9.14 Solved Problems Based on Motion Diagram (Graphical Solution)

Note : Refer theory from article 9.4 and 9.4.1

Problem 95

Figure 9.81(a) shows a diagram of acceleration versus time for a particle moving along *X*-axis for a time interval of 0 to 40 seconds. For the same time interval plot **(a)** the velocity time diagram **(b)** the displacement time diagram and hence find the maximum speed attained and maximum distance covered by the particle during the interval.

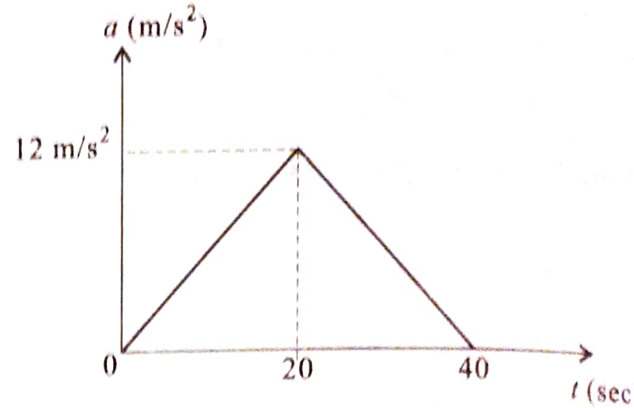

Fig. 9.81(a)

Solution : **(a) Velocity Time Diagram**

Change in velocity = Area under a-t diagram

(i) At $t = 20$ sec

$$v_{20} - v_0 = \frac{1}{2} \times 20 \times 12$$

$$v_{20} = 120 \text{ m/s} \qquad (\because v_0 = 0)$$

(ii) At $t = 40$ sec

$$v_{40} - v_{20} = \frac{1}{2} \times 20 \times 12$$

$$v_{40} = 120 + 120$$

$$v_{40} = 240 \text{ m/s}$$

(b) Displacement Time Diagram

Method I : Finding Displacement

Change in displacement = Area under v-t diagram

(i) At $t = 20$ s

$$s_{20} - s_0 = \frac{1}{3} \times 20 \times 120$$

$$s_{20} = 800 \text{ m} \qquad (\because s_0 = 0)$$

(ii) At $t = 40$ s

$$s_{40} - s_{20} = 20 \times 120 + \frac{2}{3} \times 20 \times 120$$

$$s_{40} = 800 + 20 \times 120 + \frac{2}{3} \times 20 \times 120$$

$$s_{40} = 800 + 2400 + 1600$$

$$s_{40} = 4800 \text{ m}$$

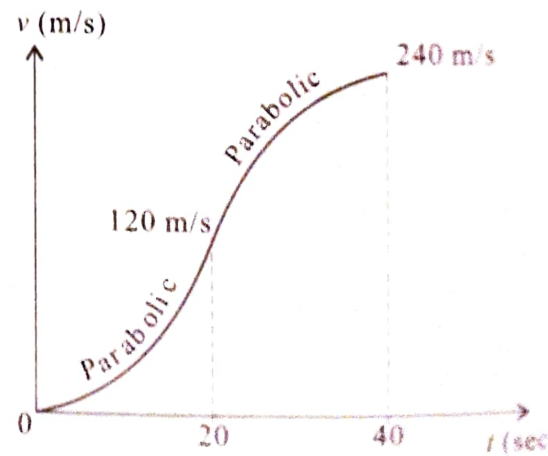

Fig. 9.81(b)

Method II : Finding Displacement Area Moment

Change in displacement = $v_0 \times t$ + Moment of area under a-t diagram

(i) At $t = 20$ sec

$$s_{20} - s_0 = v_0 \times t + \frac{1}{2} \times 20 \times 12 \times \frac{1}{3} \times 20$$

$$s_{20} = 800 \text{ m} \qquad (\because s_0 = 0,\ v_0 = 0)$$

(ii) At $t = 40$ sec

$$s_{40} - s_{20} = v_{20} \times t + \frac{1}{2} \times 20 \times 12 \times \frac{2}{3} \times 20$$

$$s_{40} = 800 + 120 \times 20 + 1600$$

$$s_{40} = 800 + 2400 + 1600$$

$s_{40} = 4800$ m ***Ans.***

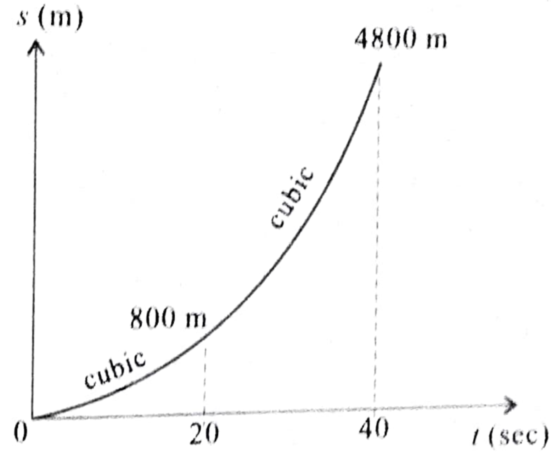

Fig. 9.81(c)

Problem 96

The a-t curve for a particle performing rectilinear motion is shown. At $t = 0$ the particles velocity is 6 m/s and the particle is located at 30 m to the lift of the origin. Determine **(a)** the velocity and **(b)** position value at $t = 9$, 18 and 27 seconds.

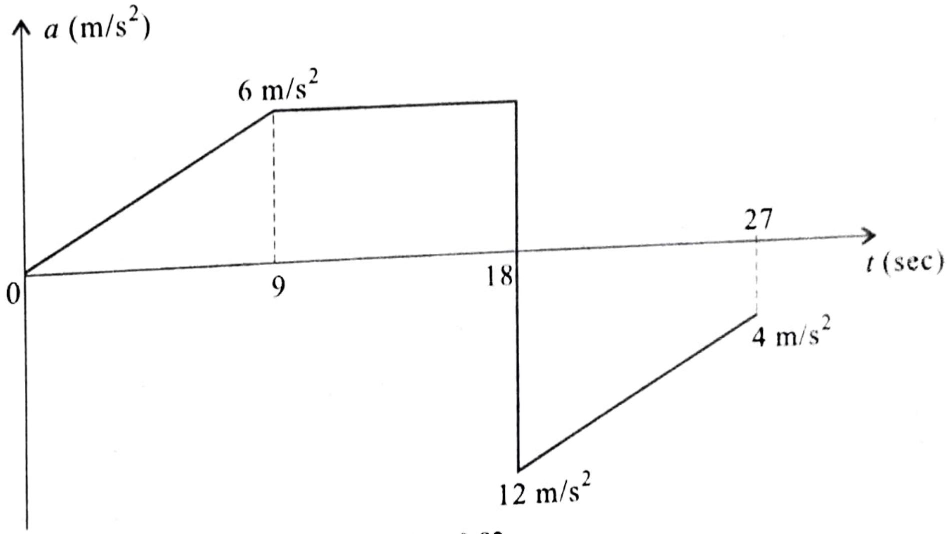

Fig. 9.82

Solution : (a) Velocities at $t = 9$, 18 and 27 seconds

Change in velocity = Area under a-t diagram

(i) At $t = 9$ sec

$$v_9 - v_0 = \frac{1}{2} \times 9 \times 6$$

$$v_9 = 6 + \frac{1}{2} \times 9 \times 6 \qquad (\because v_0 = 6 \text{ m/s})$$

$$v_9 = 33 \text{ m/s}$$

(ii) At $t = 18$ sec

$$v_{18} - v_9 = 9 \times 6$$

$$v_{18} = 33 + 9 \times 6$$

$$v_{18} = 87 \text{ m/s}$$

(iii) At $t = 27$ sec

$$v_{27} - v_{18} = \frac{1}{2}(-12 - 4)9$$
$$v_{27} = 87 - 72 = -15 \text{ m/s} \quad \textbf{Ans.}$$

(b) Position at at $t = 9$, 18 and 27 seconds

Method : Area Moment

Change in displacement = $v_0 \times t$ + Moment of area under a-t diagram

(i) At $t = 9$ sec

$$s_9 - s_0 = v_0 \times t + \frac{1}{2} \times 9 \times 6 \times \frac{1}{3} \times 9$$
$$s_9 = 30 + 6 \times 9 + 81 = 165 \text{ m}$$

(ii) At $t = 18$ sec

$$s_{18} - s_9 = v_9 \times t + 9 \times 6 \times \frac{9}{2}$$
$$s_{18} = 165 + 33 \times 9 + 243 = 705 \text{ m}$$

(iii) At $t = 27$ sec

$$s_{27} - s_{18} = v_{18} \times t - \text{Moment of area of } \Delta - \text{Moment of area of rectangle}$$
$$s_{27} = 705 + 87 \times 9 - \frac{1}{2} \times 9 \times 8 \times 6 - 9 \times 4 \times 4.5 = 1110 \text{ m} \quad \textbf{Ans.}$$

Problem 97

Figure 9.83(a) shows acceleration versus time diagram for a particle moving along x-axis. Draw velocity-time diagram and displacement-time diagram. Find the speed and distance covered by the particle after 50 second. Also find the maximum speed and the time at which the speed is attained by the particle.

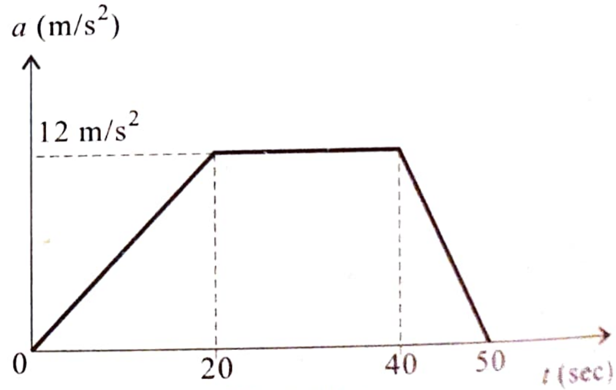

Fig. 9.83(a)

Solution : (a) Velocity Time Diagram

Change in velocity = Area under a-t diagram

(i) Velocity at 20 seconds

$$v_{20} - v_0 = \frac{1}{2} \times 20 \times 12 \quad (\because v_0 = 0)$$
$$v_{20} = 120 \text{ m/s}$$

(ii) Velocity at 40 seconds

$$v_{40} - v_{20} = 20 \times 12$$
$$v_{40} = 120 + 20 \times 12 = 360 \text{ m/s}$$

(iii) Velocity at 50 seconds

$$v_{50} - v_{40} = \frac{1}{2} \times 10 \times 12$$
$$v_{50} = 360 + 60 = 420 \text{ m/s} \quad \textbf{Ans.}$$

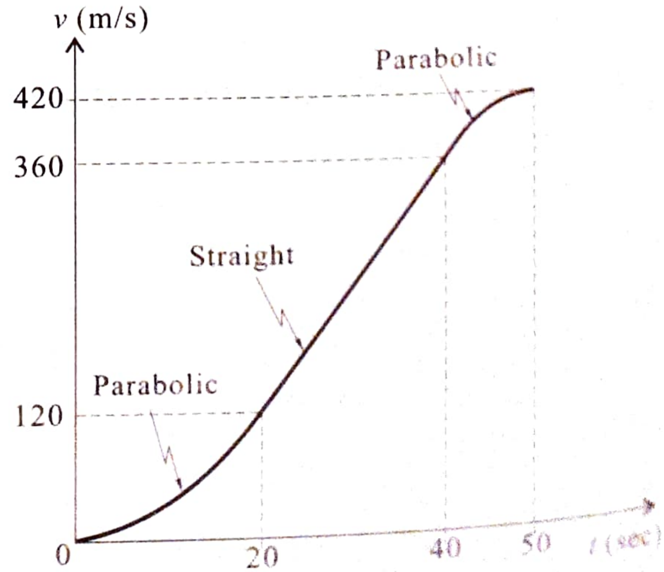

Fig. 9.83(b)

(b) Displacement Time Diagram

Method I : Finding Displacement

Change in displacement = Area under v-t diagram

(i) At t = 20 sec

$$s_{20} - s_0 = \frac{1}{3} \times b \times h \quad (\because s_0 = 0)$$

$$s_{20} = \frac{1}{3} \times 20 \times 120$$

$$s_{20} = 800 \text{ m}$$

(ii) At t = 40 sec

$$s_{40} - s_{20} = \frac{1}{2}(120 + 360) \times 20$$

$$s_{40} = 800 + \frac{1}{2} \times 480 \times 20$$

$$s_{40} = 5600 \text{ m}$$

(iii) At t = 50 sec

$$s_{50} - s_{40} = (10 \times 360) + \left(\frac{2}{3} \times 10 \times 60\right)$$

$$s_{50} = 5600 + 3600 + 400$$

$$s_{50} = 9600 \text{ m}$$

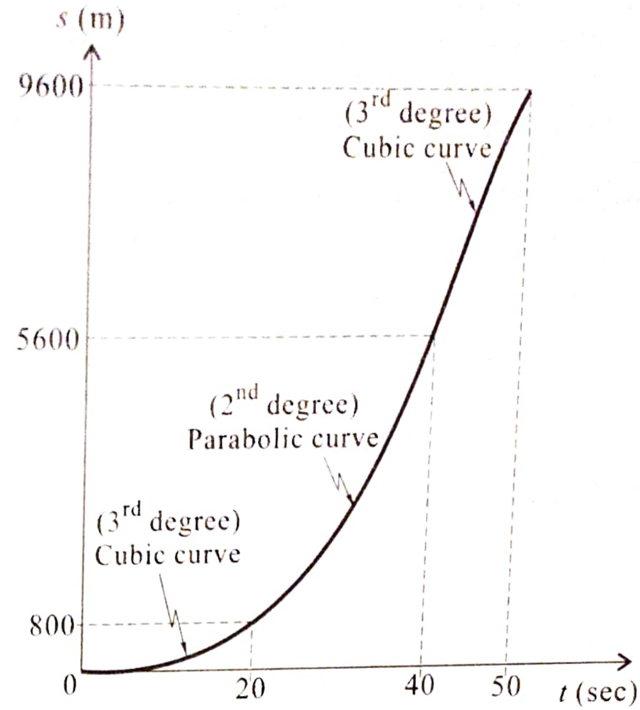

Fig. 9.83(c)

Ans. Maximum speed v_{50} = 420 m/s and maximum distance s_{50} = 9600 m

Method II : Finding Displacement - Area Moment

Change in displacement = $v_0 \times t$ + Moment of area under a-t diagram

(i) At t = 20 sec

$s_{20} - s_0 = v_0 \times t$ + Moment of area under a-t diagram between 0 to 20 about t = 20 seconds.

$$s_{20} - 0 = 0 \times 20 + \frac{1}{2} \times 20 \times 12 \times \frac{20}{3}$$

$$s_{20} = 800 \text{ m}$$

(ii) At t = 40 sec

$s_{40} - s_{20} = v_{20} \times 20$ + Moment of area under a-t diagram between 20 to 40 about t = 40 seconds.

$$s_{40} = 800 + 120 \times 20 + 20 \times 12 \times 10$$

$$s_{40} = 800 + 2400 + 2400 = 5600 \text{ m}$$

(iii) At t = 50 sec

$s_{50} - s_{40} = v_{40} \times 10$ + Moment of area under a-t diagram between 40 to 50 about t = 50 seconds.

$$s_{50} = 5600 + 360 \times 10 + \frac{1}{2} \times 10 \times 12 \times \frac{2}{3} \times 10$$

$$s_{50} = 5600 + 3600 + 400 = 9600 \text{ m}$$ ***Ans.***

Problem 98

The acceleration-time diagram for the linear motion in *figure 9.84(a)* is shown. Construct velocity time and displacement time diagrams for the motion assuming that the motion starts with initial velocity of 5 m/s from the starting point.

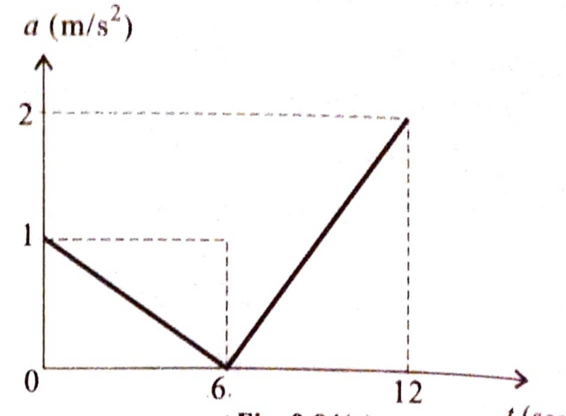

Fig. 9.84(a)

Solution : (a) Velocity Time Diagram

Change in velocity = Area under *a-t* diagram

(i) At t = 6 sec

$$v_6 - v_0 = \frac{1}{2} \times 6 \times 1 \quad (\because v_0 = 5 \text{ m/s})$$

$$v_6 = 5 + 3 = 8 \text{ m/s}$$

(ii) At t = 12 sec

$$v_{12} - v_6 = \frac{1}{2} \times 6 \times 2$$

$$v_{12} = 8 + \frac{1}{2} \times 6 \times 2 = 8 + 6$$

$$v_{12} = 14 \text{ m/s}$$

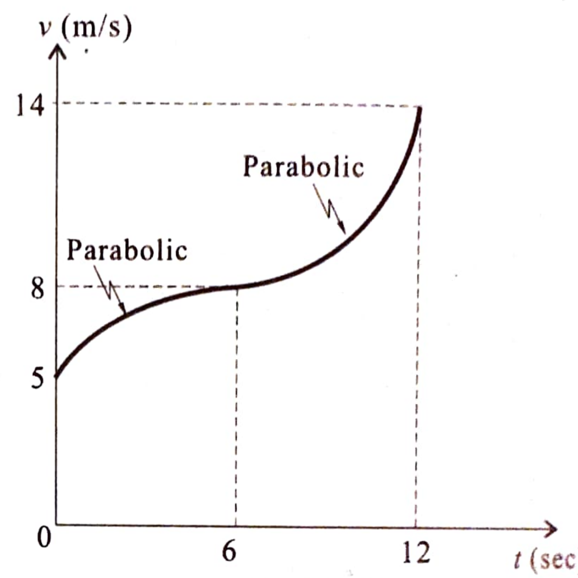

Fig. 9.84(b)

(b) Displacement Time Diagram

Method I : Finding Displacement

Change in displacement = Area under *v-t* diagram

(i) At t = 6 sec

$$s_6 - s_0 = 6 \times 5 + \frac{2}{3} \times 6 \times 3 \quad (\because s_0 = 0)$$

$$s_6 = 30 + 12 = 42 \text{ m}$$

(ii) At t = 12 sec

$$s_{12} - s_6 = 6 \times 8 + \frac{1}{3} \times 6 \times 6$$

$$s_{12} = 42 + 48 + 12 = 102 \text{ m}$$

Method II : Finding Displacement - Area Moment

Change in displacement = $v_0 \times t$ + Moment of area under *a-t* diagram

(i) At t = 6 sec

$s_6 - s_0 = v_0 \times t$ + Moment of area under *a-t* diagram between 0 to 6 about t = 6 seconds.

$$s_6 = 5 \times 6 + \frac{1}{2} \times 6 \times 1 \times \frac{2}{3} \times 6 \qquad (\because s_0 = 0, \; v_0 = 5 \text{ m/s})$$

$$s_6 = 30 + 12 = 42 \text{ m}$$

(ii) At t = 12 sec

$s_{12} - s_6 = v_6 \times t$ + Moment of area under *a-t* diagram between 6 to 12 about t = 12 seconds.

$$s_{12} = 42 + 8 \times 6 + \frac{1}{2} \times 6 \times 2 \times \frac{1}{3} \times 6$$

$$s_{12} = 42 + 48 + 12 = 102 \text{ m} \quad \textbf{Ans.}$$

Problem 99

The acceleration-time diagram for the linear motion is shown in *figure 9.85(a)*. Construct velocity time and displacement time diagrams for the motion assuming that the motion starts from rest.

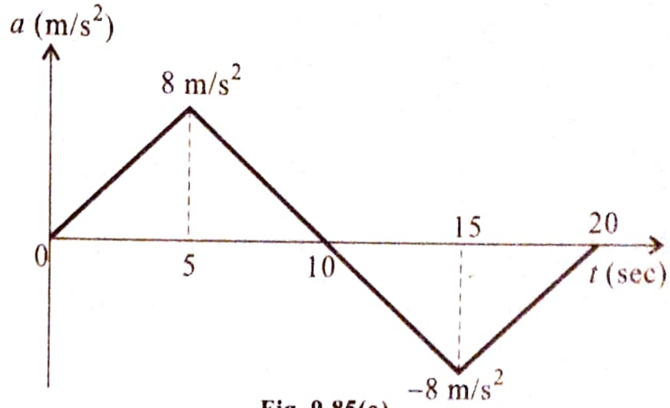

Fig. 9.85(a)

Solution : (a) Velocity Time Diagram

Change in velocity = Area under a-t diagram

(i) At t = 5 sec

$$v_5 - v_0 = \frac{1}{2} \times 5 \times 8 \quad (\because v_0 = 0)$$

$$v_5 = 20 \text{ m/s}$$

(ii) At t = 10 sec

$$v_{10} - v_5 = \frac{1}{2} \times 5 \times 8$$

$$v_{10} = 20 + 20 = 40 \text{ m/s}$$

(iii) At t = 15 sec

$$v_{15} - v_{10} = \frac{1}{2} \times 5 \times (-8)$$

$$v_{15} = 40 - 20 = 20 \text{ m/s}$$

(iv) At t = 20 sec

$$v_{20} - v_{15} = \frac{1}{2} \times 5 \times (-8)$$

$$v_{15} = 20 - 20$$

$$v_{15} = 0 \text{ m/s}$$

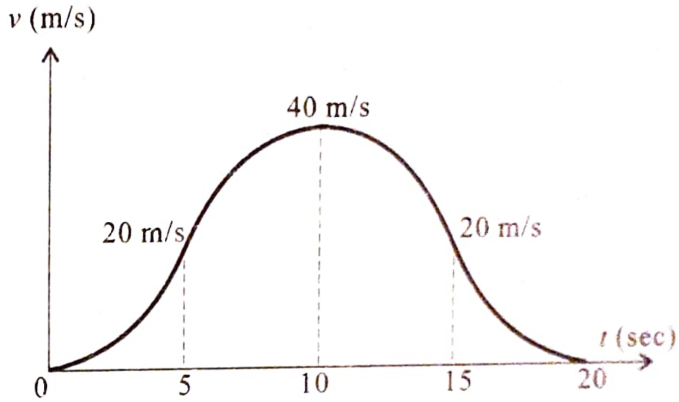

Fig. 9.85(b)

(b) Displacement Time Diagram

Method I : Finding Displacement

Change in displacement = Area under v-t diagram

(i) At t = 5 sec

$$s_5 - s_0 = \frac{1}{3} \times 5 \times 20$$

$$s_5 = 33.33 \text{ m}$$

(ii) At t = 10 sec

$$s_{10} - s_5 = 5 \times 20 + \frac{2}{3} \times 5 \times 20$$

$$s_{10} = 33.33 + 100 + 66.67$$

$$s_{10} = 200 \text{ m}$$

(iii) At t = 15 sec

$$s_{15} - s_{10} = 5 \times 20 + \frac{2}{3} \times 5 \times 20$$

$$s_{15} = 200 + 100 + 66.67$$

$$s_{15} = 366.67 \text{ m}$$

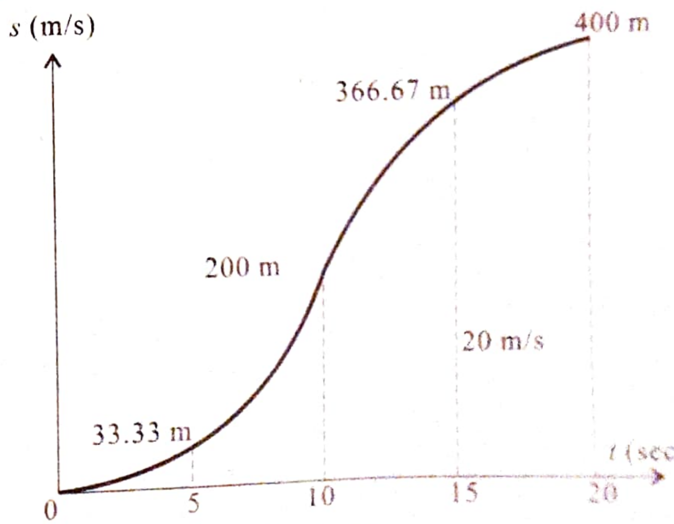

Fig. 9.85(c)

(iv) At $t = 20$ sec

$$s_{20} - s_{15} = \frac{1}{2} \times 5 \times 20$$

$$s_{20} = 366.67 + 33.33 = 400 \text{ m}$$

Method II : Finding Displacement - Area Moment

Change in displacement $= v_0 \times t +$ Moment of area under a-t diagram

(i) At $t = 5$ sec

$$s_5 - s_0 = v_0 \times t + \text{Moment of area under } a\text{-}t \text{ diagram between 0 to 5 about } t = 5 \text{ seconds.}$$

$$s_5 = \frac{1}{2} \times 5 \times 8 \times \frac{1}{3} \times 5$$

$$s_5 = 33.33 \text{ m}$$

(ii) At $t = 10$ sec

$$s_{10} - s_5 = v_5 \times t + \frac{1}{2} \times 5 \times 8 \times \frac{2}{3} \times 5$$

$$s_{10} = 33.33 + 20 \times 5 + 66.67$$

$$s_{10} = 200 \text{ m}$$

(iii) At $t = 15$ sec

$$s_{15} - s_{10} = v_{10} \times t + \frac{1}{2} \times 5 \times (-8) \times \frac{1}{3} \times 5$$

$$s_{15} = 200 + 40 \times 5 - 33.33$$

$$s_{15} = 366.67 \text{ m}$$

(iv) At $t = 20$ sec

$$s_{20} - s_{15} = v_{15} \times t + \frac{1}{2} \times 5 \times (-8) \times \frac{2}{3} \times 5$$

$$s_{20} = 366.67 + 20 \times 5 - 66.67$$

$$s_{20} = 400 \text{ m}$$

Method III : Finding Displacement - Area Moment Method when Initial Velocity is Zero

(i) At $t = 5$ sec

$$s_5 = \frac{1}{2} \times 5 \times 8 \times \frac{1}{3} \times 5$$

$$s_5 = 33.33 \text{ m}$$

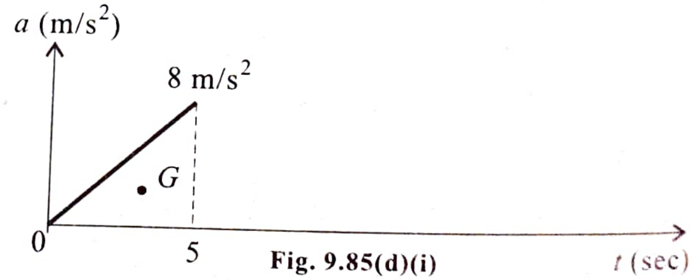

Fig. 9.85(d)(i)

(ii) At $t = 10$ sec

$$s_{10} = \frac{1}{2} \times 10 \times 8 \times 5$$

$$s_{10} = 200 \text{ m}$$

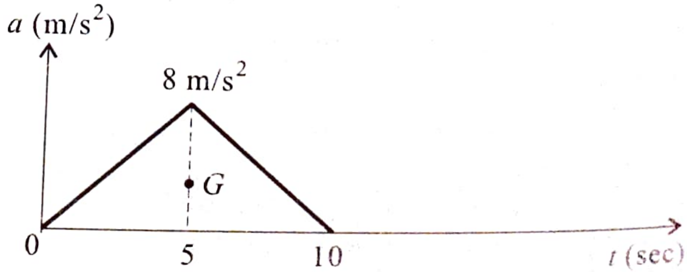

Fig. 9.85(d)(ii)

(iii) At $t = 15$ sec

$$s_{15} = \frac{1}{2} \times 10 \times 8 \times 10 + \frac{1}{2} \times 5 \times (-8) \times \frac{1}{3} \times 5$$

$$s_{15} = 400 - 33.33$$

$$s_{15} = 366.67 \text{ m}$$

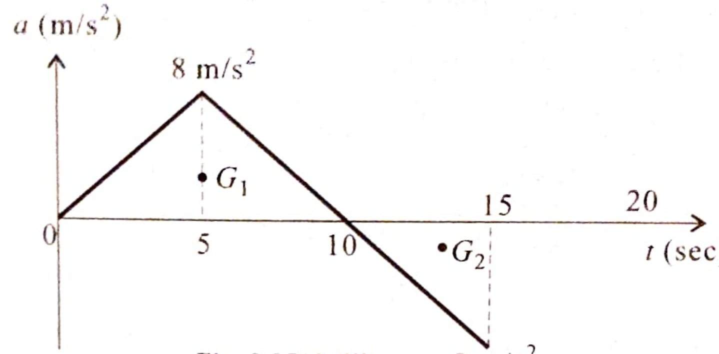

Fig. 9.85(d)(iii)

(iv) At $t = 20$ sec

$$s_{20} = \frac{1}{2} \times 10 \times 8 \times 15 + \frac{1}{2} \times 10 \times (-8) \times 5$$

$$s_{20} = 400 \text{ m}$$

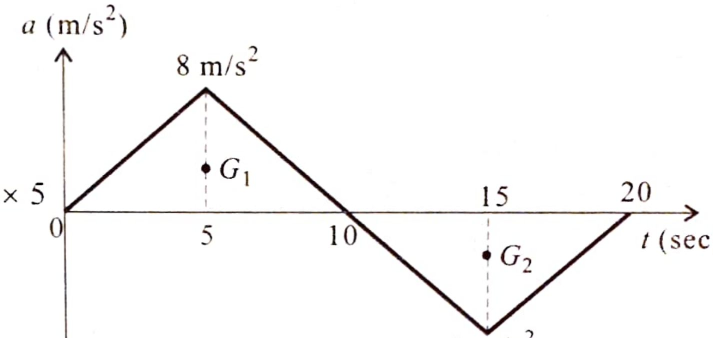

Fig. 9.85(d)(iv)

Problem 100

The motion of jet plane while traveling along a runway is defined by the v-t graph shown. Construct the s-t and a-t graph for the motion. The plane starts from rest.

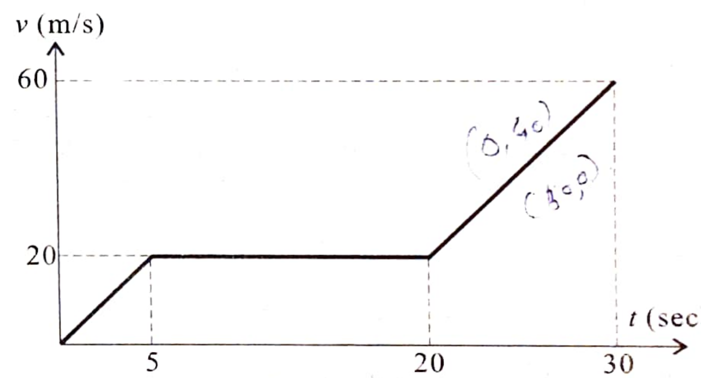

Fig. 9.86(a)

Solution : (a) Acceleration Time Diagram

Slope of v-t diagram = Acceleration

(i) At $t = 0$

$$\text{Slope} = a = \frac{20}{5} = 4 \text{ m/s}^2$$

(ii) Just before $t = 5$ sec

$$\text{Slope} = a = \frac{20}{5} = 4 \text{ m/s}^2$$

Just after $t = 5$ sec

$$\text{Slope} = a = 0 \text{ (Horizontal line)}$$

(iii) Just before $t = 20$ sec

$$\text{Slope} = a = 0$$

Just after $t = 20$ sec

$$\text{Slope} = a = \frac{40}{10} = 4 \text{ m/s}^2$$

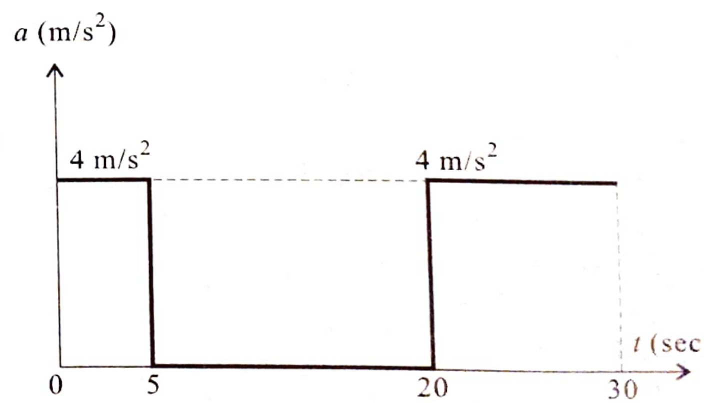

Fig. 9.86(b)

(iv) At $t = 30$ sec

$$\text{Slope} = a = \frac{40}{10} = 4 \text{ m/s}^2$$

(b) Displacement Time Diagram

Change in displacement = Area under v-t diagram

(i) At $t = 0$, $s = 0$

(ii) At $t = 5$ sec

$$s_5 - s_0 = \frac{1}{2} \times 5 \times 20$$
$$s_5 = 50 \text{ m}$$

(iii) At $t = 20$ sec

$$s_{20} - s_5 = 15 \times 20$$
$$s_{20} = 50 + 300$$
$$s_{20} = 350 \text{ m}$$

(iv) At $t = 30$ sec

$$s_{30} - s_{20} = \frac{1}{2} \times (20 + 60) \times 10$$
$$s_{30} = 350 + 400$$
$$s_{30} = 750 \text{ m}$$

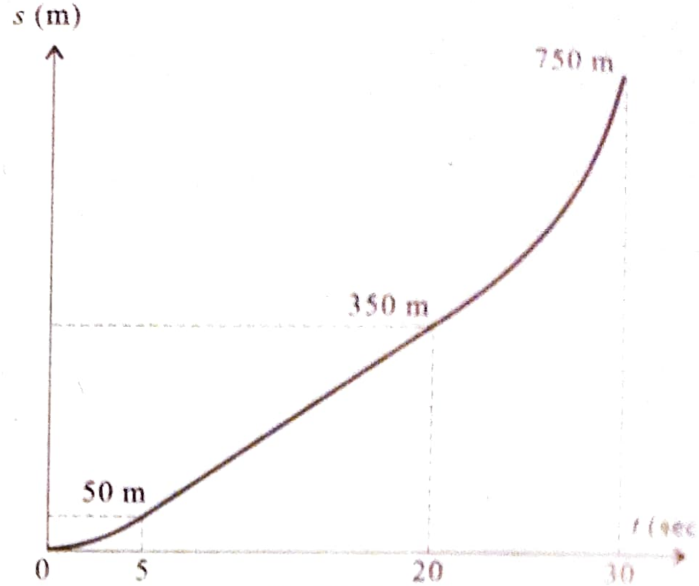

Fig. 9.86(c)

Problem 101

A particle moving with a velocity of 7.5 m/s is subjected to a retarding force which gives it a negative acceleration varying with time as shown in *figure 9.87(a)*. For the first 3 seconds, after 3 seconds the acceleration remaining constant. Plot the v-t diagram for 6 seconds of the travel of particle. Determine the distance travelled by the particle from its position $t = 0$ to $t = 6$ seconds.

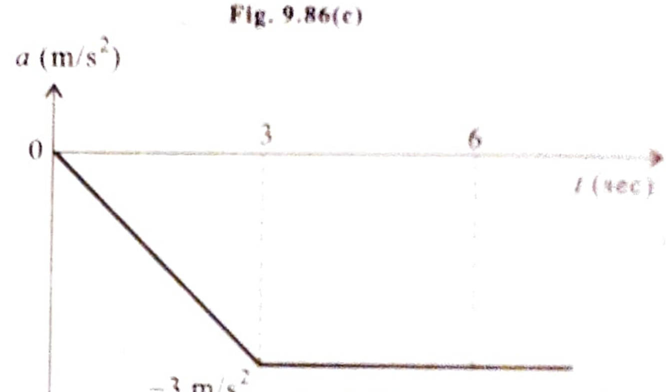

Fig. 9.87(a)

Solution : (a) Velocity Time Diagram

Change in velocity = Area under a-t diagram

(i) At t = 3 sec

$$v_3 - v_0 = \frac{1}{2} \times 3 \times (-3)$$
$$v_3 = 7.5 - 4.5 \quad (\because v_0 = 7.5 \text{ m/s})$$
$$v_3 = 3 \text{ m/s}$$

(ii) At t = 6 sec

$$v_6 - v_3 = 3 \times (-3)$$
$$v_6 = 3 - 9 = -6 \text{ m/s}$$

By property of similar Δ, we have

$$\frac{3}{6} = \frac{d}{3 - d}$$
$$\therefore \; 3(3 - d) = 6d$$
$$9 - 3d = 6d$$
$$9 = 9d \qquad \therefore \; d = 1 \quad \textbf{Ans.}$$

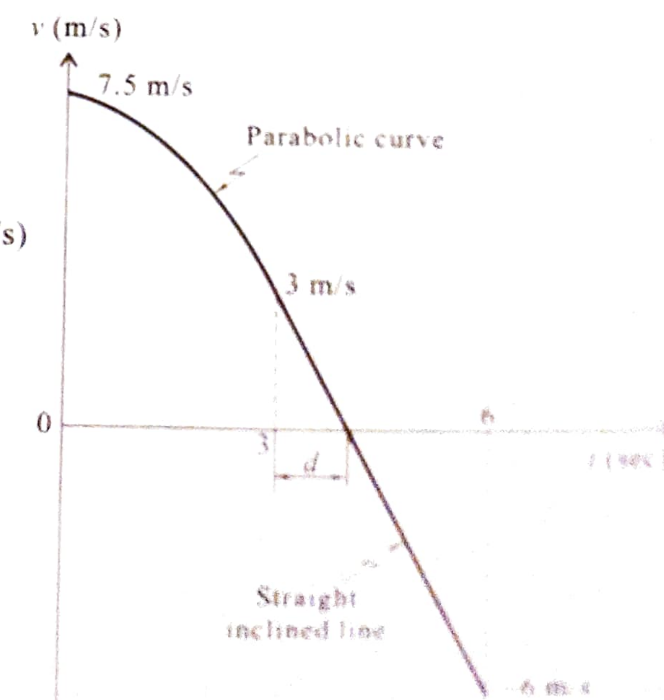

Fig. 9.87(b)

(b) Displacement Time Diagram

Change in displacement = Area under v-t diagram

(i) At $t = 3$ sec

$$s_3 - s_0 = 3 \times 3 + \frac{2}{3} \times 3 \times 4.5$$

$$s_3 = 9 + 9 = 18 \text{ m}$$

(ii) At $t = 4$ sec (Straight inclined line intersecting t axis $\because d = 1$)

$$s_4 - s_3 = \frac{1}{2} \times 1 \times 3$$

$$s_4 = 18 + 1.5$$

$$s_4 = 19.5 \text{ m}$$

(iii) At $t = 6$ sec

$$s_6 - s_4 = \frac{1}{2} \times 2 \times (-6)$$

$$s_6 = 19.5 - 6$$

$$s_6 = 13.5 \text{ m}$$

Distance travelled by particle form $t = 0$ to $t = 6$ sec

$$D = 19.5 + \frac{1}{2} \times 2 \times 6$$

$$D = 19.5 + 6$$

$$D = 25.5 \text{ m}$$

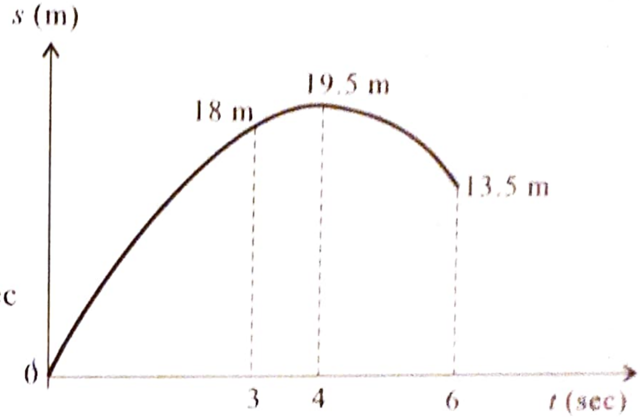

Fig. 9.87(c)

Problem 102

A particle is moving along a straight path variation of its position with respect to time is shown in s-t graph. Draw v-t and a-t curve.

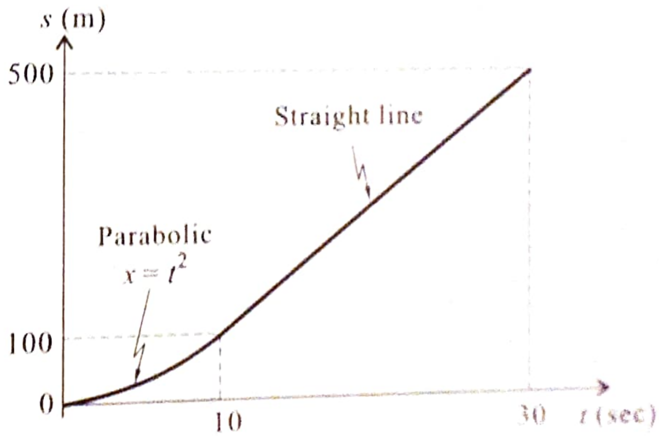

Fig. 9.88(a)

Solution

(i) $t = 0$ to $t = 10$ sec

$$x = t^2$$

Differentiating w.r.t. t

$$v = 2t \,; \quad v_0 = 0 \,, \; v_{10} = 20 \text{ m/s}$$

Differentiating w.r.t. t

$$a = 2 \text{ m/s}^2$$

(ii) $t = 10$ sec to $t = 30$ sec

Straight line equation

$$y = m\,x + c$$

$$s = m\,t + c$$

$$\text{Slope } m = \frac{400}{20} = 20$$

We know at $t = 10$ sec, $s = 100$ m

$$\therefore 100 = 20 \times 10 + c$$

$$\therefore c = -100$$

From straight line equation, we get

$$s = 20t - 100$$

Differentiating w.r.t. t

$$v = 20 \text{ m/s}$$

Again, differentiating w.r.t. t

$$a = 0 \text{ m/s}^2$$

(iii) *v-t* Graph

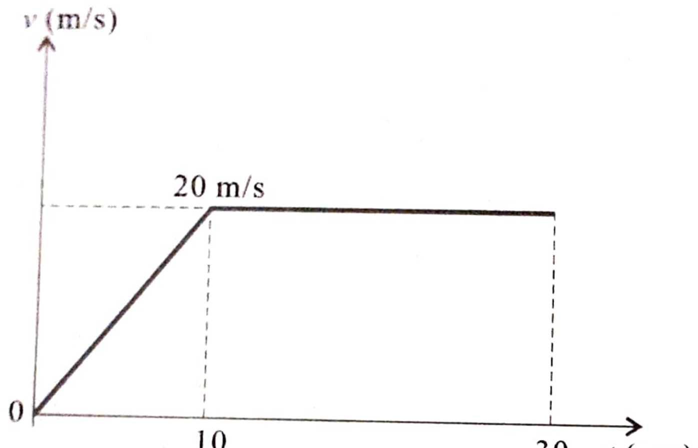

Fig. 9.88(b)

(iv) *a-t* Graph

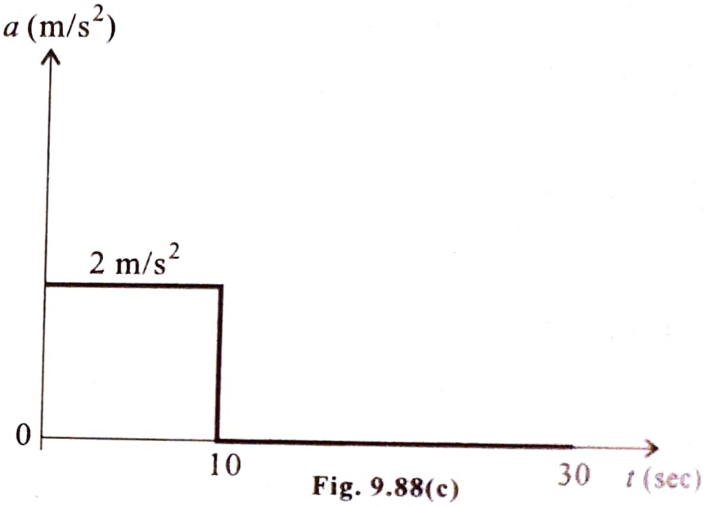

Fig. 9.88(c)

Problem 103

A particle is moving along a straight path. With an acceleration shown in the *a-t* graph. Draw *v-t* and *s-t* graph. Find maximum velocity in the time interval, distance traveled by the particle in the time interval.

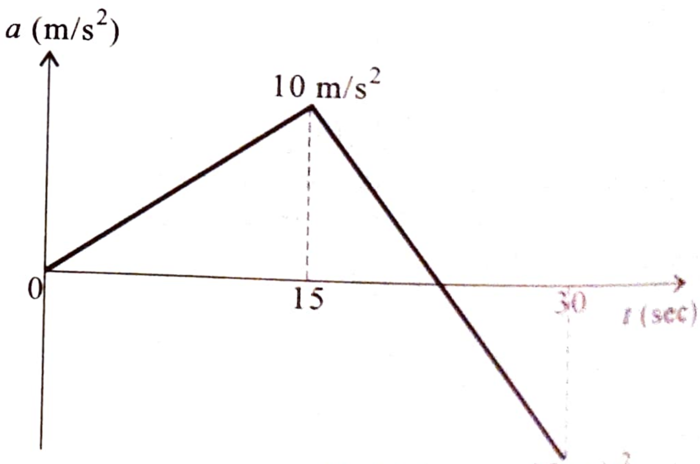

Fig. 9.89(a)

Solution : (a) Velocity Time Diagram

Change in velocity = Area under *a-t* diagram

(i) At t = 15 sec

$$v_{15} - v_0 = \frac{1}{2} \times 15 \times 10$$

$$v_{15} = 75 \text{ m/s} \quad \textbf{Ans.} \quad (\because v_0 = 0)$$

(ii) Intersecting of inclined line with t axis will be the mid point between 15 to 30 by property of triangle.

$\therefore$ t = 15 + 7.5 = 22.5 sec at t axis intercept.

At t = 22.5 sec

$$v_{22.5} - v_{15} = \frac{1}{2} \times 7.5 \times 10$$

$$v_{22.5} = 75 + 37.5$$

$$v_{22.5} = 112.5 \text{ m/s} \quad \textbf{Ans.}$$

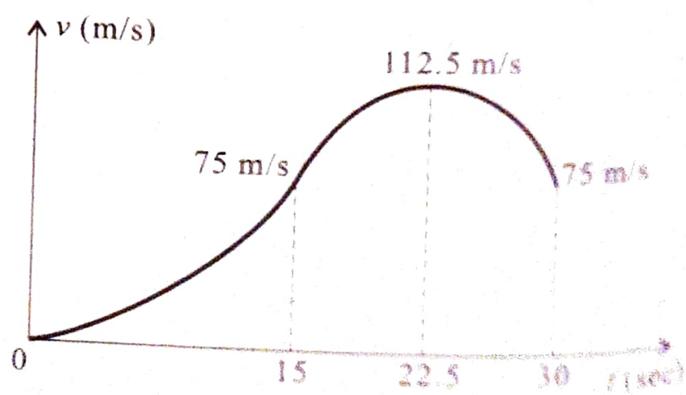

Fig. 9.89(b)

(iii) At $t = 30$ sec

$$v_{30} - v_{22.5} = \frac{1}{2} \times 7.5 \times (-10)$$

$$v_{30} = 112.5 - 37.5 = 75 \text{ m} \quad \textbf{Ans.}$$

(b) Displacement Time Diagram

Change in displacement = Area under v-t diagram

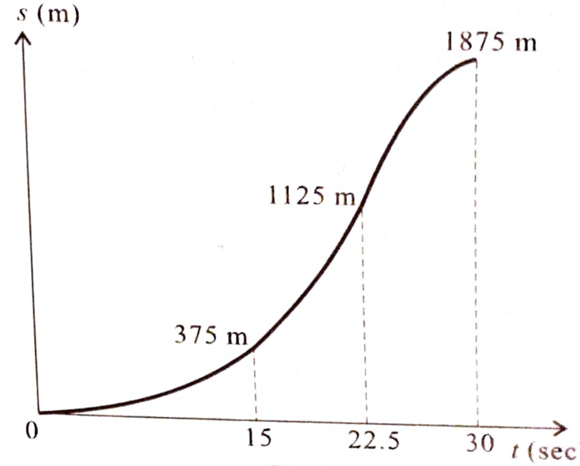

Fig. 9.89(c)

(i) At $t = 15$ sec

$$s_{15} - s_0 = \frac{1}{3} \times 15 \times 75 \quad (\because s_0 = 0)$$

$$s_{15} = 375 \text{ m} \quad \textbf{Ans.}$$

(ii) At $t = 22.5$ sec

$$s_{22.5} - s_{15} = 7.5 \times 75 + \frac{2}{3} \times 7.5 \times 37.5$$

$$s_{22.5} = 375 + 562.5 + 187.5$$

$$s_{22.5} = 1125 \text{ m} \quad \textbf{Ans.}$$

(iii) At $t = 30$ sec

$$s_{30} - s_{22.5} = 7.5 \times 75 + \frac{2}{3} \times 7.5 \times 37.5$$

$$s_{30} = 1125 + 562.5 + 187.5 = 1875 \text{ m} \quad \textbf{Ans.}$$

Ans : Maximum velocity attained by particle $v_{max} = 112.5$ m/s

Displacement = Distance travelled because velocity is throughout position.

So particle is moving in the same direction. ∴ Distance travelled $d = 1875$ m.

Problem 104

v-s graph is given in *figure 9.90(a)*. Find the velocity and acceleration at $s = 50$ m and $s = 150$ m.

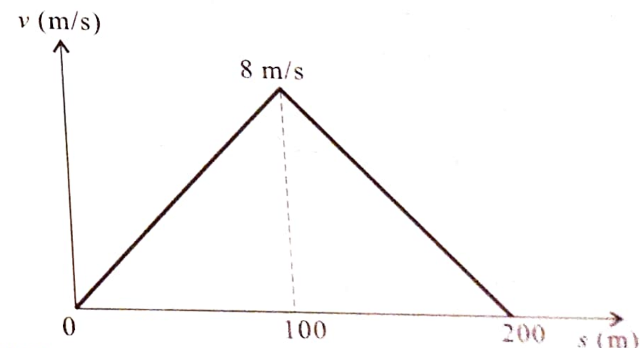

Fig. 9.90(a)

Solution

From property of similarity of triangle shown in *figure 9.90(b)*.

At $s = 50$ m and $s = 150$ m ; $v = 4$ m/s

We know $a = v\dfrac{dv}{ds}$. $\dfrac{dv}{ds}$ is the slope of v-s diagram

At $s = 50$ m ; Slope $= \dfrac{dv}{ds} = \dfrac{4}{50}$

$$a = v\frac{dv}{ds} = 4 \times \frac{4}{50}$$

$$\therefore a = 0.32 \text{ m/s}^2$$

At $s = 150$ m ; Slope $= \dfrac{dv}{ds} = \dfrac{-4}{50}$

$$a = v\frac{dv}{ds} = 4 \times \frac{-4}{50}$$

$$\therefore a = -0.32 \text{ m/s}^2 \quad \textbf{Ans.}$$

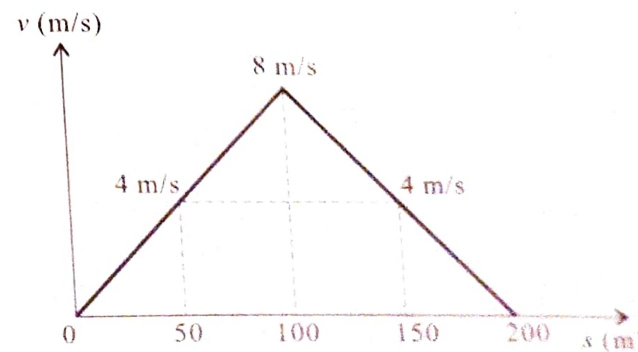

Fig. 9.90(b)

Problem 105

A particle moves in straight line with a velocity-time diagram shown in *figure 9.91(a)*. Knowing that $s = -25$ m and $t = 0$, draw s-t and a-t diagram for $0 < t < 24$.

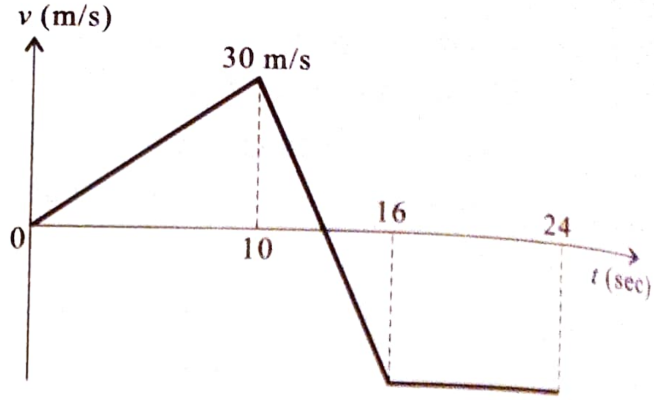

Fig. 9.91(a)

Solution : (a) Acceleration Time Diagram

Acceleration = Slope of v-t diagram

(i) $0 < t < 10$ sec;

$$\text{Slope} = a = \frac{30}{10} = 3 \text{ m/s}^2 \quad \textbf{Ans.}$$

(ii) 10 sec $< t <$ 16 sec ;

$$\text{Slope} = a = -\frac{30}{3}$$

$$= -10 \text{ m/s}^2 \quad \textbf{Ans.}$$

(By geometry, slope is intersecting as a mid point between $t = 10$ sec to $t = 16$ sec i.e. $t = 13$ sec)

(iii) From $t = 16$ sec onwards ;

$$\text{Slope} = a = 0 \quad \textbf{Ans.}$$

(b) Method for Finding Displacement

Change in displacement = Area under v-t diagram

(i) At $t = 10$ sec

$$s_{10} - s_0 = \frac{1}{2} \times 10 \times 30 \quad (\because s_0 = -25 \text{ m})$$

$$s_{10} = -25 + \frac{1}{2} \times 10 \times 30$$

$$s_{10} = 125 \text{ m} \quad \textbf{Ans.}$$

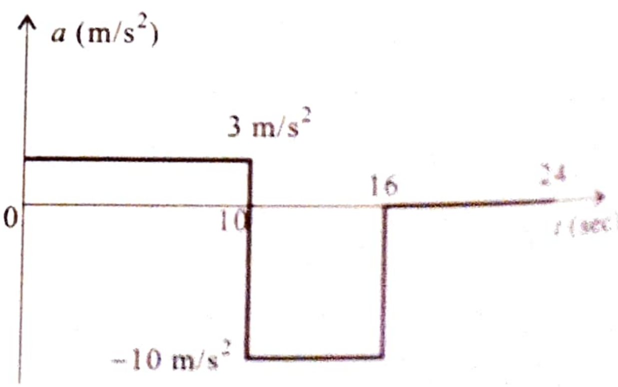

Fig. 9.91(b)

(ii) At $t = 13$ sec (t-axis intercept)

$$s_{13} - s_{10} = \frac{1}{2} \times 3 \times 30$$

$$s_{13} = 125 + 45$$

$$s_{13} = 170 \text{ m} \quad \textbf{Ans.}$$

(iii) At $t = 16$ sec

$$s_{16} - s_{13} = \frac{1}{2} \times 3 \times (-30)$$

$$s_{16} = 170 - 45$$

$$s_{16} = 125 \text{ m} \quad \textbf{Ans.}$$

(iv) Due to negative velocity particle has reversed its direction and will reach again to its origin.

Let t be the time taken by particle to reach the origin from $t = 16$ seconds.

$$s_0 - s_{16} = (-30) \times t \qquad (s_0 = 0)$$
$$0 - 125 = -30\,t$$
$$t = 4.17 \text{ sec} \quad \textbf{Ans.}$$

$\therefore$ At $t = 16 + 4.17 = 20.17$ sec the particle will again pass through origin.

(v) At $t = 24$ sec

$$s_{24} - s_{16} = 8 \times (-30)$$
$$s_{24} = 125 - 240$$
$$s_{24} = -115 \text{ m} \quad \textbf{Ans.}$$

(vi) Initially particle is having negative displacement, $s = -25$ m.

Let t be the time taken to cross origin

we know,

$$25 = \frac{1}{3} \times t \times 30$$
$$\therefore\ t = 1.67 \text{ sec} \quad \textbf{Ans.}$$

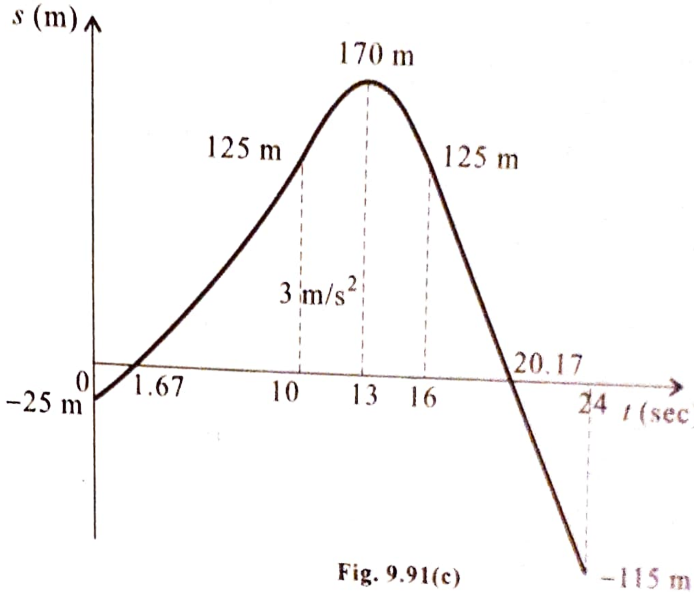

Fig. 9.91(c)

Problem 106

The motion of a particle from rest is given by the acceleration time diagram given in *figure 9.92(a)*. Sketch velocity-time diagram and hence calculate velocity at $t = 12$ sec.

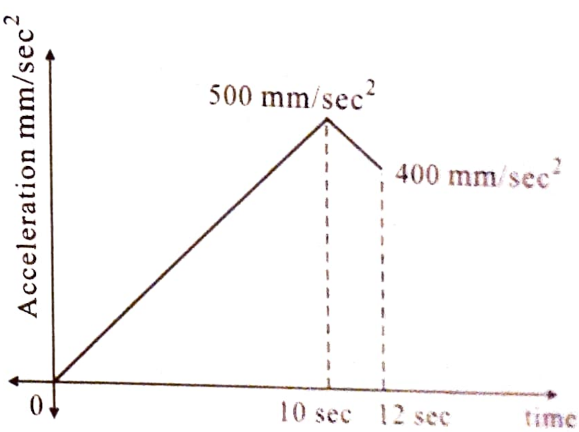

Fig. 9.92(a)

Solution

$$v_{10} - v_0 = \frac{1}{2} \times 500 \times 10$$
$$= 2500 \text{ mm/sec}$$
$$v_{12} - v_{10} = \frac{1}{2} \times 2 \times 100 + 2 \times 400$$
$$= 900 \text{ mm/sec}$$
$$\therefore\ v_{12} = 2500 + 900$$
$$v_{12} = 3400 \text{ mm/sec} \quad \textbf{Ans.}$$

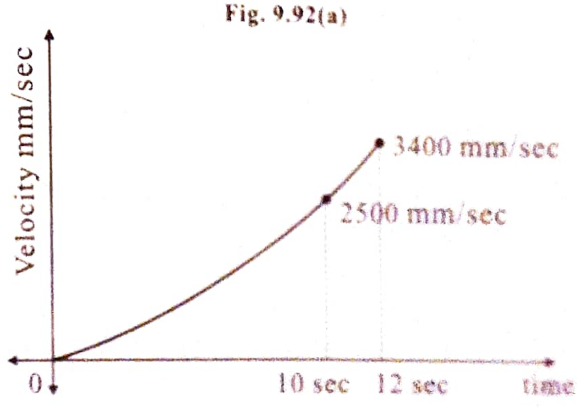

Fig. 9.92(b)