9.5 Solved Problems Based on Rectilinear Motion with Uniform (Constant) Acceleration and Uniform (Constant) Velocity

Problem 1

A particle starts moving along a straight line with initial velocity of 25 m/s, from O under a uniform acceleration of $-2.5\ m/s^2$. Determine

(i) Velocity, displacement and the distance travelled at $t = 5$ sec.

(ii) How long the particle moves in the same direction ? What are its velocity, displacement and distance covered then ?

(iii) The instantaneous velocity, displacement and the distance covered at $t = 15$ sec.

(iv) The time required to come back to O, velocity displacement and distance covered then.

(v) Instantaneous velocity, displacement and distance covered at $t = 25$ sec.

Solution

Given : $u = 25$ m/s, $a = -2.5\ m/s^2$, O is the origin.

(i) $t = 5$ sec. Refer *figure 9.1(a)*

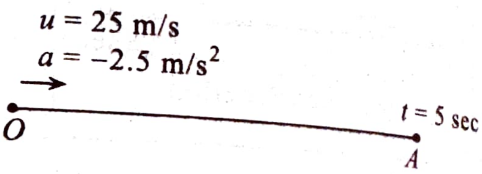

Fig. 9.1(a)

$$v = u + at$$
$$v = 25 + (-2.5) \times 5$$
$$v = 12.5 \text{ m/s } (\rightarrow)$$

Now, $$s = ut + \frac{1}{2}at^2$$
$$s = 25 \times 5 + \frac{1}{2} \times (-2.5) \times 5^2$$
$$s = 93.75 \text{ m } (\rightarrow)$$

Since velocity is positive, the particle is moving in same direction and therefore displacement is equal to the distance travelled.

$\therefore\ d = s = 93.75$ m. ***Ans.***

(ii) The particle moves in same direction till it comes to rest because of negative acceleration and then its direction will reverse.

At the above said instant velocity of particle will be zero

$v = 0$ (Point of reversal)

Let t be the time taken by the particle to move in same direction [*figure 9.1(b)*], we have

Fig. 9.1(b)

$$v = u + at$$
$$0 = 25 + (-2.5)t$$
$$t = 10 \text{ sec}$$ ***Ans.***

For displacement, we have

$$s = ut + \frac{1}{2}at^2$$
$$s = 25 \times 10 + \frac{1}{2} \times (-2.5) \times 5^2$$
$$s = 218.75 \text{ m } (\rightarrow)$$

As particle has not reversed its direction, we have

Displacement = Distance travelled

$s = d = 218.75$ m **Ans.**

(iii) $t = 15$ sec. Refer *figure 9.1(c)*

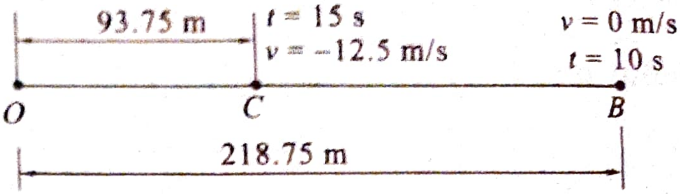

Fig. 9.1(c)

$$v = u + at$$

$$v = 25 + (-2.5) \times 15 = -12.5 \text{ m/s}$$

$$v = 12.5 \text{ m/s } (\leftarrow)$$

Now, $$s = ut + \frac{1}{2}at^2$$

$$s = 25 \times 15 + \frac{1}{2} \times (-2.5) \times 15^2 = 93.75 \text{ m } (\rightarrow)$$

Particle is moving from O to B and then from B to C in $t = 15$ sec.

Distance travelled $d = OB + BC \quad (BC = OB - OC)$

$$d = 218.75 + (218.75 - 93.75)$$

$d = 343.75$ m **Ans.**

(iv) Let t be the time taken by particle to reach origin. Refer *figure 9.1(d)*.

Displacement = 0

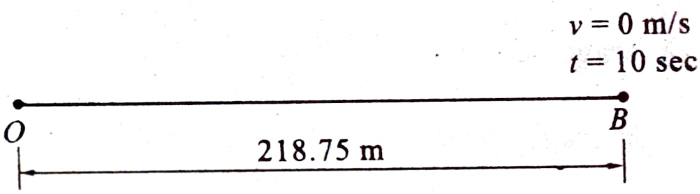

Fig. 9.1(d)

$$s = ut + \frac{1}{2}at^2$$

$$0 = 25 \times t + \frac{1}{2} \times (-2.5) \times t^2$$

$$t = 20 \text{ sec}$$

$$v = u + at$$

$$v = 25 + (-2.5) \times 20 = -25 \text{ m/s}$$

$$v = 25 \text{ m/s } (\leftarrow)$$

Distance covered $d = OB + BO = 218.75 + 218.75$

$d = 437.5$ m **Ans.**

(v) At $t = 25$ sec. Refer *figure 9.1(e)*

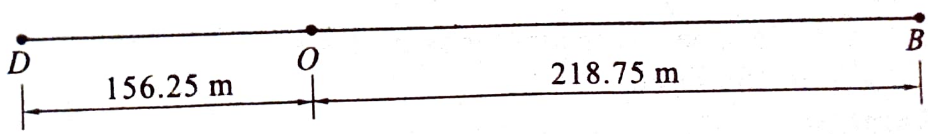

Fig. 9.1(e)

$$v = u + at$$

$$v = 25 + (-2.5) \times 25$$

$$v = -37.5 \text{ m/s}$$

$v = 37.5$ m/s $(\leftarrow)$ **Ans.**

Now, $$s = ut + \frac{1}{2}at^2$$

$$s = 25 \times 25 + \frac{1}{2} \times (-2.5) \times 25^2 = -156.25 \text{ m}$$

$s = 156.25$ m $(\leftarrow)$ **Ans.**

Distance covered $d = OB + BO + OD = 218.75 + 218.75 + 156.25$

$d = 593.75$ m **Ans.**

Problem 2

A particle travels along a straight line path such that in 4 seconds it moves from an initial position $S_A = -8$ m to position $S_B = +3$ m. Then in another 5 seconds it moves from S_B to $S_C = -6$ m. Determine the particles average velocity and average speed during 9 seconds interval.

Solution

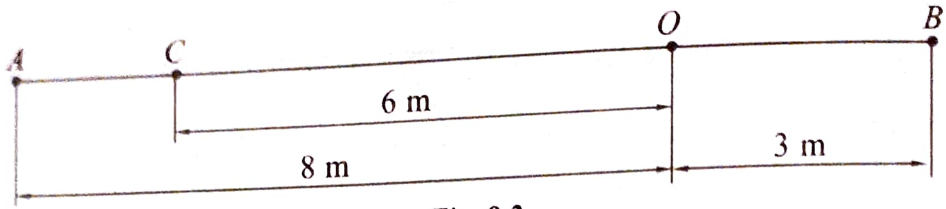

Fig. 9.2

(i) Average velocity $= \dfrac{\text{Final displacement} - \text{Initial displacement}}{\text{Time interval}}$

Average velocity $= \dfrac{-6-(-8)}{9} = \dfrac{2}{9}$

$v_{ave} = 0.2222$ m/s **Ans.**

(ii) Average speed $= \dfrac{\text{Total distance travelled}}{\text{Time interval}}$

Average velocity $= \dfrac{AO + OB + BO + OC}{\text{Time interval}} = \dfrac{8+3+3+6}{9} = \dfrac{20}{9}$

Average speed $= 2.222$ m/s **Ans.**

Problem 3

A motorist is travelling at 90 kmph, when he observes a traffic light 250 m ahead of him turns red. The traffic light is timed to stay red for 12 sec. If the motorist wishes to pass the light without stopping, just as it turns green. Determine

(i) The required uniform deceleration of the motor.

(ii) The speed of the motor as it passes the traffic light.

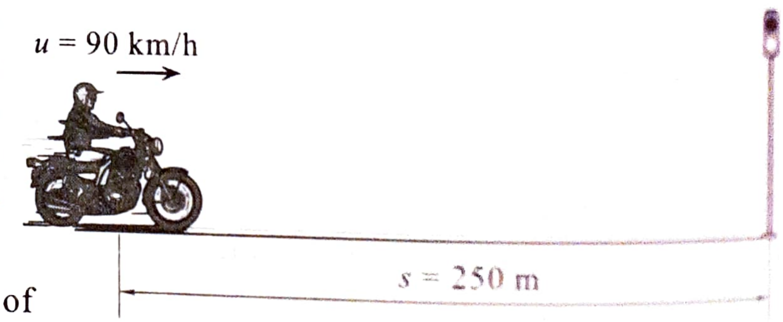

Fig. 9.3

Solution

Given : Initial velocity $u = 90$ km/hr $\therefore u = \dfrac{90 \times 5}{18} = 25$ m/s

$\dfrac{90 \times 1000}{3600}$

Time $t = 12$ s

Displacement $s = 250$ m

(i)
$$s = ut + \frac{1}{2}at^2$$

$$250 = 25 \times 12 + \frac{1}{2} \times a \times 12^2$$

$$a = -0.6944 \text{ m/s}^2$$ **Ans.**

(Negative sign indicates deceleration)

(ii)

$$v = u + at$$

$$v = 25 + (-0.6944) \times 12 = 16.67 \text{ m/s } (\rightarrow)$$

$$v = 16.67 \times \frac{18}{5}$$

$v = 60$ kmph **Ans.** (The speed of the motor cycle as it passes the traffic light)

Problem 4

Determine the time required for a car to travel 1 km along a road if the car starts from rest, reaches a maximum speed at some intermediate point, and then stops at the end of the road. The car can accelerate or decelerate at 1.5 m/s^2.

Solution : Method I - By Using Equation of Motion

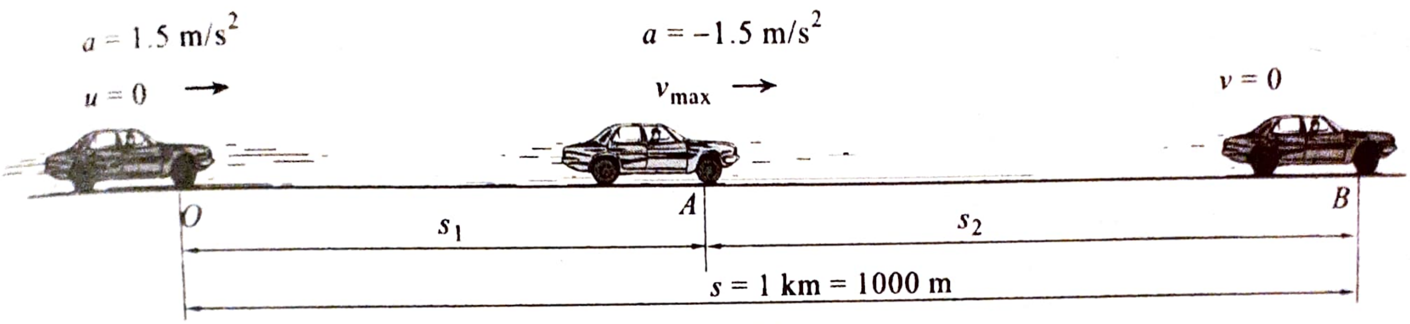

Fig. 9.4(a)

(i) Consider the motion of car from O to A [*figure 9.4(a)*]

Initial velocity $u = 0$; Final velocity $v = v_{max}$;

Acceleration $a = 1.5$ m/s^2 ; Time $t = t_1$;

Displacement $s = s_1$;

$$v = u + at$$

$$v_{max} = 0 + 1.5 \times t_1 \quad \therefore \; t_1 = \frac{v_{max}}{1.5}$$

$$s = ut + \frac{1}{2}at^2$$

$$s_1 = (0)(t_1) + \frac{1}{2} \times 1.5 \times t_1^2$$

$$s_1 = 0.75 \left(\frac{v_{max}}{1.5}\right)^2$$

(ii) Consider the motion of car from A to B

Initial velocity $u = v_{max}$; Final velocity $v = 0$;

Acceleration $a = -1.5$ m/s^2 ; Time $t = t_2$;

Displacement $s = s_2$;

$$v = u + at$$

$$0 = v_{max} + (-1.5) \times t_2 \quad \therefore \; t_2 = \frac{v_{max}}{1.5}$$

$$s = ut + \frac{1}{2}at^2$$

$$s_2 = v_{max}\left(\frac{v_{max}}{1.5}\right) + \frac{1}{2} \times (-1.5)\left(\frac{v_{max}}{1.5}\right)^2$$

$$s_2 = v_{max}\left(\frac{v_{max}}{1.5}\right) - 0.75\left(\frac{v_{max}}{1.5}\right)^2$$

(iii) Total displacement $s = s_1 + s_2$

$$1000 = 0.75\left(\frac{v_{max}}{1.5}\right)^2 + \left[\frac{v_{max}^2}{1.5} - 0.75\left(\frac{v_{max}}{1.5}\right)^2\right]$$

$$1000 = \frac{v_{max}^2}{1.5}$$

$$v_{max} = 38.73 \text{ m/s} \quad \textbf{Ans.}$$

(iv) Total time $t = t_1 + t_2$

$$t = \frac{v_{max}}{1.5} + \frac{v_{max}}{1.5}$$

$$t = \frac{38.73}{1.5} + \frac{38.73}{1.5}$$

$$t = 51.64 \text{ sec} \quad \textbf{Ans.}$$

Method II - By Using *v-t* Diagram

(i) Consider the *v-t* diagram shown in *figure 9.4(b)*

Since magnitude of acceleration is same, therefore slope is same

$$\therefore t_1 = t_2 \qquad \therefore t = 2t_1$$

$$\text{Slope} = \text{Acceleration} = 1.5 = \frac{v_{max}}{t_1}$$

$$\therefore v_{max} = 1.5t_1 \qquad .....(I)$$

Area under *v-t* diagram is displacement

$$\therefore 1000 = \frac{1}{2} \times 2t_1 \times v_{max}$$

$$v_{max} = \frac{1000}{t_1} \qquad .....(II)$$

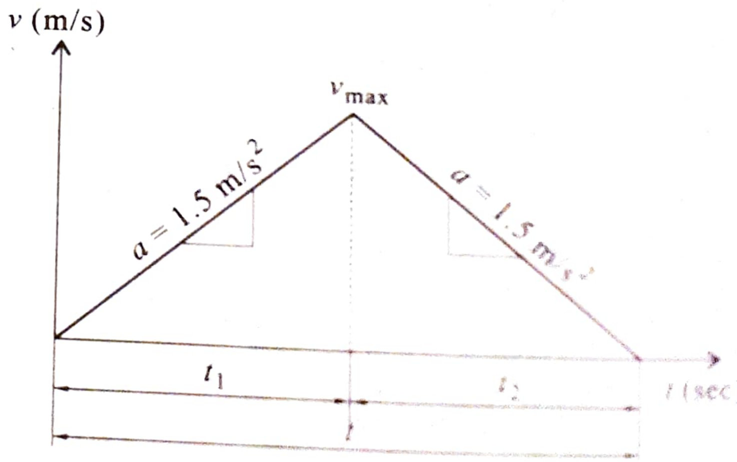

Fig. 9.4(b)

(ii) Equating equation (I) and (II)

$$1.5t_1 = \frac{1000}{t_1}$$

$$t_1 = 25.82 \text{ sec}$$

$$\therefore t = 2 \times t_1 = 51.64 \text{ sec}$$

$$v_{max} = 1.5 \times 25.82$$

$$v_{max} = 38.73 \text{ m/s} \quad \textbf{Ans.}$$

Problem 5

In Asian games, for 100 m event an athlete accelerates uniformly from the start to his maximum velocity in a distance of 4 m and runs the remaining distance with that velocity. If the athlete finishes the race in 10.4 sec, determine **(i)** his initial acceleration **(ii)** his maximum velocity.

Solution : **Method I - By Using Equation of Motion**

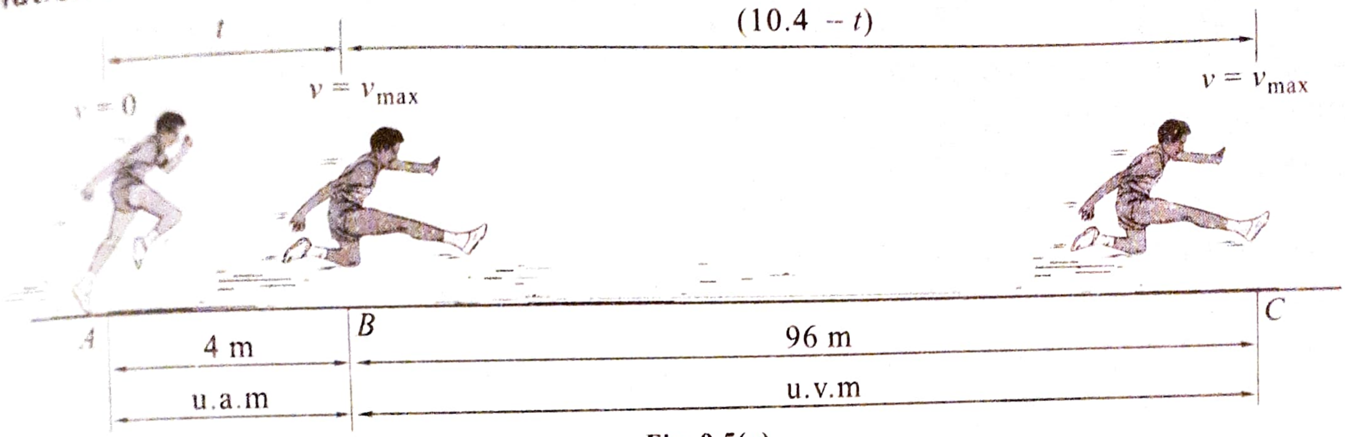

Fig. 9.5(a)

(i) Consider uniform acceleration motion from A to B [*figure 9.5(a)*]

Initial velocity $u = 0$; Final velocity $v = v_{max}$;

Displacement $s = 4$ m ;

Acceleration $a = ?$; Time $t = ?$

$$v = u + at$$

$$v_{max} = 0 + at = at \qquad \text{..... (I)}$$

$$s = ut + \frac{1}{2}at^2$$

$$4 = 0 + \frac{1}{2}at^2$$

$$8 = (at)t$$

$$8 = (v_{max})t$$

$$v_{max} = \frac{8}{t} \qquad \text{..... (II)}$$

(ii) Consider uniform velocity motion from B to C

Velocity $= v_{max}$; Time $= (10.4 - t)$

Displacement = Velocity × Time

$$96 = v_{max}(10.4 - t)$$

Substituting $v_{max} = \frac{8}{t}$ from equation (II)

$$96 = \frac{8}{t}(10.4 - t)$$

$$96t = 8 \times 10.4 - 8t$$

$$104t = 8 \times 10.4$$

$$t = 0.8 \text{ sec}$$

(iii) From equation (II)

$$v_{max} = \frac{8}{t} = \frac{8}{0.8} \qquad \therefore \; v_{max} = 10 \text{ m/s} \quad \textbf{Ans.}$$

(iv) From equation (I), $v_{max} = at$

$$a = \frac{v_{max}}{t} = \frac{10}{0.8} \qquad \therefore \; a = 12 \text{ m/s}^2 \quad \textbf{Ans.}$$

Method II - By Using v-t Diagram

Consider the v-t diagram shown in *figure 9.5(b)*

(i) Let t be the time taken by athlete to attain maximum velocity (v_{max}) in a distance of 4 m.

Time taken for remaining distance 96 m will be $(10.4 - t)$.

Area under v-t diagram is displacement, so we have

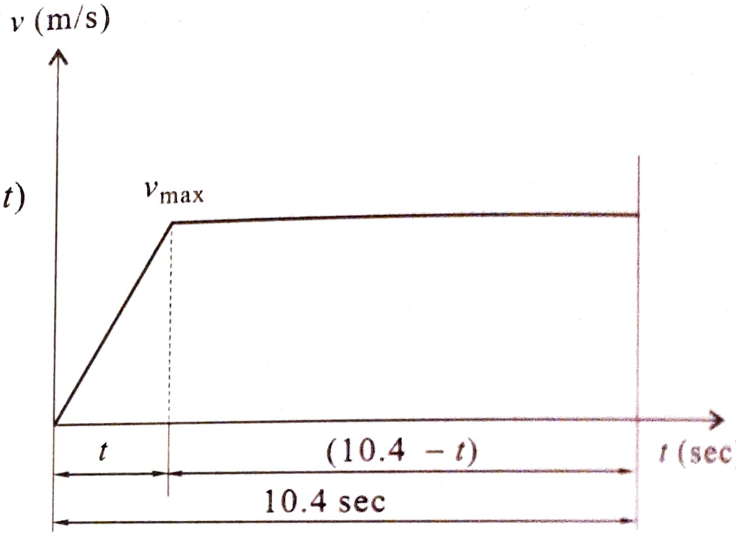

Fig. 9.5(b)

$$4 = \frac{1}{2} \times t \times v_{max} \quad \text{and} \quad 96 = v_{max}(10.4 - t)$$

$$\therefore \; v_{max} = \frac{8}{t} \quad \text{and} \quad v_{max} = \frac{96}{10.4 - t}$$

Equating v_{max}, we have

$$\therefore \; \frac{8}{t} = \frac{96}{10.4 - t}$$

$$\therefore \; t = 0.8 \text{ sec}$$

(ii) $v_{max} = \dfrac{8}{t} = \dfrac{8}{0.8} = 10 \text{ m/s}$ **Ans.**

(iii) Acceleration = Slope of v-t diagram

$$a = \frac{v_{max}}{t} = \frac{10}{0.8} \qquad \therefore \; a = 12.5 \text{ m/s}^2 \quad \textbf{Ans.}$$

Problem 6

An automobile starts from rest travelling on a straight path at 2 m/s² for sometime. After which it decelerates at 1 m/s² till it comes to a halt. If the total distance covered is 300 m, find the maximum velocity of the automobile and the time of travel.

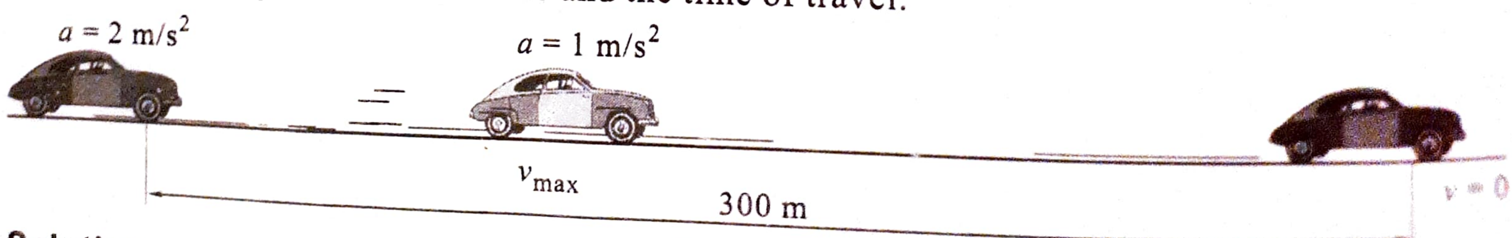

Fig. 9.6(a)

Solution

Consider the v-t diagram shown in *figure 9.6(a)*

(i) **Slope of v-t diagram is an acceleration.**

$$\frac{v_{max}}{t_1} = 2 \quad \text{and} \quad \frac{v_{max}}{t - t_1} = 1$$

$$v_{max} = 2t_1 \quad \text{and} \quad v_{max} = 1(t - t_1)$$

$$2t_1 = t - t_1 \qquad \therefore t = 3t_1$$

(ii) **Area under *v-t* diagram is displacement**

$$\frac{1}{2} \times t \times v_{max} = 300$$

$$v_{max} = \frac{600}{t}$$

$$v_{max} = \frac{600}{3t_1} \quad \text{but } v_{max} = 2t_1$$

$$\frac{600}{3t_1} = 2t_1$$

$$100 = t_1^2$$

$$t_1 = 10 \text{ sec}$$

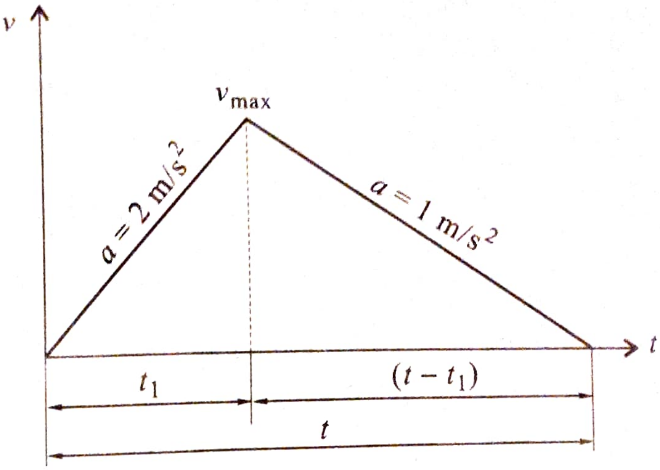

Fig. 9.6(b)

(iii) $v_{max} = 2t_1$

$$\therefore v_{max} = 20 \times 10 \qquad \therefore v_{max} = 200 \text{ m/s} \quad \textbf{Ans.}$$

(iv) **Total time of travel**

$$t = 3t_1 = 3 \times 10 \qquad \therefore t = 30 \text{ sec} \quad \textbf{Ans.}$$

Problem 7

A radar equipped police car observes a truck travelling 110 kmph. The police car starts pursuit 30 seconds after the observation, accelerates to 160 kmph in 20 seconds. Assuming the speeds are maintained constant on a straight road, how far from the observation point, will the chase end ?

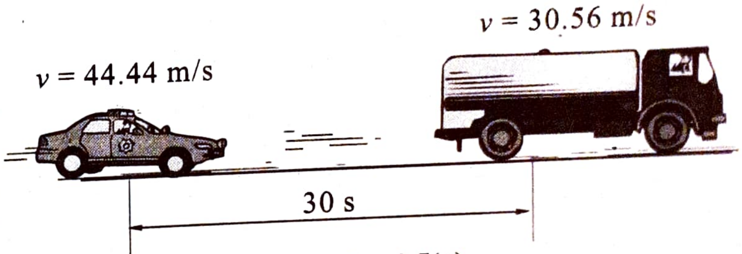

Fig. 9.7(a)

Solution

Observe the *v-t* diagram of truck and police car shown in *figure 9.7(b)* and *(c)*.

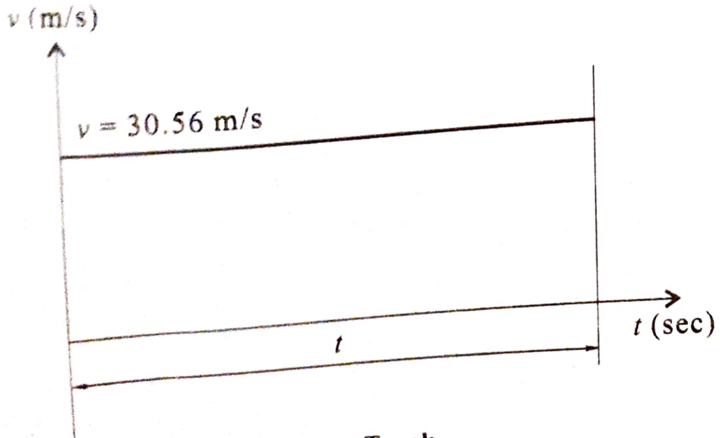

Fig. 9.7(b) : Truck

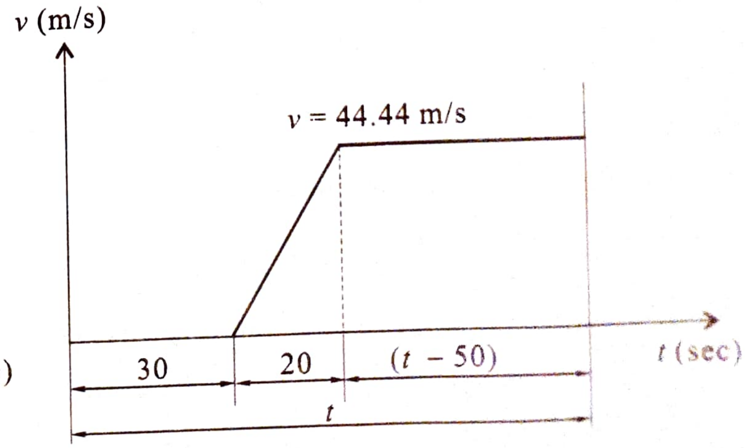

Fig. 9.7(c) : Police car

Let t be the time interval from the observation point to the point where chase ends.

Distance covered by truck and police car will be same. We know, area under v-t diagram is distance covered.

Area under v-t diagram of truck = Area under v-t diagram of police car

$$(30.56)(t) = \frac{1}{2} \times 20 \times 44.44 + 44.44\,(t - 50)$$

$$\therefore\ t = 128.07 \text{ sec} \quad \textbf{Ans.}$$

$$\text{Distance covered} = 30.56 \times 128.07$$

$$s = 3913.82 \text{ m} \quad \textbf{Ans.}$$

Problem 8

A train travelling with a speed of 90 kmph slow down on account of work in progress, at a retardation of 1.8 kmph per second to 36 kmph. With this, it travels 600 m. Thereafter it gains further speed with 0.9 kmph per second till getting original speed. Find the delay caused.

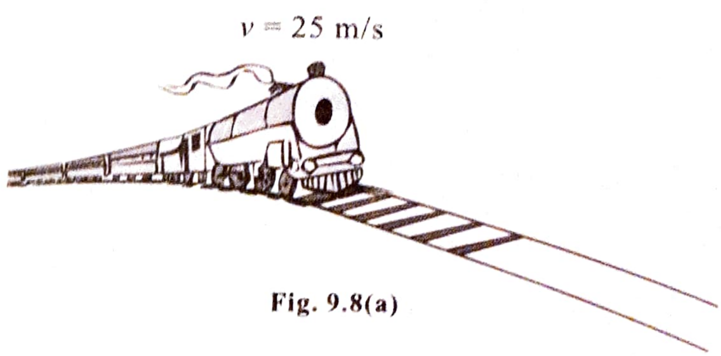

Fig. 9.8(a)

Solution

Observe the v-t diagram of train shown in *figure 9.8(b)*.

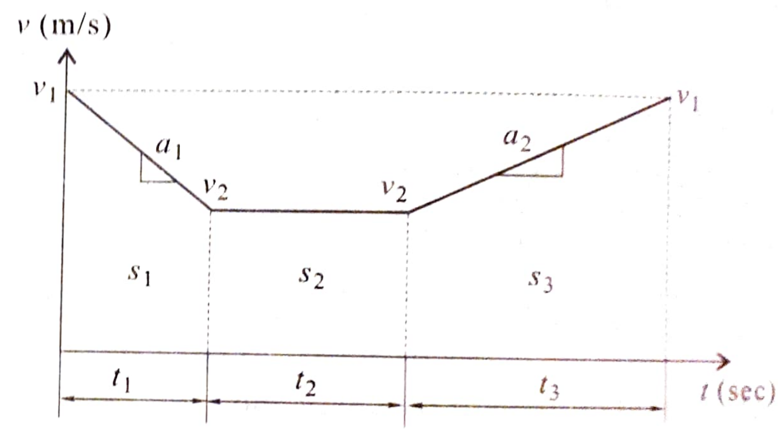

Fig. 9.8(b)

(i) Given : $v_1 = 90 \times \frac{5}{18} = 25 \text{ m/s}$

$$a_1 = 1.8 \times \frac{5}{18} = 0.5 \text{ m/s}^2$$

$$v_2 = 36 \times \frac{5}{18} = 10 \text{ m/s}$$

$$a_2 = 0.9 \times \frac{5}{18} = 0.25 \text{ m/s}^2$$

$$s_2 = 600 \text{ m}$$

(ii) First phase of motion (constant acceleration)

$$v = u + at$$

$$v_2 = v_1 + a_1 t_1$$

$$10 = 25 + (-0.5) \times t_1 \qquad \therefore\ t_1 = 30 \text{ sec}$$

Distance covered = Area under v-t diagram

$$s_1 = \frac{1}{2}(v_1 + v_2)\,t_1 = \frac{1}{2}(25 + 10)\,30 \qquad \therefore\ s_1 = 525 \text{ m}$$

(iii) Second phase of motion (constant velocity)

$$s_2 = v_2 \times t_2$$

$$600 = 10 \times t_2 \qquad \therefore\ t_2 = 60 \text{ sec}$$

(iv) Third phase of motion (constant acceleration)

$$v = u + at$$

$$v_3 = v_2 + a_2 t_3$$

$$25 = 10 + (0.25) \times t_3 \qquad \therefore\ t_3 = 60 \text{ sec}$$

Distance covered = Area under v-t diagram

$$s_3 = \frac{1}{2}(v_2 + v_1)\,t_3 = \frac{1}{2}(10 + 25)\,60$$

$$\therefore\ s_3 = 1050 \text{ m}$$

(v) **Total distance covered** $= s_1 + s_2 + s_3$

$$d = 525 + 600 + 1050$$

$$\therefore\ d = 2175 \text{ m}$$

(vi) **Total time taken** $= t_1 + t_2 + t_3$

$$t = 30 + 60 + 60 \qquad \therefore\ t = 150 \text{ sec}$$

(vii) If there would have been no '*work in progress*' the speed would have been constant

$$v_1 = 25 \text{ m/s}$$

$\therefore$ Time required to travel would have been

$$t' = \frac{\text{Distance}}{\text{Speed}} = \frac{2175}{25}$$

$$t' = 87 \text{ sec}$$

(viii) The delay caused $= t - t' = 150 - 87 = 63$ sec ***Ans.***

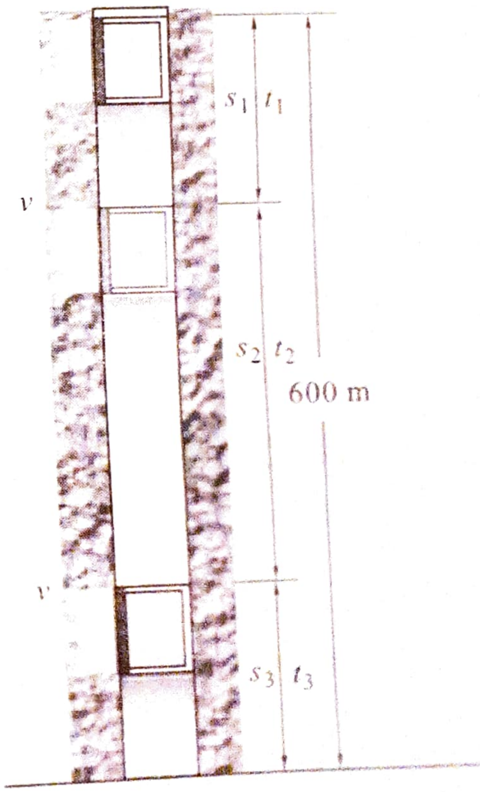

Fig. 9.9(a)

Problem 9

An elevator goes down a mine shaft 600 m deep in 60 sec. For first quarter of distance only the speed is being uniformly accelerated and during the last quarter uniformly retarded, the acceleration and retardation being equal. Find the uniform speed of the elevator while travelling central portion of shaft.

Solution

(i) Observe the v-t diagram of the elevator shown in *figure 9.9(b)*.

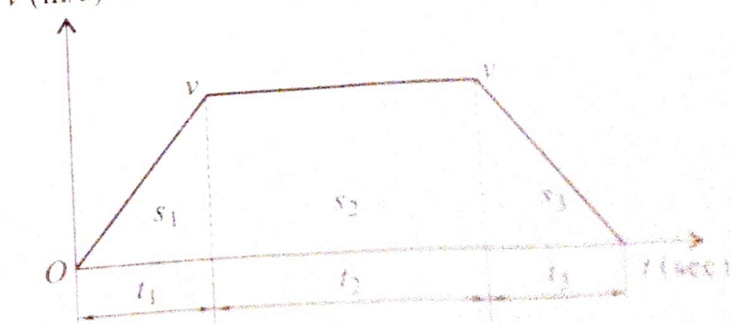

Fig. 9.9(b)

As per given condition in problem, we have

Distance travelled in first quarter = Distance travelled in last quarter

$$s_1 = s_3 = \frac{600}{4} = 150 \text{ m}$$

$$\therefore\ s_2 = 300 \text{ m}$$

(ii) From v-t diagram, we have

Distance = Area under v-t diagram

$$s_1 = \frac{1}{2} \times t_1 \times v \quad ; \quad s_2 = v \times t_2 \quad ; \quad s_3 = \frac{1}{2} \times t_3 \times v$$

$$150 = \frac{vt_1}{2} \quad ; \quad s_2 = vt_2 \quad ; \quad 150 = \frac{vt_3}{2}$$

$$150 = \frac{vt_1}{2} = \frac{vt_3}{2} \quad \therefore t_1 = t_3$$

$$s_2 = 2s_1$$

$$vt_2 = 2 \times \frac{1}{2} \times vt_1 \quad \therefore t_1 = t_2$$

$$\therefore t_1 = t_2 = t_3 = \frac{60}{3} = 20 \text{ sec}$$

(iii) Uniform velocity motion during central portion of shaft

$$v = \frac{\text{Displacement}}{\text{Time}} = \frac{300}{20}$$

$v = 15$ m/s ***Ans.***

Problem 10

A car accelerates from rest at a constant rate α for sometime after which it decelerates at a constant rate β to come to rest. If the total lapse is t seconds evaluate **(i)** the maximum velocity reached and **(ii)** the total distance travelled.

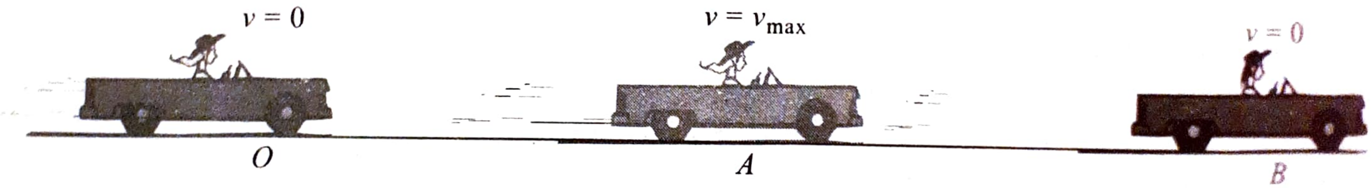

Fig. 9.10(a)

Solution : Method I - By Using Equation of Motion

(i) Motion of the car from *O* to *A*

Initial velocity $u = 0$; Final velocity $v = v_{max}$;

Acceleration $a = \alpha$; Time $t = t_1$

Displacement $s = s_1$

$$v = u + at$$

$$v_{max} = 0 + \alpha t_1$$

$$v_{max} = \alpha t_1 \qquad \text{..... (I)}$$

$$v^2 = u^2 + 2as$$

$$v_{max}^2 = 0 + 2\alpha s_1$$

$$s_1 = \frac{v_{max}^2}{2\alpha} \qquad \text{..... (II)}$$

(ii) **Motion of the car from A to B**

Initial velocity $u = v_{max}$; Final velocity $v = 0$;

Acceleration $a = -\beta$ (deceleration); Time $= (t - t_1)$

Displacement $s = s_2$

$$v = u + at$$

$$0 = v_{max} + (-\beta)(t - t_1)$$

$$\alpha t_1 = \beta t - \beta t_1$$

$$t_1(\alpha + \beta) = \beta t$$

$$t_1 = \frac{\beta t}{\alpha + \beta}$$

(iii) From equation (I)

$$v_{max} = \frac{\alpha \beta t}{\alpha + \beta} \quad \textbf{Ans.} \quad \text{..... (III)}$$

$$v^2 = u^2 + 2as$$

$$0 = v_{max}^2 + 2(-\beta)s_2$$

$$s_2 = \frac{v_{max}^2}{2\beta} \quad \text{..... (IV)}$$

(iv) $s = s_1 + s_2$

From equation (II) and (IV)

$$s = \frac{v_{max}^2}{2\alpha} + \frac{v_{max}^2}{2\beta} = \frac{v_{max}^2}{2}\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$$

$$s = \frac{v_{max}^2}{2}\left(\frac{\alpha + \beta}{\alpha\beta}\right)$$

From equation (III)

$$s = \frac{\alpha^2\beta^2 t^2}{2(\alpha + \beta)^2}\left(\frac{\alpha + \beta}{\alpha\beta}\right) = \frac{\alpha\beta t^2}{2(\alpha + \beta)} \quad \textbf{Ans.}$$

Method II - By Using v-t Diagram

(i) Consider the v-t diagram shown in *figure 9.10(b)*

$$\text{Slope} = \text{Acceleration} = \alpha = \frac{v_{max}}{t_1}$$

$$\therefore\ v_{max} = \alpha t_1$$

$$\text{Slope} = \text{Acceleration} = \beta = \frac{v_{max}}{(t - t_1)}$$

$$\therefore\ v_{max} = \beta(t - t_1)$$

Equating v_{max}, we have

$$\alpha t_1 = \beta(t - t_1)$$

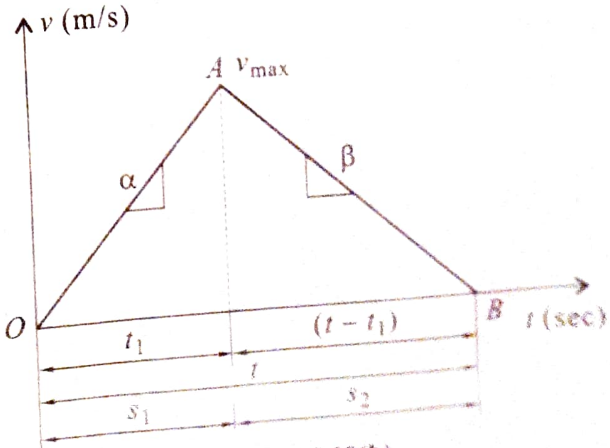

Fig. 9.10(b)

$$\alpha t_1 = \beta t - \beta t_1$$

$$t_1(\alpha + \beta) = \beta t$$

$$t_1 = \frac{\beta t}{\alpha + \beta}$$

$$v_{max} = \alpha t_1 = \frac{\alpha \beta t}{\alpha + \beta} \quad \textbf{Ans.}$$

(ii) Displacement = Area under *a-t* diagram

$$s = \frac{1}{2} \times t \times v_{max} = \frac{1}{2} \times t \times \frac{\alpha \beta t}{\alpha + \beta}$$

$$s = \frac{\alpha \beta t^2}{2(\alpha + \beta)} \quad \textbf{Ans.}$$

Problem 11

A car A is travelling on a straight level road with a uniform speed of 60 km/hr. It is followed by another car B which is moving with a speed of 70 km/hr. When the distance between them is 2.5 km, the car B is given a deceleration of 20 km/hr^2. After what distance and time will B catch up with A.

Solution : Refer *figure 9.11.*

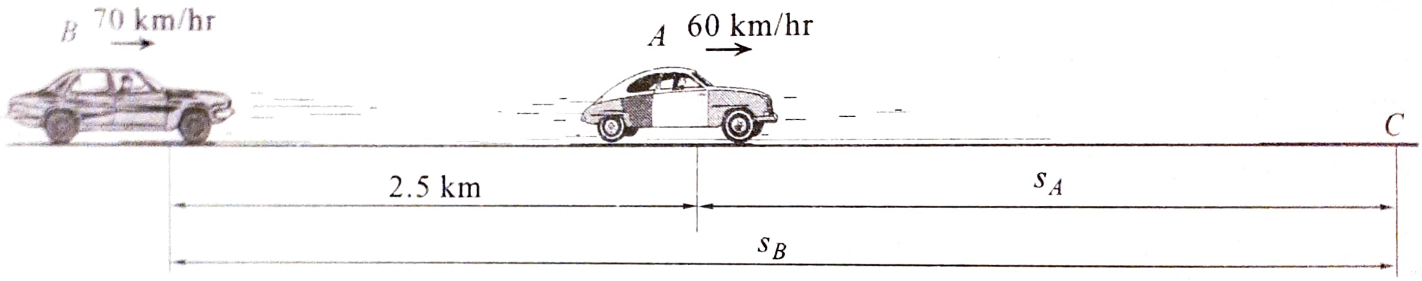

Fig. 9.11

(i) Let t be the time duration in which car B catches up with car A at point C.

Motion of car A (uniform velocity)

$$s = v \times t$$

$$s_A = 60 \times t$$

Motion of car B (uniform acceleration)

$$s = ut + \frac{1}{2} at^2$$

$$s_B = 70t + \frac{1}{2}(-20)\, t^2$$

(ii) From diagram, we have

$$s_B - s_A = 2.5$$

$$\left(70\, t - \frac{1}{2} \times 20 \times t^2\right) - (60\, t) = 2.5$$

$$-10t^2 + 10t - 2.5 = 0$$

$$10t^2 - 10t + 2.5 = 0$$

$$4t^2 - 4t + 1 = 0$$

$$t = 0.5 \text{ hours i.e. } t = 30 \text{ minutes} \quad \textbf{Ans.}$$

$$s_A = 60 \times 0.3 = 30 \text{ km}$$

$$s_B = 30 + 2.5 = 32.5 \text{ km} \quad \textbf{Ans.}$$

Problem 12

A burglar's car had a start with an acceleration 2 m/s^2. A police vigilant came in van to the spot at a velocity of 20 m/s after 3.75 seconds and continued to chase the burglar's car with uniform velocity. Find the time in which the police van will overtake the burglar's car.

Solution

Given : Burglar's car $\Rightarrow t = 0 \,;\, a = 2 \text{ m/s}^2$

Police van $\Rightarrow t = 3.75 \text{ sec} \,;\, v = 20 \text{ m/s (constant)}$

Fig. 9.12

(i) Let t be the time duration for motion of burglar's car. Motion of police van starts after 3.75 seconds from the given spot. Therefore, time interval will be $(t - 3.75)$ sec

Motion of Burglar's car
(constant acceleration)

$$s = ut + \frac{1}{2}at^2$$

$$s = 0 + \frac{1}{2} \times 2 \times t^2$$

$$s = t^2 \qquad \text{..... (I)}$$

Motion of Police Van
(constant velocity)

$$s = v \times t$$

$$s = 20(t - 3.75) \qquad \text{..... (II)}$$

(ii) Equating equation (I) and (II), we have

$$t^2 = 20(t - 3.75)$$

$$t^2 = 20t - 75$$

$$t^2 - 20t + 75 = 0$$

Solving the quadratic equation, we get

$$t = 5 \text{ sec} \text{ and } t = 15 \text{ sec}$$

(iii) In the beginning, the velocity of burglar's car is less than police van. Therefore at $t = 5$ sec the police van overtakes burglar's car.

Since burglar's car is moving with constant acceleration, thus as time progresses velocity of car increases, but velocity of police van remains same. At $t = 15$ sec burglar's car will overtake police van.