Module 4.1 Transformers (No numerical's)

4	**Electrical Machines**		**12**	**CO4**
	4.1	Single phase transformer construction and principle of working, emf equation of a transformer, losses in transformer, equivalent circuit of Ideal and practical transformer, voltage regulation and efficiency of transformer, phasor diagram at various loading condition (no numerical expected)		

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Basic Electrical Engineering

THIRD EDITION

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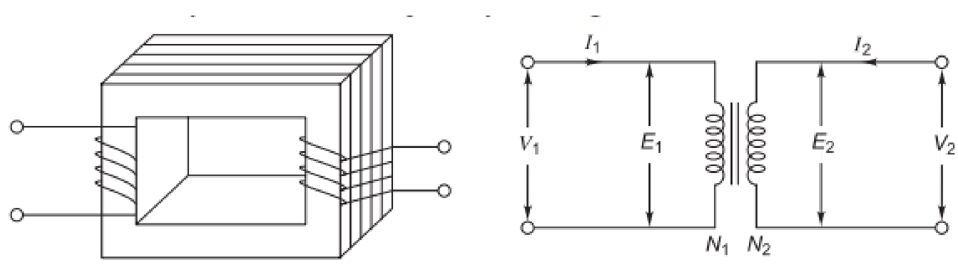
6.1 SINGLE-PHASE TRANSFORMERS

A transformer is a static device which can transfer electrical energy from one circuit to another circuit without change of frequency. It can increase or decrease the voltage but with a corresponding decrease or increase in current. It works on the principle of mutual induction. It must be used with an input voltage that varies in amplitude, i.e., an ac voltage. A major application of transformers is to increase voltage before transmitting electrical energy over long distances through wires and to reduce voltage at places where it is to be used. Transformers are also used in electronic circuits to step down the supply voltage to a level suitable for the low-voltage circuits they contain. Signal and audio transformers are used to couple stages of amplifiers and to match devices such as microphones to the input of amplifiers.

6.2 CONSTRUCTION

A transformer mainly consists of two coils or windings placed on a common core. With the increase in size (capacity) and operating voltage, it also needs other parts such as a suitable tank, bushing, conservator, breather, etc. We will discuss two basic parts—core and windings.



6.2.1 Core

The composition of a transformer core depends on voltage, current and frequency. Commonly used core materials are soft iron and steel. Generally, air-core transformers are used when the voltage source has a high frequency (above 20 kHz). Iron-core transformers are usually used when the source frequency is low (below 20 kHz). In most transformers, the core is constructed of laminated steel to provide a continuous magnetic path. The steel used for constructing the core is high-grade silicon steel where hysteresis loss is very low. Such steel is called soft steel. Due to alternating flux, certain currents are induced in the core, called as eddy currents. These currents cause considerable loss in the core, called eddy current loss. Silicon content in the steel increases its resistivity to eddy-current loss, thereby reducing eddy-current losses. To reduce eddy-current losses further, the core is laminated by a light coat of varnish or by an oxide layer on the surface. There are two main shapes of cores used in laminated steel-core transformers as shown in Fig 6.1 and Fig 6.2.

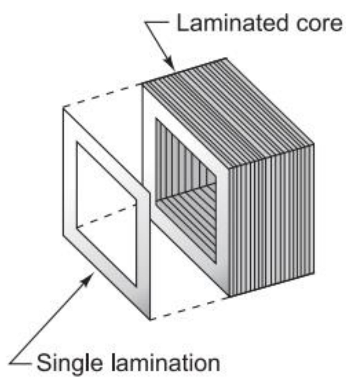


Fig. 6.1 *Hollow-core construction*

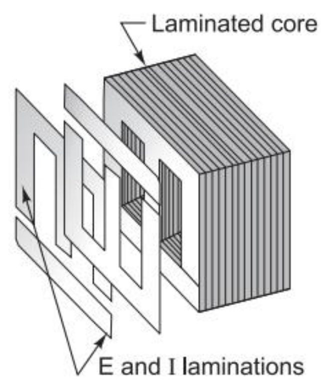


Fig. 6.2 *Shell-type core construction*

6.2.2 Transformer Windings

A transformer consists of two coils, called windings, which are wrapped around a core. The winding in which electrical energy is fed is called the *primary winding*. The winding which is connected to the load is called the *secondary winding*.

The primary and secondary windings are made up of an insulated copper conductor in the form of a round wire or strip. These windings are then placed around the limbs of the core. The windings are insulated from each other and the core, using cylinders of insulating material such as a press board or Bakelite.

For simplicity, the primary and secondary windings are shown on separate limbs of the core. If such an arrangement is used in actual practice, all the flux produced in the primary winding will not link with the secondary winding. Some of the flux will leak out through the air. Such flux is known as *leakage flux*. The more the value of leakage flux, poorer is the performance of the transformer. Hence, to reduce leakage flux, the windings are placed together on the same limb in actual transformers.

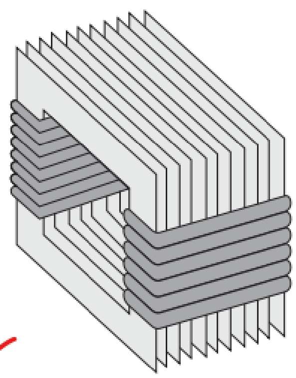


Fig. 6.3 *Core-type transformer*

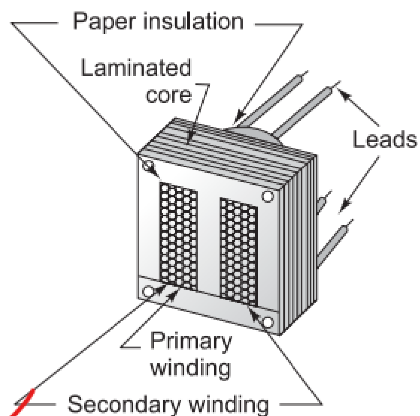
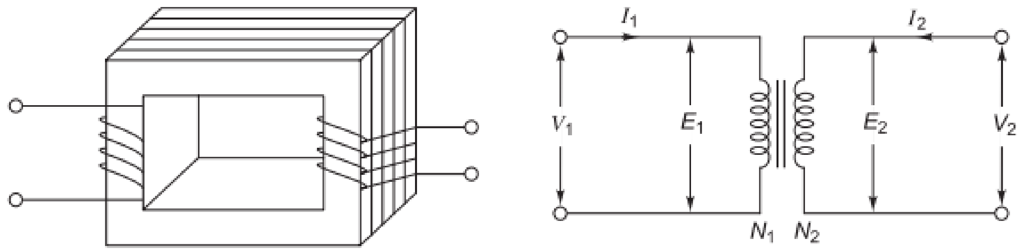


Fig. 6.4 *Shell-type transformer*

6.3 WORKING PRINCIPLE

[May 2013, Dec 2014]

When an alternating voltage V_1 is applied to a primary winding, an alternating current I_1 flows in it producing an alternating flux in the core. As per Faraday's laws of electromagnetic induction, an emf e_1 is induced in the primary winding.



(a) (b)

Fig. 6.5 *Working principle of a transformer*

$$e_1 = -N_1 \frac{d\phi}{dt}$$

where N_1 is the number of turns in the primary winding. The induced emf in the primary winding is nearly equal and opposite to the applied voltage V_1.

Assuming leakage flux to be negligible, almost the whole flux produced in primary winding links with the secondary winding. Hence, an emf e_2 is induced in the secondary winding.

$$e_2 = -N_2 \frac{d\phi}{dt}$$

where N_2 is the number of turns in the secondary winding. If the secondary circuit is closed through the load, a current I_2 flows in the secondary winding. Thus, energy is transferred from the primary winding to the secondary winding. The symbol of transformer is shown in Fig. 6.5(b). The lines between two windings represent iron core. If there is no line between two windings, then it represents air core transformer. If number of turns in the secondary winding N_2 is greater than the number of turns in the primary winding N_1, the transformer is called a step-up transformer. If N_2 is less than N_1, the transformer is called a step-down transformer. Thus, step-up transformer is used to increase the voltage at the output whereas step-down transformer is used to decrease the voltage at the output.

6.4 EMF EQUATION

[Dec 2012, 2014, May 2013, 2015]

As the primary winding is excited by a sinusoidal alternating voltage, an alternating current flows in the winding producing a sinusoidally varying flux ϕ in the core.

$$\phi = \phi_m \sin \omega t$$

As per Faraday's laws of electromagnetic induction, an emf e_1 is induced in the primary winding.

$$\begin{aligned} e_1 &= -N_1 \frac{d\phi}{dt} \\ &= -N_1 \frac{d}{dt} (\phi_m \sin \omega t) \\ &= -N_1 \phi_m \omega \cos \omega t \\ &= N_1 \phi_m \omega \sin (\omega t - 90°) \\ &= 2\pi f \phi_m N_1 \sin (\omega t - 90°) \end{aligned}$$

Maximum value of induced emf $= 2\pi f \phi_m N_1$

Hence, rms value of induced emf in primary winding is given by

$$E_1 = \frac{E_{max}}{\sqrt{2}} = \frac{2\pi f \phi_m N_1}{\sqrt{2}} = 4.44 f \phi_m N_1$$

Similarly, rms value of induced emf in the secondary winding is given by

$$E_2 = 4.44 f \phi_m N_2$$

Also,
$$\frac{E_1}{N_1} = \frac{E_2}{N_2} = 4.44 f \phi_m$$

Thus, emf per turn is same in primary and secondary windings and an equal emf is induced in each turn of the primary and secondary windings.

6.5 TRANSFORMATION RATIO (K)

We know that

$$E_1 = 4.44\, f\, \phi_m\, N_1$$
$$E_2 = 4.44\, f\, \phi_m\, N_2$$
$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = K$$

where K is called the *transformation ratio*.

Neglecting small primary and secondary voltage drops,

$$V_1 \approx E_1$$
$$V_2 \approx E_2$$
$$\frac{E_2}{E_1} = \frac{V_2}{V_1} = \frac{N_2}{N_1} = K$$

In a transformer, losses are negligible. Hence, input and output can be approximately equated.

$$V_1 I_1 = V_2 I_2$$

$$\frac{I_1}{I_2} = \frac{V_2}{V_1} = K$$

For step-up transformers,

$N_2 > N_1$ $\qquad\qquad$ $K > 1$

For step-down transformers,

$N_2 < N_1$ $\qquad\qquad$ $K < 1$

6.6 RATING OF A TRANSFORMER

[May 2014]

Rating of a transformer indicates the output power from it. But for a transformer, load is not fixed and its power factor goes on changing. Hence, rating is not expressed in terms of power but in terms of product of voltage and current, called VA rating. This rating is generally expressed in kVA.

$$\text{kVA rating of a transformer} = \frac{V_1 I_1}{1000} = \frac{V_2 I_2}{1000}$$

We can calculate full-load currents of primary and secondary windings from kVA rating of a transformer. *Full-load current* is the maximum current which can flow through the winding without damaging it.

Full-load primary current $\quad I_1 = \dfrac{\text{kVA rating} \times 1000}{V_1}$

Full-load secondary current $\quad I_2 = \dfrac{\text{kVA rating} \times 1000}{V_2}$

6.7 LOSSES IN A TRANSFORMER

[May 2014]

There are two types of losses in a transformer:

(i) Iron or core loss
(ii) Copper loss

Iron Loss This loss is due to the reversal of flux in the core. The flux set-up in the core is nearly constant. Hence, iron loss is practically constant at all the loads, from no load to full load. The losses occuring under no-load condition are the iron losses because the copper losses in the primary winding due to no-load current are negligible. Iron losses can be subdivided into two losses:

(i) Hysteresis loss
(ii) Eddy-current loss

(1) *Hysteresis Loss* This loss occurs due to setting of an alternating flux in the core. It depends on the following factors:

(i) Area of the hysteresis loop of magnetic material which again depends upon the flux density
(ii) Volume of the core
(iii) Frequency of the magnetic flux reversal

(2) *Eddy–Current Loss* This loss occurs due to the flow of eddy currents in the core caused by induced emf in the core. It depends on the following factors:

(i) Thickness of lamination of core
(ii) Frequency of the magnetic flux reversal
(iii) Maximum value of flux density in the core
(iv) Volume of the core
(v) Quality of magnetic material used

Eddy-current losses are reduced by decreasing the thickness of lamination and by adding silicon to steel.

Copper Loss This loss is due to the resistances of primary and secondary windings.

$$W_{Cu} = I_1^2 R_1 + I_2^2 R_2$$

where R_1 = primary winding resistance

R_2 = secondary winding resistance

Copper loss depends upon the load on the transformer and is proportional to square of load current of kVA rating of the transformer.

6.8 IDEAL AND PRACTICAL TRANSFORMERS

[Dec 2013]

For an ideal transformer, (i) there will be no core loss and copper loss, and (ii) winding resistance and leakage flux are zero. But in a practical transformer, the windings have some resistance and there is always some leakage flux.

In an ideal transformer, it is assumed that all the flux produced by the primary winding links both the primary and secondary windings. In practice, it is impossible to realize this condition. However, all the flux produced by the primary winding does not link with the secondary winding. Some part of the primary flux ϕ_{L_1} links with primary winding only. The flux ϕ_{L_1} is called primary leakage flux which links to primary winding and does not link to secondary winding. Similarly, some of the flux produced by the secondary winding links to secondary winding and does not link to primary winding. This flux is called secondary leakage flux and is represented by ϕ_{L_2}. The flux which does not pass completely through the core and links both the windings is known as the mutual flux and is represented by ϕ.

The primary leakage flux ϕ_{L_1} is in phase with I_1 and produces self-induced emf E_{L_1} in primary winding. Similarly, the secondary leakage flux ϕ_{L_2} is in phase with I_2 and produces self-induced emf E_{L_2} in secondary winding. The induced voltage E_{L_1} and E_{L_2} due to leakage fluxes ϕ_{L_1} and ϕ_{L_2} are different from induced voltages E_1 and E_2 caused by the main or mutual flux ϕ. Leakage fluxes produce self-induced emfs in their respective windings. It is, therefore, equivalent to an inductive coil in series with the respective winding such that voltage drop in each series coil is equal to that produced by leakage flux (Fig. 6.6).

$$E_{L_1} = I_1 X_1 \text{ and } E_{L_2} = I_2 X_2$$

The terms X_1 and X_2 are called primary and secondary leakage reactances respectively.

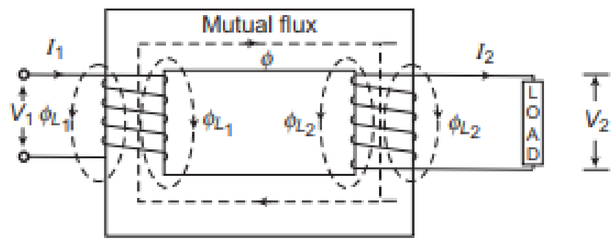


Fig. 6.6 *Magnetic fluxes in a transformer*

A transformer with winding resistance and magnetic leakage is equivalent to an ideal transformer (having no resistance and leakage reactance) having winding resistances and leakage reactances connected in series with each winding as shown in Fig. 6.7.

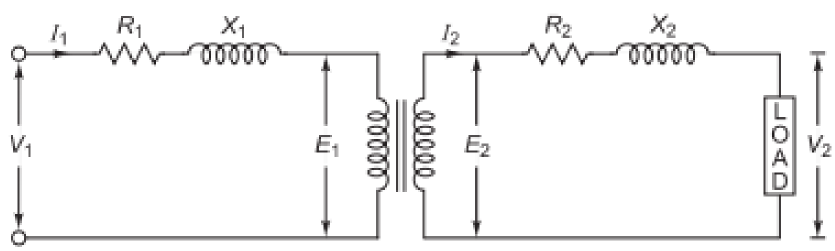


Fig. 6.7 *Winding resistances and leakage reactances of a practical transformer*

The following points should be kept in mind:

1. The leakage flux links one or the other winding but not both, hence, it in no way contributes to the transfer of energy from the primary to the secondary winding.
2. The primary voltage V_1 will have to supply reactive drop I_1X_1 in addition to I_1R_1. Similarly, the induced emf in the secondary winding E_2 will have to supply I_2R_2 and I_2X_2.

6.9 PHASOR DIAGRAM OF A TRANSFORMER ON NO LOAD

[Dec 2013, 2015, May 2015, 2016]

When the transformer is operating at no load, there is iron loss in the core and copper loss in the primary winding. Thus, primary input current I_0 has to supply iron loss in the core and a very small amount of copper loss in primary. Hence, the current I_0 has two components:

(i) a magnetising or reactive component I_μ, and

(ii) power or active component I_w.

The magnetising component I_μ is responsible for setting up flux in the core. It is in phase with the flux ϕ.

$$I_\mu = I_0 \sin \phi_0$$

The active component I_w is responsible for power loss in the transformer. It is in phase with V_1.

$$I_w = I_0 \cos \phi_0$$

Hence, no-load current I_0 is the phasor sum of I_μ and I_w.

$$\bar{I}_0 = \bar{I}_\mu + \bar{I}_w$$

$$I_0 = \sqrt{I_\mu^2 + I_w^2}$$

The no-load current I_0 is very small as compared to full-load current I_1. Hence, copper loss is negligible and no-load input power is practically equal to iron loss or core loss in the transformer.

Iron loss $W_i = V_1 I_0 \cos \phi_0$ where $\cos \phi_0$ is power factor at no load.

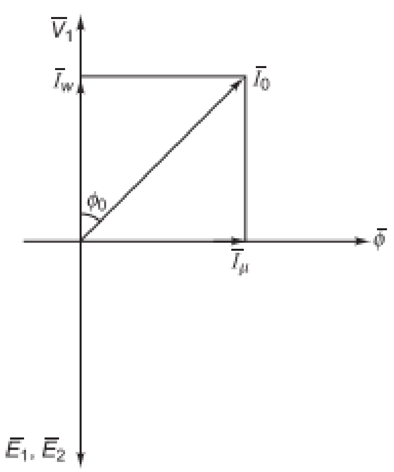


Fig. 6.8 *Phasor diagram*

Phasor Diagram Since the flux ϕ is common to both the windings, ϕ is chosen as a reference phasor. From emf equation of the transformer, it is clear that E_1 and E_2 lag the flux by 90°. Hence, emfs E_1 and E_2 are drawn such that these lag behind the flux ϕ by 90°. The magnetising component I_μ is drawn in phase with the flux ϕ. The applied voltage V_1 is drawn equal and opposite to E_1 as $V_1 \approx E_1$. The active component I_w is drawn in phase with voltage V_1. The phasor sum of I_μ and I_w gives the no-load current I_0.

6.10 PHASOR DIAGRAM OF A TRANSFORMER ON LOAD

[Dec 2012, 2013, May 2014]

When the transformer is loaded, a current I_2 will flow in the secondary winding. The secondary current I_2 sets up a secondary flux ϕ_2 that tends to reduce the flux ϕ produced by the primary current. Hence, induced emf E_1 in primary reduces. This causes more current to flow in the primary. Let the additional current in the primary be I_2'. This current I_2' is anti-phase with I_2 and sets up its own flux ϕ_2' which cancels the flux ϕ_2 produced by I_2.

Hence, the primary current I_1 is the phasor sum of the no-load current I_0 and the current I_2'.

$$\frac{N_2}{N_1} = \frac{I_1}{I_2} = \frac{I_0 + I_2'}{I_2} = \frac{I_2'}{I_2} = K$$

$$I_2' = KI_2$$

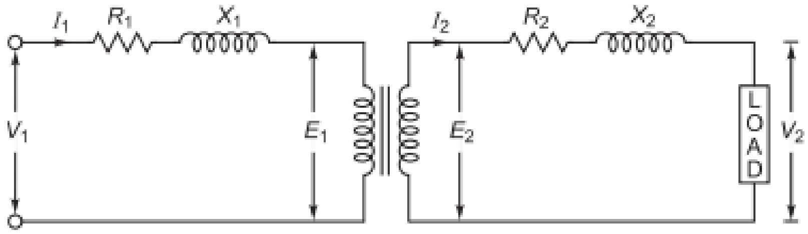


Fig. 6.9 *Practical transformer on load condition*

Figure 6.9 shows a practical transformer on load condition. When a transformer is loaded, the current I_2 flows in the secondary winding and the voltage V_2 appears across the load. Current I_2 is in phase with voltage V_2, if the load is resistive; it lags behind it, if load is inductive, and it leads, if load is capacitive.

Writing vector equations for primary and secondary sides,

$$\overline{V_1} = \overline{I_1R_1} + \overline{I_1X_1} + (-\overline{E_1})$$

$$\overline{E_2} = \overline{I_2R_2} + \overline{I_2X_2} + \overline{V_2}$$

where $$\overline{I_1} = \overline{I_0} + \overline{I_2'}$$

The phasor diagram of a transformer on load condition is drawn with the help of the above expressions.

Steps for Drawing Phasor Diagrams

1. First draw $\overline{V_2}$ and then $\overline{I_2}$. The phase angle between $\overline{I_2}$ and $\overline{V_2}$ will depend on the type of load.
2. To $\overline{V_2}$, add the resistive drop $\overline{I_2R_2}$, parallel to $\overline{I_2}$ and the inductive drop $\overline{I_2X_2}$, leading $\overline{I_2}$ by 90° such that
$$\overline{E_2} = \overline{V_2} + \overline{I_2R_2} + \overline{I_2X_2}$$
3. Draw $\overline{E_1}$ on the same side such that $E_1 = \dfrac{E_2}{K}$
4. Draw $-\overline{E_1}$ equal and opposite to $\overline{E_1}$.
5. For drawing $\overline{I_1}$, first draw $\overline{I_0}$ and $\overline{I_2'}$ such that
$$I_2' = K\,I_2$$
6. Add $\overline{I_0}$ and $\overline{I_2'}$ using the parallelogram law of vector addition.
$$\overline{I_1} = \overline{I_0} + \overline{I_2'}$$
7. To $-\overline{E_1}$, add the resistive drop $\overline{I_1R_1}$, parallel to $\overline{I_1}$ and the inductive drop $\overline{I_1X_1}$, leading $\overline{I_1}$ by 90° such that
$$\overline{V_1} = -\overline{E_1} + \overline{I_1R_1} + \overline{I_1X_1}$$

8. Draw flux ϕ such that ϕ leads $\overline{E_1}$ and $\overline{E_2}$ by 90°.

Case (i) Resistive load (unity power factor) *Case (ii)* Inductive load (lagging power factor)

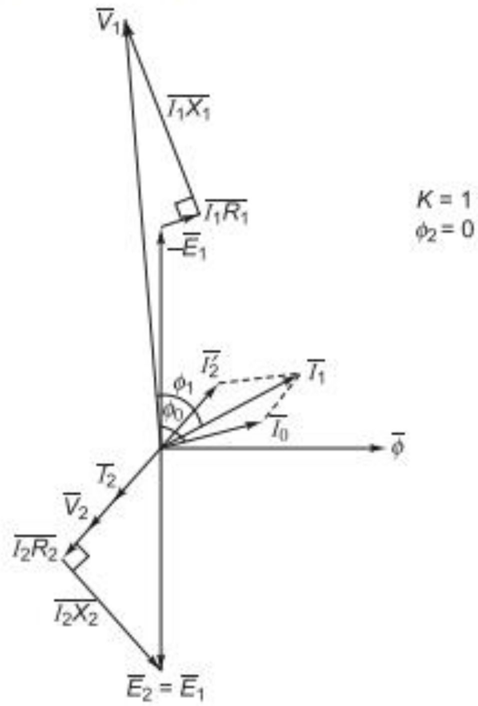


Fig. 6.10 *Phasor diagram for resistive load*

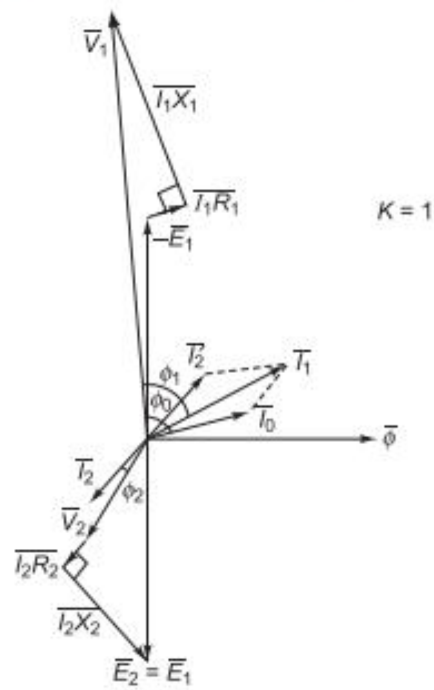


Fig. 6.11 *Phasor diagram for inductive load*

Case (iii) Capacitive load (leading power factor)

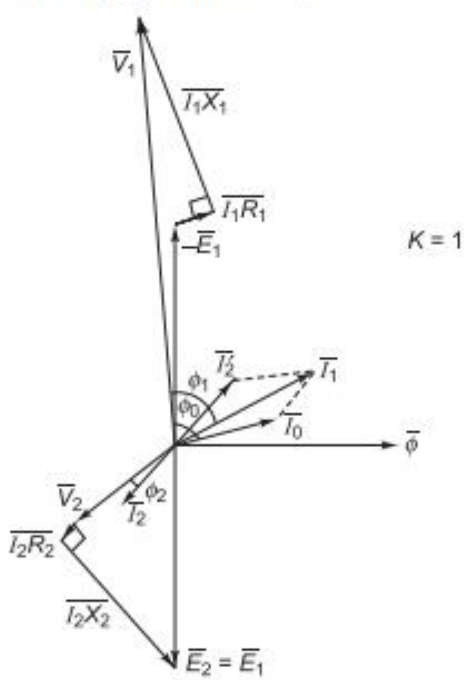


Fig. 6.12 *Phasor diagram for capacitive load*

6.11 EQUIVALENT CIRCUIT

Figure 6.13 shows a practical transformer. R_1 and R_2 represent the resistances of primary and secondary windings respectively. Similarly, X_1 and X_2 represent the leakage reactances of primary and secondary windings respectively.

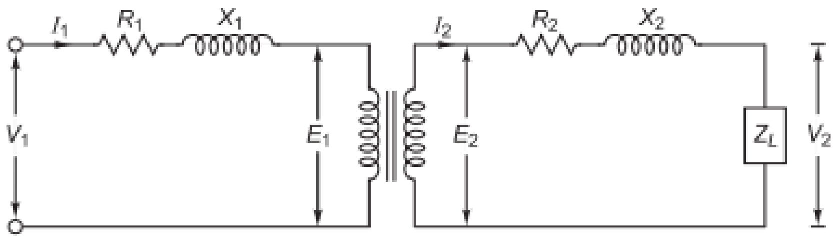


Fig. 6.13 *Practical transformer*

Figure 6.13 can be further modified to represent the no-load current I_0 and its component. The current I_0 is the phasor sum of currents I_w and I_μ. Hence, the current I_0 is simulated by the resistance R_0 taking working component I_w and inductance X_0, taking magnetising component I_μ connected in parallel across the primary circuit.

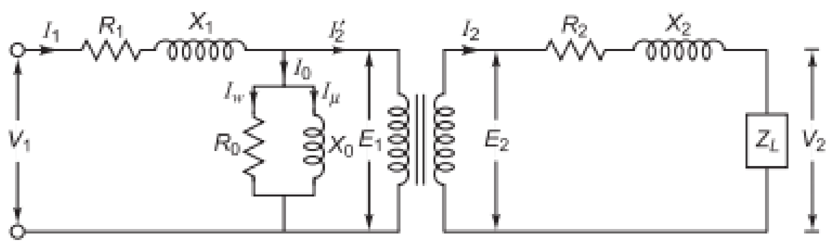


Fig. 6.14 *Practical transformer showing no-load current I_0 and its component*

For convenience, all the quantities can be shown on only one side by transferring the quantities from one side to other without any power loss. The power loss in the secondary is $I_2^2\, R_2$. If R_2' is the resistance referred to primary which would have caused the same power loss as R_2 is secondary,

$$I_1^2 R_2' = I_2^2 R_2$$

$$R_2' = \left(\frac{I_2}{I_1}\right)^2 R_2 = \frac{R_2}{K^2}$$

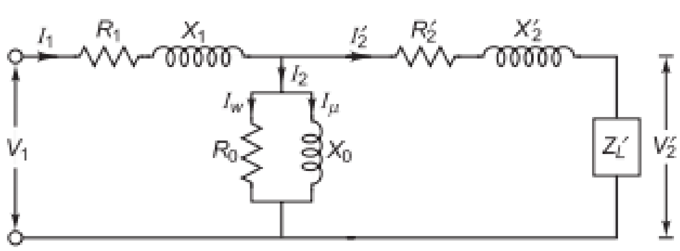


Fig. 6.15 *Modified circuit for primary winding*

Similarly, $X_2' = \dfrac{X_2}{K^2}$

Since all quantities are transferred to primary, the transformer need not be shown. The no-load current I_0 is very small compared to the full-load current I_1. Hence, drop across R_1 and X_1 due to I_0 can be neglected. Therefore, transferring R_0 and X_0 to the extreme left,

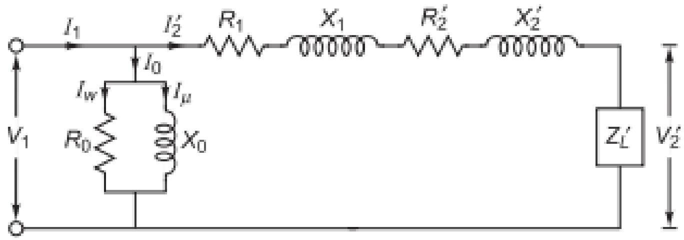


Fig. 6.16 *Modified circuit for primary winding*

The equivalent resistance referred to primary $R_{01} = R_1 + R_2' = R_1 + \dfrac{R_2}{K^2}$

The equivalent leakage reactance referred to primary $X_{01} = X_1 + X_2' = X_1 + \dfrac{X_2}{K^2}$

The equivalent impedance referred to primary $Z_{01} = \sqrt{R_{01}^2 + X_{01}^2}$

The equivalent circuit referred to primary is as shown in Fig. 6.18.

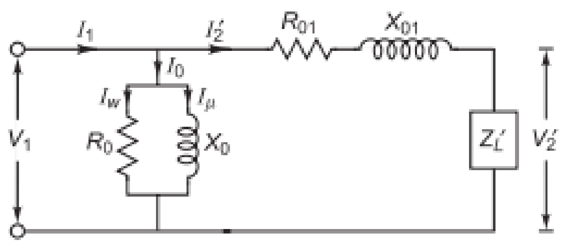


Fig. 6.17 *Equivalent circuit referred to primary winding*

Similarly, the equivalent circuit referred to secondary is as shown in Fig. 6.19.

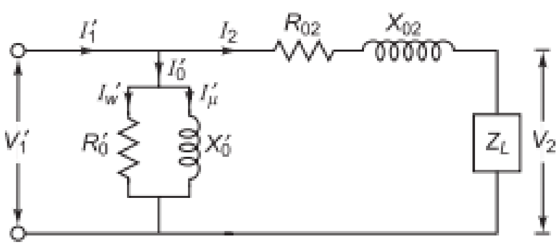


Fig. 6.18 *Equivalent circuit referred to secondary winding*

The equivalent resistance referred to secondary $R_{02} = R_2 + R_1' = R_2 + K^2R_1 = K^2R_{01}$

The equivalent leakage reactance referred to secondary $X_{02} = X_2 + X_1' = X_2 + K^2X_1 = K^2X_{01}$

The equivalent impedance referred to secondary $Z_{02} = \sqrt{R_{02}^2 + X_{02}^2} = K^2 Z_{01}$

Note:

(i) While shifting any primary resistance or reactance to the secondary, multiply it by K^2.

(ii) While shifting any secondary resistance or reactance to the primary, divide it by K^2.

6.12 VOLTAGE REGULATION

When a transformer is loaded, the secondary terminal voltage decreases due to a drop across secondary winding resistance and leakage reactance. This change in secondary terminal voltage from no load to full load conditions, expressed as a fraction of the no-load secondary voltage is called *regulation of the transformer*.

$$\text{Regulation} = \frac{\begin{pmatrix}\text{Secondary terminal}\\ \text{voltage on no load}\end{pmatrix} - \begin{pmatrix}\text{Secondary terminal voltage}\\ \text{on full-load condition}\end{pmatrix}}{\text{Secondary terminal voltage on no load}}$$

$$= \frac{E_2 - V_2}{E_2}$$

$$\text{Percentage regulation} = \frac{E_2 - V_2}{E_2} \times 100$$

6.12.1 Expression for Voltage Regulation

Consider a phasor diagram of transformer referred to secondary side on load condition (load is assumed to be inductive). With O as centre and radius OC, draw an arc cutting OA produced at M. From the point B, draw BD perpendicular on OA produced. Draw CN perpendicular to OM and draw BL parallel to OM.

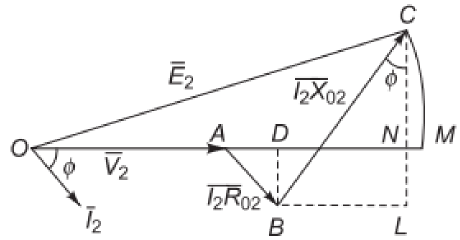


Fig. 6.19

$$\text{Total voltage drop} = E_2 - V_2 = OC - OA = OM - OA = AM = AN + NM$$

$$\text{Approximate voltage drop} \approx AN \qquad (\because NM \text{ is very small})$$

$$= AD + DN$$

$$= AD + BL$$

$$= I_2 R_{02} \cos\phi + I_2 X_{02} \sin\phi$$

$$\%\ \text{regulation} = \frac{I_2 R_{02} \cos\phi + I_2 X_{02} \sin\phi}{E_2} \times 100$$

For leading pf,

$$\text{Approximate voltage drop} = I_2 R_{02} \cos\phi - I_2 X_{02} \sin\phi$$

$$\%\ \text{regulation} = \frac{I_2 R_{02} \cos\phi - I_2 X_{02} \sin\phi}{E_2} \times 100$$

Hence, in general,

$$\%\ \text{regulation} = \frac{I_2 R_{02} \cos\phi \pm I_2 X_{02} \sin\phi}{E_2} \times 100$$

'+' sign is used for lagging pf and '–' sign is used for leading pf.

On primary side, we can express regulation as,

$$\%\ \text{regulation} = \frac{I_1 R_{01} \cos\phi \pm I_1 X_{01} \sin\phi}{V_1} \times 100$$

We can also express percentage regulation as

$$\% \text{ regulation} = \frac{100\, I_2\, R_{02}}{E_2} \cos\phi \pm \frac{100\, I_2\, X_{02}}{E_2} \sin\phi$$

$$= v_r \cos\phi \pm v_x \sin\phi$$

where

$$v_r = \frac{100\, I_2\, R_{02}}{E_2} = \text{percentage resistive drop}$$

$$v_x = \frac{100\, I_2\, X_{02}}{E_2} = \text{percentage reactive drop}$$

6.13 EFFICIENCY

[Dec 2013]

Efficiency is defined as the ratio of output power to input power.

Efficiency
$$\eta = \frac{\text{Output}}{\text{Input}} = \frac{\text{Output}}{\text{Output} + \text{Losses}} = \frac{\text{Output}}{\text{Output} + \text{Copper loss} + \text{Iron loss}}$$

Also,
$$\eta = \frac{\text{Input} - \text{Losses}}{\text{Input}} = \frac{\text{Input} - \text{Copper loss} - \text{Iron loss}}{\text{Input}}$$

Condition for Maximum Efficiency We know that,

$$\eta = \frac{\text{Output}}{\text{Output} + \text{Losses}}$$

Considering secondary side of the transformer,

$$\eta = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + W_i + I_2^2 R_{02}}$$

Differentiating both the sides w.r.t. I_2,

$$\frac{d\eta}{dI_2} = \frac{(V_2 I_2 \cos\phi_2 + W_i + I_2^2 R_{02}) V_2 \cos\phi_2 - V_2 I_2 \cos\phi_2 (V_2 \cos\phi_2 + 2 I_2 R_{02})}{(V_2 I_2 \cos\phi_2 + W_i + I_2^2 R_{02})^2}$$

For maximum efficiency, $\frac{d\eta}{dI_2} = 0$

$$(V_2 I_2 \cos\phi_2 + W_i + I_2^2 R_{02}) V_2 \cos\phi_2 = V_2 I_2 \cos\phi_2 (V_2 \cos\phi_2 + 2 I_2 R_{02})$$

$$V_2 I_2 \cos\phi_2 + W_i + I_2^2 R_{02} = V_2 I_2 \cos\phi_2 + 2 I_2^2 R_{02}$$

$$W_i = I_2^2 R_{02}$$

Similarly on primary side,

$$W_i = I_1^2 R_{01}$$

Thus when copper loss = iron loss, the efficiency of the transformer is maximum.

Load Corresponding to Maximum Efficiency For maximum efficiency,

$$W_i = I_2^2 R_{02}$$

$$I_{2\,(\text{max. efficiency})} = \sqrt{\frac{W_i}{R_{02}}}$$

Multiplying both the sides by V_2,

$$V_2 I_{2\,(\text{max. efficiency})} = V_2 \sqrt{\frac{W_i}{R_{02}}}$$

$$\text{Load } VA_{(\text{max. efficiency})} = V_2 I_2 \sqrt{\frac{W_i}{I_2^2 R_{02}}} = V_2 I_2 \sqrt{\frac{W_i}{W_{Cu}}}$$

$$\text{Load kVA}_{(\text{max. efficiency})} = \text{Full-load kVA} \sqrt{\frac{W_i}{W_{Cu}}}$$

where W_i = iron loss

W_{Cu} = full-load copper loss

Note: The efficiency at any load is given by

$$\%\eta = \frac{x \times \text{full-load kVA} \times \text{pf}}{x \times \text{full-load kVA} \times \text{pf} + W_i + x^2 W_{Cu}} \times 100$$

where x = ratio of actual to full load kVA

W_i = iron loss in kW

W_{Cu} = full-load copper loss in kW