DOUBLE INTEGRATION

Evaluate the following integrals and Sketch the area of integration.

1. $\int_0^{\pi/2}\int_0^{3(1-\cos t)} x^2 \sin t\, dx\, dt$
2. $\int_0^1\int_0^x xy(x^2y+xy^2)dy\, dx$
3. $\int_0^{\pi/2}\int_{\pi/2}^{\pi} \cos(x+y)dy\, dx$
4. $\int_0^1\int_0^{\sqrt{\frac{1}{2}(1-y^2)}} \frac{dx\, dy}{\sqrt{1-x^2-y^2}}$
5. $\int_1^2\int_0^x \frac{1}{x^2+y^2}dy\, dx$
6. $\int_0^2\int_0^{\sqrt{2x+x^2}} xy\, dy\, dx$
7. $\int_0^{2a}\int_0^{\sqrt{2ax-x^2}} xy\, dy\, dx$
8. $\int_0^1\int_0^{x^2} x(x^3+y^3)dy\, dx$
9. $\int_0^1\int_0^x e^{x+y}dy\, dx$
10. $\int_0^a\int_0^{\sqrt{a^2-x^2}} x^2y\, dy\, dx$
11. $\int_0^{\pi/2}\int_0^{a\cos\theta} r\sqrt{a^2-r^2}\, dr d\theta.$
12. $\int_0^{\pi}\int_0^{a\sin\theta} r\, dr\, d\theta$
13. $\int_0^{\pi/2}\int_0^{a\cos\theta} r\sin\theta\, dr\, d\theta$
14. $\int_0^{\pi/2}\int_0^{a\cos\theta} r^2\, dr\, d\theta$

Evaluate the following integrals over the region stated

14. $\iint(x^2-y^2)x\, dx\, dy$ over the positive quadrant of the circle $x^2+y^2=a^2$
15. $\iint(x+y)^2dx\, dy$ over the area bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$
16. $\iint(x^2-y^2)dx\, dy$ over the area of the triangle whose vertices are at the points $(0,1),\ (1,1),\ (1,2)$
17. $\iint x^{n-1}y^{m-1}dx\, dy$ over the area of the triangle given by $x\geq 0,\ y\geq 0,\ x+y\leq 1$
18. $\iint dx\, dy$ throughout the area bounded by $y=x^2\ and\ x+y=2$
19. $\iint(x^2+y^2)dx\, dy$ over the area of the ellilpse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$
20. $\iint x(x^2+y^2)dx\, dy$ over the positive quadrant of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$
21. $\iint x^2y^2dx\, dy$ over the area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$
22. $\iint y\, dx\, dy$ through out the area bounded by $y = x^2\ and\ x + y = 2$
23. $\iint e^{3x+4y}dx\, dy$ over the triangle $x = 0,\ y = 0,\ x + y = 1$
24. $\iint \frac{xy}{\sqrt{1-y^2}}dx\, dy$ over the positive quadrant of the circle $x^2 + y^2 = 1$

Change the order of following integrals.

25. $\int_0^2\int_{2+\sqrt{4-2y}}^{y} f(x,y)dx\, dy$
26. $\int_0^{4a}\int_x^{2\sqrt{ax}} f(x,y)dy\, dx$
27. $\int_0^a\int_{x^2/a}^{\sqrt{ax}} f(x,y)dy\, dx$
28. $\int_a^b\int_{x_1}^{x_2} f(x,y)dx\, dy$ where $x_1=a-\frac{a}{b}\sqrt{b^2-y^2},\ x_2=\frac{ay^2}{b^2}$
29. $\int_0^{a\cos\alpha}\int_{x\tan\alpha}^{\sqrt{a^2-x^2}} f(x,y)dydx$
30. $\int_0^a\int_x^{a^2/x} f(x,y)dy\, dx$
31. $\int_0^a\int_{y^2/a}^{2a-y} f(x,y)dxdy$
32. $\int_0^1\int_{x^2}^{\sqrt{2-x^2}} f(x,y)dy\, dx$
33. $\int_0^b\int_{\sqrt{b^2-y^2}}^{a\sqrt{1-(y^2/b^2)}} f(x,y)dx\, dy$
34. $\int_a^b\int_{k/x}^{mx} f(x,y)dy\, dx$

Express as a single integral

35. $\int_0^1\int_0^{\sqrt{y}} f(x,y)dx\, dy+\int_1^2\int_0^{2-y} f(x,y)dx\, dy$
36. $\int_0^2\int_0^x f(x,y)dy\, dx+\int_2^4\int_0^{(4x-x^2)/2} f(x,y)dy\, dx$

Change the order of integration and evaluate.

37. $\int_0^3\int_{y^2/9}^{\sqrt{10-y^2}} dx\, dy$
38. $\int_0^1\int_x^{2-x}\left(\frac{x}{y}\right)dy\, dx$

39. $\int_0^b \int_{x^2/b}^{x} \frac{x\,dy\,dx}{(b-y)\sqrt{by-y^2}}$

40. $\int_1^2 \int_1^{x^2} \frac{x^2}{y} dy\,dx$

41. $\int_3^5 \int_0^{4/x} xy\,dy\,dx$

42. $\int_0^1 \int_0^{\sqrt{1-x^2}} \frac{y\,dy\,dx}{(1+y^2)\sqrt{1-x^2-y^2}}$

43. $\int_0^{a/\sqrt{2}} \int_x^{\sqrt{a^2-x^2}} y^2 dy\,dx$

44. $\int_0^1 \int_{1-\sqrt{1-y}}^{1+\sqrt{1-y}} dx\,dy$

45. $\int_{-1}^2 \int_{x^2}^{x+2} dydx$

46. $\int_0^1 \int_1^{\sqrt{2-y^2}} \frac{y\,dx\,dy}{\sqrt{(2-x^2)(1-x^2y^2)}}$

47. $\int_0^{\pi/2} \int_0^y cos\,2y.\sqrt{1-a^2sin^2x}\,.dxdy$

48. $\int_0^a \int_{x/a}^{\sqrt{x/a}} (x^2+y^2)dy\,dx$

49. $\int_0^a \int_y^a \frac{x^2}{\sqrt{x^2+y^2}} dx\,dy$

50. $\int_0^1 \int_{x^2}^{2-x} xy\,dydx$

51. $\int_0^a \int_0^x x\sqrt{(a^2-y^2)(x^2-y^2)}\,dy\,dx$

52. $\int_0^\infty \int_0^\infty e^{-x^2(1+t^2)} x\,dx\,dt$

53. $\int_0^a \int_0^{2\sqrt{ax}} x^2 dy\,dx$

54. $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dydx$

Evaluate the following integrals

55. $\iint re^{-r^2/a^2} cos\,\theta\,sin\theta\,d\theta\,dr$ over the upper half of the circle $r = 2a\cos\theta$

56. $\iint \frac{r\,drd\theta}{(a^2+r^2)^2}$ over one loop of lemniscate $r^2 = a^2\cos 2\theta$

57. $\iint r\sin\theta\,d\,A$ over the cardioide $r = a(1+cos\theta)$ above the initial line.

Change to polar coordinates and evaluate.

58. $\int_0^a \int_y^a x\,dx\,dy$

59. $\int_0^{4a} \int_{y^2/4a}^{y} dy\,dx$

60. $\int_0^a \int_0^{\sqrt{a^2-x^2}} y^2\sqrt{(x^2+y^2)}dy\,dx$

61. $\int_0^4 \int_y^{4+\sqrt{16-y^2}} \frac{1}{(64+x^2+y^2)^2} dx\,dy$

62. $\int_0^a \int_y^a \frac{x}{(x^2+y^2)} dx\,dy$

63. $\int_0^1 \int_x^{\sqrt{(2x-x^2)}} \frac{1}{\sqrt{x^2+y^2}} dy\,dx$

64. $\int_0^a \int_0^x \frac{x^3}{\sqrt{(x^2+y^2)}} dydx$

65. $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\,dxdy$

66. $\int_0^{2a} \int_0^{\sqrt{(2ax-x^2)}} (x^2+y^2)\,dydx$

67. $\int_0^a \int_y^a \frac{x^2}{(x^2+y^2)^{3/2}} dx\,dy$

68. $\int_0^1 \int_0^{\sqrt{x-x^2}} \frac{4xy}{x^2+y^2} e^{-(x^2+y^2)} dy\,dx$

Evaluate the following integrals over the region stated, by changing to polar coordinates.

69. $\iint x^2y^2 dx\,dy$ over the circle $x^2+y^2=a^2$

70. $\iint sin\,(x^2+y^2)dx\,dy$ over the circle $x^2+y^2=a^2$

71. $\iint \frac{1}{(1+x^2+y^2)^{3/2}} dx\,dy$ over the region bounded by $y = 0,\ x = y,\ x = 1$

72. $\iint \frac{(x^2+y^2)^2}{x^2y^2} dx\,dy$ over the area bounded by **(i)** $x^2 + y^2 = ax, ax = by$ **(ii)** $x^2 + y^2 = by,\ ax = by$

73. $\iint (x^2 + y^2)x\,dx\,dy$ over the positive quadrant of the circle $x^2+y^2=a^2$

74. $\iint \frac{1}{(a^2+x^2+y^2)^{3/2}} dx\,dy$ over the entire xy plane.

75. $\iint e^{-(x^2+y^2)}\,dx\,dy$ over the positive quadrant of the xy plane.

ANSWERS

1. $\frac{9}{4}$ **2.** $\frac{1}{12}$ **3.** -2

4.	$\frac{\pi}{4}$	**5.**	$\frac{(\pi\log 2)}{4}$	**6.**	$\frac{2}{3}$
7.	$\frac{2a^4}{3}$	**8.**	$\frac{47}{280}$	**9.**	$\frac{(e-1)^2}{2}$
10.	$\frac{a^5}{15}$	**11.**	$\frac{a^3}{3}\left(\frac{\pi}{2}-\frac{2}{3}\right)$	**12.**	$\frac{\pi a^2}{4}$
13.	$\frac{a^2}{6}$	**14.**	$\frac{2a^3}{9}$	**14.**	$\frac{a^5}{15}$
15.	$\frac{\pi ab}{4}[a^2+b^2]$	**16.**	$\frac{-2}{3}$	**17.**	$\frac{\overline{\lvert m}\,\overline{\lvert n}}{\overline{\lvert m+n+1}}$
18.	$\frac{9}{2}$	**19.**	$\frac{\pi ab}{4}(a^2+b^2)$	**20.**	$\frac{1}{15}a^2b(2a^2+b^2)$
21.	$\frac{\pi a^3b^3}{24}$	**22.**	$\frac{16}{5}$	**23.**	$\frac{1}{2}(3e^4-4e^3+1)$
24.	$\frac{1}{6}$	**25.**	$\int_0^2\int_0^{2x-(x^2/2)}f(x,y)dx\,dy$	**26.**	$\int_0^{4a}\int_{y^2/4a}^{y}f(x,y)dx\,dy$

27. $\int_0^a\int_{y^2/a}^{\sqrt{ay}}f(x,y)dx\,dy$ **28.** $\int_0^a\int_{y_1}^{y_2}f(x,y)dy\,dx \quad where\ y_1=\sqrt{\frac{b^2x}{a}},\ y_2=\frac{b}{a}\sqrt{2ax-x^2}$

29. $\int_0^{a\sin\alpha}\int_0^{y\cot\alpha}f(x,y)dx\,dy+\int_{a\,sin\alpha}^{a}\int_0^{\sqrt{a^2-y^2}}f(x,y)dx\,dy$

30. $\int_0^a\int_0^y f(x,y)dx\,dy+\int_a^\infty\int_0^{a^2/y}f(x,y)dx\,dy$

31. $\int_0^a\int_0^{\sqrt{ax}}f(x,y)dy\,dx+\int_a^{2a}\int_0^{2a-x}f(x,y)dy\,dx$

32. $\int_0^1\int_0^{\sqrt{y}}f(x,y)dx\,dy+\int_1^{\sqrt{2}}\int_0^{\sqrt{2-y^2}}f(x,y)dx\,dy$

33. $\int_0^b\int_{\sqrt{b^2-x^2}}^{\sqrt{1-(x^2/a^2)}}f(x,y)dy\,dx+\int_b^a\int_0^{b\sqrt{1-(x^2/a^2)}}f(x,y)dy\,dx$

34. $\int_{k/b}^{k/a}\int_{k/y}^{b}f(x,y)dx\,dy+\int_{k/a}^{ma}\int_a^b f(x,y)dx\,dy+\int_{ma}^{mb}\int_{y/m}^{b}f(x,y)dy\,dx$

35.	$\int_0^1\int_{x^2}^{2-x}f(x,y)dy\,dx$	**36.**	$\int_0^2\int_y^{2+\sqrt{4-2y}}f(x,y)dx\,dy$	**37.**	$2+5sin^{-1}\left(\frac{3}{\sqrt{10}}\right)$
38.	$log\left(\frac{4}{e}\right)$	**39.**	$\frac{\pi b}{2}$	**40.**	$\frac{2}{9}(24log\,2-7)$
41.	$8log\frac{5}{3}$	**42.**	$\frac{\pi}{4}log\,2$	**43.**	$\frac{1}{32}a^4(\pi+2)$
44.	$\frac{4}{3}$	**45.**	$\frac{9}{2}$	**46.**	$1-\frac{\pi}{4}$
47.	$\frac{1}{3a^2}\left[(1-a^2)^{3/2}-1\right]$	**48.**	$\frac{a}{4}\left[\frac{a^2}{7}+\frac{1}{5}\right]$	**49.**	$\frac{a^3}{3}log(1+\sqrt{2})$
50.	$\mathbf{\frac{3}{8}}$	**51.**	$\frac{8}{45}a^5$	**52.**	$\frac{\pi}{4}$
53.	$\frac{4}{7}a^4$	**54.**	1	**55.**	$\frac{a^2}{16}(3+e^{-4})$
56.	$(\pi-2)/4a^2$	**57.**	$(4/3)a^3$	**58.**	$\frac{a^3}{3}$
59.	$\frac{8a^3}{3}$	**60.**	$\frac{\pi a^5}{20}$	**61.**	$\frac{1}{128}\left(\frac{\pi}{4}-\frac{1}{\sqrt{2}}tan^{-1}\frac{1}{\sqrt{2}}\right)$
62.	$\frac{\pi a}{4}$	**63.**	$2-\sqrt{2}$	**64.**	$\frac{a^4}{4}log(1+\sqrt{2})$
65.	$\frac{\pi}{4}$	**66.**	$\frac{\pi a^2}{4}$	**67.**	$\frac{a}{\sqrt{2}}$
68.	$\frac{1}{e}$	**69.**	$\frac{\pi}{24}a^6$	**70.**	$\pi(1-cosa^2)$
71.	$\pi/12$	**72.**	$ab\,/\,2\,;\quad ab\,/\,2$	**73.**	$a^5/5$
74.	$\frac{2\pi}{a}$	**75.**	$\frac{\pi}{4}$		

TRIPLE INTEGRATION

TYPE I : WHEN THE LIMITS OF INTEGRATION ARE GIVEN

Evaluate the following integrals.

1. $\int_{-1}^{1}\int_{-2}^{2}\int_{-3}^{3} dx\, dy\, dz$

2. $\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}} \frac{dxdydz}{\sqrt{(1-x^2-y^2-z^2)}}$

3. $\int_0^1\int_0^{1-x}\int_0^{1-x-y} \frac{1}{(x+y+z+1)^3} dz\, dy\, dx$

4. $\int_0^2\int_0^x\int_0^{2x+2y} e^{x+y+z} dz\, dy\, dx$

5. $\int_0^{log\, 2}\int_0^x\int_0^{x+y} e^{x+y+z} dx\, dy\, dz.$

6. $\int_0^2\int_0^x\int_0^{x+y} e^{x+y+z} dx\, dy\, dz$

7. $\int_0^{log\, 2}\int_0^x\int_0^{x+log\, y} e^{x+y+z} dx\, dy\, dz$

8. $\int_{-1}^1\int_0^z\int_{x-z}^{x+z} (x+y+z) dz\, dx\, dy.$

9. $\int_0^2\int_0^y\int_{x-y}^{x+y} (x+y+z) dx\, dy\, dz$

10. $\int_0^a\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}}\int_0^{mx} z^2 dx\, dy\, dz$

11. $\int_0^{\pi} 2d\theta \int_0^{a(1+cos\theta)} r\, dr \int_0^h \left[1-\frac{r}{a(1+cos\theta)}\right] dz$

12. $\int_0^{\pi/2}\int_0^{a\, sin\theta}\int_0^{(a^2-r^2)/a} r\, d\theta dr\, dz$

13. $\int_{-2}^2\int_{-\sqrt{4-x^2}/2}^{\sqrt{4-x^2}/2}\int_{x^2+3y^2}^{8-x^2-y^2} dzdydx$

14. $\int_0^1 dx \int_0^2 dy \int_1^2 x^2 yzdz$

15. $\int_0^a\int_0^a\int_0^a (yz+zx+xy) dx\, dy\, dz.$

16. $\int_0^a\int_0^{a-x}\int_0^{a-x-y} x^2 dx\, dy\, dz$

17. $\int_0^a\int_0^a\int_0^a (y^2z^2+z^2x^2+x^2y^2) dx\, dy\, dz$

18. $\int_0^a\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}}\int_0^y z^2 dx\, dy\, dz$

19. $\int_0^1\int_{y^2}^1\int_0^{1-x} x\, dz\, dx\, dy$

20. $\int_0^a\int_0^{a-x}\int_0^{a-x-y} (x^2+y^2+z^2) dz\, dy\, dx$

21. $\int_1^e\int_1^{log\, y}\int_1^{e^x} log\, z\, dx\, dy\, dz$

22. $\int_0^1\int_0^{1-x}\int_0^{x+y} e^z dx\, dy\, dz$

23. $\int_0^4\int_0^{2\sqrt{z}}\int_0^{\sqrt{4z-x^2}} dz\, dx\, dy$

24. $\int_0^a\int_0^x\int_0^{\sqrt{x+y}} z\, dx\, dy\, dz$

25. $\int_0^2\int_1^2\int_0^{yz} xyz\, dx\, dy\, dz$

26. $\int_0^{\infty}\int_0^{\infty}\int_0^{\infty} \frac{dx\, dy\, dz}{(1+x^2+y^2+z^2)^2}$

27. $\int_0^{2a}\int_{-\sqrt{2ax-x^2}}^{\sqrt{2ax-x^2}}\int_0^{\sqrt{4a^2-x^2-y^2}} dzdydx$

28. $\int_0^a\int_0^{\sqrt{a^2-z^2}}\int_0^{\sqrt{a^2-y^2-z^2}} dx\, dy\, dz$

29. $\int_0^a\int_0^{\sqrt{a^2-x^2}}\int_0^{\sqrt{a^2-x^2-y^2}} xyz\, dx\, dy\, dz$

30. $\int_0^a\int_0^{\sqrt{a^2-x^2}}\int_0^{\sqrt{a^2-x^2-y^2}} (x^2+y^2+z^2)\, dx\, dy\, dz$

ANSWERS

1. 48 **2.** $\frac{\pi^2}{8}$ **3.** $\frac{1}{2}\left(log\, 2-\frac{5}{8}\right)$ **4.** $\frac{e^{12}}{18}-\frac{e^6}{9}-\frac{e^4}{2}+e^2-\frac{4}{9}$

5. $\frac{5}{8}$ **6.** $\frac{e^8}{8}+e^2-\frac{3}{4}e^4-\frac{3}{8}$ **7.** $\frac{1}{9}(24\, log\, 2-19)$ **8.** 0

9. 16 **10.** $\frac{4m^3a^5}{45}$ **11.** $\frac{\pi a^2}{2}h$ **12.** $\frac{5a^3}{64}\pi$

13. 10π **14.** 1 **15.** $\frac{3}{4}a^5$ **16.** $\frac{a^5}{60}$

17. $\frac{a^7}{3}$ **18.** 0 **19.** $\frac{4}{35}$ **20.** $\frac{a^5}{20}$

21. $\frac{1}{4}(e^2-8e+13)$ **22.** $\frac{1}{2}$ **23.** 8π **24.** $\frac{a^3}{4}$

25. $\frac{15}{2}$ **26.** $\frac{\pi^2}{8}$ **27.** $\frac{8a^3\pi}{3}$ **28.** $\frac{\pi a^3}{6}$

29. $\frac{a^6}{48}$ **30.** $\frac{\pi a^5}{10}$

TYPE II : WHEN THE REGION OF INTEGRATION IS BOUNDED BY PLANES

1. Evaluate $\iiint x^2yz\,dx\,dy\,dz$ throughout the volume bounded by the planes $x = 0, y = 0, z = 0,$ $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

2. Evaluate $\iiint x^2\,dx\,dy\,dz$ throughout the volume of the tetrahedron $x \geq 0, y \geq 0, z \geq 0, \frac{x}{a}+\frac{y}{b}+\frac{z}{c}\leq 1.$

3. Evaluate $\iiint dx\,dy\,dz$ over the volume of the tetrahedron bounded by $x = 0, y = 0, z = 0,$ $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

4. Evaluate $\iiint z\,dx\,dy\,dz$ over the volume of the tetrahedron bounded by $x = 0, y = 0, z = 0,$ $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

5. Evaluate $\iiint \frac{dx\,dy\,dz}{(1+x+y+z)^3}$ over the volume of the tetrahedron $x = 0,\ y = 0, z = 0, x + y + z = 1$

6. Evaluate $\iiint (x+y+z)dx\,dy\,dz$ over the tetrahedron bounded by the planes $x = 0,\ y = 0, z = 0$ and $x + y + z = 1$

7. Evaluate $\iiint x^2yzdxdy\,dz$ throughout the volume bounded by $x = 0,\ y = 0,\ z = 0, x + y + z = 1$

8. Evaluate $\iiint (x^2+y^2+z^2)dxdydz$ over the volume of the solid bounded by the coordinate planes and $x+y+z=2$

9. Evaluate $\iiint z^2\,dxdydz$ over the volume of the solid bounded by the coordinate planes and$x+y+z=a$

10. Evaluate in terms of Gamma function $\iiint x^{l-1}y^{m-1}z^{n-1}\,dx\,dy\,dz$ throughout the volume of the tetrahedron $x \geq 0, y \geq 0, z \geq 0, x + y + z \leq 1.$

11. Evaluate $\iiint xyz\,dx\,dy\,dz$ throughout the volume bounded by $x = 0, y = 0, z = 0, x + y + z = 1.$

12. Evaluate $\iiint x^2y^2z^2dx\,dy\,dz$ throughout the volume bounded by $x = 0, y = 0, z = 0,$ $x + y + z = 1.$

13. Evaluate the integral $\iiint_v xyz^2\,dv$ over the region bounded by the planes $x=0, x=1, y=-1, y=2,$ $z=0, z=3$

ANSWERS

1.	$\frac{a^3b^2c^2}{2520}$	**2.**	$\frac{a^3bc}{60}$	**3.**	$\frac{abc}{6}$	**4.**	$\frac{abc^2}{24}$
5.	$\frac{1}{2}\left[log2-\frac{5}{8}\right]$	**6.**	$\frac{1}{8}$	**7.**	$\frac{1}{2520}$	**8.**	$\frac{8}{5}$
9.	$\frac{a^5}{60}$	**10.**	$\frac{1}{(l+m+n)}\cdot\frac{\lvert\bar{l}\lvert\bar{m}\lvert\bar{n}}{\lvert\overline{l+m+n}}$	**11.**	$\frac{1}{720}$	**12.**	$\frac{1}{45360}$
13.	$\frac{27}{4}$						

TYPE III : WHEN THE REGION OF INTEGRATION IS NOT BOUNDED BY PLANES, BUT BY SPHERE, ELLIPSOID ETC.

Evaluate the following integrals.

1. $\iiint_v \frac{dx\,dy\,dz}{(1+x^2+y^2+z^2)^2}$ where V is the volume in the first octant.

2. $\iiint (x^2+y^2+z^2)\,dx\,dy\,dz$ over the first octant of the sphere $x^2 + y^2 + z^2 = a^2$

3. $\iiint xyz\,dx\,dy\,dz$ over the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$

4. $\iiint xyz\,(x^2+y^2+z^2)dx\,dy\,dz$ over the first octant of the sphere $x^2 + y^2 + z^2 = a^2$

5. $\iiint \frac{dx\,dy\,dz}{x^2+y^2+z^2}$ throughout the volume of the sphere $x^2 + y^2 + z^2 = a^2$.

6. $\iiint \frac{z^2 dx\,dy\,dz}{x^2+y^2+z^2}$ over the volume of the sphere $x^2 + y^2 + z^2 = 2$

7. $\iiint_v \frac{dx\,dy\,dz}{\sqrt{a^2-x^2-y^2-z^2}}$ over the volume of sphere $x^2 + y^2 + z^2 = a^2$

8. $\iiint (x^2y^2 + y^2z^2 + z^2x^2)dx\,dy\,dz$ over the volume of the sphere $x^2 + y^2 + z^2 = a^2$

9. $\iiint e^{(x^2+y^2+z^2)^{3/2}}\, dV$ throughout the volume of the unit sphere

10. $\iiint_v \frac{z^2}{x^2+y^2+z^2} dx\,dy\,dz$ where V is the volume bounded by the sphere $x^2 + y^2 + z^2 = z$

11. $\iiint \frac{dx\,dy\,dz}{(x^2+y^2+z^2)^{1/2}}$ over the volume bounded by the spheres $x^2 + y^2 + z^2 = a^2$and $x^2 + y^2 + z^2 = b^2$ $a > b > 0$

12. $\iiint_v \frac{dx\,dy\,dz}{(x^2+y^2+z^2)^{3/2}}$ where V is the volume bounded by the spheres $x^2 + y^2 + z^2 = a^2$ and $x^2 + y^2 + z^2 = b^2, (b > a)$

13. $\iiint \frac{dx\,dy\,dz}{(x^2+y^2+z^2)^{3/2}}$ over the volume bounded by the spheres $x^2 + y^2 + z^2 = 16$ and $x^2 + y^2 + z^2 = 25$

14. $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dzdydx}{\sqrt{1-x^2-y^2-z^2}}$ by changing to spherical polar coordinates.

15. $\iiint x^2 dx\,dy\,dz$ over the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

16. $\iiint \sqrt{1 - \frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2}} dx\,dy\,dz$ throughout the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

17. $\iiint \sqrt{\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}}\, dx\,dy\,dz$ over the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

18. $\iiint \sqrt{\frac{x^2}{1^2} + \frac{y^2}{2^2} + \frac{z^2}{3^2}}\, dx\,dy\,dz$ over the volume of the ellipsoid $\frac{x^2}{1^2} + \frac{y^2}{2^2} + \frac{z^2}{3^2} = 1$

ANSWERS

1.	$\frac{\pi^2}{8}$	**2.**	$\pi.\frac{a^5}{10}$	**3.**	$\frac{a^6}{48}$	**4.**	$\frac{a^8}{64}$
5.	$4\pi a$	**6.**	$8.\frac{\pi\sqrt{2}}{9}$	**7.**	$a^2\pi^2$	**8.**	$\frac{4a^7\pi}{35}$
9.	$\frac{4}{3}\pi(e-1)$	**10.**	$\frac{\pi}{9}$	**11.**	$2\pi\,(a^2 - b^2)$	**12.**	$4\pi log\left(\frac{b}{a}\right)$
13.	$4\pi\ log\ (5\ /\ 4)$	**14.**	$\frac{\pi^2}{8}$	**15.**	$\frac{4\pi a^3 bc}{15}$	**16.**	$\frac{\pi^2}{4}abc$
17.	πabc	**18.**	6π				

TYPE IV : WHEN THE REGION OF INTEGRATION IS BOUNDED BY A CONE OR A CYLINDER OR A PARABOLOID.

Evaluate the following integrals.

1. $\iiint \sqrt{x^2+y^2}\ dx\,dy\,dz$ over the volume bounded by the right circular cone $x^2 + y^2 = z^2, z > 0$ and the planes $z = 0$ and $z = 1$.

2. $\iiint z(x^2+y^2+z^2)dx\,dy\,dz$ through the volume of the cylinder $x^2+y^2 = a^2$ intercepted by the plane $z = 0$ and $z = h$.

3. $\iiint z(x^2+y^2)dx\,dy\,dz$ over the volume of the cylinder $x^2+y^2=1$ intercepted by the planes $z = 2$ and $z = 3$

4. $\iiint_v (x^2+y^2)dx\,dy\,dz$ where v is the volume bounded by $x^2+y^2=2z \; and \; z=2$

5. $\iiint (x^2+y^2)dx\,dy\,dz$ throughout the volume bounded by the surface of the paraboloid $x^2+y^2=9-z$ and the plane $z=0$

6. $\iiint z^2\,dx\,dy\,dz$ over the volume bounded by the cylinder $x^2+y^2=a^2$ and the paraboloid $x^2+y^2=z$ and the plane $z = 0$

7. $\iiint z^2\,dx\,dy\,dz$ over the volume common to the sphere $x^2+y^2+z^2=a^2$ and the cylinder $x^2+y^2+z^2=ax.$

8. Evaluate $\iiint_V (x^2+y^2)\,dV$ where V is the solid bounded by the surface $x^2+y^2=z^2$ and the planes $z=0, z=2$

ANSWERS

1.	$\frac{\pi}{6}$	**2.**	$\frac{\pi}{4}a^2h^2(a^2+h^2)$	**3.**	$\frac{5\pi}{4}$	**4.**	$\frac{16\,\pi}{3}$
5.	$\frac{243\pi}{2}$	**6.**	$\frac{\pi a^8}{12}$	**7.**	$\frac{2a^5\pi}{15}$	**8.**	$\frac{16\pi}{5}$