Digital Image Processing

Outline

- Introduction and Fundamentals:
  - Motivation and Perspective, Applications, Components of Image Processing System,
  - Element of Visual Perception, A Simple Image Model,
  - Sampling and Quantization, Some Basic Relationships between Pixels
- Intensity Transformations and Spatial Filtering:
  - Introduction, Some Basic Intensity Transformation Functions,
  - Histogram Processing, Histogram Equalization, Histogram Specification,
  - Local Enhancement, Enhancement using Arithmetic/Logic Operations
  - Basics of Spatial Filtering, Smoothing, Sharpening
- Filtering in the Frequency Domain:
  - Fourier Transform and the Frequency Domain,
  - Basis of Filtering in Frequency Domain

Introduction and Fundamentals

Image

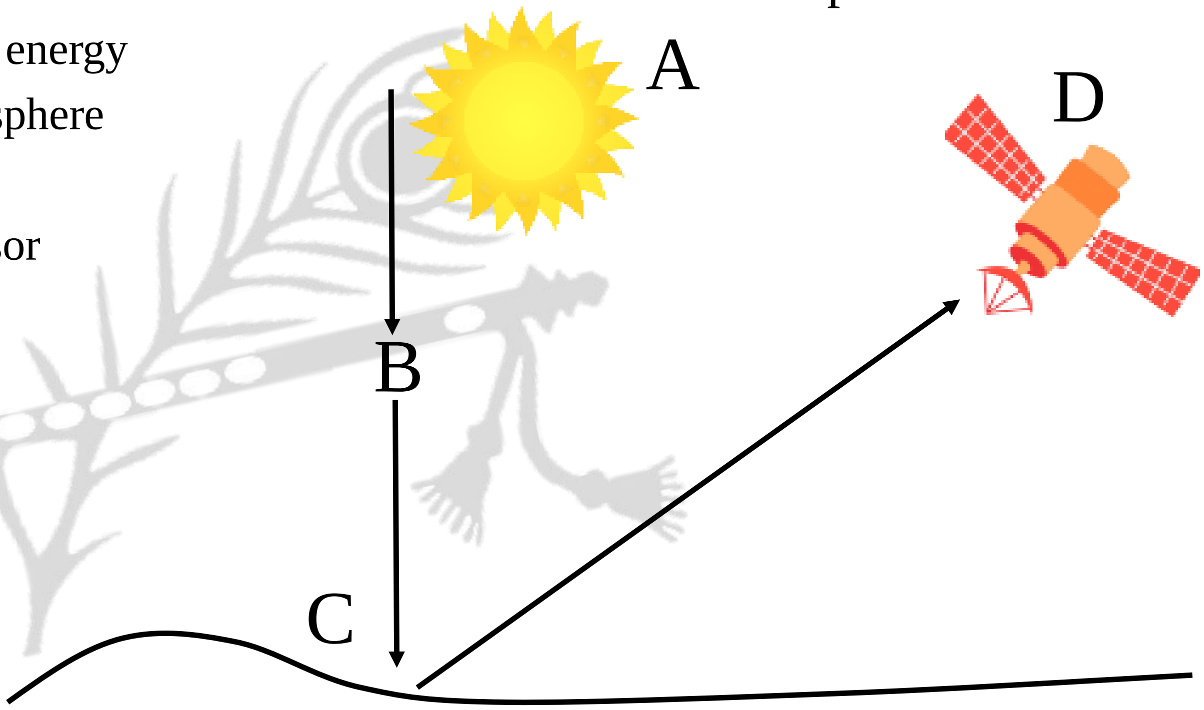
- An image is a visual representation of scene
- An image is an item that depicts visual perception such as a photograph or two-dimensional picture, which resembles a subject, usually a physical object
- In the context of signal processing, an image is a collection of distributed amplitude of color(s)
- An image is a representation of visual information, such as drawings, pictures, graphs, logos, or individual video frames

Digital Image

- A digital image is a binary representation of visual information
- A digital image is a representation in a two dimensional space as a finite set of digital values, called picture elements or pixels

Image Acquisition

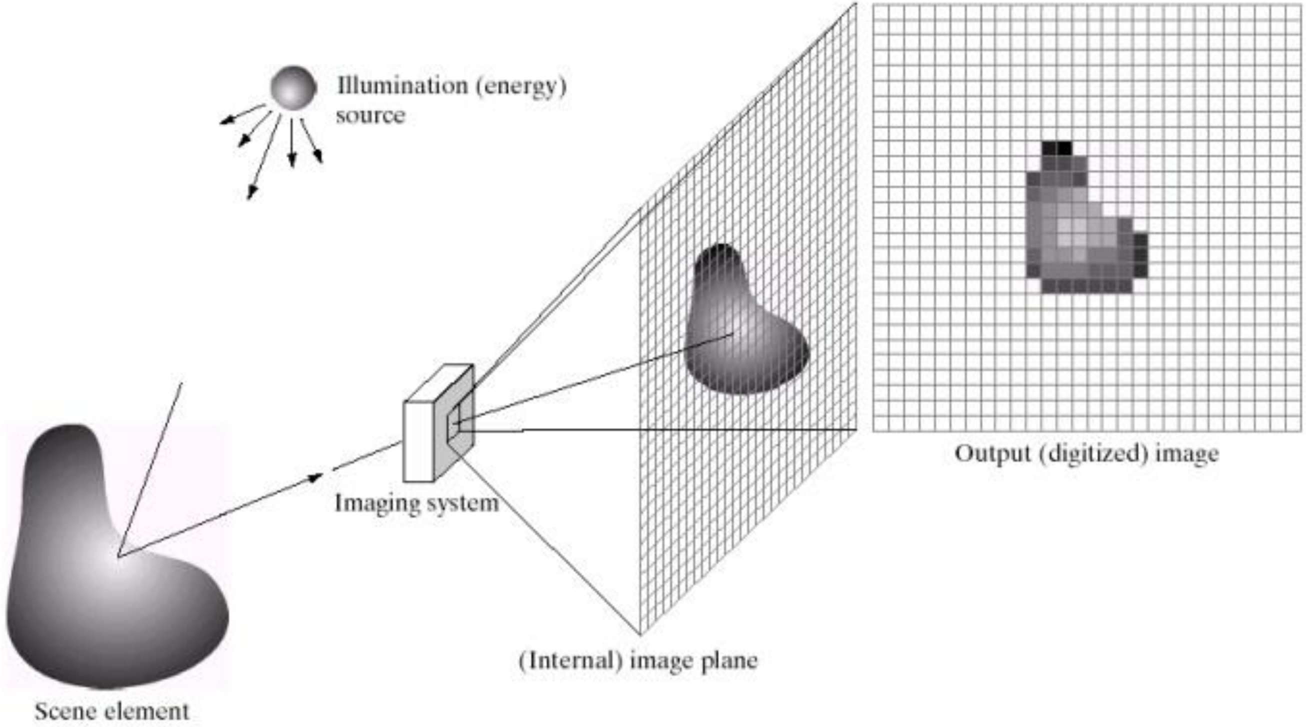
- Images captured by sensors have an involvement of various components
  - Source of electromagnetic energy
  - Interaction with the Atmosphere
  - Interaction with the Target
  - Energy Recording by Sensor



Process involved in Digital Image

- Output of most sensors is a continuous voltage waveform whose amplitude & spatial behavior are related to the physical phenomenon (e.g. brightness) being sensed
- For an image to be processed by a computer, it must be represented by an appropriate discrete data structure (e.g. Matrix)
- Two important processes to convert continuous analog image into digital image
  - Sampling
  - Quantization

Process involved in Digital Image

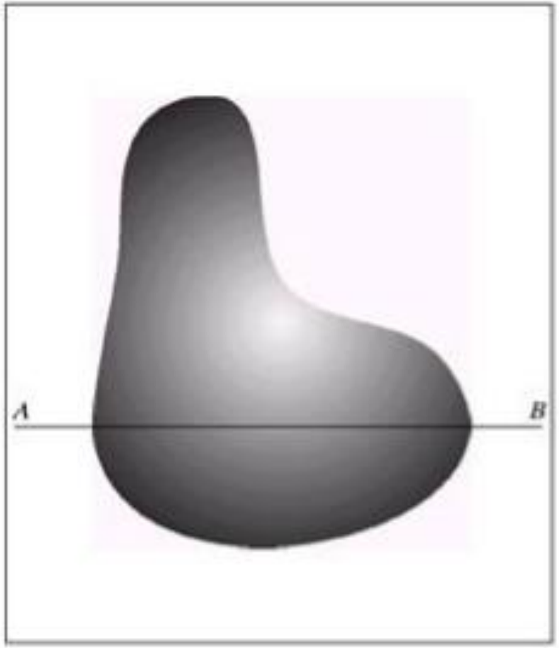


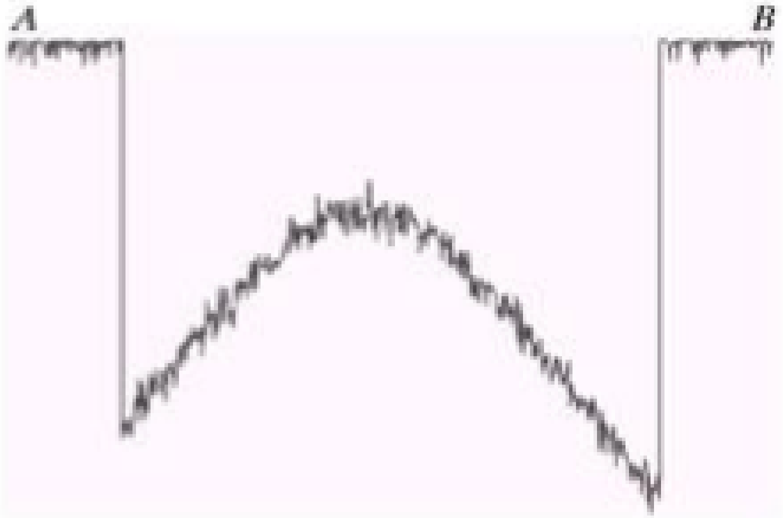
Process involved in Digital Image

- To convert an image to digital form, we have to sample the image in both coordinates (spatial domain) and in amplitude

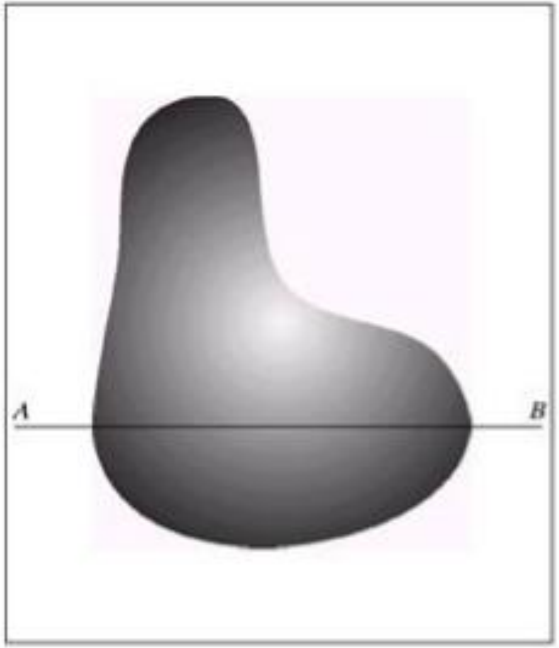
- Discretization
  - Process in which signals or data samples are considered at regular intervals
- Sampling
  - It is the discretization of image data in spatial coordinates or defining finite discrete coordinate to every part of an image
- Quantization
  - It is the discretization of image intensity (gray level) values or defining finite value to every discrete interval

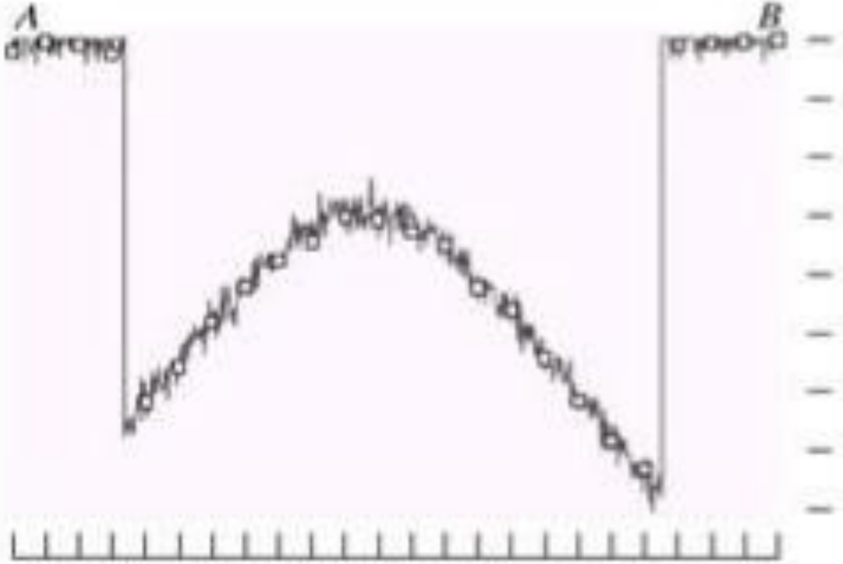
Sampling and Quantization



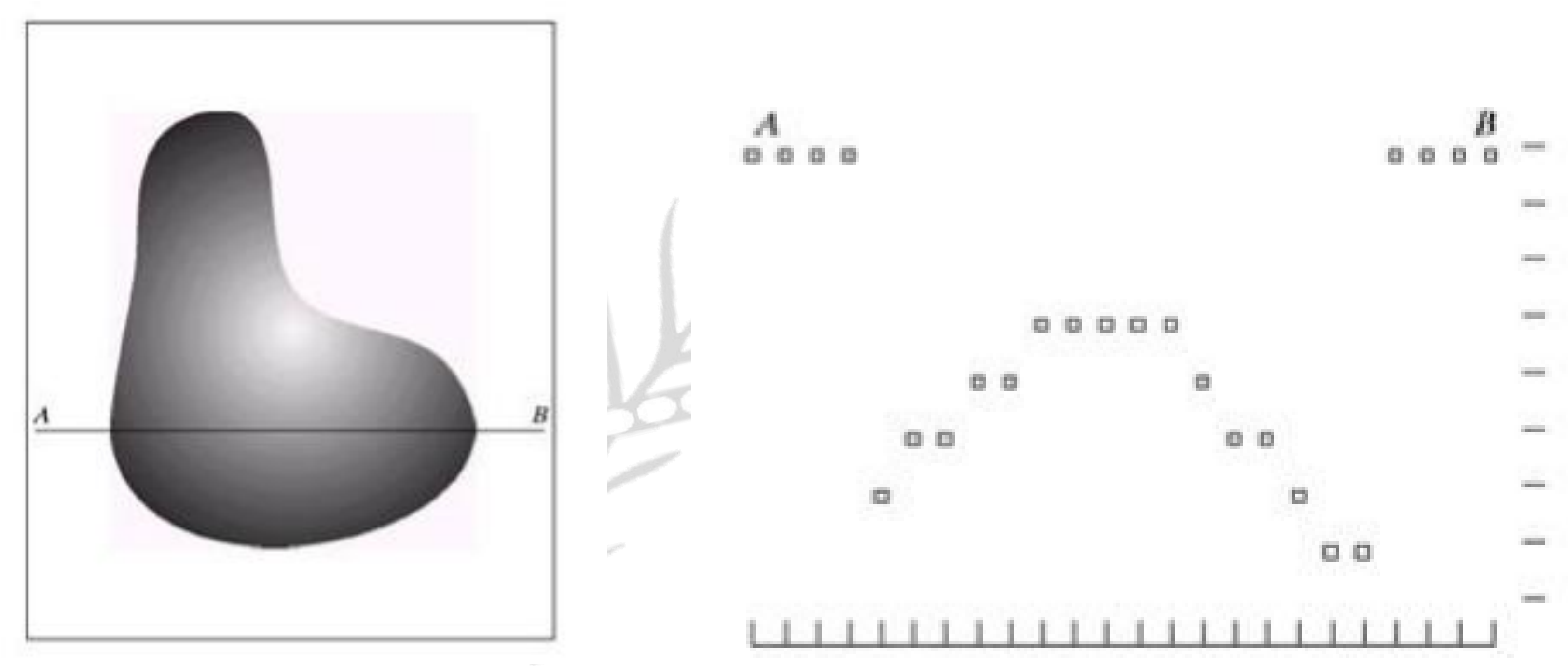


Sampling and Quantization

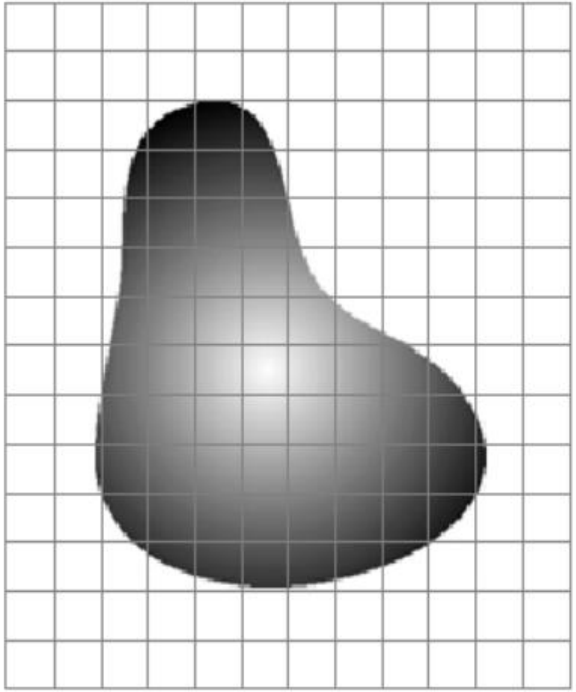




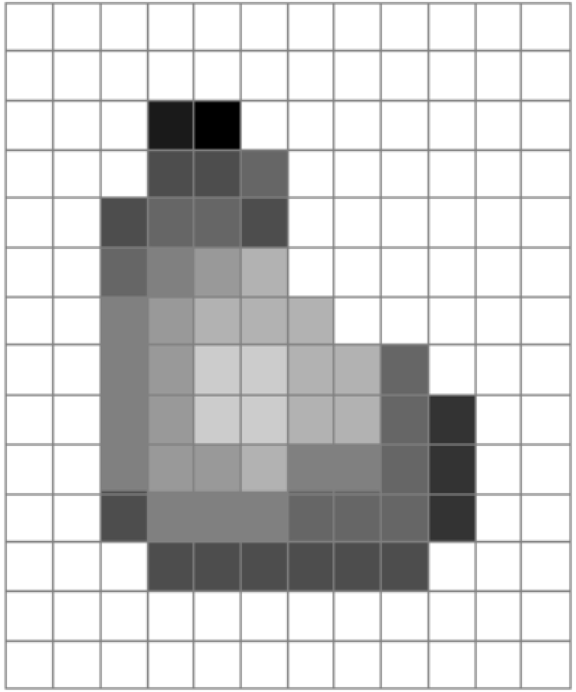
Sampling and Quantization



Sampling and Quantization



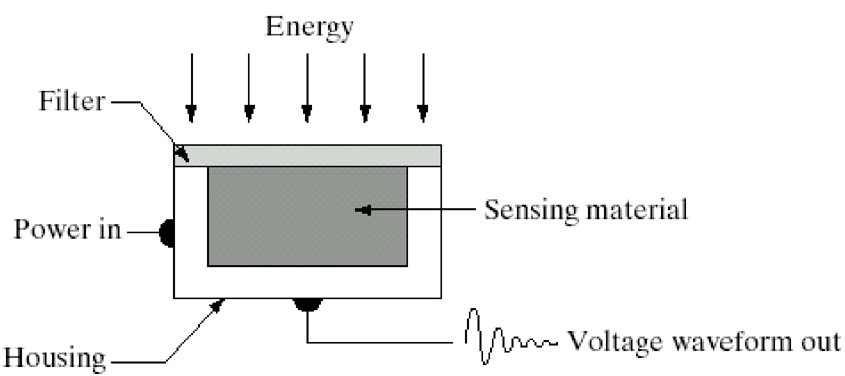
before



after

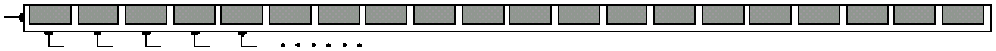
Sensor Arrangement

- Single imaging sensor



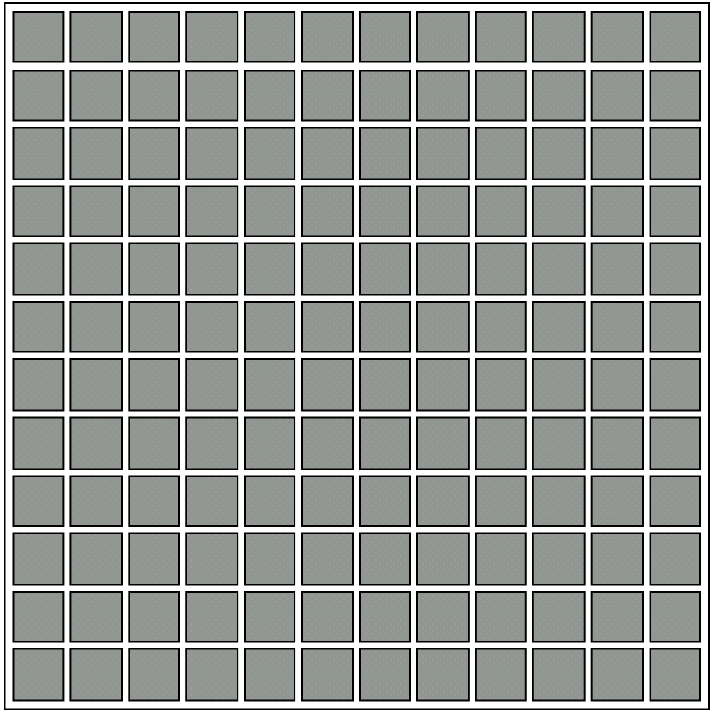
Sensor Arrangement

- Single imaging sensor
- Line sensor



Sensor Arrangement

- Single imaging sensor
- Line sensor
- Array sensor



Digital Image Representation

- Monochrome image can be represented as a two dimensional light intensity function f(x, y)
  - where
    - x & y are spatial coordinates
    - the value of f at point (x, y) is the gray level or the brightness of the image

- For computer storage, an array with the number of gray levels should be in the power of 2
- A typical gray level image contains 256 shades of gray (8 bit)
- Values are stored between 0 - 255

Digital Image Representation

- The intensity of a monochrome image at any coordinate (x_i, y_i) is called the gray level (i_i) of the image, which is given by
  - $i_i = f(x_i, y_i)$
  - $0 \le i \le L_{max} - 1$
  - $i = 0$ is considered black
  - $i = L_{max} - 1$ is considered white

Image Samples

- Generally, image is of size m x n
  - where,
    - m: number of rows and
    - n: number of columns
- The no. of gray level values (L), is taken as power of 2
- $L = 2^k$
  - where,
    - k: number of bits used for representation

Image Size

- For an image of size m x n, if k are the number of bits for representation, then

$$\text{Image size} = m \times n \times k$$

Image Size

- Eg:
  - For an image of 512 by 512 pixels, with 8 bits per pixel

Solution

$$
\begin{aligned}
\text{Size} &= 512 * 512 * 8 \text{ bits} \\
&= 2^9 * 2^9 * 2^3 \text{ bits} \\
&= 2^{21}/2^3 \text{ bytes} \\
&= 2^{18}/2^{10} \text{ K bytes} \\
&= 256 \text{ KB}
\end{aligned}
$$

Motivation

- The human brain is unequalled in 2D image analysis and image understanding

- Limitations of the human visual system
  - Quantification
  - Reconstruction of 3D scenes
  - High dimensional image spaces

- Practical applications
  - Quantitative measurements
  - Automatic analysis of images

Perspective

- Digital image processing deals with manipulation of digital images through a digital computer
- It is a subfield of signals and systems but focus particularly on images
- DIP focuses on developing a computer system that is able to perform processing on an image
- The input of that system is a digital image and the system process that image using efficient algorithms, and gives an image as an output

Applications

- Some of the major fields in which digital image processing is widely used are
  - Medical field

Applications

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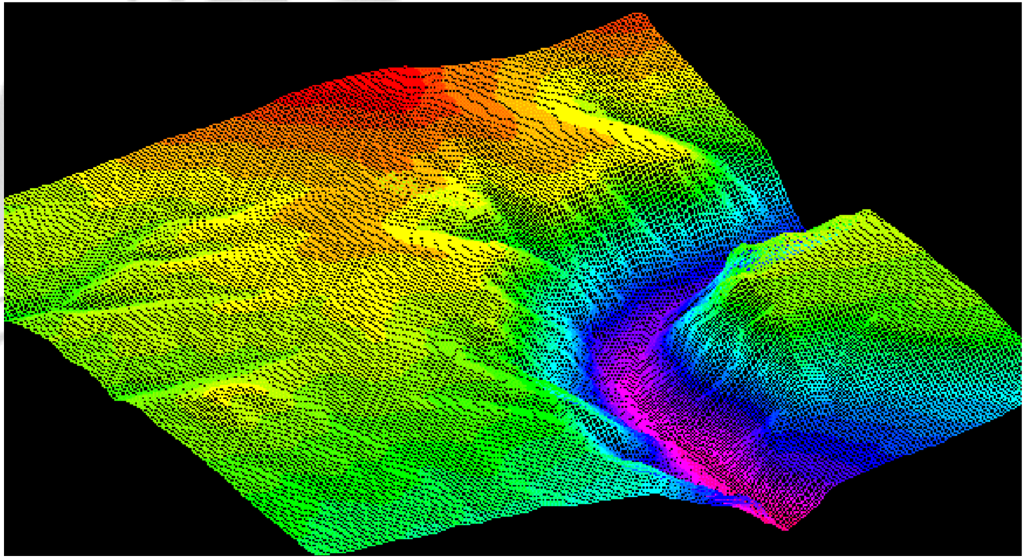
  - GIS



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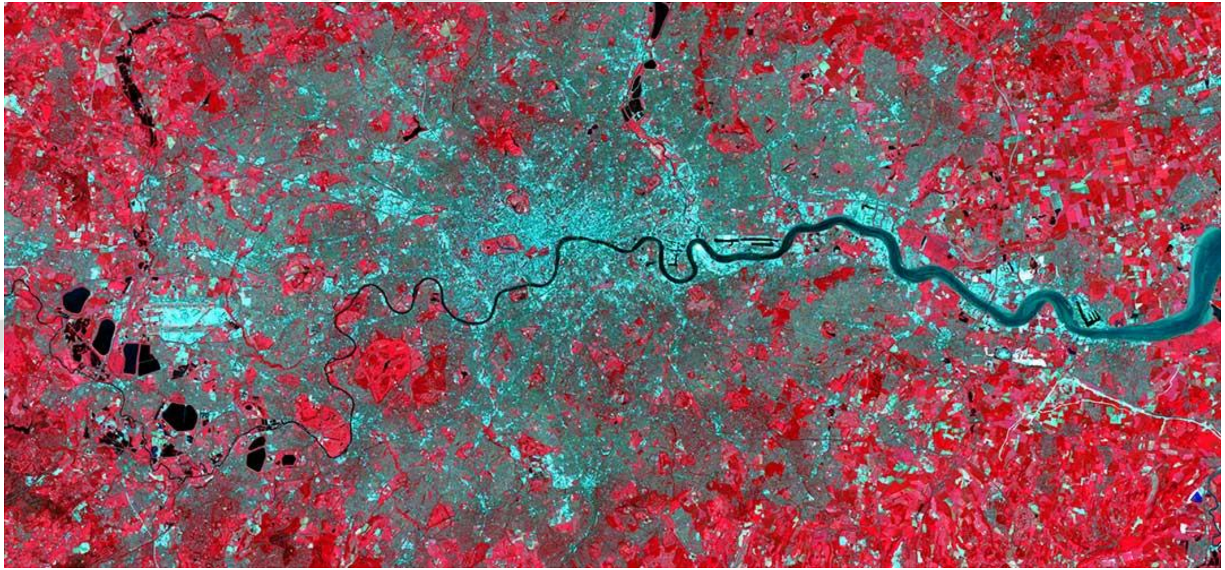
  - GIS
  - Remote sensing



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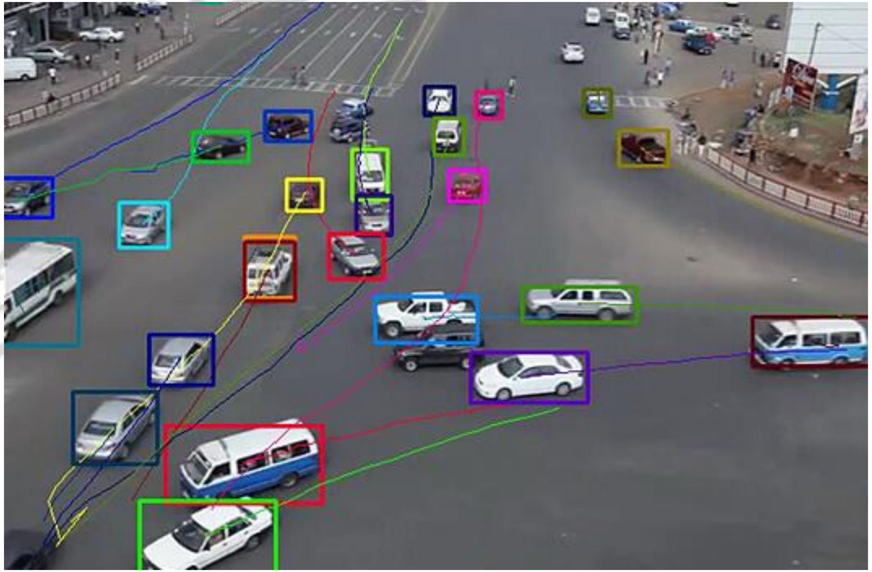
  - GIS
  - Remote sensing
  - Object tracking



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  - Object tracking
  - Classification

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  - Remote sensing
  - Object tracking
  - Classification
  - Change detection

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  - Object tracking
  - Classification
  - Change detection
  - Disaster monitoring

Applications

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  - Object tracking
  - Classification
  - Change detection
  - Disaster monitoring
  - etc

Image Sensing

- Sensing operates in various region of Electro Magnetic Spectrum

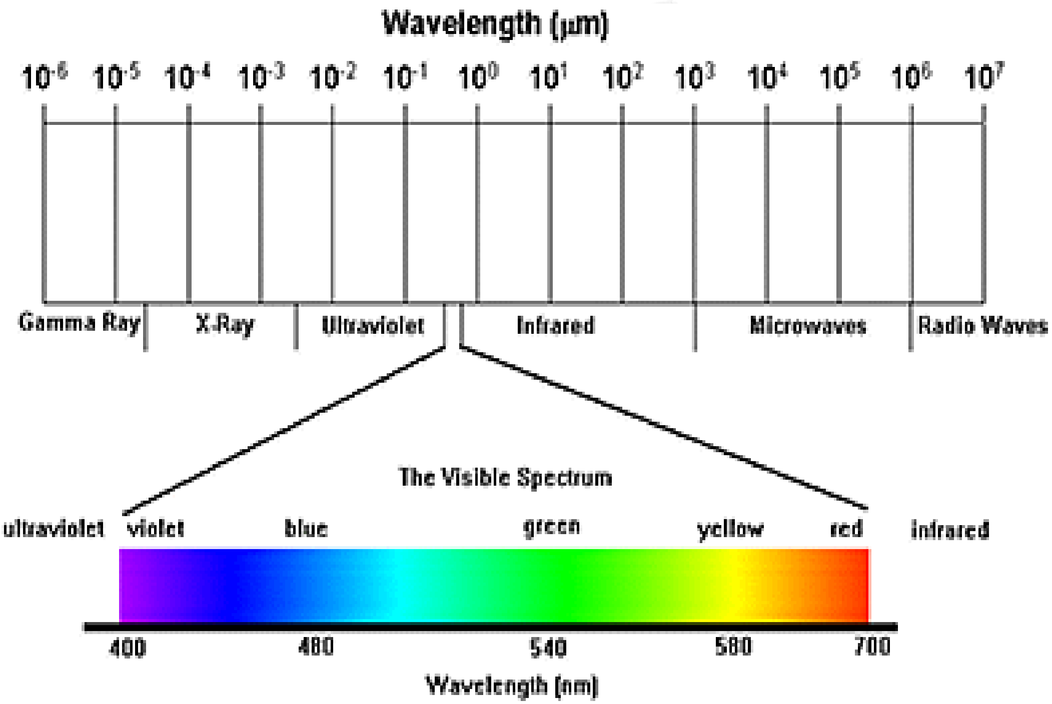


Image Resolutions

- There are four types of resolution
  - Spatial resolution
  - Spectral resolution
  - Temporal resolution
  - Radiometric resolution

Spatial Resolution

- The smallest possible feature that can be detected in an image

OR

- The size of single pixel or area covered by a single pixel (GSD)

Spatial Resolution

← 512 x 512

← 256 x 256

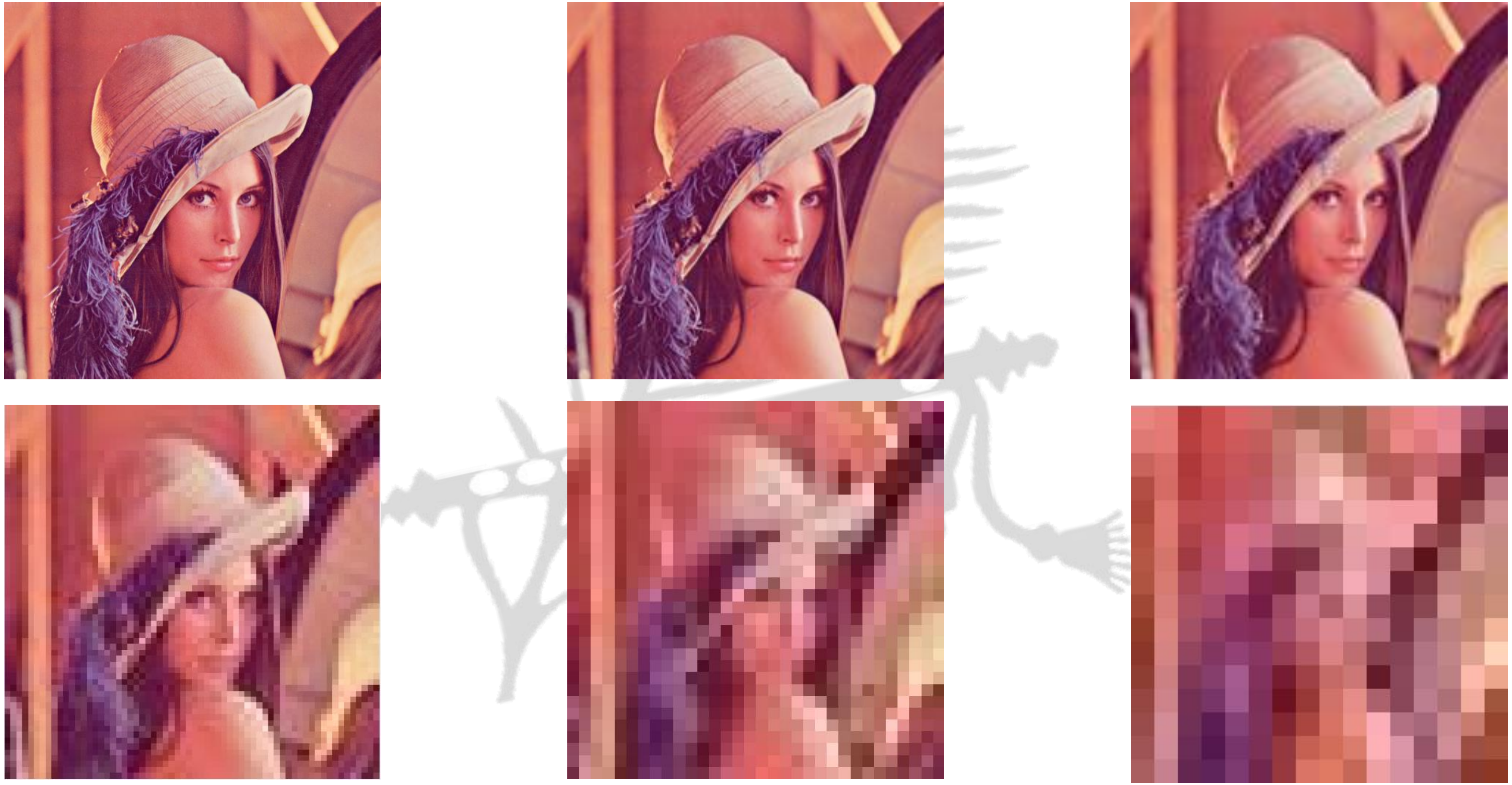
← 128 x 128

← 64 x 64

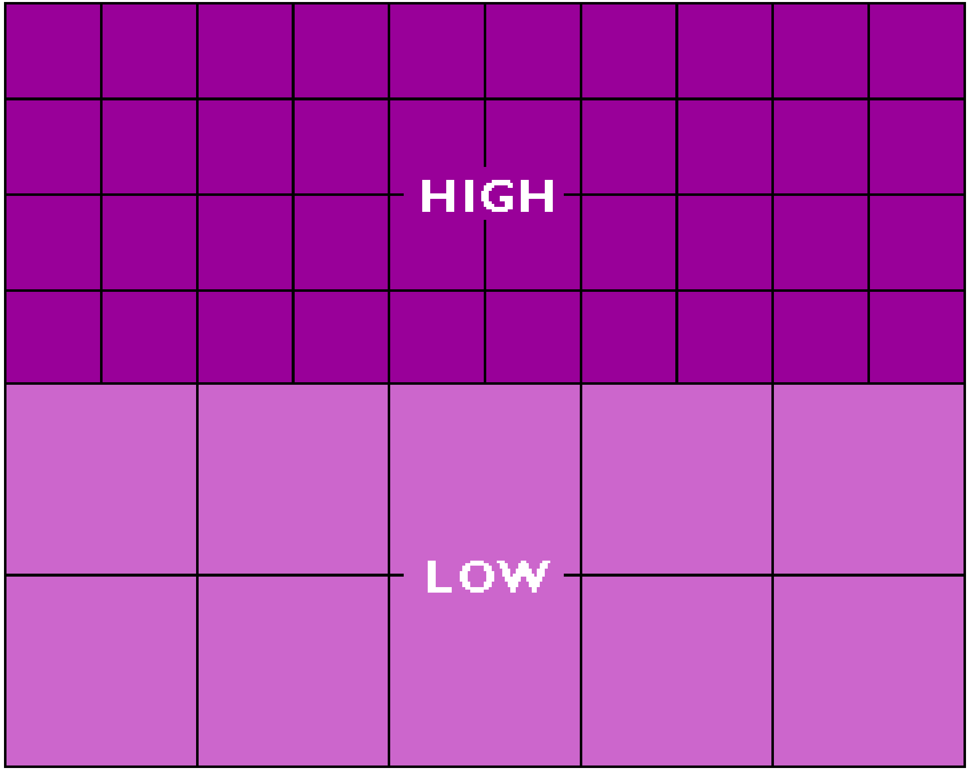
← 32 x 32

← 16 x 16

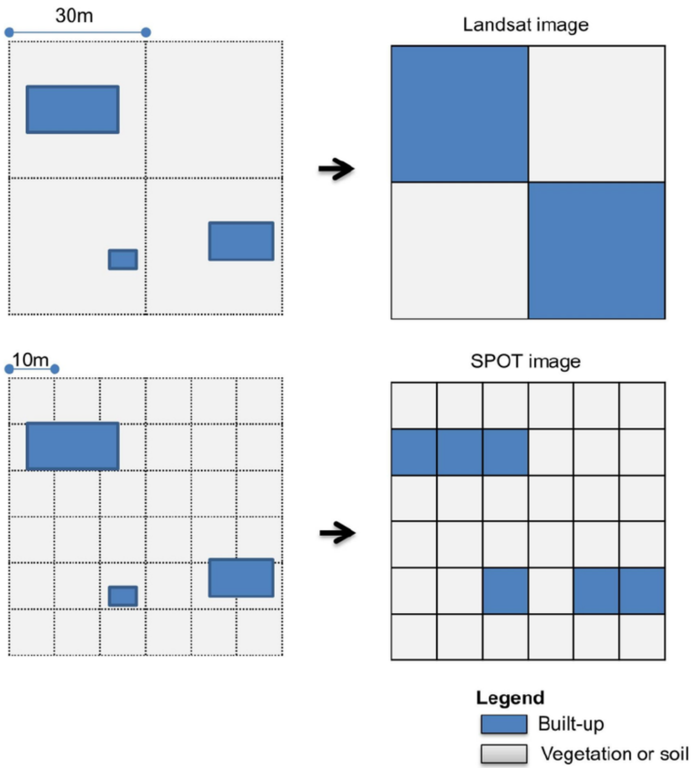
Spatial Resolution



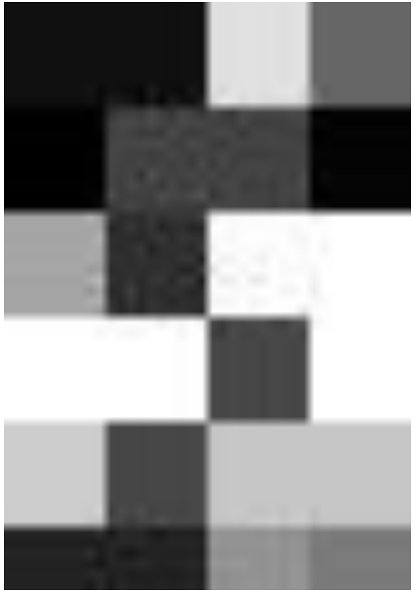
Spatial Resolution

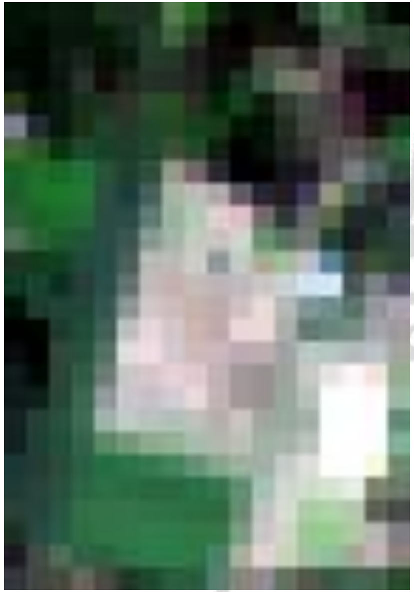


Spatial Resolution



Various sensors image


MODIS (250 m)

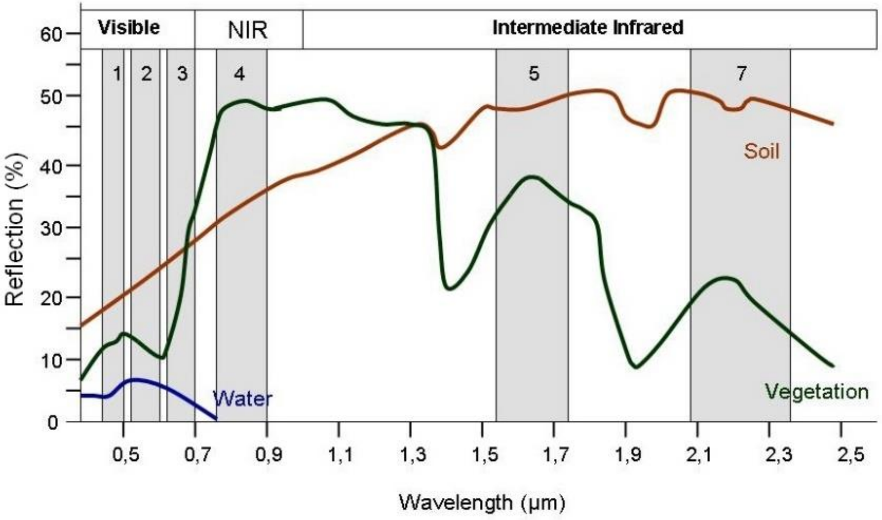

LANDSAT 8 (30 m)


SENTINEL 2 (10 m)


DRONE (0.05 m)

Spectral Resolution

- It describes the capability of a sensor to capture information in various bands which have fine wavelength intervals
- Information recorded by the sensor in various bands are referred as multi-spectral sensors



Spectral Resolution

Spectral Resolution

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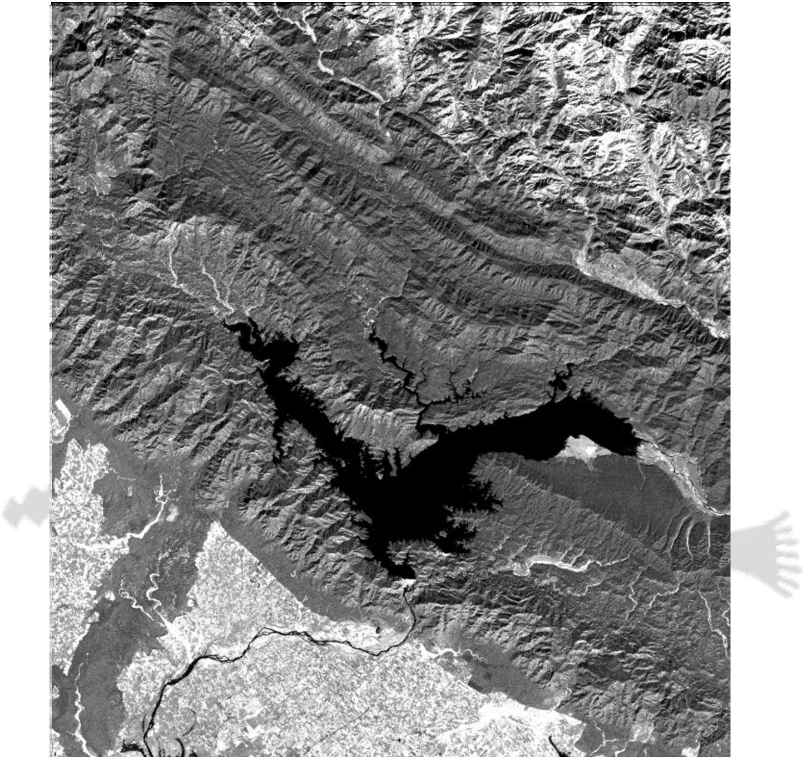
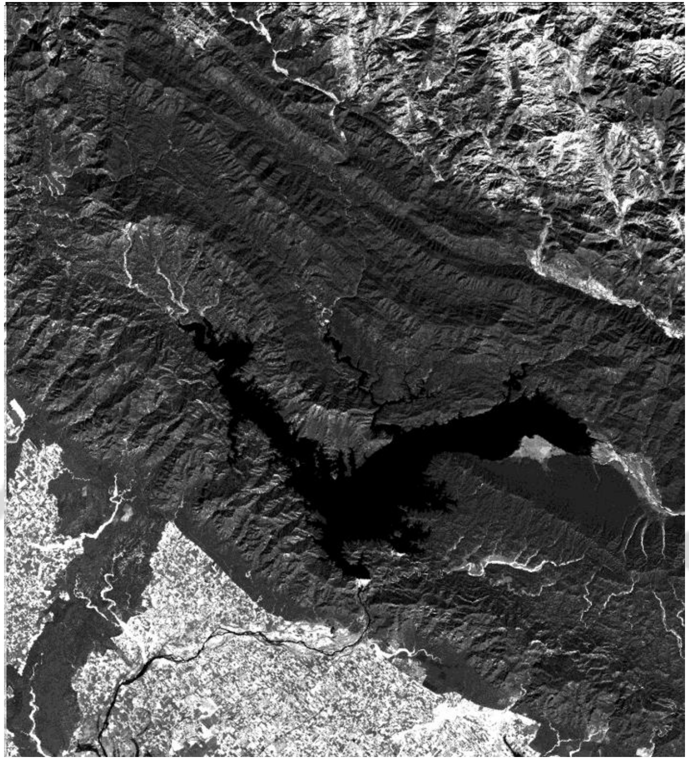
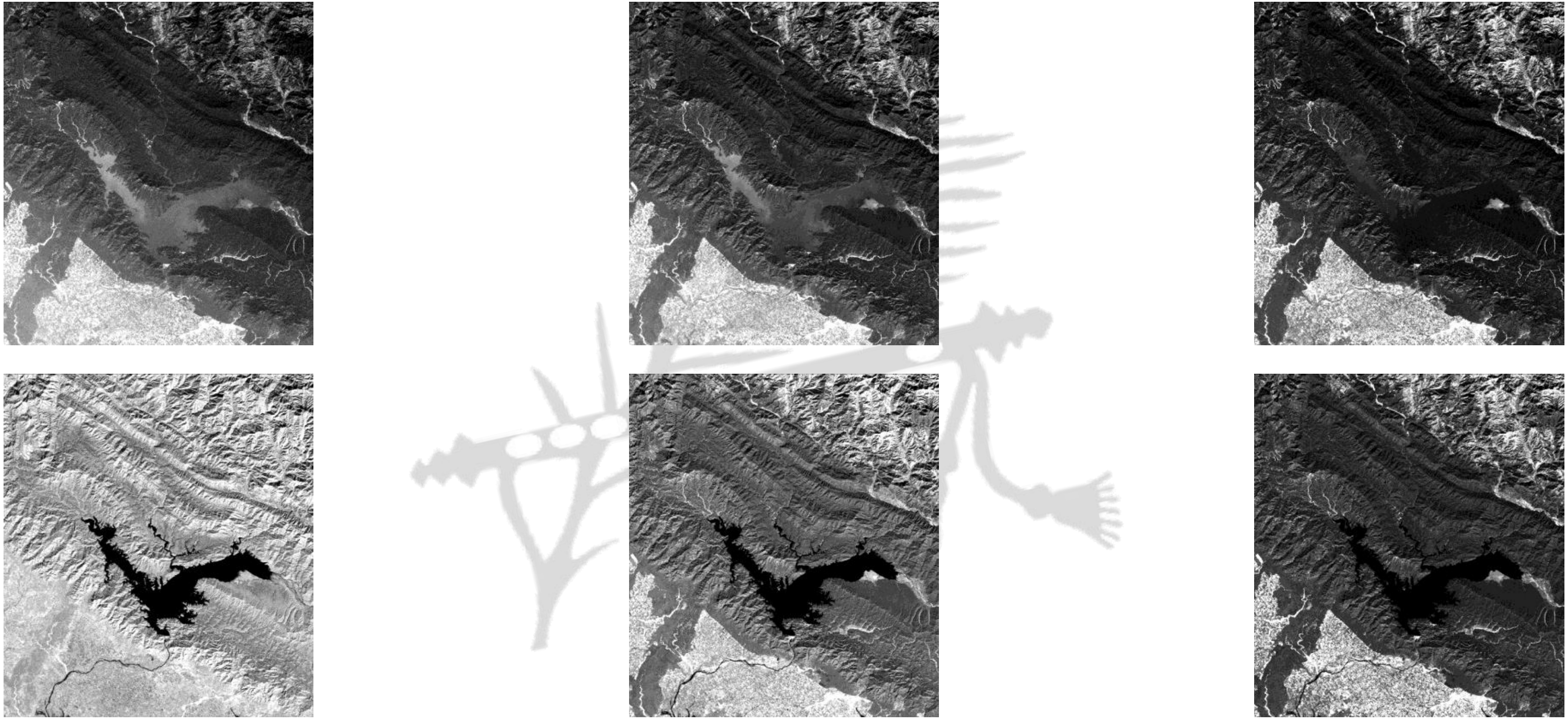
Spectral Resolution



Spectral Resolution



Spectral Resolution



Temporal Resolution

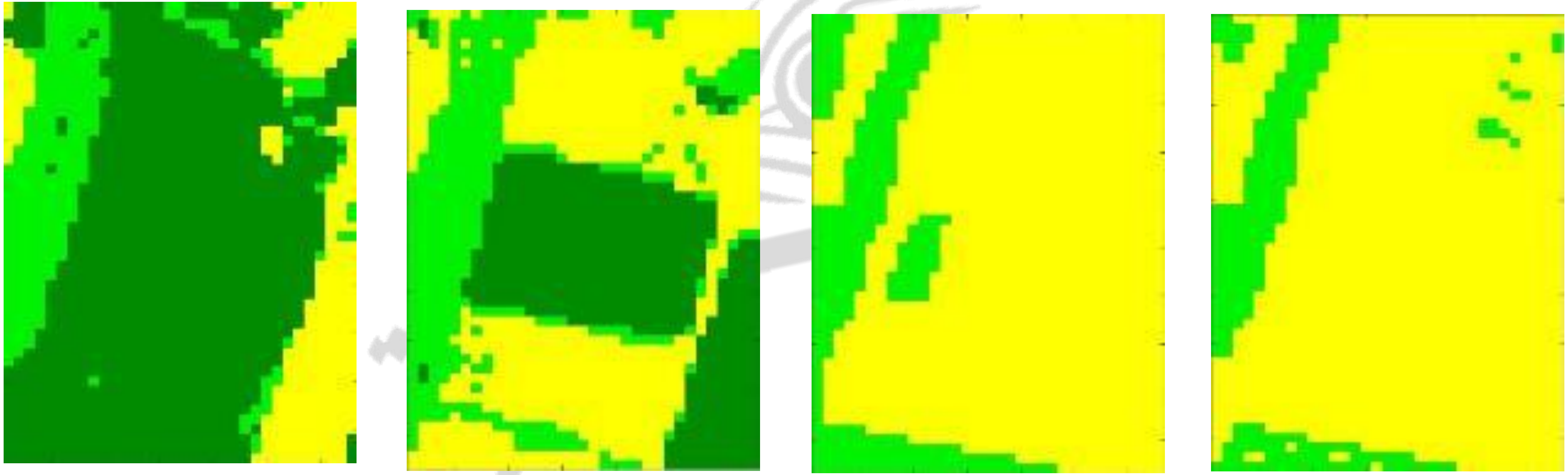
- The concept of revisit period is important for monitoring that help us to identify the changes over a period of time
- Time taken by a sensor to locate exact same area second time at same interval time is considered as temporal resolution

Temporal Resolution

Sentinel-2
RGB

Temporal Resolution

Sentinel-2
Classified



Bare Land | Dense Vegetation | Sparse Vegetation

Radiometric Resolution

- Image captured by sensor have certain characteristics which can be improved later
- This is a part of radiometric resolution which helps to enhance the pixel characteristics, which ultimately help to identify very slight difference in an image
- It depends on the ability of sensor
- Sensor having greater radiometric property is more sensitive to detect changes
- It is the smallest visible change in the gray level of an image

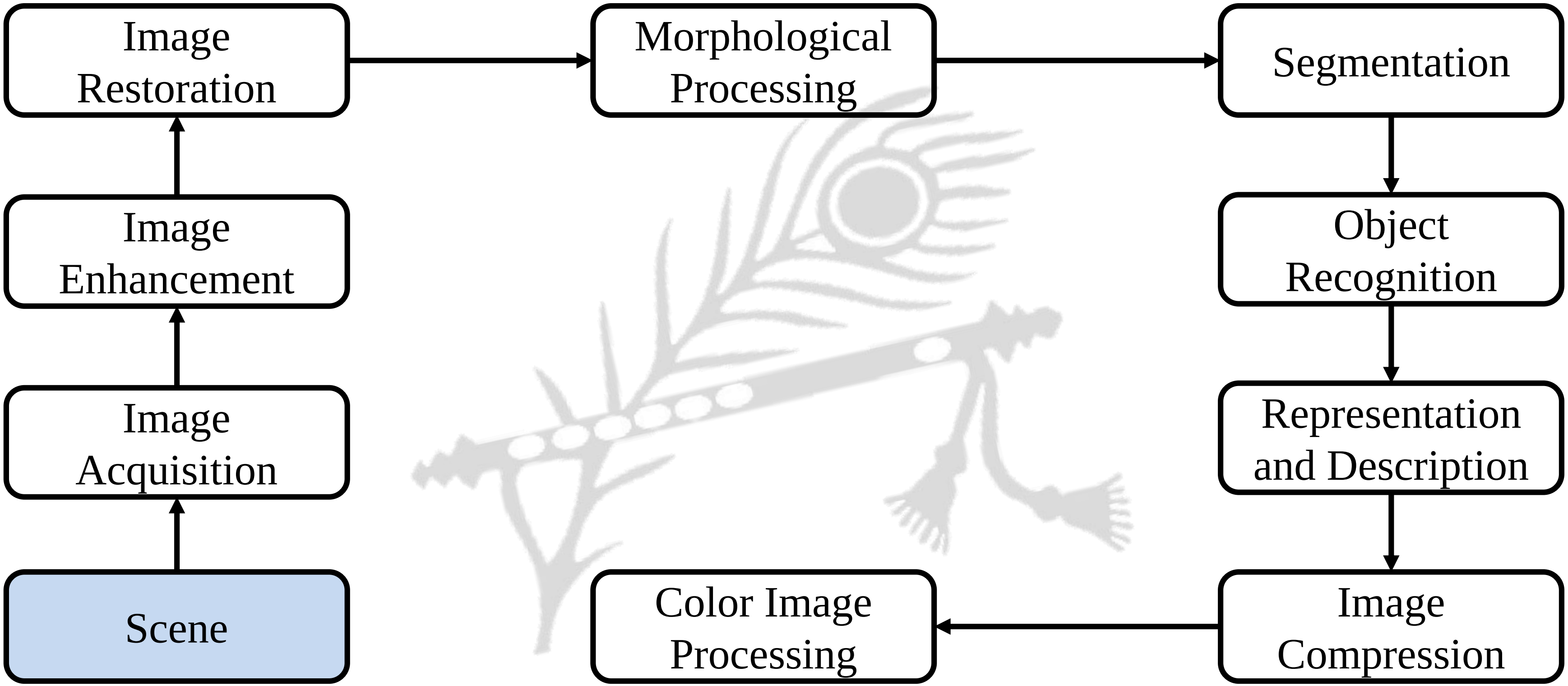
Radiometric Resolution

Type of effects

- There are two type of effects
  - Checkerboard effect
    - When the no. of pixels in an image is reduced by keeping the no. of gray levels in the image constant, fine checkerboard patterns are found at the edges of the image
  - False contouring effect
    - When the no. of gray-levels in the image is low, the foreground details of the image merge with the background details of the image, causing ridge like structures. This degradation phenomenon is known as false contouring

COMPONENTS OF
IMAGE PROCESSING SYSTEM

Key Stages in Digital Image Processing



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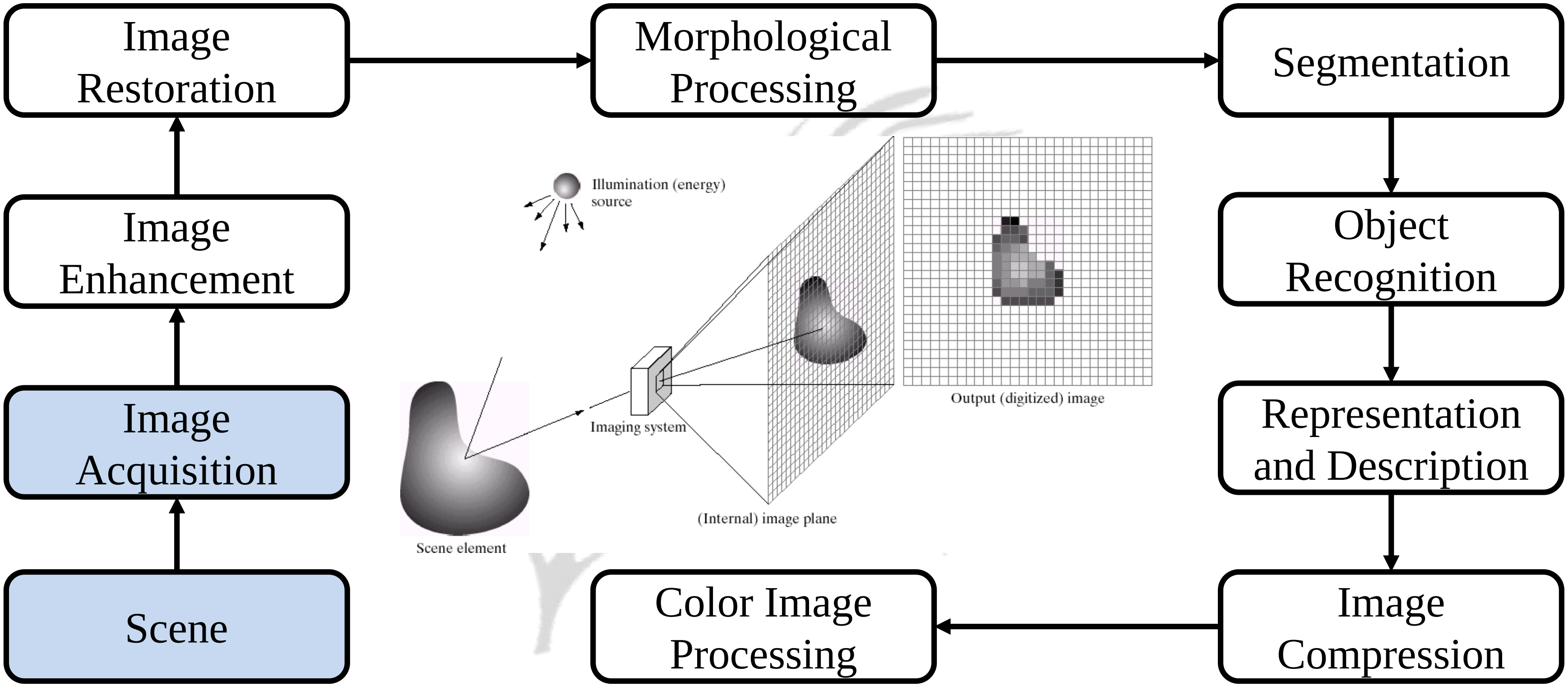

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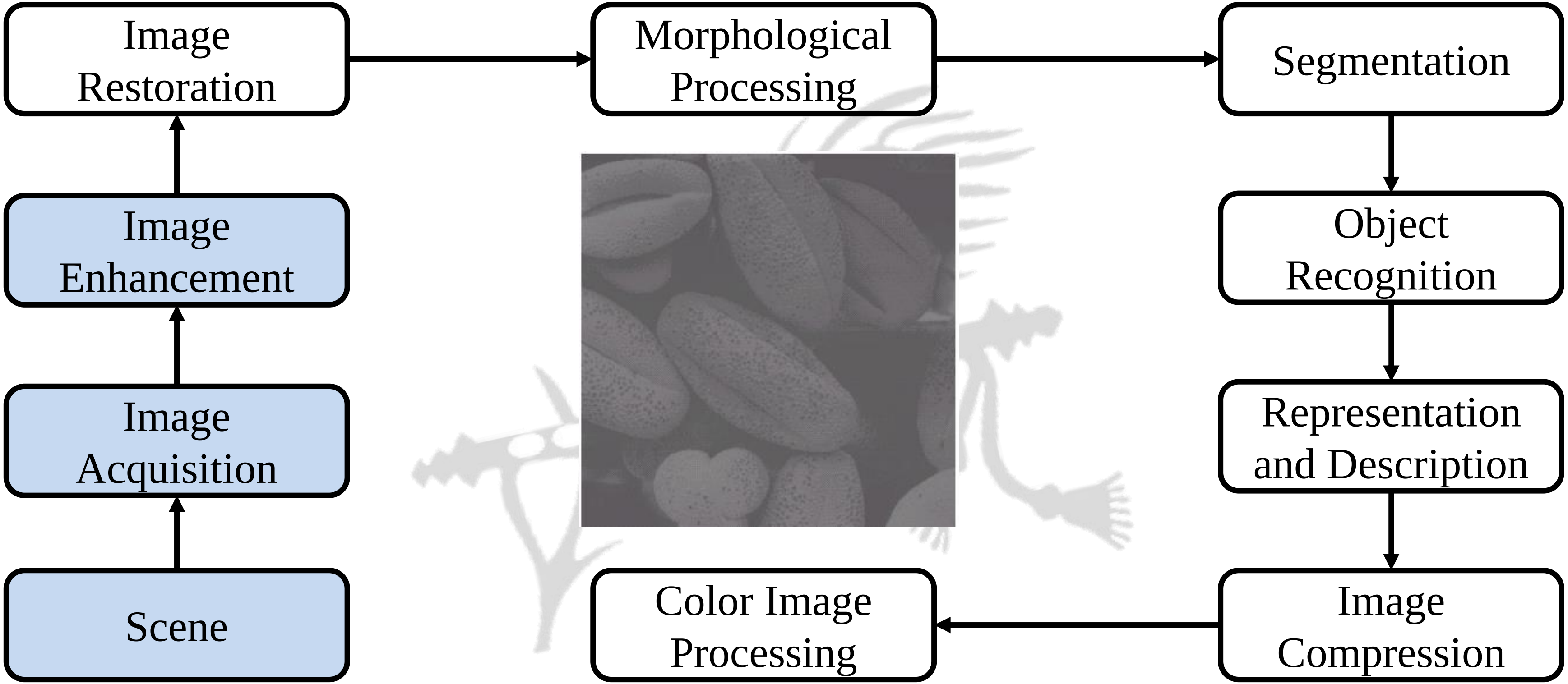

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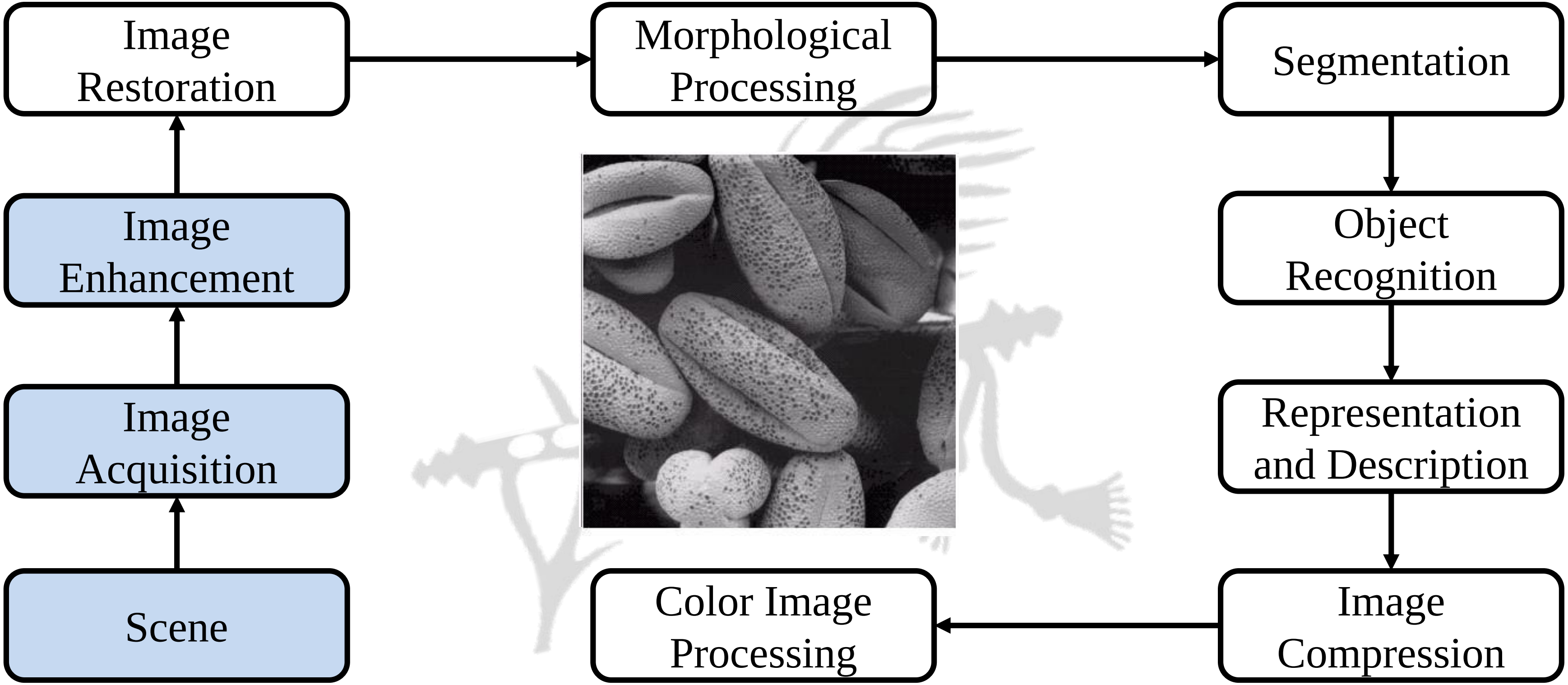

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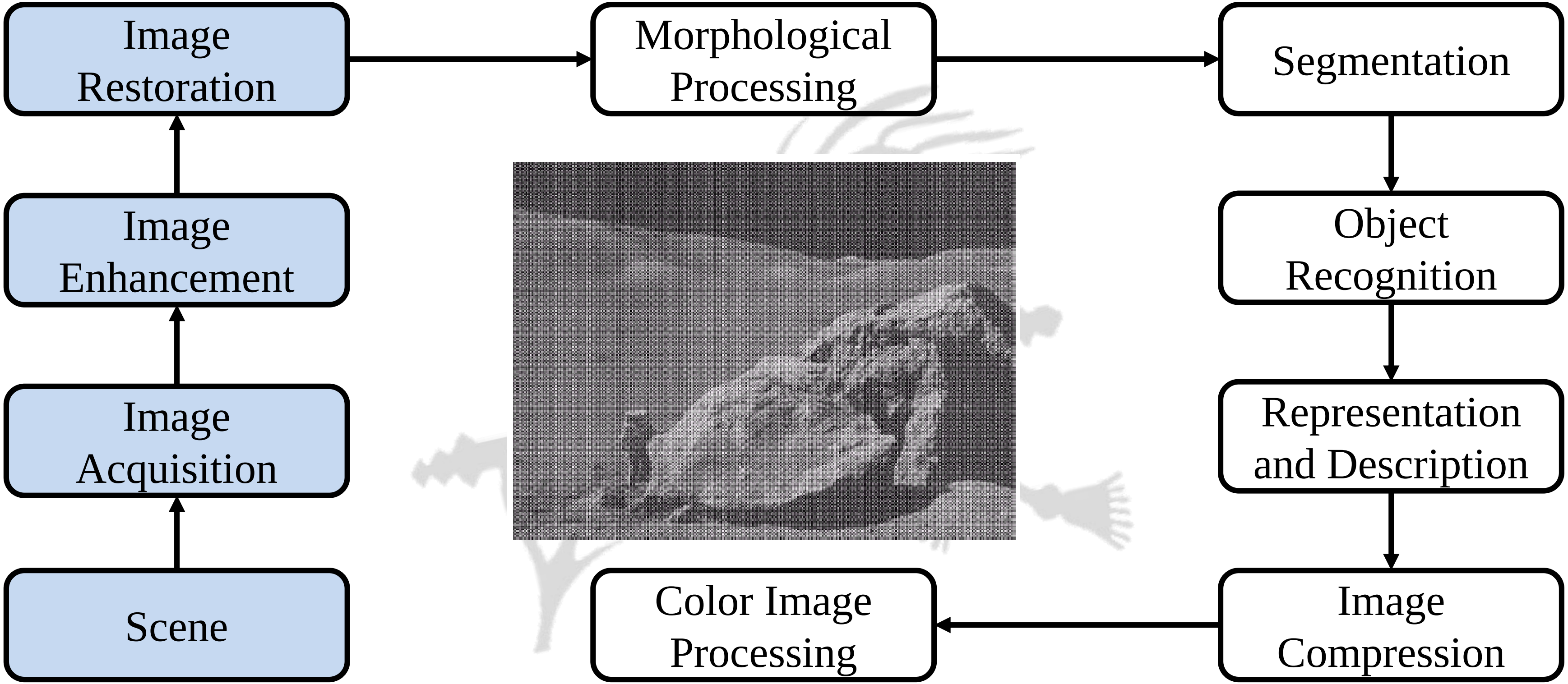

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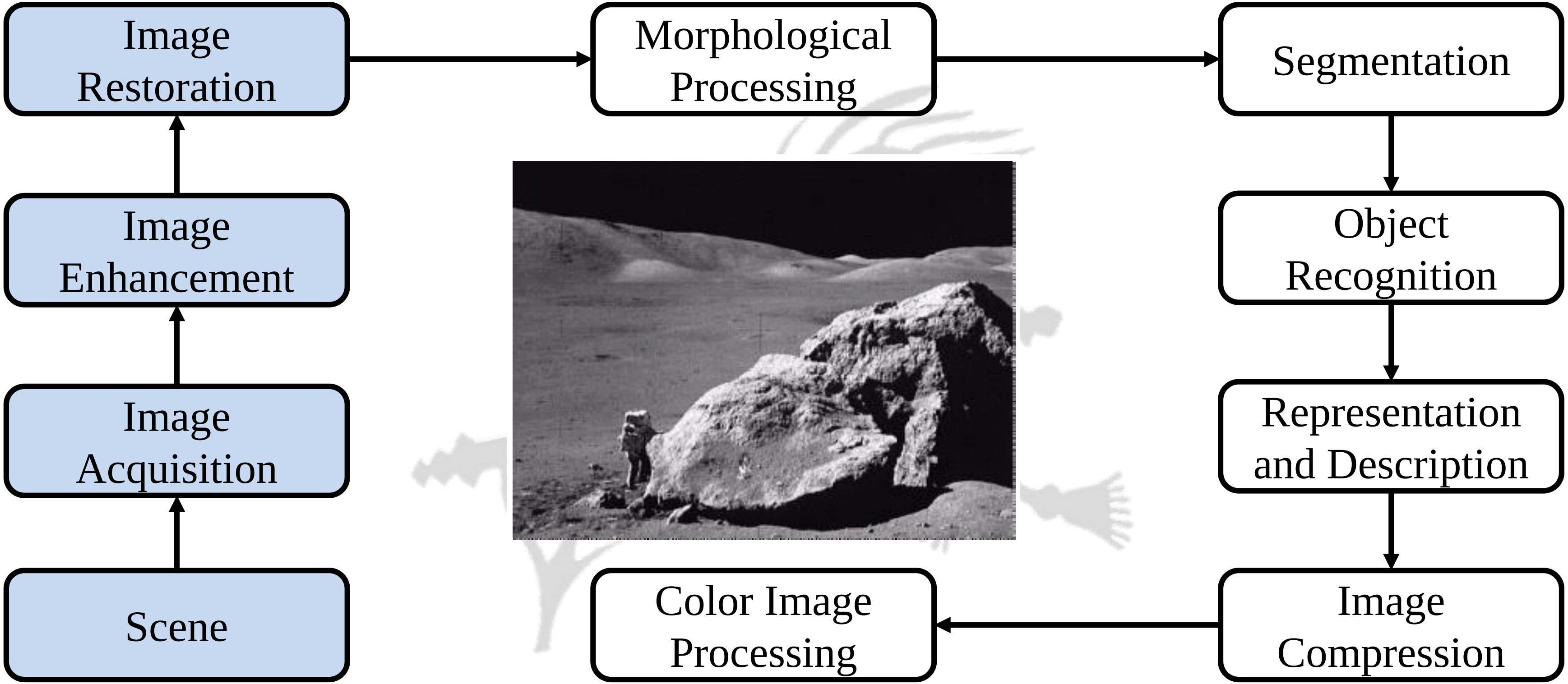

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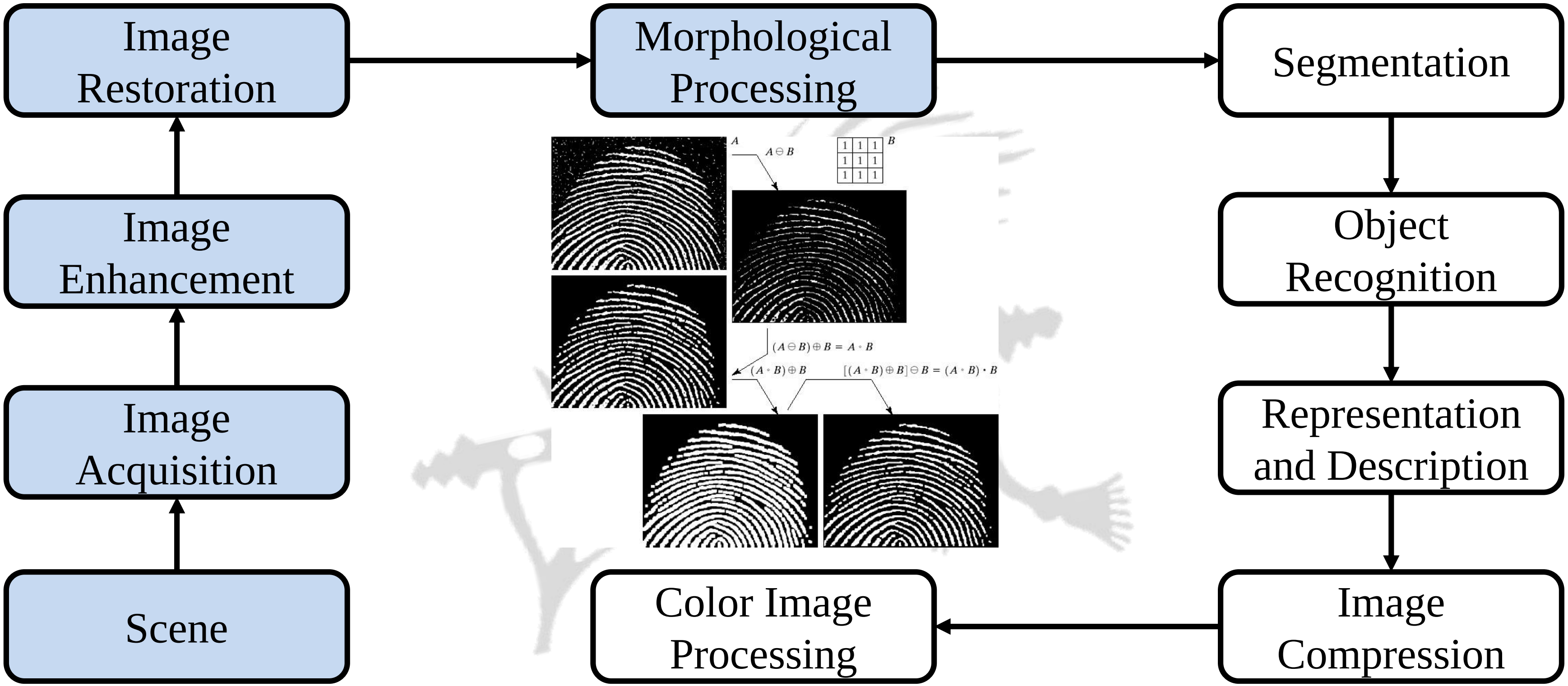

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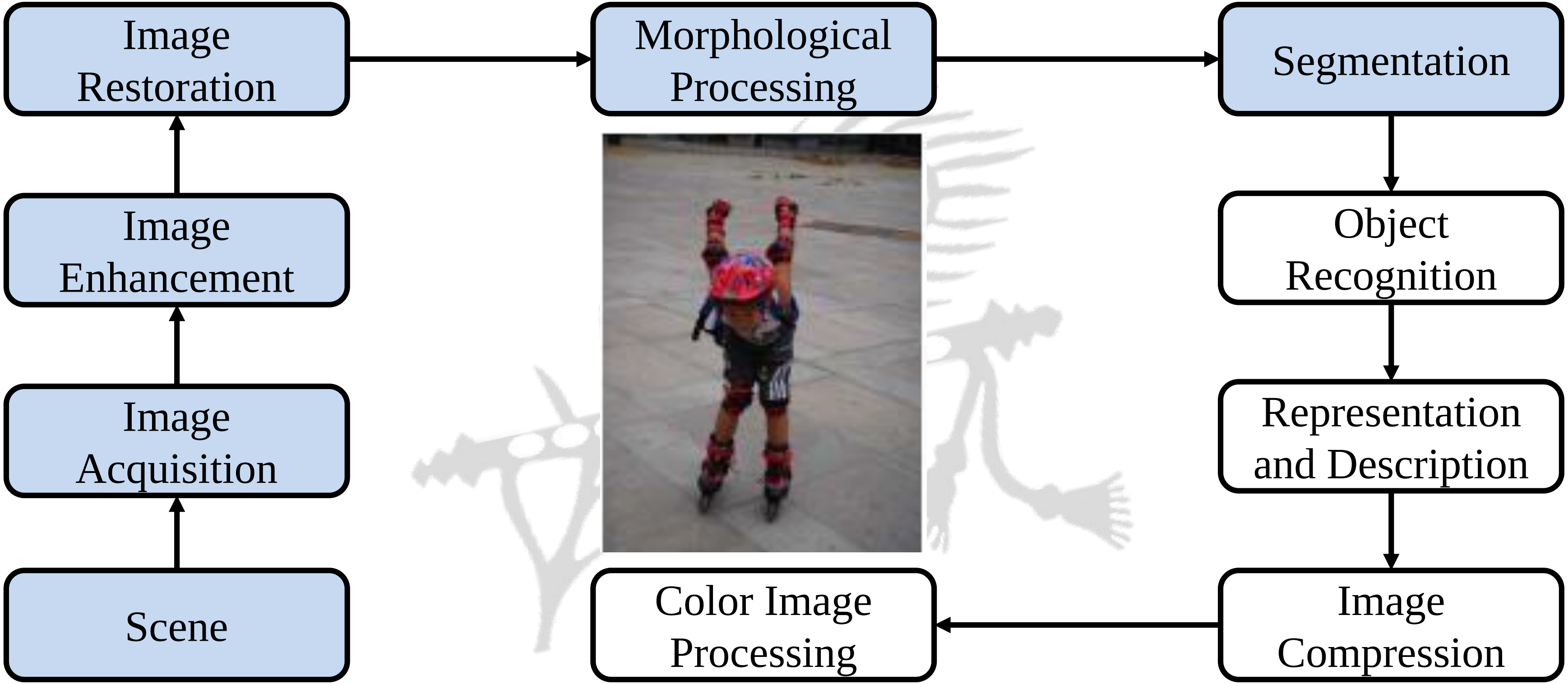

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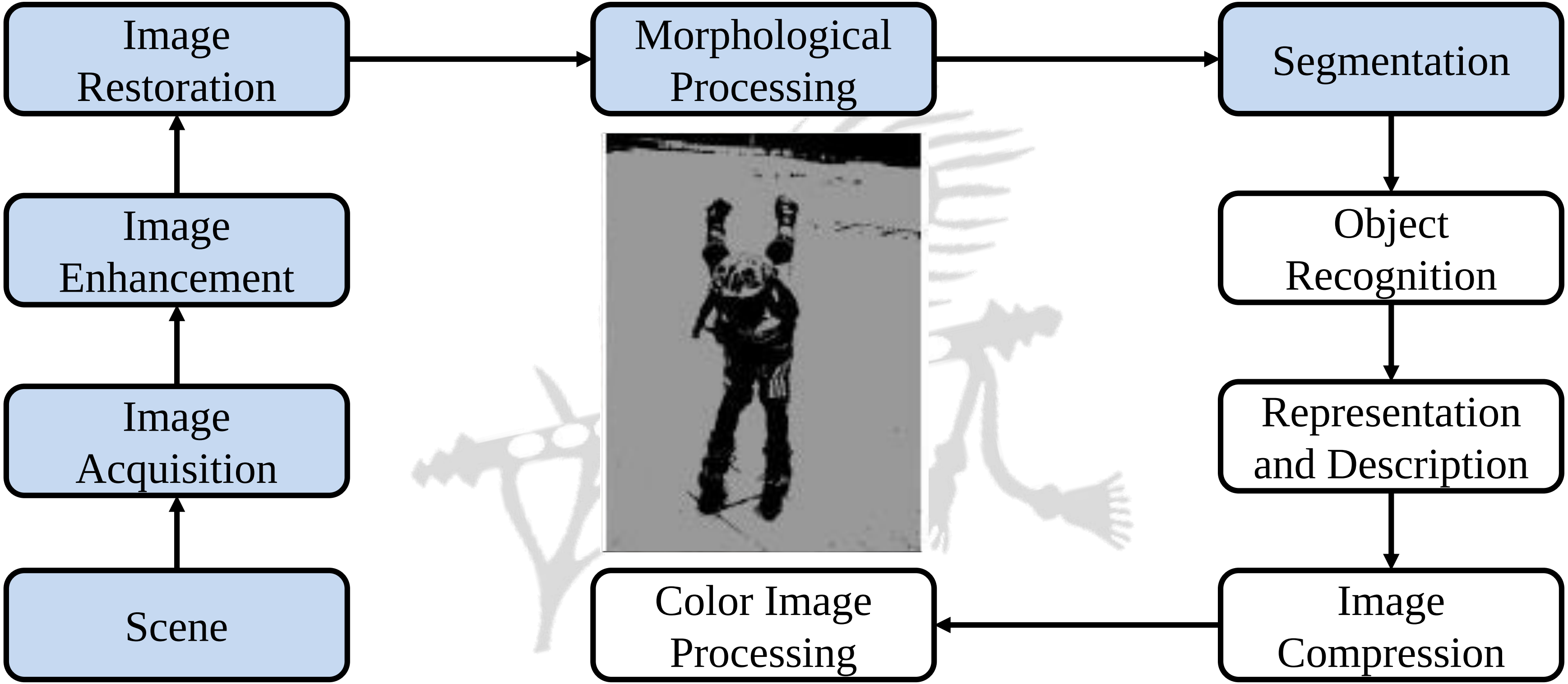

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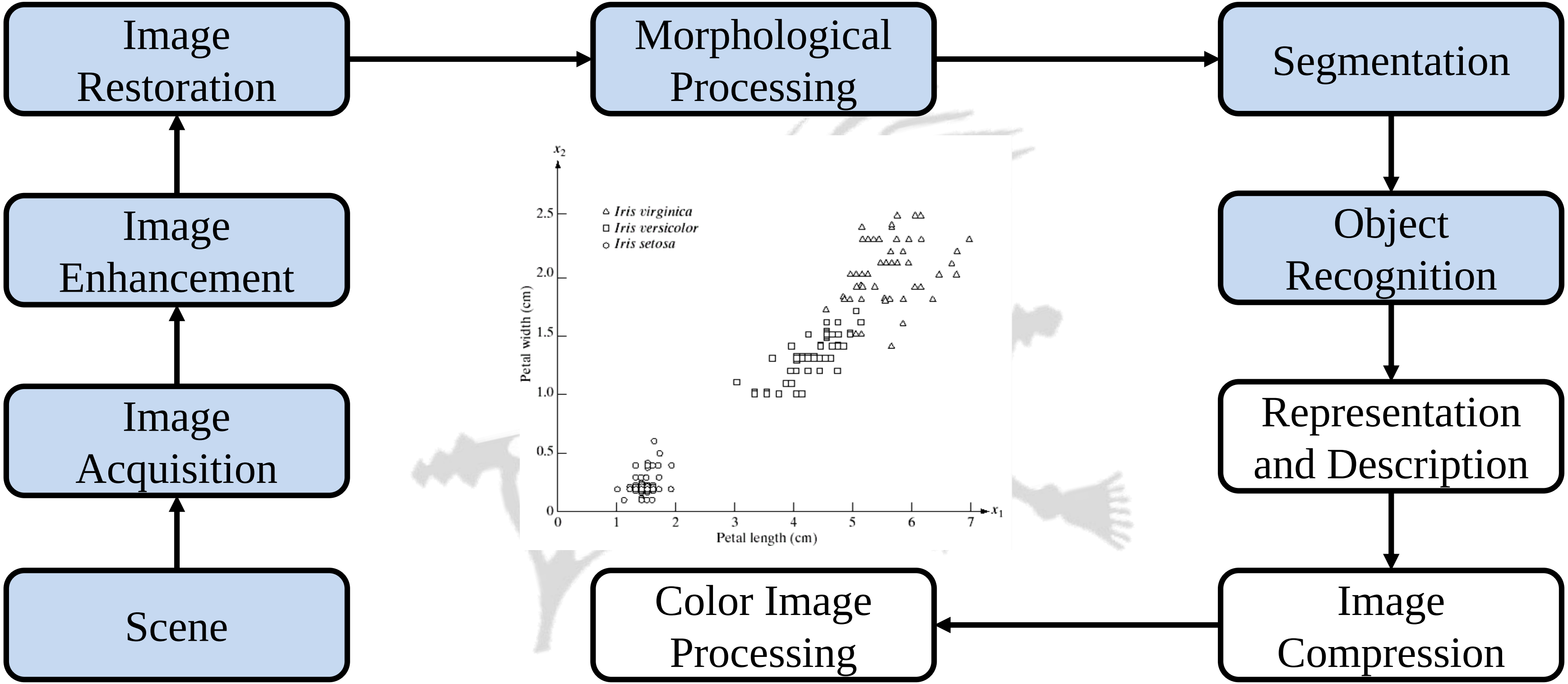

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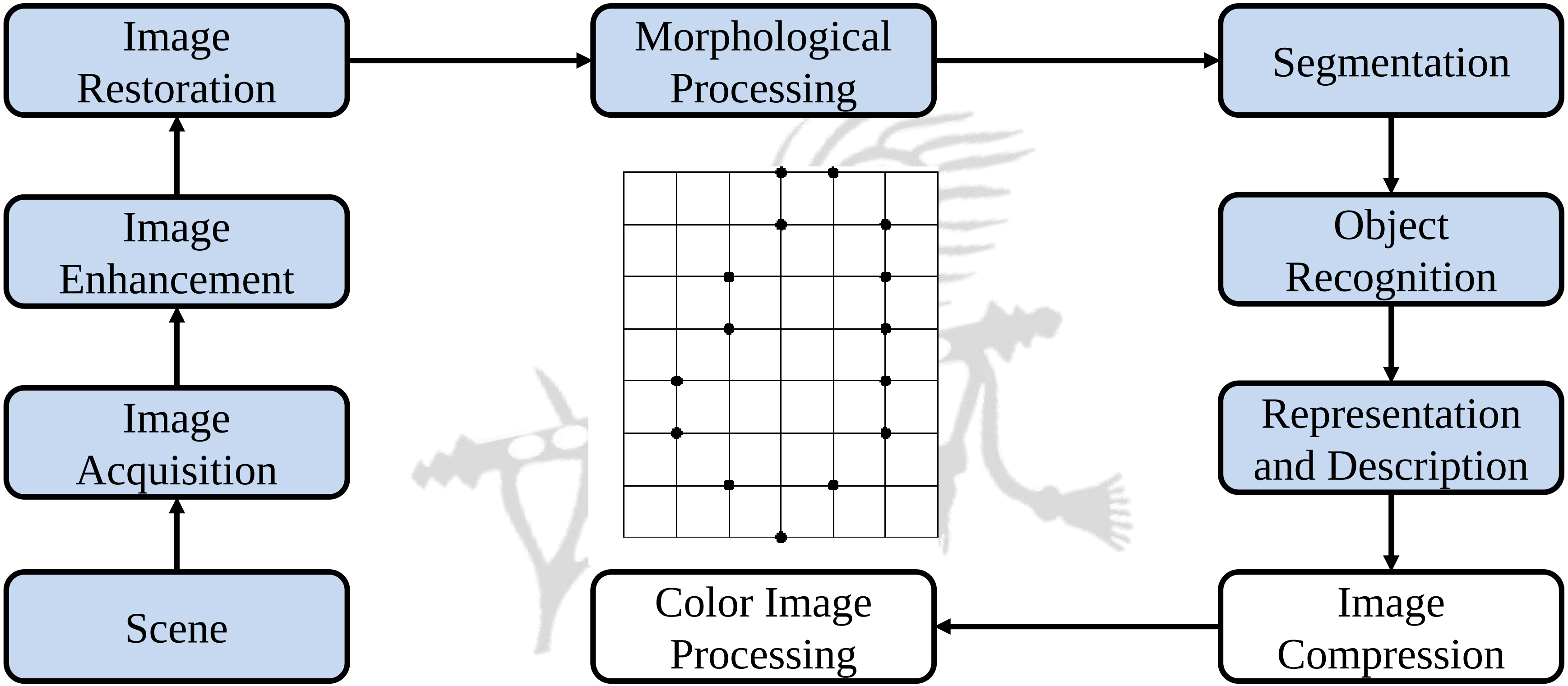

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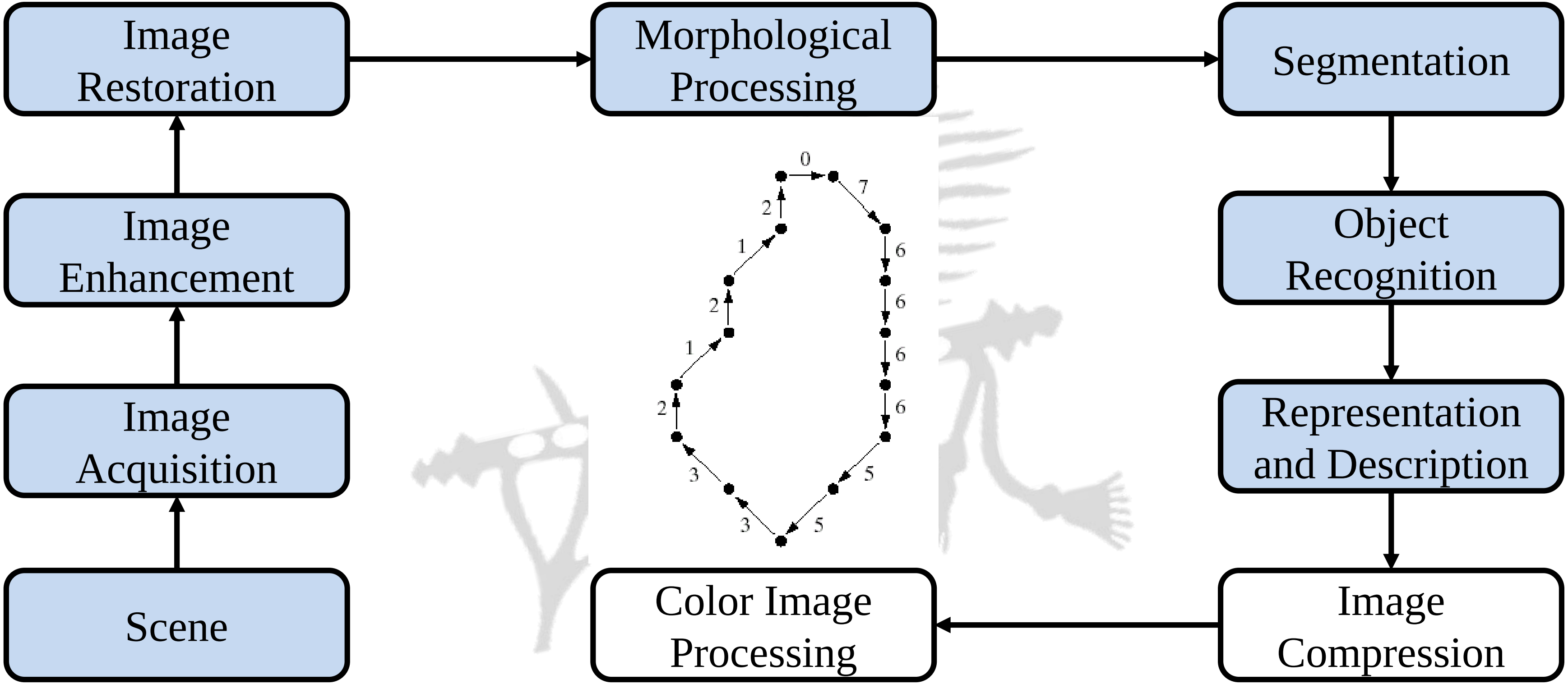

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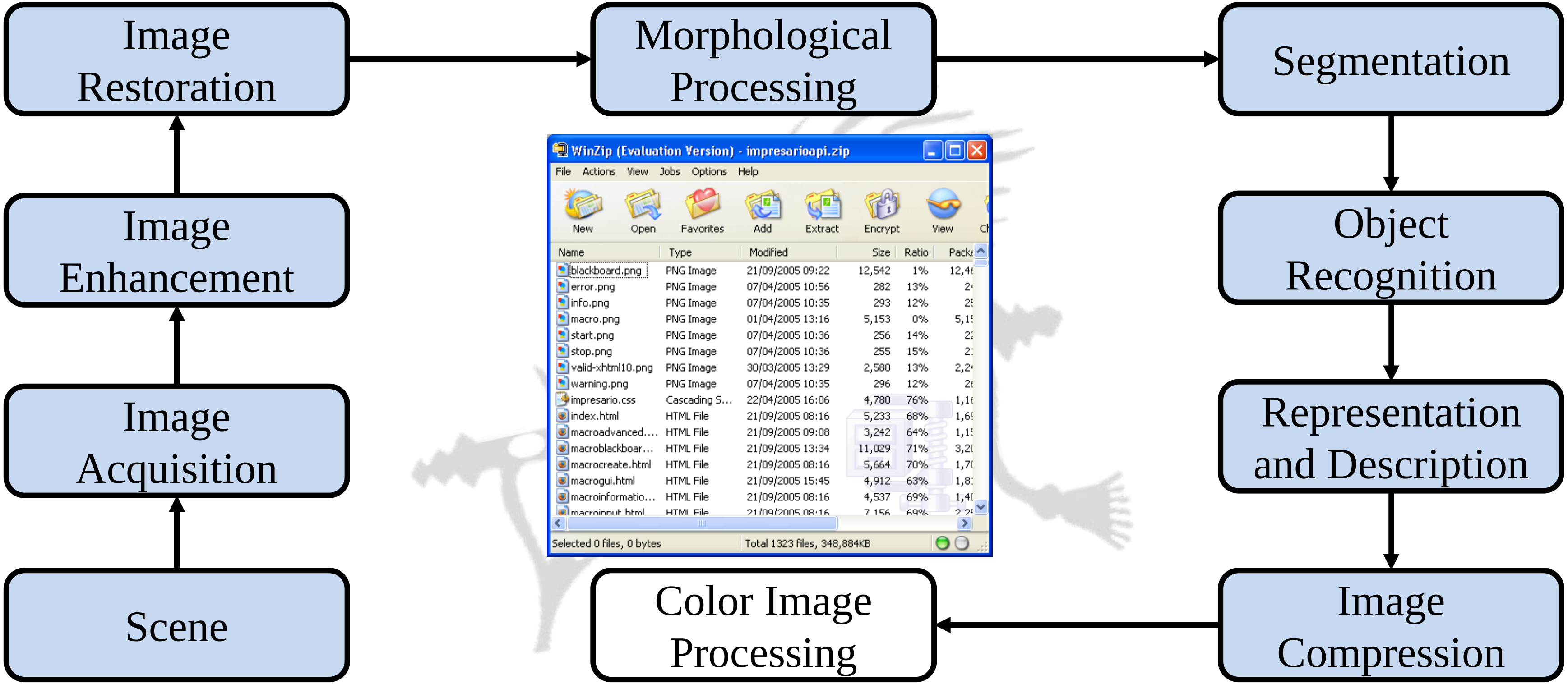

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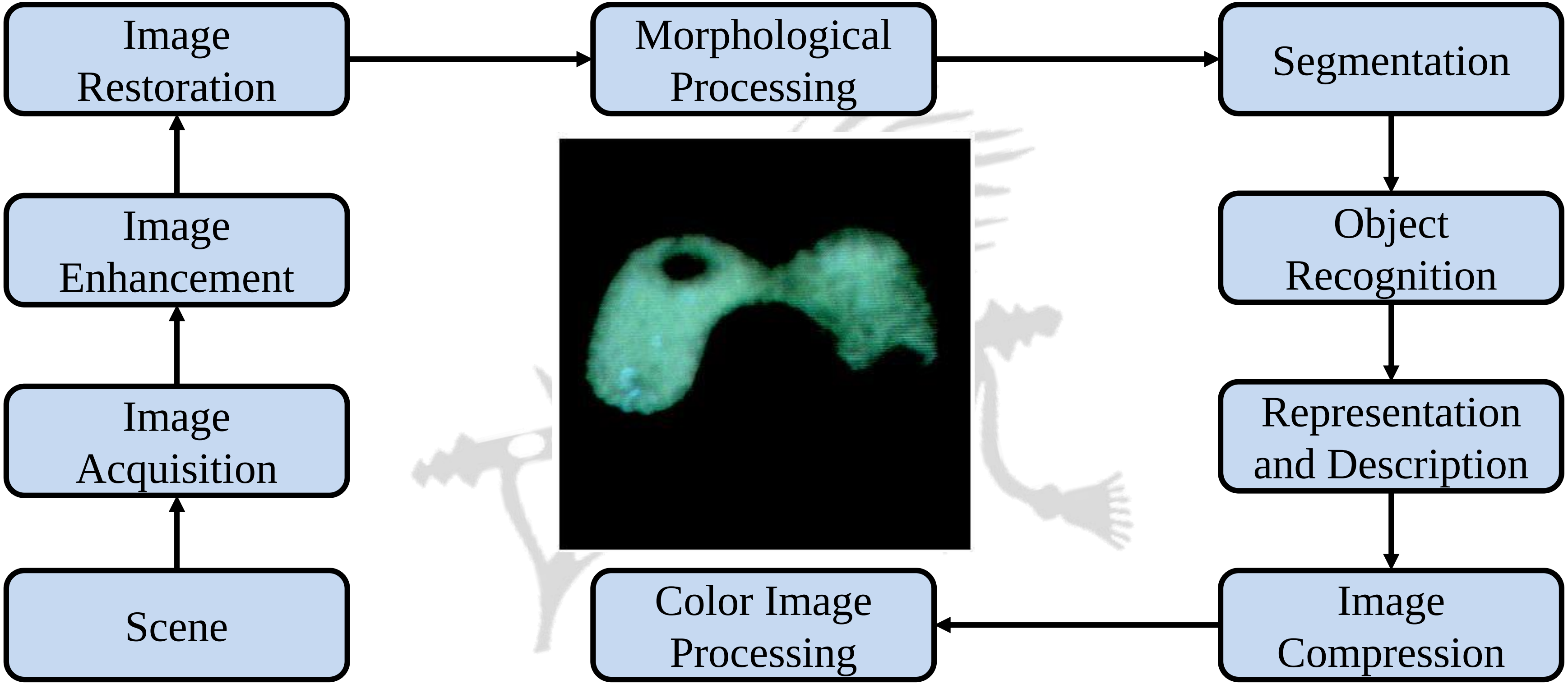

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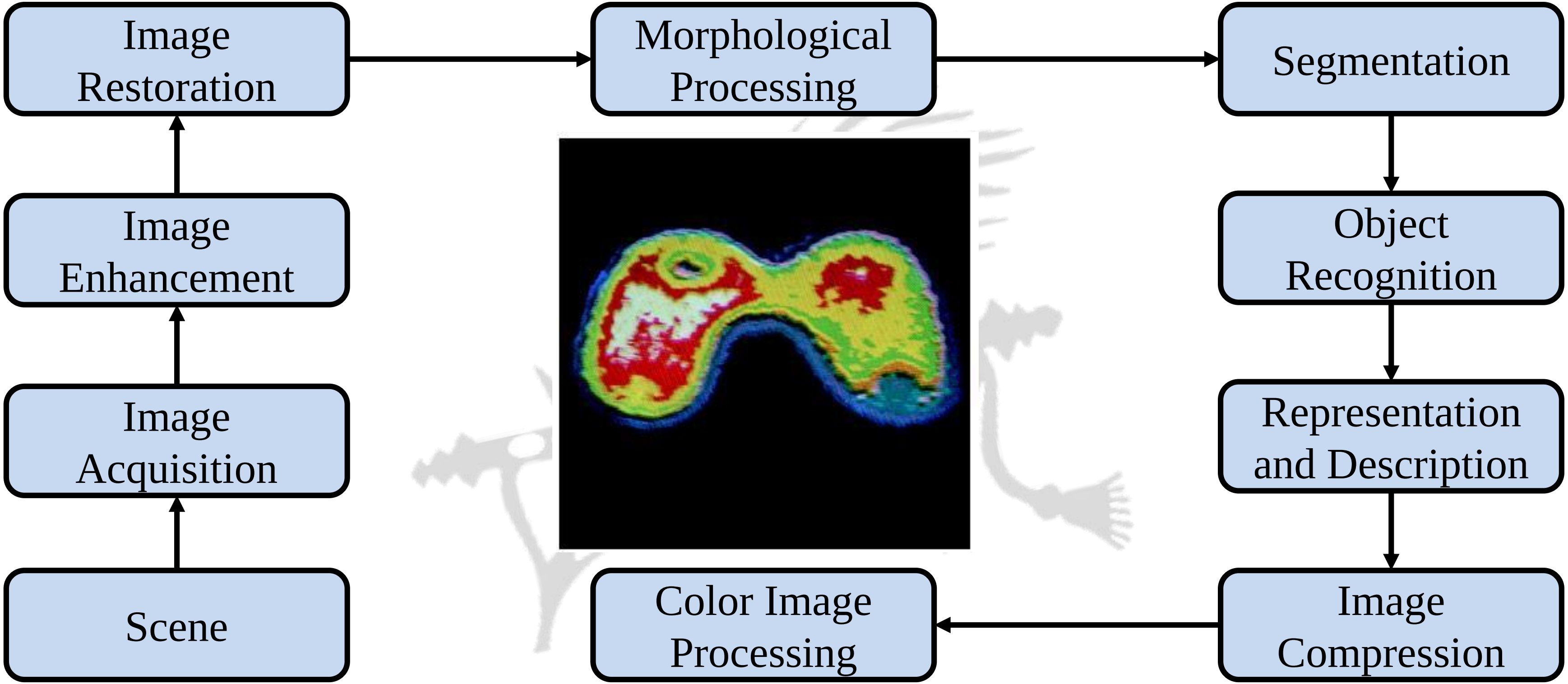


Various operations on Image

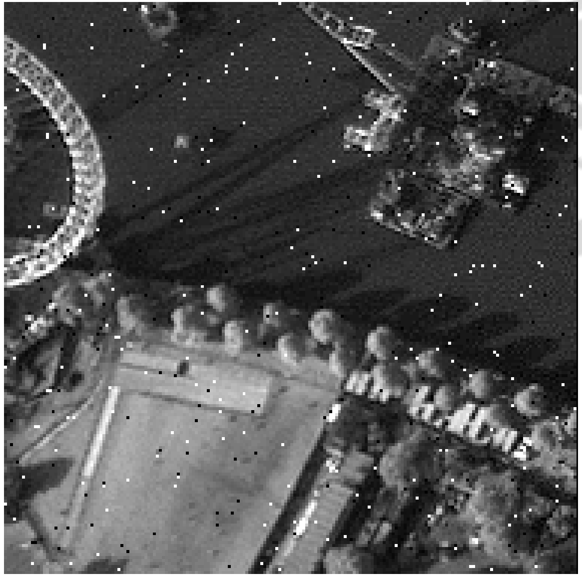
- Image Restoration
  - Image Restoration is the process of recovering the native dataset undistorted and uncorrupted by noise

Original

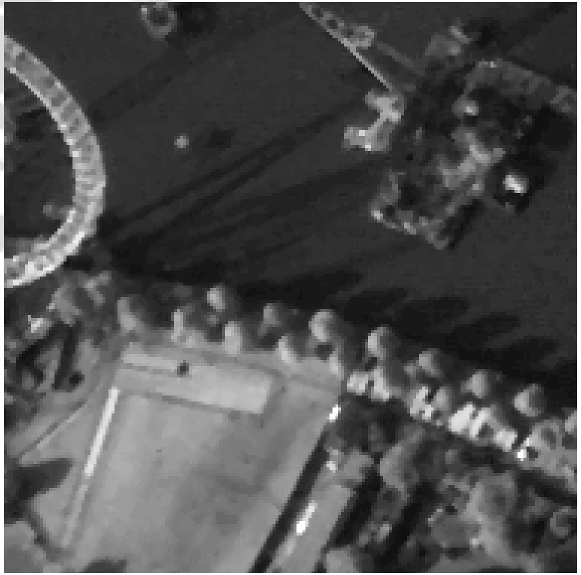
Restored

Various operations on Image

- Noise Removal
  - Various types of noise is induced in the data that needs to be corrected



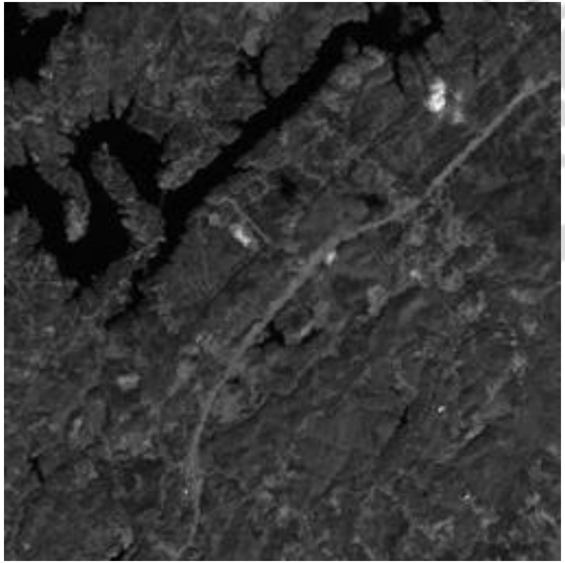
Original



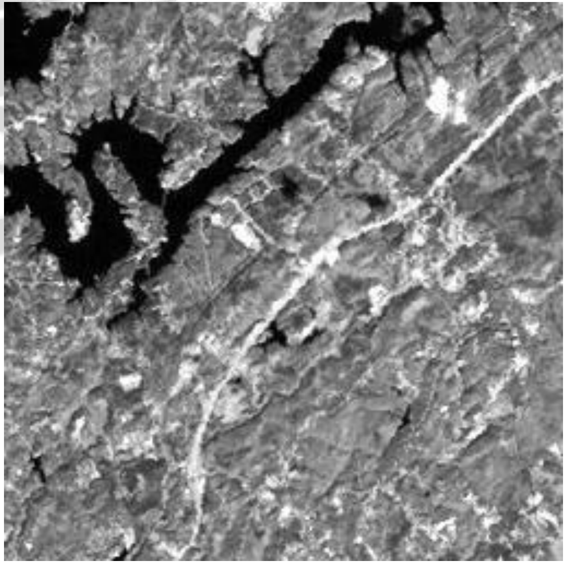
Corrected

Various operations on Image

- Image Enhancement (Min-Max stretching)
  - It is required to enhance the feature so that they can be extracted



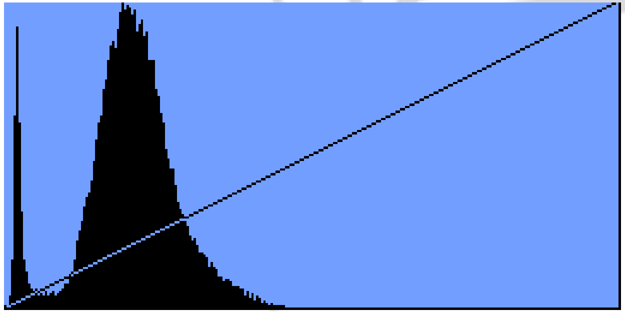
Original



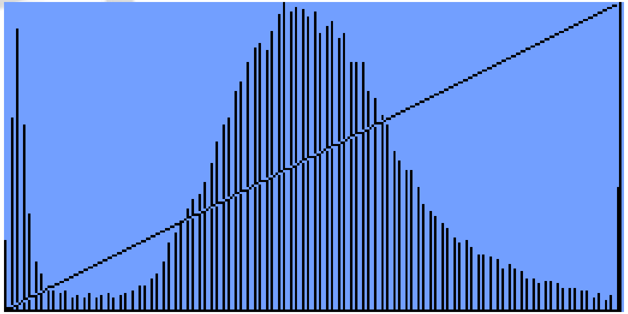
Enhanced

Various operations on Image

- Image Enhancement (Min-Max stretching)
  - It is required to enhance the feature so that they can be extracted



Original



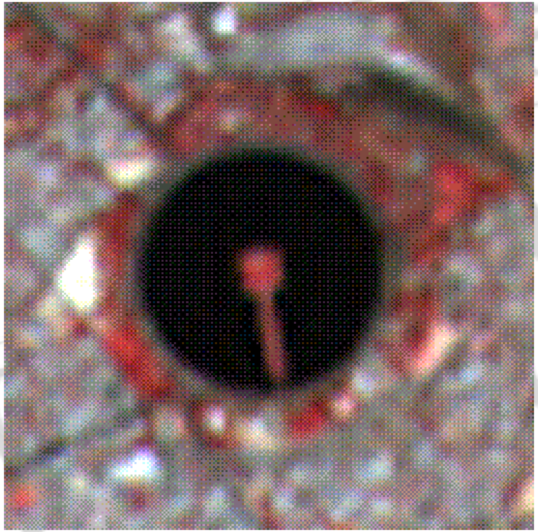
Enhanced

Various operations on Image

- Image Enhancement (data fusion)

PAN (5.8 m)

+



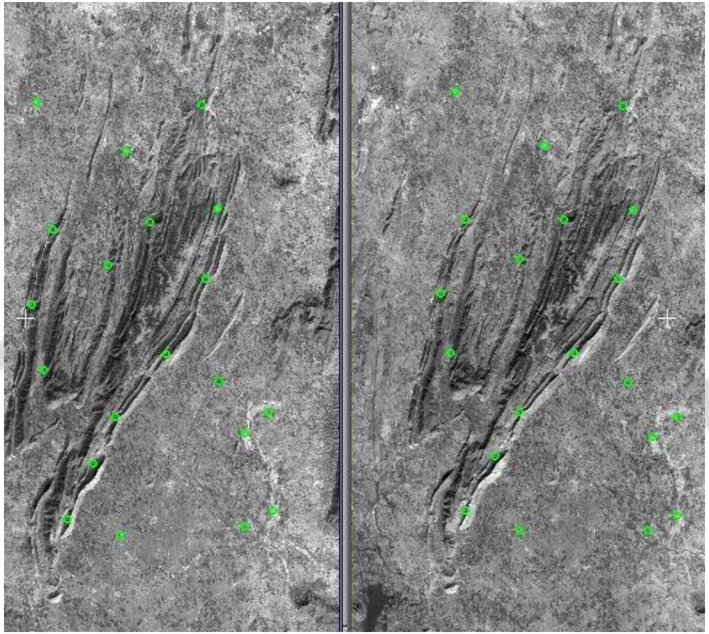
L3 (23.5 m)

→

P+L3

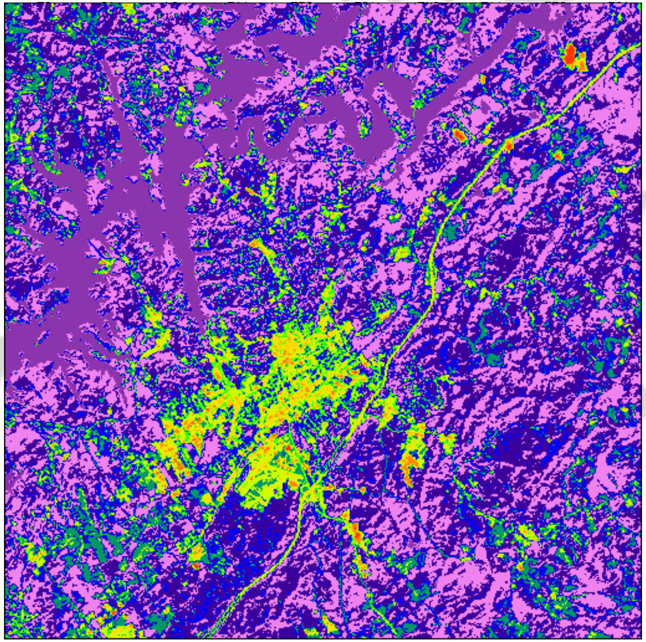
Various operations on Image

- Image Matching
  - It is used to geo-register the image by matching the features



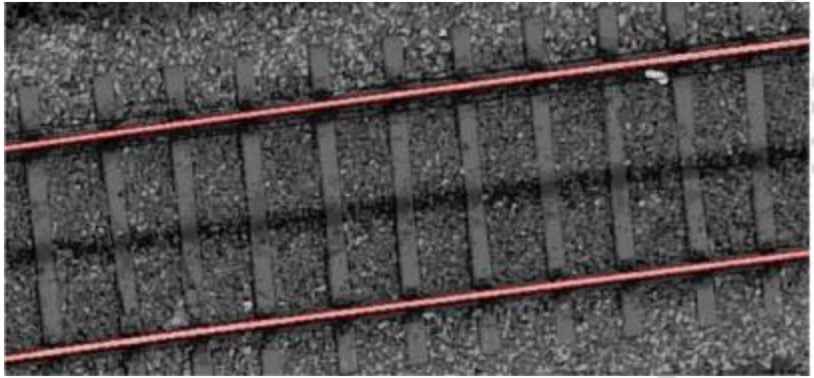
Various operations on Image

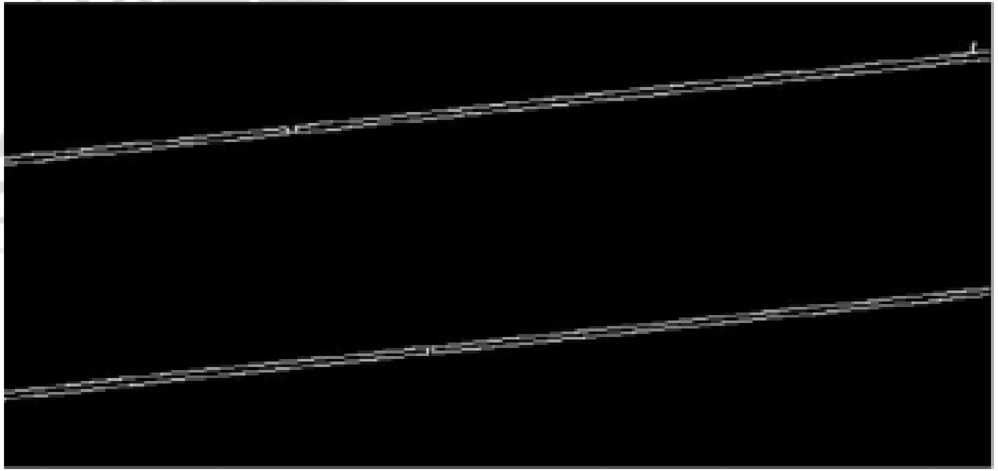
- Image Classification
  - To classify the images into different classes



Various operations on Image

- Object Extraction
  - To extract the object by using morphological operators

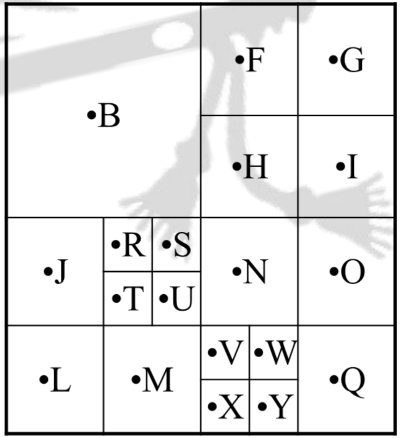




Various operations on Image

- Image Representation
  - Successive divisions of image into quadrants and recursively into sub quadrants
  - Recursion is stopped when all pixels in a sub image is found to have the same colour or gray value
  - Suitable for binary and segmented/classified images

•0	•0	•0	•0	•0	•0	•0	•0
•0	•0	•0	•0	•0	•0	•0	•0
•0	•0	•0	•0	•1	•1	•1	•1
•0	•0	•0	•0	•1	•1	•1	•1
•0	•0	•0	•1	•1	•1	•1	•1
•0	•0	•1	•1	•1	•1	•1	•1
•0	•0	•1	•1	•1	•1	•1	•1
•0	•0	•1	•1	•1	•1	•1	•1



Various operations on Image

- Image Morphology
  - It is used to extract boundary, skeleton, CCA, etc

Various operations on Image

- Image Morphology
  - It is used to extract boundary, skeleton, CCA, etc

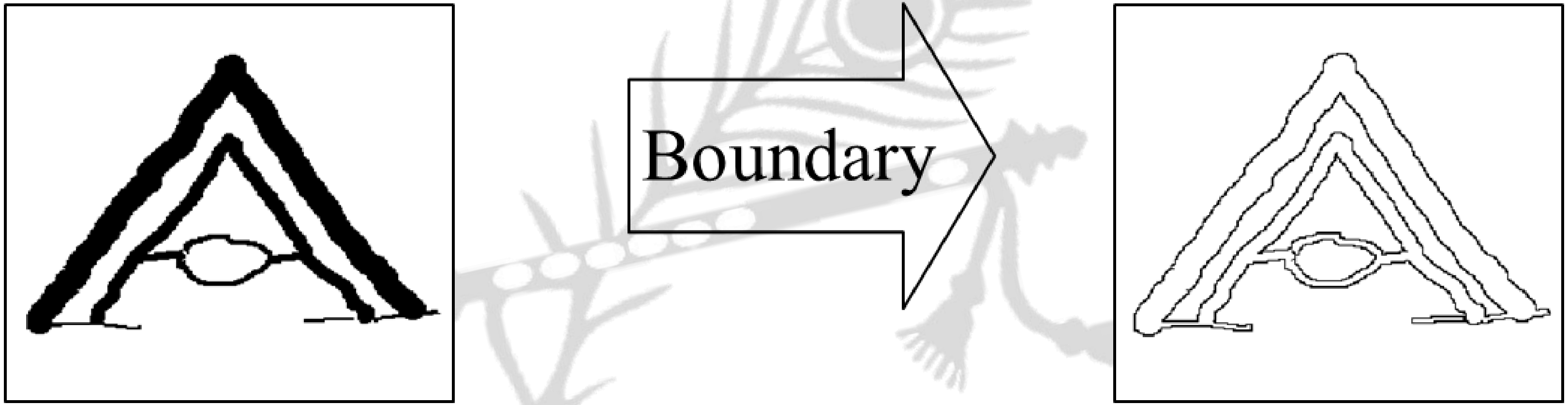


Image types

Image Examples

- X-ray Image
- Ultrasound Images
- CT Scan Images
- MR Images
- Satellite Images
- Thermal Images
- Radar Images
- etc…

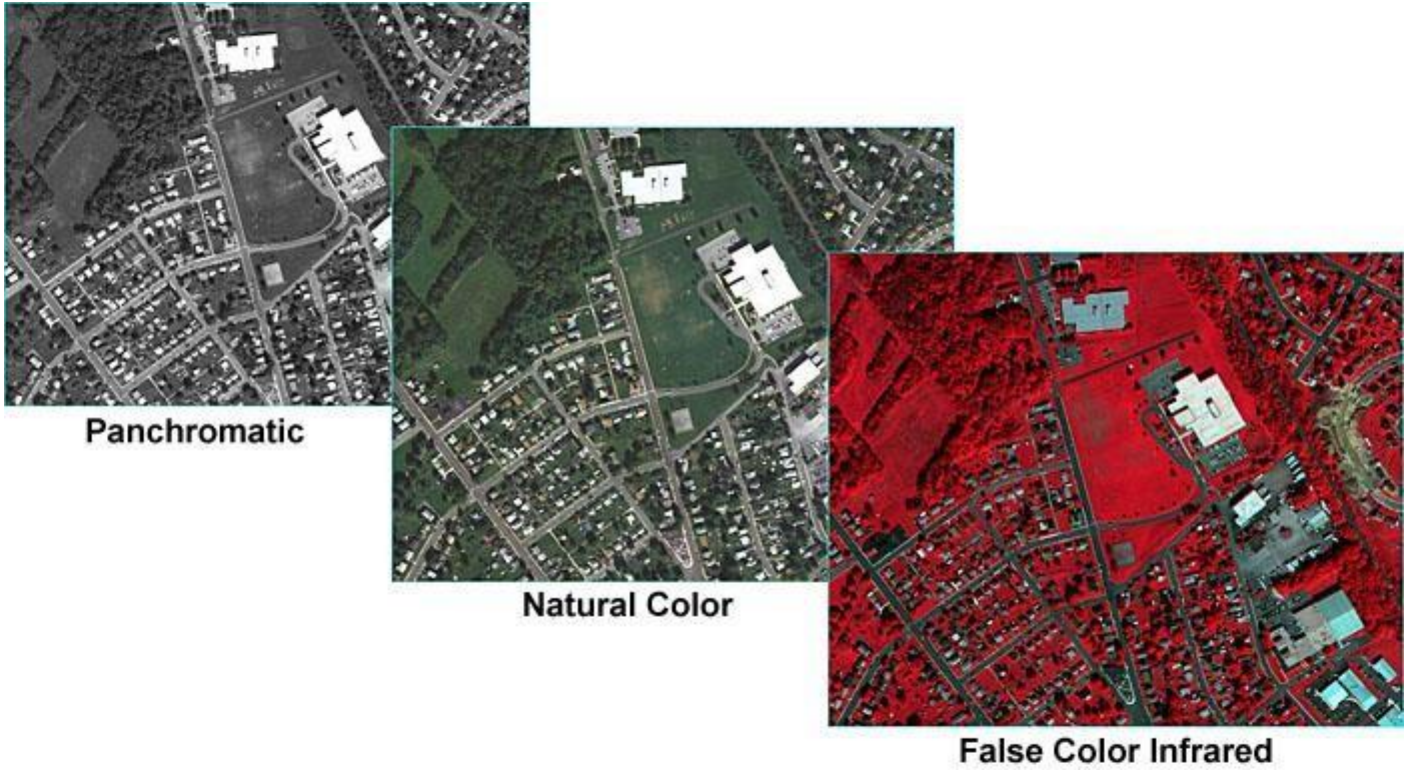
Image Formats

- RAW
- BMP
- GIF
- JPG
- TIFF
- HDF
- etc…

Image Types

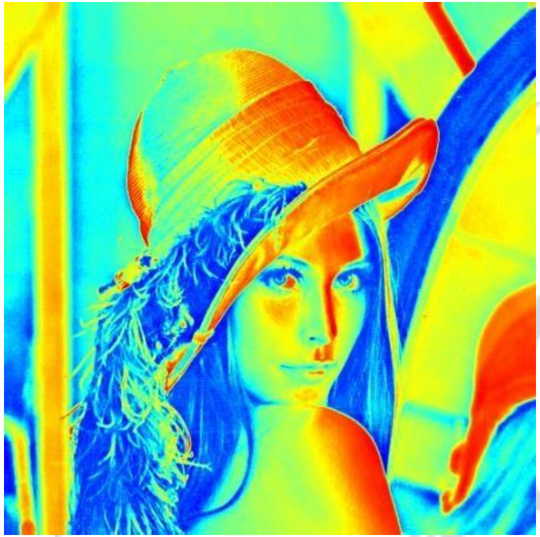
- Bilevel
- GrayScale
- True Color
- False Color
- MultiSpectral
- HyperSpectral
- etc…

Types of Images



Types of Images

Original Images



Pseudo Color

Binary Image

Aspect Ratio

- Eg:
  - If we want to resize a 1024x768 image to one that is 600 pixels wide with the same aspect ratio as the original image, what should be the height of the resized image?

Solution

$$
\begin{aligned}
\text{Aspect Ratio} &= \text{width/height} \\
&= 1024/768 \\
&= 1.33 \\
\text{Height} &= 600/1.33 \\
&= 451
\end{aligned}
$$

Baud Rate

- A common measure of transmission for digital data
- It is the no. of bits transmitted per second
- Generally transmission is accomplished in packets consisting of a start bit, a byte of information & a stop bit

Baud Rate

- Eg:
  - How many minutes would it take to transmit a 1024 x 1024 image with 256 gray levels using a 56K baud rate?

Solution

Total no. of bits needed to represent the image: 1024 x 1024 x 8

No. of packets required: 1024 x 1024

Total no. of bits that need to be transferred: 1024 x 1024 x [8 + 2]

Total time required: 1024 x 1024 x [8 + 2]/56000

= 187.25 sec or 3.1 min

RELATIONSHIP BETWEEN PIXELS

Basic relationship between pixels

- An image is denoted by a function f(x, y)
- Each element f(x, y) at location (x, y) is called a pixel
- There exist some basic and important relationships between pixels

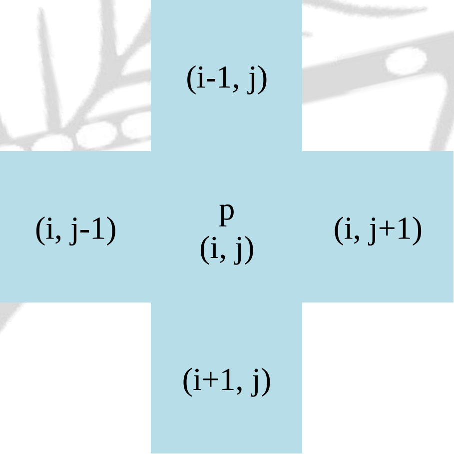
Basic relationship between pixels

- Neighborhood
- Adjacency
- Connectivity
- Paths
- Regions and boundaries

(i-1, j-1)	(i-1, j)	(i-1, j+1)
(i, j-1)	(i, j)	(i, j+1)
(i+1, j-1)	(i+1, j)	(i+1, j+1)

Basic relationship between pixels (Neighbours)

- A pixel p at location (x, y) has two horizontal and two vertical neighbours
- This set of four pixels is called 4-neighbors of p = $N_4(p)$
- Each of these neighbours is at a unit distance from p
- If p is a boundary pixel then it will have less number of neighbours

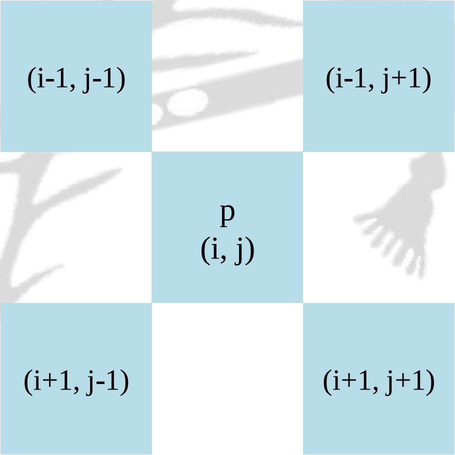


Basic relationship between pixels (Neighbours)

- A pixel p has four diagonal neighbours $N_D(p)$
- The union of $N_4(p)$ and $N_D(p)$ together are called 8-neighbors of p

$$N_8(p) = N_4(p) \cup N_D(p)$$

- If p is a boundary pixel then both $N_D(p)$ and $N_8(p)$ will have less number of pixels



Basic relationship between pixels (Adjacency)

- Two pixels are said to be connected if they are adjacent in some sense
  - They are neighbours (N_4, N_D or N_8) and
  - Their intensity values (gray levels) are similar

- Two pixels are adjacent if they are neighbour and their intensity level 'V' satisfy some specific criteria of similarity
- Let V be the set of intensity values

Basic relationship between pixels (Adjacency)

- 4-adjacency: Two pixels p and q with values from set ‘V’ are 4-adjacent if q is in the set $N_4(p)$
- Eg:
  - V = {0, 1}

1	1	0
1	1	0
1	0	1

Basic relationship between pixels (Adjacency)

- 8-adjacency: Two pixels p and q with values from set ‘V’ are 8-adjacent if q is in the set $N_8(p)$
- Eg:
  - V = {1, 2}

0	1	1
0	2	0
0	0	1

Basic relationship between pixels (Adjacency)

- m-adjacency: Two pixels p and q with values from V are m-adjacent if
  - (i) q is in the set $N_4(p)$, Eg: V = {1} OR
  - (ii) q is in the set $N_D(p)$ and the set $N_4(p) \cap N_4(q)$ is empty

- Eg:

0	1	1
0	1	0
0	0	1

Basic relationship between pixels (Adjacency)

- m-adjacency: Two pixels p and q with values from V are m-adjacent if
  - (i) q is in the set $N_4(p)$, Eg: V = {1} OR
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- Eg:

0	1	1
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0	1	1
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- Eg:

0	1	1
0	1	0
0	0	1

Basic relationship between pixels (Adjacency)

- Eg:
  - Find 4-adjacency and 8-adjacency of the center pixel
  - Note: V = {1}

0	1	1
0	1	0
0	0	1

Basic relationship between pixels (Adjacency)

- Eg:
  - Find 4-adjacency and 8-adjacency of the center pixel
  - Note: V = {1}

Solution

- 4-adjacency

0	1	1
0	1	0
0	0	1

Basic relationship between pixels (Adjacency)

- Eg:
  - Find 4-adjacency and 8-adjacency of the center pixel
  - Note: V = {1}

Solution

  - 8-adjacency

0	1	1
0	1	0
0	0	1

Basic relationship between pixels (Connectivity)

- Let S represent a subset of pixels in an image
- Two pixels p with coordinates (x_0, y_0) and q with coordinates (x_n, y_n) are said to be connected in S if there exists a path

$$(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$$

Basic relationship between pixels (Path)

- A path from pixel p with coordinates (x_0, y_0) to pixel q with coordinates (x_n, y_n) is a sequence of distinct pixels with coordinates $(x_0, y_0), (x_1, y_1), \dots, (x_n, y_n)$
  - where (x_i, y_i) and (x_{i-1}, y_{i-1}) are adjacent for $1 \le i \le n$
- Here n is the length of the path
- If $(x_0, y_0) = (x_n, y_n)$, the path is closed path
- We can define 4-, 8-, and m-paths based on the type of adjacency used

Basic relationship between pixels (Path)

- Eg:
  - Compute the length of shortest-4 path between pixels p and q, where V = {1, 2}

q

4	2	3	2
3	3	1	3
2	3	2	2
2	1	2	3

p

Basic relationship between pixels (Path)

- Eg:
  - Compute the length of shortest-4 path between pixels p and q, where V = {1, 2}

Solution

	4	2	3	2	q
	3	3	1	3	
	2	3	2	2	
p	2	1	2	3	

Basic relationship between pixels (Path)

- Eg:
  - Compute the length of shortest-8 path between pixels p and q, where V = {1, 2}

	4	2	3	2	q
	3	3	1	3	
	2	3	2	2	
p	2	1	2	3	

Basic relationship between pixels (Path)

- Eg:
  - Compute the length of shortest-8 path between pixels p and q, where V = {1, 2}

Solution

	4	2	3	2	q
	3	3	1	3	
	2	3	2	2	
p	2	1	2	3	

Basic relationship between pixels (Path)

- Eg:
  - Compute the length of shortest-m path between pixels p and q, where V = {1, 2}

	4	2	3	2	q
	3	3	1	3	
	2	3	2	2	
p	2	1	2	3	

Basic relationship between pixels (Path)

- Eg:
  – Compute the length of shortest-m path between pixels p and q, where V = {1, 2}

Solution

	4	2	3	2	q
	3	3	1	3	
	2	3	2	2	
p	2	1	2	3	

Basic relationship between pixels (Region)

- Let R be a subset of pixels in an image, two regions R_i and R_j are said to be adjacent if their union form a connected set
- Regions that are not connected are said to be disjoint
- We consider 4- and 8- adjacency when referring to the regions
- Eg:
  - V = {1}

R_i

1	1	0
1	0	1
1	1	0

R_j

0	1	1
0	1	1
1	1	1

- Regions are adjacent only if 8-adjacency is used

DISTANCE MEASURE

Distance Measure

- Given pixels p(u, v), q(w, x) and r(y, z) with coordinates, the distance function D has following properties:
  - $D(p, q) \geq 0$ [$D(p, q) = 0$, iff $p = q$]
  - $D(p, q) = D(q, p)$
  - $D(p, r) \leq D(p, q) + D(q, r)$

Distance Measure

- The following are the different Distance measures
  - Euclidean Distance:
    - $D_e(p, q) = [(u-w)^2 + (v-x)^2]^{1/2}$
  - City Block Distance:
    - $D_4(p, q) = |u-w| + |v-x|$
  - Chess Board Distance:
    - $D_8(p, q) = \max(|u-w|, |v-x|)$

Pixel	Coordinate
p	(u, v)
q	(w, x)

Distance Measure (Euclidean Distance)

- For distance measure, the pixels having a distance less than or equal to some value from p(u, v) are the points contained in a disk of radius (r) centered at p(u, v)

Distance Measure (City-block Distance)

- The pixels having a D_4 distance from p(u, v) less than or equal to some value, form a diamond centered at p(u, v)
- The pixels with D_4 =1 are the 4-neighbors of p(u, v)

		2		
	2	1	2	
2	1	0	1	2
	2	1	2	
		2		

Distance Measure (Chessboard Distance)

- The pixels with D_8 distance from p(u, v) less than or equal to some value, form a square centered at p(u, v)
- The pixels with D_8=1 are the 8-neighbors of (u, v)

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

Some Mathematical Tools

- When we multiply two images, we usually carry out array multiplication

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} p & q \\ r & s \end{bmatrix} = \begin{bmatrix} ap & bq \\ cr & ds \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} p & q \\ r & s \end{bmatrix} = \begin{bmatrix} ap + br & aq + bs \\ cp + dr & cq + ds \end{bmatrix}$$

Some Mathematical Tools (Arithmetic Operations)

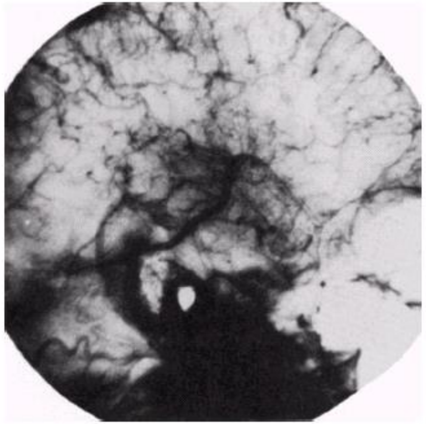
- Arithmetic operations are performed on the pixels of two or more images
- Let p and q be the pixel values at location (x, y) in first and second images respectively
  - Addition: p + q
  - Subtraction: p - q
  - Multiplication: p.q
  - Division: p/q

Some Mathematical Tools (Mask Mode Radiography)

- One of the most successful commercial applications of image subtraction

Some Mathematical Tools

- Eg:
  - Blood stream is injected with a dye and X-ray images are taken before and after the injection
  - f(x, y): image after injecting a dye
  - h(x, y): image before injecting the dye



before injection

after injection

Some Mathematical Tools (Image Averaging)

- A noisy image:

$$g(x, y) = f(x, y) + n(x, y)$$

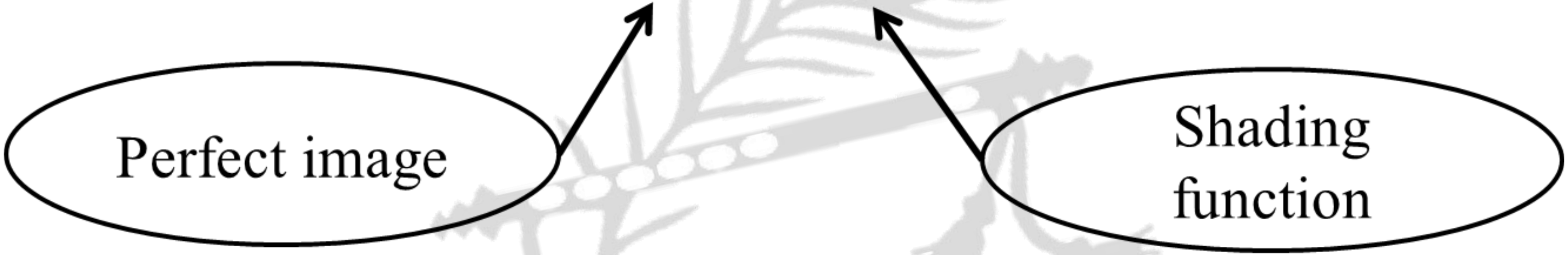
- Averaging M different noisy images:

$$\bar{g}(x, y) = \frac{1}{M}\sum_{i=1}^{M} g_i(x, y)$$

- As M increases, the variability of the pixel values at each location decreases
- This means that g(x, y) approaches f(x, y) as the number of noisy images used in the averaging process increases

Some Mathematical Tools (Shading Correction)

- An important applications of image division
- An imaging sensor produces image g(x, y), where
- $$g(x, y) = f(x, y)\, h(x, y)$$

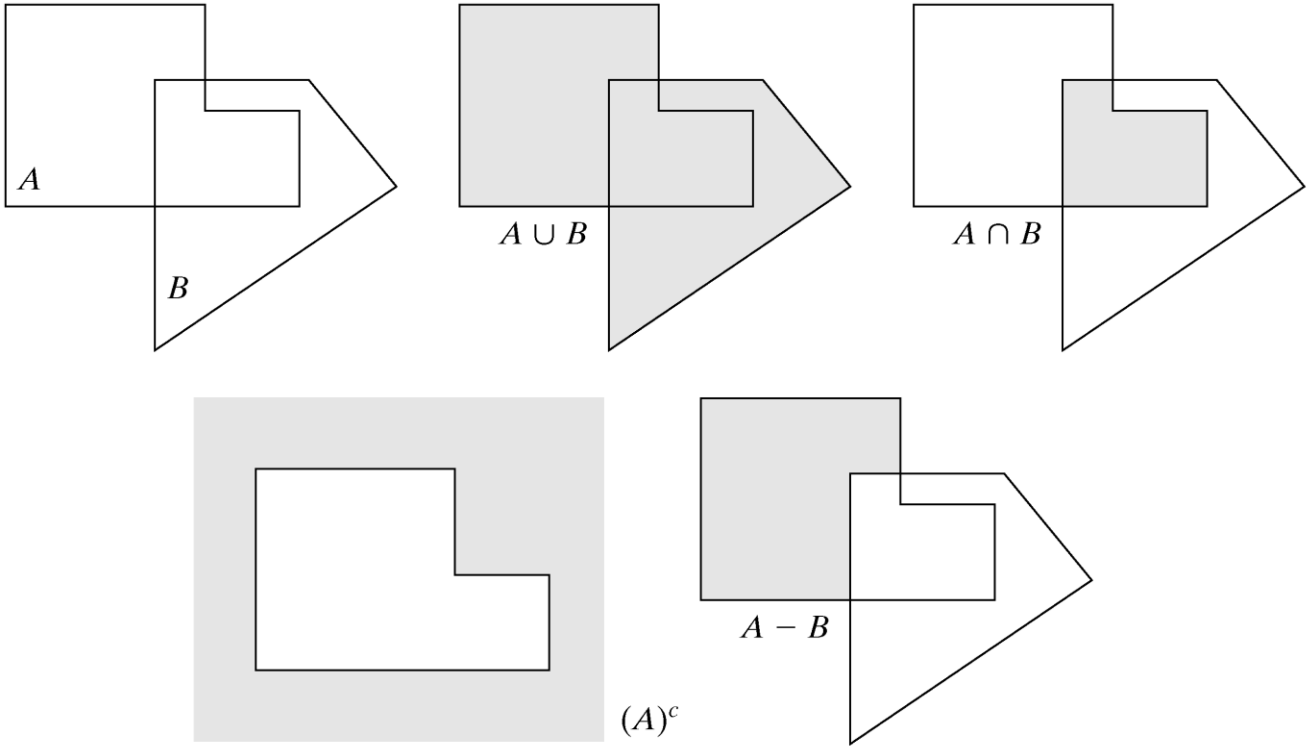


- By dividing g(x, y) by h(x, y), we can get the perfect image

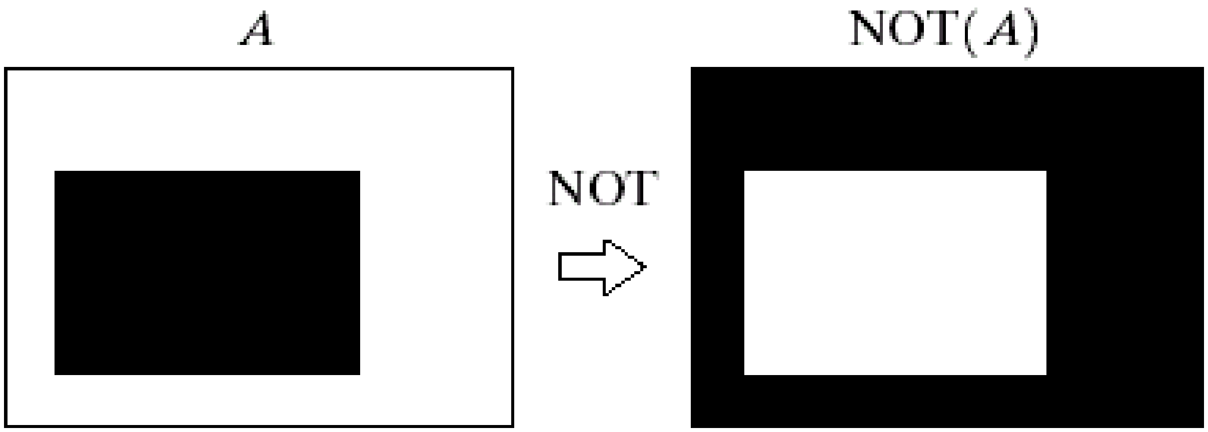
Some Mathematical Tools (Region of Interest)

- An important applications of image multiplication

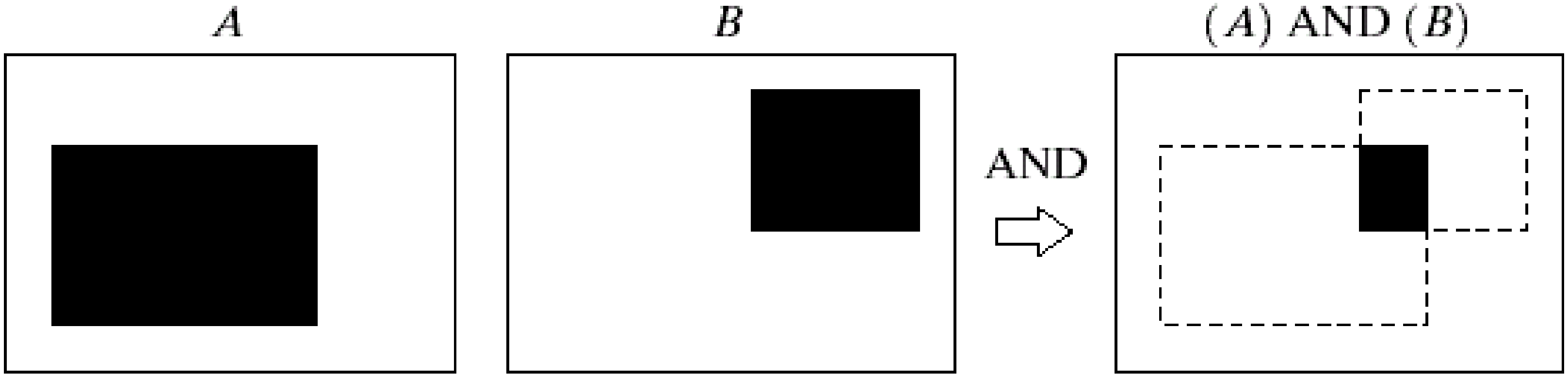
Some Mathematical Tools (Set Operations)



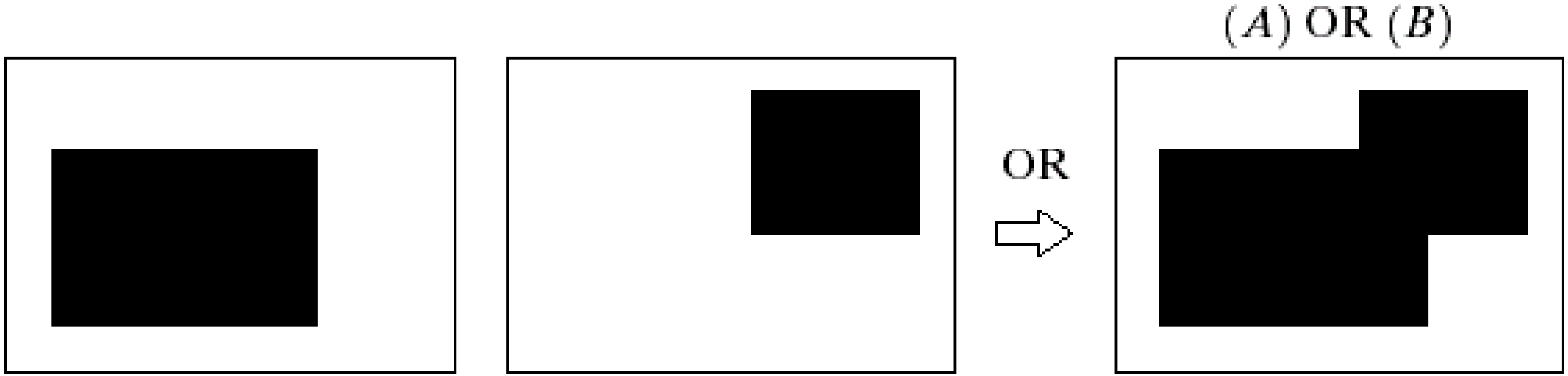
Some Mathematical Tools (Logical Operations)



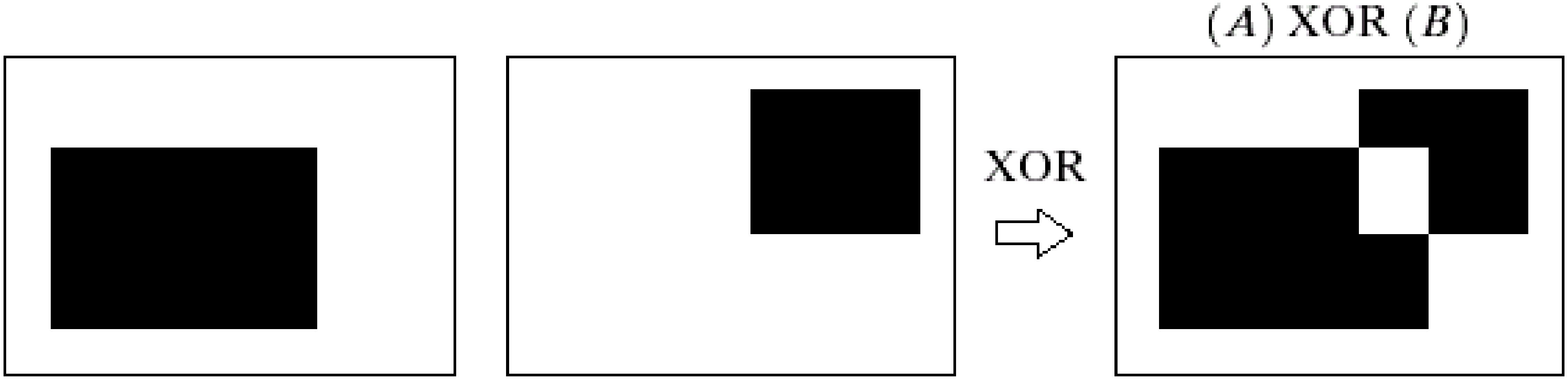
Some Mathematical Tools (Logical Operations)



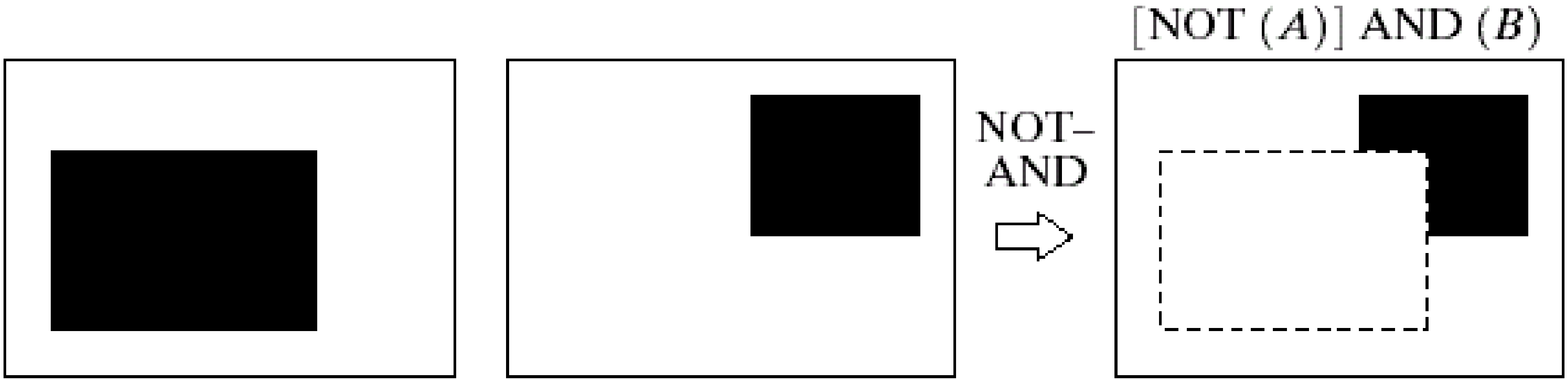
Some Mathematical Tools (Logical Operations)



Some Mathematical Tools (Logical Operations)



Some Mathematical Tools (Logical Operations)



Recap

Recap

- Digital Image
- Steps of Image Acquisition
  - (4 steps)
- Process involved in Digital image
  - (Sampling and Quantization)
- Sensor arrangement
  - (Single, Line, Array)
- Levels in image
  - ($L = 2^k$)
- Image size
  - ($m \times n \times k$)
- Applications of Image Processing
- Image sensing
  - (EMS)

Recap

- Types of resolutions
  - (Spatial, Spectral, Temporal, Radiometric)
- Types of effect
  - (Checkerboard, False contouring)
- Key stages of Image Processing
- Aspect ratio
  - (r = w/h)
- Baud rate
- Relationship between pixels
  - (Neighbour: N_4, N_D, N_8, Adjacency: 4-, 8-, m-, Connectivity, Path: 4-, 8-, m-, Region)
- Distance measure
  - (Euclidean, City block, Chess board)
- Set operations

IMAGE ENHANCEMENT

Image Enhancement

- Image enhancement is the process of making images more useful
- The reasons for doing this include:
  - Making images more visually appealing
  - Highlighting interesting details in images
  - Removing noise from images

Image Enhancement

- Images may suffer from the following degradations:
  - Poor contrast due to poor illumination
  - Electronic sensor noise or atmospheric disturbances
  - Aliasing effects due to inadequate sampling
  - Finite aperture effects or motion

Image Enhancement

- Applications of transformations
  - Contrast enhancement
  - Gray scale transformation
  - Photometric calibration
  - Display calibration
  - Contour lines

Image Enhancement

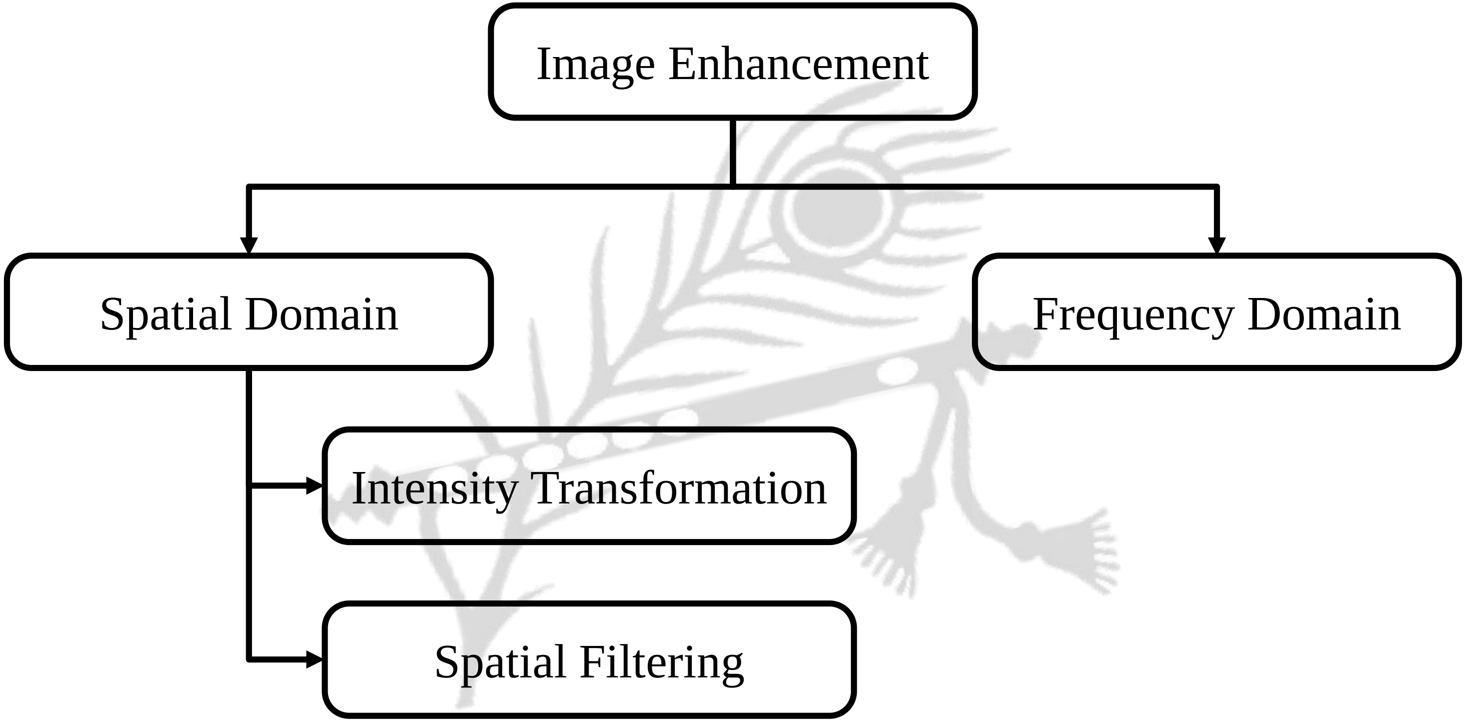


Image Enhancement

- There are two broad categories of image enhancement approaches
  - Spatial domain methods
    - Based on direct manipulation of pixels in an image
  - Frequency domain methods
    - Based on modifying the fourier transform of an image

Image Enhancement (Spatial domain)

- Intensity Transformations (Point processing)
  - Operate on single pixels of an image
  - Eg: image averaging; logic operation; contrast stretching
- Spatial Filtering (Mask processing)
  - Working in a neighborhood of every pixel in an image
  - Eg: blurring, median

Intensity Transformations

Intensity Transformations

- Linear transformations
  - Image negative
- Non-linear transformations
  - Logarithmic transformation
  - Power Law (Exponential) transformation
- Piecewise-linear transformations
  - Contrast stretching
  - Gray-level slicing
  - Bit plane slicing

Image Enhancement (Spatial →Intensity)

- T = gray level (or intensity or mapping) transformation function

$$s = T(r)$$

  - where
    - r = gray level of f(x, y)
    - s = gray level of g(x, y)

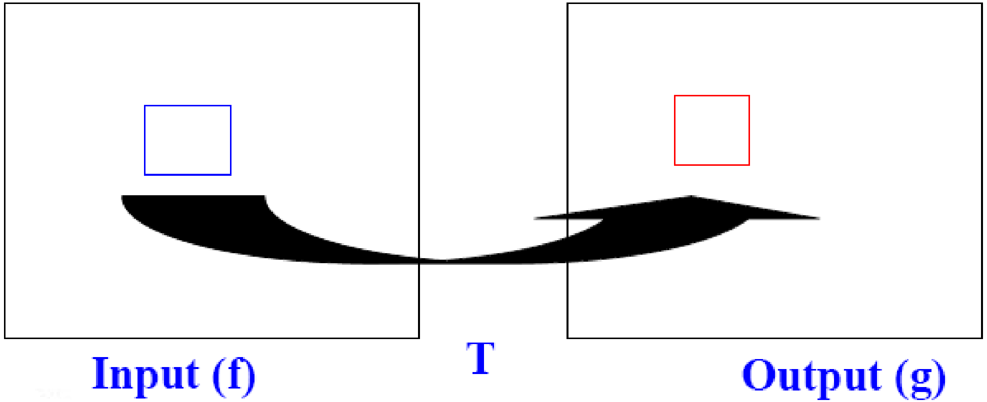
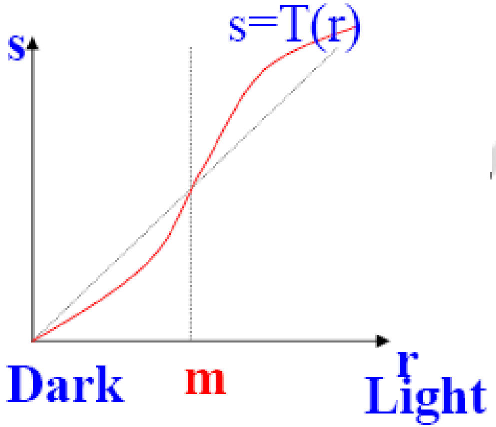


Image Enhancement (Spatial →Intensity)

- The gray level below m are darkened and the levels above m are brightened or vice-versa
- Produces a binary image from input image and the function is known as thresholding function

$$g(x, y) = \begin{cases} L & \text{if } f(x, y) > m, \\ 0 & \text{otherwise} \end{cases}$$



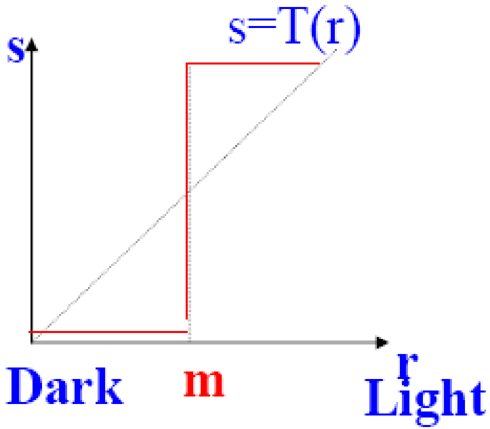
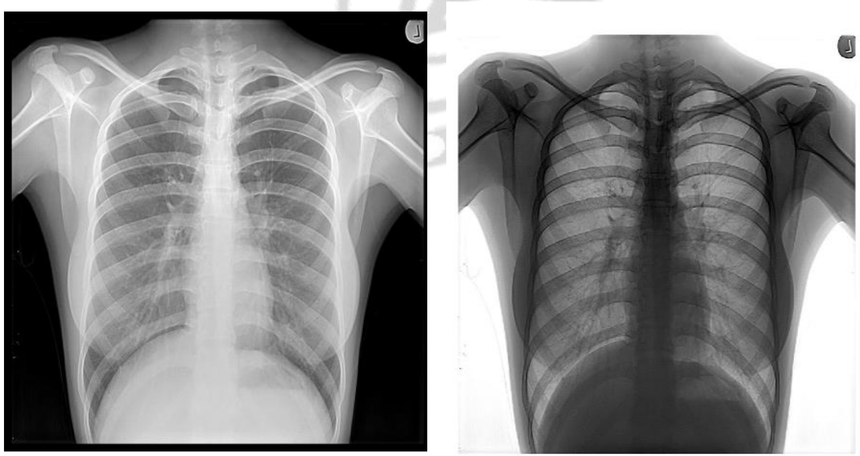


Image Enhancement (Spatial →Intensity)

Negative transformation

- Reversing the intensity levels of an image

$$s = L - 1 - r$$



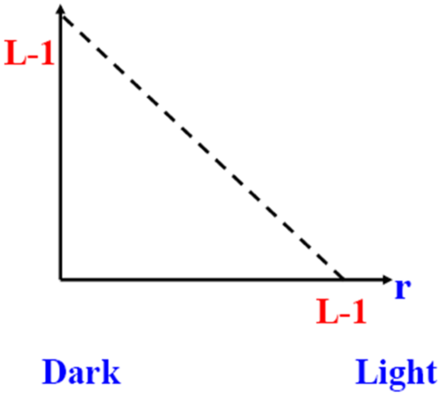
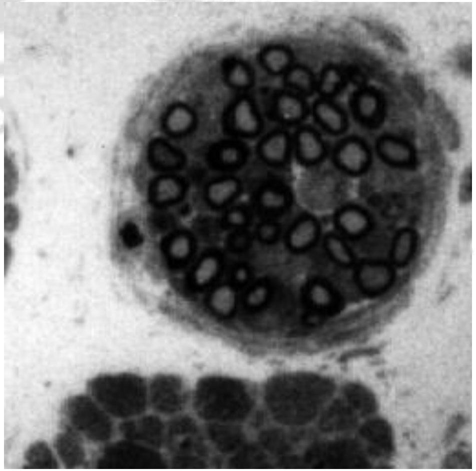


Image Enhancement (Spatial →Intensity)

Log Transformations

$$s = c \log(1+r)$$

- c is a constant and $r \geq 0$
- Used to expand the values of dark pixels in an image while compressing the higher-level values



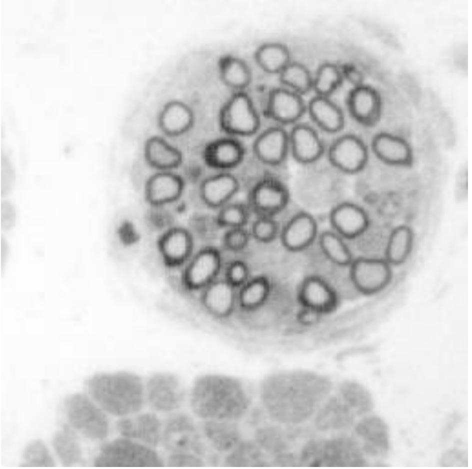


Image Enhancement (Spatial → Intensity)

Power-Law Transformations

$$s = cr^{\gamma}$$

- c and γ are positive constants
- Power-law curves with fractional values of γ map a narrow range of dark input values into a wider range of output values, with the opposite being true for higher values of input levels
- $c = \gamma = 1$ → Identity function

Image Enhancement (Spatial →Intensity)

An expansion of gray levels are desirable

- a) The picture is dark

result after power-law transformation with

- b) $\gamma = 0.6$, c=1
- c) $\gamma = 0.4$
- d) $\gamma = 0.3$

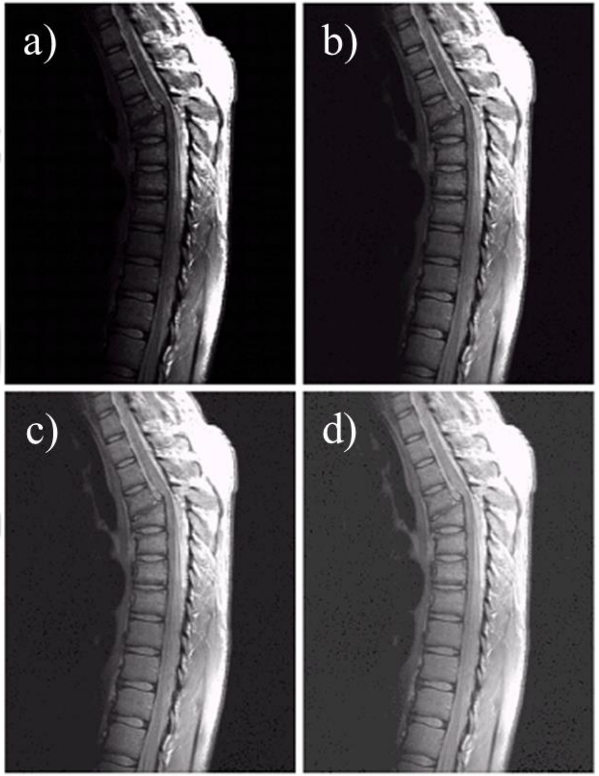
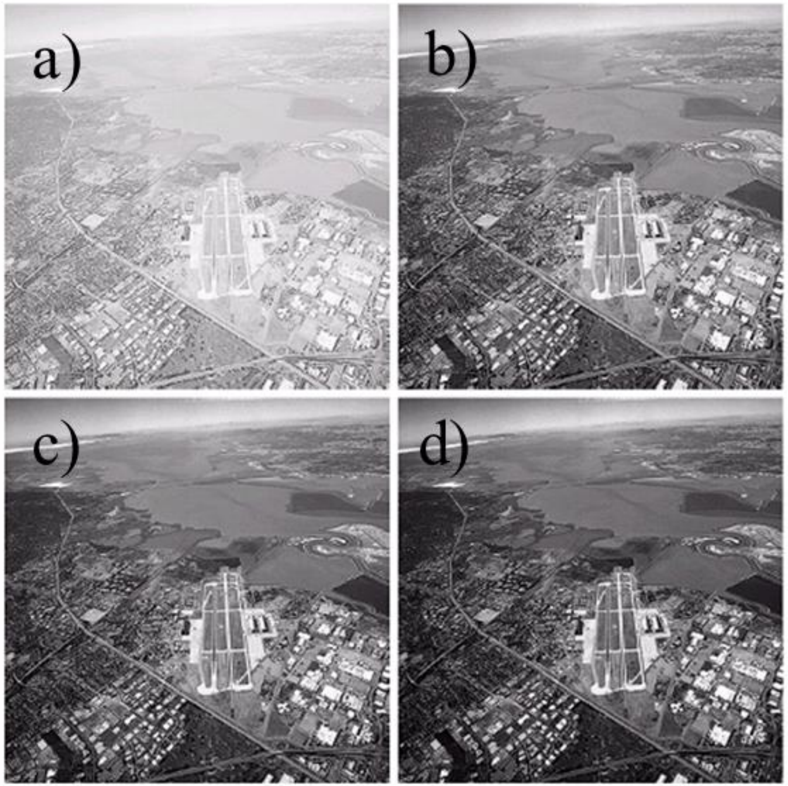


Image Enhancement (Spatial →Intensity)

- a) image has a washed-out appearance
- b) γ = 3.0 (suitable)
- c) γ = 4.0 (suitable)
- d) γ = 5.0 (some detail is lost)



Piecewise-Linear Transformation Functions

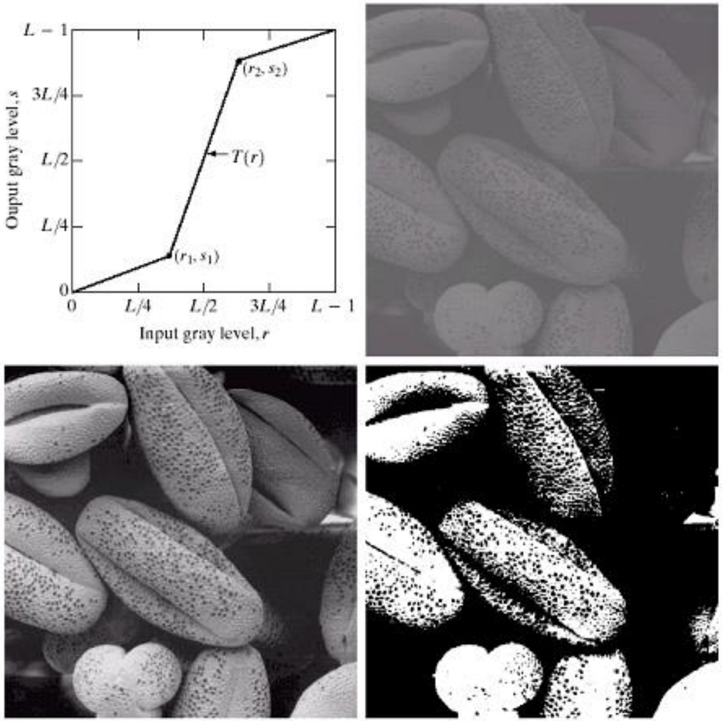
- A complementary approach to the previous methods
- Practical implementation of some important transformations can be formulated only as piecewise functions

Piecewise-Linear (Contrast Stretching)

- The simplest of piecewise linear functions is contrast stretching
- Basic idea is to increase the dynamic range of the gray levels in the image
- During image acquisition, low contrast images may result due to
  - Poor illumination
  - Lack of dynamic range in image sensor
  - Wrong setting of the lens aperture

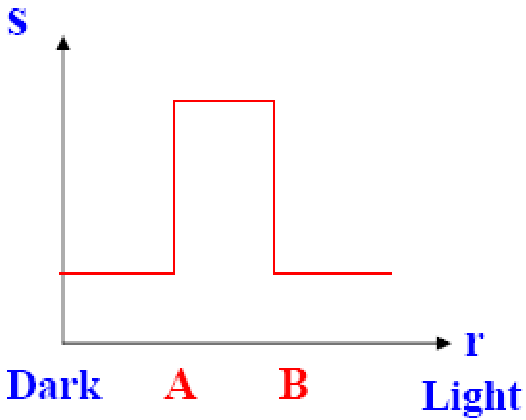
Piecewise-Linear (Contrast Stretching)

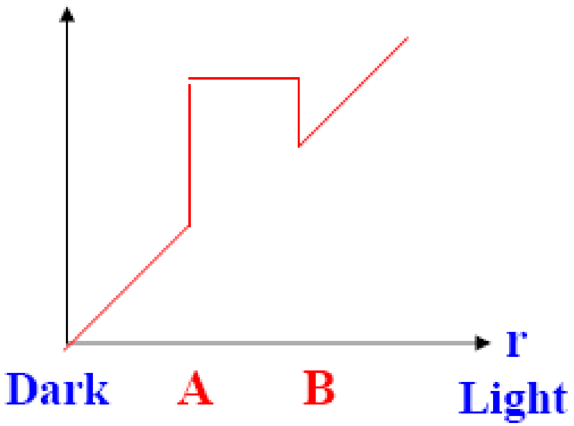
- (a) increase the dynamic range of the gray levels in the image
- (b) a low-contrast image
- (c) result of contrast stretching
  - $(r_1, s_1) = (r_{min}, 0)$
  - $(r_2, s_2) = (r_{max}, L-1)$
- (d) result of thresholding



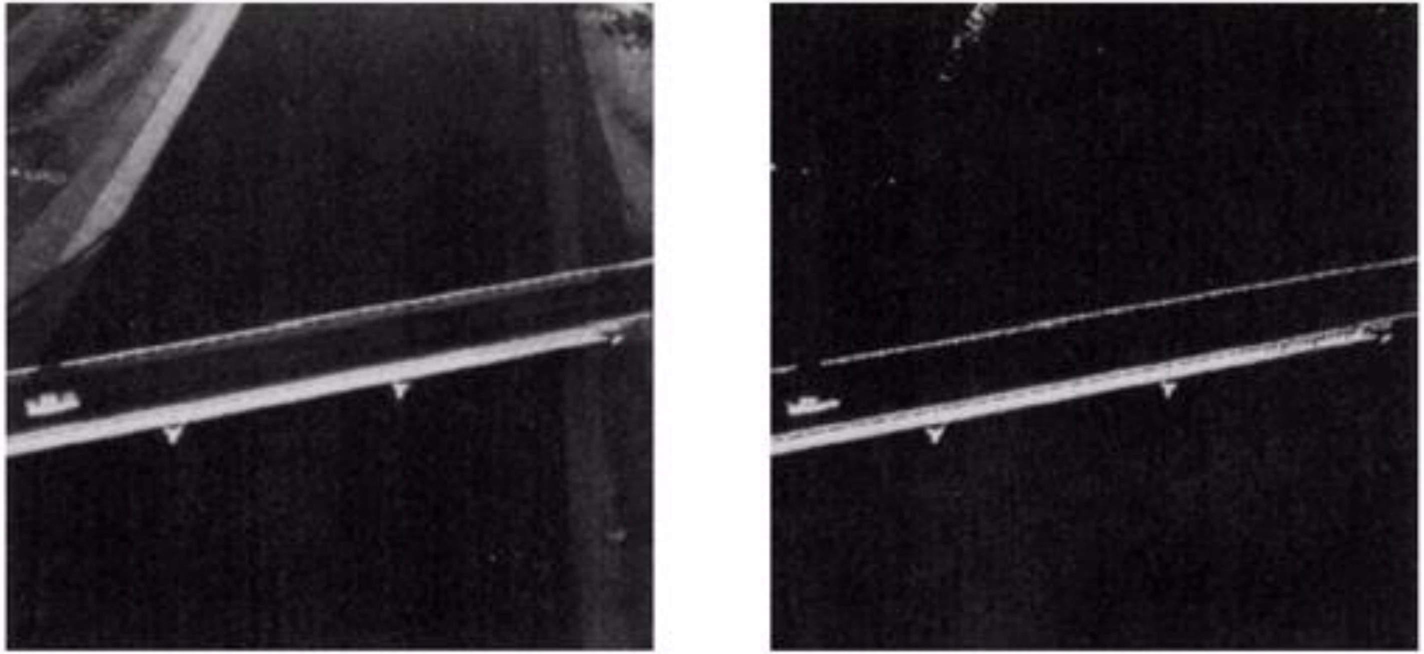
Piecewise-Linear (Gray level slicing)

- Highlight a specific range of gray values
- Two basic Methods
  - Display a high value for all gray levels in the range of interest and a low value for all other
  - Brighten the desired range of gray levels but preserve all other levels



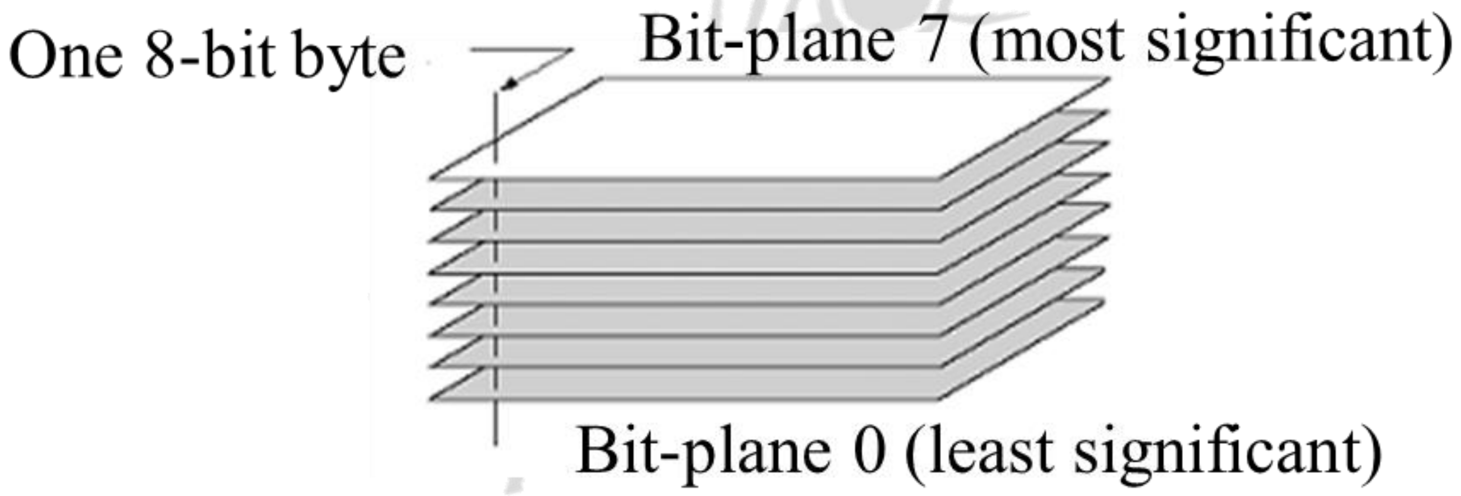


Piecewise-Linear (Gray level slicing)



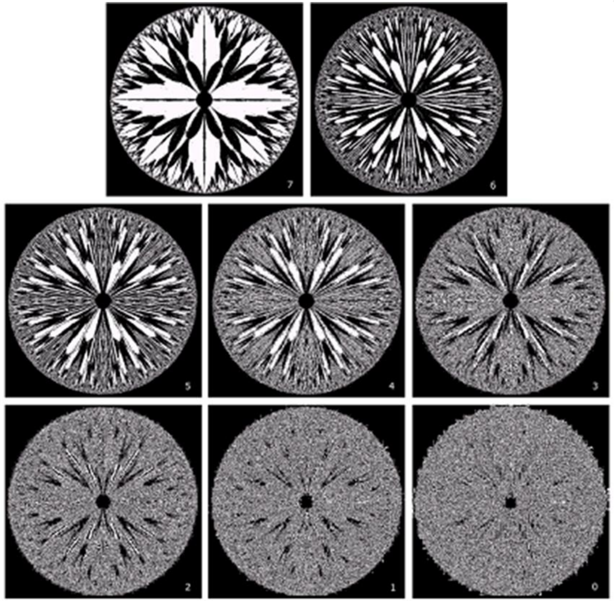
Piecewise-Linear (Bit-plane slicing)

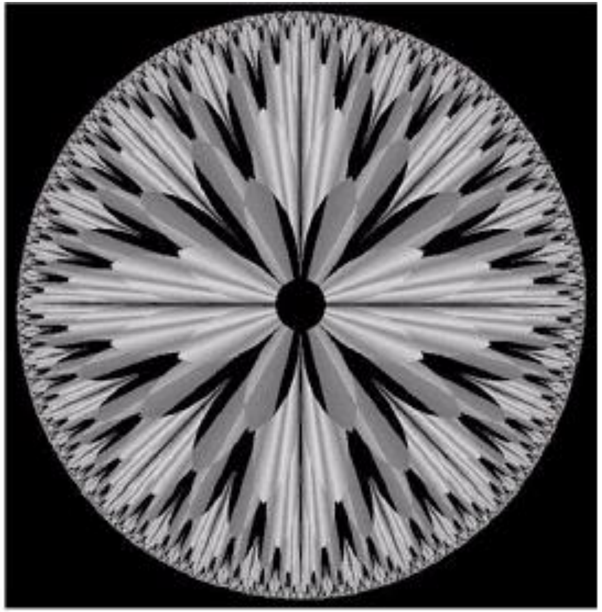
- Highlighting the contribution made to total image appearance by specific bits
- Useful for analyzing the relative importance played by each bit of the image



Piecewise-Linear (Bit-plane slicing)

- Higher order bit planes of an image carry a significant amount of details
- Lower order planes contribute more to fine details





HISTOGRAM PROCESSING

Histogram Processing

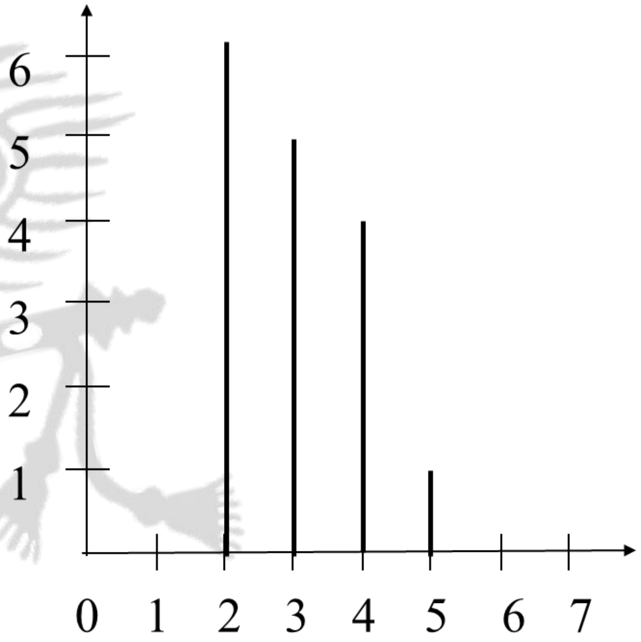
- Histogram is a discrete function formed by counting the number of pixels that have a certain gray level in the image
- Histogram of a digital image with gray levels in the range [0, L-1] is a discrete function

$$h(r_k) = n_k$$

  - where
  - r_k: the k^{th} gray level
  - n_k: the number of pixels in the image having gray level r_k
  - $h(r_k)$: histogram of a digital image with gray levels r_k

Histogram Processing

2	3	3	2
4	2	4	3
3	2	3	5
2	4	2	4



Normalized Histogram

- Dividing each of histogram at gray level (r_k) by the total number of pixels in the image (n)

$$p(r_k) = n_k/n$$

  - for k = 0, 1, …, L-1
  - $p(r_k)$ gives an estimate of the probability of occurrence of gray level r_k
  - The sum of all components of a normalized histogram is equal to 1

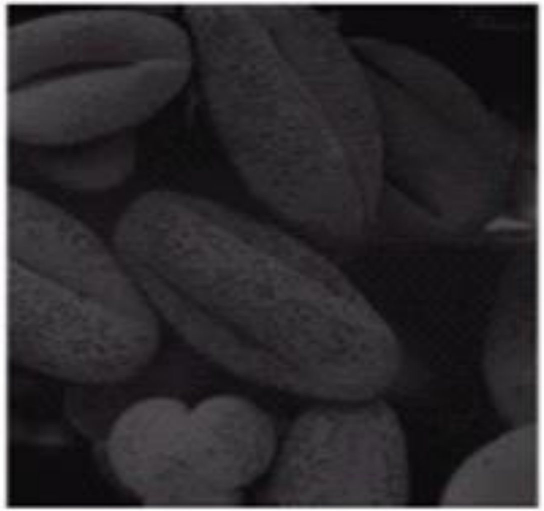
Histogram Processing

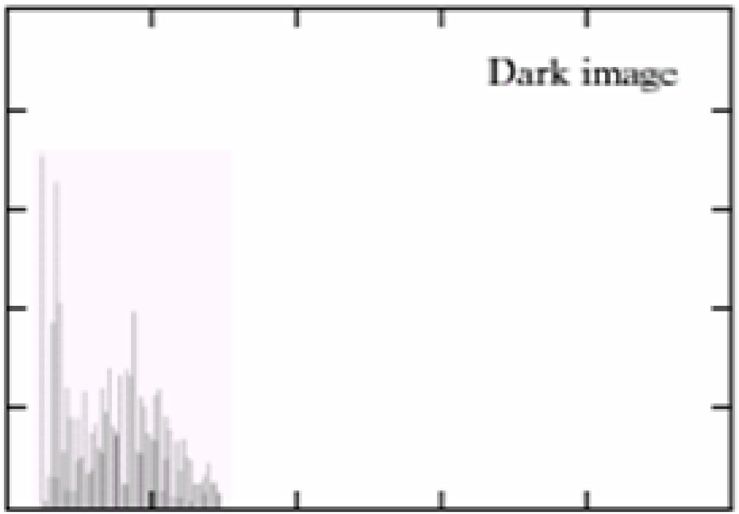
- Eg:
  - An image with gray levels between 0 to 7 is given below. Find the histogram of an image and normalize it

1	6	2	2
1	3	3	3
4	6	4	0
1	6	4	7

Histogram Processing

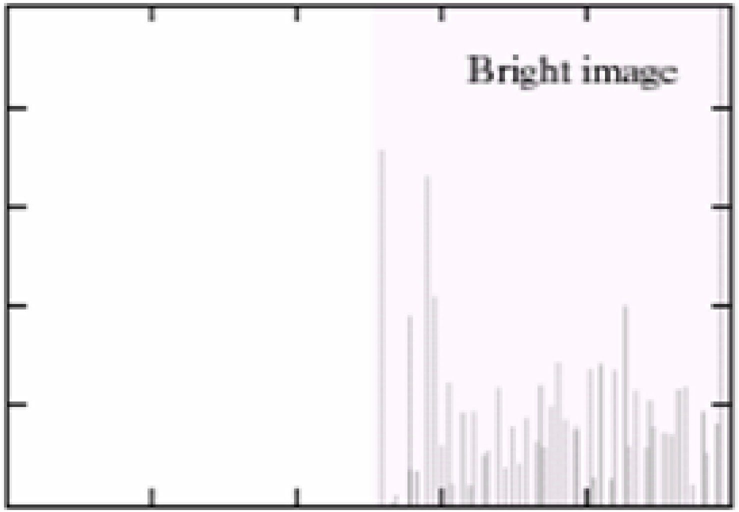
- In the dark images, components of the histogram are concentrated on the lower (dark) side of the gray scale





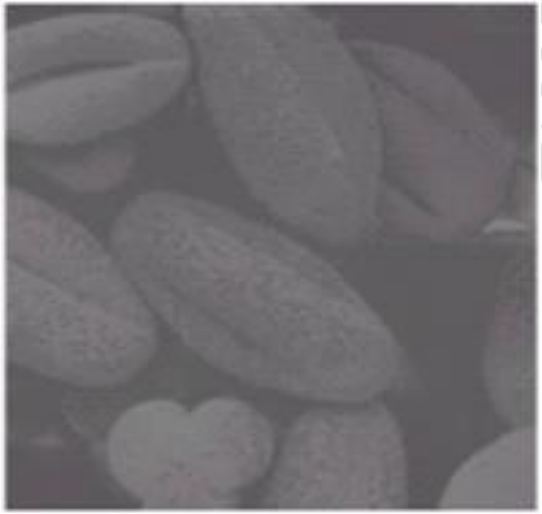
Histogram Processing

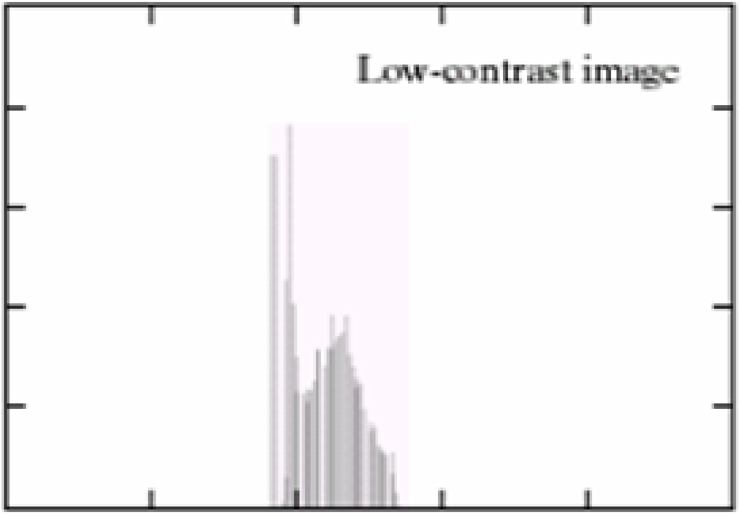
- In the dark images, components of the histogram are concentrated on the lower (dark) side of the gray scale
- In bright images, the histogram is biased towards the higher side of the gray scale



Histogram Processing

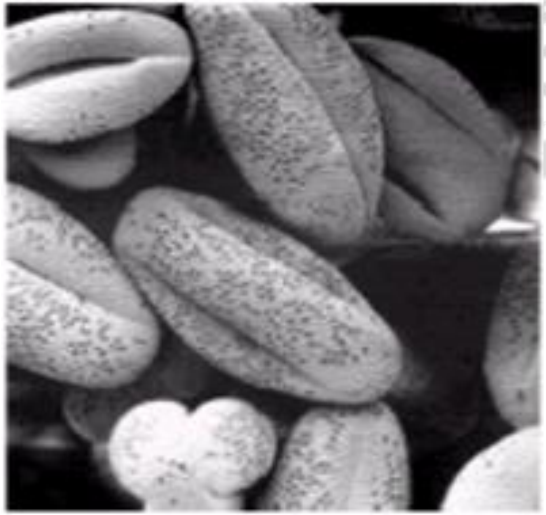
- In low contrast, the histogram will be narrow & centred towards the middle of the gray scale

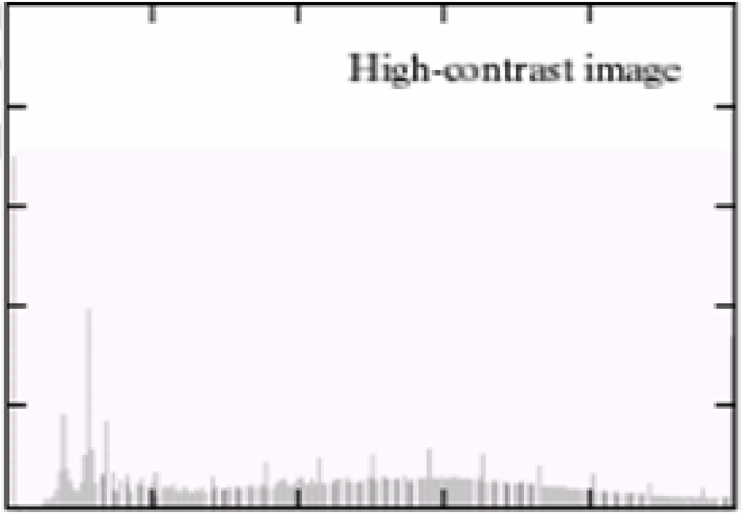




Histogram Processing

- In low contrast, the histogram will be narrow & centred towards the middle of the gray scale
- In high contrast, a large variety of gray tones occupy the entire range of possible gray levels





Histogram Equalization

- It is the process that transforms the intensity values so that the histogram of the output image approximately matches the flat (uniform) histogram
- The aim is to create an image with equally distributed brightness levels over the whole brightness scale
- We can do it by adjusting the probability density function (pdf) of the original histogram of the image so that the probability spread equally

Histogram Equalization

- Histogram equalization results are similar to contrast stretching but offer the advantage of full automation
- HE automatically determines a transformation function to produce a new image with a uniform histogram
- Goal
  - find a transform $s = T(r)$ such that the transformed image has a flat (equalized) histogram
  - where $T(r)$ satisfies the following conditions:
    - $T(r)$ is single-valued and monotonically increasing in interval $[0, 1]$
    - $0 \leq T(r) \leq 1 \quad \text{for } 0 \leq r \leq 1$

Histogram Equalization

- The discrete transformation function is given by

$$s_j = T(\mathrm{r}_j)$$

$$= \sum_{j=0}^{k} p(r_j)$$

$$= \sum_{j=0}^{k} \frac{n_j}{n} \qquad k = 0,1,\ldots,L-1$$

Histogram Equalization

- The following equations bring back the gray levels in the range [0, L-1]

$$
\begin{aligned}
s_j &= T(\mathrm{r}_j) \\
&= (\mathrm{L}-1)\sum_{j=0}^{k} p(r_j) \\
&= (\mathrm{L}-1)\sum_{j=0}^{k} \frac{n_j}{MN} \\
&= \frac{(\mathrm{L}-1)}{MN}\sum_{j=0}^{k} n_j \qquad k = 0,1,\ldots,L-1
\end{aligned}
$$

- M – no. of rows
- N – no. of columns
- MN – total no. of pixels in the image

Histogram Equalization

- Eg:
  - Suppose that a 3-bit image ($L = 8$) of size 64×64 pixels ($MN = 4096$) has the intensity distribution shown in following table. Get the histogram equalization transformation function and give the $p(s_k)$ for each s_k

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

Histogram Equalization

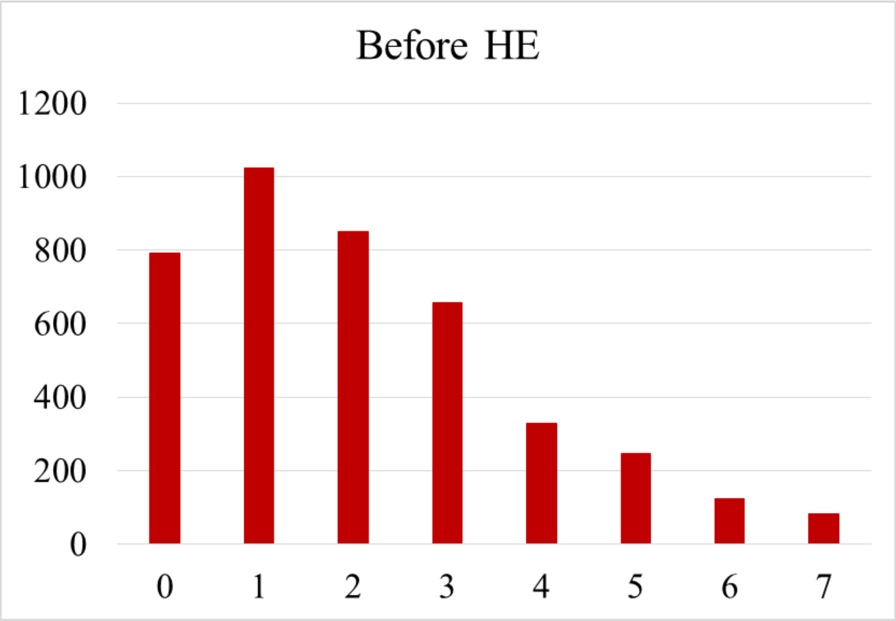
i/p Gray Level (r_k)	no. of pixels (n_k)	$p(r_k) = n_k/MN$	Σ	$(L-1)\Sigma$	o/p Gray Level (s)
0	790	0.19	0.19	1.33	1
1	1023	0.25	0.44	3.08	3
2	850	0.21	0.65	4.55	5
3	656	0.16	0.81	5.67	6
4	329	0.08	0.89	6.23	6
5	245	0.06	0.95	6.65	7
6	122	0.03	0.98	6.86	7
7	81	0.02	1.00	7.00	7

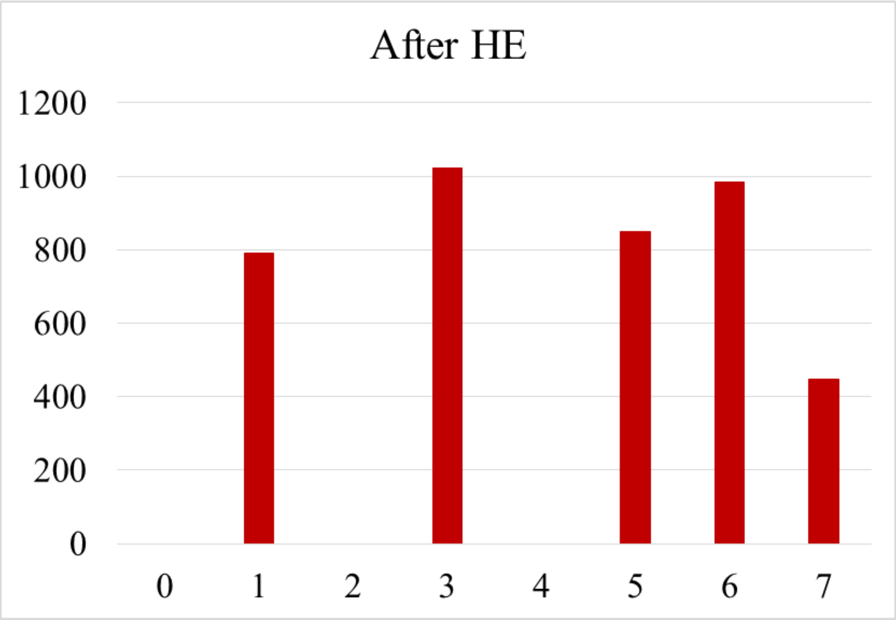
Histogram Equalization

(r_k)	(n_k)	$p(r_k)$	Σ	$(L-1)\Sigma$	(s_k)
0	790	0.19	0.19	1.33	1
1	1023	0.25	0.44	3.08	3
2	850	0.21	0.65	4.55	5
3	656	0.16	0.81	5.67	6
4	329	0.08	0.89	6.23	6
5	245	0.06	0.95	6.65	7
6	122	0.03	0.98	6.86	7
7	81	0.02	1.00	7.00	7

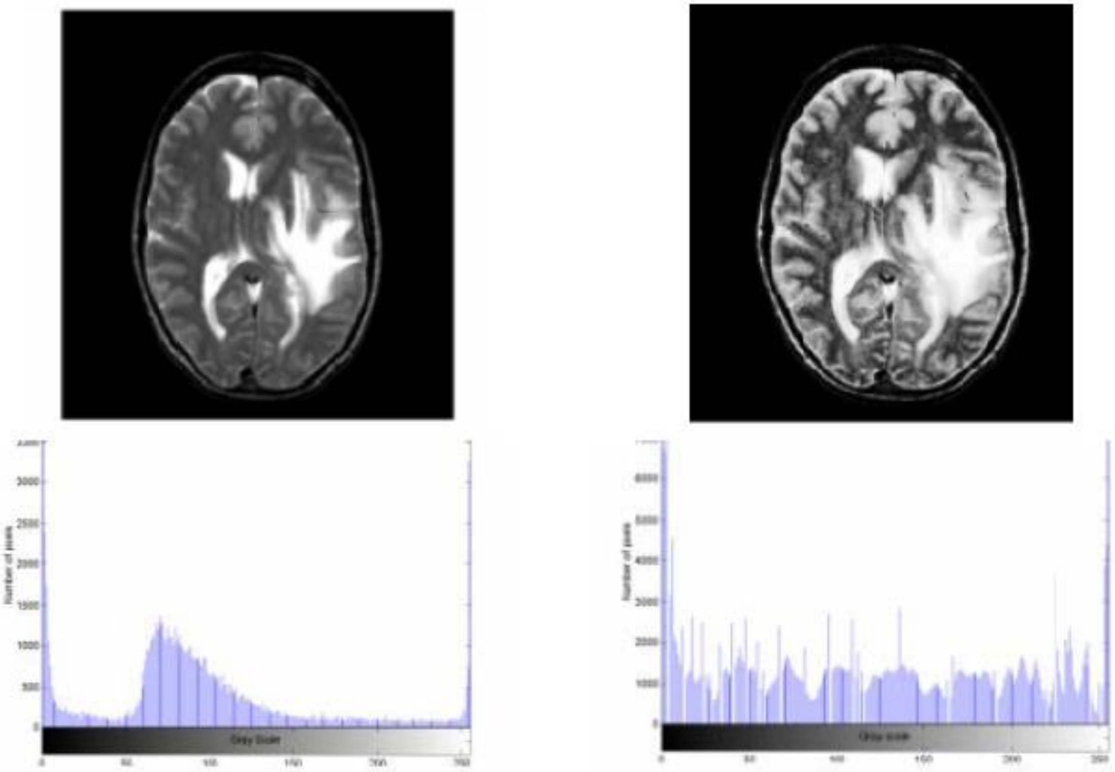
Equalized	n_k
0	0
1	790
2	0
3	1023
4	0
5	850
6	656 + 329 = 985
7	245 + 122 + 81 = 448

Histogram Equalization





Histogram Equalization (Results)



Histogram Matching (Specification)

- Histogram equalization has a disadvantage that it can generate only one type of output image
- Sometimes it is useful to specify the shape of the histogram
- With histogram specification, we can specify the shape of the histogram
- It doesn’t have to be a uniform histogram

- For an image, whose enhancement is to be done, we are given an histogram, $G(z_k)$, that actually shows how the processed image’s histogram should look after applying the transformation function to the i/p image

Histogram Matching (Specification)

- Step 1: Find histogram of input image $p(r_j)$, and find histogram equalization mapping
- Step 2: Specify the desired histogram, and find histogram equalization mapping
- Step 3: Build lookup table:
  - For each gray level k, find s_k and then find a level such that s_k best matches z_q:

$$\min|s_k - z_q|$$

  - setup a lookup entry lookup[k] = q

Histogram Matching (Specification)

- Eg:
  - Suppose that a 3-bit image of size 64 × 64 pixels has the intensity distribution shown in the table (on the left). Get the histogram transformation function and make the output image with the specified histogram, listed in the table on the right

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

z_q	Specified $p_z(z_q)$
$z_0 = 0$	0.00
$z_1 = 1$	0.00
$z_2 = 2$	0.00
$z_3 = 3$	0.15
$z_4 = 4$	0.20
$z_5 = 5$	0.30
$z_6 = 6$	0.20
$z_7 = 7$	0.15

Histogram Matching (Specification)

(r_k)	(n_k)	$p(r_k)$	Σ	$(L-1)\Sigma$	(s_k)
0	790	0.19	0.19	1.33	1
1	1023	0.25	0.44	3.08	3
2	850	0.21	0.65	4.55	5
3	656	0.16	0.81	5.67	6
4	329	0.08	0.89	6.23	6
5	245	0.06	0.95	6.65	7
6	122	0.03	0.98	6.86	7
7	81	0.02	1.00	7.00	7

(z_k)	$p(z_k)$	Σ	$(L-1)\Sigma$	$G(z_k)$
0	0.00	0	0	0
1	0.00	0	0	0
2	0.00	0	0	0
3	0.15	0.15	1.05	1
4	0.20	0.35	2.45	2
5	0.30	0.65	4.55	5
6	0.20	0.85	5.95	6
7	0.15	1	7	7

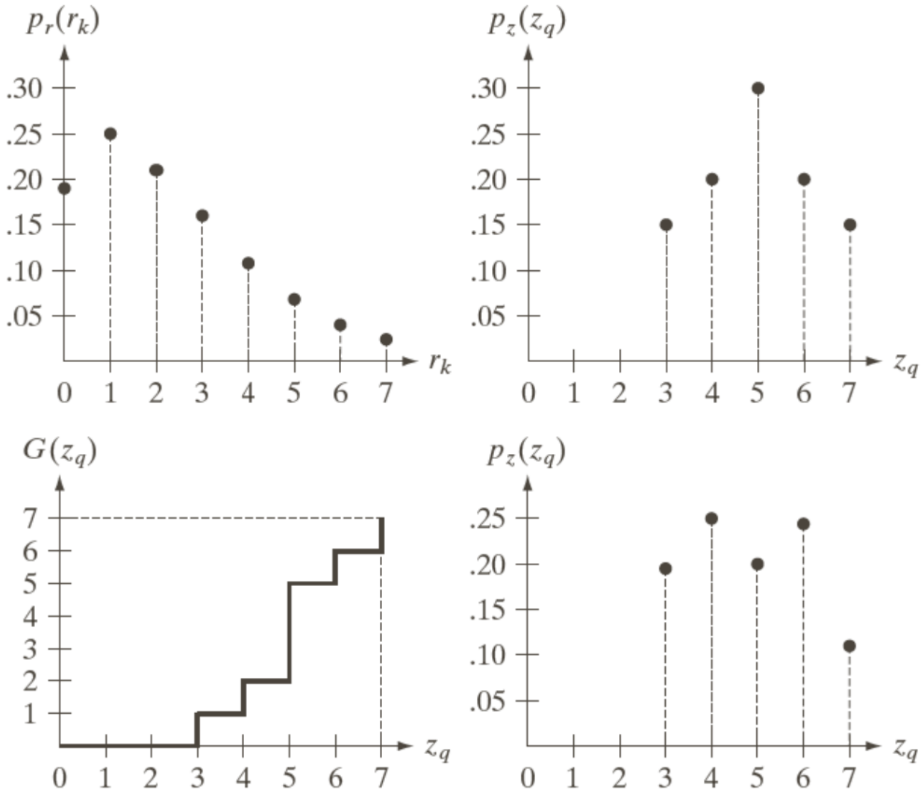
Histogram Matching (Specification)

(r_k)	(s_k)
0	1
1	3
2	5
3	6
4	6
5	7
6	7
7	7

$G(z_k)$	(z_k)
0	0
0	1
0	2
1	3
2	4
5	5
6	6
7	7

(r_k)		(z_k)
0		0
1		1
2		2
3		3
4		4
5		5
6		6
7		7

Histogram Matching (Specification)



Histogram

- Question:
  – What will be the effect on the histogram if we shuffle the pixels within the image?

Answer:

- It’s histogram won’t change. No point processing will be affected

RECAP

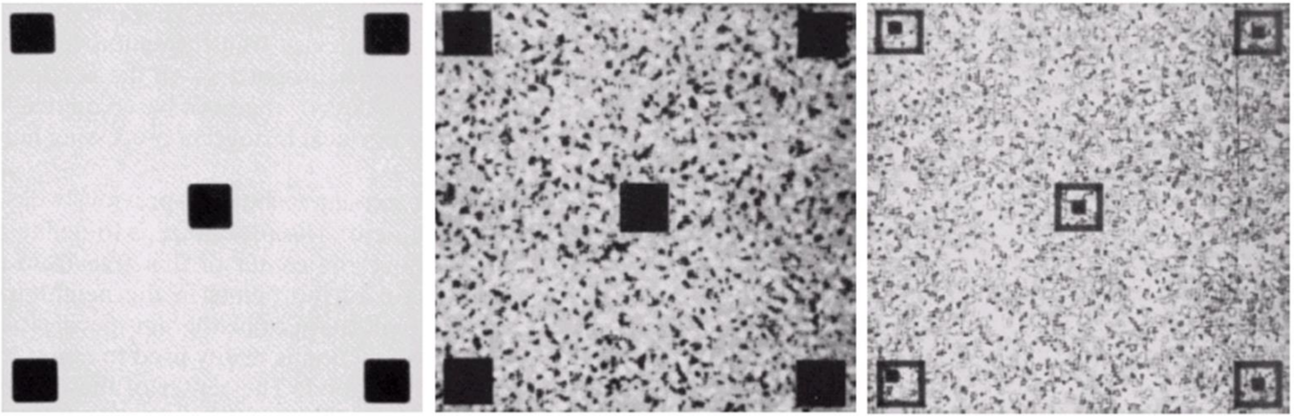
Recap

- Image Enhancement
  - Spatial domain
    - Intensity transformation (based on manipulation of one pixel)
      - Linear transformations
        - » Image negative
      - Non-linear transformations
        - » Logarithmic transformation
        - » Power Law (Exponential) transformation
      - Piecewise-linear transformations
        - » Contrast stretching
        - » Gray-level slicing
        - » Bit plane slicing
    - Spatial filtering (based on neighbor)
  - Frequency domain

SPATIAL FILTERING

Local Enhancement

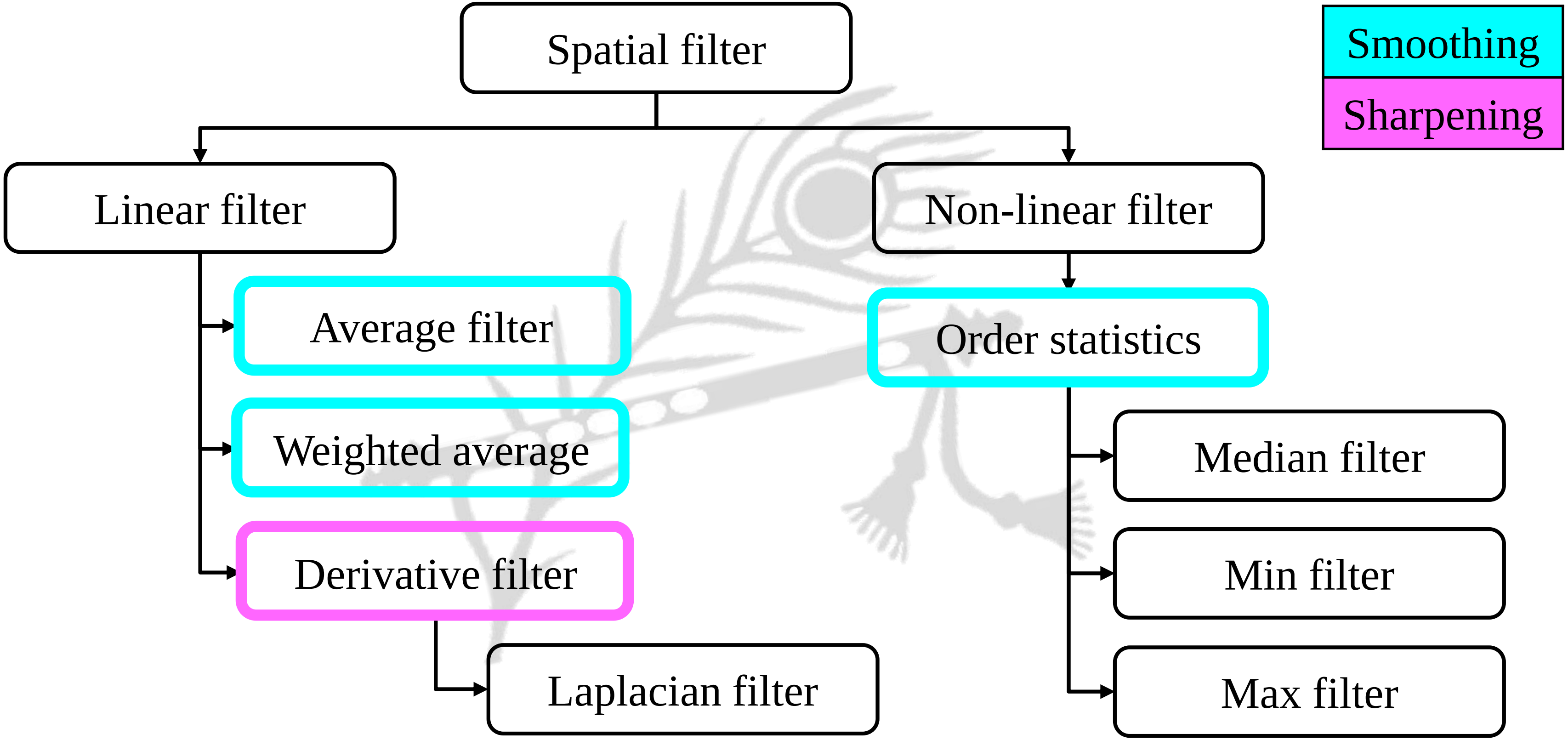
- Histogram processing methods are global processing in the sense that pixels are modified by a transformation function based on the gray level content of an entire image
- Sometimes, we may need to enhance details over small areas in an image, which is known as local enhancement



Spatial Filtering

- Enhancement techniques based on the neighbours of pixel of an image are often referred to as Spatial filtering
- Filter term in “Digital image processing” is referred to the sub-image
- Sub-image is also termed as mask, kernel, template, or window
- The value in a filter sub-image are referred as coefficients, rather than pixels

Spatial Filtering



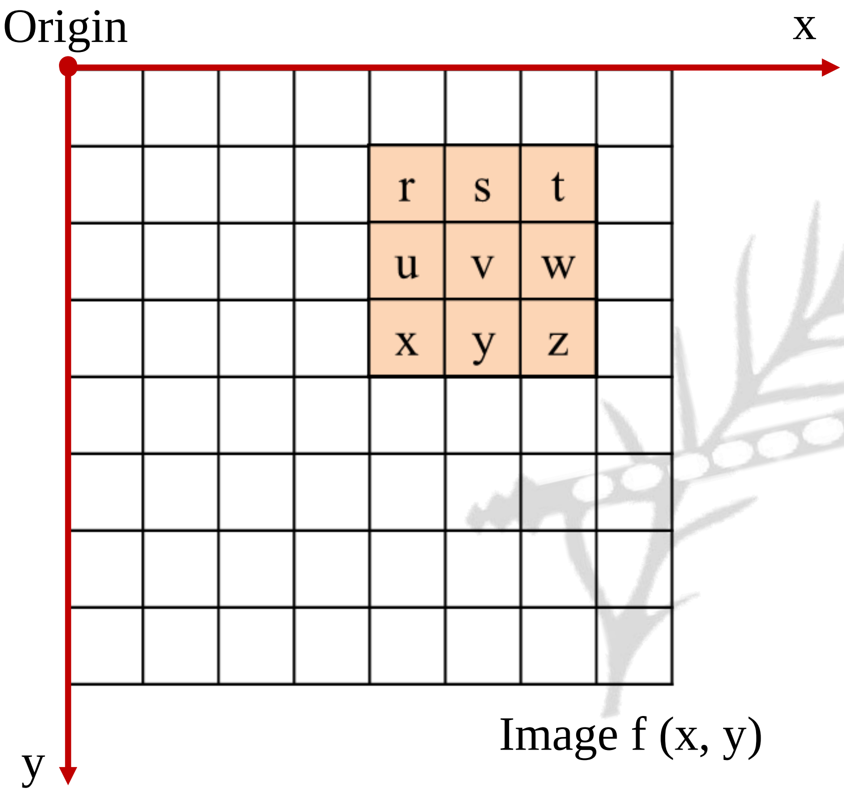
Mechanics of Spatial filtering

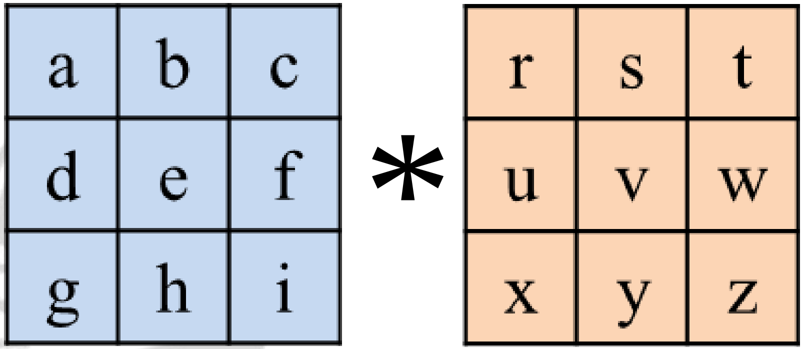
- The process consists simply of moving the filter mask from point to point in an image
- At each point (x, y), the response of the filter at that point is calculated using a predefined relationship

$$R = w_1 z_1 + w_2 z_2 + \ldots + w_{mn} z_{mn}$$

$$= \sum_{i=i}^{mn} w_i z_i$$

Mechanics of Spatial filtering





Original Image Pixels

Filter

$$e_{processed} = r*a + s*b + t*c + u*d + v*e + w*f + x*g + y*h + z*i$$

The above is repeated for every pixel in the original image to generate the filtered image

Smoothing Spatial Filters

- Used for blurring and for noise reduction

- Blurring is used in preprocessing steps, such as
  - removal of small details from an image prior to object extraction
  - bridging of small gaps in lines or curves

- Noise reduction can be accomplished by blurring with a linear filter and also by a nonlinear filter
- There are 2 types of smoothing spatial filters
  - Linear filters
  - Order statistics filters

Linear filters → Average filter

- Output is simply the average of the pixels contained in the neighbourhood of the filter mask
- Also called averaging filters or Lowpass filters and is useful for highlighting gross detail

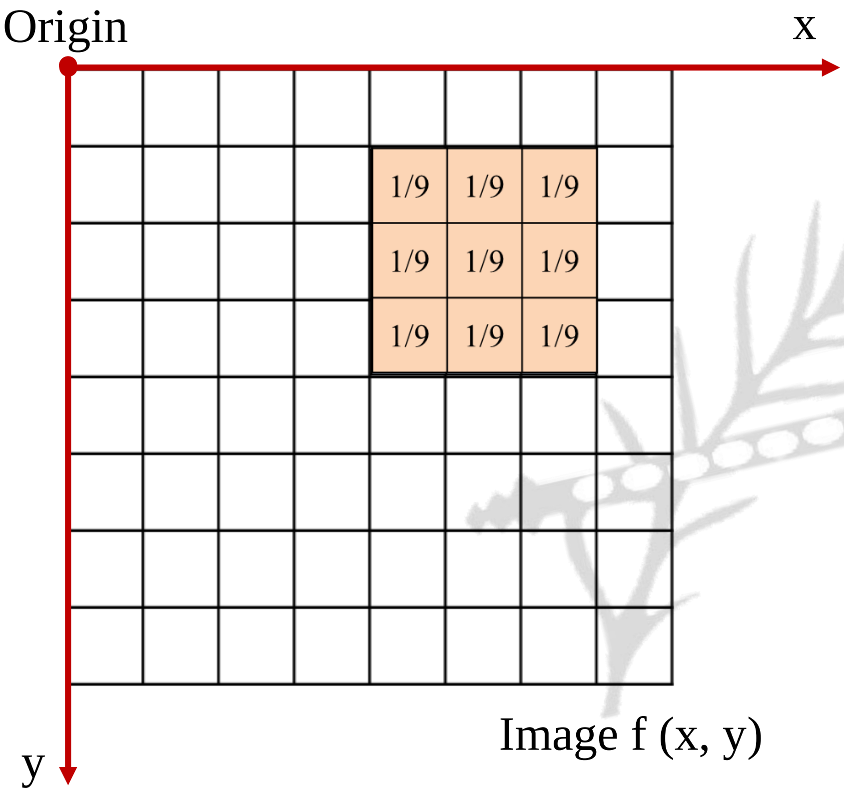
$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Average filter

$$\frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

Weighted average filter

Linear filters → Average filter



$$\begin{bmatrix} 104 & 100 & 108 \\ 99 & 106 & 98 \\ 95 & 90 & 85 \end{bmatrix} * \begin{bmatrix} 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \end{bmatrix}$$

$$\begin{aligned} e_{processed} &= (104*1 + 100*1 + 108*1 + \\ &\quad 99*1 + 106*1 + 98*1 + \\ &\quad 95*1 + 90*1 + 85*1)/9 \\ &= 98.33 \end{aligned}$$

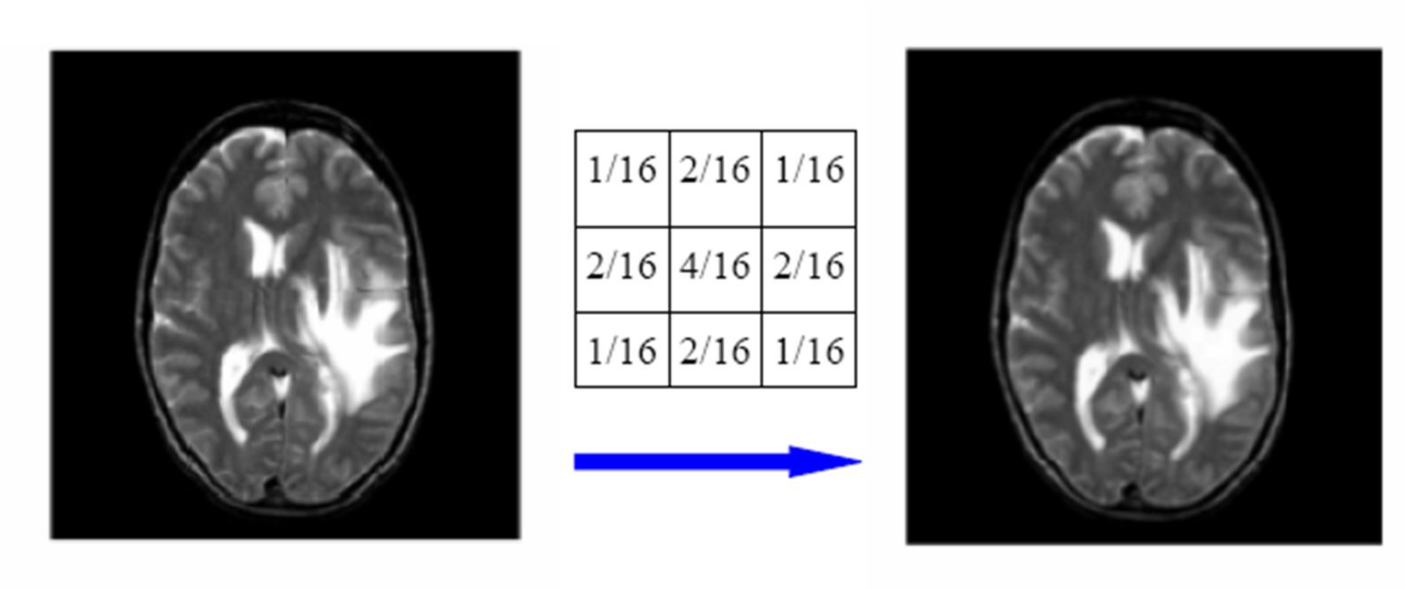
The above is repeated for every pixel in the original image to generate the filtered image

Linear filters → Average filter

- The general implementation for filtering an M x N image with a average/weighted average filter of size m x n is given by the expression

$$g(x,y) = \frac{\sum_{s=-a}^{a}\sum_{t=-b}^{b} w(s,t) f(x+s, y+t)}{\sum_{s=-a}^{a}\sum_{t=-b}^{b} w(s,t)}$$

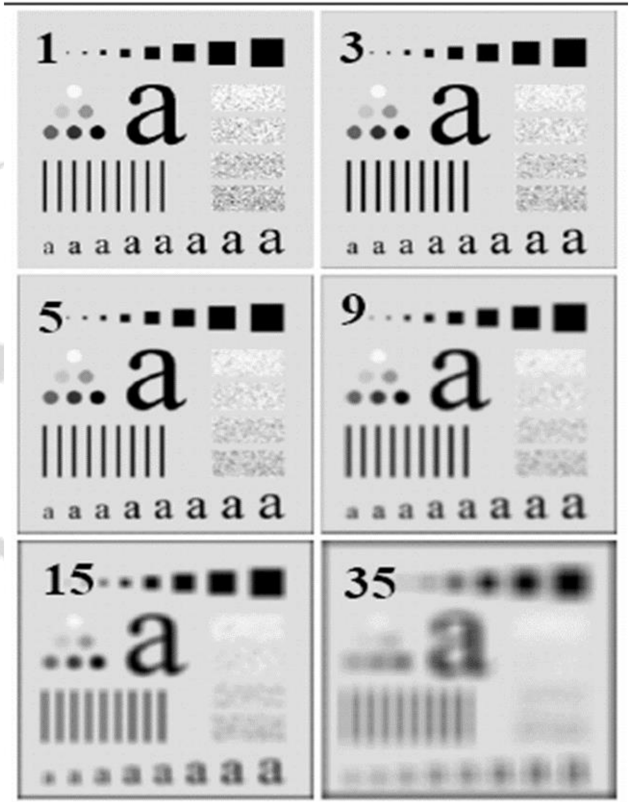
Linear filters → Average filter (Results)



Before smoothing **After smoothing**

Linear filters → Average filter (Results)

- Original image of 500 x 500 pixels
- Results of smoothing with averaging filter masks of size m = 3, 5, 9, 15 and 35
- Note:
  – big mask is used to eliminate small objects from an image



Linear filters → Weighted average filter

- More effective smoothing filters can be generated by allowing different pixels in the neighbourhood having different weights in the averaging function
- Pixels closer to the central pixel are more important
- Often referred to as a weighted averaging

$$\frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

Non-linear filters → Order statistics filters

- Order-statistics filters are nonlinear spatial filters whose response is based on ordering (ranking) the pixels contained in the image area encompassed by the filter,
- Types:
  - median filter : $R = \text{median}\{z_k \mid k = 1, 2, \ldots, n \times n\}$
  - max filter : $R = \max\{z_k \mid k = 1, 2, \ldots, n \times n\}$
  - min filter : $R = \min\{z_k \mid k = 1, 2, \ldots, n \times n\}$

Non-linear filters → Order statistics filters → Median filter

- Sort the values of the pixel in the region
- In the M x N mask, the median is the middle element
- Eg:

10	15	20
20	100	20
20	20	25

Solution:

- 10, 15, 20, 20, 20, 20, 20, 25, 100

Non-linear filters → Order statistics filters → Min filter

- Sort the values of the pixel in the region
- In the M x N mask, the min is the first element
- Eg:

10	15	20
20	100	20
20	20	25

Solution:

- 10, 15, 20, 20, 20, 20, 20, 25, 100

Non-linear filters → Order statistics filters → Max filter

- Sort the values of the pixel in the region
- In the M x N mask, the max is the last element
- Eg:

10	15	20
20	100	20
20	20	25

Solution:

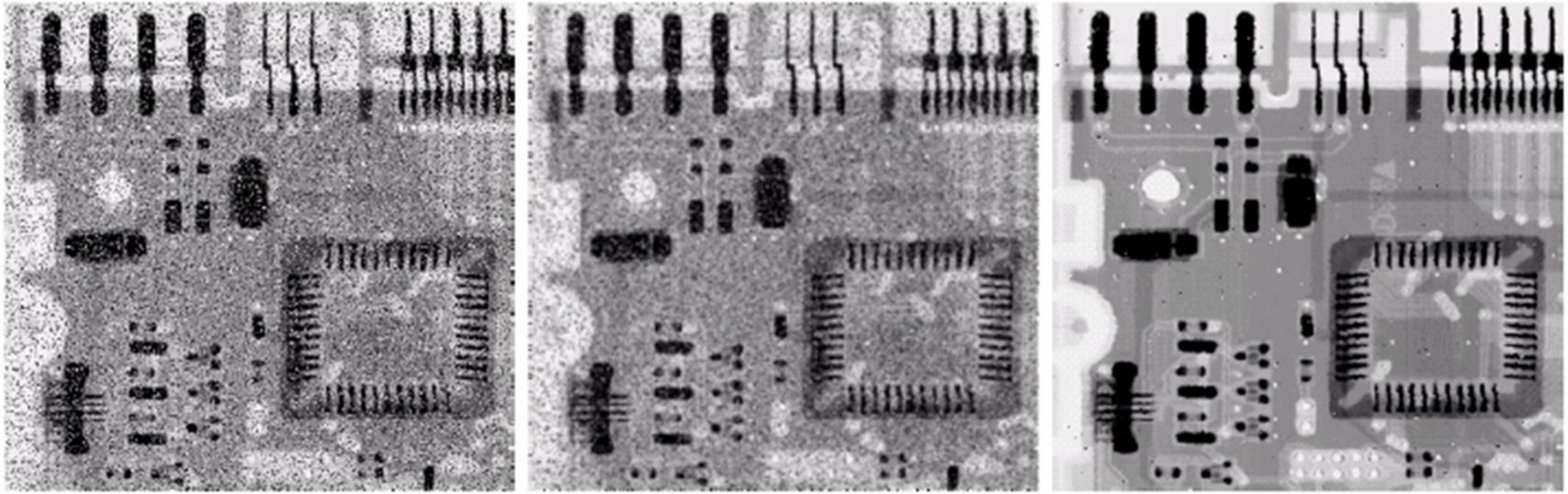
- 10, 15, 20, 20, 20, 20, 20, 25, 100

Non-linear filters → Order statistics filters → Median filter

- Median filtering is particularly effective in the presence of impulse noise (salt and pepper noise)
- Unlike average filtering, median filtering does not blur too much image details

- Advantages:
  - Removes impulsive noise
  - Preserves edges
- Disadvantages:
  - performance poor when # of noise pixels in the window is greater than 1/2 # in the window
  - performs poorly with Gaussian noise

Non-linear filters → Order statistics filters → Median filter



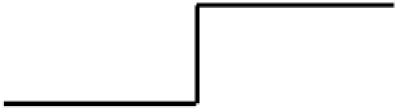
a) X-ray image of circuit board corrupted by salt-pepper noise
b) Noise reduction with a 3 x 3 averaging mask
c) Noise reduction with a 3 x 3 median filter

Blurring v/s Sharpening

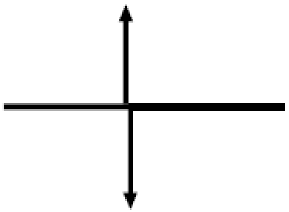
- As we know that blurring can be done in spatial domain by pixel averaging in a neighbours
- Averaging is analogous to integration
- Therefore, logically it can be concluded that the sharpening must be accomplished by spatial differentiation

Sharpening Spatial Filters

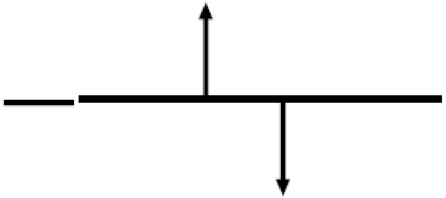
- The principal objective of sharpening is to highlight fine detail in an image or to enhance detail that has been blurred
- First and second order derivatives are commonly used for sharpening



$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$



$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$



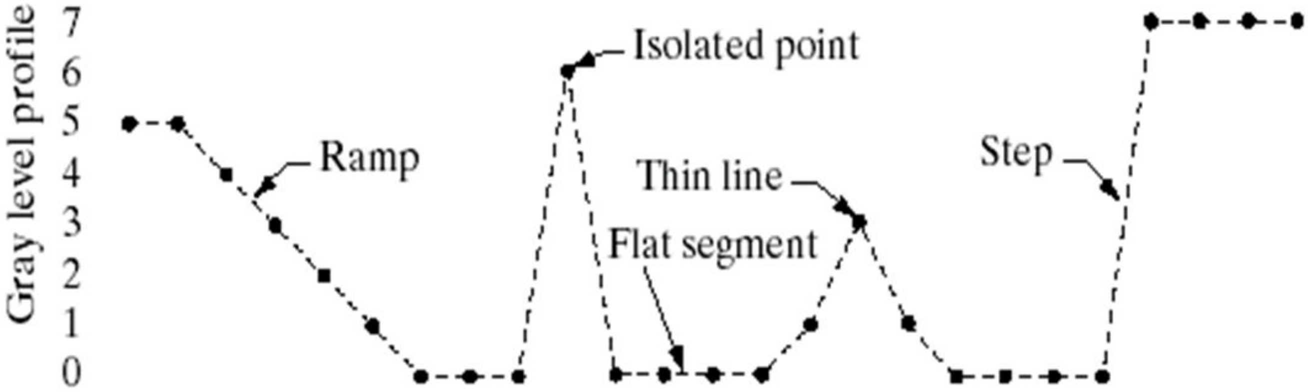
Derivative operator

- Response of first order derivative is:
  - zero in flat segments
  - Non zero at the onset (beginning) of a ramp
  - Non zero along ramps

- Response of second order derivative is:
  - Zero in flat areas
  - Non zero at the onset (beginning) of a ramp
  - Zero along ramps of constant slope

Derivative operator

- Eg:



5	5	4	3	2	1	0	0	0	6	0	0	0	0	1	3	1	0	0	0	0	7	7	7	.	.	.

First Derivative	−1	−1	−1	−1	−1	0	0	6	−6	0	0	0	1	2	−2	−1	0	0	0	7	0	0	0
Second Derivative	−1	0	0	0	0	1	0	6	−12	6	0	0	1	1	−4	1	1	0	0	7	−7	0	0

Derivative operator

- First-order derivatives generally produce thicker edges in an images
- Second-order derivatives have a stronger response to fine detail (e.g. thin lines or isolated points).
- First-order derivatives generally have a stronger response to a gray-level step
- Second-order derivatives produce a double response at step changes in gray level

Derivative operator → Laplacian (Second derivative)

- The filter is expected to be isotropic: response of the filter is independent of the direction of discontinuities in an image
- The digital implementation of the 2-Dimensional Laplacian is obtained by summing 2 components

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

0	1	0
1	-4	1
0	1	0

$$\nabla^2 f = f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y)$$

Derivative operator → Laplacian (Second derivative)

0	1	0
1	-4	1
0	1	0

0	-1	0
-1	4	-1
0	-1	0

← 90° isotropic

45° isotropic →

1	1	1
1	-8	1
1	1	1

-1	-1	-1
-1	8	-1
-1	-1	-1

Derivative operator → Laplacian (Second derivative)

- As it is a derivative operator,
  - it highlights gray-level discontinuities in an image
  - it deemphasizes regions with slowly varying gray levels
- Tends to produce images that have
  - grayish edge lines and other discontinuities, all superimposed on a dark
  - featureless background

Derivative operator → Laplacian (Second derivative)

Original Image

Laplacian Filtered Image

Sharpened Image

FOURIER TRANSFORM

Fourier Transform

- Any function that periodically repeats itself
- It can be expressed as a sum of sines/cosines with different amplitudes A, frequencies f, and phases p

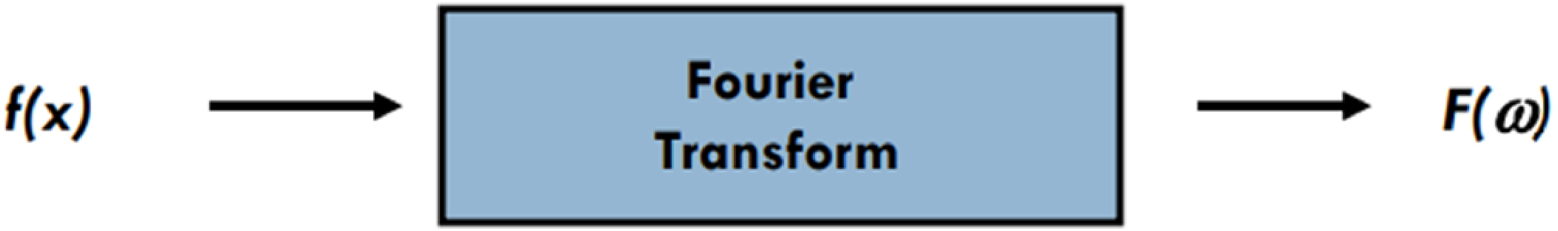
$$y(x) = A \sin(fx + p)$$

- This sum is called a Fourier Series

- Even functions that are not periodic (but whose area under the curve is finite) can be expressed as the integral of sines and/or cosines multiplied by a weighting function

Fourier Transform

- We want to understand the frequency w of our signal. So, let’s reparametrize the signal by w instead of x:



Fourier Transform

- The Fourier transform of f(x) is:

$$F(u) = \int_{-\infty}^{\infty} f(x) \mathrm{e}^{-j2\pi ux} \, dx$$

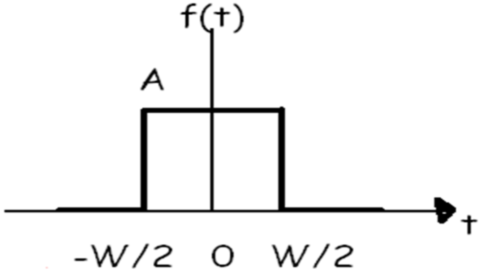
$$= \int_{-\infty}^{\infty} f(\mathrm{x})[\cos 2\pi \, \mathrm{ux} - \mathrm{j}\sin 2\pi \, \mathrm{ux}] dx$$

- The inverse Fourier transform of f(x) is:

$$f(x) = \int_{-\infty}^{\infty} F(u) \mathrm{e}^{j2\pi ux} \, du$$

Fourier Transform

- Eg:
  - Find its Fourier Transform



Fourier Transform

Solution

$$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

$$F(u) = \int_{-\infty}^{\infty} f(t) e^{-j2\pi ut} dt$$

$$F(u) = \int_{-\frac{W}{2}}^{\frac{W}{2}} A e^{-j2\pi ut} dt$$

$$F(u) = \frac{A}{\pi u} \sin(\pi u W)$$

Fourier Transform (Discrete)

- The Fourier transform of a discrete function of one variable, f(x), x = 0, 1, …, M-1, is given by the following equation

$$F(u) = \frac{1}{M}\sum_{x=0}^{M-1} f(x)\,e^{-j2\pi ux/M}$$

For u = 0, 1, 2, …, M-1

Where $j = \sqrt{-1}$

- Inverse

$$f(x) = \frac{1}{M}\sum_{u=0}^{M-1} F(u)\,e^{j2\pi ux/M}$$

Fourier Transform (Discrete)

- Eg:
  - Find the DFT of f(x) = {0, 1, 2, 1}

Euler's Formula:

$$e^{j\theta} = \cos\theta + j\sin\theta$$

Fourier Transform (Discrete)

$$F(u)=\frac{1}{M}\sum_{x=0}^{M-1} f(x)\mathrm{e}^{-j2\pi ux/M}$$

Solution

$$F(0)=\frac{1}{4}\left[0+1+2+1\right]$$

$$=1$$

Fourier Transform (Discrete)

Solution

$$F(1)=\frac{1}{4}\sum_{x=0}^{3}f(x)e^{\frac{-j2\pi ux}{M}}$$

$$=\frac{1}{4}\left[0+1e^{\frac{-j2\pi 1.1}{4}}+2e^{\frac{-j2\pi 1.2}{4}}+1e^{\frac{-j2\pi 1.3}{4}}\right]$$

$$=\frac{1}{4}\left[e^{\frac{-j\pi}{2}}+2e^{-j\pi}+e^{\frac{-j3\pi}{2}}\right]$$

$$=\frac{1}{4}\left[\cos\left(\frac{-\pi}{2}\right)+j\sin\left(\frac{-\pi}{2}\right)+2(\cos(-\pi)+j\sin(-\pi))+\cos\left(\frac{-3\pi}{2}\right)+j\sin\left(\frac{-3\pi}{2}\right)\right]$$

$$=\frac{1}{4}[0-j+2(-1+0)+0+j]=\frac{-1}{2}$$

Fourier Transform (Discrete)

Solution

$$F(2) = \frac{1}{4}\left[0 + 1e^{-\frac{j2\pi 2.1}{4}} + 2e^{-\frac{j2\pi 2.2}{4}} + 1e^{-\frac{j2\pi 2.3}{4}}\right]$$

$$= \frac{1}{4}\left[\cos(-\pi) + j\sin(-\pi) + 2(\cos(-2\pi) + j\sin(-2\pi)) + \cos(-3\pi) + j\sin(-3\pi)\right]$$

$$= \frac{1}{4}\left[-1 + 0 + 2\,.\,1 + 0 + -1 + 0\right]$$

$$= 0$$

Fourier Transform (Discrete)

Solution

$$F(3)=\frac{1}{4}\left[0+1e^{-\frac{j2\pi 3.1}{4}}+2e^{-\frac{j2\pi 3.2}{4}}+1e^{-\frac{j2\pi 3.3}{4}}\right]$$

$$=\frac{1}{4}\left[\cos\left(-\frac{3\pi}{2}\right)+j\sin\left(-\frac{3\pi}{2}\right)+2\left(\cos(-3\pi)+j\sin(-3\pi)\right)+\cos\left(-\frac{9\pi}{2}\right)+j\sin\left(-\frac{9\pi}{2}\right)\right]$$

$$=\frac{1}{4}\left[0+j-2+0+0-j\right]$$

$$=\frac{-1}{2}$$

Fourier Transform (Discrete)

Solution

$$F(0) = 1$$

$$F(1) = -1/2$$

$$F(2) = 0$$

$$F(3) = -1/2$$

Fourier Transform (Discrete)

- Twiddle factor
  - A new factor defined as

$$W_M = e^{-j2\pi/M}$$

- Now

$$F(u) = \frac{1}{M}\sum_{x=0}^{M-1} f(x)\,\mathrm{W}_M^{ux}$$

- The equation now reduces to

F = W.f

Fourier Transform (Discrete)

- Eg:
  - If M = 4, then W_M will be a 4x4 matrix

$$
W_4 = \begin{array}{c|c}
u/x & \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\
\begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \end{array} &
\begin{bmatrix}
W_M^0 & W_M^0 & W_M^0 & W_M^0 \\
W_M^0 & W_M^1 & W_M^2 & W_M^3 \\
W_M^0 & W_M^2 & W_M^4 & W_M^6 \\
W_M^0 & W_M^3 & W_M^6 & W_M^9
\end{bmatrix}
\end{array}
$$

Fourier Transform (Discrete)

Solution

$$\mathbf{W_M^0 = e^{\frac{-j2\pi.0}{4}}}$$
$$\mathbf{= 1}$$

$$\mathbf{W_M^1 = e^{\frac{-j2\pi.1}{4}}}$$
$$\mathbf{= \cos\left(-\frac{\pi}{2}\right) + j\sin\left(-\frac{\pi}{2}\right)}$$
$$\mathbf{= -j}$$

$$W_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

Fourier Transform (Discrete)

Find the DFT of f(x) = {0, 1, 2, 1}

$$\begin{bmatrix} F(0) \\ F(1) \\ F(2) \\ F(3) \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}\begin{bmatrix} f(0) \\ f(1) \\ f(2) \\ f(3) \end{bmatrix}$$

$$\begin{bmatrix} F(0) \\ F(1) \\ F(2) \\ F(3) \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 2 \\ 1 \end{bmatrix}$$

Fourier Transform (Discrete)

- Eg:
  - Find the DFT of the following image.

0	1	2	1
1	2	3	2
2	3	4	3
1	2	3	2

$$\begin{bmatrix} 1 & \frac{-1}{2} & 0 & \frac{-1}{2} \\ 2 & \frac{-1}{2} & 0 & \frac{-1}{2} \\ 3 & \frac{-1}{2} & 0 & \frac{-1}{2} \\ 2 & \frac{-1}{2} & 0 & \frac{-1}{2} \end{bmatrix}$$

$$\begin{bmatrix} 2 & \frac{-1}{2} & 0 & \frac{-1}{2} \\ \frac{-1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{-1}{2} & 0 & 0 & 0 \end{bmatrix}$$

Fourier Transform (Discrete)

- Low pass

$$H(u,v) = e^{-D^2(u,v)/2\sigma^2}$$

- High Pass

$$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}$$