 2\,(I_2 - I_3) - 2\,I_2 = 0$$

$$4I_2 - 2I_3 = -1 \qquad (2)$$

Applying KVL to Mesh 3,

$$-2\,(I_3 - I_1) - 2\,(I_3 - I_2) = 0$$

$$-2I_1 - 2I_2 + 4I_3 = 0 \qquad (3)$$

Solving Eqs. (1), (2) and (3),

$$I_1 = -0.64\text{ A}$$

$$I_2 = -0.55\text{ A}$$

$$I_3 = -0.59\text{ A}$$

$$I_N = I_3 = -0.59\text{ A}$$

Step II: Calculation of R_N

Replacing the voltage source by a short circuit and the current source by an open circuit,

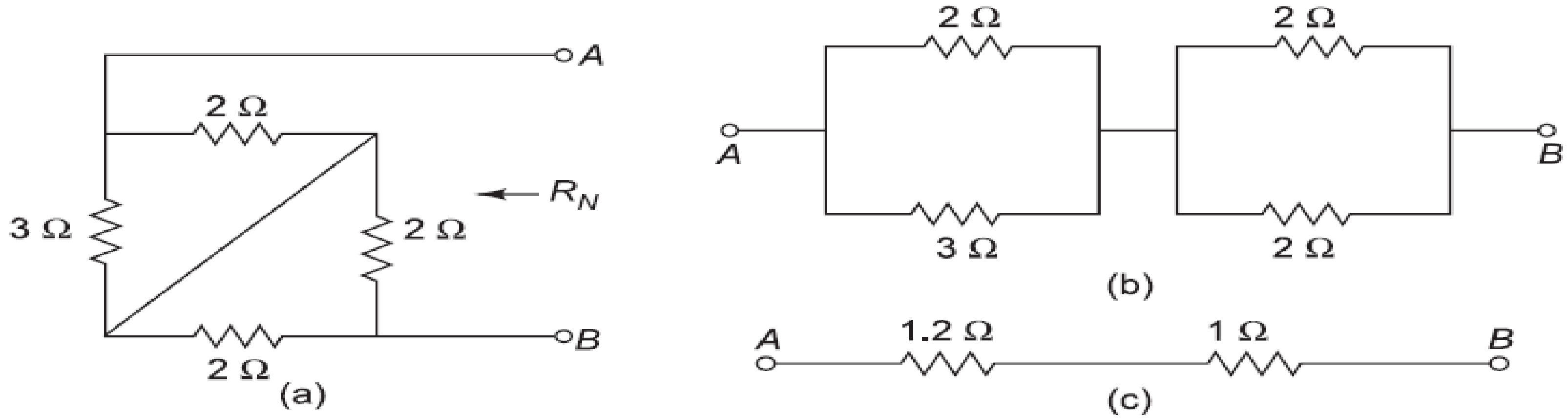


Fig. 2.510

$$R_N = 2.2\ \Omega$$

Step III: Calculation of I_L

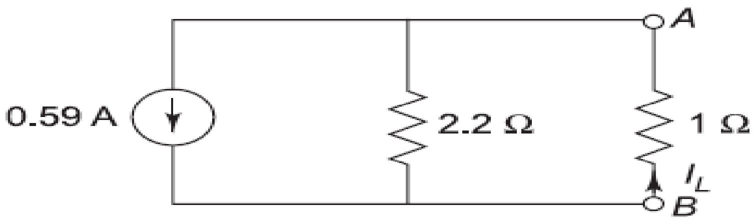


Fig. 2.511

$$I_L = 0.59 \times \frac{2.2}{2.2+1} = 0.41\ \text{A}$$

2.11 MAXIMUM POWER TRANSFER THEOREM

[Dec 2012, 2015, May 2013, 2014]

It states that *'the maximum power is delivered from a source to a load when the load resistance is equal to the source resistance.'*

$$I = \frac{V}{R_S + R_L}$$

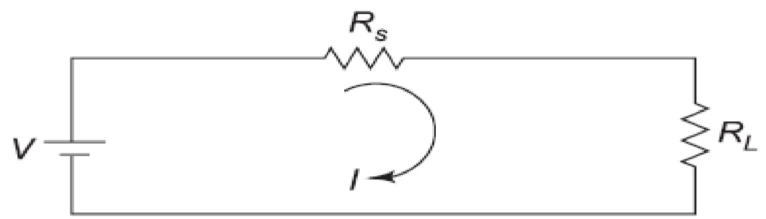


Fig. 2.519 *Maximum power transfer theorem*

$$I = \frac{V}{R_S + R_L}$$

Power delivered to the load $R_L = P = I^2 R_L = \dfrac{V^2 R_L}{(R_S + R_L)^2}$

Power delivered to the load $R_L = P = I^2 R_L = \dfrac{V^2 R_L}{(R_S + R_L)^2}$

To determine the value of R_L for maximum power to be transferred to the load,

$$\frac{dP}{dR_L} = 0$$

$$\frac{dP}{dR_L} = \frac{d}{dR_L} \frac{V^2}{(R_S + R_L)^2} R_L$$

$$\frac{dP}{dR_L} = \frac{d}{dR_L} \frac{V^2}{(R_S + R_L)^2} R_L$$

$$= \frac{V^2[(R_S + R_L)^2 - (2R_L)(R_S + R_L)]}{(R_S + R_L)^4}$$

$$(R_S + R_L)^2 - 2\, R_L\, (R_S + R_L) = 0$$

$$R_S{}^2 + R_L{}^2 + 2R_S\, R_L - 2R_L\, R_S - 2R_L{}^2 = 0$$

$$R_L = R_S$$

Hence, the maximum power will be transferred to the load when load resistance is equal to the source resistance.

2.11.1 Steps to be followed in Maximum Power Transfer Theorem

1. Remove the variable load resistor R_L.
2. Find the open circuit voltage V_{Th} across points A and B.
3. Find the resistance R_{Th} as seen from points A and B with voltage sources and current sources replaced by internal resistances.
4. Find the resistance R_L for maximum power transfer.

$$R_L = R_{Th}$$

5. Find the maximum power.

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{V_{Th}}{2R_{Th}}$$

$$P_{max} = I_L^2 R_L = \frac{V_{Th}^2}{4R_{Th}^2} \times R_{Th} = \frac{V_{Th}^2}{4R_{Th}}$$

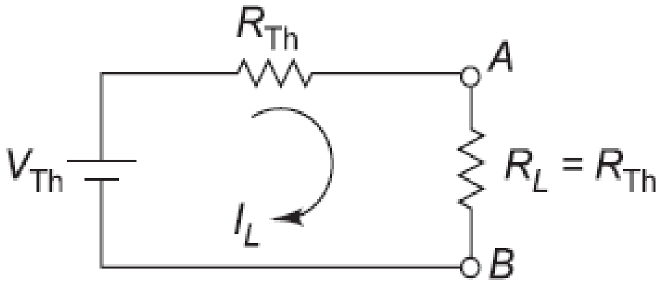


Fig. 2.520 *Equivalent circuit*

Example 1

Find the value of resistance R_L for maximum power transfer calculate maximum power.

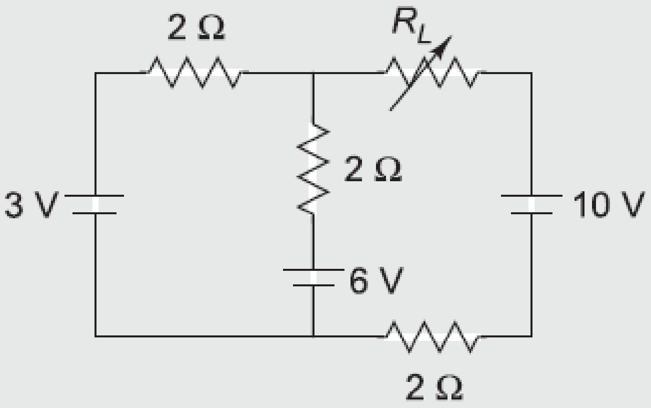


Fig. 2.521

Solution

Step I: Calculation of V_{Th}

Removing the variable resistor R_L from the network,

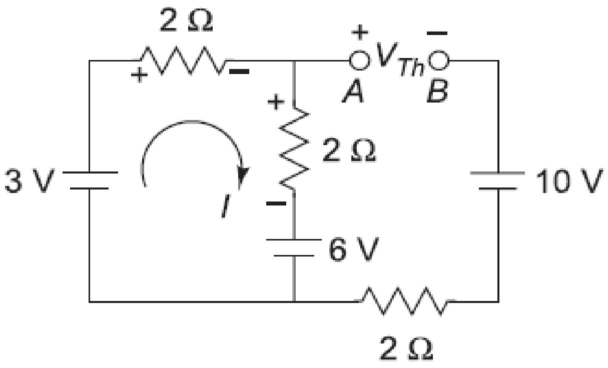


Fig. 2.522

Applying KVL to the mesh,

$$3 - 2I - 2I - 6 = 0$$

$$I = -0.75 \text{ A}$$

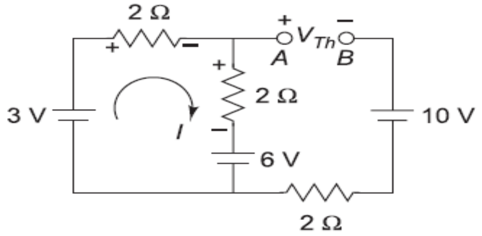


Fig. 2.522

Writing V_{Th} equation,

$$6 + 2I - V_{Th} - 10 = 0$$

$$V_{Th} = 6 + 2I - 10$$

$$= 6 + 2(-0.75) - 10$$

$$= -5.5 \text{ V}$$

$$= 5.5 \text{ V (terminal } B \text{ is positive w.r.t } A)$$

Step II: Calculation of R_{Th}

Replacing voltage sources by short circuits,

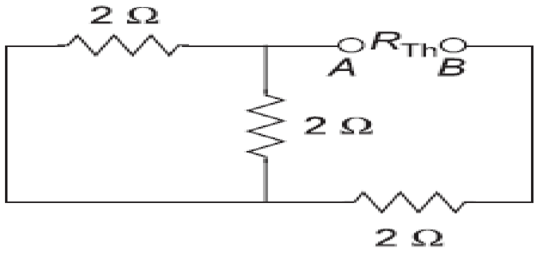


Fig. 2.523

$$R_{Th} = (2 \parallel 2) + 2 = 3\ \Omega$$

Step III: Value of R_L

For maximum power transfer

$$R_L = R_{Th} = 3\ \Omega$$

Step IV: Calculation of P_{max}

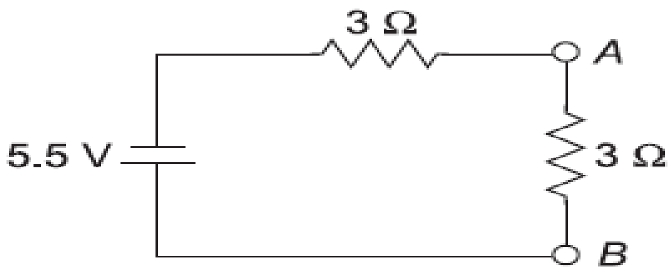


Fig. 2.524

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(5.5)^2}{4 \times 3} = 2.52\ \text{W}$$

Example 2

Find the value of resistance R_L for maximum power transfer and calculate maximum power.

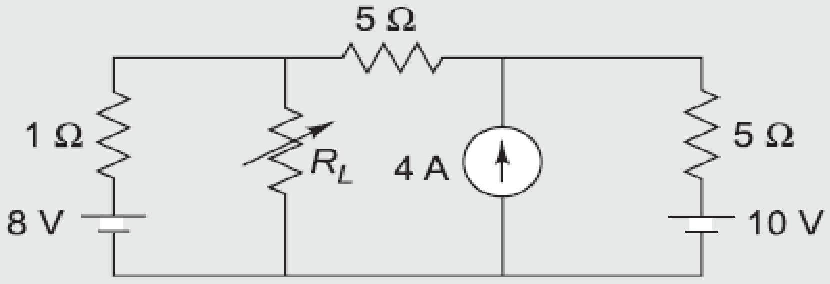


Fig. 2.525

Solution

Step I: Calculation of V_{Th}

Removing the variable resistor R_L from the network,

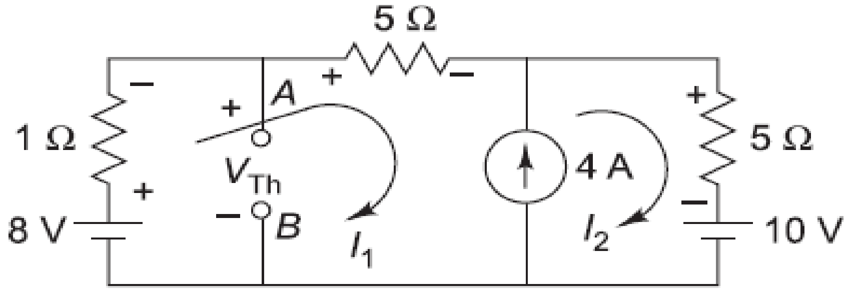


Fig. 2.526

Meshes 1 and 2 will form a supermesh.

Writing current equation for the supermesh,

$$I_2 - I_1 = 4 \qquad (1)$$

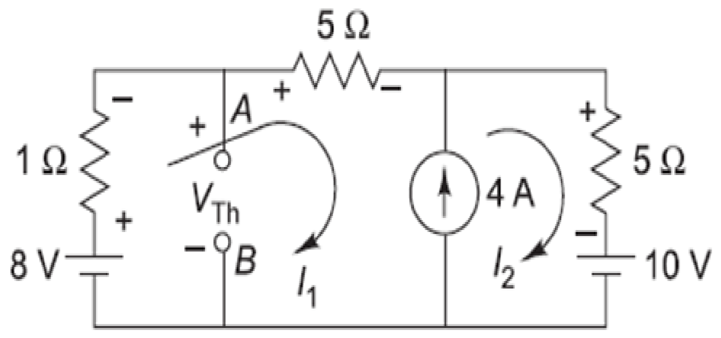


Fig. 2.526

Applying KVL to the supermesh,

$$8 - 1I_1 - 5I_1 - 5I_2 - 10 = 0$$

$$-6I_1 - 5I_2 = 2 \qquad (2)$$

Solving Eqs. (1) and (2),

$$I_1 = -2\text{ A}$$

$$I_2 = 2\text{ A}$$

Step II: Calculation of R_{Th}

Writing V_{Th} equation,

$$8 - 1I_1 - V_{Th} = 0$$
$$V_{Th} = 8 - I_1$$
$$= 8 - (-2)$$
$$= 10 \text{ V}$$

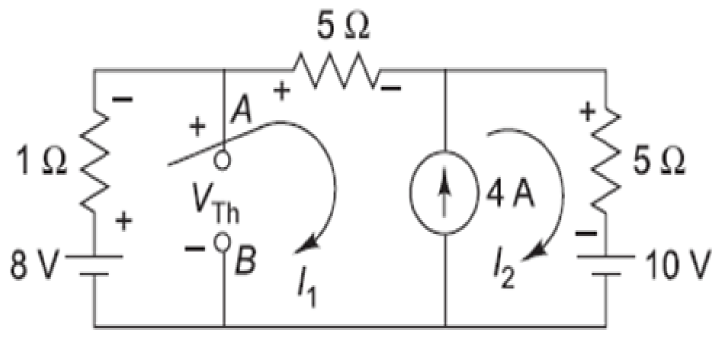


Fig. 2.526

Step II: Calculation of R_{Th}

Step II: Calculation of R_{Th}

Replacing the voltage sources by short circuits and current source by an open circuit,

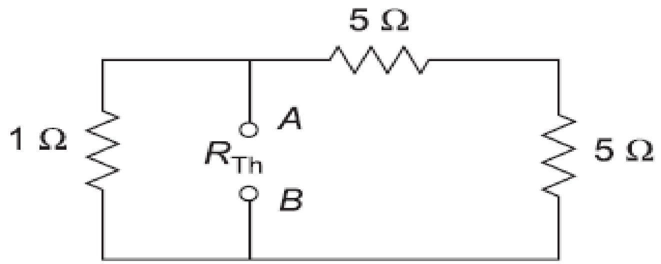


Fig. 2.527

$$R_{\text{Th}} = 10 \,||\, 1 = 0.91\ \Omega$$

Step III: Value of R_L

For maximum power transfer

$$R_L = R_{\text{Th}} = 0.91\ \Omega$$

Step IV: Calculation of P_{max}

$$P_{\max} = \frac{V_{\text{Th}}^2}{4R_{\text{Th}}} = \frac{(10)^2}{4 \times 0.91} = 27.47\ \text{W}$$

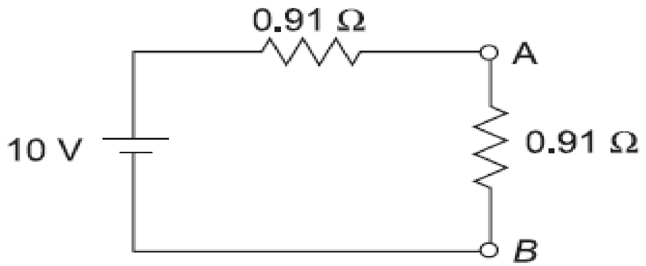


Fig. 2.528

2.8 SUPERPOSITION THEOREM

It states that '*In a linear network containing more than one independent sources, the resultant current in any element is the algebraic sum of the currents that would be produced by each independent source acting alone, all the other independent sources being represented meanwhile by their respective internal resistances.*'

The independent voltage sources are represented by their internal resistances if given or simply with zero resistances, i.e., short circuits if internal resistances are not mentioned.

The independent current sources are represented by infinite resistances, i.e., open circuits.

A linear network is one whose parameters are constant, i.e., they do not change with voltage and current.

Explanation Consider the circuit shown in Fig. 2.261. Suppose we have to find current I_4 flowing through R_4.

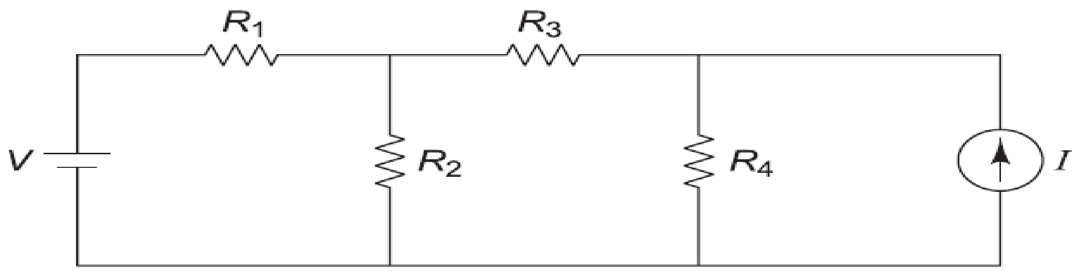


Fig. 2.261 *Superposition theorem*

2.8.1 Steps to be followed in Superposition Theorem

1. Find the current I'_4 flowing through R_4 due to independent voltage source 'V', representing independent current source with infinite resistance, i.e., open circuit.

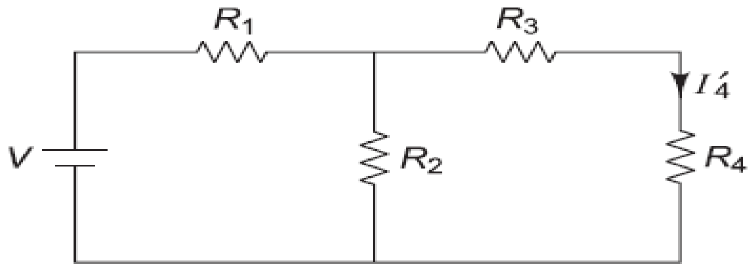


Fig. 2.262 *Step 1*

2. Find the current I''_4 flowing through R_4 due to independent current source 'I', representing the independent voltage source with zero resistance or short circuit.

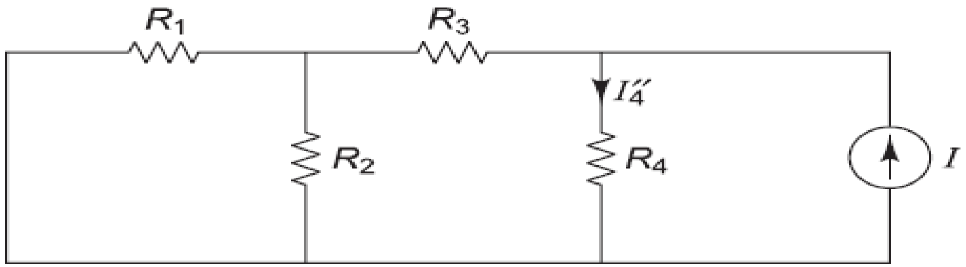


Fig. 2.263 *Step 2*

3. Find the resultant current I_4 through R_4 by the superposition theorem.

$$I_4 = I'_4 + I''_4$$

Example 1

Find the value of current flowing through the 2 Ω resistor.

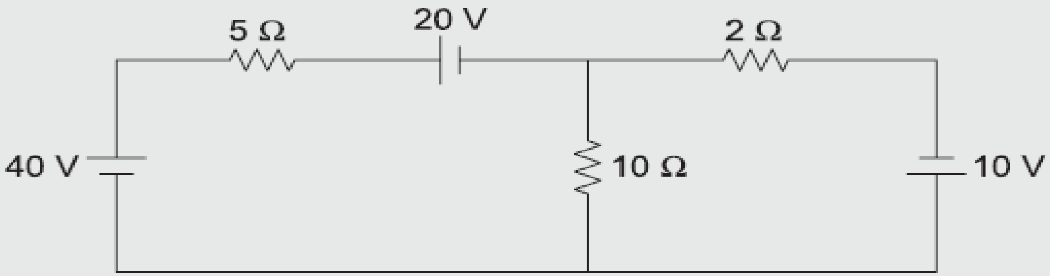


Fig. 2.264

Solution *Step I: When the 40 V source is acting alone*

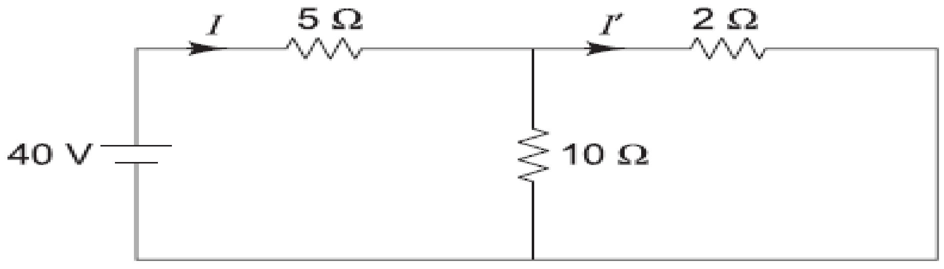


Fig. 2.265

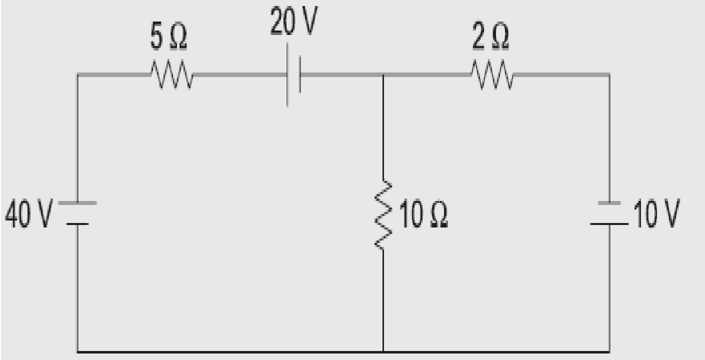


Fig. 2.264

By series–parallel reduction technique,

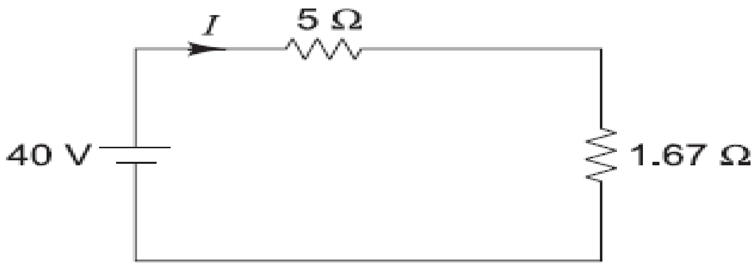


Fig. 2.266

$$I = \frac{40}{5+1.67} = 6\text{ A}$$

From Fig. 2.265, by current-division rule,

$$I' = 6 \times \frac{10}{10+2} = 5\text{ A} (\rightarrow)$$

Step II: When the 20 V source is acting alone

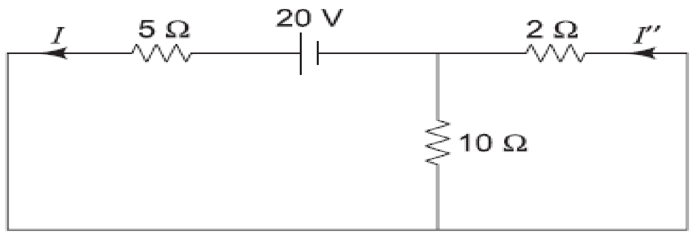


Fig. 2.267

By series–parallel reduction technique,

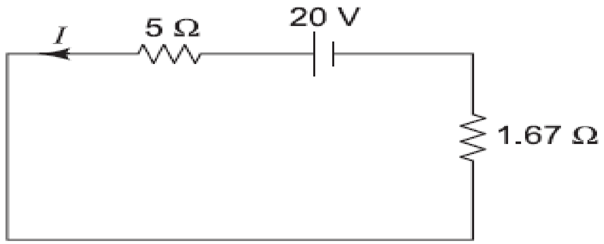


Fig. 2.268

$$I = \frac{20}{5+1.67} = 3 \text{ A}$$

From Fig. 2.267, by current-division rule,

$$I'' = 3 \times \frac{10}{10+2} = 2.5 \text{ A} (\leftarrow) = -2.5 \text{A} (\rightarrow)$$

Step III: When the 10 V source is acting alone

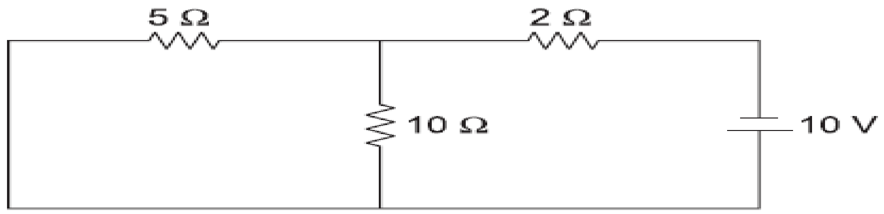


Fig. 2.269

By series–parallel reduction technique,

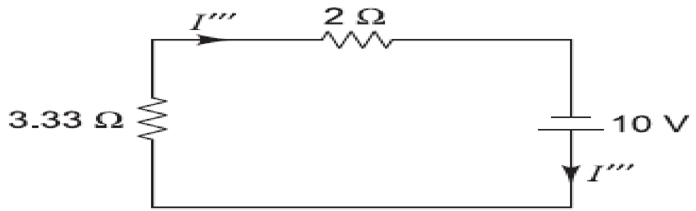


Fig. 2.270

$$I''' = \frac{10}{3.33 + 2} = 1.88 \text{ A} (\rightarrow)$$

Step IV: By superposition theorem,

$$I = I' + I'' + I'''$$
$$= 5 - 2.5 + 1.88$$
$$= 4.38 \text{ A} (\rightarrow)$$