Somaiya TRUST

Batch: A1-2 Roll No.: 16010622050
Experiment / assignment / tutorial No. 7
Grade: AA/ AB / BB / BC / CC / CD / DD

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Tutorial 7

1. Using De Moivre's Theorem, prove that $\frac{\sin 6\theta}{\sin 2\theta} = 16\cos^4\theta - 16\cos^2\theta + 3$

By De Moivres Theorem,

$$\cos 6\theta + i\sin 6\theta = (\cos\theta + i\sin\theta)^6$$

$$= \cos^6\theta + 6\cos^5\theta(i\sin\theta) + 15\cos^4\theta(i\sin\theta)^2 + 20\cos^3\theta(i\sin\theta)^3 + 15\cos^2\theta(i\sin\theta)^4 + 6\cos\theta(i\sin\theta)^5 + (i\sin\theta)^6$$

$$= \cos^6\theta + 6i\cos^5\theta\sin\theta - 15\cos^4\theta\sin^2\theta - 20i\cos^3\theta\sin^3\theta + 15\cos^2\theta\sin^4\theta + 6i\cos\theta\sin^5\theta - \sin^6\theta$$

Upon comparing real and imaginary parts,

$$\sin 6\theta = 6\cos^5\theta\sin\theta - 20\cos^3\theta\sin^3\theta + 6\cos\theta\sin^5\theta$$

We know, $\sin 2\theta = 2\cos\theta \cdot \sin\theta$

$$\therefore \frac{\sin 6\theta}{\sin 2\theta} = \frac{6\cos^5\theta\sin\theta - 20\cos^3\theta\sin^3\theta + 6\cos\theta\sin^5\theta}{2\cos\theta\sin\theta}$$

$$= \cancel{6\cos^5\theta} \quad 3\sin^4\theta + 3\cos^4\theta - 10\cos^2\theta\sin^2\theta$$

$$= 3(1-\cos^2\theta)^2 + 3\cos^4\theta - 10\cos^2\theta(1-\cos^2\theta)$$

$$= \cancel{3 - 3\cos^2\theta}$$

(not cancelled)

$$= 3\cos^4\theta - 10\cos^2\theta + 10\cos^4\theta + 3\cos^4\theta - 3\cos^2\theta + 3 - 3\cos^2\theta$$

$$= 16\cos^4\theta - 16\cos^2\theta + 3$$

As seen above, LHS = RHS and hence,

$$\frac{\sin 6\theta}{\sin 2\theta} = 16\cos^4\theta - 16\cos^2\theta + 3.$$

2. Prove that $\cos^8\theta + \sin^8\theta = \frac{1}{64}\left[\cos 8\theta + 28\cos 4\theta + 35\right]$.

Let $x = \cos\theta + i\sin\theta$ —— (i).

then, $\frac{1}{x} = \cos\theta - i\sin\theta$ —— (ii).

$\therefore\ x + \frac{1}{x} = 2\cos\theta$ (iii). & $x - \frac{1}{x} = 2i\sin\theta$ (iv).

$$\therefore\ \sin^8\theta = \frac{1}{(2i)^8}\left(x - \frac{1}{x}\right)^8 \quad \text{(From equation (iv).)}$$

$$= \frac{1}{2^8}\left(x^8 - 8x^6 + 28x^4 - 56x^2 + 70 - \frac{56}{x^2} + \frac{28}{x^4} - 8/x^6 + 1/x^8\right)$$

$$= \frac{1}{2^8}\left(\left(x^8 + \frac{1}{x^8}\right) + 28\left(x^4 + \frac{1}{x^4}\right) - 8\left(x^6 + \frac{1}{x^6}\right) - 56\left(x^2 + \frac{1}{x^2}\right) + 70\right)$$

$$\therefore\ \cos^8\theta = \frac{1}{2^8}\left(x + \frac{1}{x}\right)^8 \quad \text{(From equation (iii).)}$$

$$= \frac{1}{2^8}\left(\left(x^8 + \frac{1}{x^8}\right) + 28\left(x^4 + \frac{1}{x^4}\right) + 8\left(x^6 + \frac{1}{x^6}\right) + 70 + 56\left(x^2 + 1/x^2\right)\right)$$

Adding equations for $\sin^8\theta + \cos^8\theta$ gives,

$$= \frac{1}{2^8}\left[2\left(\left(x^8 + \frac{1}{x^8}\right) + 28\left(x^4 + \frac{1}{x^4}\right) + 70\right)\right]$$

$$\cancel{= \frac{1}{2^8}\left[2\left(2\left(\overset{\cos 8\theta}{x^8 + \frac{1}{x^8}}\right) + 2(28\cos 4\theta) + 70\right)\right]}$$

$$= \frac{1}{2^8}\cdot 2\cdot\left[2\cos 8\theta + 28(2\cos 4\theta) + 70\right]$$

$$= \frac{1}{2^{8}_{6}}\cdot\cancel{2}\cdot\cancel{2}\left[\cos 8\theta + 28\cos 4\theta + 35\right] = \frac{1}{64}\left[\cos 8\theta + 28\cos 4\theta + 35\right]$$

As seen LHS = RHS & hence, $\cos^8\theta + \sin^8\theta = \frac{1}{64}\left(\cos 8\theta + 28\cos 4\theta + 35\right)$.

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3. Show that adj (adj A) of $A = \frac{1}{9}\begin{bmatrix} -1 & -8 & 4 \\ -4 & 4 & 7 \\ -8 & -1 & -4 \end{bmatrix}$ is A itself.

We know, $A = \frac{1}{9}\begin{bmatrix} -1 & -8 & 4 \\ -4 & 4 & 7 \\ -8 & -1 & -4 \end{bmatrix}$

To prove $adj(adj(A)) = A$.

$adj(A) \Rightarrow A_{11} = C_{11}(-1)^2 = -9$

$A_{12} = C_{12}(-1)^3 = -72$; $A_{13} = C_{13}(-1)^4 = 36$

$A_{21} = C_{21}(-1)^3 = -36$; $A_{22} = C_{22}(-1)^4 = 36$

$A_{23} = C_{23}(-1)^5 = 63$; $A_{31} = C_{31}(-1)^4 = -72$

$A_{32} = C_{32}(-1)^5 = -9$; $A_{33} = C_{33}(-1)^6 = -36$

Hence, $adj(A) = \frac{1}{81}\begin{bmatrix} -9 & -72 & 36 \\ -36 & 36 & 63 \\ -72 & -9 & -36 \end{bmatrix}$

$\Rightarrow Adj(adj(A)) \Rightarrow A_{11} = C_{11}(-1)^2 = -729$

$A_{21} = C_{21}(-1)^3 = -2916$; $A_{31} = C_{31}(-1)^4 = -5832$

$A_{12} = C_{12}(-1)^3 = -5832$; $A_{22} = C_{22}(-1)^4 = 2916$

$A_{13} = C_{13}(-1)^4 = 2916$; $A_{32} = C_{32}(-1)^5 = -729$

$A_{23} = C_{23}(-1)^5 = -5103$; $A_{33} = C_{33}(-1)^6 = -2916$

Hence, $adj(A(\alpha)) = \begin{bmatrix} \cancel{\cos\alpha} & \cancel{-\sin\alpha} & \cancel{0} \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$

and $[Adj(A(\alpha))]^T = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cancel{\cos\alpha} & 0 \\ 0 & 0 & 1 \end{bmatrix}$

LHS = RHS

As seen, $\frac{[Adj(A(\alpha))]^T}{|A|} = [A(\alpha)]^{-1}$

$\therefore [A(\alpha)]^{-1} = A(-\alpha) \Rightarrow$ Hence shown.

④

4. If $A(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, prove that $[A(\alpha)]^{-1}$

$= A(-\alpha)$.

$$A(-\alpha) = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

We have to prove that $[A(\alpha)]^{-1} = A(-\alpha)$

$$LHS = [A(\alpha)]^{-1} = \frac{1}{|A|} \, adj\,(A(\alpha))^T$$

We know A^{-1} exists as, ~~expanding~~ determinat of A gives,

$$= 0 - 0 + 1\,(\cos^2\alpha + \sin^2\alpha)$$
$$= 1 \neq 0$$

Now, to find $adj\,A = (A_{ij})^{AT}$,

$$(A_{ij})^T = \begin{bmatrix} \cos(\alpha+0) & -(\sin\alpha - 0) & 0 \\ -(\sin\alpha - 0) & \cos(\alpha - 0) & 0 \\ 0 & 0 & \cos^2\alpha + \sin^2\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^{-1} = adj\,A = (A_{ij})^T = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

We can say, $A^{-1} = [A(\alpha)]^{-1} = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$

It is given that, $A(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

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That means, $A(-\alpha) = \begin{bmatrix} \cos(-\alpha) & -\sin(-\alpha) & 0 \\ \sin(-\alpha) & \cos(-\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Now, $\cos(-\alpha) = \cos\alpha$ and $\sin(-\alpha) = -\sin(\alpha)$

$$[A(\alpha)]^{-1} = [A(-\alpha)]$$

$$\downarrow$$

$$A(-\alpha) = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Hence, LHS = RHS as seen above.
Hence proven.