11.3 D'Alemberts' Principle (Dynamic Equilibrium)

- **Dynamic Equilibrium :** The force system consisting of external forces and inertia force can be considered to keep the particle in equilibrium. Since the resultant force externally acting on the particle is not zero, the particle is said to be in *Dynamic equilibrium*.

- **D'Alemberts' Principle :** *The algebraic sum of external force (ΣF) and inertia force (−ma) is equal to zero.*

$$\Sigma F + (-ma) = 0$$

- **For Rectilinear Motion**

$$\Sigma F_x + (-ma_x) = 0 \quad \text{and} \quad \Sigma F_y + (-ma_y) = 0$$

- **For Curvilinear Motion**

$$\Sigma F_t + (-ma_t) = 0 \quad \text{and} \quad \Sigma F_n + (-ma_n) = 0$$

Note : Comparing D'Alemberts' Principle with Newton's Second Law

We understand the Newton's Law as the original and D'Alembert had expressed the same concept in a different wording with adjustment of mathematical expression. So in this book we have solved problems considering Newton's Second Law.

How to Analyze a Problem ?

1. Draw F.B.D. of particle showing all active and reactive force with all known and unknown values by considering geometrical angles if any. (*Similar to F.B.D. in Statics*)
2. Show direction of acceleration and consider +ve sign conversion along the direction of acceleration.
3. Assumption for direction of acceleration.
   - **a.** If friction is not given then one can assume any direction for acceleration. Positive answer means assumed direction is correct.
   - **b.** If friction is given then one has to care fully analyse the problem and assume the direction of acceleration. Here we must get +ve answer. In case if answer is negative then one should resolve the whole problem with change in the direction opposite to the assumed direction.
4. If more than one particles are involved in a system then find the kinematic relation between the particles. (i.e. relation of displacement, velocity and acceleration)
5. For finding the kinematic relation of connected particles introduce the tension in each cord. Apply the Virtual Work Principle which says - *total virtual work done by internal force (tension) is zero*. Consider work done to be +ve if displacement and tension are in same direction and work done to be −ve if displacement and tension are in one direction

Problem 2

A 50 kg block kept on the top of a 15° sloping surface is pushed down the plane with an initial velocity of 20 m/s. If $\mu_k = 0.4$, determine the distance traveled by the block and the time it will take as it comes to rest.

Solution

(i) **Consider F.B.D. of 50 kg Block** (Refer *figure 11.4*)

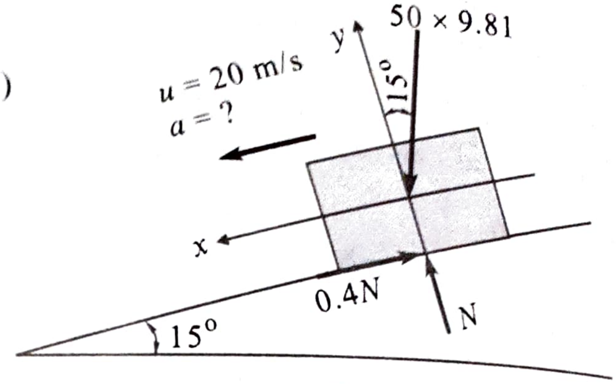

Fig. 11.4

(ii) By Newton's IInd Law, we have

$$\Sigma F_y = ma_y = 0 \quad (\because a_y = 0)$$

$$N - 50 \times 9.81 \cos 15 = 0$$

$$N = 50 \times 9.81 \cos 15$$

$$\Sigma F_x = ma_x$$

$$50 \times 9.81 \sin 15 - 0.4 \times 50 \times 9.81 \cos 15 = 50a$$

$$a = -1.25 \text{ m/s}^2 \text{ (Retardation)}$$

(iii) $u = 20$ m/s ; $v = 0$; $a = -1.25$ m/s^2 ; $s = ?$; $t = ?$

$$v = u + at$$

$$0 = 20 + (-1.25)t$$

$$t = 16 \text{ sec.}$$

$$s = ut + \frac{1}{2}at^2$$

$$s = 20 \times 16 + \frac{1}{2}(-1.25) \times (16)^2$$

$$s = 160 \text{ m}$$ **Ans.**

Problem 3

An aeroplane has a mass of 25000 kg and its engines develop a total thrust of 40 kN along the run way. The force of air resistance to motion of aeroplane is given by $R = 2.25v^2$ where v is m/s and R is in Newtons. Determine the length of runway required if the plane takes off and becomes airborne at a speed of 240 km/hr.

Solution

(i) **Consider F.B.D. of Plane** (Refer *figure 11.5*)

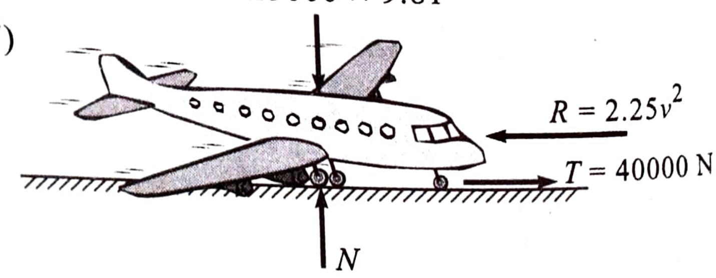

Fig. 11.5 : F.B.D. of Plane

By Newton's IInd Law, we have

$$\Sigma F_x = ma_x$$

$$40000 - 2.25v^2 = 25000a$$

$$40000 - 2.25v^2 = 25000 \times v\frac{dv}{ds}$$

$$\therefore ds = 25000\left(\frac{v\,dv}{40000 - 2.25v^2}\right)$$

Integrating both the sides, we get

$$\int_0^s ds = 25000\int_0^{66.67}\left(\frac{v\,dv}{40000 - 2.25v^2}\right) \qquad \left[v = 240 \times \frac{5}{18} = 66.67 \text{ m/s}\right]$$

$$s = \frac{25000}{-2.25 \times 2}\left[\log_e(40000 - 2.25\,v^2)\right]_0^{66.67}$$

$$\therefore s = 1598.3 \text{ m (Runway length)}$$ **Ans.**

Problem 4

Two blocks A and B are held stationary 10 m apart on a 20° inclined plane. The coefficient of kinetic friction between plane and body A is 0.3, where as between the plane and body B is 0.1. If the blocks are released simultaneously, calculate the time taken and distance travelled by each block before they are at the verge of collision.

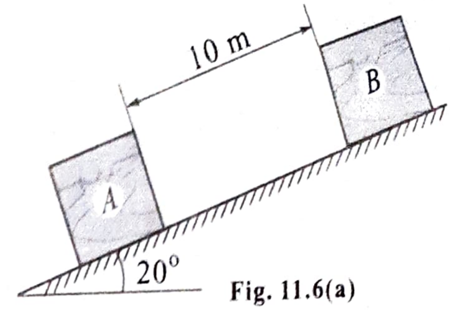

Fig. 11.6(a)

Solution

(i) **Consider F.B.D. of Block A.** Refer *figure 11.6(b)*

By Newton's II[nd] Law, we have

$$\Sigma F_x = ma_x$$

$$mg \sin 30 - 0.3 \times N_A = ma_A$$

$$mg \sin 30 - 0.3 \times mg \cos 20 = ma_A$$

$$a_A = 0.588 \text{ m/s}^2 \; (20°)$$

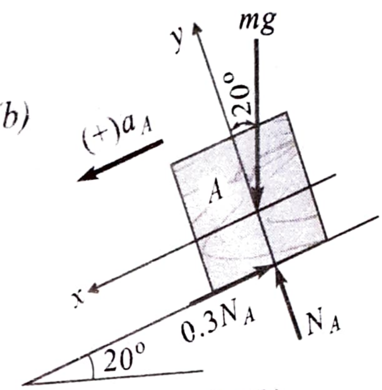

Fig. 11.6(b)

(ii) **Consider F.B.D. of Block B.** Refer *figure 11.6(c)*

By Newton's II[nd] Law, we have

$$\Sigma F_x = ma_x$$

$$mg \sin 30 - 0.1 \times mg \cos 30 = ma_B$$

$$a_B = 2.431 \text{ m/s}^2$$

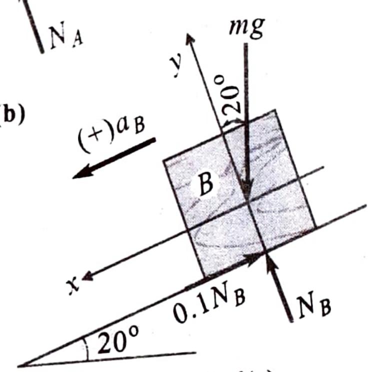

Fig. 11.6(c)

(iii) Let s_A be the distance travelled by block A. For block B to collide block A, it has to travel $s_B = (s_A + 10)$ m.

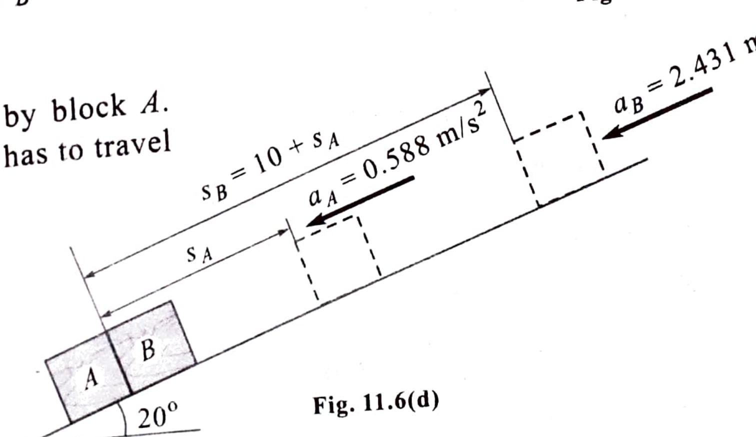

Fig. 11.6(d)

Motion of block A

$$s = ut + \frac{1}{2}at^2$$

$$s_A = 0 + \frac{1}{2} \times 0.588 \times t^2$$

$$s_A = 0.294t^2 \quad \text{... (i)}$$

Motion of block B

$$s = ut + \frac{1}{2}at^2$$

$$s_A + 10 = 0 + \frac{1}{2} \times 2.431 \times t^2$$

$$s_A = 1.2155t^2 - 10 \quad \text{... (ii)}$$

Equating (i) and (ii)

$$0.294t^2 = 1.2155t^2 - 10$$

$t = 3.294$ sec **Ans.**

$s_A = 3.19$ m and $s_B = (s_A + 10) = 13.19$ m **Ans.**

Problem 9

Two blocks A and B of masses $M_A = 280$ kg and $M_B = 420$ kg are joined by an inextensible cable as shown in figure *11.11(a)*. Assume that the pulley is frictionless and $\mu = 0.3$ between block A and the surface. The system is initially at rest. Determine **(a)** acceleration of block A. **(b)** velocity after it has moved 3.5 m and **(c)** velocity after 1.5 sec.

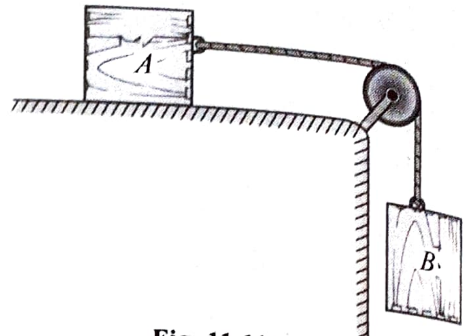

Fig. 11.11(a)

Solution

Since both the blocks are connected by single rope, acceleration of both the blocks will be same.

(i) Consider F.B.D. of Block A

By Newton's II[nd] Law, we have

$\Sigma F_y = ma_y = 0 \qquad (\because a_y = 0)$

$N_A - 280 \times 9.81 = 0$

$N_A = 280 \times 9.81$

$\Sigma F_x = ma_x$

$T - \mu N_A = 280a$

$T - 0.3 \times 280 \times 9.81 = 280a$

$T = 824.04 + 280a \qquad .....\ \text{(I)}$

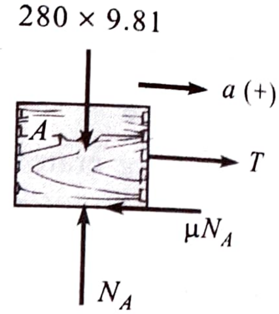

Fig. 11.11(b) : F.B.D. of Block A

(ii) Consider F.B.D. of Block B

By Newton's II[nd] Law, we have

$\Sigma F_y = ma_y$

$420 \times 9.81 - T = 420\,a$

$T = 4120.2 - 420\,a \qquad .....\ \text{(II)}$

Solving equation (I) and (II), we get

$a = 4.709$ m/s^2; $\quad T = 2142.56$ N ***Ans.***

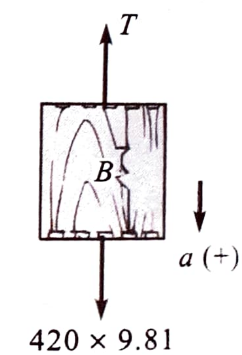

Fig. 11.11(c) : F.B.D. of Block B

(iii) Velocity of Block A After it has Moved a Distance of 3.5 m

$v^2 = u^2 + 2as$

$v^2 = 0 + 2 \times 4.709 \times 3.5$

$v = 5.741$ m/s ***Ans.***

(iv) Velocity of Block A After 1.5 sec

$v = u + at$

$v = 0 + 4.709 \times 1.5$

$v = 7.064$ m/s ***Ans.***

Problem 10

A body of mass 25 kg resting on a horizontal table is connected by string passing over a smooth pulley at the edge of the table to another body of mass 3.75 kg and hanging vertically as shown. Initially, the friction between 25 kg mass and the table is just sufficient to prevent the motion. If an additional 1.25 kg is added to the 3.75 kg mass, find the acceleration of the masses.

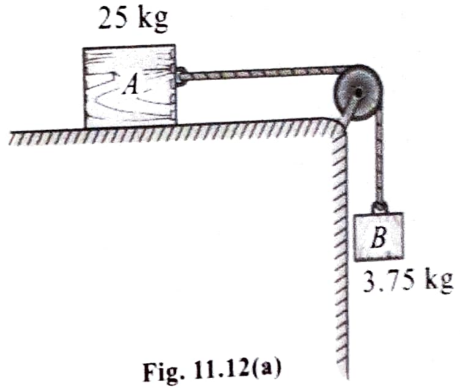

Fig. 11.12(a)

Solution

(i) **Static Equilibrium Analysis**

Consider F.B.D. of block A

$\sum F_y = 0$

$N - 25 \times 9.81 = 0$

$N = 245.25\text{N}$

$\sum F_x = 0$

$T - \mu N = 0$

$3.75 \times 9.81 - \mu \times 245.25 = 0$

$\mu = 0.15$

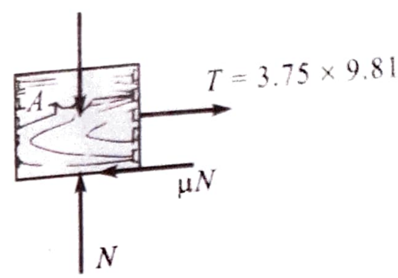

Fig. 11.12(b) : F.B.D. of Block A

(ii) **Dynamic Equilibrium Analysis**

Assume $\mu_s = \mu_k = 0.15$

By Newton's II$^{\text{nd}}$ Law, we have

Consider the F.B.D. of block A

$\sum F_y = ma_y = 0 \quad (\because a_y = 0)$

$N - 25 \times 9.81 = 0$

$N = 245.25\text{N}$

$\sum F_x = ma_x$

$T - \mu N = 25a$

$T - 0.15 \times 245.25 = 25a$

$T = 36.79 + 25a$ ..... (I)

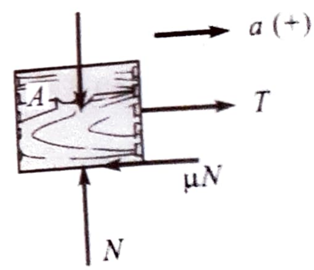

Fig. 11.12(c) : F.B.D. of Block A

(iii) **Consider the F.B.D. of Block B**

By Newton's II$^{\text{nd}}$ Law, we have

$\sum F_y = ma_y$

$5 \times 9.81 - T = 5 \times a$

$T = 49.05 - 5a$ ..... (II)

Equating equation (I) and (II) we get

$a = 0.409 \text{ m/s}^2$ and $T = 47.005$ N ***Ans.***

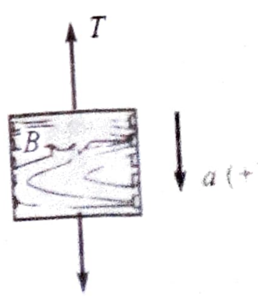

Fig. 11.12(d) : F.B.D. of Block B

Problem 13

Masses A and B are 7.5 kg and 27.5 kg respectively as shown in *figure 11.15(a)*. The coefficient of friction between A and the plane is 0.25 and between B and the plane is 0.1. What is the force between the two as they slide down the incline?

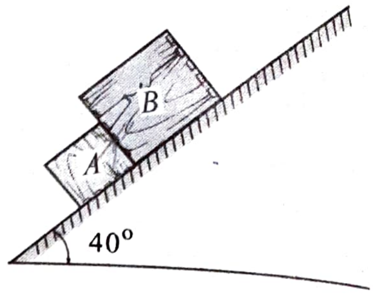

Fig. 11.15(a)

Solution

(i) Consider F.B.D. of Block A

By Newton's IInd Law, we have

$$\Sigma F_y = ma_y = 0 \qquad (\because\ a_y = 0)$$

$$N_A - 7.5 \times 9.81 \cos 40 = 0$$

$$N_A = 56.36 \text{ N}$$

$$\Sigma F_x = ma_x$$

$$P + 7.5 \times 9.81 \sin 40 - 0.25 \times 56.36 = 7.5a$$

$$33.2 + P = 7.5a \qquad \text{..... (I)}$$

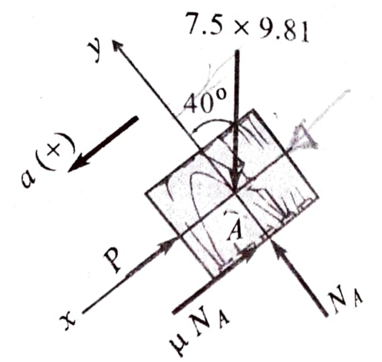

Fig. 11.15(b) : F.B.D. of Block A

(ii) Consider F.B.D. of Block B

By Newton's IInd Law, we have

$$\Sigma F_y = ma_y = 0 \qquad (\because\ a_y = 0)$$

$$N_B - 27.5 \times 9.81 \cos 40 = 0$$

$$N_B = 206.66 \text{ N}$$

$$\Sigma F_x = ma_x$$

$$27.5 \times 9.81 \sin 40 - P - 0.1 \times 206.66 = 27.5a$$

$$152.74 - P = 27.5a \qquad \text{..... (II)}$$

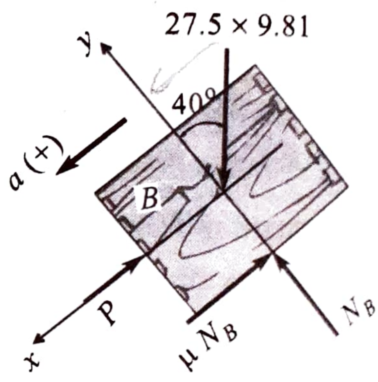

Fig. 11.15(c) : F.B.D. of Block B

Solving (I) and (II)

$$P = 6.625 \text{ and } a = 5.31 \text{ m/s}^2$$

Problem 14

In the system of pulleys, the pulleys are massless and string are inextensible. Mass of A = 2 kg, mass of B = 4 kg and mass = 6 kg. If the system is released from rest, find **(i)** tension in each of the three string **(ii)** acceleration of each of the three masses.

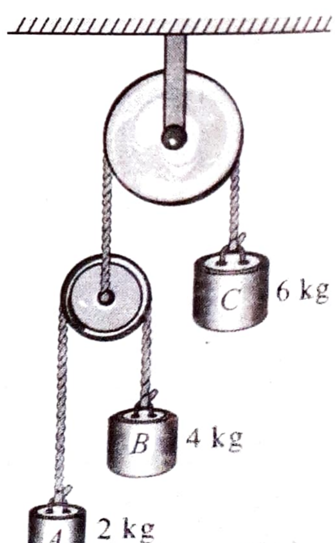

Fig. 11.16(a)

Solution

Assume the direction of motion of all block as above.

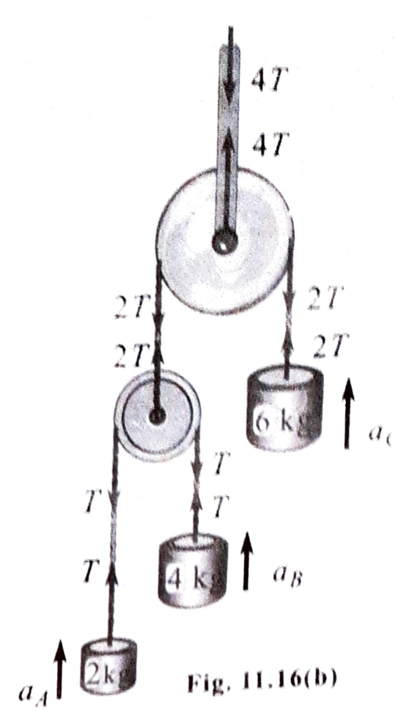

Fig. 11.16(b)

(i) Kinematic Relation

$$Tx_A + Tx_B + 2Tx_C = 0$$

$$x_A + x_B + 2x_C = 0$$

Differentiating w.r.t. t

$$v_A + v_B + 2v_C = 0$$

Differentiating w.r.t. t again,

$$a_A + a_B + 2a_C = 0 \quad \text{..... (I)}$$

(ii) Consider F.B.D. of Block *A*

$$\Sigma F_y = ma_y$$

$$T = 2 \times 9.81 = 2a_A$$

$$a_A = 0.5T - 9.81 \quad \text{..... (II)}$$

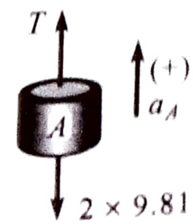

Fig. 11.16(c) : F.B.D. of Block *A*

(iii) Consider F.B.D. of Block *B*

$$\Sigma F_y = ma_y$$

$$T - 4 \times 9.81 = 4a_B$$

$$a_B = 0.25T - 9.81 \quad \text{..... (III)}$$

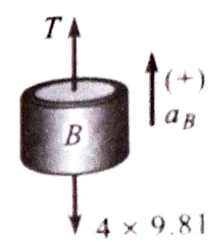

Fig. 11.16(d) : F.B.D. of Block *B*

(iv) Consider F.B.D. of Block *C*

$$\Sigma F_y = ma_y$$

$$2T - 6 \times 9.81 = 6a_C$$

$$a_C = 0.33T - 9.81 \quad \text{..... (IV)}$$

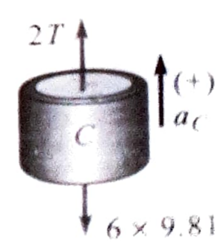

Fig. 11.16(e) : F.B.D. of Block *C*

(v) Putting equation (II) and (III) and (IV) in equation (I)

$$a_A + a_B + a_C = 0$$

$$(0.5T - 9.81) + (0.25T - 9.81) + (0.33T - 9.81) = 0$$

$$0.5T + 0.25T + 0.33T - 9.81 - 9.81 - 9.81 = 0$$

$$1.08T - 29.43 = 0$$

$$T = 27.25 \text{ N} \quad \textbf{Ans.}$$

(vi) From equation (I)

$$a_A = 0.5 \times 27.25 - 9.81$$

$$a_A = 3.82 \text{ m/s}^2 \ (\uparrow) \quad \textbf{Ans.}$$

(vii) From equation (II)

$$a_B = 0.25 \times 27.25 - 9.81$$

$$a_B = -3 \text{ m/s}^2 \text{ (Wrong assumed direction)}$$

$$a_B = 3 \text{ m/s}^2 \ (\downarrow) \quad \textbf{Ans.}$$

(viii) From equation (III)

$$a_C = 0.33 \times 27.25 - 9.81$$

$$a_C = -0.82 \text{ m/s}^2 \text{ (Wrong assumed direction)}$$

$$a_C = 0.82 \text{ m/s}^2 \ (\downarrow) \quad \textbf{Ans.}$$

Problem 15

Block A = 100 kg shown in the *figure 11.17(a)* is observed to move upward with an acceleration of 1.8 m/s². Determine **(i)** mass of block B **(ii)** corresponding tension in the cable.

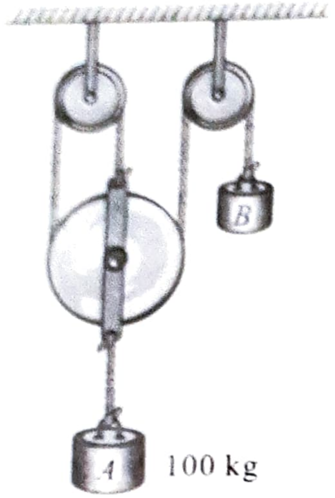

Fig. 11.17(a)

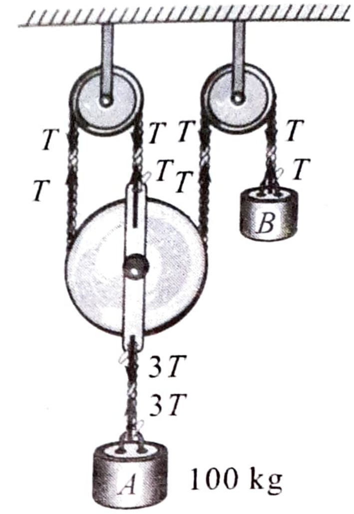

Fig. 11.17(b)

Solution

(i) Kinematic Relation

Work done by internal forces = 0

$3Tx_A - Tx_B = 0$

$3x_A = x_B$

Differentiating w.r.t. t

$3v_A = v_B$

Differentiating w.r.t. t

$3a_A = a_B$

$\therefore\ a_B = 3 \times 1.8 \qquad (\because\ a_A = 1.8 \text{ m/s}^2)$

$a_B = 5.4 \text{ m/s}^2$

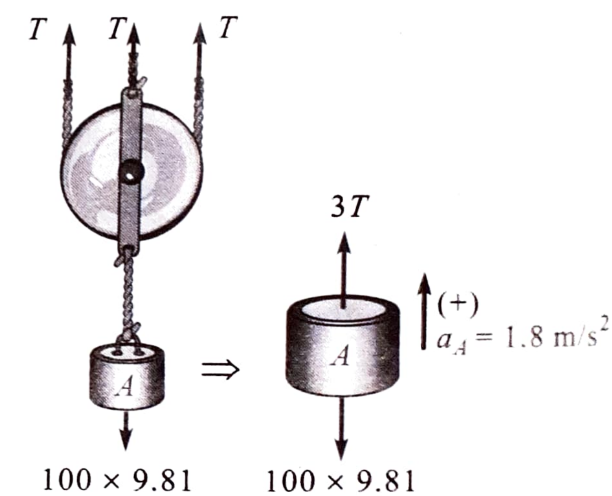

Fig. 11.17(c) : F.B.D. of Block A

(ii) Consider F.B.D. of Block A

By Newton's II[nd] Law, we have

$\sum F_y = ma_y$

$3T - 100 \times 9.81 = 100 \times 1.8$

$T = 387$ N ***Ans.***

(iii) Consider the F.B.D. of Block B

By Newton's II[nd] Law, we have

$\sum F_y = ma_y$

$m_B \times 9.81 - T = m_B \times a_B$

$m_B (9.81 - 5.4) = 387$

$m_B = 87.76$ kg ***Ans.***

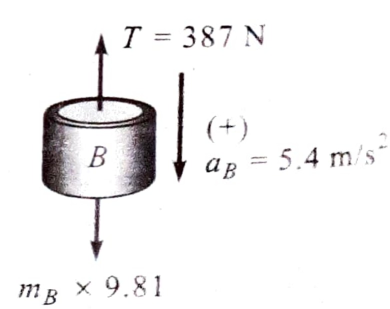

Fig. 11.17(d) : F.B.D. of Block B

Problem 19

Three blocks A, B and C are connected as shown in the figure 11.21(a). Determine the acceleration of each weight and tension in the cord. $W_A = 150$ N, $W_B = 450$ N and $W_C = 300$ N.

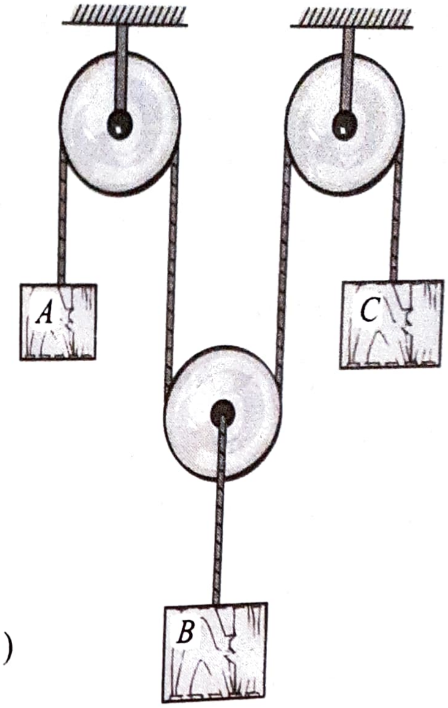

Fig. 11.21(a)

Solution

(i) **Kinematic Relation**

Assume direction of acceleration $a_A(\uparrow)$, $a_B(\downarrow)$, $a_C(\uparrow)$

By virtual work principle, we have

Total virtual work done by internal force (Tension) = zero

$$Tx_A - 2Tx_B + Tx_C = 0$$

$$x_A + x_C = 2x_B$$

Differentiating w.r.t. t

$$v_A + v_C = 2v_B$$

Differentiating w.r.t. t

$$a_A + a_C = 2a_B \quad \text{..... (I)}$$

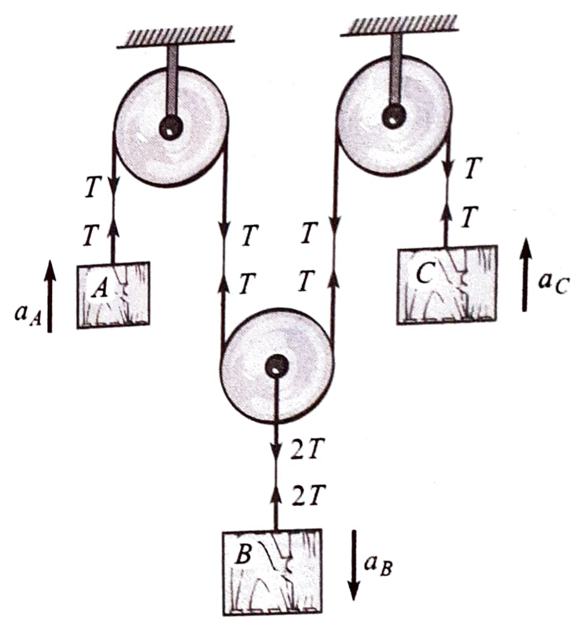

Fig. 11.21(b)

(ii) **Consider F.B.D. of Block A**

By Newton's IInd Law, we have

$$T - 150 = \frac{150}{9.81} a_A$$

$$a_A = \frac{9.81}{150} T - 9.81 \quad \text{..... (II)}$$

(iii) **Consider the F.B.D. of Block B**

By Newton's IInd Law, we have

$$450 - 2T = \frac{450}{9.81} a_B$$

$$a_B = 9.81 - \frac{2 \times 9.81}{450} T \quad \text{..... (III)}$$

(iv) **Consider F.B.D. of Block C**

By Newton's IInd Law, we have

$$T - 300 = \frac{300}{9.81} a_C$$

$$a_C = \frac{9.81}{150} T - 9.81 \quad \text{..... (IV)}$$

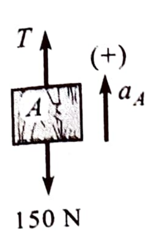

Fig. 11.21(c) : F.B.D. of Block A

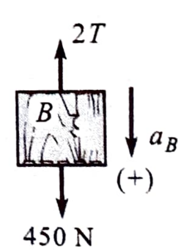

Fig. 11.21(d) : F.B.D. of Block B

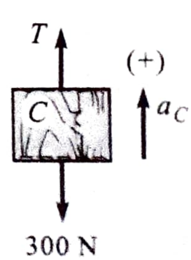

Fig. 11.21(e) : F.B.D. of Block C

(iv) Putting values of equation (II), (III), (IV) in equation (I)

$$\frac{9.81}{150}T - 9.81 + \frac{9.81}{150}T - 9.81 = 2\left[9.81 - \frac{2 \times 9.81}{450}T\right]$$

$$\frac{3T + 1.5T + 4T}{450} = 4$$

$$8.5T = 1800$$

$T = 211.75$ N **Ans.**

(v) Putting value of T in equation (II), (III) and (IV), we get

$a_A = 4.04\ m/s^2\ (\uparrow)$ **Ans.**

$a_B = 0.577\ m/s^2\ (\uparrow)$ **Ans.**

$a_C = -2.885\ m/s^2$

$a_C = 2.885\ m/s^2\ (\downarrow)$ **Ans.**

Problem 20

Determine the weight W required to be attached to 150 N block to bring the system in the *figure 11.22(a)* to stop in 5 sec if at any stage 500 N is moving down at 3 m/s. Assume pulley to be frictionless and mass less.

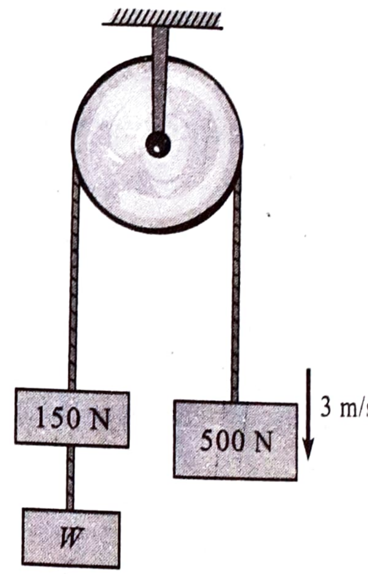

Fig. 11.22(a)

Solution

(i) $v = u + at$

$0 = 3 + a \times 5$

$\therefore a = -0.6\ m/s^2$

(ii) **Consider F.B.D. of Block 500 N**

By Newton's II[nd] Law, we have

$$T - 500 = \frac{500}{9.81}a$$

$$T = \frac{500 \times 0.6}{9.81} + 500$$

$$T = 530.58\ N$$

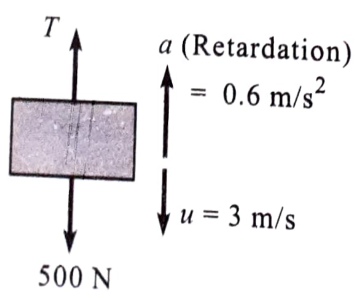

Fig. 11.22(b) : F.B.D. of 500 N Block

(ii) **Consider F.B.D. of Block (150 + W) Together**

By Newton's II[nd] Law, we have

$$(150 + W) - T = \left[\frac{150 + W}{9.81}\right]a$$

$$(150 + W) - 530.58 = \left[\frac{150 + W}{9.81}\right]0.6$$

$W = 415.15$ N **Ans.**

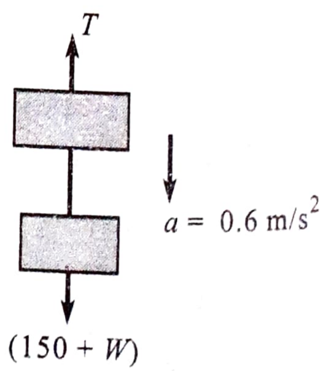

Fig. 11.22(c) : F.B.D. of Block (150 + W)