

### 7.3 Mechanism of Friction

Consider a block of weight  $W$  resting on a horizontal surface as shown in figure 7.1. The underlying surface possesses a certain amount of roughness. Let  $P$  be the horizontal force applied which will vary continuously from zero to a value sufficient to just move the block and that is to maintain the motion. The free body diagram of the block shows active forces (i.e. applied force  $P$  and weight of block  $W$ ) and reactive forces (i.e. Normal reaction  $N$  and impulsive frictional force  $F$ ).

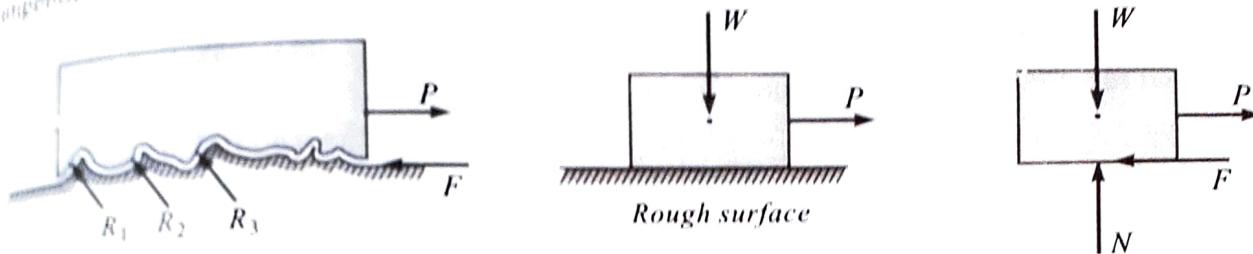


Fig. 7.1 : Mechanism of Friction

Frictional force  $F$  has the remarkable property of adjusting itself in magnitude equal to the applied force  $P$  till the limiting equilibrium condition.

**Limiting Equilibrium Condition :** As applied force  $P$  increases, the frictional force  $F$  is equal in magnitude and opposite in direction. However, there is a limit beyond which the magnitude of the frictional force cannot be increased. If the applied force is more than this maximum frictional force, there will be movement of one body over the other. Once the body begins to move, there is decrease in frictional force  $F$  from maximum value observed under static condition. The frictional force between the two surfaces when the body is in motion is called as *Kinetic or Dynamic friction*  $F_K$ .

**Limiting Frictional Force ( $F_{max}$ ) :** It is the maximum frictional force developed at the surface when the block is at the verge of motion (impending motion).

By experimental evidence it is proved that limiting frictional force is directly proportional to normal reaction.

$$F_{max} \propto N$$

$$F_{max} = \mu_s N$$

where  $\mu_s$  = Coefficient of static friction

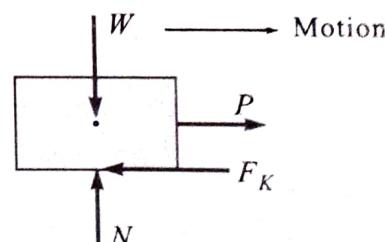
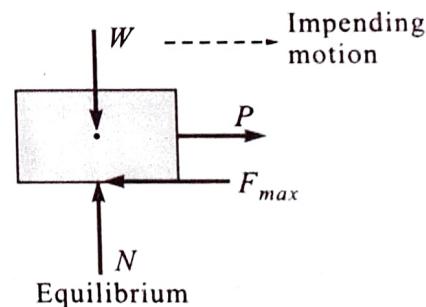
If body is in motion, we have

$$F_K \propto N$$

$$F_K = \mu_K N$$

where  $\mu_K$  = Coefficient of kinetic friction

Kinetic friction is always less than limiting friction.



## 7.4 Laws of Friction

- The frictional force is always tangential to the contact surface and acts in a direction opposite to that in which the body tends to move.
- The magnitude of frictional force is self adjusting to the applied force till the limiting frictional force is reached and at the limiting frictional force the body will have impending motion.
- Limiting frictional force  $F_{max}$  is directly proportional to normal reactions (i.e.  $F_{max} = \mu_s N$ ).
- For body in motion, kinetic frictional force  $F_K$  developed is less than that of limiting frictional force  $F_{max}$  and the relation  $F_K = \mu_k N$  is applicable.
- Frictional force depends upon the roughness of the surface and the material in contact.
- Frictional force is independent of the area of contact between the two surfaces.
- Frictional force is independent of the speed of the body.
- Coefficient of static friction  $\mu_s$  is always greater than the coefficient of kinetic friction  $\mu_k$ .

**Angle of Friction :** It is the angle made by the resultant of the limiting frictional force  $F_{max}$  and the normal reaction  $N$  with the normal reactions.

Consider the block with weight  $W$  and applied force  $P$ .

When the block is at the verge of motion, limiting frictional force  $F_{max}$  will act in opposite direction of applied force and normal reaction  $N$  will act perpendicular to surface as shown in figure 7.2. We can replace the  $F_{max}$  and  $N$  by resultant reaction  $R$  which acts at an angle  $\phi$  to the normal reaction.

This angle  $\phi$  is called as the *angle of friction*.

From figure 7.2, we have

$$F_{max} = R \sin \phi$$

$$\mu_s N = R \sin \phi \quad \dots (I) \quad (\because F_{max} = \mu_s N)$$

$$N = R \cos \phi \quad \dots (II)$$

(I)  $\div$  (II), we get

$$\tan \phi = \mu_s \text{ or } \phi = \tan^{-1} \mu_s \quad \dots (7.3)$$

**Angle of Repose :** It is the minimum angle of inclination of a plane with the horizontal at which the body kept will just slide down on it without the application of any external force (due to self weight).

Consider the block with weight  $W$  resting on an inclined plane, which makes an angle  $\theta$  with horizontal as shown in figure 7.3. When  $\theta$  is small the block will rest on the plane. If  $\theta$  is increased gradually a slope is reached at which the block is about to start sliding. This angle  $\theta$  is called as *angle of repose*.

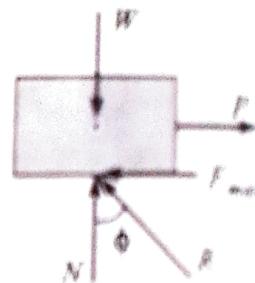


Fig. 7.2

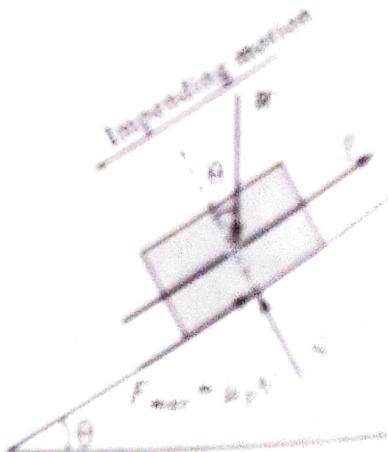


Fig. 7.3

for limiting equilibrium condition, we have

$$\begin{aligned}\sum F_x &= 0 \\ \mu_s N - W \sin \theta &= 0 \\ W \sin \theta &= \mu_s N\end{aligned} \quad \dots (I)$$

$$\begin{aligned}\sum F_y &= 0 \\ N - W \cos \theta &= 0 \\ W \cos \theta &= N\end{aligned} \quad \dots (II)$$

(I)  $\div$  (II), we get

$$\tan \theta = \mu_s \quad \dots (7.3)$$

In previous discussion, we had  $\tan \phi = \mu_s$  which shows  
Angle of friction  $\phi$  = Angle of repose  $\theta$

The above relation also shows that the angle of repose is independent of the weight of the body and it depends on  $\mu$ .

**Cone of Friction :** When the applied force  $P$  is just sufficient to produce the impending motion of given body, angle of friction  $\phi$  is obtained which is the angle made by resultant of limiting friction force and normal reaction with normal reaction as shown in figure 7.4. If the direction of applied force  $P$  is gradually changed through  $360^\circ$ , the resultant  $R$  generates a right circular cone with semi vertex angle equal to  $\phi$ . This is called as *cone of friction*.

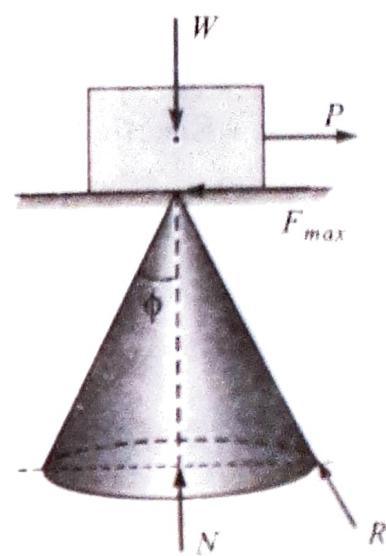


Fig. 7.4

## 7.5 Types of Friction Problems

The above discussion can be represented by a graph with applied force  $P$  v/s frictional force  $F$  as shown in figure 7.5.

Referring to the graph we may now recognize three distinct types of problems. Here, we have  $F_{max}$  static friction, limiting friction and kinetic  $F_K$  friction.

### 1. First Type - Static Friction

If in the problem there is neither the condition of impending motion nor that of motion then to determine the actual force, we first assume static equilibrium and take  $F$  as a frictional force required to maintain the equilibrium condition.

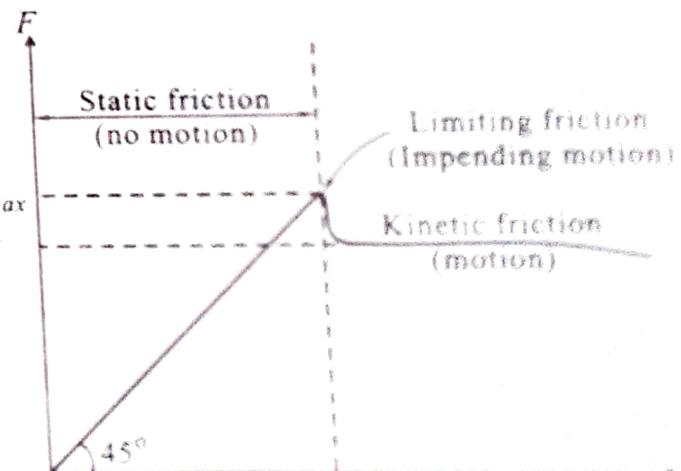


Fig. 7.5

Here, we have three possibilities

- (i)  $F < F_{max}$   $\Rightarrow$  Body is in static equilibrium condition which means body is purely at rest.
- (ii)  $F = F_{max}$   $\Rightarrow$  Body is in limiting equilibrium condition which means impending motion and hence  $F = F_{max} = \mu_S N$  is valid equation.
- (iii)  $F > F_{max}$   $\Rightarrow$  Body is in motion which means  $F = F_K = \mu_K N$  is valid equation (this condition is impossible, since the surfaces cannot support more force than the maximum frictional force. Therefore, the assumption of equilibrium is invalid, the motion occurs).

## 2. Second Type - Limiting Friction

The condition of impending motion is known to exist. Here a body which is in equilibrium is on the verge of slipping which means the body is in limiting equilibrium condition.

$F_{max} = \mu_S N$  is valid equation.

## 3. Third Type - Kinetic Friction

The condition of relative motion is known to exist between the contacting surfaces. So, the body is in motion.

Kinetic friction takes place  $F_K = \mu_K N$  is valid equation.

## 7.6 Solved Problems

### 7.6.1 Body Placed on Horizontal Plane

#### Problem 1

Determine the frictional force developed on the block shown in figure 7.6 when (i)  $P = 40$  N,

(ii)  $P = 80$  N. Coefficient of static friction between block and floor is  $\mu_S = 0.3$  and  $\mu_K = 0.25$ ,

(iii) Also find the value of  $P$  when the block is about to move.

#### Solution

##### (i) When $P = 40$ N

Consider the F.B.D. of block.

Let  $F$  be the frictional force required to maintain the static equilibrium condition.

$$\Sigma F_y = 0$$

$$N - 100 - 40 \sin 30 = 0$$

$$N = 120 \text{ N}$$

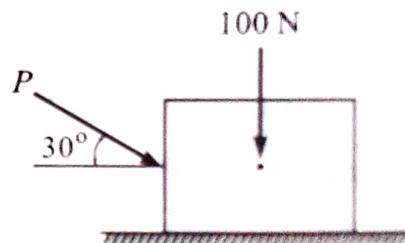
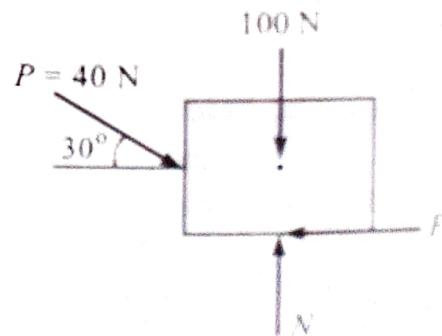


Fig. 7.5



F.B.D. of Block

$$\sum F_x = 0$$

$$40 \cos 30 - F = 0$$

$$F = 34.64 \text{ N}$$

For limiting equilibrium condition, we have

$$F_{max} = \mu_s \times N$$

$$F_{max} = 0.3 \times 120$$

$$F_{max} = 36 \text{ N}$$

Since  $F < F_{max}$  therefore block is in static equilibrium condition.

Therefore, actual frictional force is  $F = 34.64 \text{ N}$  **Ans.**

Here block is not moving.

(ii) When  $P = 80 \text{ N}$

Consider the F.B.D. of block.

Let  $F$  be the frictional force required to maintain the static equilibrium condition.

$$\sum F_y = 0$$

$$N - 100 - 80 \sin 30 = 0$$

$$N = 140 \text{ N}$$

$$\sum F_x = 0$$

$$80 \cos 30 - F = 0$$

$$F = 69.28 \text{ N}$$

For limiting equilibrium condition, we have

$$F_{max} = \mu_s \times N$$

$$F_{max} = 0.3 \times 140$$

$$F_{max} = 42 \text{ N}$$

$$\therefore F > F_{max}$$

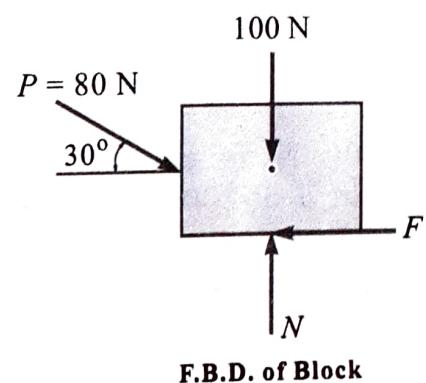
$\therefore$  The block is in motion and kinetic friction is considered.

$\therefore F_K = \mu_K N$  is applicable and  $F_{max} = \mu_s N$  is not applicable.

$$F_K = 0.25 \times 140$$

$$F_K = 35 \text{ N}$$

$\therefore$  Actual frictional force acting at surface is  $F_K = 35 \text{ N}$  and block is in motion. **Ans.**



**(iii) Find  $P = ?$**

For limiting equilibrium condition.

Consider the F.B.D. of block.

$$\sum F_x = 0$$

$$N - 100 - P \sin 30 = 0$$

$$N = 100 + P \sin 30$$

$$\sum F_y = 0$$

$$P \cos 30 - \mu_s N = 0$$

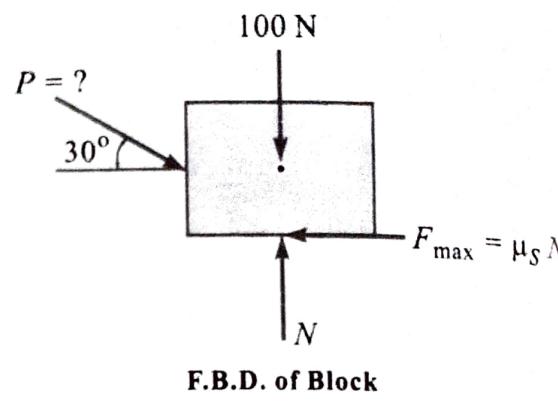
From equation (I)

$$P \cos 30 - 0.3 (100 + P \sin 30) = 0$$

$$P \cos 30 - 0.3 P \sin 30 = 0.3 \times 100$$

$$P = 41.9 \text{ N}$$

When  $P = 41.9 \text{ N}$  the block is about to move (Impending motion). **Ans.**

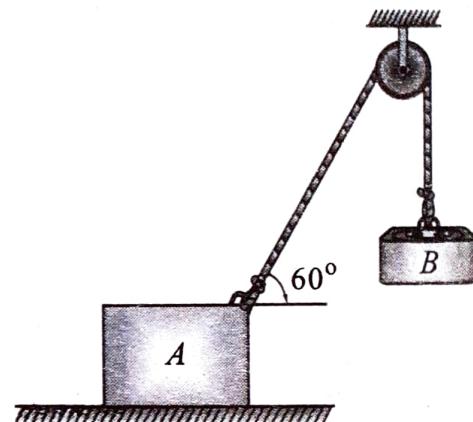


**F.B.D. of Block**

**Problem 2**

Block  $A$  of  $500 \text{ N}$  is connected to a suspended weight  $B$  of  $200 \text{ N}$  as shown in figure 7.7.

- (i) Determine whether  $A$  moves if  $\mu = 0.7$ .
- (ii) Find  $\mu$  if the block  $A$  is on the point of motion (Assume the pulley to be frictionless).



**Fig. 7.7**

**Solution**

- (i) When  $\mu = 0.7$

**Consider the F.B.D. of Block  $A$**

Let  $F$  be the frictional force required to maintain the static equilibrium condition.

$$\sum F_y = 0$$

$$N + 200 \sin 60 - 500 = 0$$

$$N = 326.8 \text{ N}$$

$$\sum F_x = 0$$

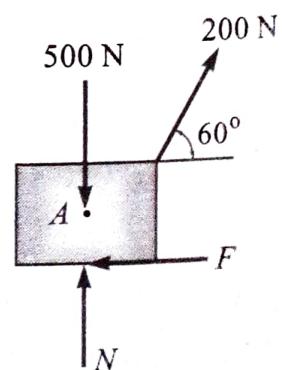
$$200 \cos 60 - F = 0$$

$$F = 100 \text{ N}$$

For limiting equilibrium condition, we have

$$F_{\max} = \mu N$$

$$F_{\max} = 0.7 \times 326.8$$



**F.B.D. of Block  $A$**

$$F_{max} = 228.76 \text{ N}$$

$$F < F_{max}$$

∴ block A is in static equilibrium condition.

∴ block A will not move. **Ans.**

(ii) Find  $\mu = ?$

When block A is on the point of the motion.

This is the limiting equilibrium condition and  
 $F_{max} = \mu N$  is applicable.

Consider the F.B.D. of Block A

$$\sum F_y = 0$$

$$N + 200 \sin 60 - 500 = 0$$

$$N = 326.8 \text{ N}$$

$$\sum F_x = 0$$

$$200 \cos 60 - \mu N = 0$$

$$\mu = \frac{200 \cos 60}{326.8}$$

$$\therefore \mu = 0.31 \quad \text{Ans.}$$

### Problem 3

Find the maximum height at which P should be applied so that the body would just slide without tipping. Also state magnitude of P. Refer figure 7.8.

### Solution

Consider the F.B.D. of Block

$$\sum F_y = 0$$

$$N - 2 = 0$$

$$N = 2 \text{ kN}$$

$$\sum F_x = 0$$

$$P - \mu N = 0$$

$$P = 0.3 \times 2$$

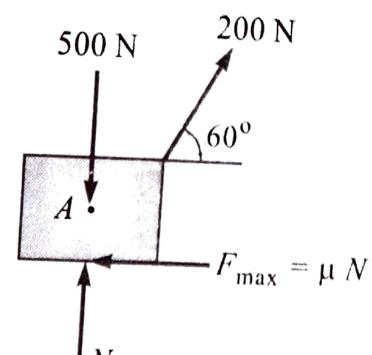
$$P = 0.6 \text{ kN} \quad \text{Ans.}$$

$$\sum M_A = 0$$

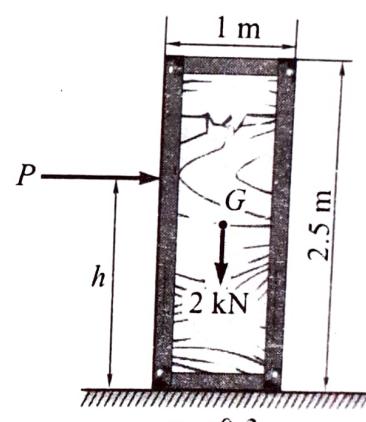
$$2 \times 0.5 - P \times h = 0$$

$$h = \frac{2 \times 0.5}{0.6}$$

$$h = 1.67 \text{ m} \quad \text{Ans.}$$

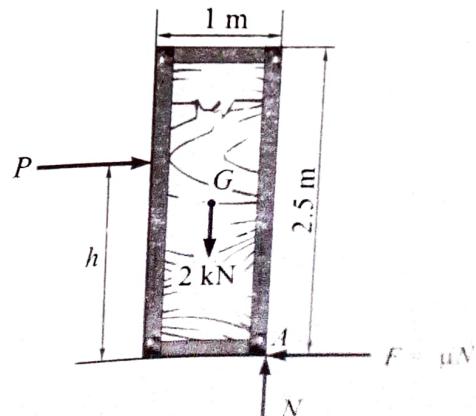


F.B.D. of Block A



$$\mu = 0.3$$

Fig. 7.8



F.B.D. of Block

### Problem 4

A homogeneous block  $A$  of weight  $W$  rests upon an inclined plane as shown in figure 7.9.  $\mu = 0.3$ . Determine the greatest height at which a force  $P$  parallel to the inclined plane may be applied so that the block will slide up the plane without tipping over.

### Solution

#### Consider the F.B.D. of Block A

$$\sum F_y = 0$$

$$N - W \cos \theta = 0$$

$$N = W \cos 36.87$$

$$\sum F_x = 0$$

$$P - \mu N - W \sin \theta = 0$$

$$P = 0.3 \times W \cos 36.87 + W \sin 36.87$$

$$P = 0.84 W \text{ N}$$

$$\sum M_A = 0$$

$$W \cos \theta \times 30 + W \sin \theta \times 40 - P \times h = 0$$

$$W \cos 36.87 \times 30 + W \sin 36.87 \times 40 - 0.84 W \times h = 0$$

$$h = 57.14 \text{ cm } \textbf{Ans.}$$

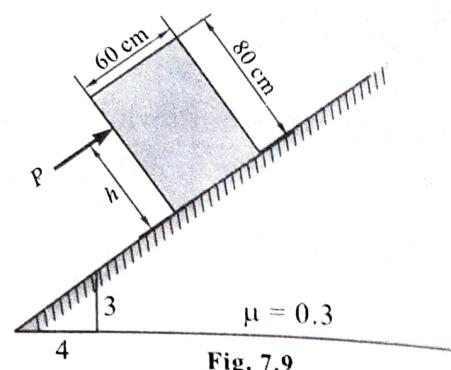
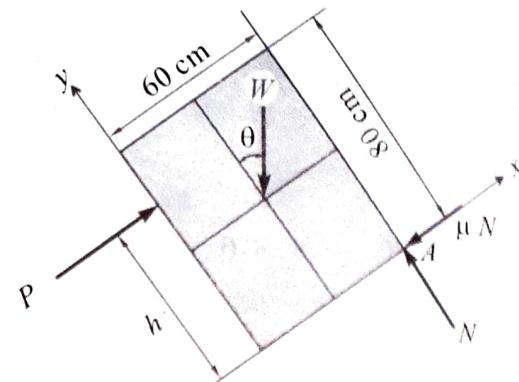


Fig. 7.9



F.B.D. of Block A

### Problem 5

Find the minimum weight  $W_3$  for limiting equilibrium shown in figure 7.10. Take  $\mu = 0.2$  for all rubbing surfaces.

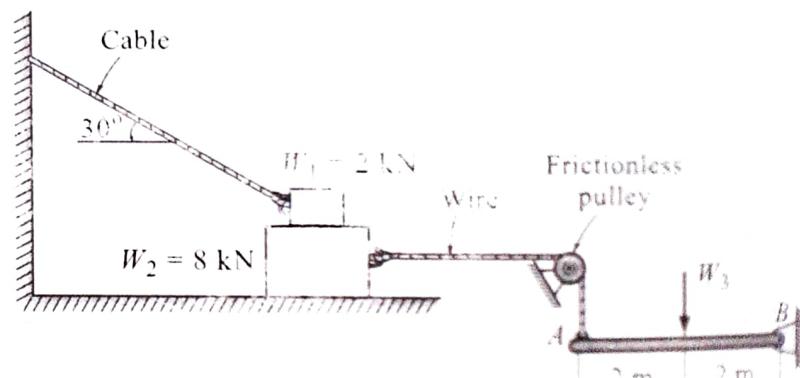


Fig. 7.10

### Solution

#### Consider F.B.D. of Block $W_1$

$$\sum F_y = 0$$

$$N_1 - 2 + T_1 \sin 30 = 0$$

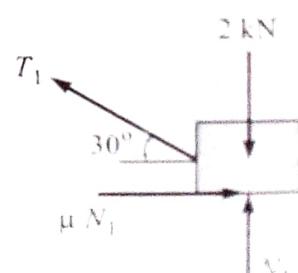
$$N_1 = 2 - T_1 \sin 30 \quad \dots (\text{I})$$

$$\sum F_x = 0$$

$$\mu N_1 - T_1 \cos 30 = 0$$

$$0.2(2 - T_1 \sin 30) - T_1 \cos 30 = 0$$

$$T_1 = 0.414 \text{ kN}$$



F.B.D. of Block  $W_1$

From equation (I)  
 $N_1 = 1.793 \text{ kN}$

Consider F.B.D. of Block  $W_2$

$$\sum F_y = 0$$

$$N_2 - N_1 - 8 = 0$$

$$N_2 = 9.793 \text{ N}$$

$$\sum F_x = 0$$

$$T_2 - \mu N_1 - \mu N_2 = 0$$

$$T_2 = 0.2(1.793 + 9.793)$$

$$T_2 = 2.317 \text{ kN}$$

Consider F.B.D. of Rod  $AB$

$$\sum M_B = 0$$

$$W_3 \times 2 - T_2 \times 4 = 0$$

$$W_3 = \frac{2.317 \times 4}{2}$$

$$W_3 = 4.634 \text{ kN} \quad \text{Ans.}$$

### Problem 6

A wooden block of mass 40 kg is on rough inclined plane as shown in figure 7.9. Find the frictional force at surface in contact if  $\mu_s = 0.4$  and  $\mu_k = 0.35$ .

**Solution**

$$\tan \phi = \mu_s$$

$$\therefore \phi = 21.8^\circ$$

We know angle of friction is equal to angle of repose for limiting equilibrium condition where self weight of block is just sufficient to slide down without any external force acting on it.

In the above case inclination of surface  $15^\circ$  is less than angle of friction  $\phi = 21.8^\circ$ . Therefore, the block will be in static equilibrium condition (i.e. stationary).

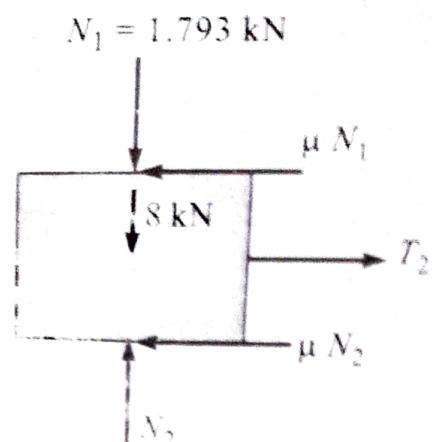
**Consider F.B.D. of Block**

Let  $F$  be the frictional force required to maintain the static equilibrium condition.

$$\sum F_x = 0$$

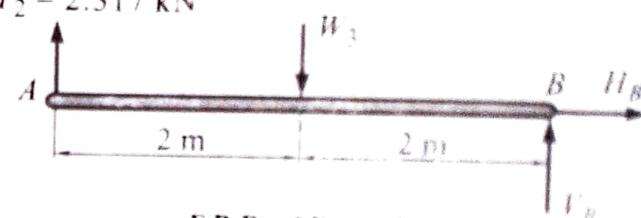
$$F - 40 \times 9.81 \sin 15^\circ = 0$$

$$F = 101.56 \text{ N} \quad \text{Ans.}$$



F.B.D. of Block  $W_2$

$$T_2 = 2.317 \text{ kN}$$



F.B.D. of Rod  $AB$

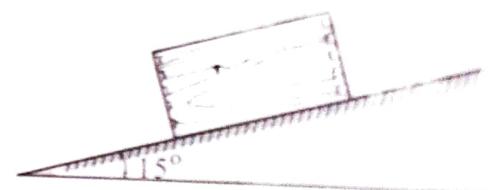
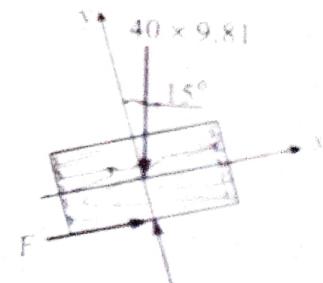


Fig. 7.9



F.B.D. of Block

$$P = 220.86 \text{ N} \quad \text{Ans.}$$

### Problem 16

Calculate the force  $P$  required to cause the block  $A$  of weight 600 N to slide under the block  $B$  of weight 200 N as shown in figure 7.18. What will then be the tension in the string? Assume the coefficient of friction for all surfaces of contact as 0.2.

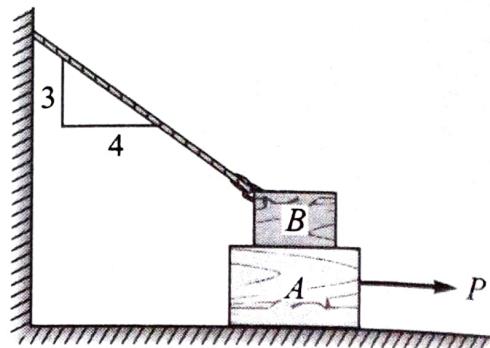


Fig. 7.18

### Solution

#### Consider the F.B.D. of Block B

$$\sum F_y = 0$$

$$N_B - 200 + T \sin 36.87 = 0$$

$$N_B = 200 - T \sin 36.87 \quad \dots (\text{I})$$

$$\sum F_x = 0$$

$$\mu N_B - T \cos 36.87 = 0$$

$$0.2(200 - T \sin 36.87) - T \cos 36.87 = 0$$

$$T = 43.48 \text{ N}$$

From equation (I)

$$N_B = 173.91 \text{ N}$$

#### Consider the F.B.D. of Block A

$$\sum F_y = 0$$

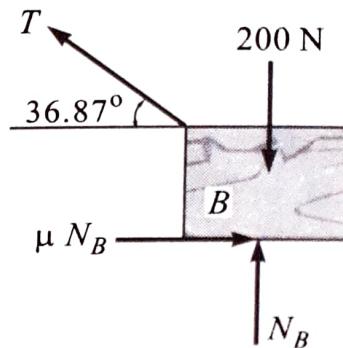
$$N_A - 600 - N_B = 0$$

$$N_A = 773.91 \text{ N}$$

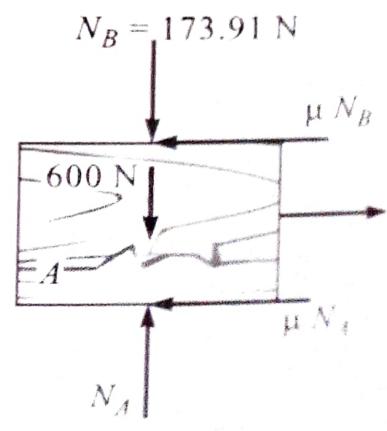
$$\sum F_x = 0$$

$$P - \mu N_A - \mu N_B = 0$$

$$P = 189.56 \text{ N} \quad \text{Ans.}$$



F.B.D. of Block B



F.B.D. of Block A

**Problem 18**

Find the value of  $\theta$  if the block  $A$  and  $B$  shown in figure 7.20 have impending motion. Given block  $A = 20 \text{ kg}$ , block  $B = 20 \text{ kg}$ ,  $\mu_A = \mu_B = 0.25$ .

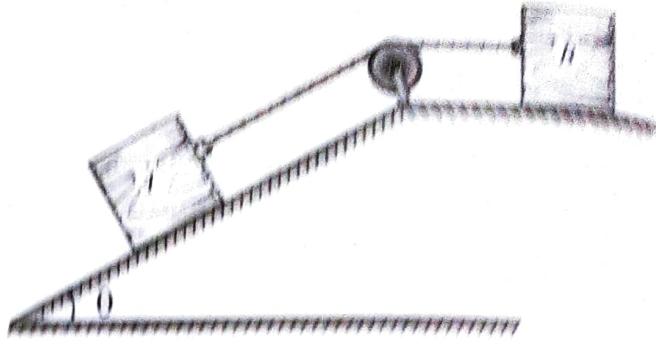


Fig. 7.20

**Solution****Consider the F.B.D. of Block B**

$$\Sigma F_y = 0$$

$$N_B - (20 \times 9.81) = 0$$

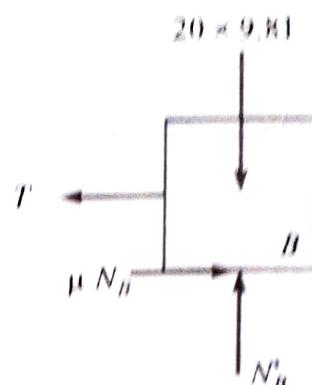
$$N_B = 20 \times 9.81$$

$$\Sigma F_x = 0$$

$$\mu N_B - T = 0$$

$$(0.25 \times 20 \times 9.81) - T = 0$$

$$T = 0.25 \times 20 \times 9.81$$



F.B.D. of Block B

**Consider the F.B.D. of Block A**

$$\Sigma F_y = 0$$

$$N_A - (20 \times 9.81) \cos \theta = 0$$

$$N_A = (20 \times 9.81) \cos \theta$$

$$\Sigma F_x = 0$$

$$\mu N_A + T - (20 \times 9.81) \sin \theta = 0$$

$$(0.25 \times 20 \times 9.81) \cos \theta + (0.25 \times 20 \times 9.81) - (20 \times 9.81) \sin \theta = 0$$

$$0.25 \cos \theta + 0.25 - \sin \theta = 0 \quad (\text{Multiply by 4})$$

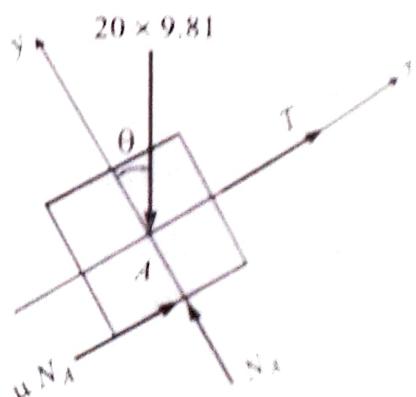
$$\cos \theta + 1 = 4 \sin \theta$$

$$2 \cos^2(\theta/2) = 4 [2 \sin(\theta/2), \cos(\theta/2)]$$

$$\frac{1}{4} = \tan \frac{\theta}{2}$$

$$\frac{\theta}{2} = \tan^{-1} 0.25$$

$$\theta = 28.07^\circ \quad \text{Ans.}$$



F.B.D. of Block A

**problem 19**  
Determine the minimum value and the direction of a force  $P$  required to cause motion of a 100 kg block to impend upon a  $30^\circ$  plane shown in figure. The coefficient of friction is 0.2.

**Solution**

Consider the F.B.D. of Block

$$\tan \phi = \mu = 0.2$$

$$\phi = 11.31^\circ$$

By Lami's theorem,

$$\frac{100 \times 9.81}{\sin(48.69 + 30 + \alpha)} = \frac{-P_{\min}}{\sin(180 + 30 + \phi)}$$

$$P_{\min} = \frac{-981 \sin(221.31)}{\sin(78.69 + \alpha)} \quad \dots (I)$$

For  $P$  to be minimum, denominator

$$\sin(78.69 + \alpha) = 1 \quad (\text{i.e. maximum})$$

$$78.69 + \alpha = 90$$

$$\therefore \alpha = 11.31^\circ \quad \text{Ans.}$$

From equation (I)

$$P_{\min} = 647.59 \text{ N} \quad \text{Ans.}$$

**Problem 20**

Two blocks  $A = 100 \text{ N}$  and  $B = 150 \text{ N}$  are resting on ground as shown in figure 7.22. Coefficient of friction between ground and block  $B$  is 0.10 and that between block  $B$  and  $A$  is 0.30. Find the minimum value of weight  $P$  in the pan so that motion starts. Find whether  $B$  is stationary w.r.t. ground and  $A$  moves or  $B$  is stationary w.r.t.  $A$ .

**Solution**

**Case I :**  $B$  is stationary w.r.t. ground and  $A$  moves.

Consider given  $A$  is in limiting equilibrium which means block  $A$  moves over the surface of  $B$ .

**Consider F.B.D. of Block A**

$$\sum F_y = 0$$

$$N_1 + P \sin 30 - 100 = 0$$

$$N_1 = 100 - P \sin 30$$

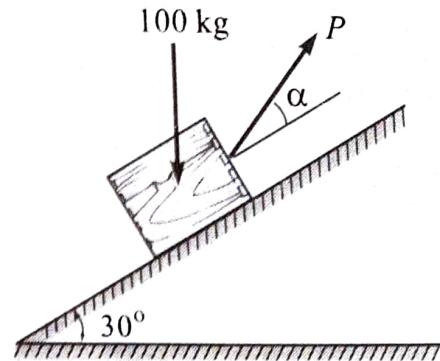
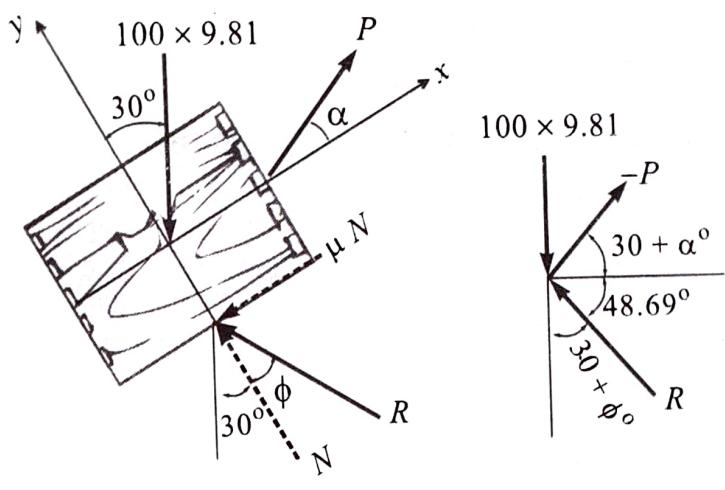


Fig. 7.21



F.B.D. of Block B

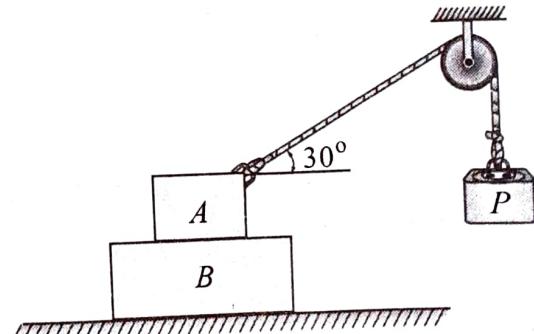
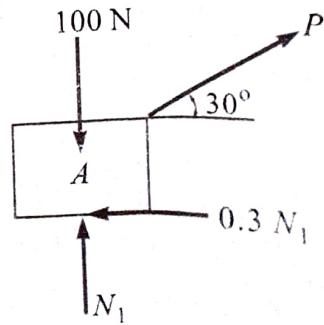


Fig. 7.22



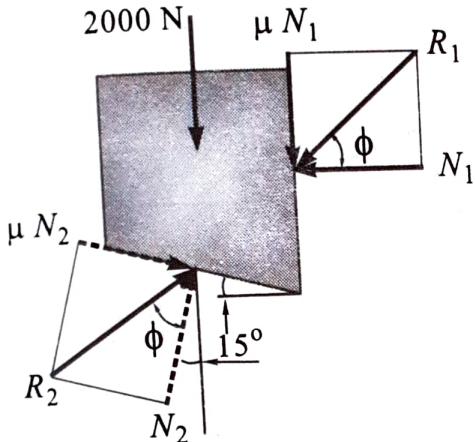
F.B.D. of Block A

### problem 25

To raise a heavy stone block weighing 2000 N, the arrangement shown in figure 7.27 is used. What horizontal force  $P$  will be necessary to apply to the wedge in order to raise the block if the coefficient of friction for all continuous surfaces is  $\mu = 0.25$ ? Neglect the weight of the wedge.

**Solution**

**Consider F.B.D. of Upper Block**



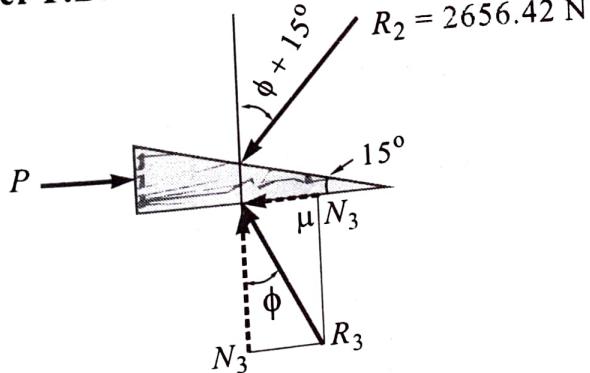
F.B.D. of Upper Block

By Lami's theorem, we have

$$\frac{2000}{\sin 133.08} = \frac{R_2}{\sin 75.96}$$

$$\therefore R_2 = 2656.42 \text{ N}$$

**Consider F.B.D. of Lower Block (Wedge)**



F.B.D. of Lower Block (Wedge)

By Lami's theorem, we have

$$\frac{P}{\sin 136.92} = \frac{2656.42}{\sin 104.04}$$

$$\therefore P = 1870.26 \text{ N } \text{Ans.}$$

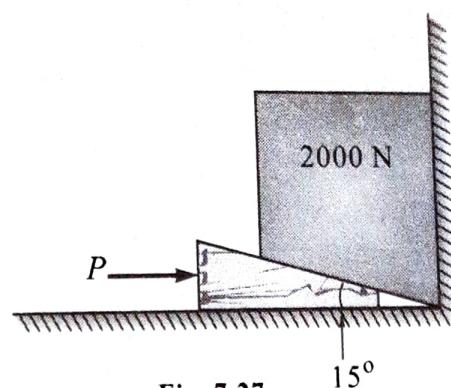
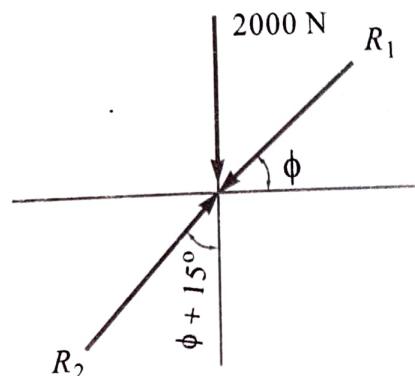
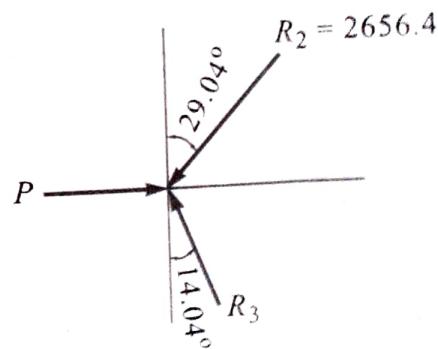


Fig. 7.27



$$\tan \phi = \mu = 0.25$$

$$\therefore \phi = 14.04^\circ$$

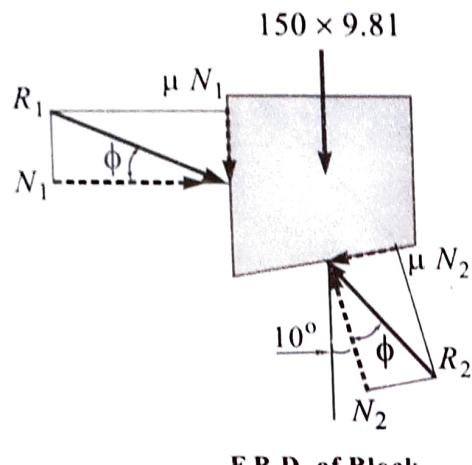


### Problem 26

A block of mass 150 kg is raised by a  $10^\circ$  wedge weighing 50 kg under it and by applying a horizontal force at it as shown in figure 7.28. Taking coefficient of friction between all surfaces of contact as 0.3, find what minimum force should be applied to raise the block.

### Solution

#### Consider F.B.D. of 150 kg Block



F.B.D. of Block

By Lami's theorem, we have

$$\frac{R_2}{\sin(90 - 16.7)} = \frac{150 \times 9.81}{\sin(90 + 16.7 + 26.7)}$$

$$\therefore R_2 = 1939.84 \text{ N}$$

#### Consider F.B.D. of Wedge

$$\sum F_y = 0$$

$$N_2 - (50 \times 9.81) - 1939.84 \cos 26.7 = 0$$

$$N_2 = 2223.5 \text{ N}$$

$$\sum F_x = 0$$

$$\mu N_2 + 1939.84 \sin 26.7 - P = 0$$

$$P = (0.3 \times 2223.5) + 1939.84 \sin 26.7$$

$$P = 1538.66 \text{ N} \quad \text{Ans.}$$

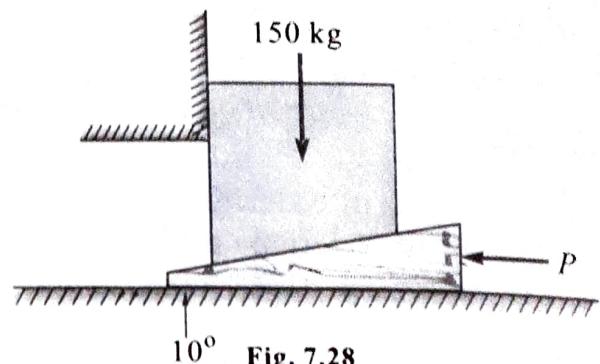
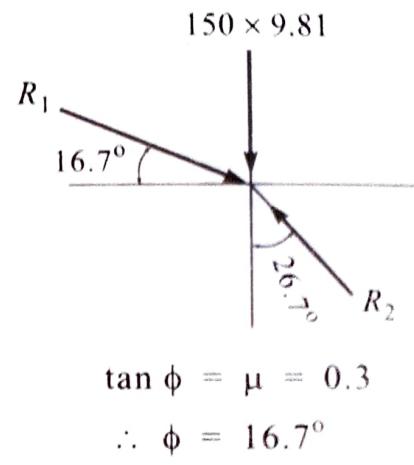
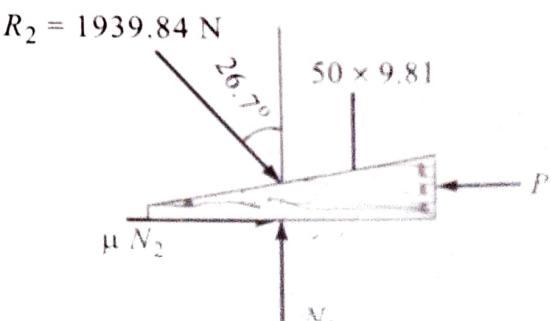


Fig. 7.28



$$\tan \phi = \mu = 0.3$$

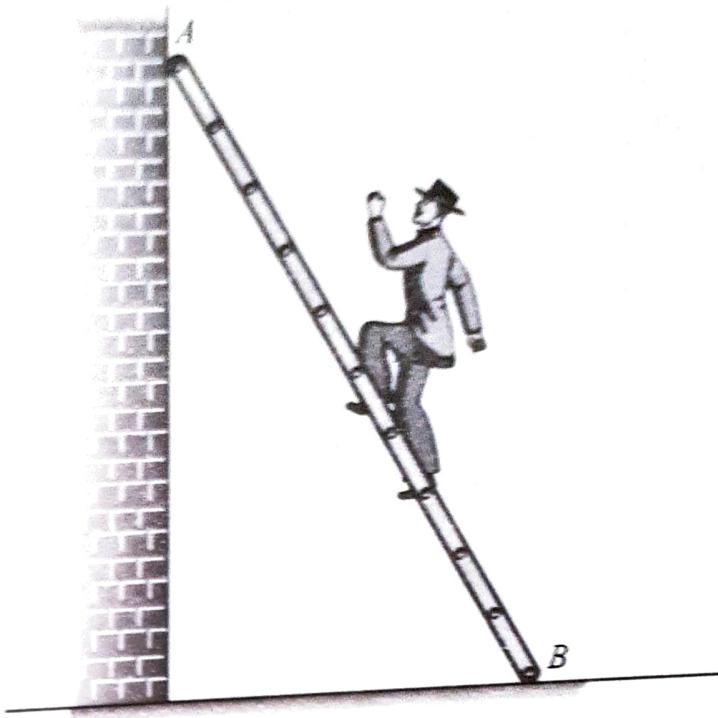
$$\therefore \phi = 16.7^\circ$$



F.B.D. of Wedge

**problem 31**

A uniform ladder weighing 100 N and 5 meters long has lower end *B* resting on the ground and upper end *A* resting against a vertical wall as shown in figure 7.32. The inclination of the ladder with horizontal is  $60^\circ$ . If the coefficient of friction at all surfaces of contact is 0.25, determine how much distance up along the ladder a man weighing 600 N can ascent without causing it to slip.



**Solution**

**Consider the F.B.D. of Ladder**

$$\sum F_x = 0$$

$$N_A - \mu N_B = 0$$

$$N_B = 4 N_A$$

$$\sum F_y = 0$$

$$\mu N_A + N_B - 100 - 600 = 0$$

$$0.25 N_A + 4 N_A = 700$$

$$N_A = 164.71$$

$$\sum M_B = 0$$

$$100 \times 2.5 \cos 60 + 600 \times d \cos 60$$

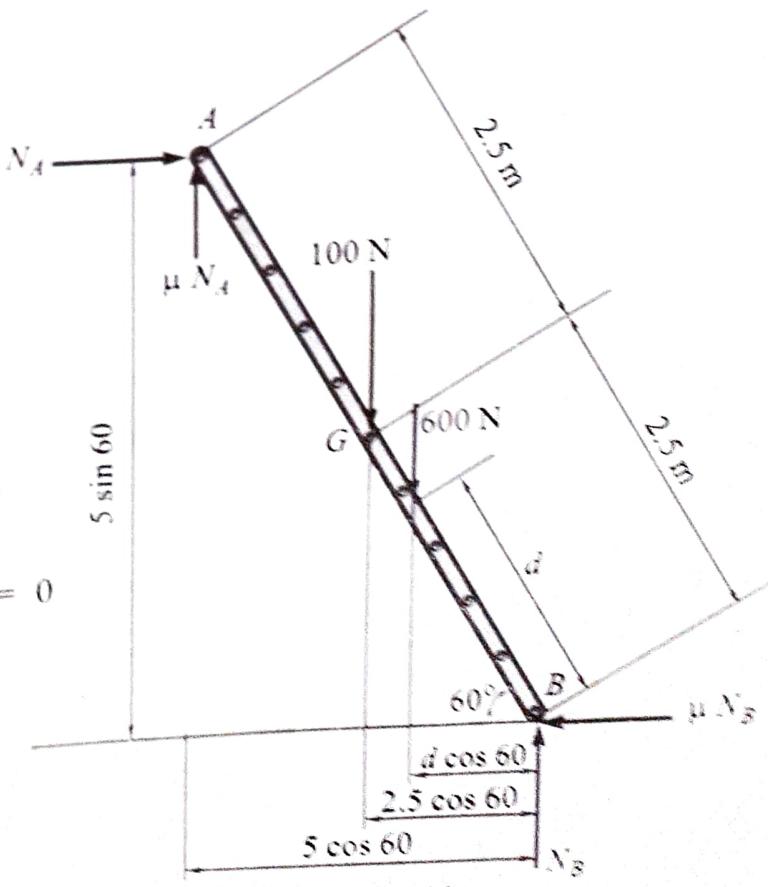
$$- N_A \times 5 \sin 60 - \mu N_A \times 5 \cos 60 = 0$$

$$100 \times 2.5 \cos 60 + 600 \times d \cos 60 - 164.71$$

$$\times 5 \sin 60 - 0.25 \times 164.71 \times 5 \cos 60 = 0$$

$$d = 2.304 \text{ m} \quad \text{Ans.}$$

Fig. 7.33



F.B.D. of Ladder

**Problem 32**

A weightless ladder of length 8 meter is resting against a smooth vertical wall and rough horizontal ground as shown in figure 7.34. The coefficient of friction between ground and ladder is 0.25. A man of weight 500 N wants to climb up the ladder. The man can climb without slip. A second person weighting 800 N wants to climb up the same ladder. Would he climb less than the earlier person ? Find his distance covered.

**Solution****Case I**

$$\Sigma F_x = 0$$

$$N_A - \mu N_B = 0$$

$$N_A = 0.25 N_B$$

$$\Sigma F_y = 0$$

$$N_B - 500 = 0$$

$$N_B = 500 \text{ N}$$

$$\therefore N_A = 125 \text{ N}$$

$$\Sigma M_B = 0$$

$$500 \times d_1 \cos 60 - N_A \times 8 \sin 60 = 0$$

$$d_1 = \frac{125 \times 8 \sin 60}{500 \cos 60}$$

$$d_1 = 3.464 \text{ m} \quad \text{Ans.}$$

**Case II**

$$\Sigma F_x = 0$$

$$N_A - \mu N_B = 0$$

$$N_A = 0.25 N_B$$

$$\Sigma F_y = 0$$

$$N_B - 800 = 0$$

$$N_B = 800 \text{ N}$$

$$\therefore N_A = 200 \text{ N}$$

$$\Sigma M_B = 0$$

$$800 \times d_2 \cos 60 - N_A \times 8 \sin 60 = 0$$

$$d_2 = \frac{200 \times 8 \sin 60}{800 \cos 60}$$

$$d_2 = 3.464 \text{ m} \quad \text{Ans.}$$

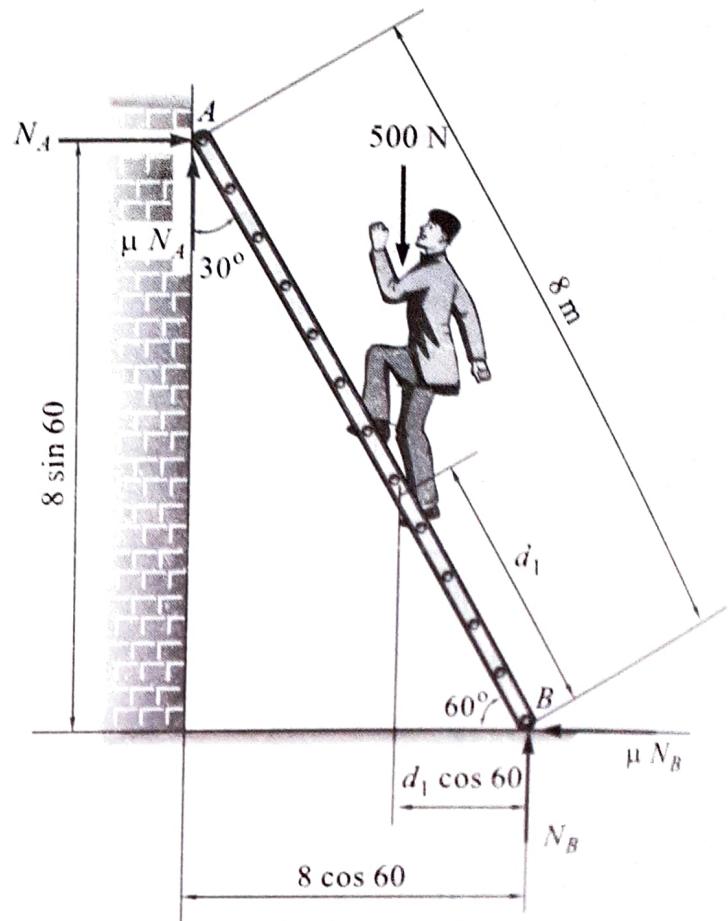


Fig. 7.34 (Case I)

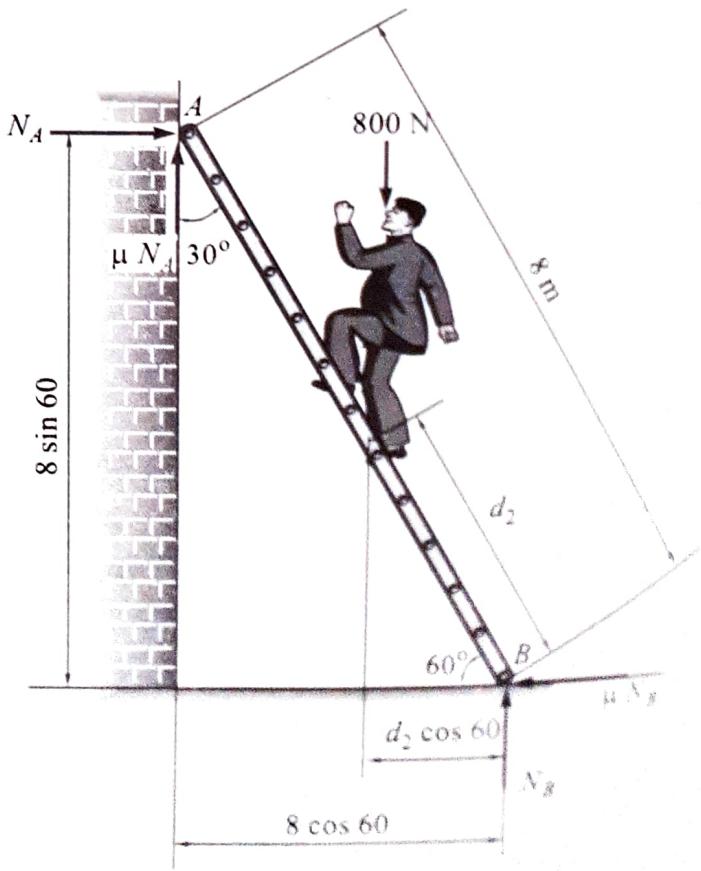


Fig. 7.34 (Case II)