3. AC circuits

3.1 Generation of alternating voltage, average value, RMS value, form factor, crest factor, phasor representation in rectangular and polar form.

3.2 Steady state behavior of single phase AC circuits with pure R, L, and C, concept of inductive and capacitive reactance, phasor diagram of impedance, phase relationship in voltage and current.

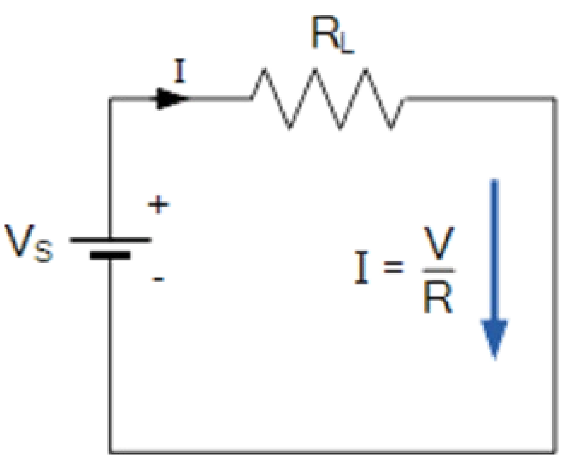
3.3 RL, RC and RLC series and parallel circuits, concept of impedance and admittance, power triangle, power factor, active, reactive and apparent power, concept of power factor improvement.

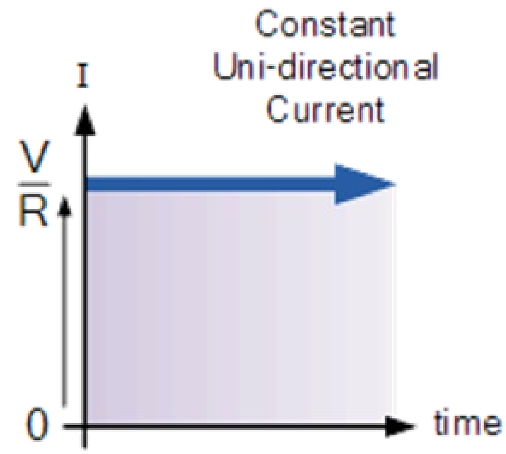
3.4 Series and parallel resonance, Q-factor and bandwidth

3.5 Three-phase balanced circuits, voltage and current relations in star and delta connections.

3.6 Measurement of power in 3-phase system using two wattmeter method

DC Circuit and Waveform





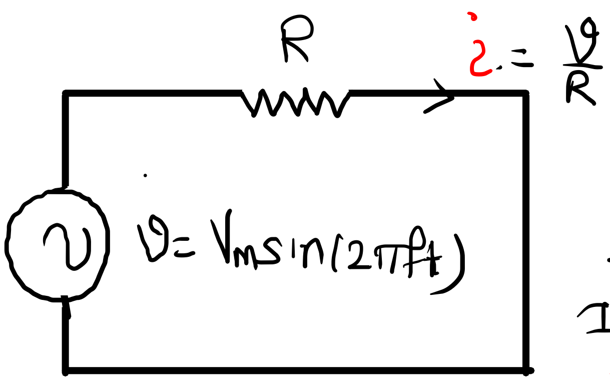
An alternating function or AC Waveform on the other hand is defined as one that varies in both magnitude and direction in more or less an even manner with respect to time making it a "Bi-directional" waveform.

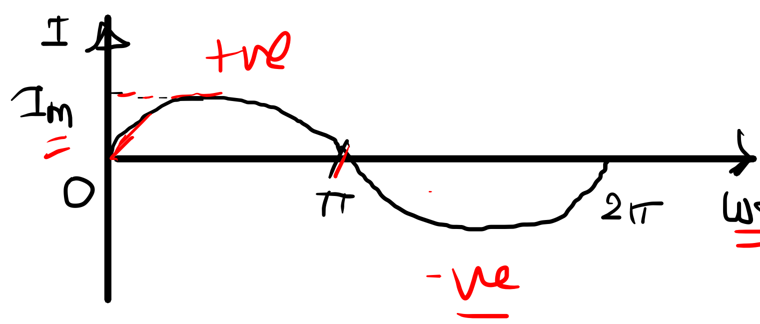
$$\vartheta = V_m \sin(2\pi f t)$$

→ frequency

↳ maximum or peak value

↳ Instantenous value

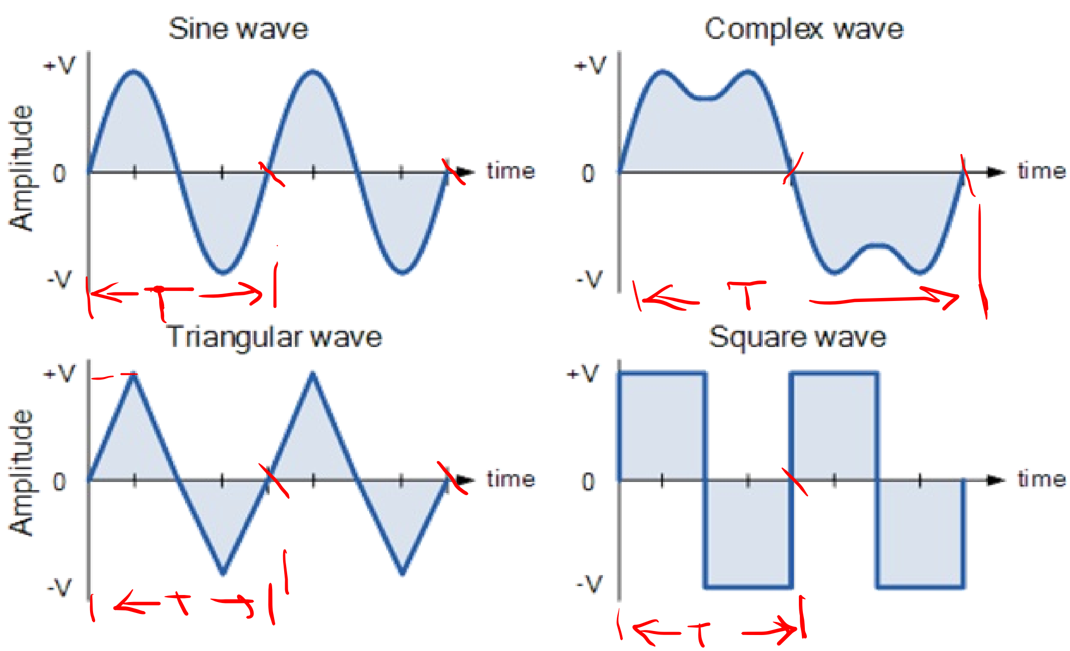




$$i = \frac{\vartheta}{R} = \frac{V_m \sin(2\pi f t)}{R}$$

$$i = I_m \sin(2\pi f t)$$

Types of AC Waveform



$f = \frac{1}{T}$

$v = 10 \sin(100\pi t) = 10 \sin(2\pi f t)$

amplitude $\Rightarrow 10$

freqn =

$2\pi f = 100$

$f = \frac{100}{2\pi}$

AC Waveform Characteristics

- The Period, (T) is the length of time in seconds that the waveform takes to repeat itself from start to finish. This can also be called the Periodic Time of the waveform for sine waves, or the Pulse Width for square waves.

- The Frequency, (f) is the number of times the waveform repeats itself within a one second time period. Frequency is the reciprocal of the time period, (f = 1/T) with the unit of frequency being the Hertz, (Hz).

- The Amplitude (A) is the magnitude or intensity of the signal waveform measured in volts or amp.

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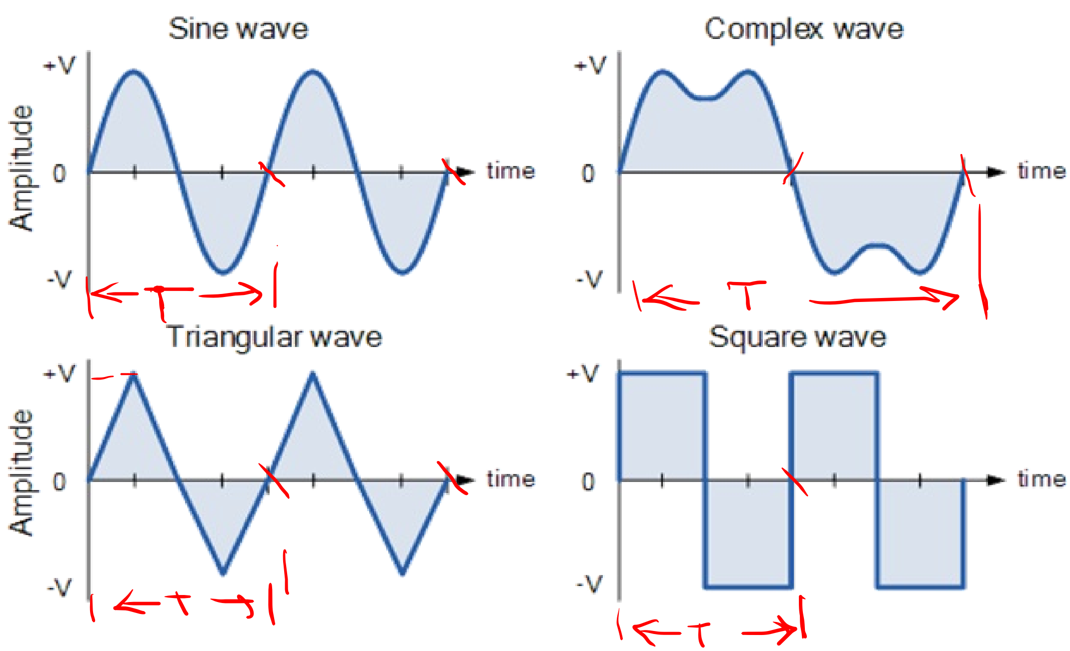

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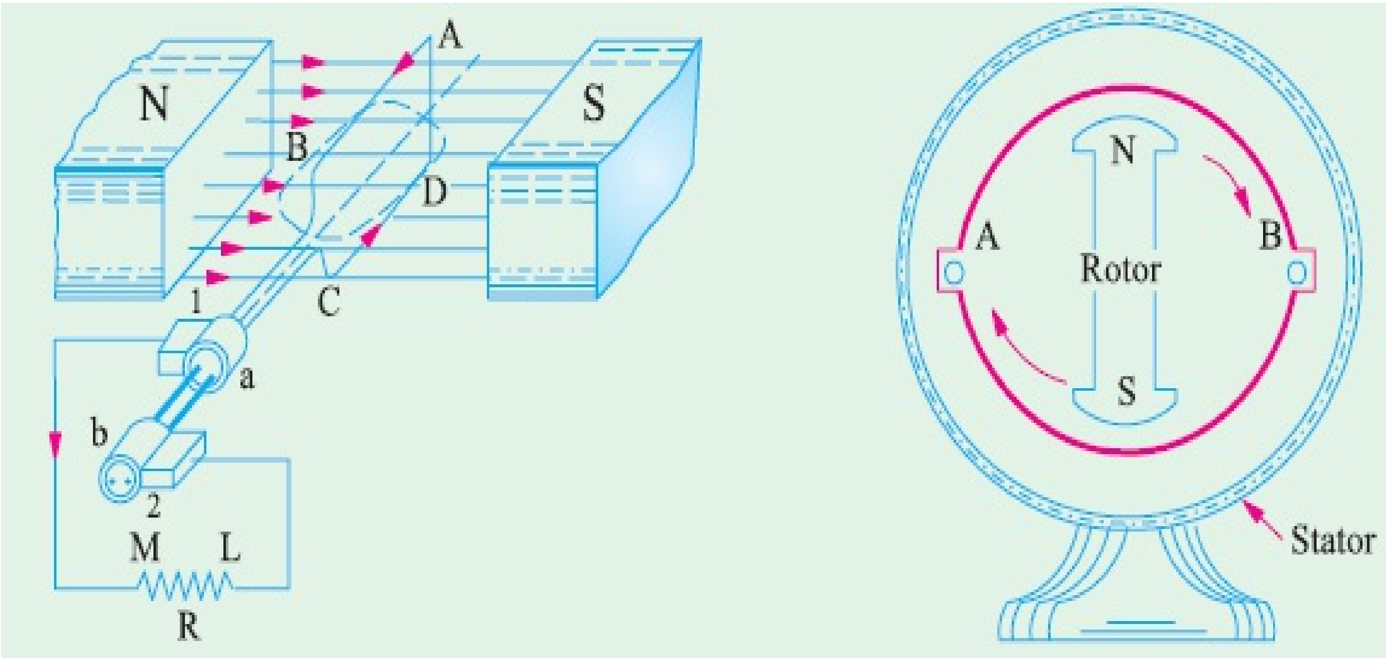
Generation of Alternating Voltages and Currents

Alternating voltage may be generated by rotating a coil in a magnetic field



$$e = -N \frac{d\phi}{dt}$$

No. of turns of coil

flux

The amount of EMF induced into a coil cutting the magnetic lines of force is determined by the three factors.

- Speed – the speed at which the coil rotates inside the magnetic field.
- Strength – the strength of the magnetic field.
- Length – the length of the coil or conductor passing through the magnetic field.

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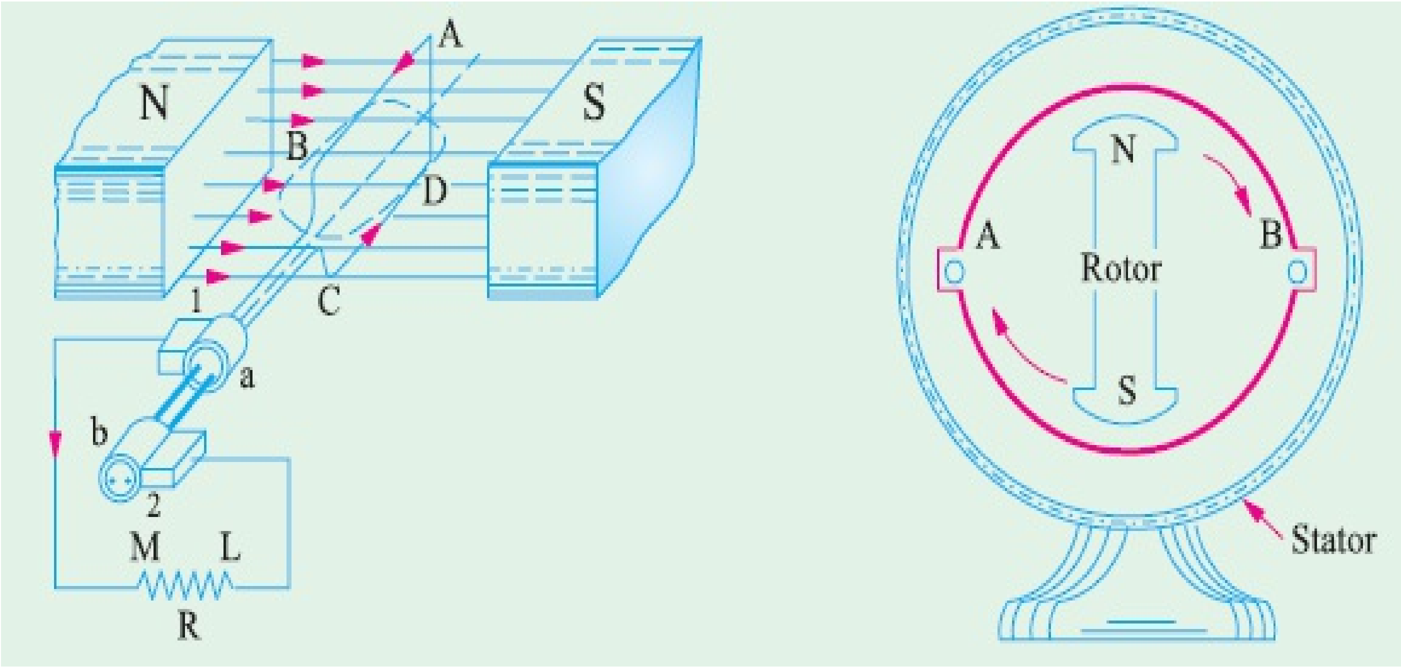

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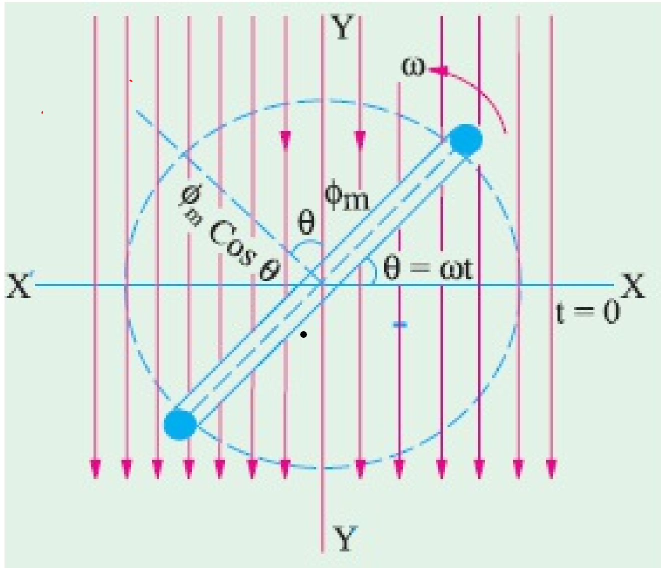
Equations of the Alternating Voltages and Currents



Rectangular coil, having N turns and rotating in a uniform magnetic field, with an angular velocity of ω radian/second.

Maximum flux Φm is linked with the coil, when its plane coincides with the X-axis.
In time t seconds, this coil rotates through an angle θ = ω t.
In this deflected position, the component of the flux which is perpendicular to the plane of the coil, is Φ = Φm cos ω t. Hence, flux linkages of the coil at any time are N Φ = N Φm cos ω t.
According to Faraday's Laws of Electromagnetic Induction, the e.m.f. induced in the coil is given by the rate of change of flux-linkages of the coil.
Hence, the value of the induced e.m.f. at this instant (i.e. when θ = ω t) or the instantaneous value of the induced e.m.f. is

$$e = -\frac{d}{dt}(N\Phi) \text{ volt} = -N.\frac{d}{dt}(\Phi_m \cos \omega t) \text{ volt} = -N\Phi_m\ \omega(-\sin \omega t) \text{ volt}$$

$$= \omega N \Phi_m \sin \omega t \text{ volt} = \omega N \Phi_m \sin \theta \text{ volt} \qquad ...(i)$$

When the coil has turned through 90° *i.e.* when θ = 90°, then sin θ = 1, hence *e* has maximum value, say E_m. Therefore, from Eq. *(i)* we get

$$E_m = \omega N \Phi_m = \omega N B_m A = 2\pi f N B_m A \text{ volt} \qquad ...(ii)$$

where B_m = maximum flux density in Wb/m^2 ; A = area of the coil in m^2

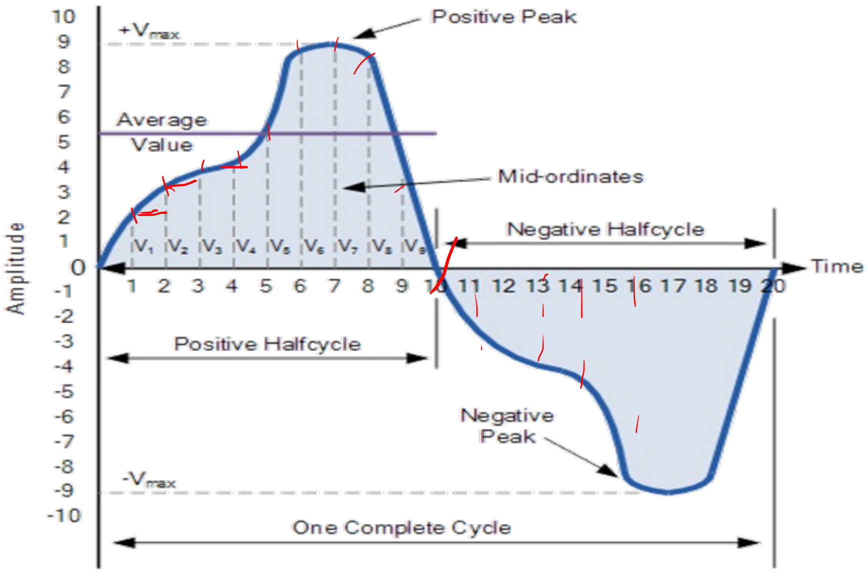
f = frequency of rotation of the coil in rev/second

Substituting this value of E_m in Eq. *(i)*, we get $e = E_m \sin\theta = E_m \sin \omega t$ ...*(iii)*

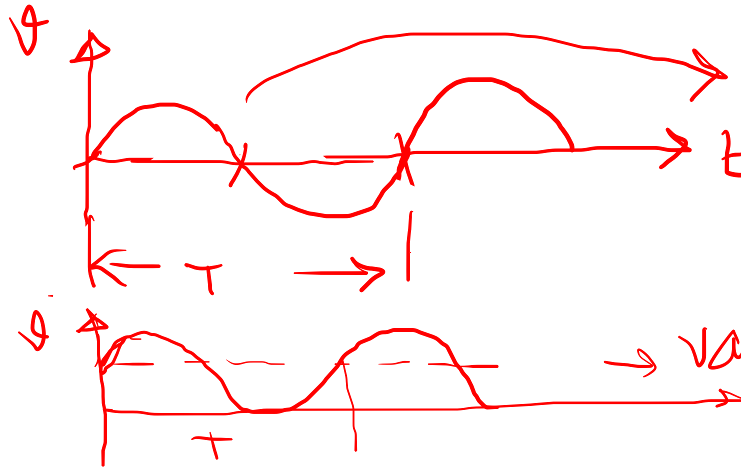
$\frac{e}{R} = \frac{E_m \sin\theta}{R}$ $\quad$ $i = I_m \sin\theta$

Average Value:

The average value of an alternating current is expressed by that steady current which transfers across any circuit the same charge as is transferred by that alternating current during the same time.



$$V_{average} = \frac{V_1 + V_2 + V_3 + V_4 + \dots + V_n}{n}$$

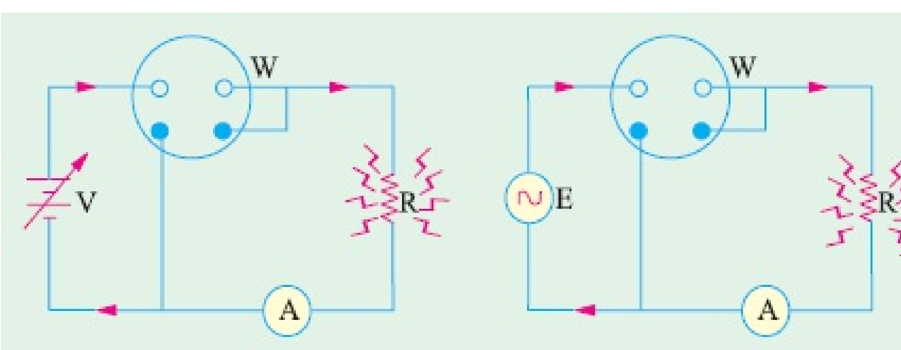


$$V_{av} = \frac{1}{T/2}\int_0^{T/2} v\,dt$$

$$V_{av} = \frac{1}{T}\int_0^{T} v\,dt$$

Root-Mean-Square (R.M.S.) Value

The r.m.s. value of an alternating current is given by that steady (d.c.) current which when flowing through a given circuit for a given time produces the same heat as produced by the alternating current when flowing through the same circuit for the same time.



$$V_{RMS} = \sqrt{\frac{V_1^2 + V_2^2 + V_3^2 + V_4^2 + \dots + V_n^2}{n}}$$

$$\boxed{v_{rms} = \sqrt{\frac{1}{T}\int_0^{T} v^2\,dt}}$$

Form Factor and Crest Factor :

Both Form Factor and Crest Factor can be used to give information about the actual shape of the AC waveform.

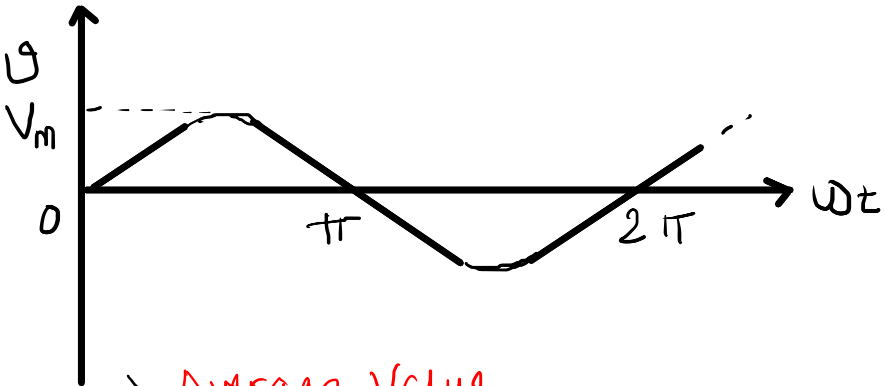
Form Factor is the ratio between the average value and the RMS value and is given as. For a pure sinusoidal waveform the Form Factor will always be equal to 1.11.

$$\text{Form Factor} = \frac{\text{R.M.S value}}{\text{Average value}} = \frac{0.707 \times \text{Vmax}}{0.637 \times \text{Vmax}}$$

Crest Factor is the ratio between the R.M.S. value and the Peak value of the waveform and is given as.

$$\text{Crest Factor} = \frac{\text{Peak value}}{\text{R.M.S. value}} = \frac{\text{Vmax}}{0.707 \times \text{Vmax}}$$

Example :- ① Find Average and Rms Value, Form Factor, peak factor



⇒ Average Value

$$V_{av} = \frac{1}{\pi}\int_0^{\pi} V_m \sin\omega t \, d\omega t$$

$$V_{av} = \frac{V_m}{\pi}\left[-\cos\omega t\right]_0^{\pi}$$

$$V_{av} = \frac{V_m}{\pi}\left[-\cos\pi + \cos 0\right]$$

$$\boxed{V_{av} = \frac{2V_m}{\pi}}$$

⇒ Rms Value

$$V_{rms} = \left[\frac{1}{2\pi}\int_0^{2\pi} V_m^2 \sin^2\omega t \, d\omega t\right]^{1/2}$$

$$= \left[\frac{V_m^2}{2\pi}\int_0^{2\pi}\left(\frac{1-\cos 2\omega t}{2}\right)d\omega t\right]^{1/2}$$

$$= \left[\frac{V_m^2}{4\pi}\left(\omega t - \frac{\sin 2\omega t}{2}\right)_0^{2\pi}\right]^{\frac{1}{2}}$$

$$= \left[\frac{V_m^2}{\cancel{4\pi}_{\,2}}\left(2\cancel{\pi} - 0 - 0 + 0\right)\right]^{\frac{1}{2}}$$

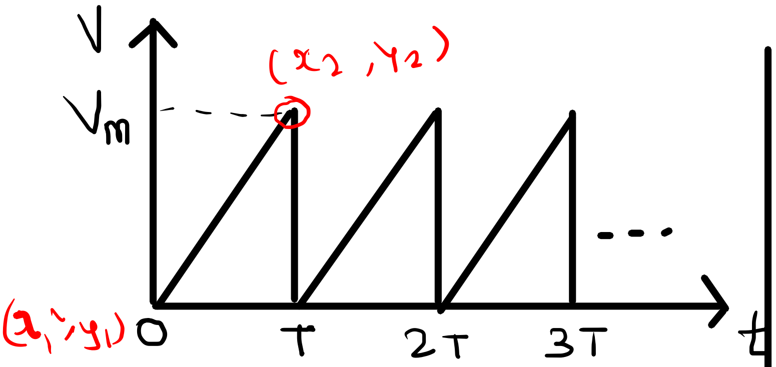
$$= \left[\frac{V_m^2}{2}\right]^{\frac{1}{2}}$$

$$\boxed{V_{rms} = \frac{V_m}{\sqrt{2}}}$$

$$\text{Form factor} = \frac{V_{rms}}{V_{av}} = \frac{\cancel{V_m}/\sqrt{2}}{2\cancel{V_m}/\pi} = \frac{\pi}{2\sqrt{2}} = 1.11$$

$$\text{Peak factor} = \frac{\text{peak value}}{\text{rms value}} = \frac{V_m}{V_m/\sqrt{2}} = \sqrt{2}$$

Example-2 : Find Average & RMS Value



$\Rightarrow$ Period $= T$

$$\frac{y - y_1}{y_1 - y_2} = \frac{x - x_1}{x_1 - x_2}$$

$$\frac{v - 0}{0 - V_m} = \frac{t - 0}{0 - T}$$

$$\frac{v}{V_m} = \frac{t}{T}$$

$$\boxed{v = \frac{V_m}{T} t}$$

$$V_{av} = \frac{1}{T}\int_0^T v\,dt$$

$$= \frac{1}{T}\int_0^T \frac{V_m}{T} \cdot t\,dt$$

$$= \frac{V_m}{T^2}\int_0^T t \cdot dt$$

$$= \frac{V_m}{T^2}\left[\frac{t^2}{2}\right]_0^T$$

$$= \frac{V_m}{\cancel{T^2}}\left[\frac{\cancel{T^2}}{2}\right]$$

$$\boxed{V_{av} = \frac{V_m}{2}}$$

$$V_{rms} = \left[\frac{1}{T}\int_0^T v^2\,dt\right]^{\frac{1}{2}}$$

$$= \left[\frac{1}{T}\int_0^T \left(\frac{V_m}{T} t\right)^2 dt\right]^{\frac{1}{2}}$$

$$= \left[\frac{1}{T}\int_0^T \frac{V_m^2 t^2}{T^2}\,dt\right]^{\frac{1}{2}}$$

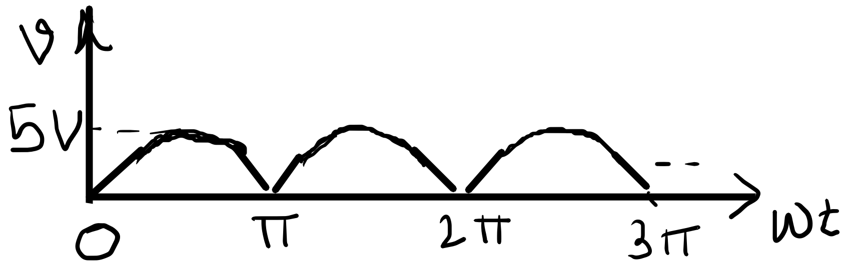
$$= \left[\frac{V_m^2}{T^3}\int_0^T t^2\,dt\right]^{\frac{1}{2}}$$

$$= \left[\frac{V_m^2}{T^3}\left(\frac{t^3}{3}\right)_0^T\right]^{\frac{1}{2}}$$

$$= \left[\frac{V_m^2}{\cancel{T^3}}\left(\frac{\cancel{T^3}}{3}\right)\right]^{\frac{1}{2}}$$

$$\boxed{V_{rms} = \frac{V_m}{\sqrt{3}}}$$

Example-3 : Find Average, RMS Value, Form Factor & peak Factor.



$$\Rightarrow \quad \vartheta = V_m \sin \omega t = 5 \sin \omega t$$

period $= \pi$

$$V_{av} = \frac{1}{\pi} \int_0^{\pi} 5 \sin \omega t \, d\omega t$$

$$V_{av} = \frac{5}{\pi} \left[-\cos \omega t \right]_0^{\pi}$$

$$V_{av} = \frac{5}{\pi} \left[-(-1) + 1 \right]$$

$$\boxed{V_{av} = \frac{10}{\pi}}$$

$$V_{rms} = \left[\frac{1}{\pi} \int_0^{\pi} (\vartheta)^2 \, d\omega t \right]^{\frac{1}{2}}$$

$$= \left[\frac{1}{\pi} \int_0^{\pi} (5 \sin \omega t)^2 \, d\omega t \right]^{\frac{1}{2}}$$

$$= \left[\frac{1}{\pi} \int_0^{\pi} 25 \sin^2 \omega t \, d\omega t \right]^{\frac{1}{2}}$$

$$= \left[\frac{25}{\pi} \int_0^{\pi} \left(\frac{1 - \cos 2\omega t}{2} \right) d\omega t \right]^{\frac{1}{2}}$$

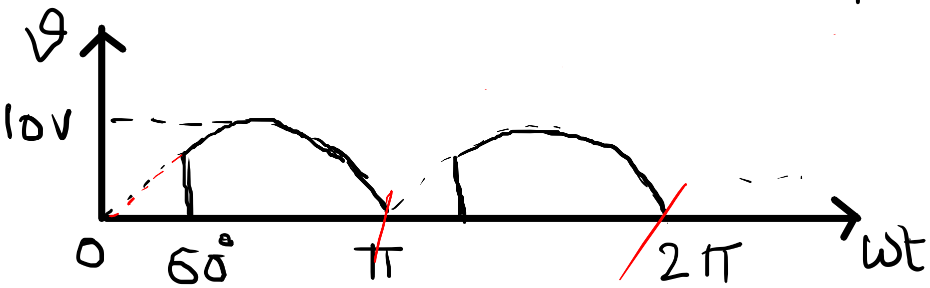
$$= \left[\frac{25}{2\pi} \left(\omega t - \frac{\sin 2 \omega t}{2} \right)_0^{\pi} \right]^{\frac{1}{2}}$$

$$V_{rms} = \left[\frac{25}{2\cancel{\pi}} \times \cancel{\pi} \right]^{\frac{1}{2}} = \frac{5}{\sqrt{2}}$$

$$\text{Form factor} = \frac{V_{rms}}{V_{av}} = \frac{5/\sqrt{2}}{10/\pi} = 1.1$$

$$\text{Peak factor} = \frac{5}{5/\sqrt{2}} = \sqrt{2}$$

Example 4: Find Average & Rms Value of full wave Rectified waveform delayed by $60°$.



period $= \pi$ $\quad 60° = \frac{\pi}{3}$

$$V_{av} = \frac{1}{\pi}\int_0^{\pi} 10\sin\omega t \, d\omega t$$

$$= \frac{1}{\pi}\left[\int_0^{\pi/3} 0 \, d\omega t + \int_{\pi/3}^{\pi} 10\sin\omega t \, d\omega t\right]$$

$$= \frac{1}{\pi}\int_{\pi/3}^{\pi} 10\sin\omega t \, d\omega t$$

$$= \frac{10}{\pi}\left(-\cos\omega t\right)_{\pi/3}^{\pi}$$

$$= \frac{10}{\pi}\left[-\cos\pi + \cos\frac{\pi}{3}\right]$$

$$V_{av} = \frac{10}{\pi}(1+0.5) = \frac{15}{\pi}$$

$$V_{rms} = \left[\frac{1}{\pi}\int_{\pi/3}^{\pi}(10\sin\omega t)^2 \, d\omega t\right]^{\frac{1}{2}}$$

$$= \left[\frac{100}{\pi}\int_{\pi/3}^{\pi}\sin^2\omega t \, d\omega t\right]^{\frac{1}{2}}$$

$$= \left[\frac{100}{\pi}\int_{\pi/3}^{\pi}\left(\frac{1-\cos 2\omega t}{2}\right)d\omega t\right]^{\frac{1}{2}}$$

$$= \left[\frac{50}{\pi}\left[\omega t - \frac{\sin 2\omega t}{2}\right]_{\pi/3}^{\pi}\right]^{1/2}$$

$$= \left[\frac{50}{\pi}\left(\pi - 0 - \frac{\pi}{3} + \frac{\sin(2\pi/3)}{2}\right)\right]^{\frac{1}{2}}$$

$$= \left[\frac{50}{\pi}\left(\pi - \frac{\pi}{3} + \frac{0.866}{2}\right)\right]$$

$$V_{rms} = 6.34V$$

Example- 5: An ac current of frequency 60 Hz has maximum value of 120 A. Write equation for its instanteneous value. Find (i) instanteneous value of current after (1/360) seconds (ii) time taken to reach 96 A for the first time.

$\Rightarrow$ $f = 60HZ$, $I_m = 120A$

$i(t) = I_m \sin \omega t = I_m \sin(2\pi f \cdot t)$

$i(t) = 120 \sin(2\pi \times 60\, t)$

$\underline{i(t) = 120 \sin(120\pi t)}$

(i) i after $t = \frac{1}{360}$ seconds

$i\left(\frac{1}{360}\right) = 120 \sin\left(120 \times \pi \times \frac{1}{360}\right)$

$i(t)\Big|_{t=\frac{1}{360}} = 120 \times \frac{\sqrt{3}}{2}$

$$\boxed{i(t)\Big|_{t=\frac{1}{360}} = 103.9A}$$

(ii) $i(t) = 96A$

$i(t) = 120 \sin(120\pi t)$

$96 = 120 \sin(120\pi t)$

$\sin(120\pi t) = \frac{96}{120}$

$120\pi t = \sin^{-1}\left(\frac{96}{120}\right)$

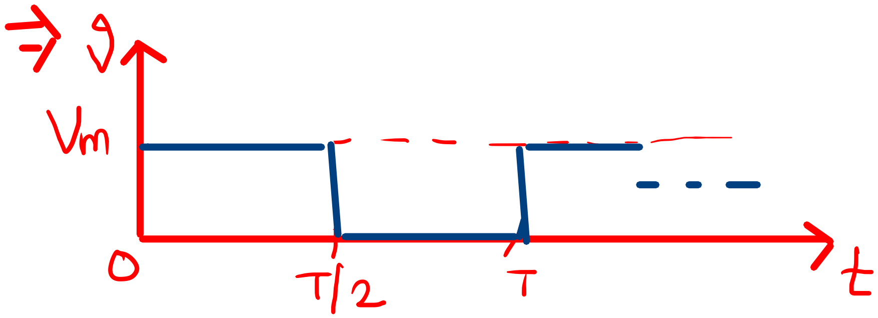
$t = \frac{0.927}{120\pi}$

$t = 2.45 \times 10^{-3}$ sec

$$\boxed{t = 2.45 \text{ m seconds}}$$

Example-6: Find Average and RMS value of the voltage expressed as

v =Vm , 0< t< T/2
= 0 , T/2 <t <T



period = T

$$V_{av} = \frac{1}{T}\int_0^{T/2} V_m \, dt + \int_{T/2}^{T} 0 \, dt$$

$$V_{av} = \frac{1}{T}\int_0^{T/2} V_m \, dt$$

$$= \frac{V_m}{T}\left[t \right]_0^{T/2}$$

$$V_{av} = \frac{V_m}{T} \times \frac{T}{2}$$

$$\boxed{V_{av} = \frac{V_m}{2}}$$

$$V_{rms} = \left[\frac{1}{T}\int_0^{T/2} (v^2)\, dt \right]^{1/2}$$

$$= \left[\frac{1}{T}\int_0^{T/2} V_m^2 \, dt \right]^{\frac{1}{2}}$$

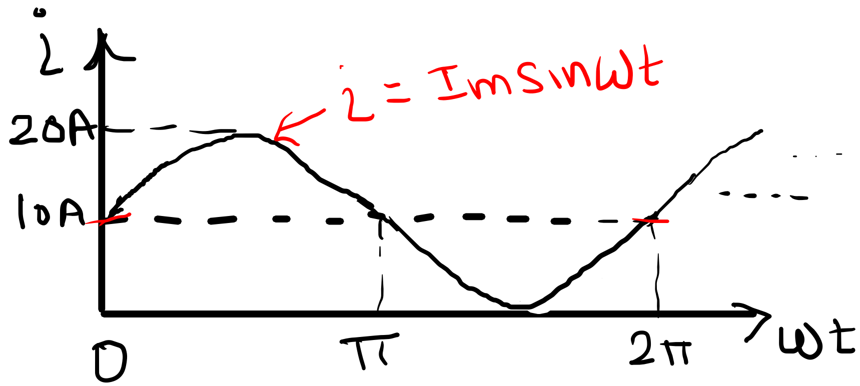
$$= \left[\frac{V_m^2}{T} \left(t \right)_0^{T/2} \right]^{1/2}$$

$$= \left[\frac{V_m^2}{T} \cdot \frac{T}{2} \right]^{1/2}$$

$$= \left[\frac{V_m^2}{2} \right]^{\frac{1}{2}}$$

$$\boxed{V_{rms} = \frac{V_m}{\sqrt{2}}}$$

Example-7: Find the RMS value of the current indicated in the following waveform.



$$i(t) = 10 + 10\sin\omega t$$

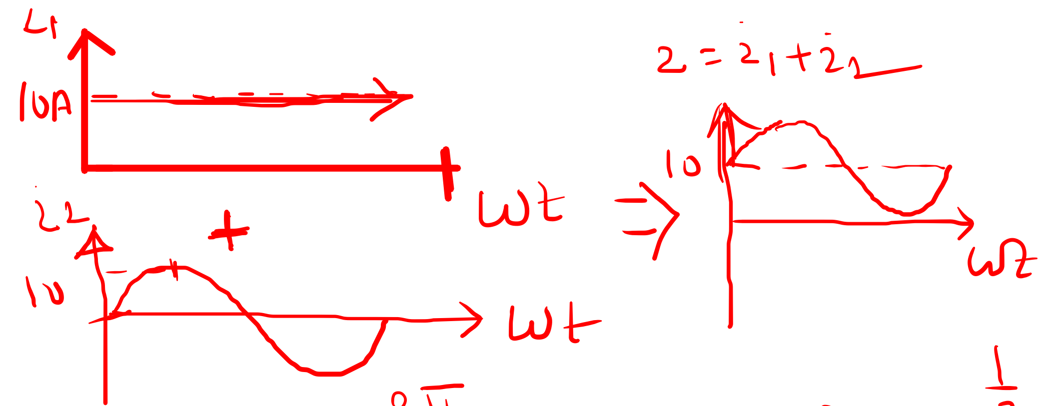
$$i_{av} = \frac{1}{2\pi}\int_0^{2\pi} (10 + 10\sin\omega t)\, d\omega t$$

$$= \frac{10}{2\pi}\Big[\omega t - \cos\omega t\Big]_0^{2\pi}$$

$$= \frac{10}{2\pi}\Big[2\pi - \cos 2\pi - 0 + \cos 0\Big]$$

$$= \frac{10}{2\pi}\Big[2\pi - \cancel{1} - 0 + \cancel{1}\Big]$$

$$i_{av} = \frac{10}{2\pi} \times 2\pi = 10\,A$$



$$I_{rms} = \left[\frac{1}{2\pi}\int_0^{2\pi} (10 + 10\sin\omega t)^2\, d\omega t\right]^{\frac{1}{2}}$$

$$= \left[\frac{1}{2\pi}\int_0^{2\pi} (100 + 200\sin\omega t + 100\sin^2\omega t)\, d\omega t\right]^{1/2}$$

intergrating & solving

$$I_{rms} = 12.25\,A$$