Review of Complex Numbers

❖ Standard (Cartesian) form of Complex Number

A number of the form $x + iy$, where x and y are real numbers and $i = \sqrt{-1}$, is called a complex number. A complex number is generally denoted by z.

$z = x + iy$: Standard form of complex number

x : Real part of z, $\mathrm{Re}(z)$

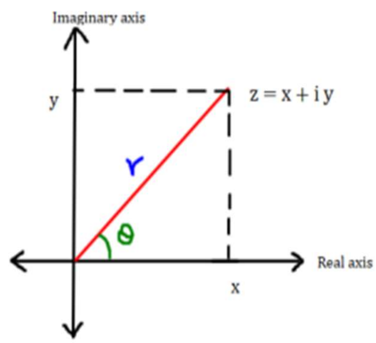
y : Imaginary part of z, $\mathrm{Im}(z)$

→ If $x = 0$ and $y \neq 0$, then $z = iy$ is called purely imaginary.

→ If $x \neq 0$ and $y = 0$, then $z = x$ is called purely real.

→ There is no order relation between two complex numbers.

❖ Argand's Diagram



The plot of a complex number $z = x + iy$ as the point (x, y) in the XY-plane is known as the Argand's diagram. X-axis is called real axis and Y-axis is called imaginary axis. The XY- plane is called complex plane.

❖ Conjugate of a Complex Number

Complex conjugate of a complex number $z = x + iy$ is defined as $\bar{z} = x - iy$.

Also, $x = \text{Re}(z) = \frac{z+\bar{z}}{2}$, $\quad y = \text{Im}(z) = \frac{z-\bar{z}}{2i}$, $\quad z \cdot \bar{z} = x^2 + y^2$

❖ Polar form of Complex Number

Modulus of $z = x + iy,\ r = |z| = \sqrt{x^2 + y^2}$

Argument of $z = x + iy,\ \theta = \arg(z) = \tan^{-1}\left(\frac{y}{x}\right)$

(r, θ) are called the polar co-ordinates of the point (x, y).

The polar form of z is $z = r(\cos\theta + i\sin\theta)$ where $x = r\cos\theta\,,\ y = r\sin\theta$.

❖ Working Rule to calculate Argument of Complex Number

Find $\alpha = \tan^{-1}\left|\frac{y}{x}\right|$,

Then, for $z = x + iy$,

Sr. No.	Quadrant Position of z	Value of θ
1	I	$\theta = \alpha$
2	II	$\theta = \pi - \alpha$
3	III	$\theta = \pi + \alpha$
4	IV	$\theta = -\alpha$ or $2\pi - \alpha$

❖ Exponential form of Complex Number

The exponential form of z is $re^{i\theta}$ where $e^{i\theta} = \cos\theta + i\sin\theta$.

❖ Algebra of Complex Number

Let $z_1 = x_1 + iy_1 = r_1 e^{i\theta_1}$ and $z_2 = x_2 + iy_2 = r_2 e^{i\theta_2}$ be two complex numbers.

(a) **Equality** : $z_1 = z_2 \Leftrightarrow x_1 = x_2$ and $y_1 = y_2$

(b) **Addition** : $z_1 + z_2 = (x_1 + x_2) + i(y_1 + y_2)$

(c) **Subtraction :** $z_1 - z_2 = (x_1 - x_2) + i(y_1 - y_2)$

(d) **Multiplication :** $z_1 \cdot z_2 = (x_1x_2 - y_1y_2) + i(x_1y_2 + x_2y_1) = r_1e^{i\theta_1} . r_2e^{i\theta_2} = r_1r_2e^{i\,(\theta_1+\theta_2)}$

(e) **Division :** $\frac{z_1}{z_2} = \frac{x_1x_2+y_1y_2}{x_2^2+y_2^2} + i\,\frac{x_2y_1-x_1y_2}{x_2^2+y_2^2} = \frac{r_1e^{i\theta_1}}{r_2e^{i\theta_2}} = \frac{r_1}{r_2}e^{i\,(\theta_1-\theta_2)}$