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3 PHASE AC Circuits

Presented by:

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Three-phase AC Circuits

(iii) Generation of three-phase voltages A three-phase system utilizes three separate but identical windings that are displaced by 120 electrical degrees from each other. When these three windings are rotated in an anticlockwise direction with constant angular velocity in a uniform magnetic field, the voltages are induced in each winding which have the same magnitude and frequency but are displaced 120 electrical degrees from one another.

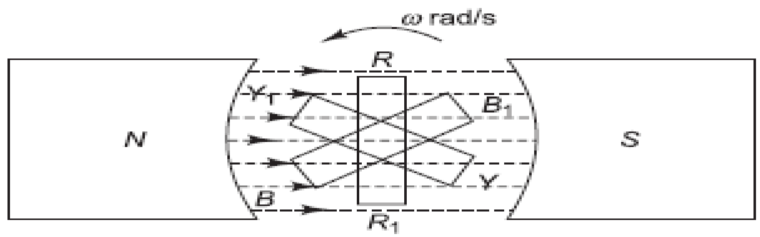

Fig. 5.7 *Three-phase system*

The instantaneous values of induced voltages in three windings RR_1, YY_1 and BB_1 are given by

$$v_R = V_m \sin \theta$$
$$v_Y = V_m \sin (\theta - 120°)$$
$$v_B = V_m \sin (\theta - 240°)$$

The induced voltage in winding YY_1 lags behind that in winding RR_1 by 120° and the induced voltage in winding BB_1 lags behind that in winding RR_1 by 240°. The waveforms of these three voltages are shown in Fig. 5.8.

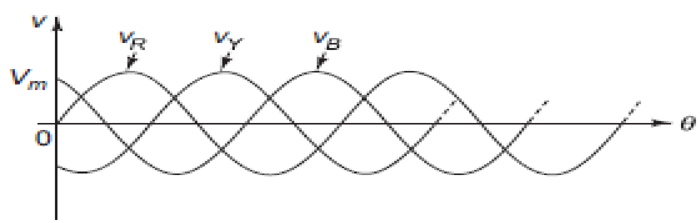

Fig. 5.8 *Voltage waveforms*

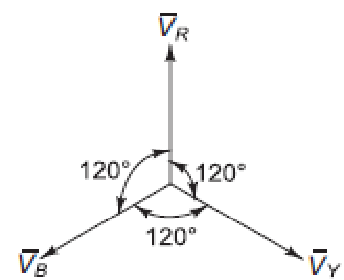

Fig. 5.9 *Phasor diagram*

Figure 5.9 shows the phasor diagram of these induced voltages.

5.3 ADVANTAGES OF A THREE-PHASE SYSTEM

1. In a single-phase system, the instantaneous power is fluctuating in nature. However, in a three-phase system, it is constant at all times.
2. The output of a three-phase system is greater than that of a single-phase system.
3. Transmission and distribution of a three-phase system is cheaper than that of a single-phase system.
4. Three-phase motors are more efficient and have higher power factors than single-phase motors of the same frequency.
5. Three-phase motors are self-starting whereas single-phase motors are not self-starting.

5.4 SOME DEFINITIONS

Phase Sequence The sequence in which the voltages in the three phases reach the maximum positive value is called the *phase sequence* or *phase order*. From the phasor diagram of a three-phase system, it is clear that the voltage in the coil R attains maximum positive value first, next in the coil Y and then in the coil B. Hence, the phase sequence is R-Y-B.

Phase Voltage The voltage induced in each winding is called the *phase voltage*.

Phase Current The current flowing through each winding is called the *phase current*.

Line Voltage The voltage available between any pair of terminals or lines is called the *line voltage*.

Line Current The current flowing through each line is called the *line current*.

Symmetrical or Balanced System A three-phase system is said to be balanced if the

(a) voltages in the three phases are equal in magnitude and differ in phase from one another by 120°, and

(b) currents in the three phases are equal in magnitude and differ in phase from one another by 120°.

Balanced Load The load is said to be balanced if loads connected across the three phases are identical, i.e., all the loads have the same magnitude and power factor.

5.6 STAR OR WYE CONNECTION

In this method, similar terminals (start or finish) of the three windings are joined together as shown in Fig. 5.11. The common point is called *star* or *neutral point*.

Figure 5.12 shows a three-phase system in star connection.

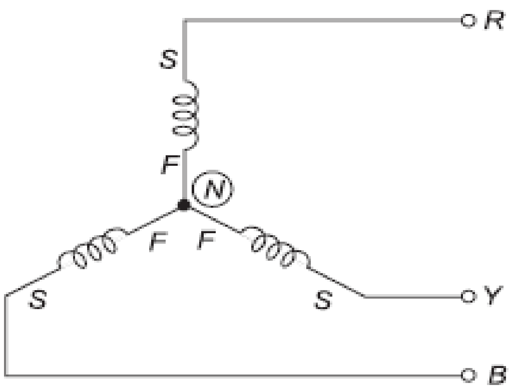

Fig. 5.11 *Three-phase star connection*

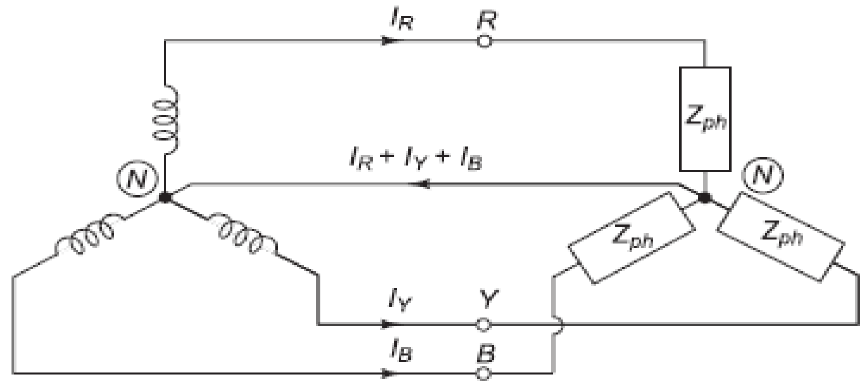

Fig. 5.12 *Three-phase, four-wire system*

This system is called a three-phase, four-wire system. If three identical loads are connected to each phase, the current flowing through the neutral wire is the sum of the three currents I_R, I_Y and I_B. Since the impedances are identical, the three currents are equal in magnitude but differ in phase from one another by 120°.

$$i_R = I_m \sin \theta$$

$$i_Y = I_m \sin (\theta - 120°)$$

$$i_B = I_m \sin (\theta - 240°)$$

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$$i_R + i_Y + i_B = I_m \sin\theta + I_m \sin(\theta - 120°) + I_m \sin(\theta - 240°) = 0$$

Therefore, the neutral wire can be removed without any way affecting the voltages or currents in the circuit as shown in Fig. 5.13. This constitutes a three-phase, three-wire system. If the load is not balanced, the neutral wire carries some current.

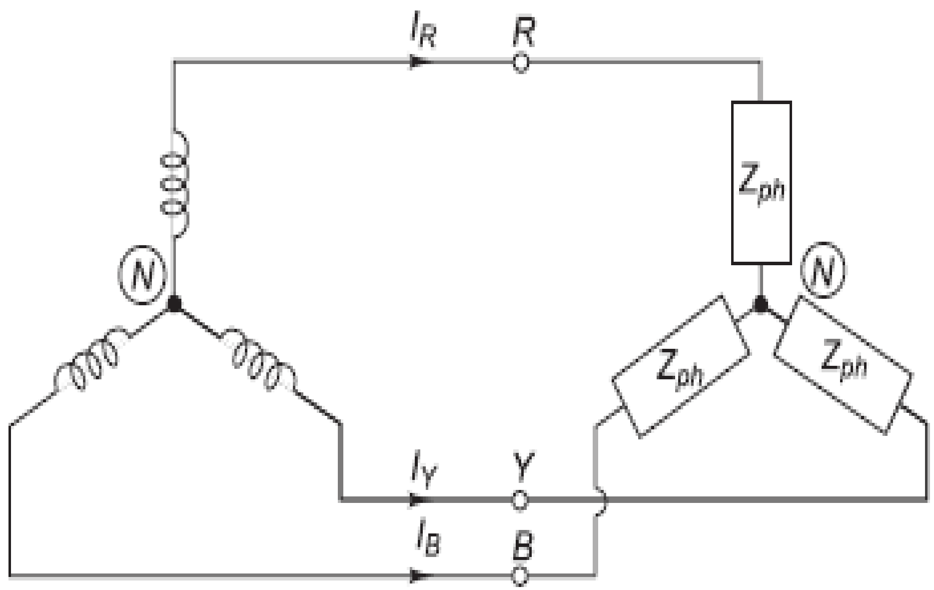

Fig. 5.13 Three-phase, three-wire system

5.7 DELTA OR MESH CONNECTION

In this method, dissimilar terminals of the three windings are joined together, i.e., the 'finish' terminal of one winding is connected to the 'start' terminal of the other winding, and so on, as shown in Fig. 5.14. This system is also called a three-phase, three-wire system.

For a balanced system, the sum of the three phase voltages round the closed mesh is zero. The three emfs are equal in magnitude but differ in phase from one another by 120°.

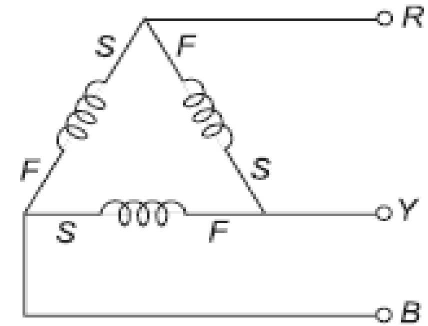

Fig. 5.14 *Three-phase delta connection*

$$v_R = V_m \sin \theta$$

$$v_Y = V_m \sin (\theta - 120°)$$

$$v_B = V_m \sin (\theta - 240°)$$

$$v_R + v_Y + v_B = V_m \sin \theta + V_m \sin (\theta - 120°) + V_m \sin (\theta - 240°) = 0$$

5.8 VOLTAGE, CURRENT AND POWER RELATIONS IN A BALANCED STAR-CONNECTED LOAD

[May 2013, Dec 2013]

5.8.1 Relation between Line Voltage and Phase Voltage

Since the system is balanced, the three-phase voltages V_{RN}, V_{YN} and V_{BN} are equal in magnitude and differ in phase from one another by 120°.

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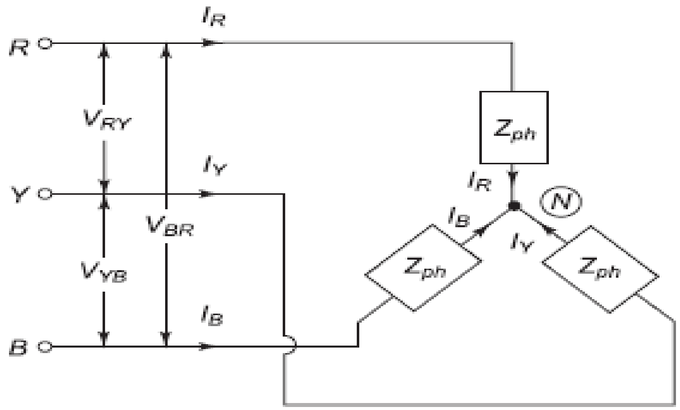

Fig. 5.15 *Star connection*

Let $\quad V_{RN} = V_{YN} = V_{BN} = V_{ph}$

where V_{ph} indicates the rms value of phase voltage.

$$\overline{V}_{RN} = V_{ph}\angle 0°$$
$$\overline{V}_{YN} = V_{ph}\angle -120°$$
$$\overline{V}_{BN} = V_{ph}\angle -240°$$

Let $\quad V_{RY} = V_{YB} = V_{BR} = V_L$

where V_L indicates the rms value of line voltage.

Applying Kirchhoff's voltage law,

$$\begin{aligned}\overline{V}_{RY} &= \overline{V}_{RN} + \overline{V}_{NY} \\ &= \overline{V}_{RN} - \overline{V}_{YN} \\ &= V_{ph}\angle 0° - V_{ph}\angle -120° \\ &= (V_{ph} + j0) - (-0.5\,V_{ph} - j0.866\,V_{ph}) \\ &= 1.5\,V_{ph} + j0.866\,V_{ph} \\ &= \sqrt{3}\,V_{ph}\angle 30°\end{aligned}$$

Similarly,

$$\overline{V}_{YB} = \overline{V}_{YN} + \overline{V}_{NB} = \sqrt{3}\,V_{ph}\angle 30°$$
$$\overline{V}_{BR} = \overline{V}_{BN} + \overline{V}_{NR} = \sqrt{3}\,V_{ph}\angle 30°$$

Thus, in a star-connected, three-phase system, $V_L = \sqrt{3}\,V_{ph}$ and line voltages lead respective phase voltages by 30°.

5.8.2 Relation between Line Current and Phase Current

From Fig. 5.15, it is clear that line current is equal to the phase current.

$$I_L = I_{ph}$$

5.8.3 Phasor Diagram (Lagging Power Factor)

Steps for Drawing Phasor Diagram

1. First draw $\overline{V}_{RN}$ as reference voltage.
2. Since three phase voltages are equal in magnitude and differ in phase from one another by 120°, draw $\overline{V}_{YN}$ and $\overline{V}_{BN}$ lagging 120° behind w.r.t. each other.
3. Draw $\overline{V}_{NY}$ equal and opposite to $\overline{V}_{YN}$.
4. Add $\overline{V}_{RN}$ and $\overline{V}_{NY}$ using the parallelogram law of vector addition such that

$$\overline{V}_{RY} = \overline{V}_{RN} + \overline{V}_{NY}$$

5. Draw $\overline{V}_{NB}$ equal and opposite to $\overline{V}_{BN}$.

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6. Add $\overline{V}_{YN}$ and $\overline{V}_{NB}$ using the parallelogram law of vector addition such that

$$\overline{V}_{YB} = \overline{V}_{YN} + \overline{V}_{NB}$$

7. Draw $\overline{V}_{NR}$ equal and opposite to $\overline{V}_{RN}$.
8. Add $\overline{V}_{BN}$ and $\overline{V}_{NR}$ using the parallelogram law of vector addition such that

$$\overline{V}_{BR} = \overline{V}_{BN} + \overline{V}_{NR}$$

9. Assuming inductive load, draw three phase currents $\overline{I}_R$, $\overline{I}_Y$ and $\overline{I}_B$ lagging behind its respective phase voltages by an angle ϕ. The phase currents are equal in magnitude and differ in phase from one another by 120°.
10. Line currents are same as the phase currents in star connected load. Hence, separate line currents are not drawn.
11. Since $\overline{V}_{NY}$ is antiphase with $\overline{V}_{YN}$, angle between $\overline{V}_{RN}$ and $\overline{V}_{NY}$ is 60°. The line voltage $\overline{V}_{RY}$ leads phase voltage $\overline{V}_{RN}$ by 30°. Similarly, line voltage $\overline{V}_{YB}$ leads phase voltage $\overline{V}_{YN}$ by 30° and line voltage $\overline{V}_{BR}$ leads phase voltage $\overline{V}_{BN}$ by 30°.

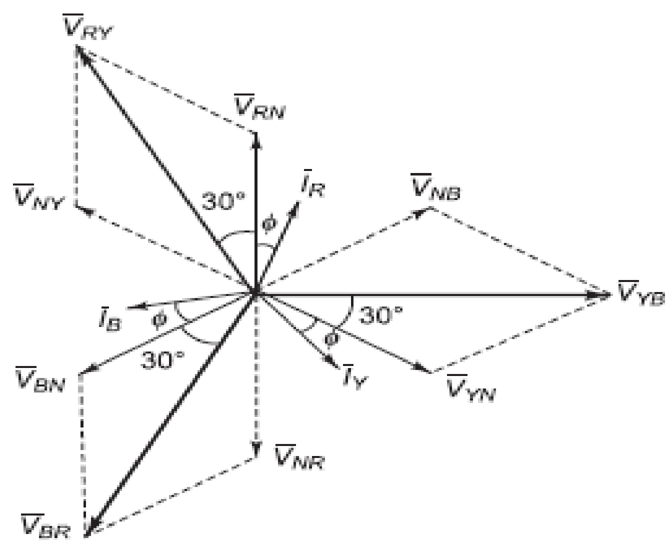

Fig. 5.16 *Phasor diagram*

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5.8.4 Power

The total power in a three-phase system is the sum of powers in the three phases. For a balanced load, the power consumed in each load phase is the same.

Total active power $P = 3 \times \text{power in each phase} = 3\, V_{ph}\, I_{ph} \cos\phi$

In a star-connected, three-phase system,

$$V_{ph} = \frac{V_L}{\sqrt{3}}$$

$$I_{ph} = I_L$$

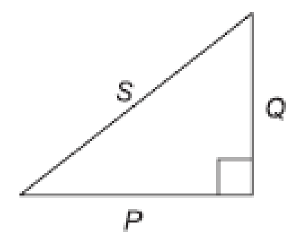
$$P = 3 \times \frac{V_L}{\sqrt{3}} \times I_L \times \cos\phi = \sqrt{3}\, V_L\, I_L \cos\phi$$

where ϕ is the phase difference between phase voltage and corresponding phase current.

Similarly, total reactive power $Q = 3\, V_{ph}\, I_{ph} \sin\phi$

$$= \sqrt{3}\; V_L\, I_L \sin\phi$$

Total apparent power $S = 3 V_{ph}\, I_{ph} = \sqrt{3}\; V_L\, I_L$



The power triangle for a three-phase system is shown in Fig. 5.17.

Fig. 5.17

5.9 VOLTAGE, CURRENT AND POWER RELATIONS IN A BALANCED DELTA-CONNECTED LOAD

[Dec 2012, 2013]

5.9.1 Relation between Line Voltage and Phase Voltage

From Fig. 5.18, it is clear that line voltage is equal to phase voltage.

$$V_L = V_{ph}$$

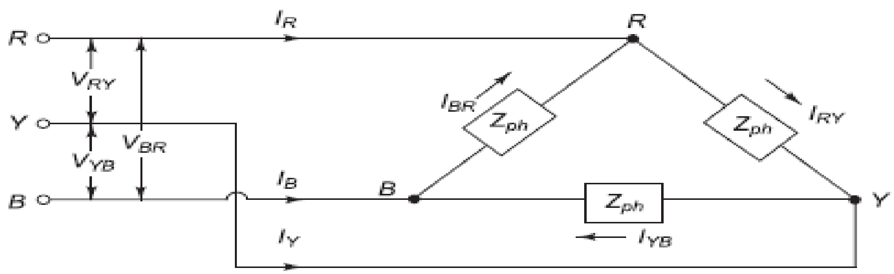


Fig. 5.18 *Delta connection*

5.9.2 Relation between Line Current and Phase Current

Since the system is balanced, the three-phase currents I_{RY}, I_{YB} and I_{BR} are equal in magnitude but differ in phase from one another by 120°.

Let $\quad I_{RY} = I_{YB} = I_{BR} = I_{ph}$

where I_{ph} indicates rms value of the phase current.

$$\bar{I}_{RY} = I_{ph}\angle 0°$$

$$\bar{I}_{YB} = I_{ph}\angle -120°$$

$$\bar{I}_{BR} = I_{ph}\angle -240°$$

Let $\quad I_R = I_Y = I_B = I_L$

where I_L indicates rms value of the line current.

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Applying Kirchhoff's current law,

$$\bar{I}_R + \bar{I}_{BR} = \bar{I}_{RY}$$

$$\bar{I}_R = \bar{I}_{RY} - \bar{I}_{BR} = I_{ph}\angle 0° - I_{ph}\angle -240°$$

$$= (I_{ph} + j0) - (-0.5\, I_{ph} + j0.866\, I_{ph})$$

$$= 1.5\, I_{ph} - j0.866\, I_{ph}$$

$$= \sqrt{3}\; I_{ph}\angle -30°$$

Similarly, $$\bar{I}_Y = \bar{I}_{YB} - \bar{I}_{RY} = \sqrt{3}\; I_{ph}\angle -30°$$

$$\bar{I}_B = \bar{I}_{BR} - \bar{I}_{YB} = \sqrt{3}\; I_{ph}\angle -30°$$

Thus, in a delta-connected, three-phase system, $I_L = \sqrt{3}\; I_{ph}$ and line currents are 30° behind the respective phase currents.

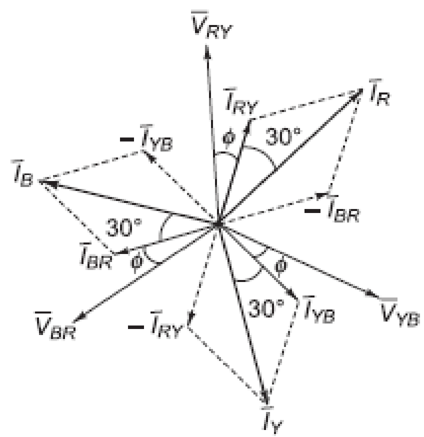

Fig. 5.19 *Phasor diagram*

Three-phase AC Circuits

5.9.4 Power

$$P = 3\ V_{ph}\ I_{ph} \cos\phi$$

In a delta-connected, three-phase system,

$$V_{ph} = V_L$$

$$I_{ph} = \frac{I_L}{\sqrt{3}}$$

$$P = 3 \times V_L \times \frac{I_L}{\sqrt{3}} \times \cos\phi = \sqrt{3}\ V_L\ I_L \cos\phi$$

Total reactive power $Q = 3\ V_{ph}\ I_{ph} \sin\phi = \sqrt{3}\ V_L\ I_L \sin\phi$

Total apparent power $S = 3\ V_{ph}\ I_{ph} = \sqrt{3}\ V_L\ I_L$

The power triangle for a three-phase system is shown in Fig. 5.20.

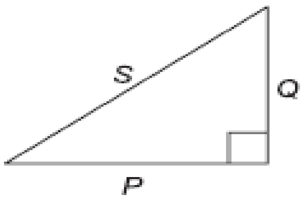

Fig. 5.20

RELATION BETWEEN POWER IN DELTA AND STAR SYSTEMS

$$P_Y = \frac{1}{3} P_\Delta$$

Thus, power consumed by a balanced star-connected load is one-third of that in the case of a delta-connected load.

5.12 COMPARISON BETWEEN STAR AND DELTA CONNECTIONS

Star Connection	*Delta Connection*
1. $V_L = \sqrt{3}\, V_{ph}$	1. $V_L = V_{ph}$
2. $I_L = I_{ph}$	2. $I_L = \sqrt{3}\, I_{ph}$
3. Line voltage leads the respective phase voltage by 30°.	3. Line current lags behind the respective phase current by 30°.
4. Power in star connection is one-third of power in delta connection.	4. Power in delta connection is 3 times of the power in star connection.
5. Three-phase, three-wire and three-phase, four-wire systems are possible.	5. Only three-phase, three-wire system is possible.
6. The phasor sum of all the phase currents is zero.	6. The phasor sum of all the phase voltages is zero.

Example 1

Three identical coils each of [4.2 + j5.6] ohms are connected in star across a 415 V, 3-phase, 50 Hz supply. Determine (i) V_{ph}, (ii) I_{ph}, and (iii) power factor. **[May 2014]**

Solution

$$\overline{Z}_{ph} = 4.2 + j5.6 = 7\ \angle 53.13°\ \Omega$$

$$V_L = 415\ \text{V}$$

$$f = 50\ \text{Hz}$$

For a star-connected load,

(i) $$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{415}{\sqrt{3}} = 239.6\ \text{V}$$

(ii) $$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{239.6}{7} = 34.23\ \text{A}$$

(iii) $$\text{pf} = \cos\phi = \cos(53.13°) = 0.6\ \text{(lagging)}$$

Example 2

Three equal impedances, each of 8 + j10 ohms, are connected in star. This is further connected to a 440 V, 50 Hz, three-phase supply. Calculate (i) phase voltage, (ii) phase angle, (iii) phase current, (iv) line current, (v) active power, and (vi) reactive power.

Solution

$$\bar{Z}_{ph} = 8 + j10\ \Omega$$

$$V_L = 440\text{ V}$$

$$f = 50\text{ Hz}$$

$$V_{ph} = \frac{V_L}{\sqrt{3}}$$

$$I_L = I_{ph}$$

Three-phase AC Circuits

Solution $\overline{Z}_{ph} = 8 + j10\ \Omega$

$V_L = 440$ V

$f = 50$ Hz

For a star-connected load,

(i) Phase voltage

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{440}{\sqrt{3}} = 254.03 \text{ V}$$

(ii) Phase angle

$$\overline{Z}_{ph} = 8 + j10 = 12.81 \angle 51.34^\circ\ \Omega$$

$$Z_{ph} = 12.81\ \Omega$$

$$\phi = 51.34^\circ$$

(iii) Phase current

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{254.03}{12.81} = 19.83 \text{ A}$$

(iv) Line current

$$I_L = I_{ph} = 19.83 \text{ A}$$

(v) Active power

$$P = \sqrt{3}\ V_L I_L \cos\phi = \sqrt{3} \times 440 \times 19.83 \times \cos(51.34^\circ) = 9.44 \text{ kW}$$

(vi) Reactive power

$$Q = \sqrt{3}\ V_L I_L \sin\phi = \sqrt{3} \times 440 \times 19.83 \times \sin(51.34^\circ) = 11.81 \text{ kVAR}$$

Example 3

A balanced delta-connected load of impedance (8 – j6) ohms per phase is connected to a three-phase, 230 V, 50 Hz supply. Calculate (i) power factor, (ii) line current, and (iii) reactive power.

Solution

$$\bar{Z}_{ph} = 8 - j6\ \Omega$$
$$V_L = 230\ \text{V}$$
$$f = 50\ \text{Hz}$$

For a delta-connected load,

(i) Power factor

$$\bar{Z}_{ph} = 8 - j6 = 10\ \angle -36.87°\ \Omega$$
$$Z_{ph} = 10\ \Omega$$
$$\phi = 36.87°$$
$$\text{pf} = \cos\phi = \cos(36.87°) = 0.8\ \text{(leading)}$$

(ii) Line current

$$V_{ph} = V_L = 230\ \text{V}$$
$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{230}{10} = 23\ \text{A}$$
$$I_L = \sqrt{3}\ I_{ph} = \sqrt{3} \times 23 = 39.84\ \text{A}$$

(iii) Reactive power

$$Q = \sqrt{3}\ V_L I_L \sin\phi = \sqrt{3} \times 230 \times 39.84 \times \sin(36.87°) = 9.52\ \text{kVAR}$$

Example 4

Three coils, each having a resistance and an inductance of 8 Ω and 0.02 H respectively, are connected in star across a three-phase, 230 V, 50 Hz supply. Find the (i) power factor, (ii) line current, (iii) power, (iv) reactive volt-amperes, and (v) total volt-amperes.

Solution

$$R = 8\ \Omega$$

$$L = 0.02\ \text{H}$$

$$V_L = 230\ \text{V}$$

$$f = 50\ \text{Hz}$$

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For a star-connected load,

(i) Power factor

$$X_L = 2\pi f L = 2\pi \times 50 \times 0.02 = 6.28\ \Omega$$

$$\overline{Z}_{ph} = R + jX_L = 8 + j6.28 = 10.17\ \angle 38.13^\circ\ \Omega$$

$$Z_{ph} = 10.17\ \Omega$$

$$\phi = 38.13^\circ$$

$$\text{pf} = \cos\phi = \cos(38.13^\circ) = 0.786 \text{ (lagging)}$$

(ii) Line current

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{230}{\sqrt{3}} = 132.79\ \text{V}$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{132.79}{10.17} = 13.05\ \text{A}$$

$$I_L = I_{ph} = 13.05\ \text{A}$$

(iii) Power

$$P = \sqrt{3}\ V_L I_L \cos\phi = \sqrt{3} \times 230 \times 13.05 \times 0.786 = 4.088\ \text{kW}$$

(iv) Reactive volt-amperes

$$Q = \sqrt{3}\ V_L I_L \sin\phi = \sqrt{3} \times 230 \times 13.05 \times \sin(38.13^\circ) = 3.21\ \text{kVAR}$$

(v) Total volt-ampere

$$S = \sqrt{3}\ V_L I_L = \sqrt{3} \times 230 \times 13.05 = 5.198\ \text{kVA}$$

Example 6

Three coils, each having a resistance of 8 Ω and an inductance of 0.02 H, are connected in delta to a three-phase, 400 V, 50 Hz supply. Calculate the (i) line current, and (ii) power absorbed.

Solution

$$R = 8\ \Omega$$
$$L = 0.02\ \text{H}$$
$$V_L = 400\ \text{V}$$
$$f = 50\ \text{Hz}$$

For a delta-connected load,

Example 6

Three coils, each having a resistance of 8 Ω and an inductance of 0.02 H, are connected in delta to a three-phase, 400 V, 50 Hz supply. Calculate the (i) line current, and (ii) power absorbed.

Solution

$$R = 8\ \Omega$$
$$L = 0.02\ \text{H}$$
$$V_L = 400\ \text{V}$$
$$f = 50\ \text{Hz}$$

For a delta-connected load,

(i) Line current

$$V_L = V_{ph} = 400\ \text{V}$$
$$X_L = 2\pi f L = 2\pi \times 50 \times 0.02 = 6.28\ \Omega$$
$$\bar{Z}_{ph} = R + jX_L = 8 + j6.28 = 10.17\ \angle 38.13°\ \Omega$$
$$Z_{ph} = 10.17\ \Omega$$
$$\phi = 38.13°$$
$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{400}{10.17} = 651639.33\ \text{A}$$
$$I_L = \sqrt{3}\ I_{ph} = \sqrt{3} \times 39.33 = 68.12\ \text{A}$$

(ii) Power absorbed

$$P = \sqrt{3}\ V_L I_L \cos\phi = \sqrt{3} \times 400 \times 68.12 \times \cos(38.13°) = 37.12\ \text{kW}$$

Example 7

The three equal impedances of each of 10 $\angle 60^\circ\ \Omega$, are connected in star across a three-phase, 400 V, 50 Hz supply. Calculate the (i) line voltage and phase voltage, (ii) power factor and active power consumed, (iii) If the same three impedances are connected in delta to the same source of supply, what is the active power consumed?

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Solution $\bar{Z}_{ph} = 10 \angle 60° \ \Omega$

$V_L = 400$ V

$f = 50$ Hz

For a star-connected load,

(i) Line voltage and phase voltage

$$V_L = 400 \text{ V}$$

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94 \text{ V}$$

(ii) Power factor and active power consumed

$$\phi = 60°$$

$$\text{pf} = \cos\phi = \cos(60°) = 0.5 \text{ (lagging)}$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{230.94}{10} = 23.094 \text{ A}$$

$$I_L = I_{ph} = 23.094 \text{ A}$$

$$P = \sqrt{3}\, V_L\, I_L \cos\phi = \sqrt{3} \times 400 \times 23.094 \times 0.5 = 8 \text{ kW}$$

Three-phase AC Circuits

(iii) Active power consumed for delta-connected load

$$V_L = 400\ \text{V}$$

$$Z_{ph} = 10\ \Omega$$

$$V_{ph} = V_L = 400\ \text{V}$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{400}{10} = 40\ \text{A}$$

$$I_L = \sqrt{3}\ I_{ph} = \sqrt{3} \times 40 = 69.28\ \text{A}$$

$$P = \sqrt{3}\ V_L I_L \cos\phi = \sqrt{3} \times 400 \times 69.28 \times \cos(60°) = 24\ \text{kW}$$

Example 9

A balanced 3-phase load consists of 3 coils, each of resistance 4 Ω and inductance 0.02 H. It is connected to a 440 V, 50 Hz, 3 φ supply. Find the total power consumed when the load is connected in star and the total reactive power when the load is connected in delta. **[Dec 2014]**

Solution

$$R = 4\ \Omega$$

$$L = 0.02\ \text{H}$$

$$V_L = 440\ \text{V}$$

$$f = 50\ \text{Hz}$$

Three-phase AC Circuits

For a star-connected load,

(i) Total power consumed

$$X_L = 2\pi f L = 2\pi \times 50 \times 0.02 = 6.28\ \Omega$$

$$\bar{Z}_{ph} = R + j X_L = 4 + j\, 6.28 = 7.45 \angle 57.51^\circ\ \Omega$$

$$Z_{ph} = 7.45\ \Omega$$

$$\phi = 57.51^\circ$$

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{440}{\sqrt{3}} = 254.03\ \text{V}$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{254.03}{7.45} = 34.1\ \text{A}$$

$$I_L = I_{ph} = 34.1\ \text{A}$$

$$P = \sqrt{3}\, V_L I_L \cos\phi = \sqrt{3} \times 440 \times 34.1 \times \cos(57.51^\circ) = 13.96\ \text{kW}$$

$$Q = \sqrt{3}\, V_L I_L \sin\phi = \sqrt{3} \times 440 \times 34.1 \times \sin(57.51^\circ) = 21.92\ \text{kVAR}$$

(ii) When the load is connected in delta across same supply

$$Q_\Delta = 3 Q_Y = 3 \times 21.92 \times 10^3 = 65.76\ \text{kVAR}$$

5.13 MEASUREMENT OF THREE-PHASE POWER

[May 2013]

In a three-phase system, total power is the sum of powers in three phases. The power is measured by wattmeter. It consists of two coils: (i) Current coil, and (ii) Voltage coil. Current coil is connected in series with the load and it senses current. Voltage coil is connected across supply terminals and it senses voltages.

There are three methods to measure three-phase power:

1. Three-wattmeter method
2. Two-wattmeter method
3. One-wattmeter method

5.13.2 Two-Wattmeter Method

This method is used for balanced as well as unbalanced loads. The current coils of the two wattmeters are inserted in any two lines and the voltage coil of each wattmeter is joined to a third line. The load may be star or delta connected as shown in Fig. 5.25. The sum of the two wattmeter readings gives three-phase power.

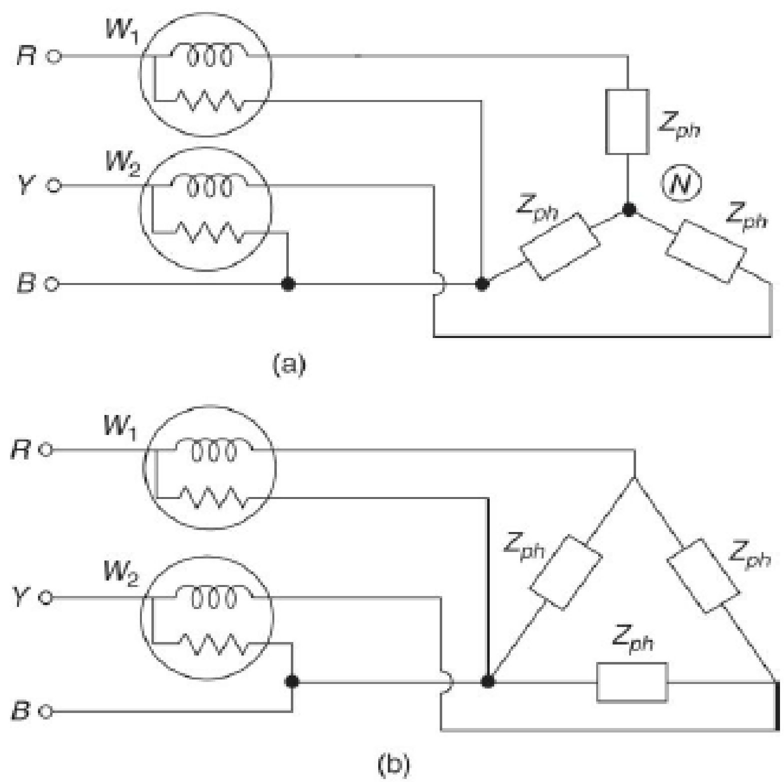


Fig. 5.25 *Two-wattmeter method*

Total power $P = W_1 + W_2$

5.15.1 Measurement of Active Power for Star Connected Load

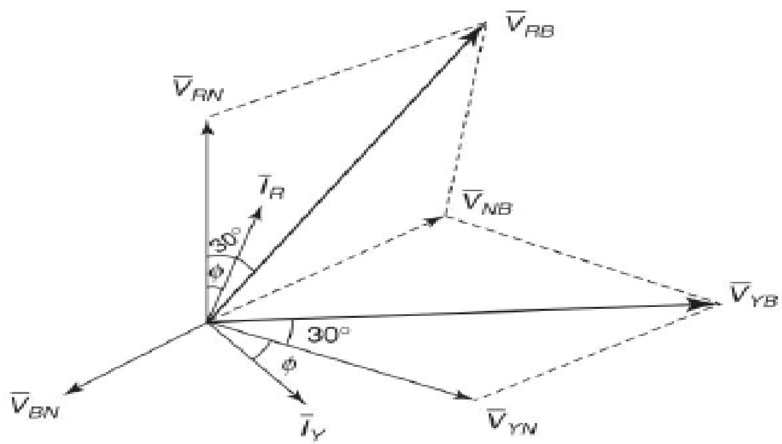
[Dec 2012, 2015, May 2014, 2015]

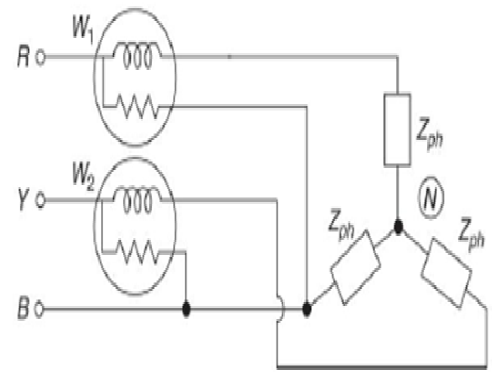
Figure 5.24 shows a balanced star-connected load and this load may be assumed to be inductive. Let V_{RN}, V_{YN} and V_{BN} be the three phase voltages and I_R, I_Y and I_B be the phase currents. The phase currents will lag behind their respective phase voltages by angle ϕ.

Current through current coil of $W_1 = I_R$

Voltage across voltage coil of $W_1 = V_{RB} = V_{RN} + V_{NB} = V_{RN} - V_{BN}$

Figure 5.29 shows the phasor diagram of a balanced star-connected inductive load.





(a)

Fig. 5.29 *Phasor diagram*

From the phasor diagram, it is clear that the phase angle between V_{RB} and I_R is $(30° - \phi)$.

$$W_1 = V_{RB} I_R \cos(30° - \phi)$$

Current through current coil of $W_2 = I_Y$

Voltage across voltage coil of $W_2 = V_{YB} = V_{YN} + V_{NB} = V_{YN} - V_{BN}$

From the phasor diagram, it is clear that phase angle between V_{YB} and I_Y is $(30° + \phi)$.

$$W_2 = V_{YB} \; I_Y \cos(30° + \phi)$$

But, $$I_R = I_Y = I_L$$

Three-phase AC Circuits

$$V_{RB} = V_{YB} = V_L$$

$$W_1 = V_L I_L \cos(30^\circ - \phi)$$

$$W_2 = V_L I_L \cos(30^\circ + \phi)$$

$$W_1 + W_2 = V_L I_L [\cos(30^\circ + \phi) + \cos(30^\circ - \phi)]$$

$$= V_L I_L (2\cos 30^\circ \cos\phi) = \sqrt{3}\, V_L I_L \cos\phi$$

Thus, the sum of two wattmeter readings gives three-phase power.

Three-phase AC Circuits

5.15.2 Measurement of Active Power for Delta Connected Load [Dec 20

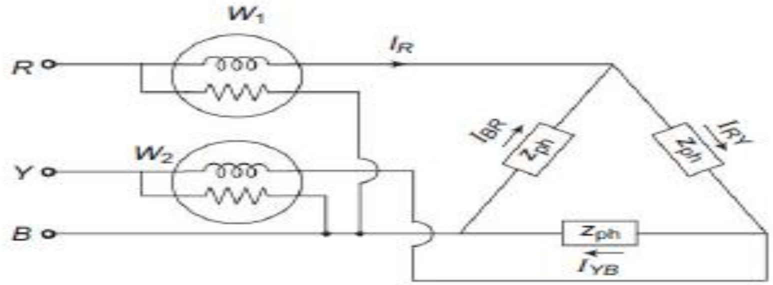


Fig. 5.30 *Two wattmeter method*

Current through current coil of $W_1 = \overline{I}_R = \overline{I}_{RY} - \overline{I}_{BR}$

Voltage across voltage coil of $W_1 = V_{RB}$

Current through current coil of $W_2 = I_Y = \overline{I}_{YB} - \overline{I}_{RY}$

Voltage across voltage coil of $W_2 = V_{YB}$

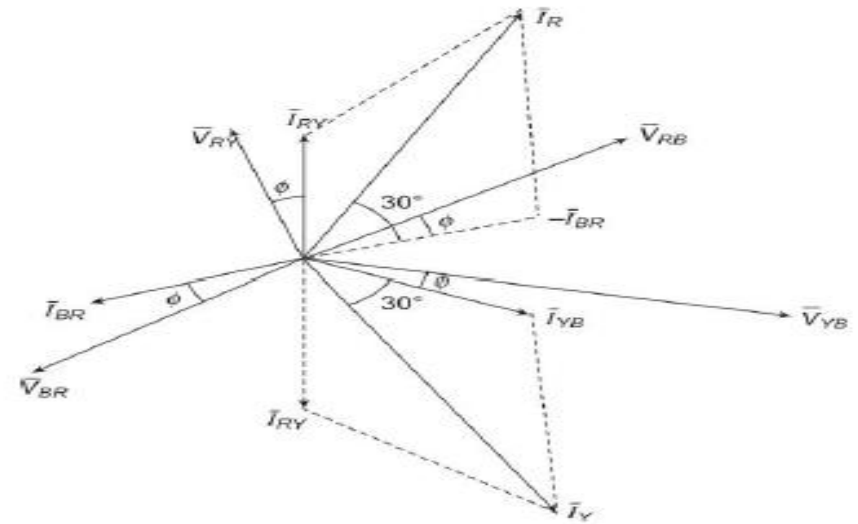


Fig. 5.31 *Phasor Diagram*

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From phasor diagram, it is clear that the phase angle between I_R and V_{RB} is $(30° - \phi)$.

$$W_1 = I_R V_{RY} \cos(30° - \phi) = I_L V_L \cos(30° - \phi)$$

From phasor diagram, it is clear that the phase angle between I_Y and V_{YB} is $(30° + \phi)$

$$W_2 = I_Y V_{YB} \cos(30° + \phi) = I_L V_L \cos(30° + \phi)$$

$$W_1 + W_2 = V_L I_L [\cos(30° - \phi) + \cos(30° + \phi)]$$

$$= V_L I_L (2 \cos 30° \cos \phi)$$

$$= \sqrt{3} V_L I_L \cos \phi$$

Thus, sum of two wattmeter readings gives three-phase power.

5.15.4 Measurement of Power Factor

1. Lagging Power Factor

$$W_1 = V_L I_L \cos(30^\circ - \phi)$$

$$W_2 = V_L I_L \cos(30^\circ + \phi)$$

$$\therefore \quad W_1 > W_2$$

$$W_1 + W_2 = \sqrt{3}\, V_L I_L \cos\phi$$

$$W_1 - W_2 = V_L I_L [\cos(30^\circ - \phi) - \cos(30^\circ + \phi) = V_L I_L \sin\phi$$

$$\frac{W_1 - W_2}{W_1 + W_2} = \frac{V_L I_L \sin\phi}{\sqrt{3} V_L I_L \cos\phi}$$

$$\tan\phi = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2}$$

$$\phi = \tan^{-1}\left(\sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2}\right)$$

$$\text{pf} = \cos\phi = \cos\left\{\tan^{-1}\left(\sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2}\right)\right\}$$

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2. *Leading Power Factor*

Figure 3.32 shows the phasor diagram of a balanced star-connected capacitive load.

$$W_1 = V_L I_L \cos(30^\circ + \phi)$$
$$W_2 = V_L I_L \cos(30^\circ - \phi)$$

$$\therefore W_1 < W_2$$

$$W_1 + W_2 = \sqrt{3} V_L I_L \cos\phi$$
$$W_1 - W_2 = -V_L I_L \sin\phi$$

$$\tan\phi = -\sqrt{3}\frac{(W_1 - W_2)}{(W_1 + W_2)}$$

$$\phi = \tan^{-1}\left(-\sqrt{3}\frac{(W_1 - W_2)}{(W_1 + W_2)}\right)$$

$$\text{pf} = \cos\phi = \cos\left\{\tan^{-1}\left(-\sqrt{3}\frac{W_1 - W_2}{W_1 + W_2}\right)\right\}$$

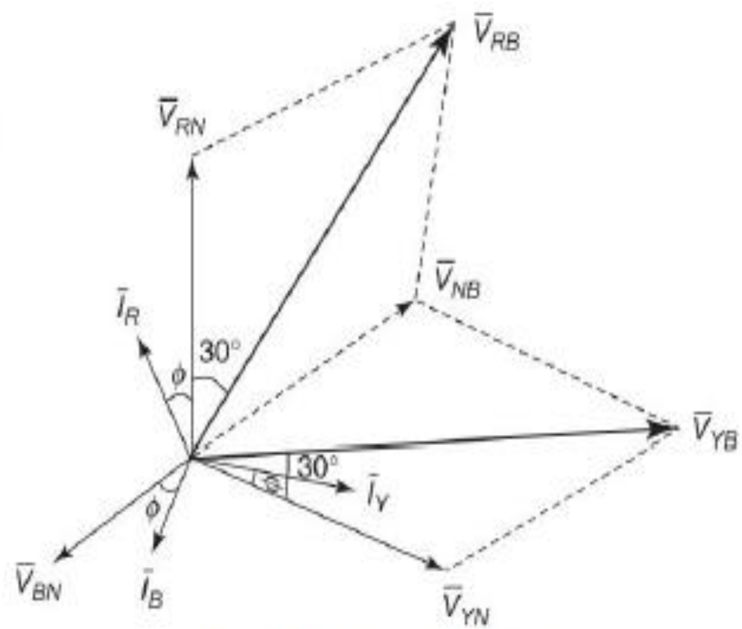

Fig. 5.32 *Phasor diagram*

Example 1

Two wattmeters are used to measure power in a three-phase balanced load. Find the power factor if (i) two readings are equal and positive, (ii) two readings are equal and opposite, and (iii) one wattmeter reads zero. **[Dec 2013]**

Solution (i) $W_1 = W_2$

(ii) $W_2 = 0\ W_1 = -W_2$

(i) Power factor if two readings are equal and positive

$$W_1 = W_2$$

$$\tan\phi = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2} = \sqrt{3}\,(0) = 0$$

$$\phi = 0^\circ$$

$$\text{Power factor} = \cos\phi = \cos(0^\circ) = 1$$

(ii) Power factor if two readings are equal and opposite

$$W_1 = -W_2$$

$$\tan\phi = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2} = \infty$$

$$\phi = 90^\circ$$

$$\text{Power factor} = \cos\phi = \cos(90^\circ) = 0$$

(iii) Power factor if one wattmeter reads zero

$$W_2 = 0$$

$$\tan\phi = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2} = \sqrt{3}\left(\frac{W_1}{W_1}\right) = \sqrt{3}$$

$$\phi = 60^\circ$$

$$\text{Power factor} = \cos\phi = \cos(60^\circ) = 0.5$$

Example 3

In a balanced three-phase circuit, power is measured by two wattmeters, the ratio of two wattmeter readings is 2 : 1. Determine the power factor of the system. **[Dec 2012]**

Solution

$$\frac{W_1}{W_2} = \frac{2}{1}$$

$$W_1 = 2W_2$$

$$\tan\phi = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2} = \sqrt{3}\,\frac{W_2}{3W_2} = \sqrt{3}\left(\frac{1}{3}\right) = 0.577$$

$$\phi = 30°$$

$$\text{pf} = \cos\phi = \cos(30°) = 0.866\ (\text{lagging})$$

Example 5

Two wattmeters are used to measure power in a 3ϕ balanced delta connected load using two wattmeter method. The readings of the 2 wattmeters are 500 W and 2500 W respectively. Calculate the total power consumed by the 3ϕ load and the power factor. **[May 2015]**

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Solution $W_1 = 500 \text{ W}$

$W_2 = 2500 \text{ W}$

(i) Total power

$$P = W_1 + W_2 = 500 + 2500 = 3 \text{ kW}$$

(ii) Power factor

The power factor is leading in nature since $W_1 < W_2$.

$$\tan\phi = -\sqrt{3}\,\frac{(W_1 - W_2)}{W_1 + W_2}$$

$$\phi = \tan^{-1}\left[-\sqrt{3}\,\frac{(W_1 - W_2)}{W_1 + W_2}\right] = \tan^{-1}\left[-\sqrt{3}\left(\frac{-2000}{3000}\right)\right] = 49.12$$

$$\text{pf} = \cos\phi = 0.65 \text{ (leading)}$$

Example 6

Find the power and power factor of the balanced circuit in which the wattmeter readings are 5 kW and 0.5 kW, the latter being obtained after the reversal of the current coil terminals of the wattmeter.

Solution $W_1 = 5\text{ kW}$

$W_2 = 0.5\text{ kW}$

(i) Power

When the latter reading is obtained after the reversal of the current coil terminals of the wattmeter,

$$W_1 = 5\text{ kW}$$

$$W_2 = -0.5\text{ kW}$$

$$\text{Power} = W_1 + W_2 = 5 + (-0.5) = 4.5\text{ kW}$$

(ii) Power factor

$$\tan\phi = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2} = \sqrt{3}\,\frac{(5+0.5)}{(5-0.5)} = 2.12$$

$$\phi = 64.72°$$

$$\text{Power factor} = \cos\phi = \cos(64.72°) = 0.43$$

Example 8

The input power of a three-phase motor was measured by the two-wattmeter method. The readings of two wattmeters are 5.2 kW and –1.7 kW and the line voltage is 415 V. Calculate the total active power, power factor and line current. [May 2013]

Solution

$$W_1 = 5.2 \text{ kW}$$

$$W_2 = -1.7 \text{ kW}$$

$$V_L = 415 \text{ V}$$

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(i) Total active power

$$P = W_1 + W_2 = 5.2 - 1.7 = 3.5 \text{ kW}$$

(ii) Power factor

$$\tan \phi = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2} = \sqrt{3}\left(\frac{5.2 + 1.7}{5.2 - 1.7}\right) = 3.41$$

$$\phi = 73.68^\circ$$

$$\text{pf} = \cos \phi = \cos (73.68^\circ) = 0.28 \text{ (lagging)}$$

(ii) Line current

$$P = \sqrt{3}\, V_L I_L \cos \phi$$

$$3.5 \times 10^3 = \sqrt{3} \times 415 \times I_L \times 0.28$$

$$I_L = 17.39 \text{ A}$$