3.9 Solved Problems

Problem 1

A circular roller of weight 1000 N and radius 20 cm hang by a tie rod AB = 40 cm and rest against a smooth vertical wall at C as shown in *figure 3.32(a)*. Determine the tension in the rod and reaction at point C.

Solution

(i) Draw F.B.D. of roller

$$\cos\theta = \frac{20}{40} \qquad \therefore \theta = 60^\circ$$

(ii) By Lami's theorem we have

$$\frac{1000}{\sin 120} = \frac{T_{AB}}{\sin 90} = \frac{R_C}{\sin 150}$$

$\therefore T_{AB} = 1154.7$ N (60°) **Ans.**

$\therefore R_C = 577.35$ N ($\rightarrow$) **Ans.**

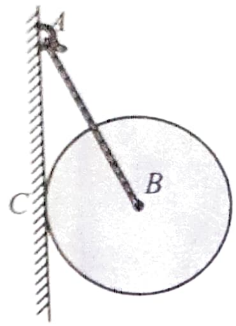

Fig. 3.32(a)

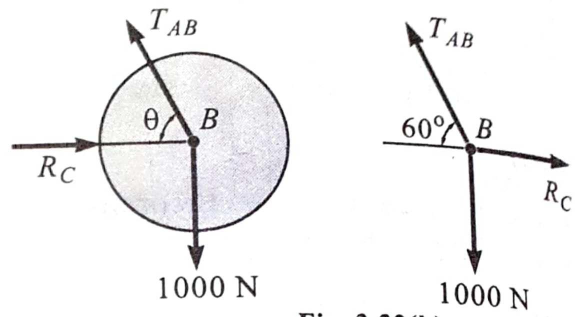

Fig. 3.32(b)

Problem 2

A roller of weight W = 1000 N rests on a smooth inclined plane. It is kept from rolling down the plane by string AC as shown in *figure 3.33(a)*. Find the tension in the string and reaction at the point of contact D.

Solution

(i) Draw F.B.D. of roller

(ii) By Lami's theorem

$$\frac{1000}{\sin 75} = \frac{R_D}{\sin 60} = \frac{-T_{AC}}{\sin 225}$$

$\therefore R_D = 896.58$ N (45°) **Ans.**

$\therefore T_{AC} = 732$ N **Ans.**

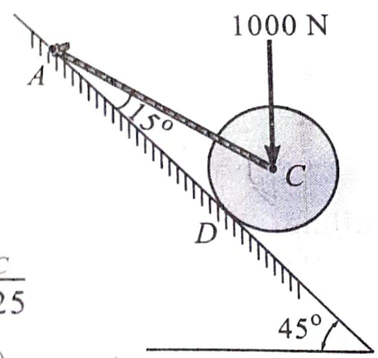

Fig. 3.33(a)

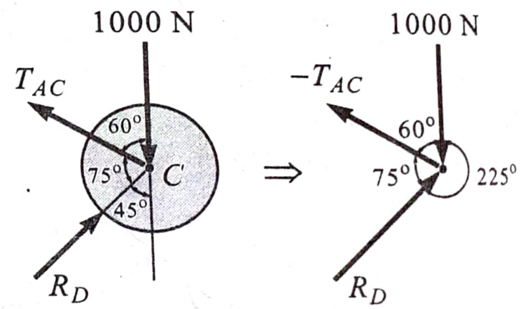

Fig. 3.33(b)

Problem 3

A frame ABC is pin joint at B and external force 1000 N is applied horizontally as shown in *figure 3.34(a)*. Determine the force exerted in bar AB and BC.

Solution

(i) By Sine Rule, in ΔABC we have

$$\frac{2}{\sin\theta} = \frac{4}{\sin 50} \qquad \therefore \theta = 22.52^\circ$$

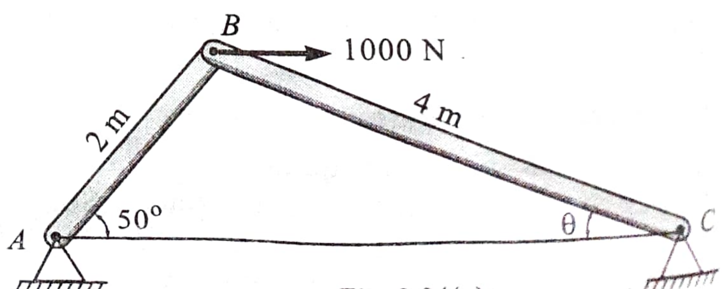

Fig. 3.34(a)

(ii) Consider the F.B.D. of joint B

(iii) By Lami's theorem we have

$$\frac{1000}{\sin 107.48} = \frac{F_{AB}}{\sin 22.52} = \frac{F_{BC}}{\sin 230}$$

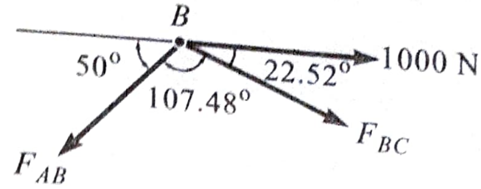

Fig. 3.34(b) : F.B.D. of B

F_{AB} = 401.55 N (Tension) **Ans.**

F_{BC} = –803.13 N (–ve sign indicates wrong assumed direction)

F_{BC} = 803.13 N (Compression) **Ans.**

Problem 4

A cylinder of mass 50 kg is resting on a smooth surface which are inclined at 30° and 60° to horizontal as shown in *figure 3.35(a)*. Determine the reaction at contact A and B.

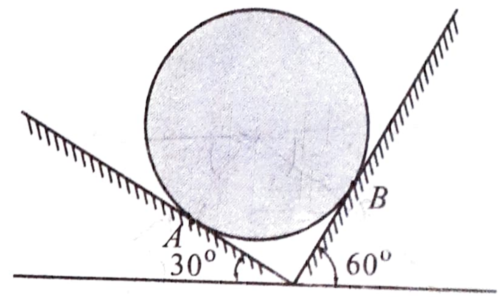

Fig. 3.35(a)

Solution

(i) Consider the F.B.D. of cylinder.

(ii) By Lami's theorem we have

$$\frac{50 \times 9.81}{\sin 90} = \frac{R_A}{\sin 120} = \frac{R_B}{\sin 150}$$

R_A = 424.79 N ; R_B = 245.25 N **Ans.**

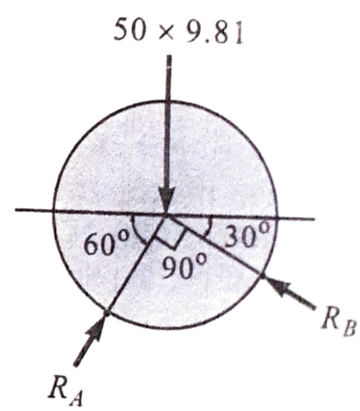

Fig. 3.35(b) : F.B.D. of Cylinder

Problem 5

Find the tension in each rope in *figure 3.36(a)*.

Solution

(i) Consider the F.B.D. of point C

(ii) By Lami's theorem

$$\frac{981}{\sin 156.87} = \frac{T_{AC}}{\sin 60} = \frac{T_{BC}}{\sin 143.13}$$

T_{AC} = 2162.76 N ; T_{BC} = 1498.41 N **Ans.**

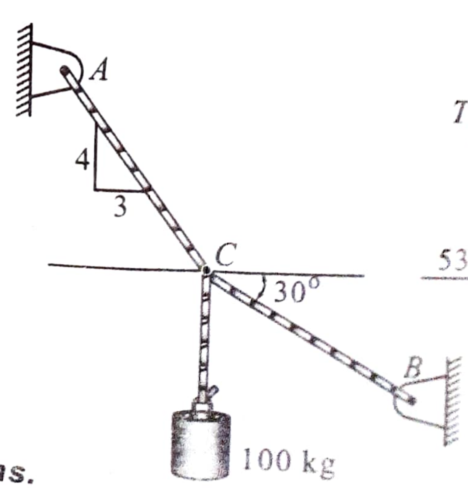

Fig. 3.36(a)

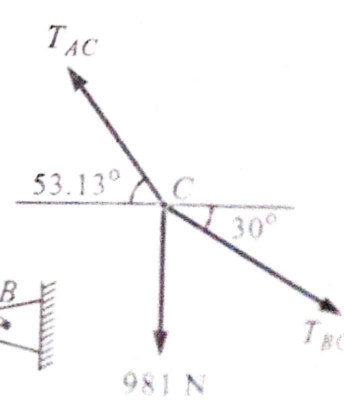

Fig. 3.36(b) : F.B.D.

Problem 10

Determine the magnitude and direction of the minimum force P required to just start that roller over the block as shown in *figure 3.41(a)*.

Mass of roller = 100 kg, radius of roller = 60 cm.

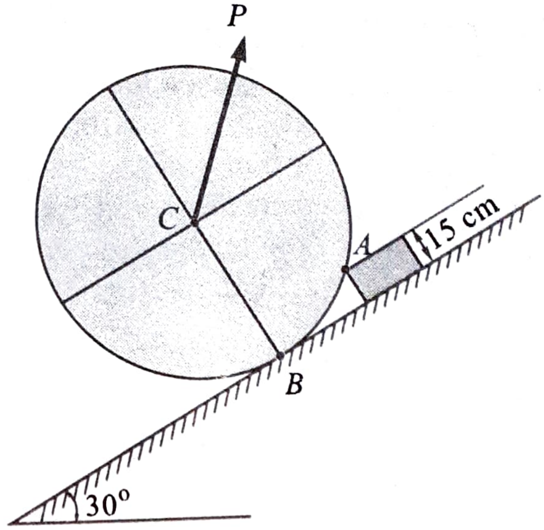

Fig. 3.41(a)

Solution

(i) For just start condition, reaction at B will be zero and roller is subjected to three non-parallel forces which must be concurrent as per three force principle.

Consider F.B.D. of roller with proper geometrical angles as shown in *figure 3.41(b)*.

$$\therefore \sin\alpha = \frac{45}{60} \qquad \therefore \alpha = 48.59^\circ$$

(ii) By Lami's theorem

$$\frac{981}{\sin(\theta+\alpha)} = \frac{-P_{min}}{\sin(30+180+90-\alpha)}$$

$$\frac{981}{\sin(\theta+48.59)} = \frac{-P_{min}}{\sin 251.41}$$

$$P_{min} = \frac{-981 \sin 251.41}{\sin(\theta+48.59)} \quad \text{... (I)}$$

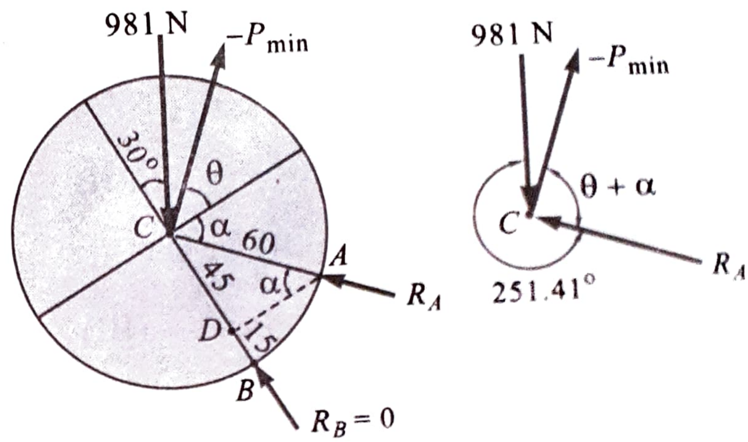

Fig. 3.41(b) : F.B.D. of Roller

(iii) For P to be minimum $\sin(\theta + 48.59)$ must be maximum.

$$\therefore \theta + 48.59 = 90 \qquad \therefore \theta = 41.41$$

(iv) From equation (I)

$$P_{min} = 929.82 \text{ N } (\measuredangle 71.41^\circ) \quad \textbf{Ans.}$$

Problem 11

A smooth collar having mass 10 kg can slide on the horizontal rod as shown in *figure 3.42(a)*. A 20 kg load is suspended from a frictionless pulley and the cable makes a 60° angle with the horizontal. Find the force P required for equilibrium.

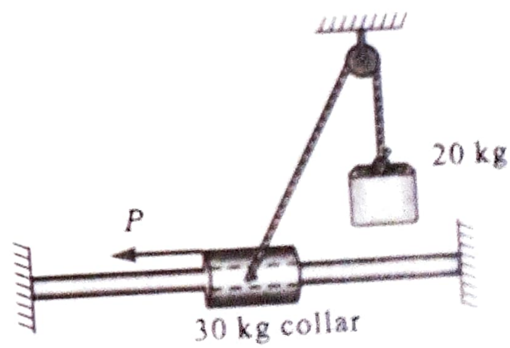

Fig. 3.42(a)

Solution

(i) F.B.D. of collar is shown in *figure 3.42(b)*.

(ii) $\Sigma F_x = 0$

$$20 \times 9.81 \cos 69 - P = 0$$

$$P = 98.1 \text{ N } (\leftarrow) \quad \textbf{Ans.}$$

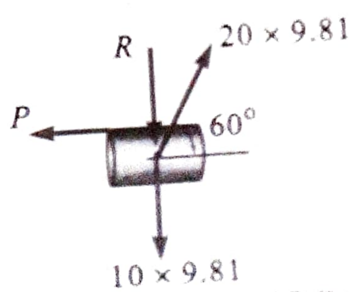

Fig. 3.42(b) : F.B.D. of Collar

Problem 12

A bar AB pinned at A carries a load $W = 5$ kN. The cord attached at B is passing through frictionless pulley as shown in *figure 3.43(a)*. Find the compression in the bar AB and also the limiting value of the tension in cord when bar approaches the vertical position.

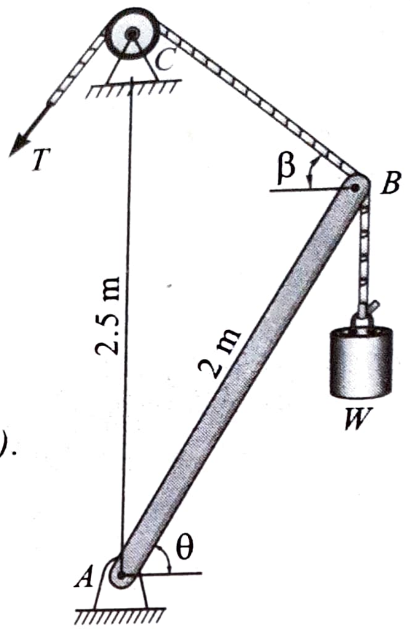

Fig. 3.43(a)

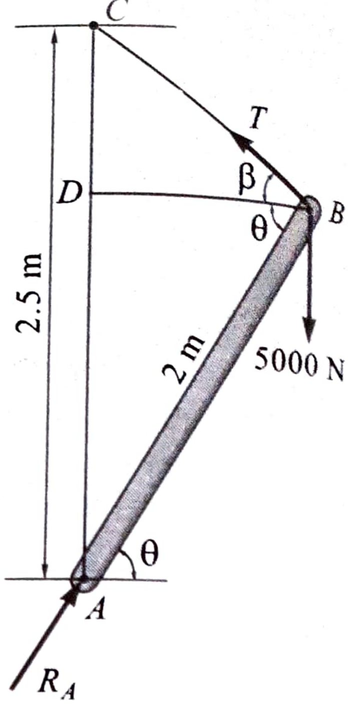

Fig. 3.43(b) : F.B.D. of Bar AB

Solution

(i) F.B.D. of bar AB is shown in *figure 3.43(b)*.

(ii) $\Sigma F_y = 0$

$$R_A \sin\theta + T\sin\beta - 5000 = 0$$

$$T\sin\beta = 5000 - R_A \sin\theta \qquad \text{...(I)}$$

(iii) $\Sigma F_x = 0$

$$R_A \cos\theta - T\cos\beta = 0$$

$$T\cos\beta = R_A\cos\theta \qquad \text{...(II)}$$

(iv) Dividing equation (I) by (II) we get

$$\tan\beta = \frac{5000 - R_A\sin\theta}{R_A\cos\theta} \qquad \text{...(III)}$$

(v) By the geometrical configuration

$$\tan\beta = \frac{CD}{BD} = \frac{AC - AD}{BD} \qquad \text{...(IV)}$$

(vi) From equation (III) and (IV) we get

$$\frac{2.5 - 2\sin\theta}{2\cos\theta} = \frac{5000 - R_A\sin\theta}{R_A\cos\theta}$$

$$2.5\,R_A - 2\,R_A\sin\theta = 10000 - 2\,R_A\sin\theta$$

$$R_A = \frac{10000}{2.5} \qquad \therefore R_A = 4000 \text{ N} \quad \textbf{Ans.}$$

While obtaining R_A in the above expression the inclination of bar AB is not influencing the solution. So we concluded that reaction at A remains 4000 N for all values of θ as shown in *figure 3.43(c)*.

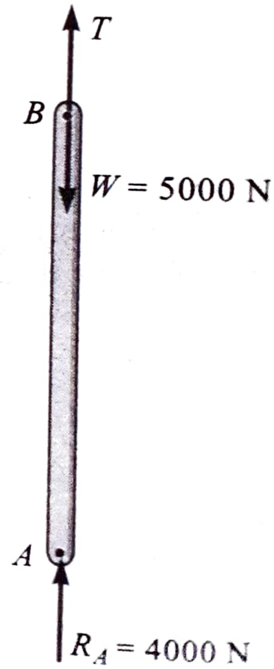

Fig. 3.43(c)

(vii) Consider the F.B.D. of bar AB in vertical position.

By equilibrium condition

$$\Sigma F_y = 0 \;\Rightarrow\; R_A + T - 5000 = 0$$

$$T = 5000 - 4000$$

$$\therefore T = 1000 \text{ N} \quad \textbf{Ans.}$$

Problem 20

Two homogenous solid cylinders of identical weight of 5000 N and radius of 0.4 m are resting against inclined wall and sloping ground as shown in *figure 3.51(a)*. Assuming all smooth surfaces, find the reactions at A, B and C of the contact points, on ground and wall.

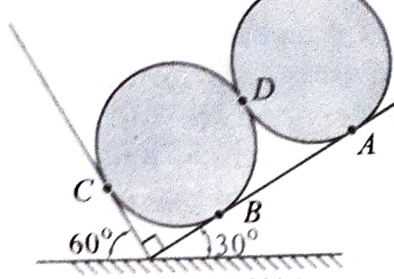

Fig. 3.51(a)

Solution

(i) Consider the F.B.D. of upper cylinder. [*figure 3.51(b)*]

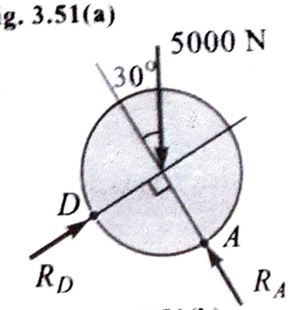

Fig. 3.51(b)

(ii) By Lami's theorem

$$\frac{5000}{\sin 90} = \frac{R_A}{\sin 120} = \frac{R_D}{\sin 150}$$

$\therefore R_A = 4330.13$ N ; $R_D = 2500$ N **Ans.**

(iii) Consider the F.B.D. of lower cylinder [*figure 3.51(c)*]

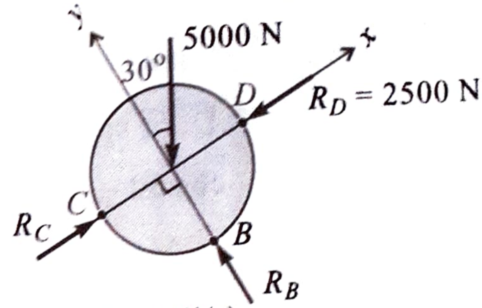

Fig. 3.51(c)

$\Sigma F_x = 0$

$R_C - 2500 + 5000 \sin 30 = 0 \quad \therefore R_C = 5000$ N **Ans.**

$\Sigma F_y = 0$

$R_B - 5000 \cos 30 = 0 \quad \therefore R_B = 4330.13$ N **Ans.**

Problem 21

Two smooth spheres A and B of weight 200 N and 100 N respectively are resting against two smooth vertical walls and smooth horizontal floor as shown in *figure 3.52(a)*. The radius of sphere A is 100 mm and the radius of sphere B is 50 mm. Find the reactions from the vertical walls and horizontal floor.

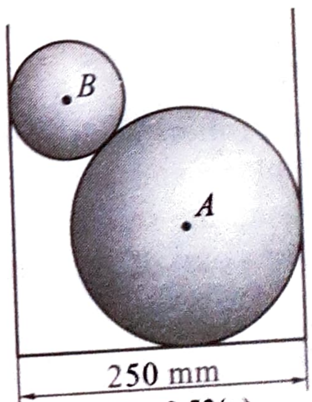

Fig. 3.52(a)

Solution

(i) Consider the F.B.D. of sphere B and A together. [*figure 3.52(b)*]

(ii) From figure, $AC = 100$ mm, $AB = 150$ mm.

$\therefore BC = \sqrt{(AB)^2 - (AC)^2} = 111.8$ mm

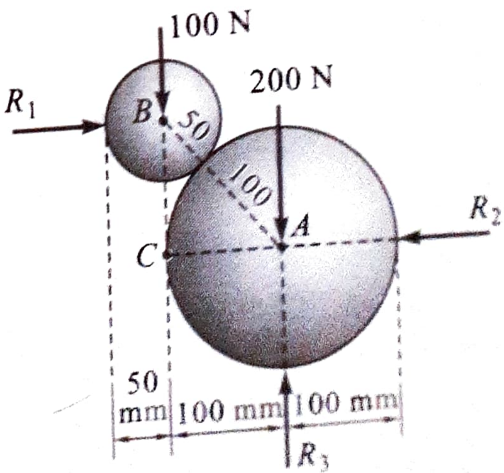

Fig. 3.52(b)

(iii) $\Sigma M_A = 0$

$100 \times (AC) - R_1 \times (BC) = 0$

$R_1 = \dfrac{100 \times 100}{111.8} \quad \therefore R_1 = 89.45$ N (→) **Ans.**

(iv) $\Sigma F_x = 0$

$R_1 - R_2 = 0 \quad \therefore R_2 = 89.45$ N (←) **Ans.**

(v) $\Sigma F_y = 0$

$R_3 - 100 - 200 = 0 \quad \therefore R_3 = 300$ N (↑) **Ans.**

Note : We can also solve this problem by considering F.B.D. of individual cylinders.

Problem 22

Two spheres A and B are resting in a smooth through as shown in *figure 3.53(a)*. Draw the free body diagrams of A and B showing all the forces acting on them, both in magnitude and direction. Radius of spheres A and B are 250 mm and 200 mm respectively.

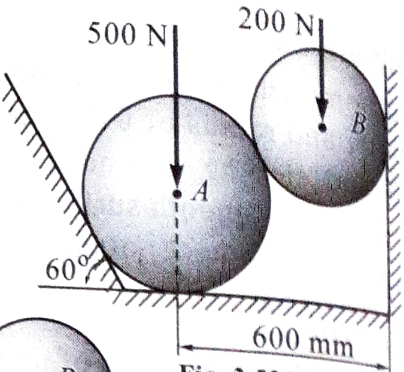

Fig. 3.53(a)

Solution

(i) From *figure 3.53(b)*. $AB = 450$ mm and $AC = 400$ mm

$$\cos\theta = \frac{AC}{AB} = \frac{400}{450} \quad \therefore \theta = 27.27^\circ$$

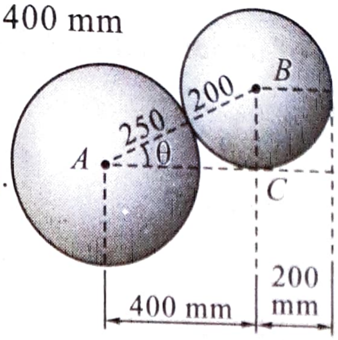

Fig. 3.53(b)

(ii) Consider F.B.D. of sphere B [*figure 3.53(c)*].
By Lami's theorem

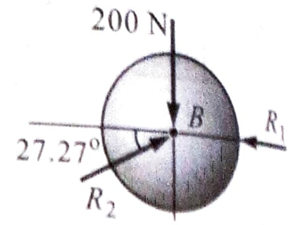

Fig. 3.53(c)

$$\frac{200}{\sin 152.73} = \frac{R_1}{\sin 117.27} = \frac{R_2}{\sin 90}$$

$\therefore R_1 = 388$ N $(\leftarrow)$ and $R_2 = 436.51$ N $(\measuredangle 27.27^\circ)$ **Ans.**

(iii) Consider the F.B.D. of sphere A [*figure 3.53(d)*]

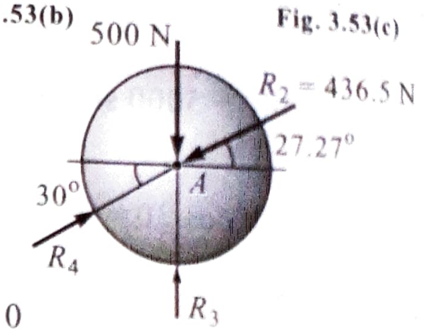

Fig. 3.53(d)

$$\Sigma F_x = 0 \Rightarrow R_4 \cos 30 - 436.51 \cos 27.27 = 0$$

$$R_4 = 448 \text{ N } (\measuredangle 30^\circ) \quad \textbf{Ans.}$$

$$\Sigma F_y = 0 \Rightarrow -500 + R_3 - 436.51 \sin 27.27 + 448 \sin 30 = 0$$

$$R_3 = 476 \text{ N } (\uparrow) \quad \textbf{Ans.}$$

Problem 23

Two identical rollers each of mass 50 kg are supported by an inclined plane and a vertical wall as shown in *figure 3.54(a)*. Assuming smooth surfaces, find the reactions induced at the point of support A, B and C.

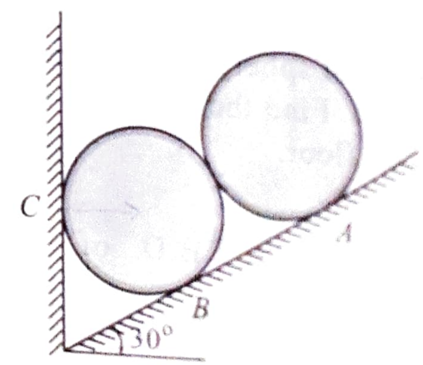

Fig. 3.54(a)

Solution

(i) Consider F.B.D. of both rollers together and let R be the radius of rollers.

(ii) $\Sigma M_O = 0$

$$R_A \times 2R - 50 \times 9.81 \cos 30 \times 2R = 0$$

$$R_A = 424.79 \text{ N } (60^\circ) \quad \textbf{Ans.}$$

(iii) $\Sigma F_y = 0$

$$R_B \cos 30 + R_A \cos 30 - 50 \times 9.81 - 50 \times 9.81 = 0$$

$$R_B = 707.97 \text{ N } (60^\circ) \quad \textbf{Ans.}$$

(iv) $\Sigma F_x = 0$

$$R_C - R_A \sin 30 - R_B \sin 30 = 0$$

$$R_C = 424.79 \sin 30 + 707.97 \sin 30$$

$$R_C = 566.38 \text{ N } (\rightarrow) \quad \textbf{Ans.}$$

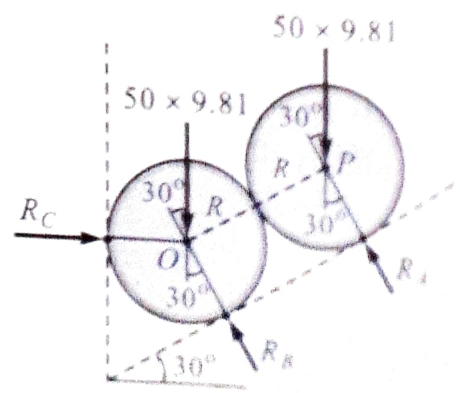

Fig. 3.54(b)

Problem 24

Three identical tubes of weights 8 kN each are placed as shown in *figure 3.55(a)*. Determine the forces exerted by the tubes on the smooth walls and floor.

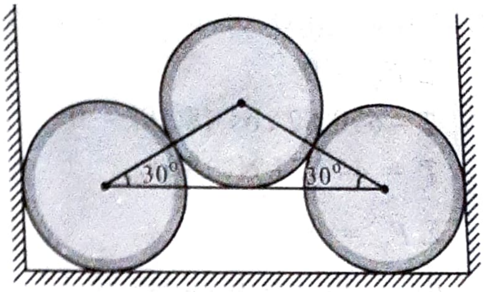

Fig. 3.55(a)

Solution

(i) Consider F.B.D. of upper tube *figure 3.55(b)*.

Since tubes are identical and placed symmetrically, reactions R at contact will be same.

(ii) By Lami's theorem

$$\frac{8}{\sin 120} = \frac{R}{\sin 120} \qquad \therefore R = 8 \text{ kN} \quad \textbf{Ans.}$$

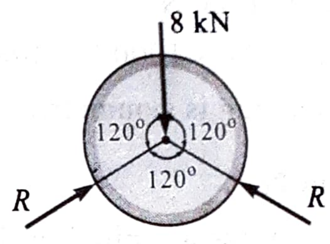

Fig. 3.55(b)

(iii) Consider the F.B.D. of any lower tube (say left) *figure 3.55(c)*.

(iv) By Lami's theorem

$$\frac{8}{\sin 90} = \frac{R_W}{\sin 120} = \frac{R_F}{\sin 150}$$

(v) $R_W = 6.928$ kN (Force exerted by the tubes on the smooth wall) ***Ans.***

$R_F = 4$ kN (Force exerted by the tubes on the smooth floor) ***Ans.***

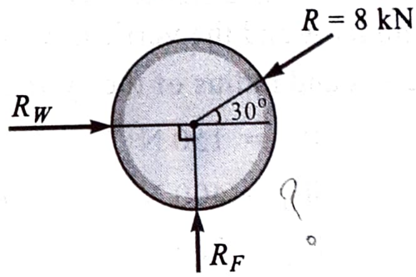

Fig. 3.55(c)

Problem 25

Two smooth circular cylinder of weight $W = 500$ N each and radius $r = 150$ mm are connected at their centre by a string of length $l = 400$ mm and rest upon a horizontal plane supporting above them a third cylinder of weight 1000 N and radius $r = 150$ mm as shown in *figure 3.56(a)*. Find the tension in the string and pressure at the point of contact D and E.

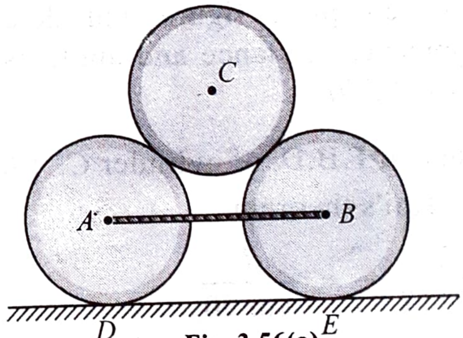

Fig. 3.56(a)

Solution

(i) Consider the Δ ABC and simplify its geometric length and angle as shown in *figure 3.56(b)*.

$$\cos\theta = \frac{200}{300} \qquad \therefore \theta = 48.19^\circ$$

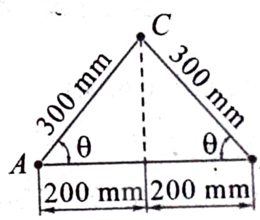

Fig. 3.56(b)

(ii) Draw F.B.D. of upper cylinder C. [*figure 3.56(c)*]

(iii) By Lami's theorem

$$\frac{1000}{\sin 83.62} = \frac{R}{\sin 138.19}$$

$$\therefore R = 670.82 \text{ N} \quad \textbf{Ans.}$$

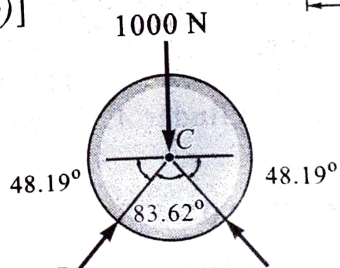

Fig. 3.56(c)

(iv) Draw F.B.D. of lower cylinder A as shown in *figure 3.56(d)*

$\Sigma F_x = 0$

$T - 670.82 \cos 48.19 = 0$

$T = 447.21$ N **Ans.**

$\Sigma F_y = 0$

$R_D - 500 - 670.82 \sin 48.19 = 0$

$R_D = 1000$ N

$R_D = R_E = 1000$ N **Ans.**

(Since loading is symmetric therefore reaction will be equal)

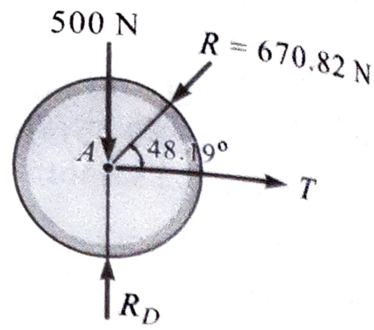

Fig. 3.56(d)

Problem 26

Three cylinders are piled up in a rectangular channel as shown in *figure 3.57(a)*. Determine the reactions between cylinder A and the vertical wall of the channel.

Weight and radius of the cylinder are as follows :

$W_A = 150$ N $\quad r_A = 4$ cm

$W_B = 400$ N $\quad r_B = 6$ cm

$W_C = 200$ N $\quad r_C = 5$ cm

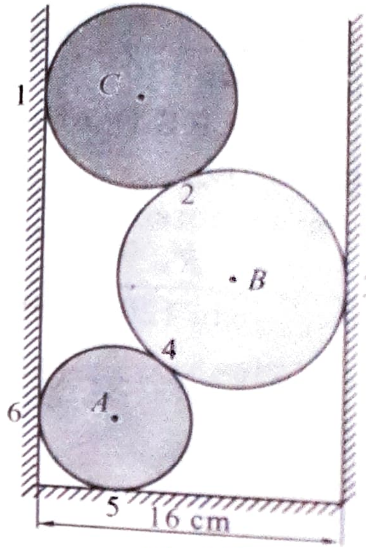

Fig. 3.57(a)

Solution

(i) Refer complete figure and determine the geometrical distance and angle as shown in *figure 3.57(b)*.

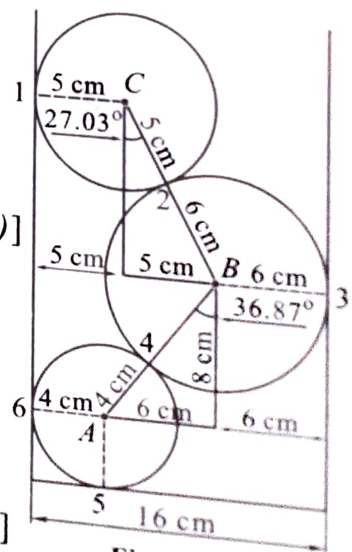

Fig. 3.57(b)

(ii) Consider F.B.D. of cylinder C. [*figure 3.57(c)*]

By Lami's theorem

$$\frac{200}{\sin 117.03} = \frac{R_2}{\sin 90}$$

$R_2 = 224.53$ N **Ans.**

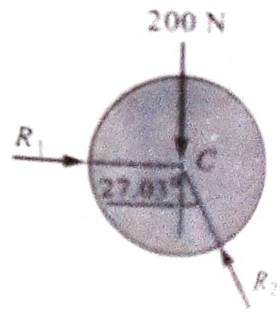

Fig. 3.57(c)

(iii) Consider F.B.D. of cylinder B. [*figure 3.57(d)*]

$\Sigma F_y = 0$

$R_4 \cos 36.87 - 224.53 \cos 27.03 - 400 = 0$

$R_4 = 750$ N **Ans.**

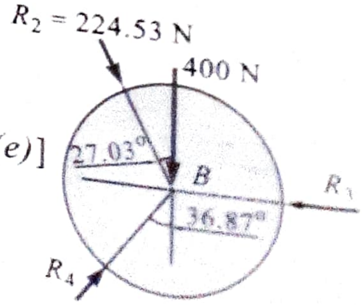

Fig. 3.57(d)

(iv) Consider F.B.D. of cylinder A. [*figure 3.57(e)*]

$\Sigma F_x = 0$

$R_6 - 750 \sin 36.87 = 0$

$R_6 = 450$ N ($\rightarrow$) **Ans.**

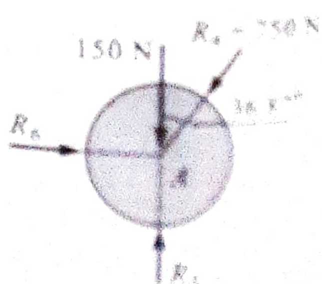

Fig. 3.57(e)