# Fast Fourier Transform

# Discrete Fourier Transform

- DFT of discrete-time signal $x[n]$ of length $N$

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- **Fast Fourier Transform (FFT) is an algorithm** that efficiently computes the Discrete Fourier Transform (DFT).
- Computational algorithms that exploit both the **symmetry and the periodicity of the sequence $W_N^{nk}$**.
- Runge (1905) and later Danielson and Lanczos (1942) described algorithms for which computation was roughly proportional to $N\log N$ **rather than** $N^2$.
    - Distinction was **not of great importance for the small values of *N***.
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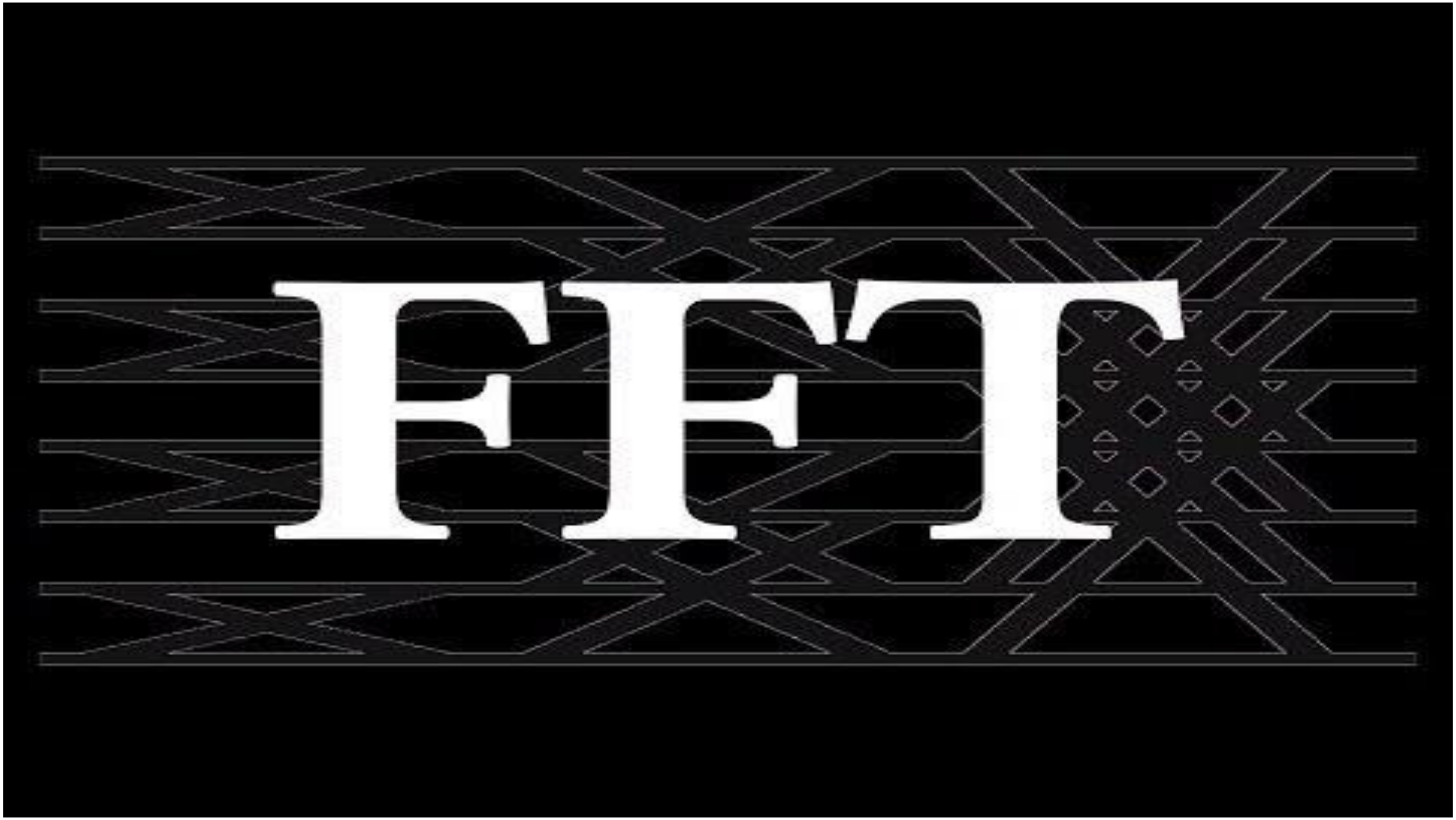
- The efficiency of FFT algorithms is so high, that in many cases the **most efficient** procedure for implementing **a convolution is through DFT and IDFT process**.

# FFT: Cooley-Tuckey algorithm



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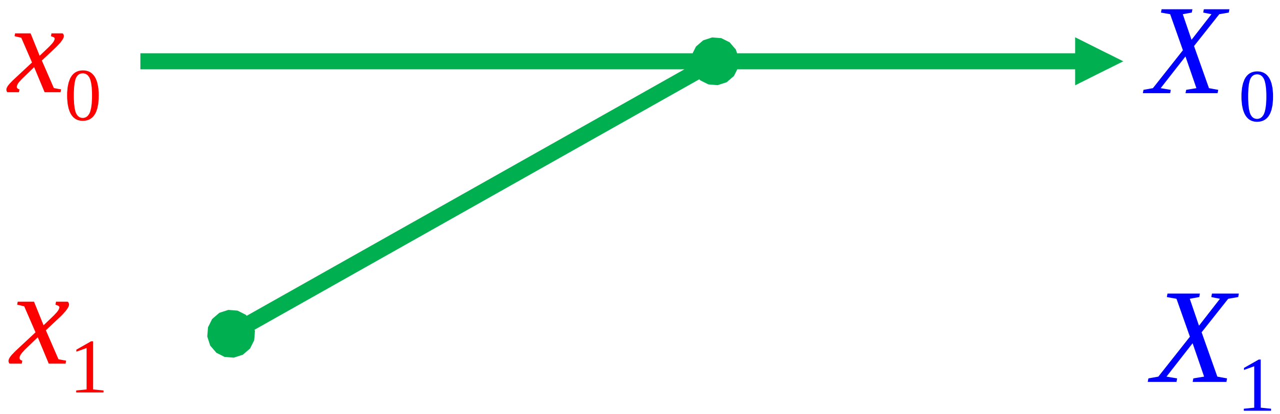

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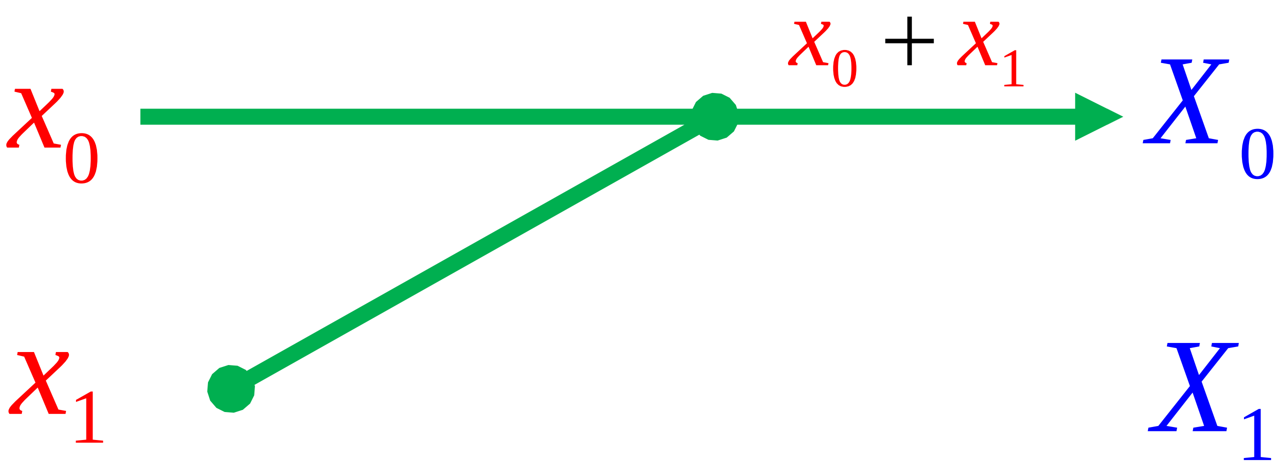

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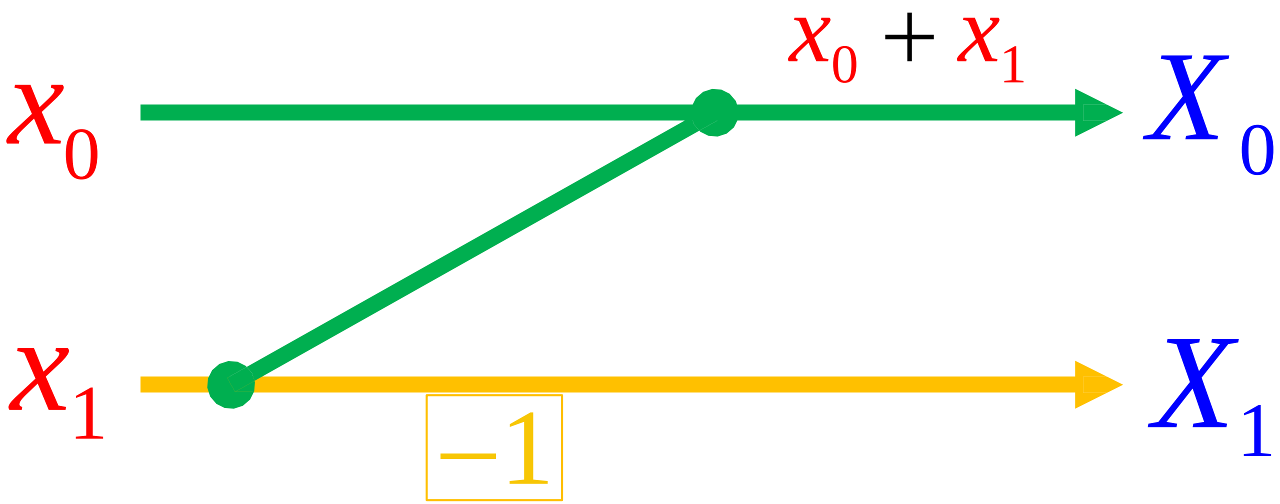

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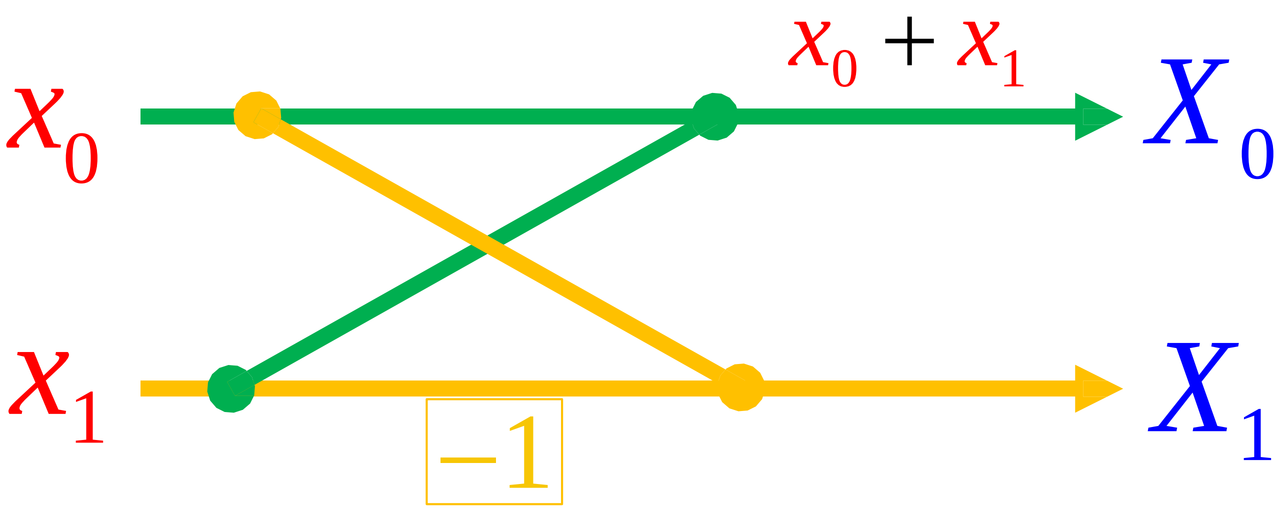

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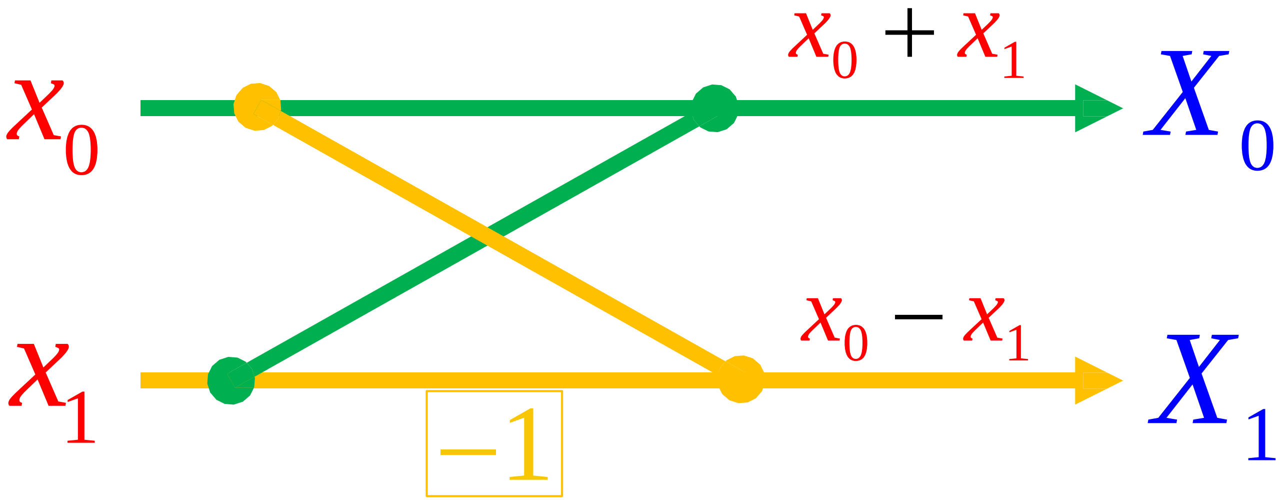

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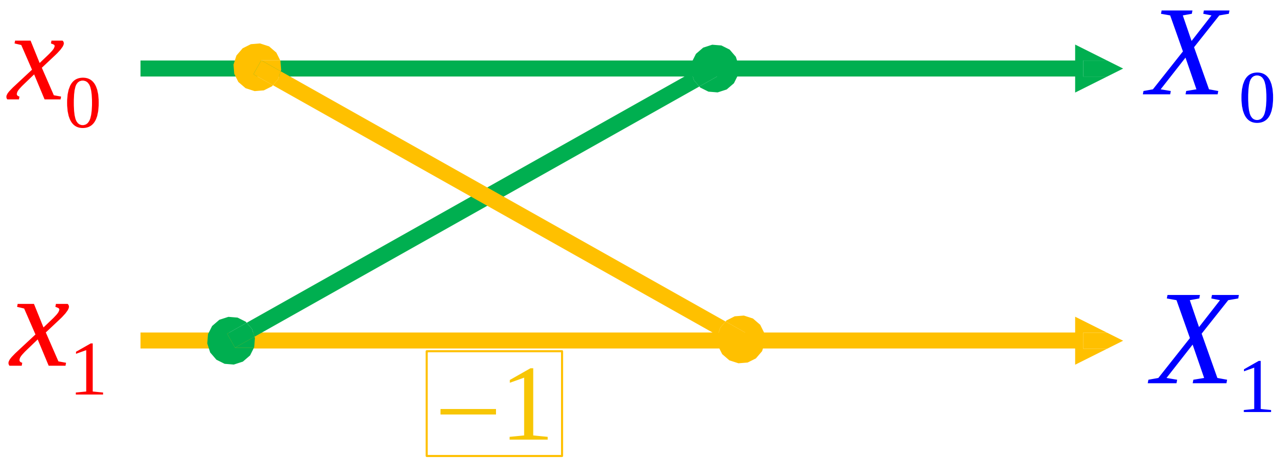
Compute DFT of $x[n] = \{1, 2\}$ using FFT algorithm



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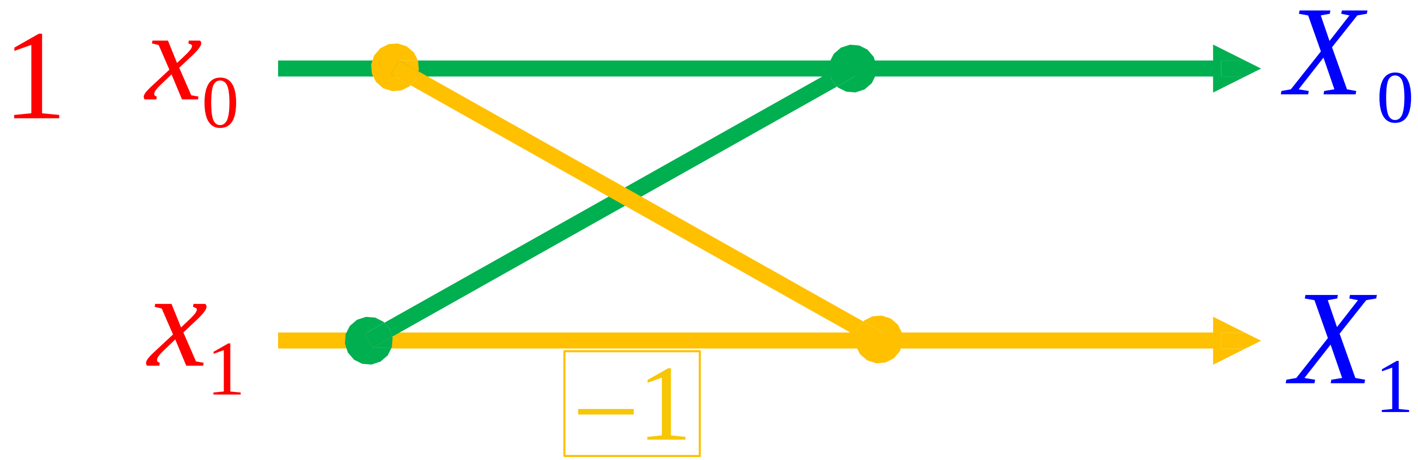

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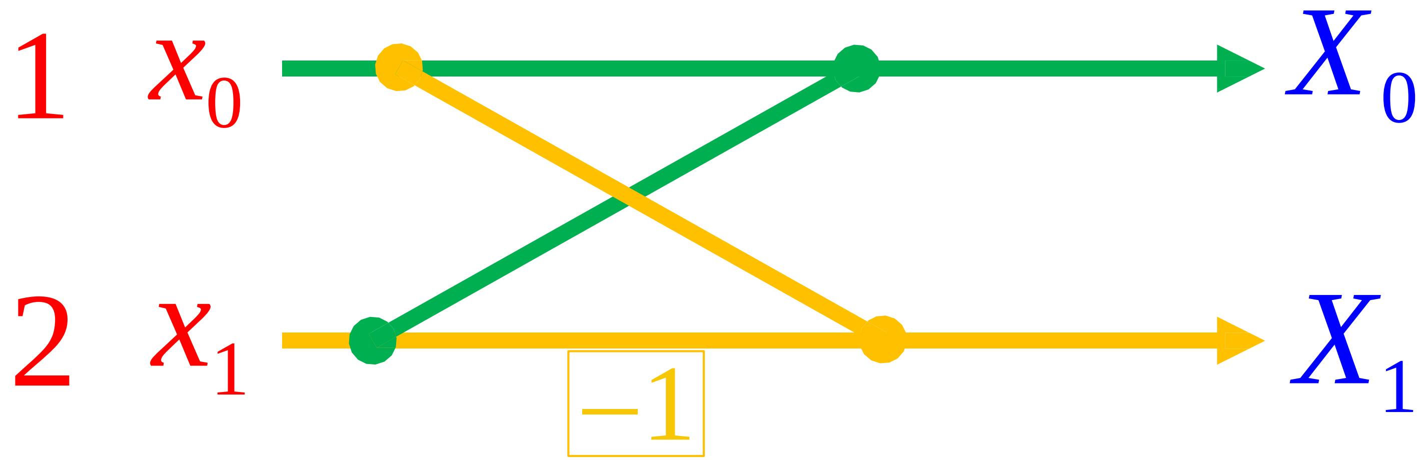

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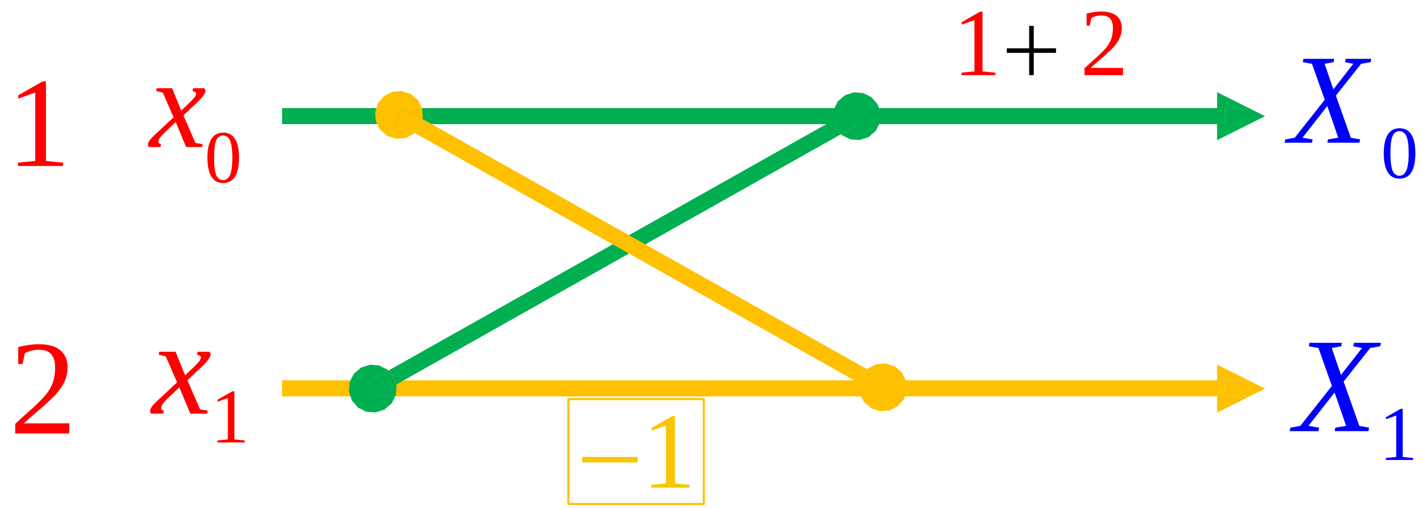

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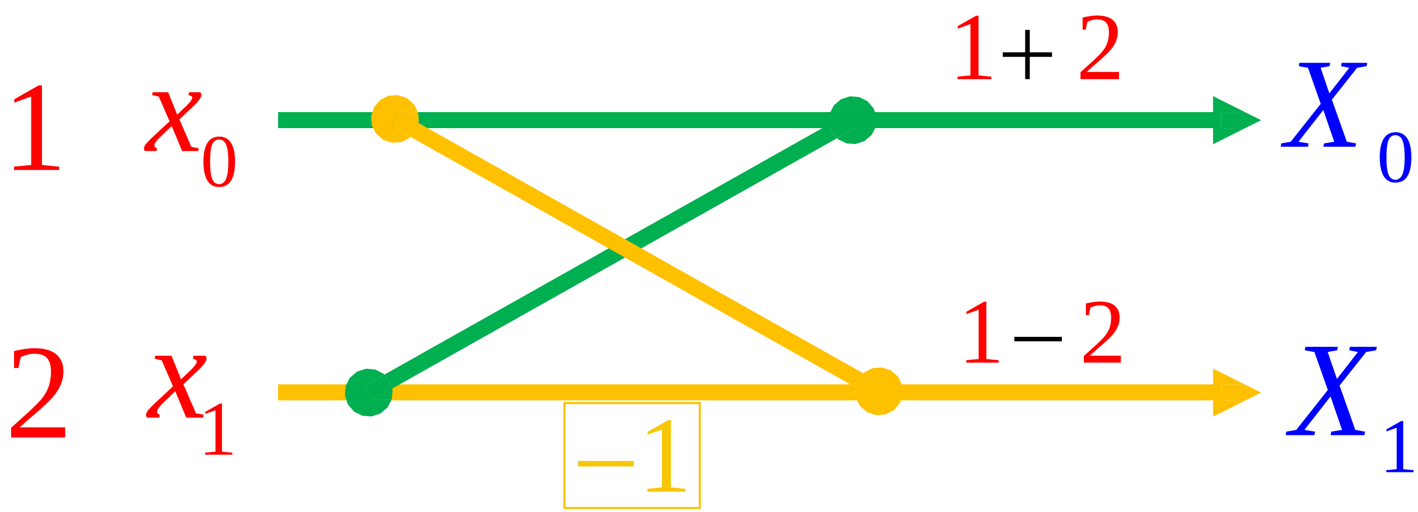

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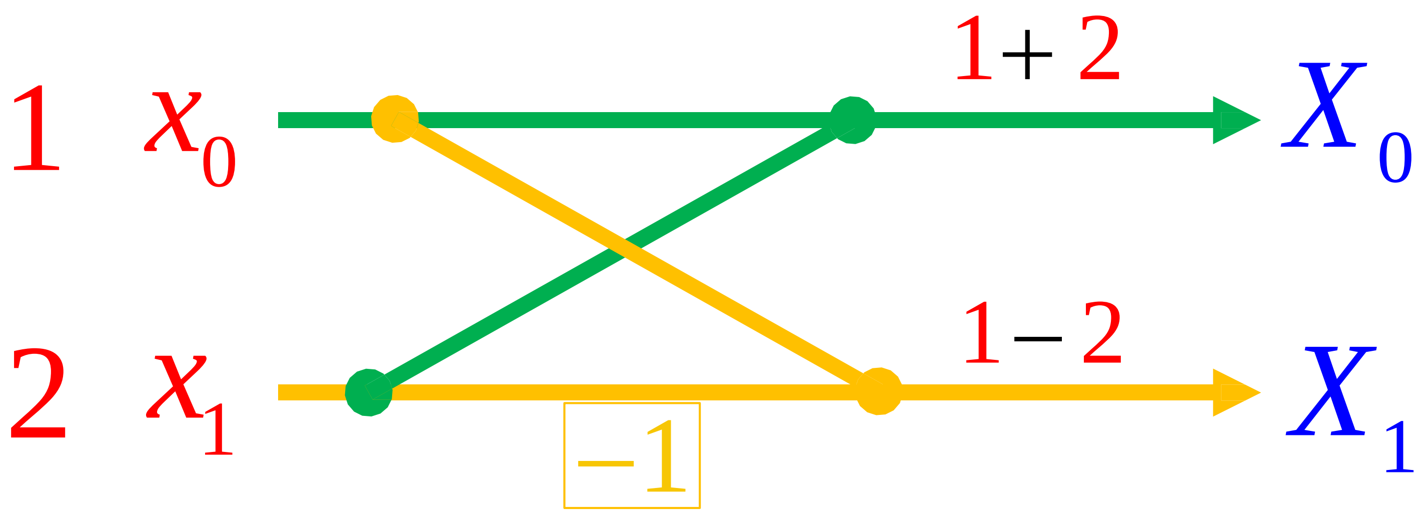


$$X[k] = \{3, \quad -1\}$$

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  - **Decimation in Frequency (DIF)**

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$$\left(\mathbf{W}_N^2\right)^{k\,m} = \left(e^{-j\frac{2\pi}{N}2}\right)^{k\,m} = \left(e^{-j\frac{2\pi}{N/2}}\right)^{k\,m} = \mathbf{W}_{N/2}^{k\,m}$$

$$X[k] = \sum_{m=0}^{N/2-1} x[2m]\, \mathbf{W}_{N/2}^{k\,m} + \mathbf{W}_N^k \sum_{m=0}^{N/2-1} x[2m+1]\, \mathbf{W}_{N/2}^{k\,m}$$

# Decimation-In-Time FFT Algorithm

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$$X[k] = \sum_{m=0}^{N/2-1} x[2m]\, W_{N/2}^{km} + W_N^k \sum_{m=0}^{N/2-1} x[2m+1]\, W_{N/2}^{km}$$

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$$X[k] = G[k] + \mathrm{W}_N^k H[k]$$

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- $G[k]$ and $H[k]$ are $N/2$-point DFTs.
- Although $k = 0, 1, \ldots, N-1$, since $G[k]$ and $H[k]$ are each periodic in $k$ with period $N/2$ sums must be computed only for $k = 0, 1, \ldots, (N/2) - 1$.

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$$X[k] = G[k] + \mathbf{W}_N^{k} H[k] \qquad k = (N/2), \ldots, N-1$$

# Decimation-In-Time FFT Algorithm

$$X[k] = \sum_{m=0}^{N/2 - 1} x[2m]\, W_{N/2}^{km} + W_N^k \sum_{m=0}^{N/2 - 1} x[2m+1]\, W_{N/2}^{km}$$

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$$X[k] = G[k] + W_N^k H[k] \qquad k = (N/2), \ldots, N-1$$

$$W_N^{(k+N/2)}$$

# Decimation-In-Time FFT Algorithm

$$X[k] = \sum_{m=0}^{N/2-1} x[2m]\, W_{N/2}^{km} + W_N^k \sum_{m=0}^{N/2-1} x[2m+1]\, W_{N/2}^{km}$$

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$$X[k] = G[k] + W_N^k H[k] \qquad k = (N/2), \ldots, N-1$$

$$W_N^{(k+N/2)} = W_N^k \; W_N^{N/2}$$

# Decimation-In-Time FFT Algorithm

$$X[k] = \sum_{m=0}^{N/2-1} x[2m]\, W_{N/2}^{km} + W_N^k \sum_{m=0}^{N/2-1} x[2m+1]\, W_{N/2}^{km}$$

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$$W_N^{(k+N/2)} = W_N^k \; W_N^{N/2} = W_N^k \; e^{-j\frac{2\pi}{N}N/2}$$

# Decimation-In-Time FFT Algorithm

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$$X[k] = G[k] + W_N^k H[k] \qquad k = (N/2), \ldots, N-1$$

$$W_N^{(k+N/2)} = W_N^k\ W_N^{N/2} = W_N^k\ e^{-j\frac{2\pi}{N}N/2} = -W_N^k$$

# Decimation-In-Time FFT Algorithm

# Decimation-In-Time FFT Algorithm

$$G[k] = \sum_{l=0}^{N/4-1} g[2l]\, W_{N/2}^{k\,2l} + \sum_{l=0}^{N/4-1} g[2l+1]\, W_{N/2}^{k\,(2l+1)}$$

# Decimation-In-Time FFT Algorithm

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$$G[k] = \sum_{l=0}^{N/4-1} g[2l]\left(W_{N/2}^{2}\right)^{k\,l} + \sum_{l=0}^{N/4-1} g[2l+1]\left(W_{N/2}^{2}\right)^{k\,l} W_{N/2}^{k}$$

# Decimation-In-Time FFT Algorithm

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$$G[k] = \sum_{l=0}^{N/4-1} g[2l]\, W_{N/4}^{k\,l} + W_{N/2}^{k} \sum_{l=0}^{N/4-1} g[2l+1]\, W_{N/4}^{k\,l}$$

# Decimation-In-Time FFT Algorithm

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$$G[k] = \sum_{l=0}^{N/4-1} g[2l]\, \left(W_{N/2}^{2}\right)^{kl} + \sum_{l=0}^{N/4-1} g[2l+1]\, \left(W_{N/2}^{2}\right)^{kl} W_{N/2}^{k}$$

$$G[k] = \sum_{l=0}^{N/4-1} g[2l]\, W_{N/4}^{kl} + W_{N/2}^{k} \sum_{l=0}^{N/4-1} g[2l+1]\, W_{N/4}^{kl}$$

Similarly

$$H[k] = \sum_{l=0}^{N/4-1} h[2l]\, W_{N/4}^{kl} + W_{N/2}^{k} \sum_{l=0}^{N/4-1} h[2l+1]\, W_{N/4}^{kl}$$

# 4-point DITFFT

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- Let $x[n] = \{x_0, x_1, x_2, x_3\}$ and corresponding DFT $X(k) = \{X_0, X_1, X_2, X_3\}$

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$$x_0$$

$$x_2$$

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$x_0$

$x_2$

$x_1$

$x_3$

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$x_0$ $X_0$

$x_2$ $X_1$

$x_1$ $X_2$

$x_3$ $X_3$

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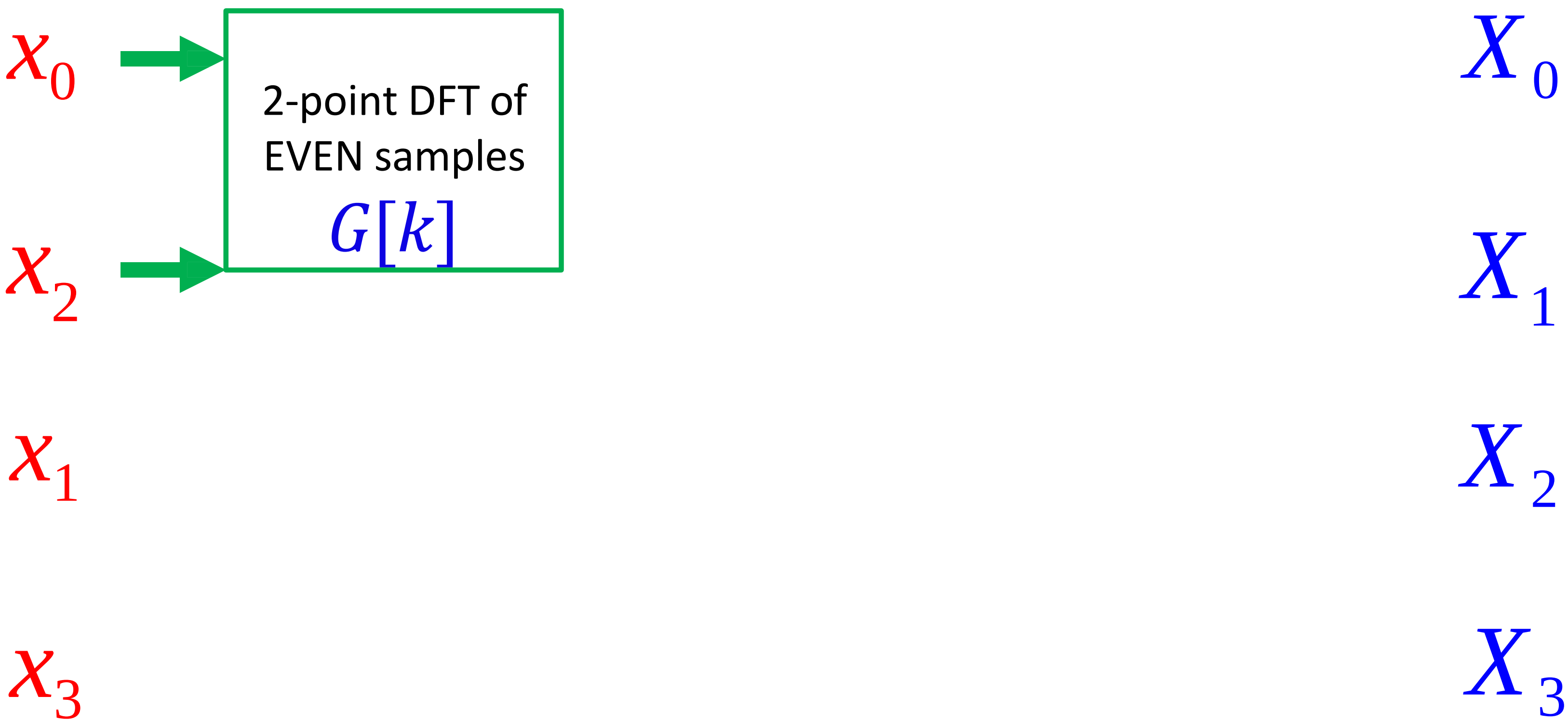

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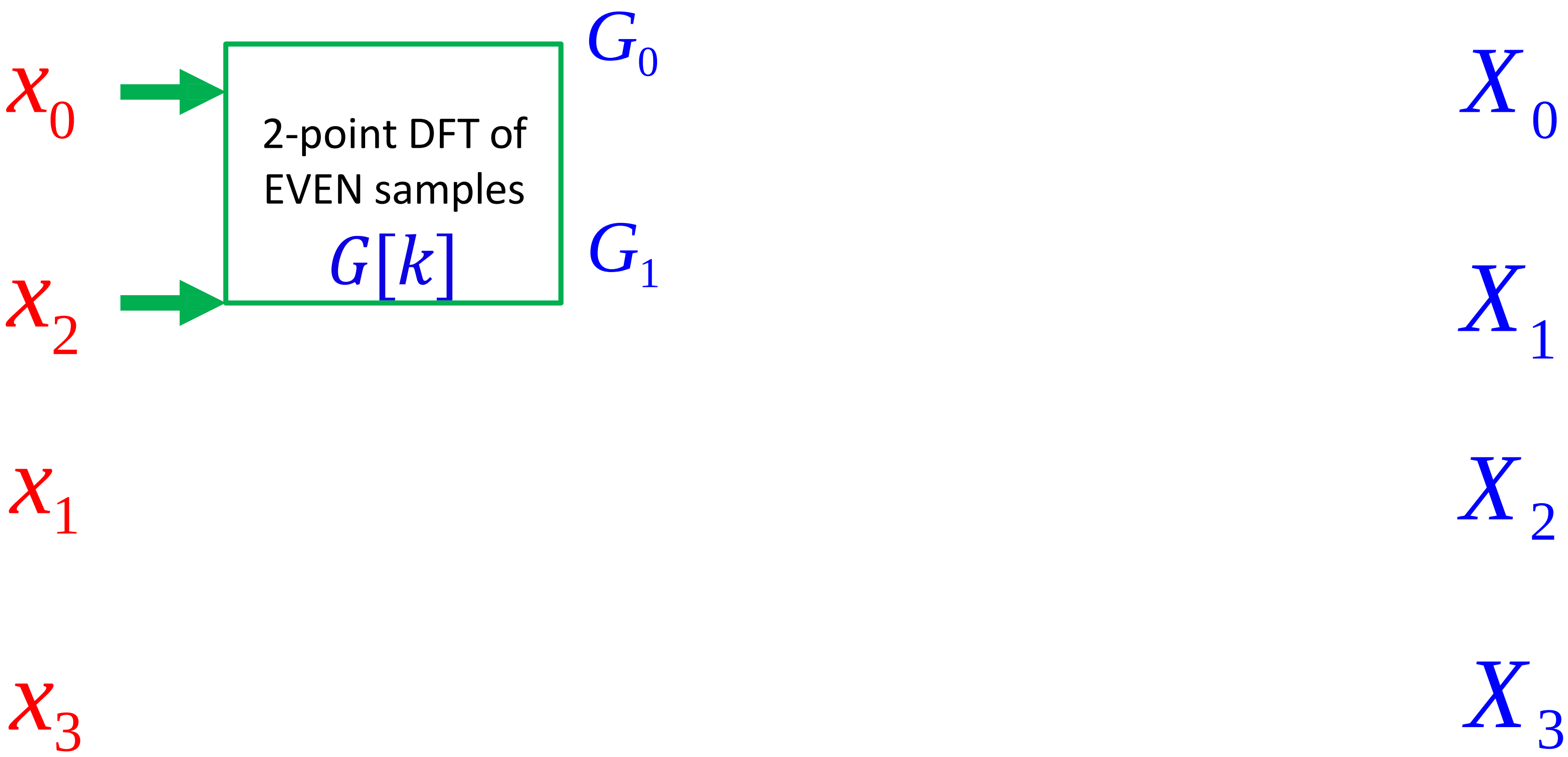

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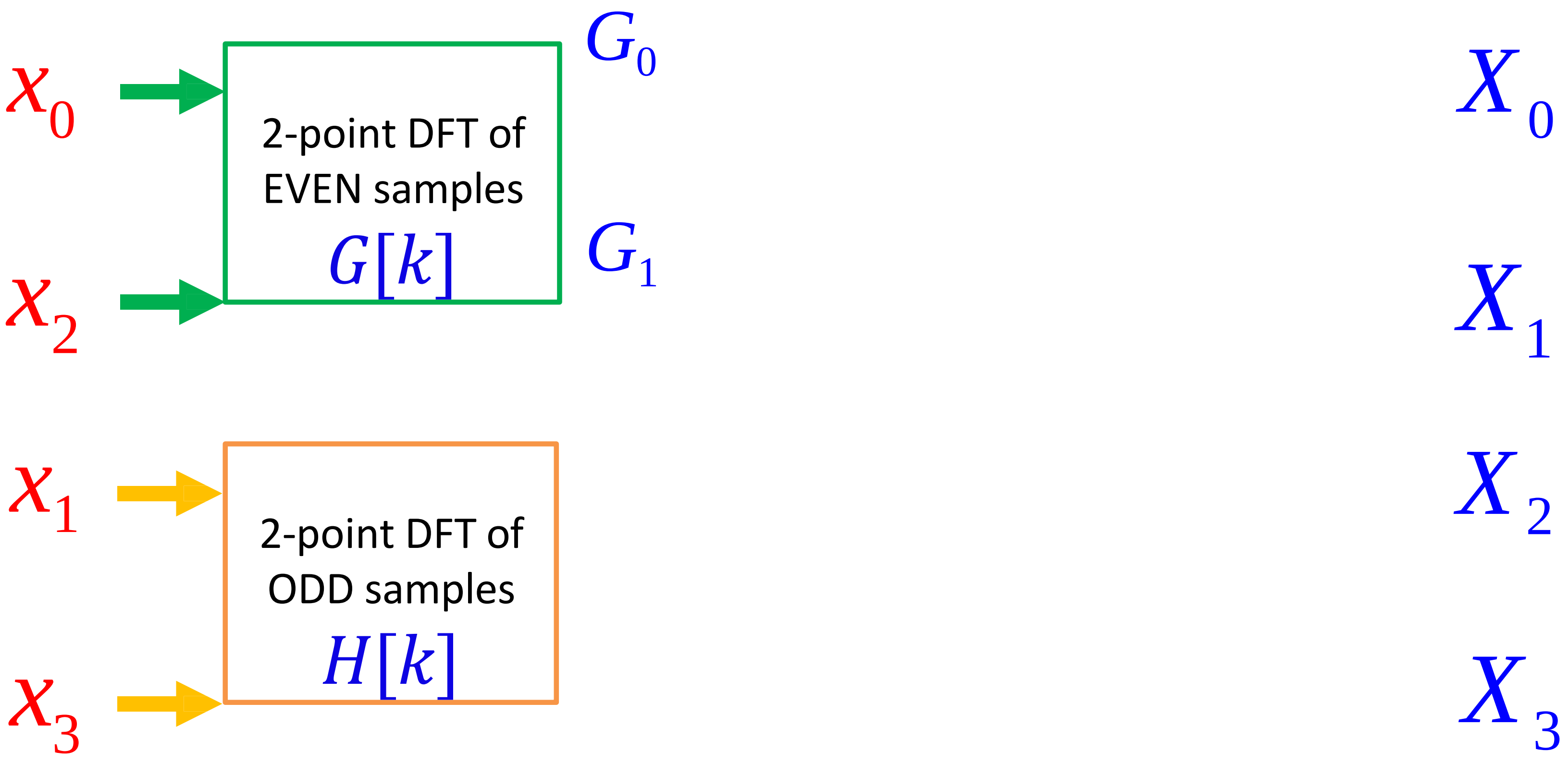

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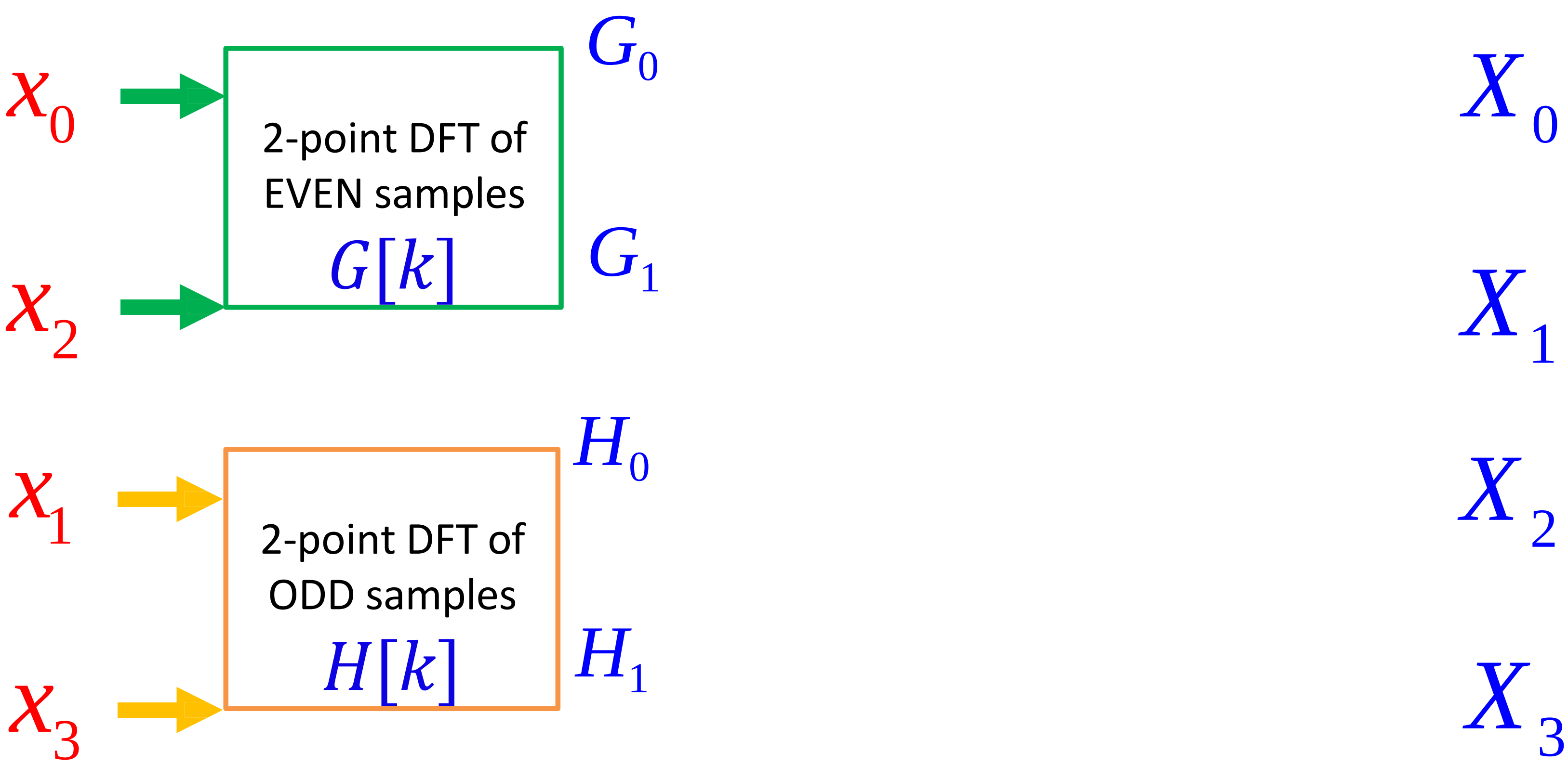

## 4-point DITFFT

$$X[k] = G[k] + W_4^k H[k]$$

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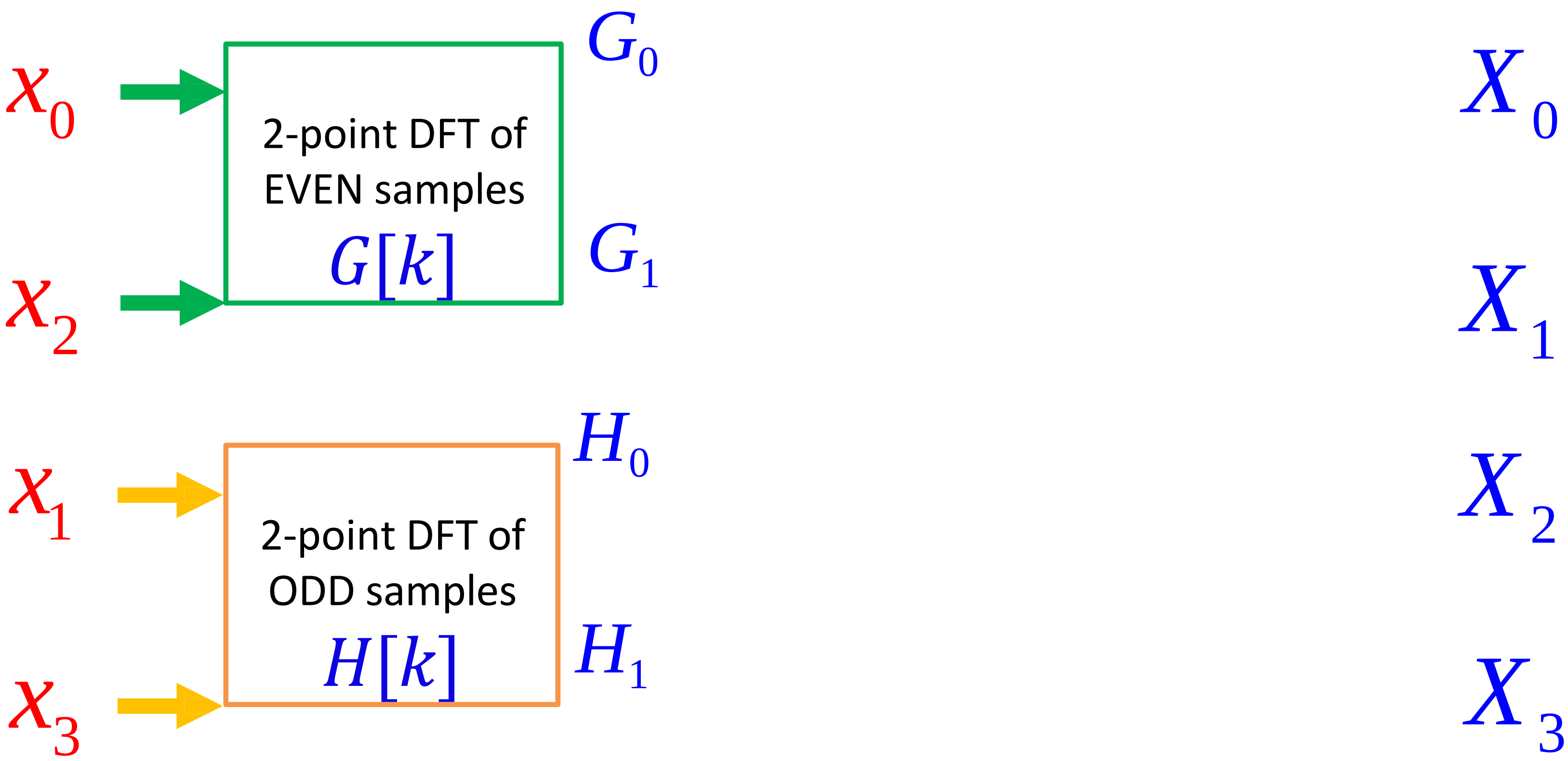

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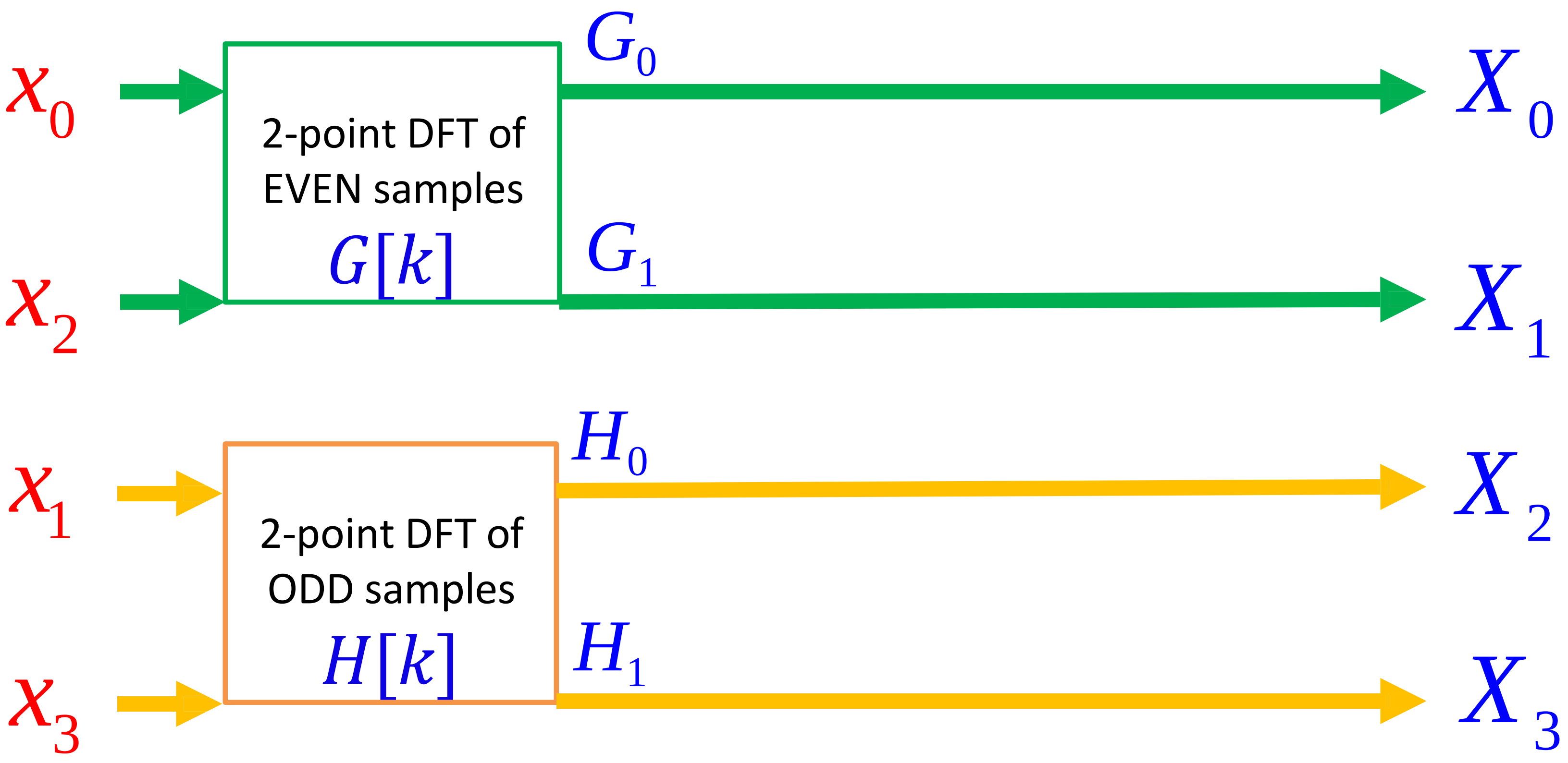

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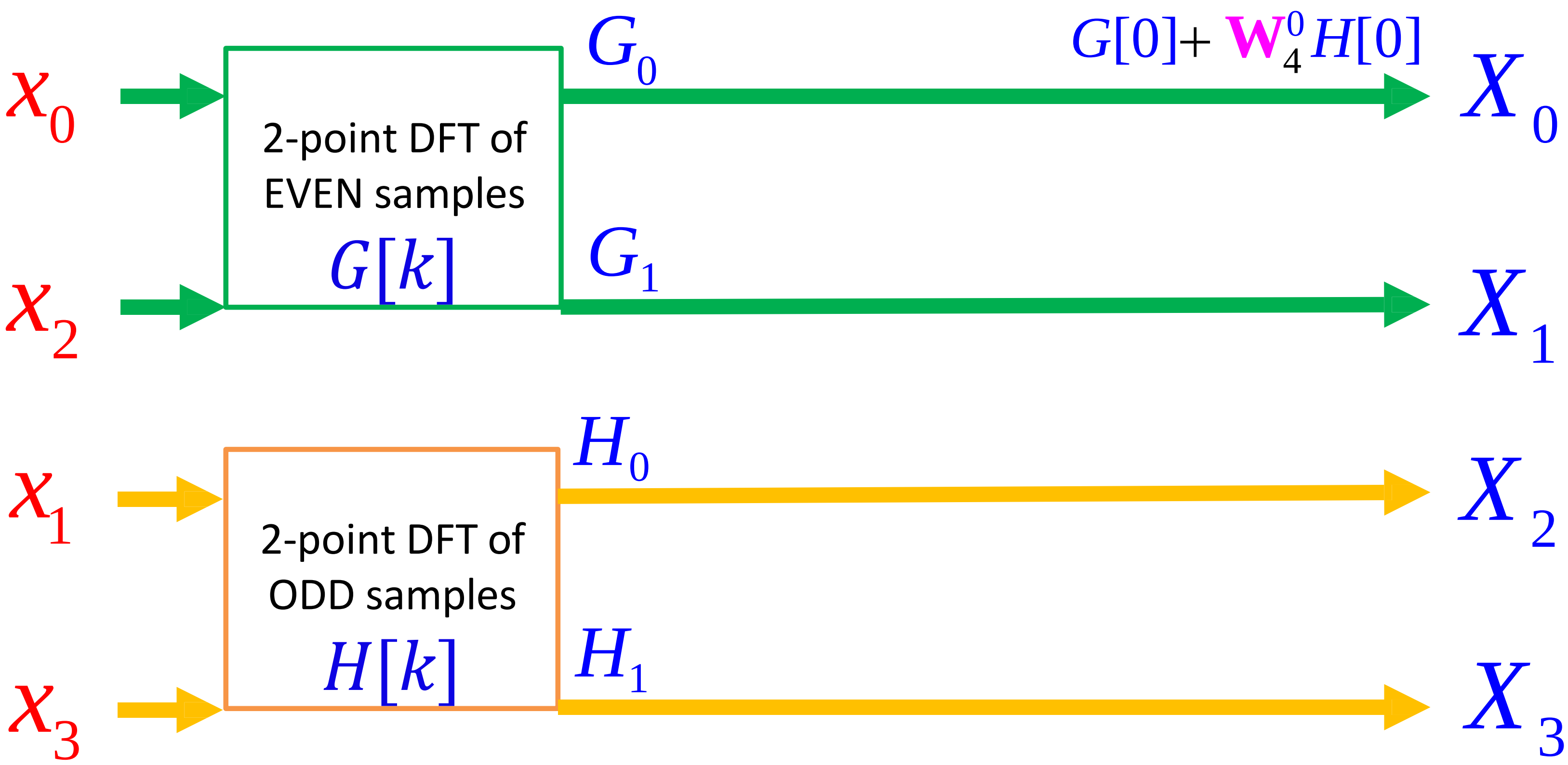

## 4-point DITFFT $X[k] = G[k] + W_4^k H[k]$

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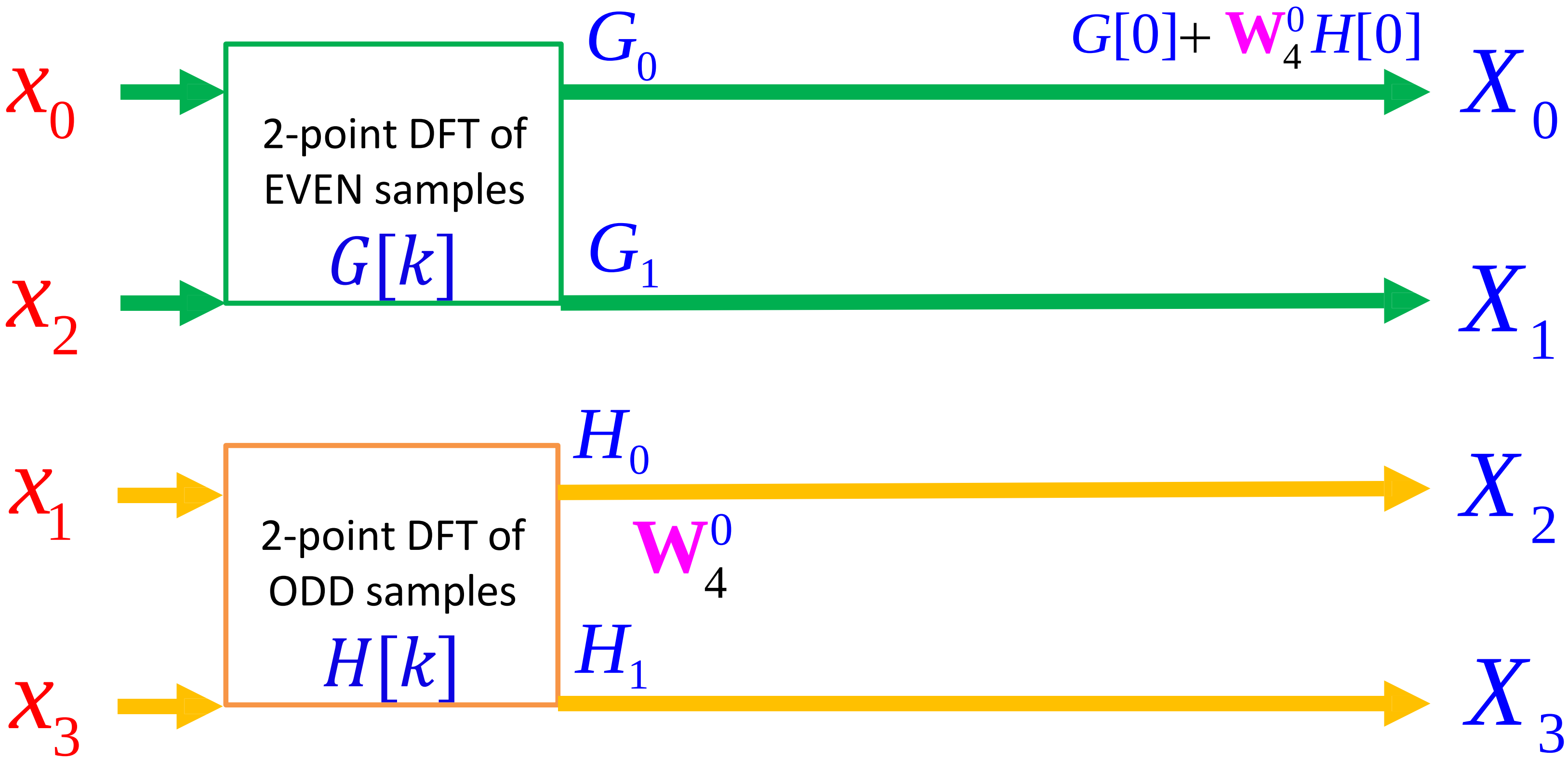

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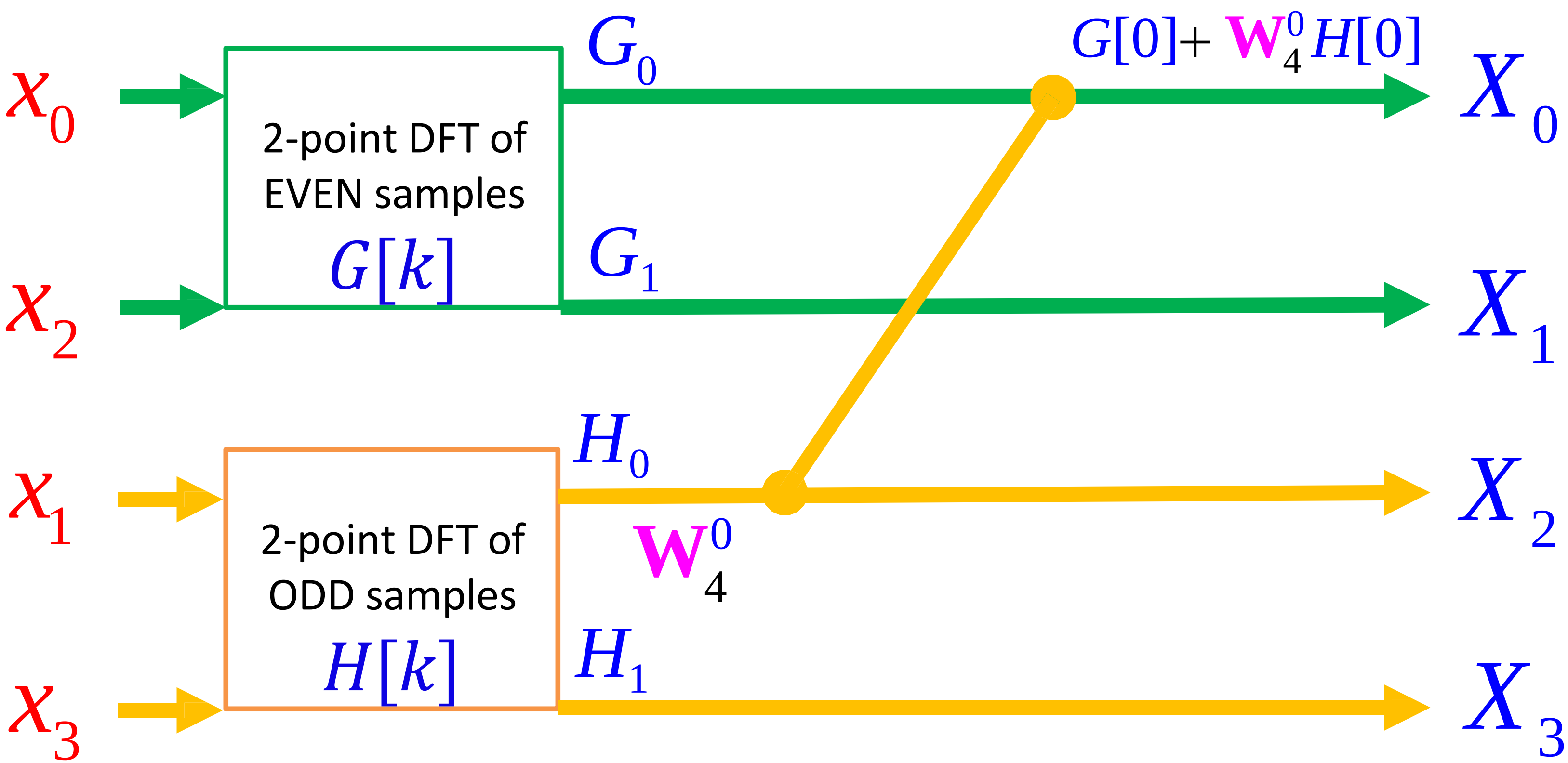

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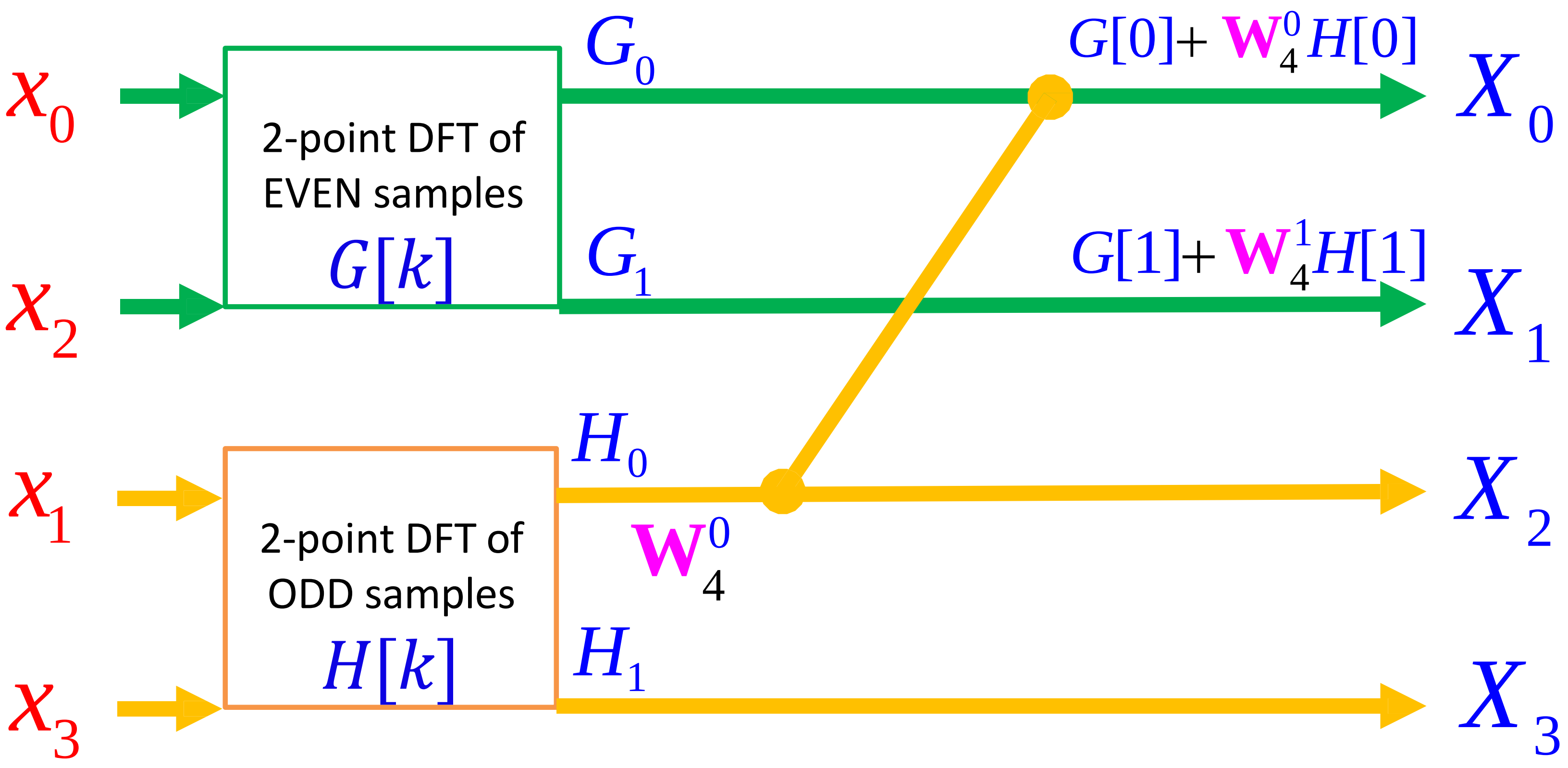

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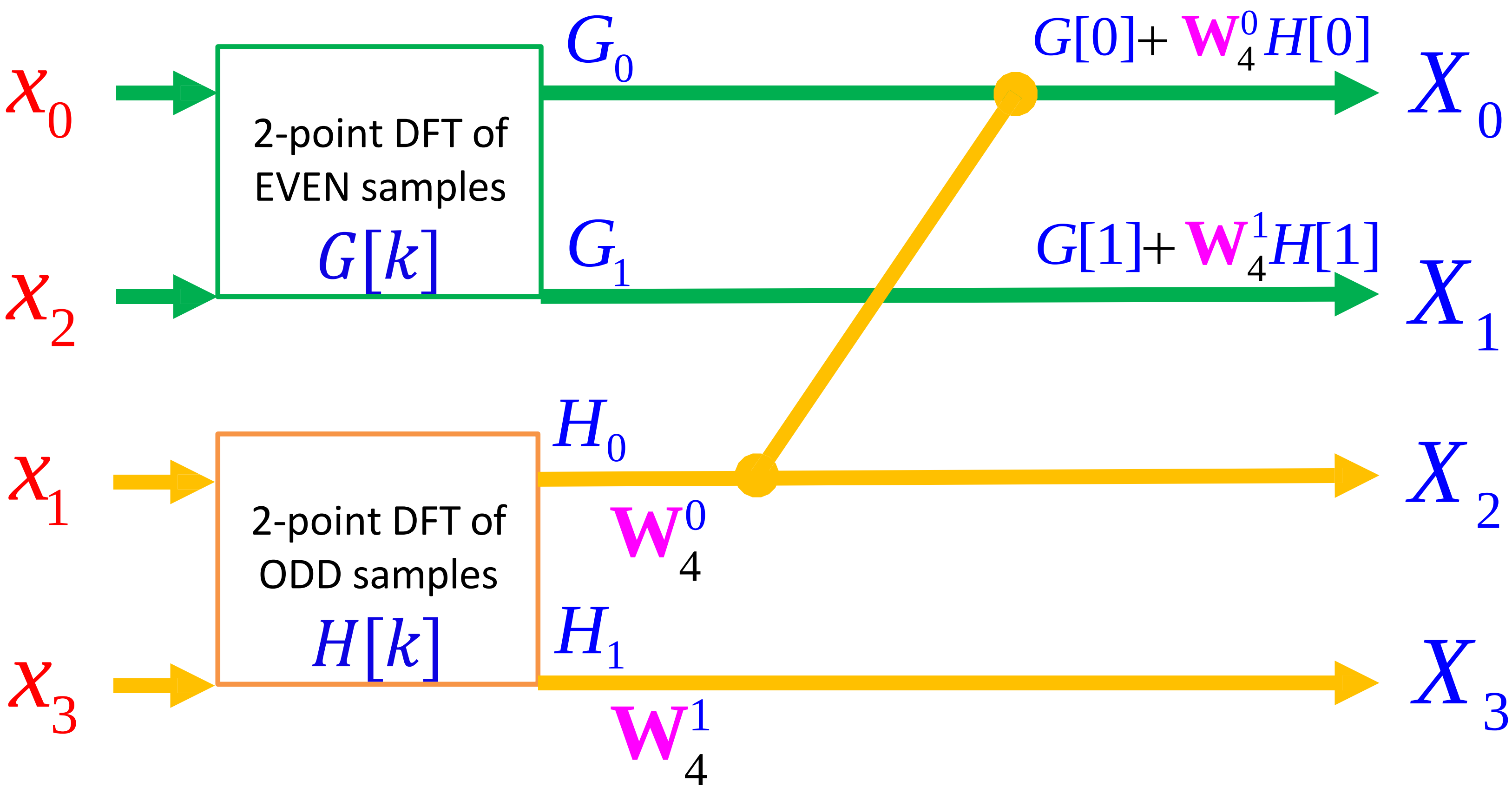

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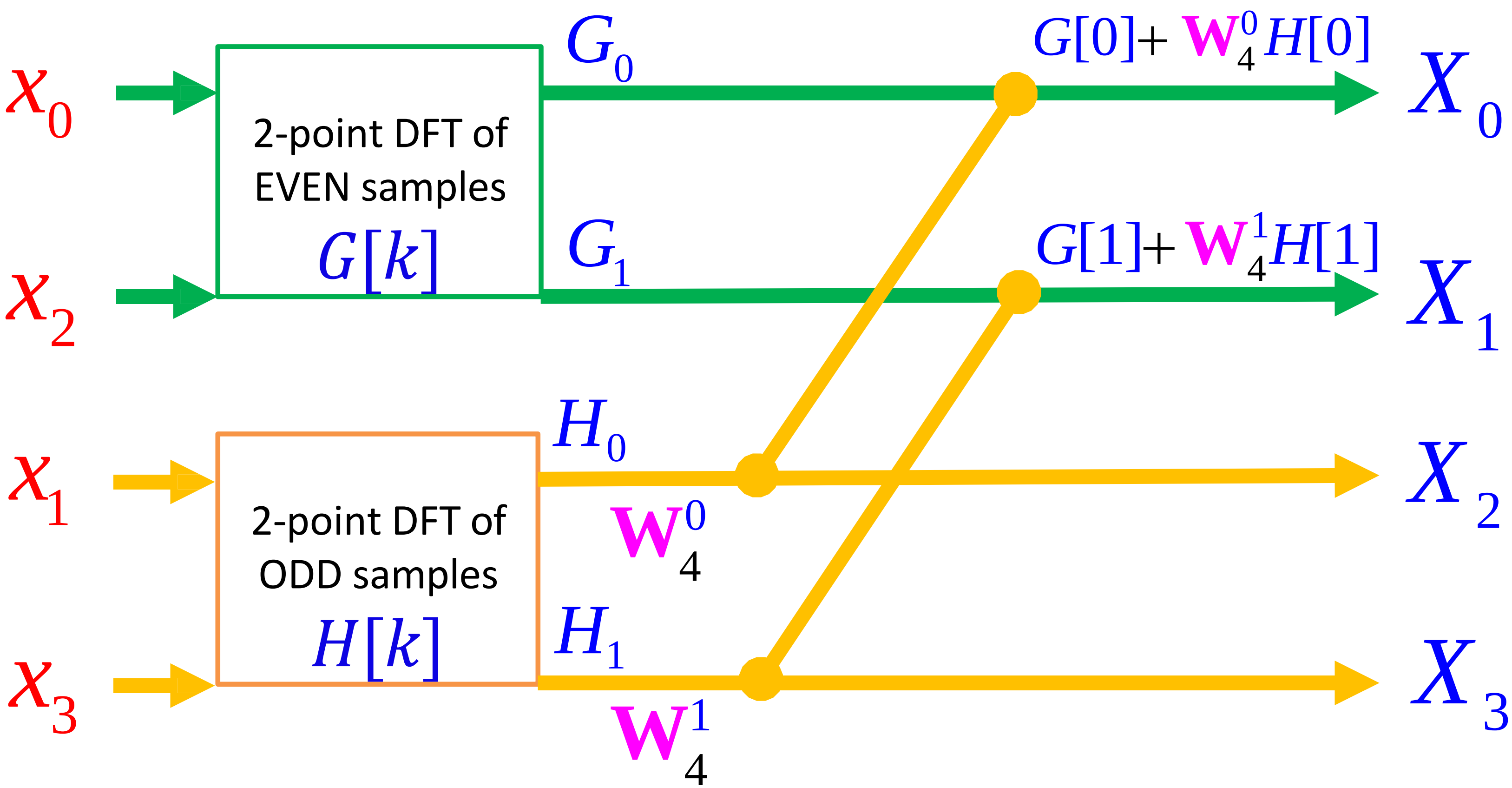

## 4-point DITFFT

$$X[k] = G[k] - \mathbf{W}_4^k H[k]$$

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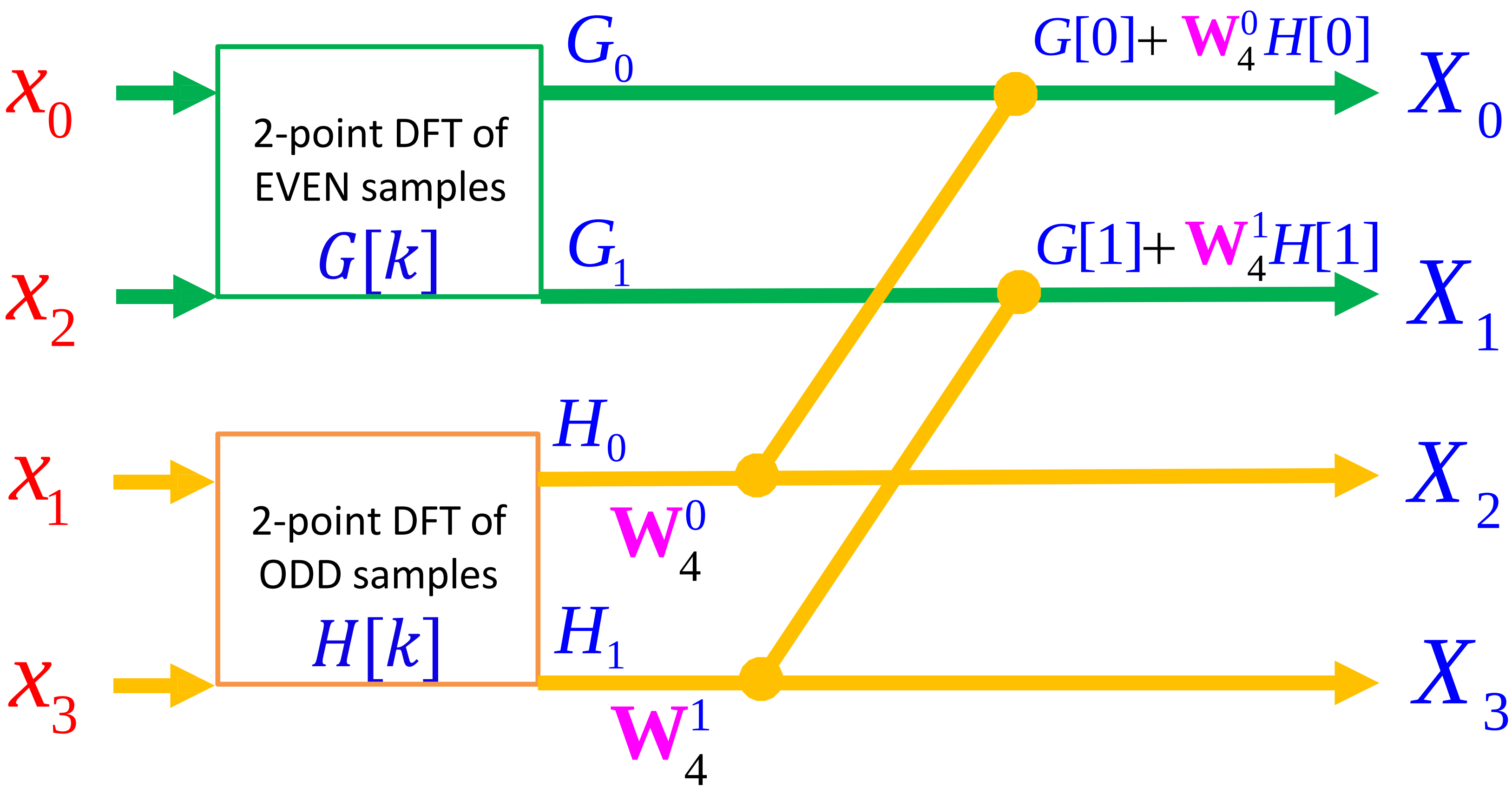

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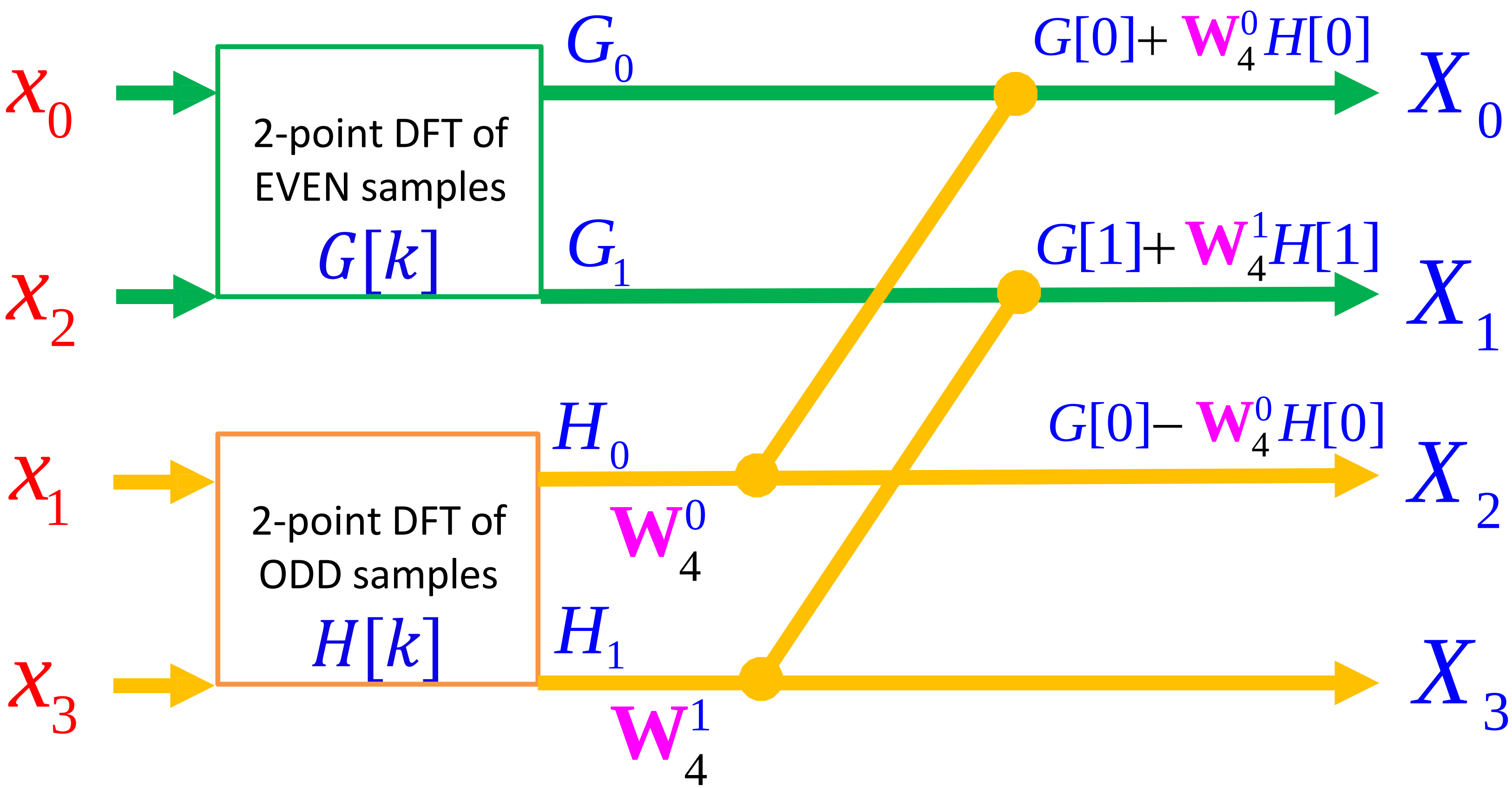

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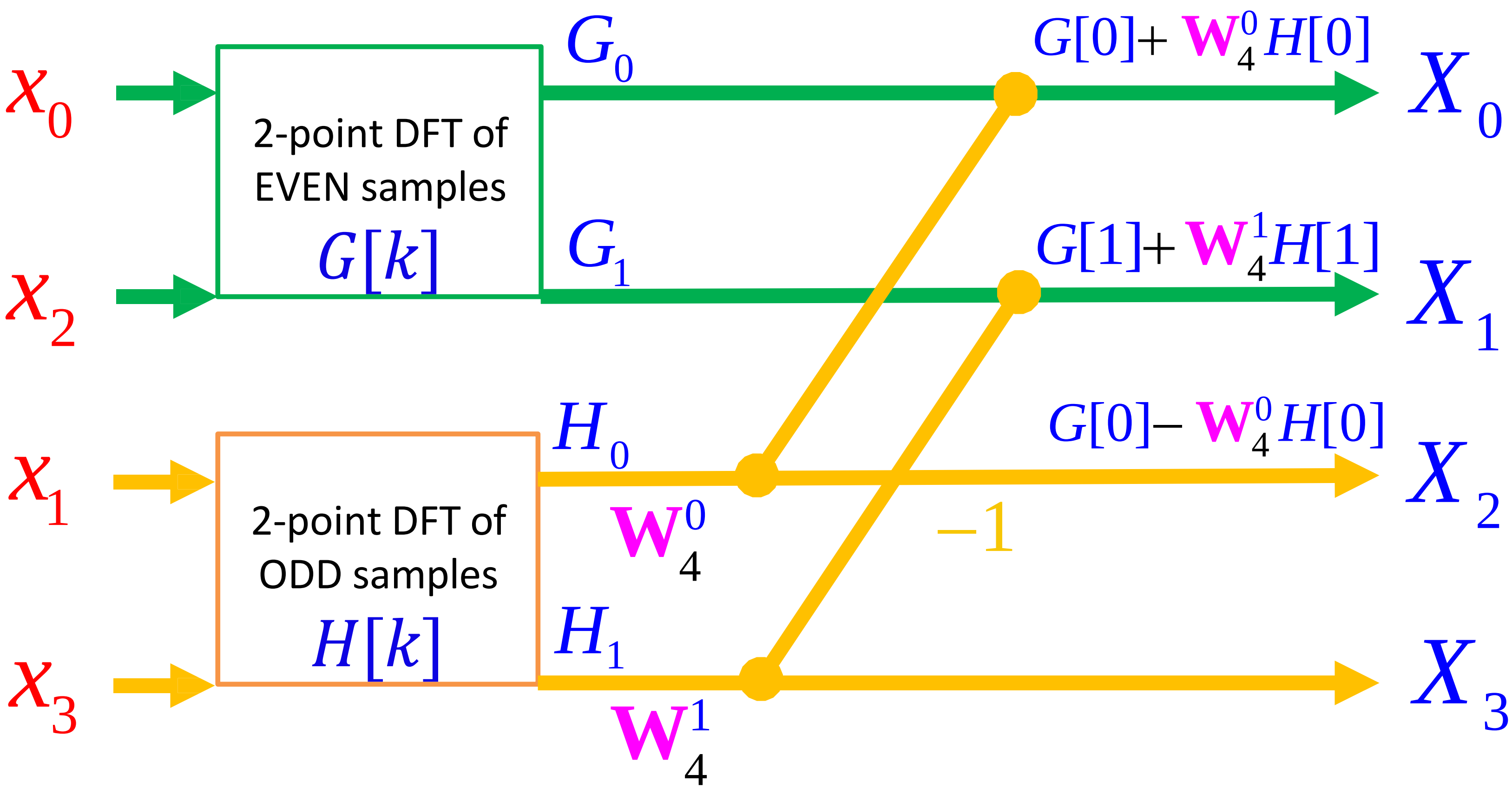

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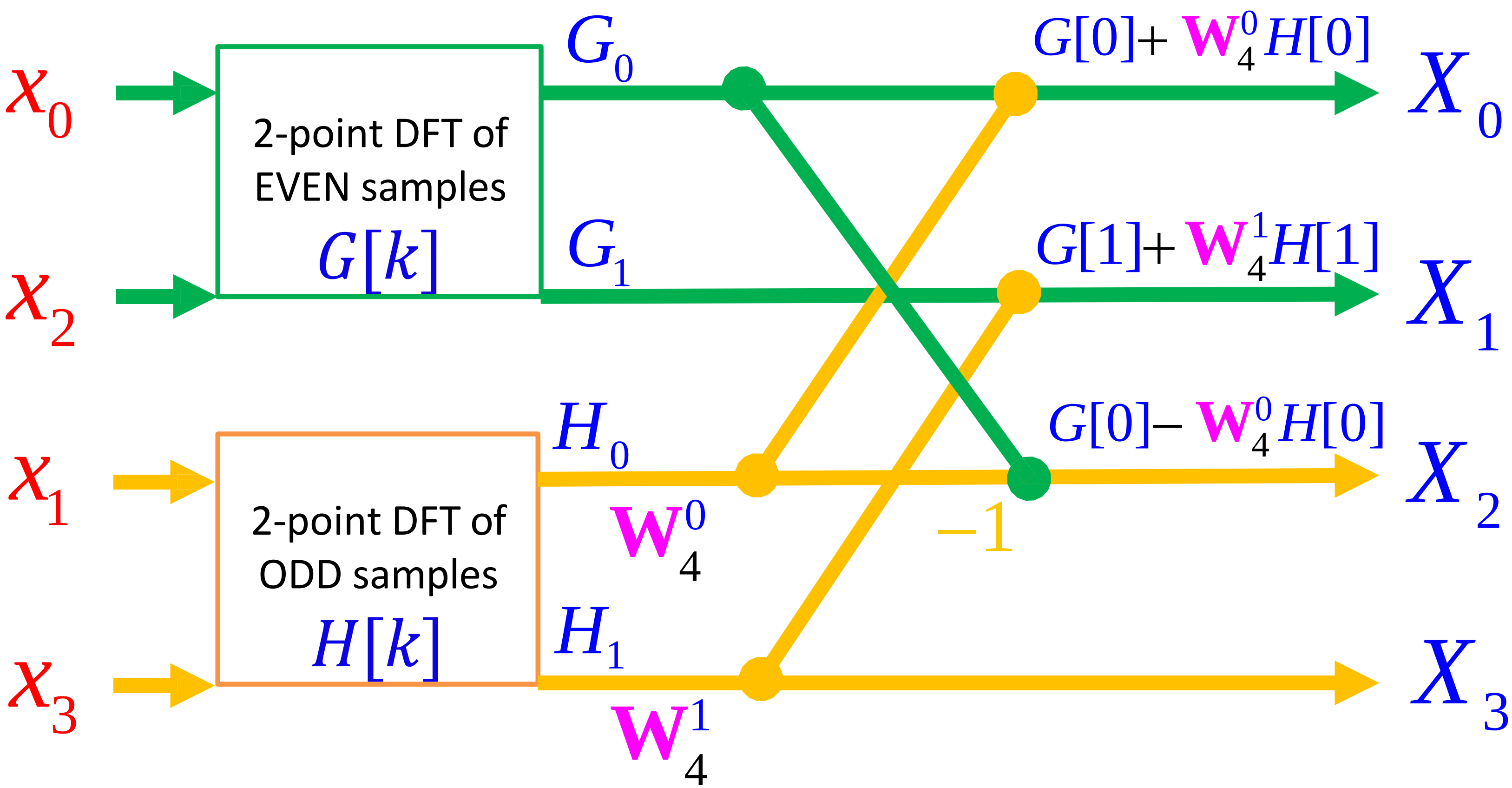

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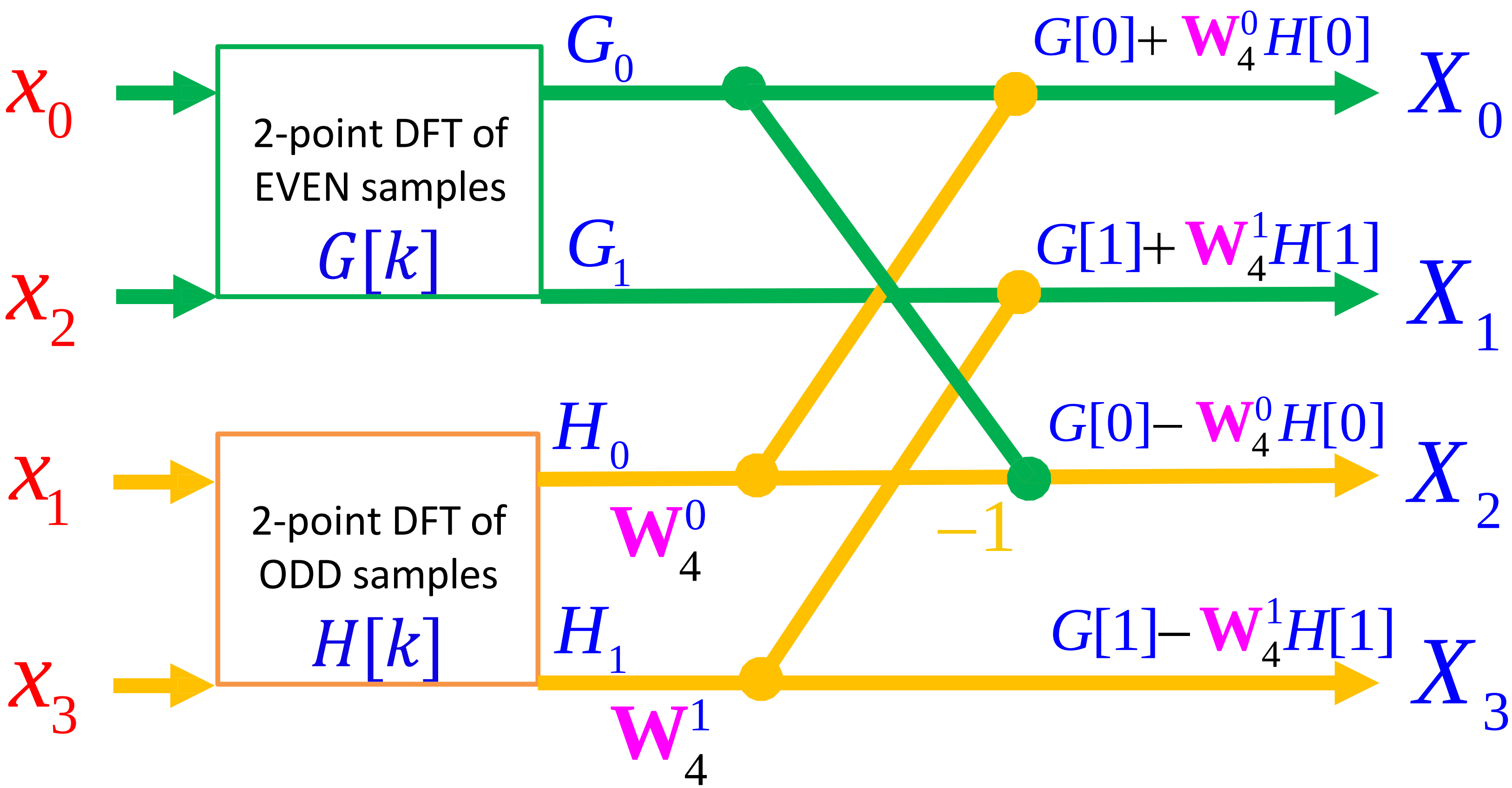

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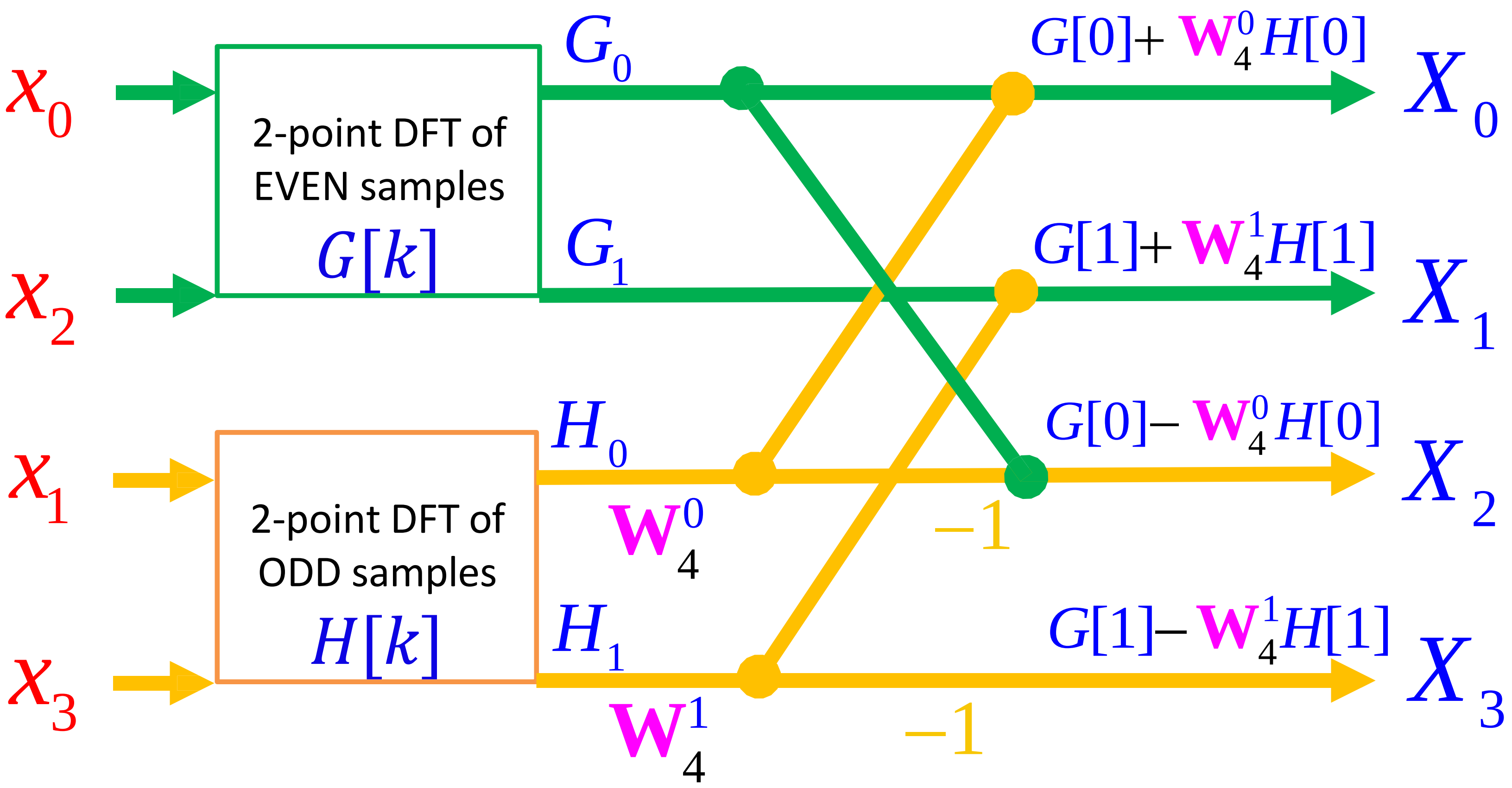

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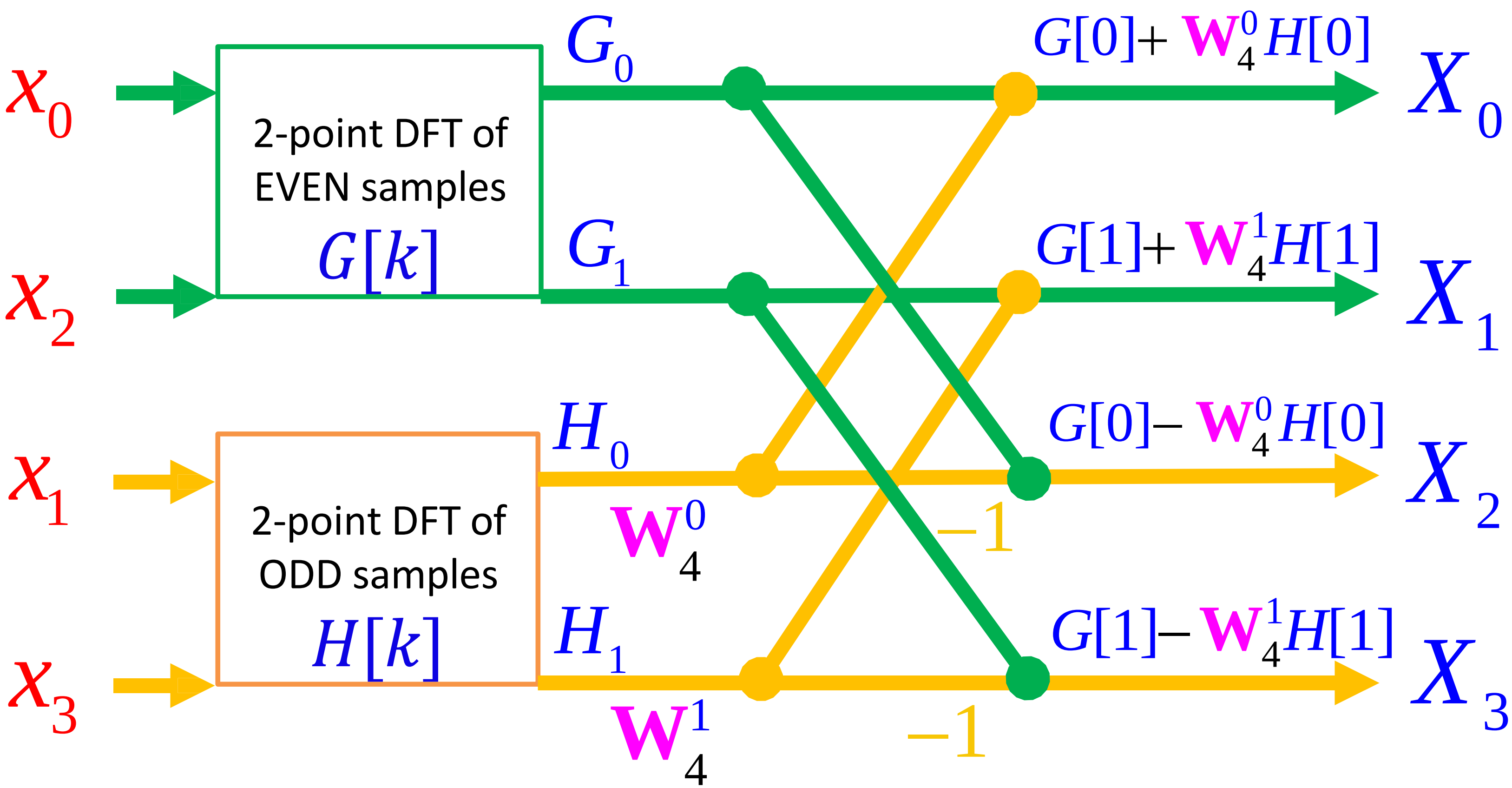


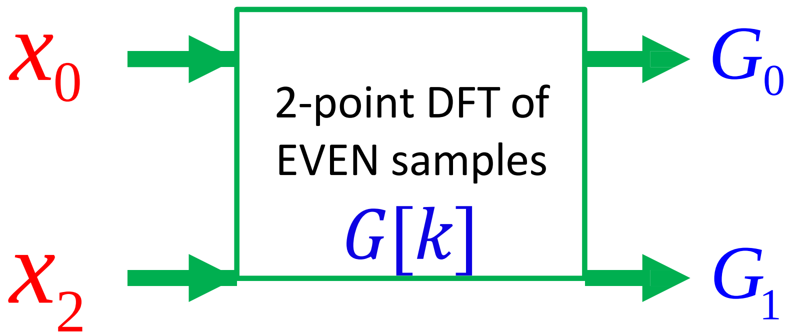
# 4-point DITFFT

# 4-point DITFFT

$x_0$

$x_2$

# 4-point DITFFT



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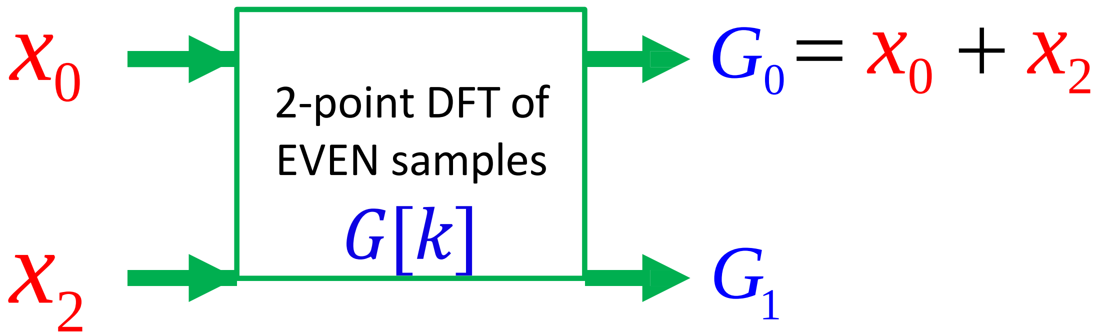

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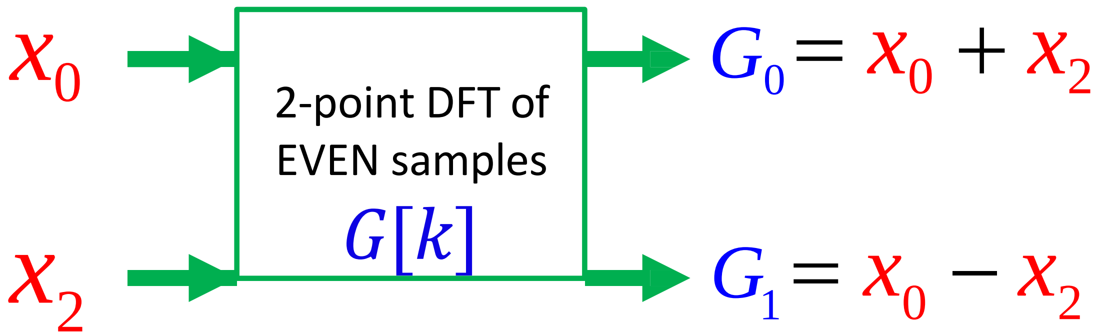

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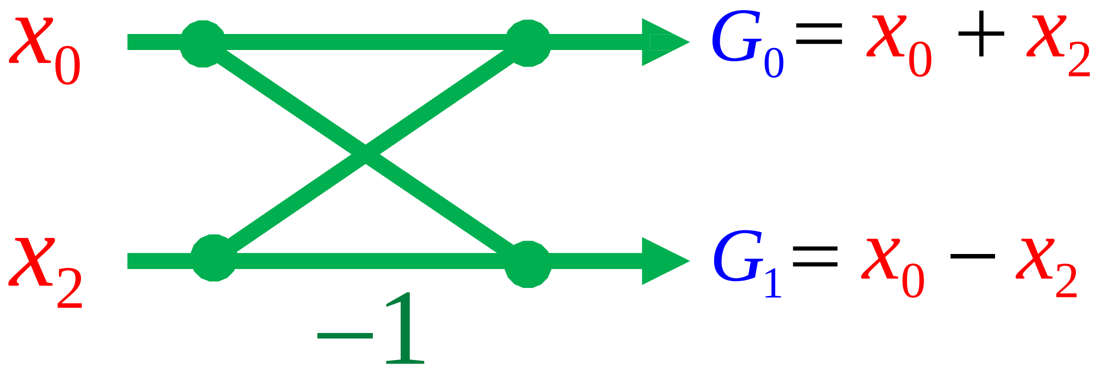
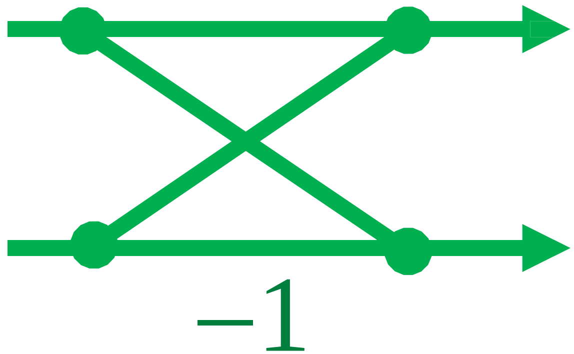


# 4-point DITFFT

$x_0$

$x_2$



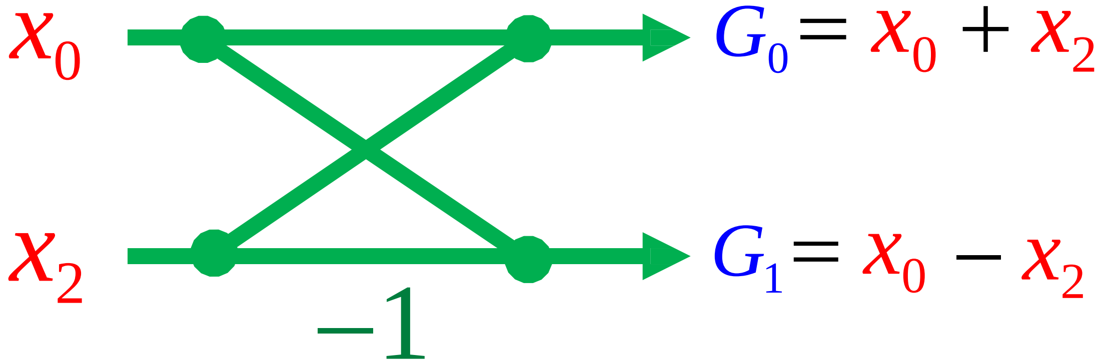

$$G_0 = x_0 + x_2$$

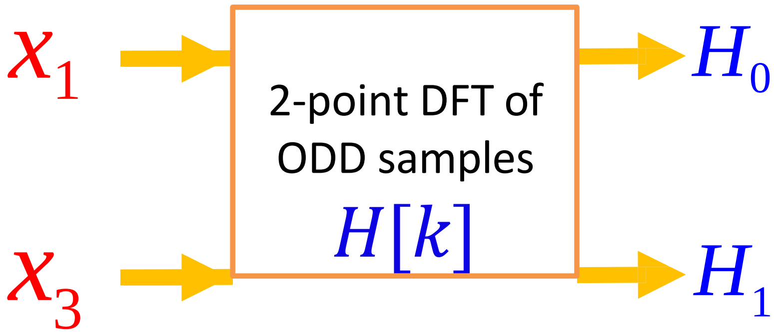
$$G_1 = x_0 - x_2$$

$x_1$

$x_3$

# 4-point DITFFT



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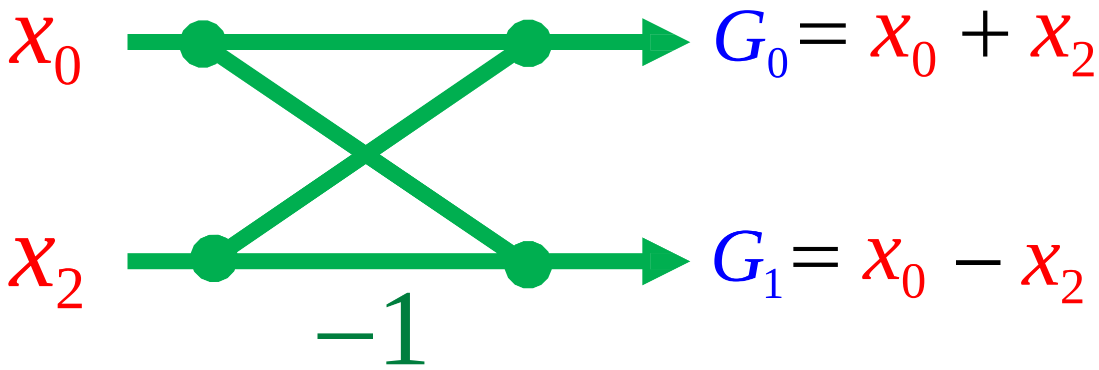
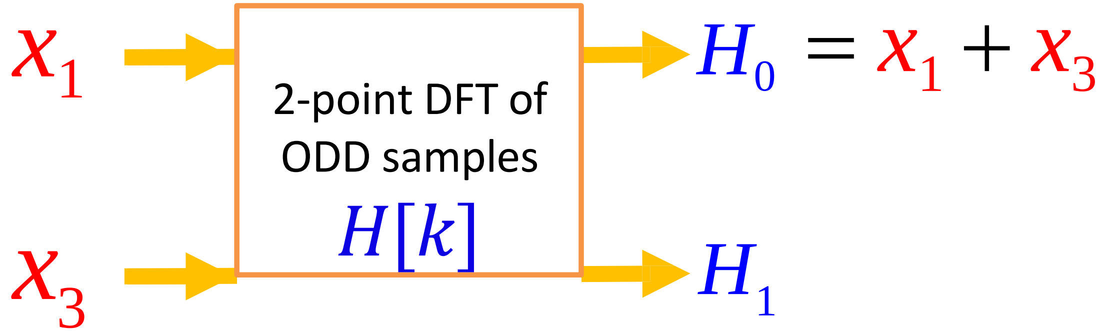
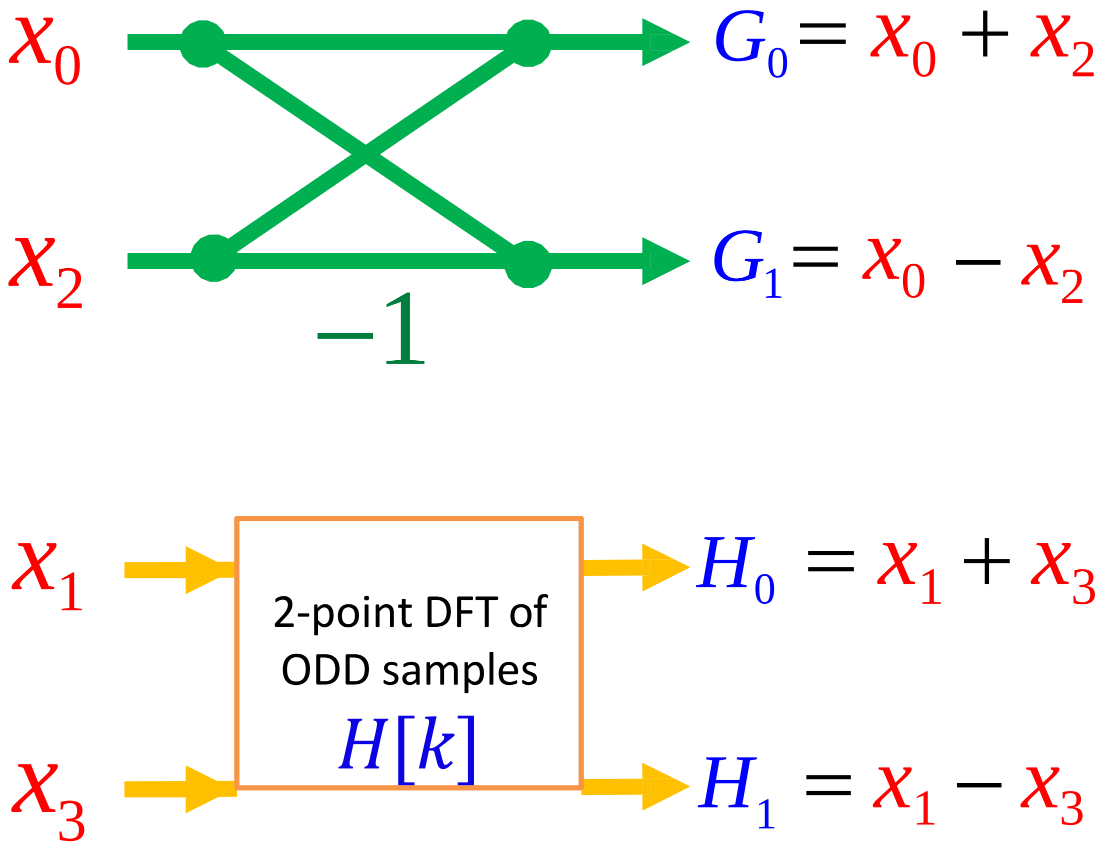
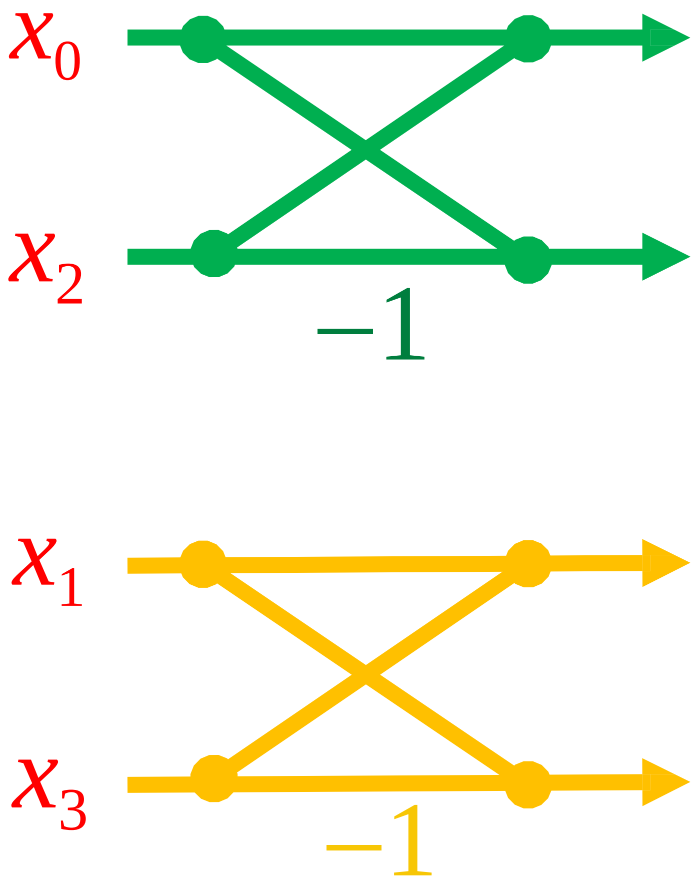


# 4-point DITFFT



# 4-point DITFFT




$$G_0 = x_0 + x_2$$

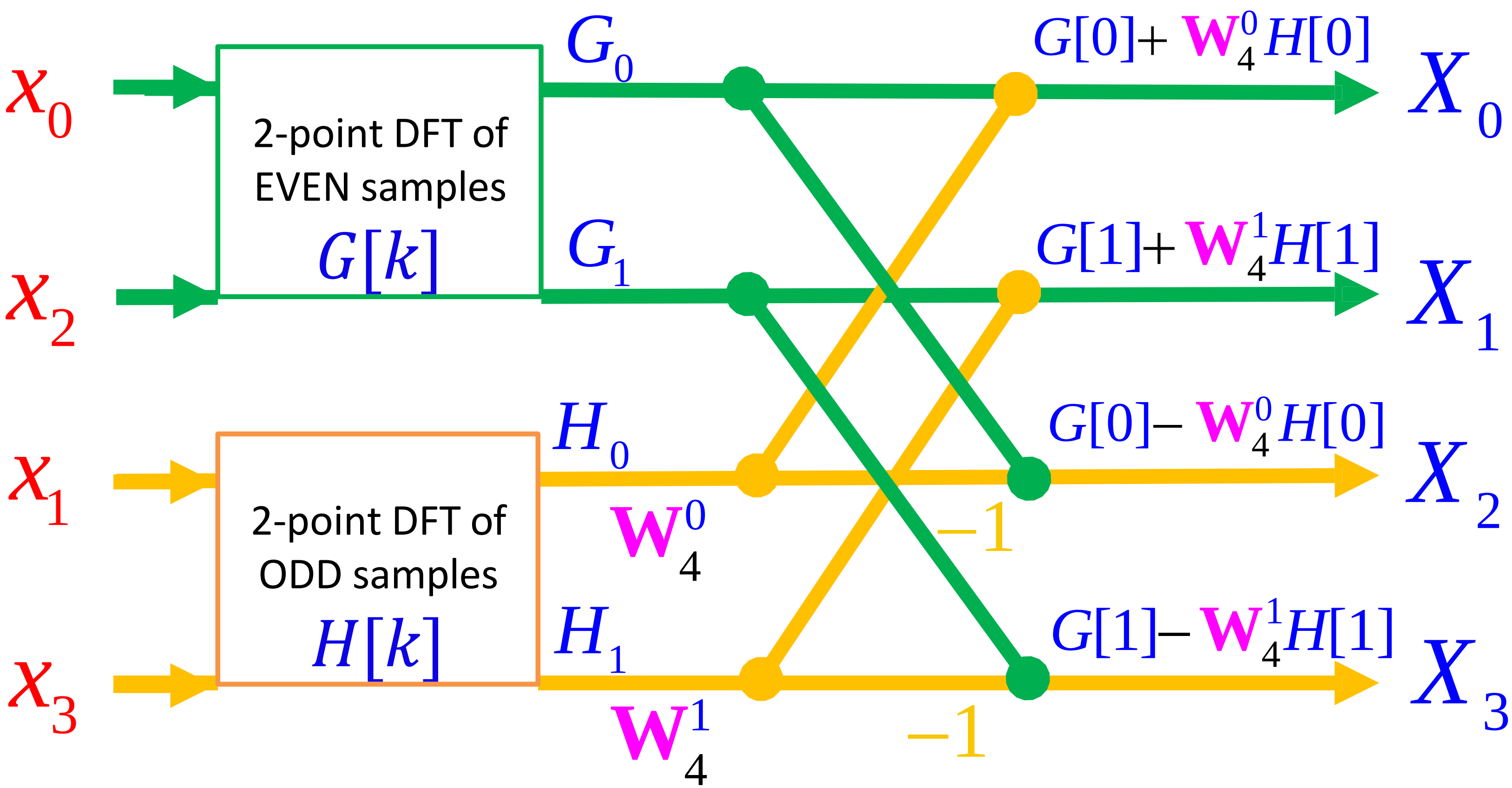
$$G_1 = x_0 - x_2$$

$$H_0 = x_1 + x_3$$

$$H_1 = x_1 - x_3$$

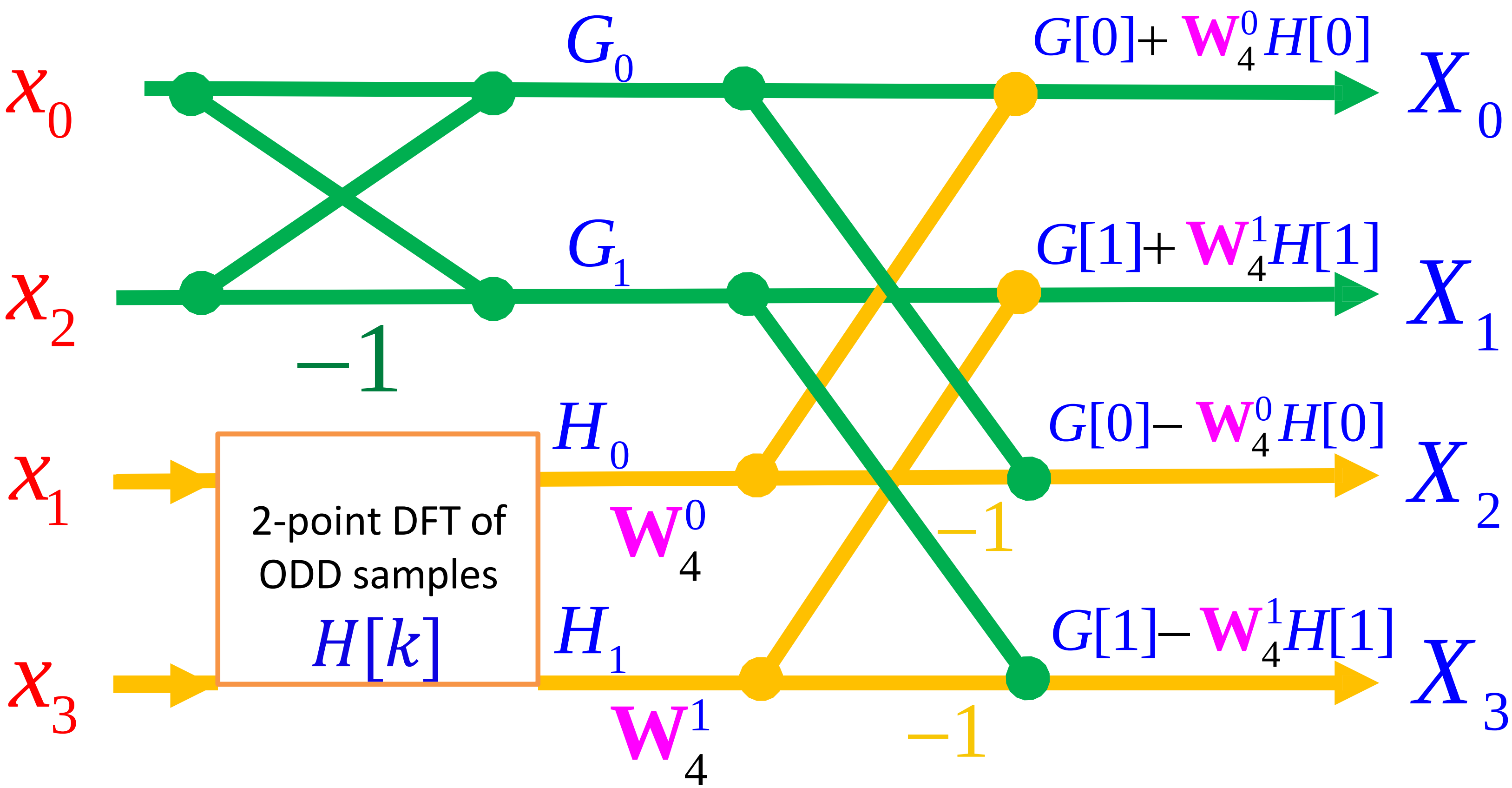
## 4-point DITFFT

- Let $x[n] = \{x_0, x_1, x_2, x_3\}$ and corresponding DFT $X(k) = \{X_0, X_1, X_2, X_3\}$



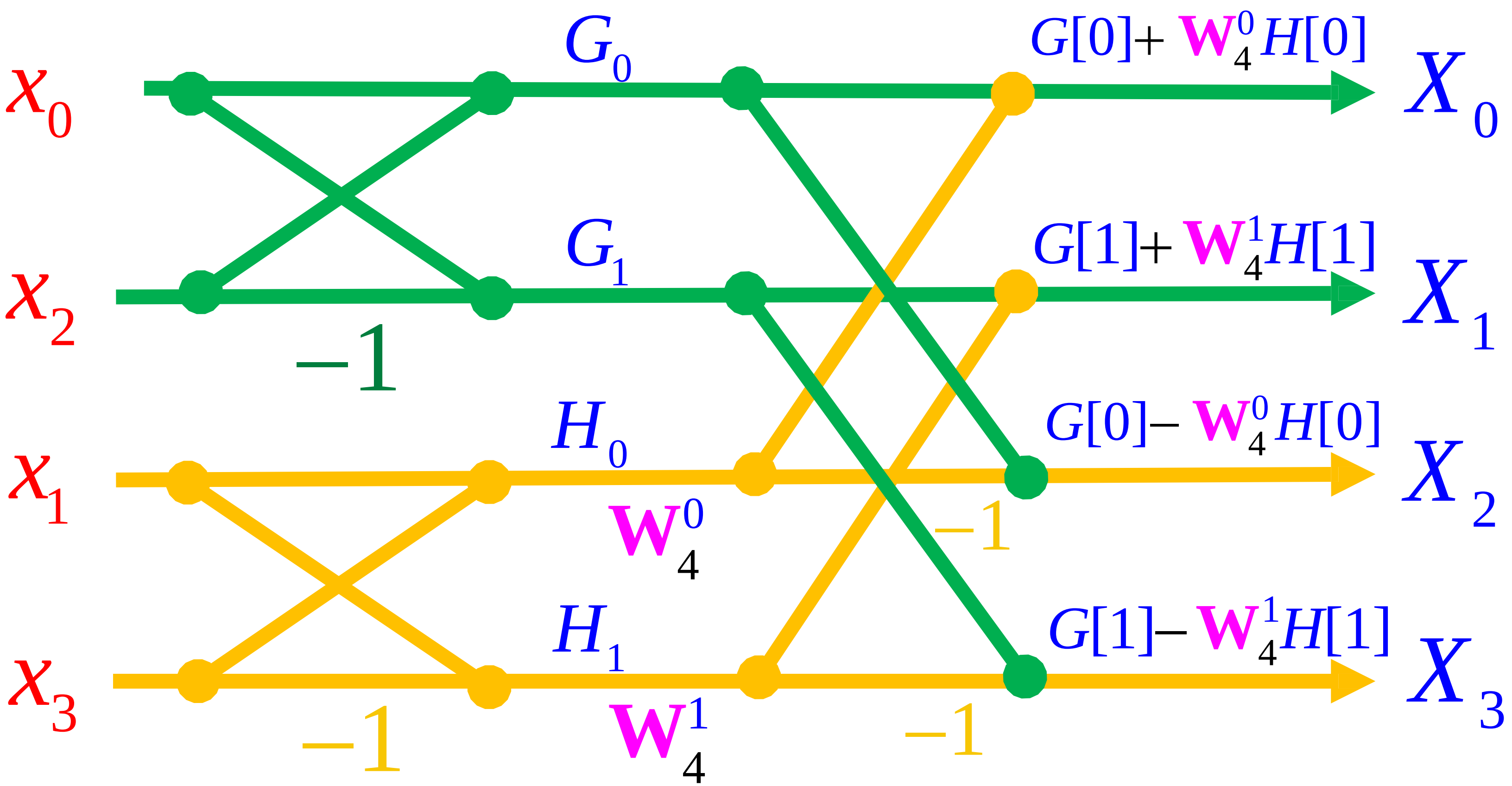
## 4-point DITFFT

- Let $x[n] = \{x_0, x_1, x_2, x_3\}$ and corresponding DFT $X(k) = \{X_0, X_1, X_2, X_3\}$



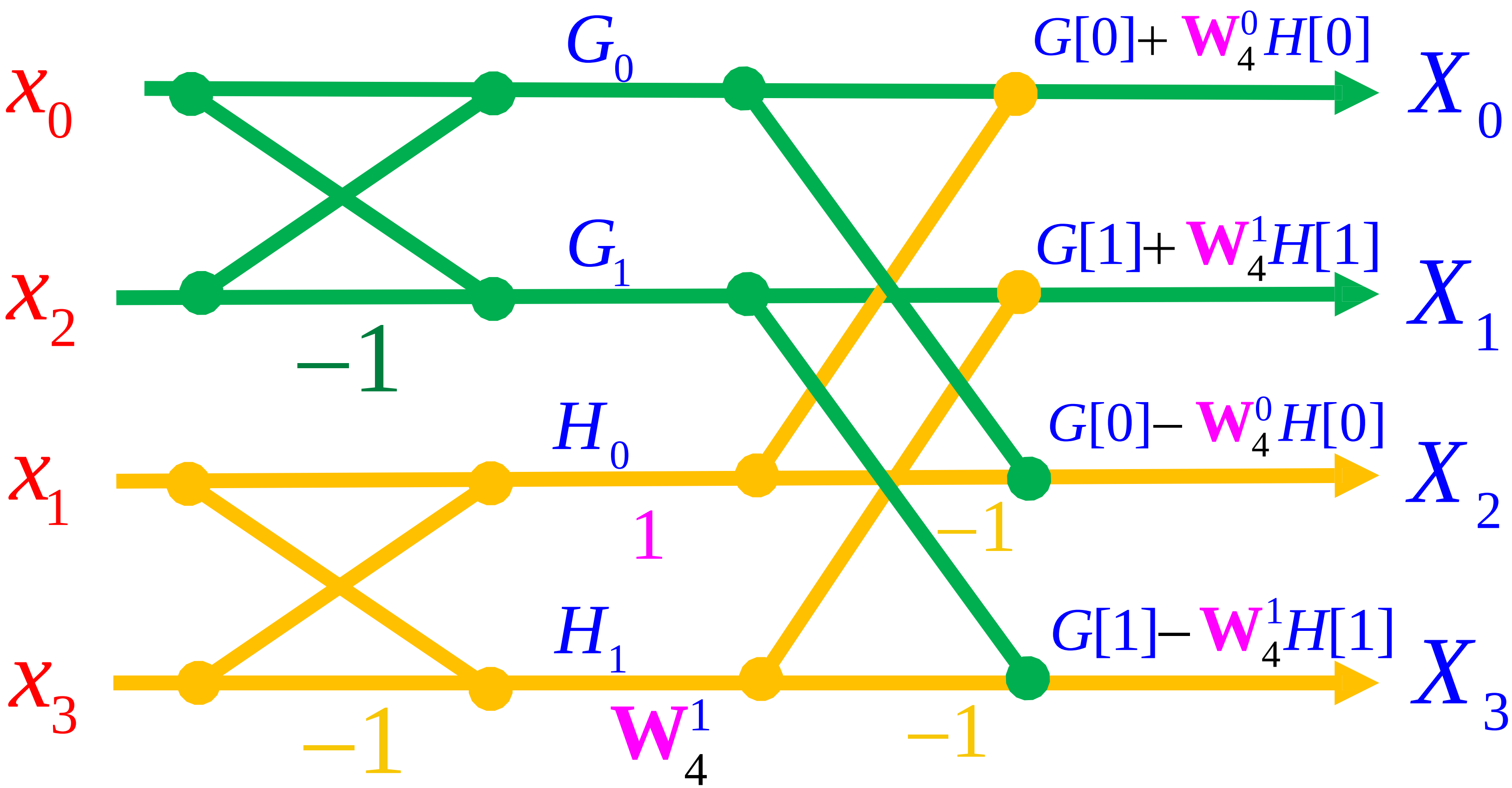
# 4-point DITFFT

- Let $x[n] = \{x_0, x_1, x_2, x_3\}$ and corresponding DFT $X(k) = \{X_0, X_1, X_2, X_3\}$



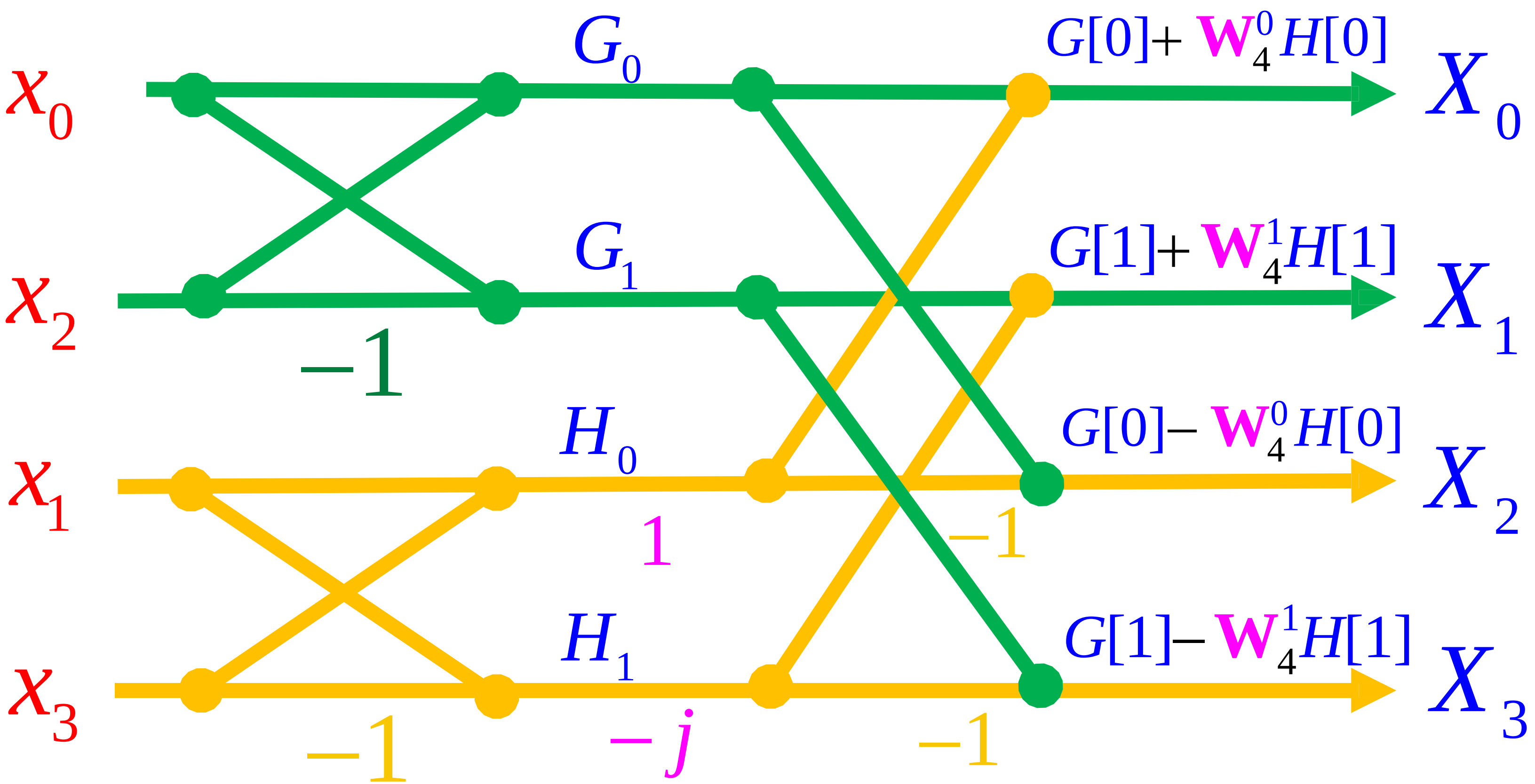
## 4-point DITFFT

- Let $x[n] = \{x_0, x_1, x_2, x_3\}$ and corresponding DFT $X(k) = \{X_0, X_1, X_2, X_3\}$



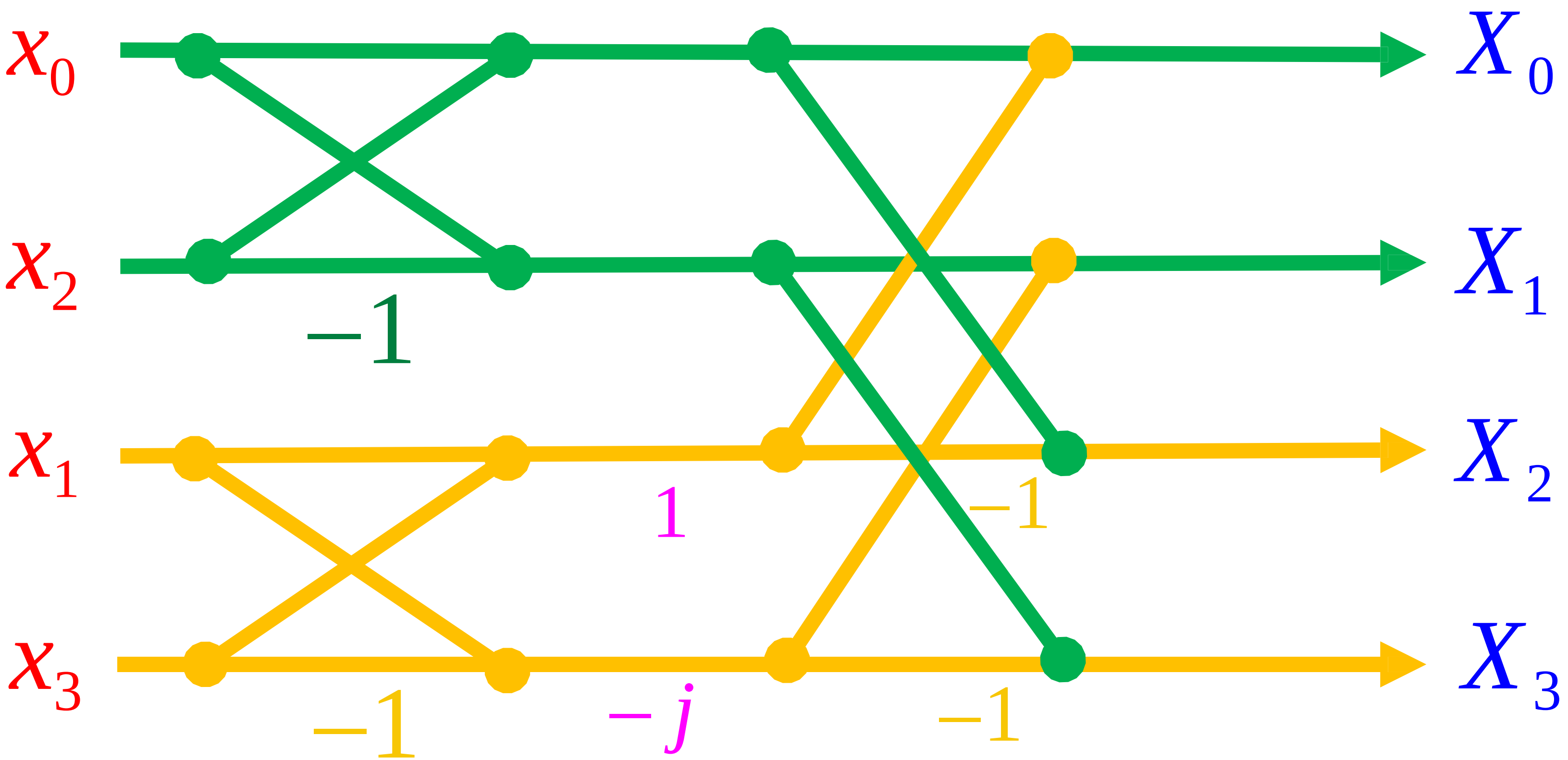
## 4-point DITFFT

- Let $x[n] = \{x_0, x_1, x_2, x_3\}$ and corresponding DFT $X(k) = \{X_0, X_1, X_2, X_3\}$



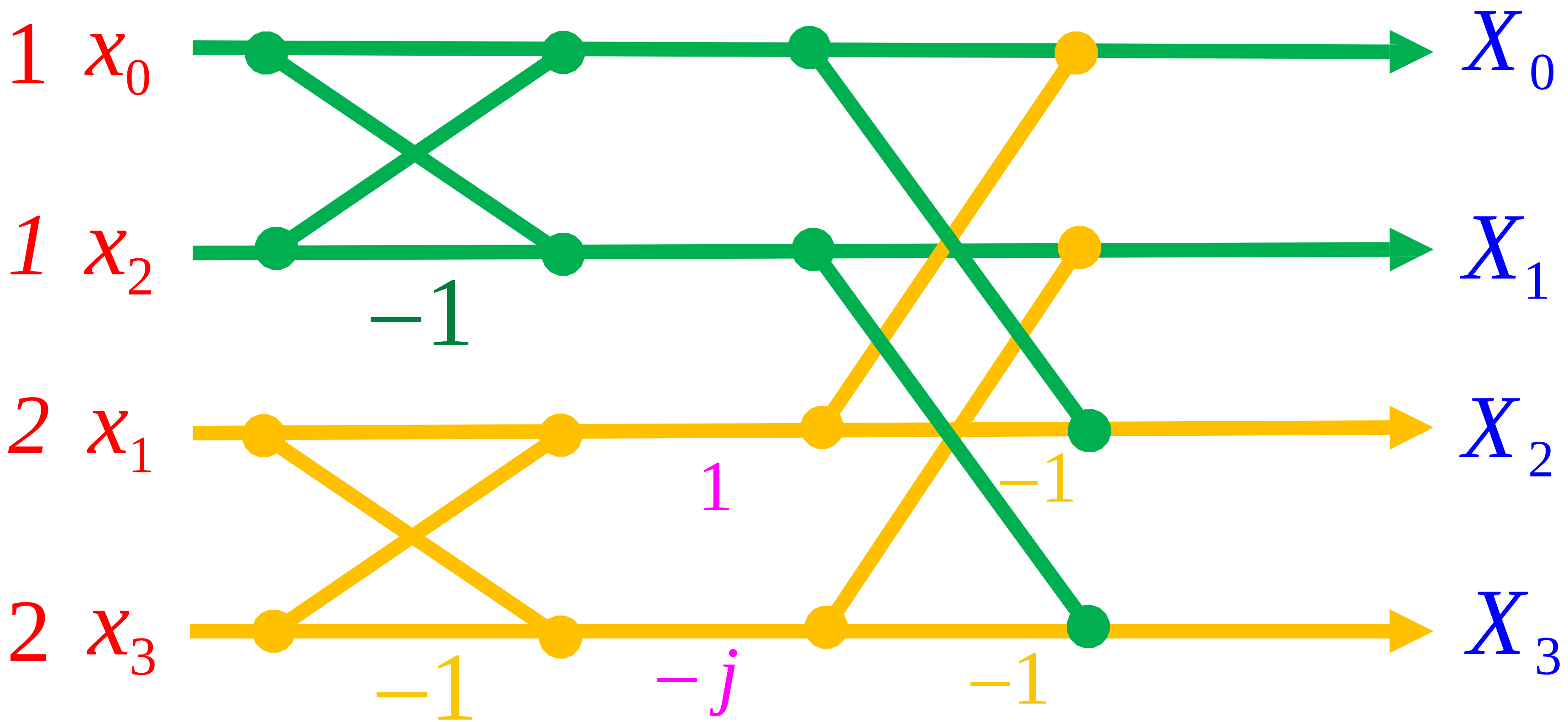
## Example

- Find DFT of $x[n] = \{1, 2, 1, 2\}$ using DITFFT (DEC18, 4marks)



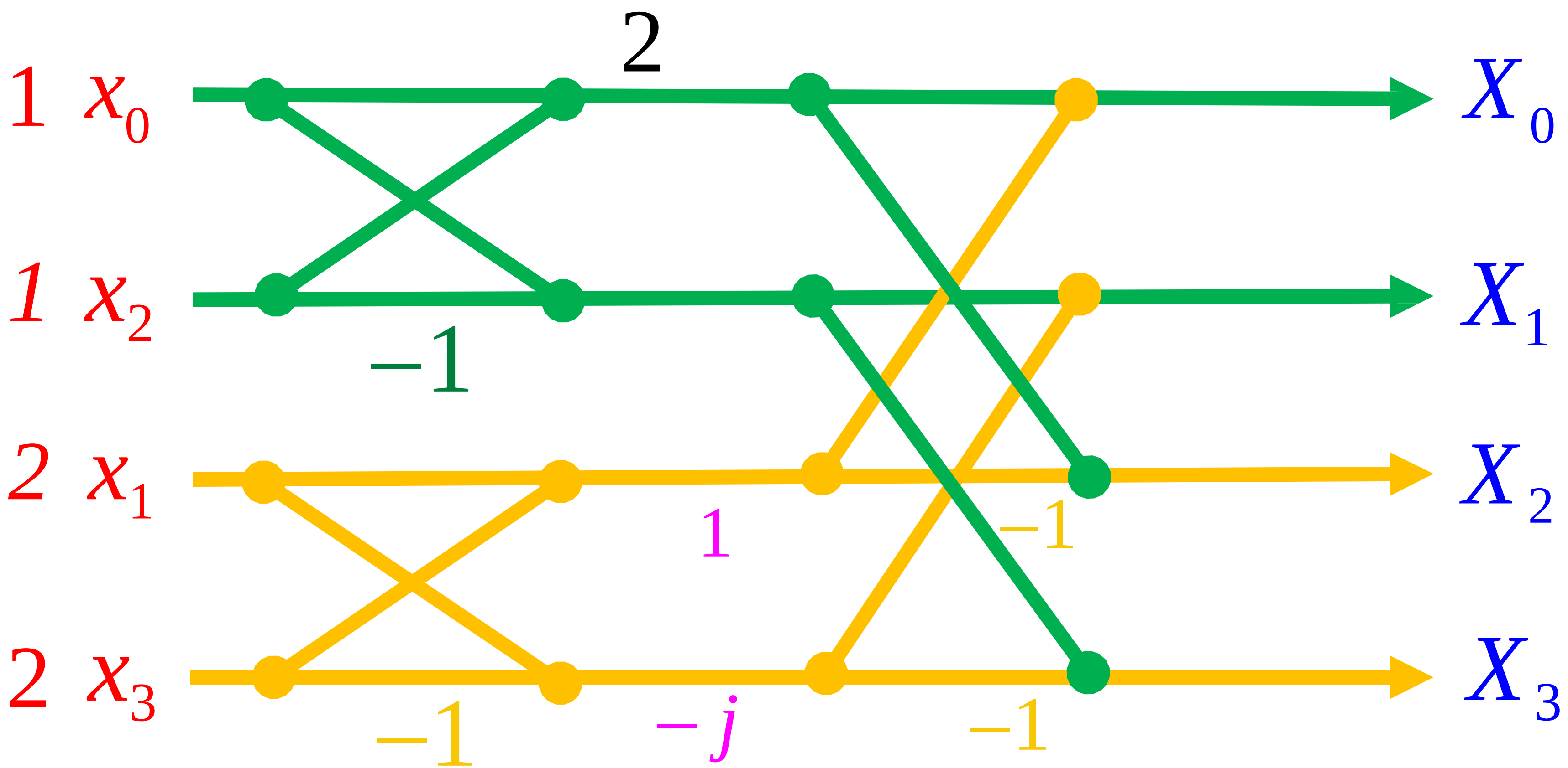
## Example

- Find DFT of $x[n] = \{1, 2, 1, 2\}$ using DITFFT



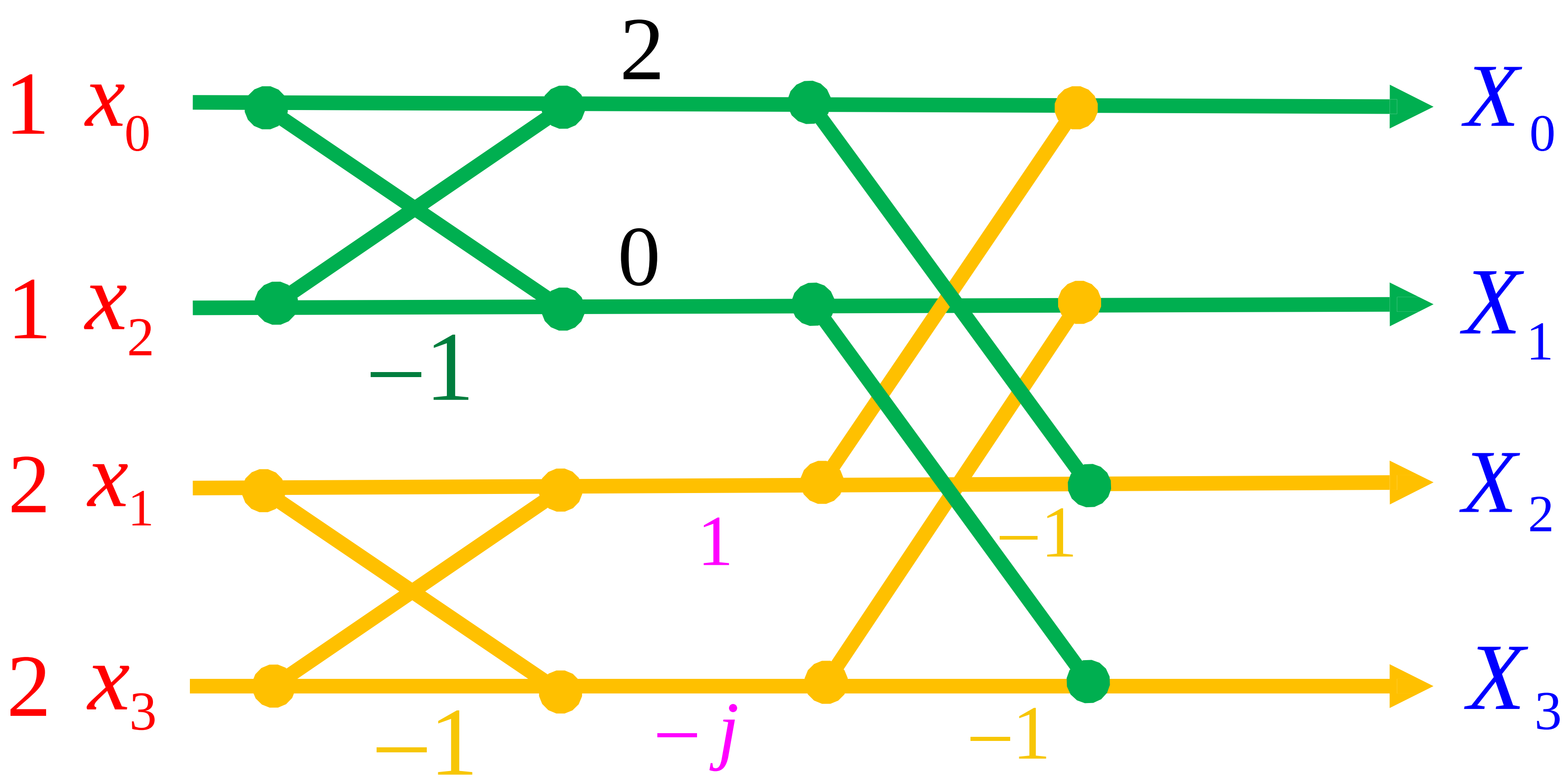
## Example

- Find DFT of $x[n] = \{1, 2, 1, 2\}$ using DITFFT



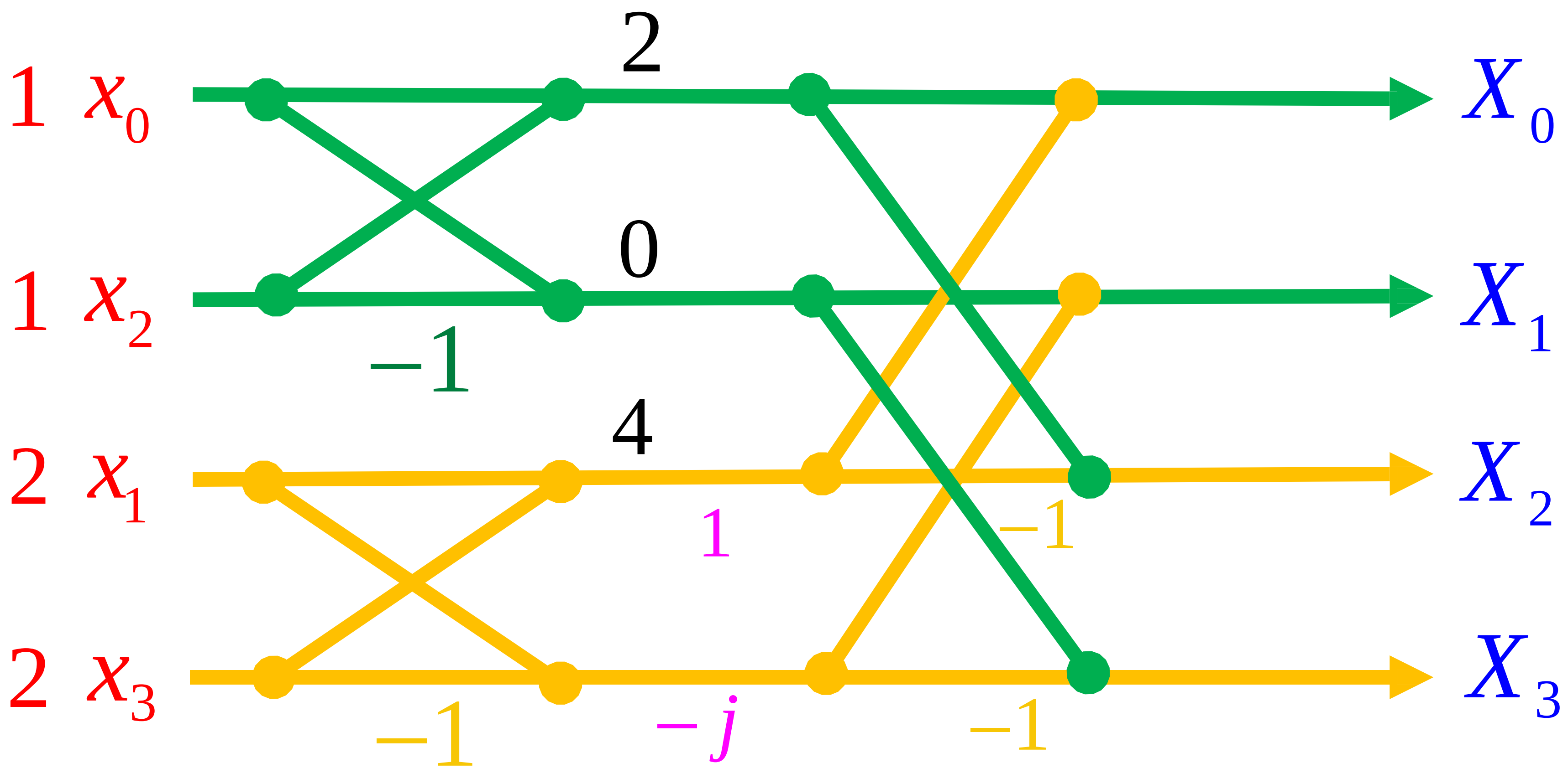
## Example

- Find DFT of $x[n] = \{1, 2, 1, 2\}$ using DITFFT

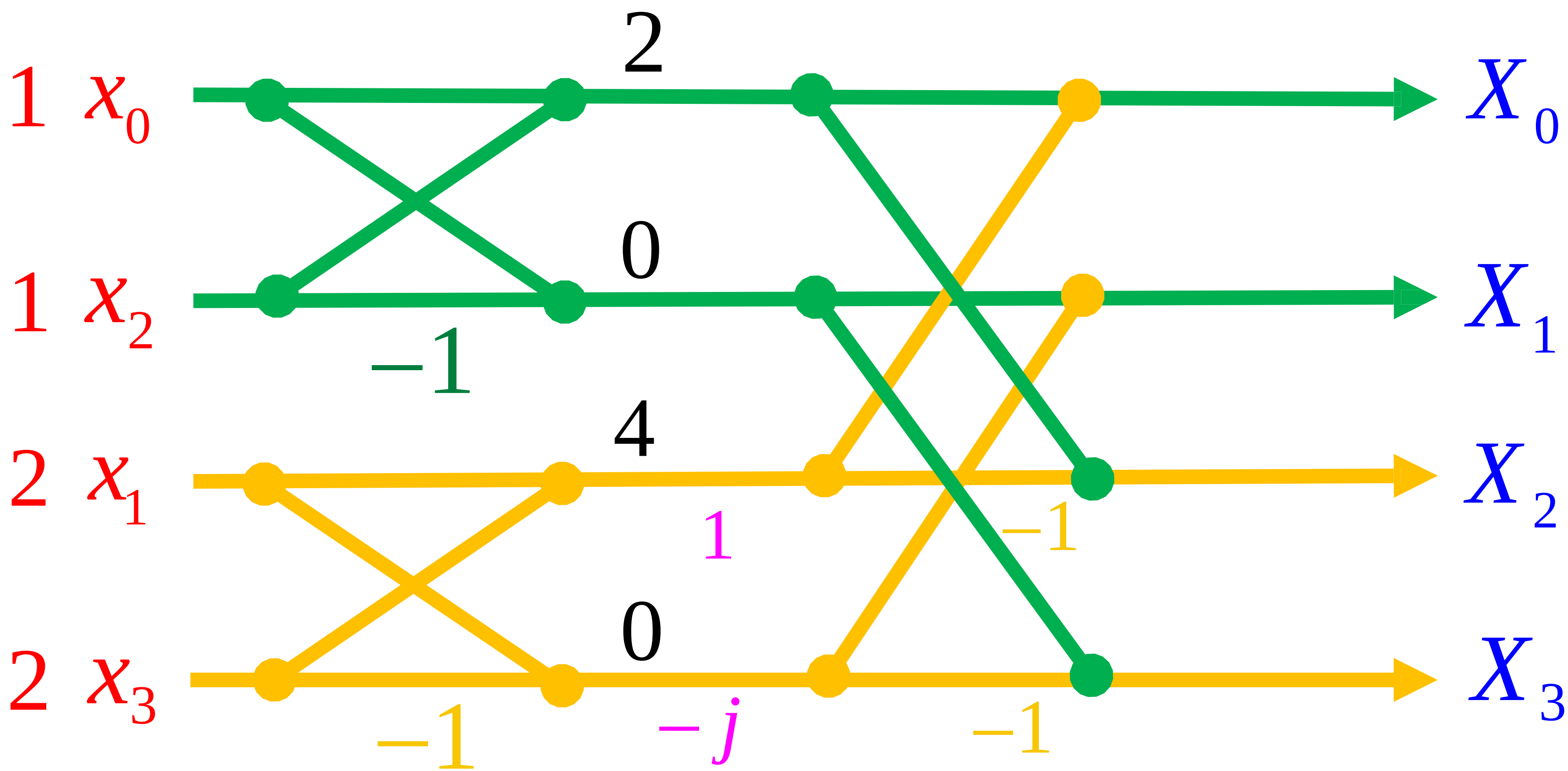


## Example

- Find DFT of $x[n] = \{1, 2, 1, 2\}$ using DITFFT

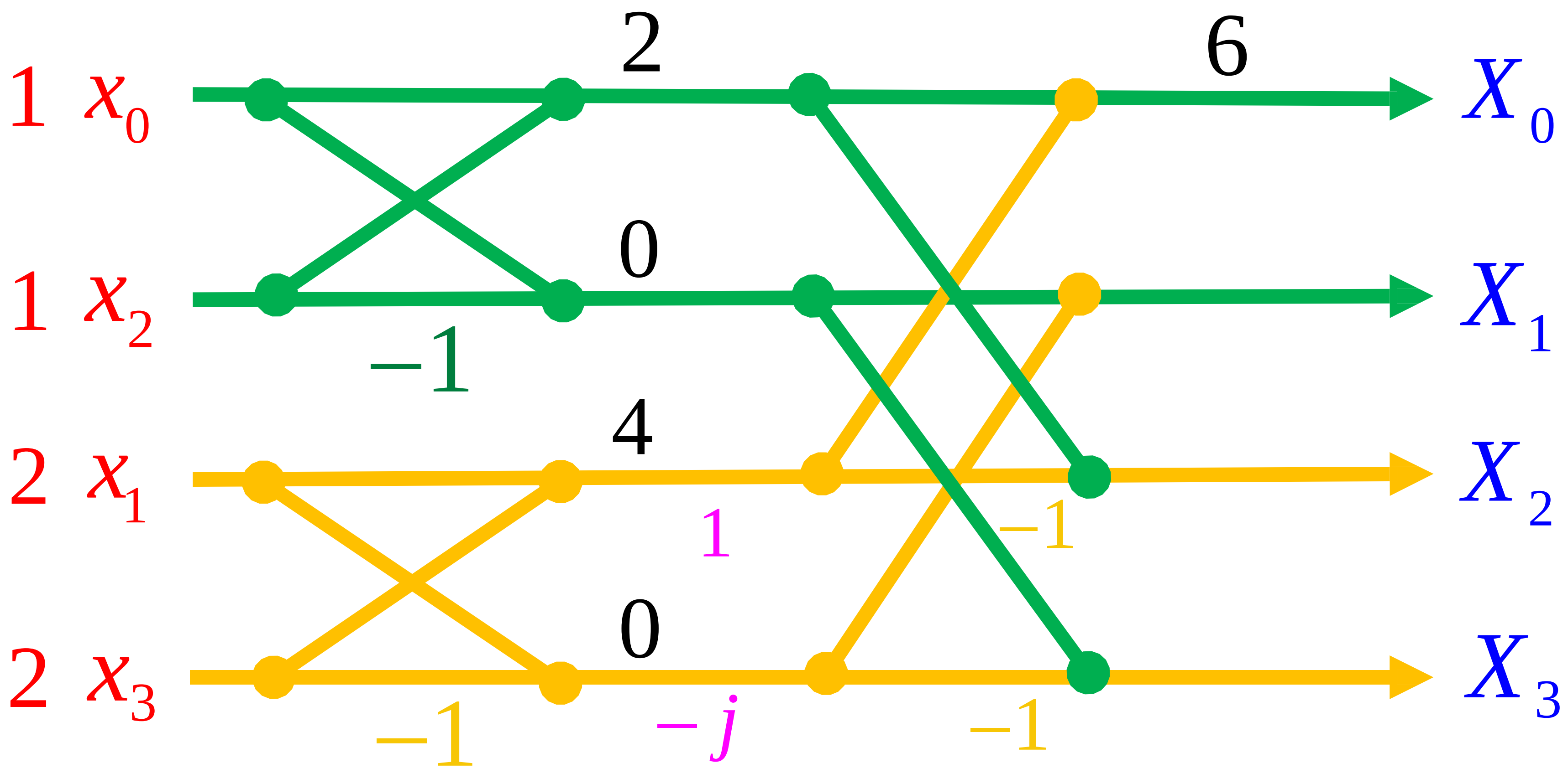


- Find DFT of $x[n] = \{1, 2, 1, 2\}$ using DITFFT



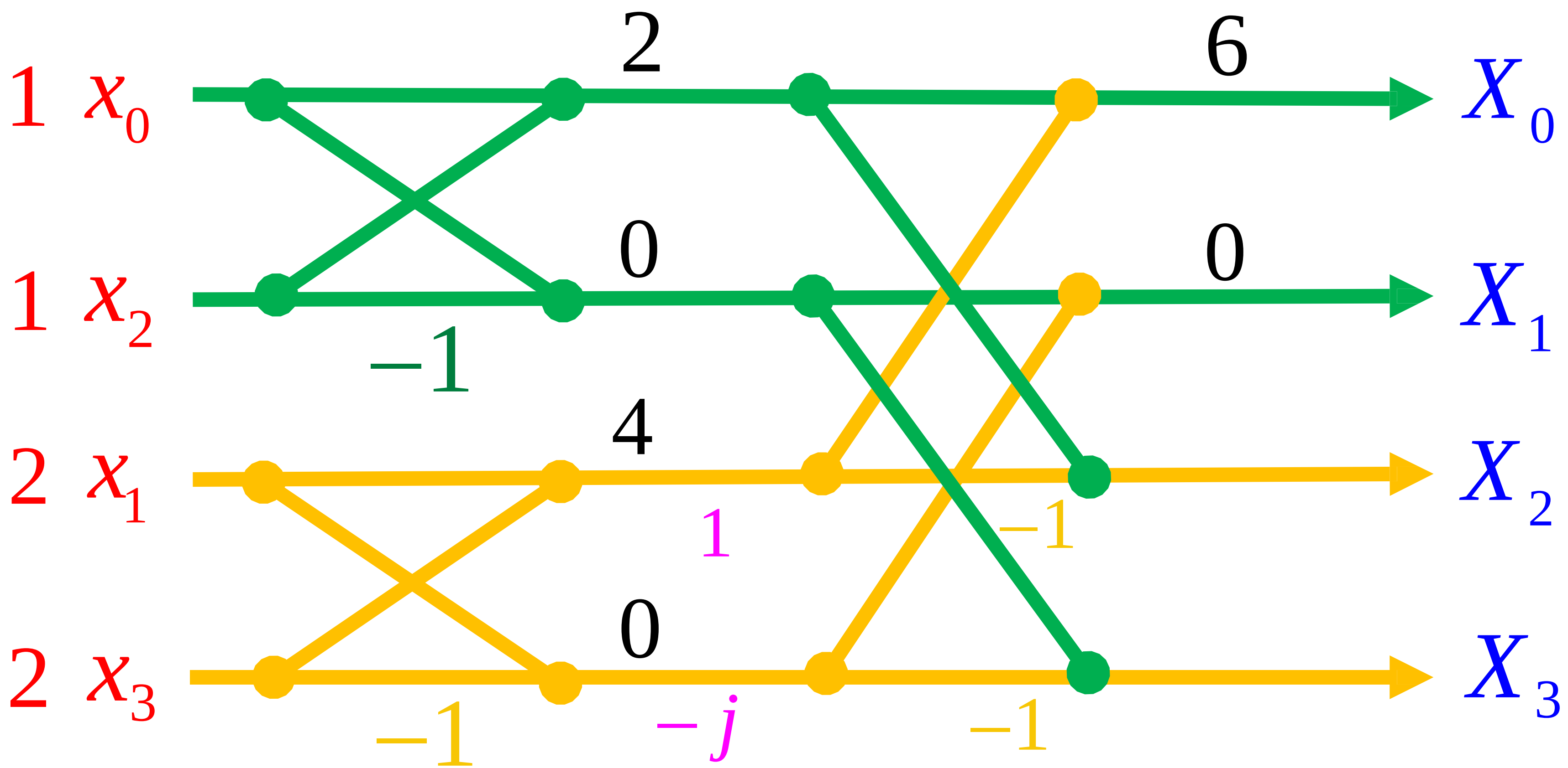
## Example

- Find DFT of $x[n] = \{1, 2, 1, 2\}$ using DITFFT



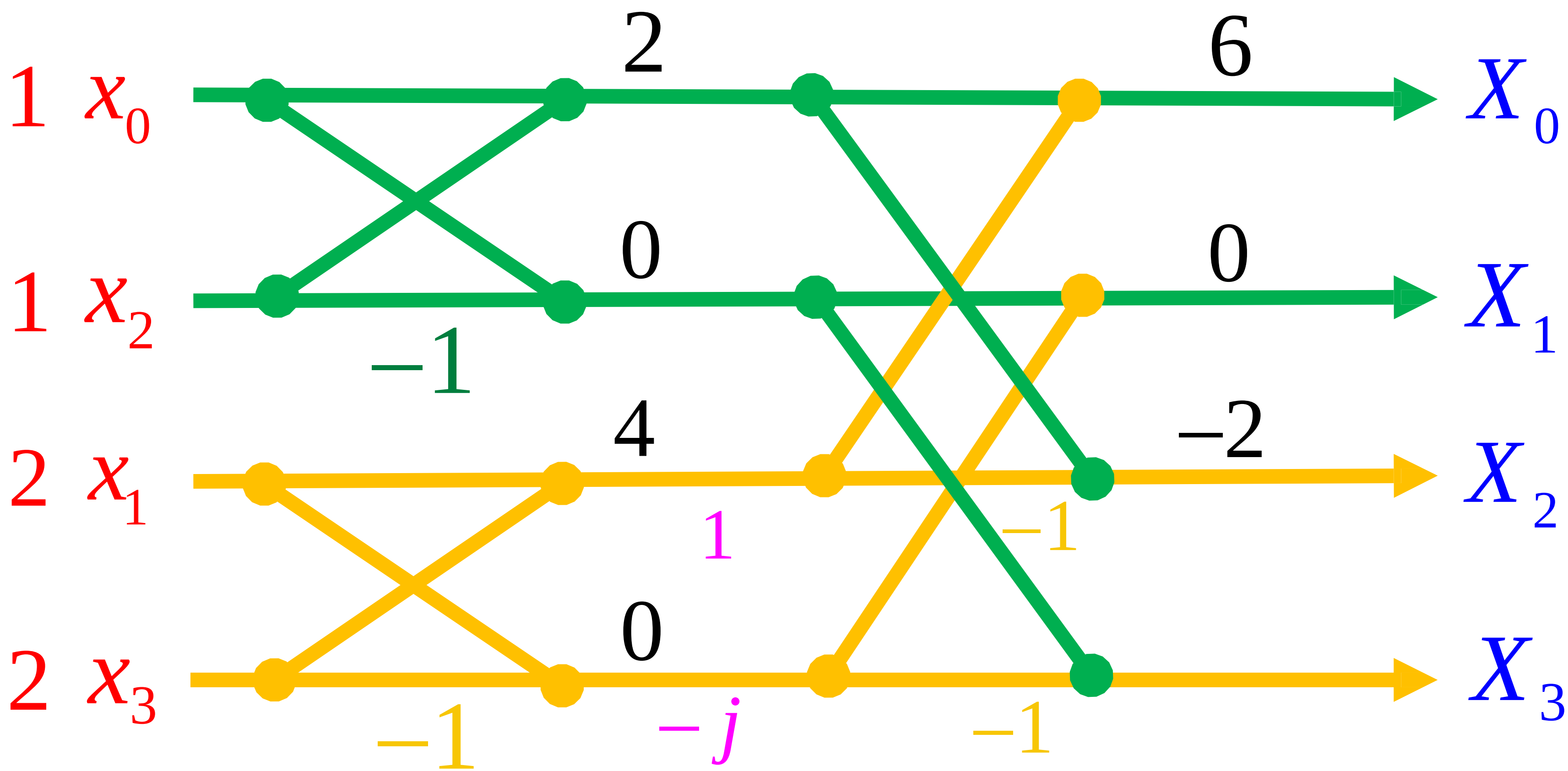
## Example

- Find DFT of $x[n] = \{1, 2, 1, 2\}$ using DITFFT)



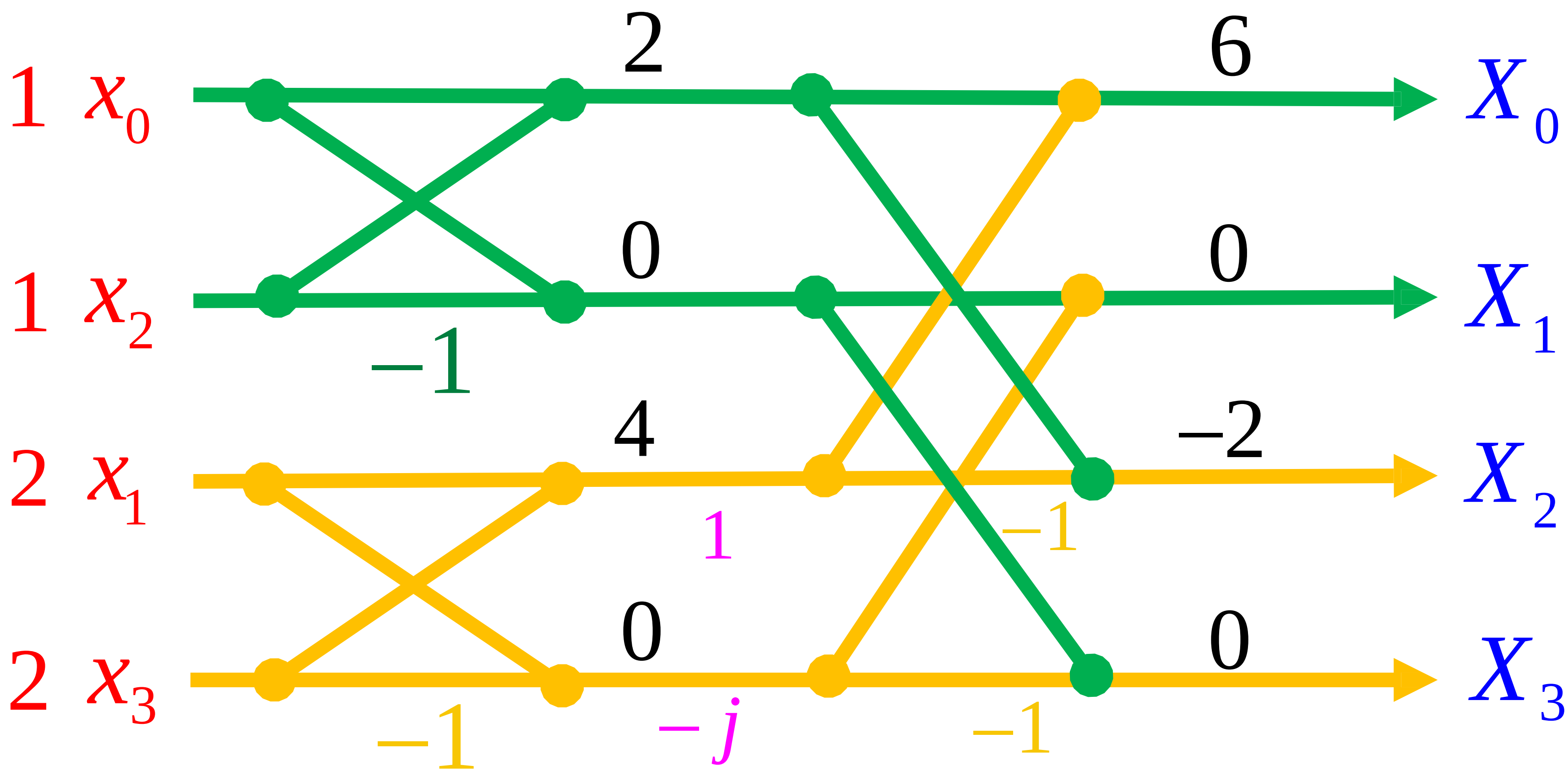
## Example

- Find DFT of $x[n] = \{1, 2, 1, 2\}$ using DITFFT

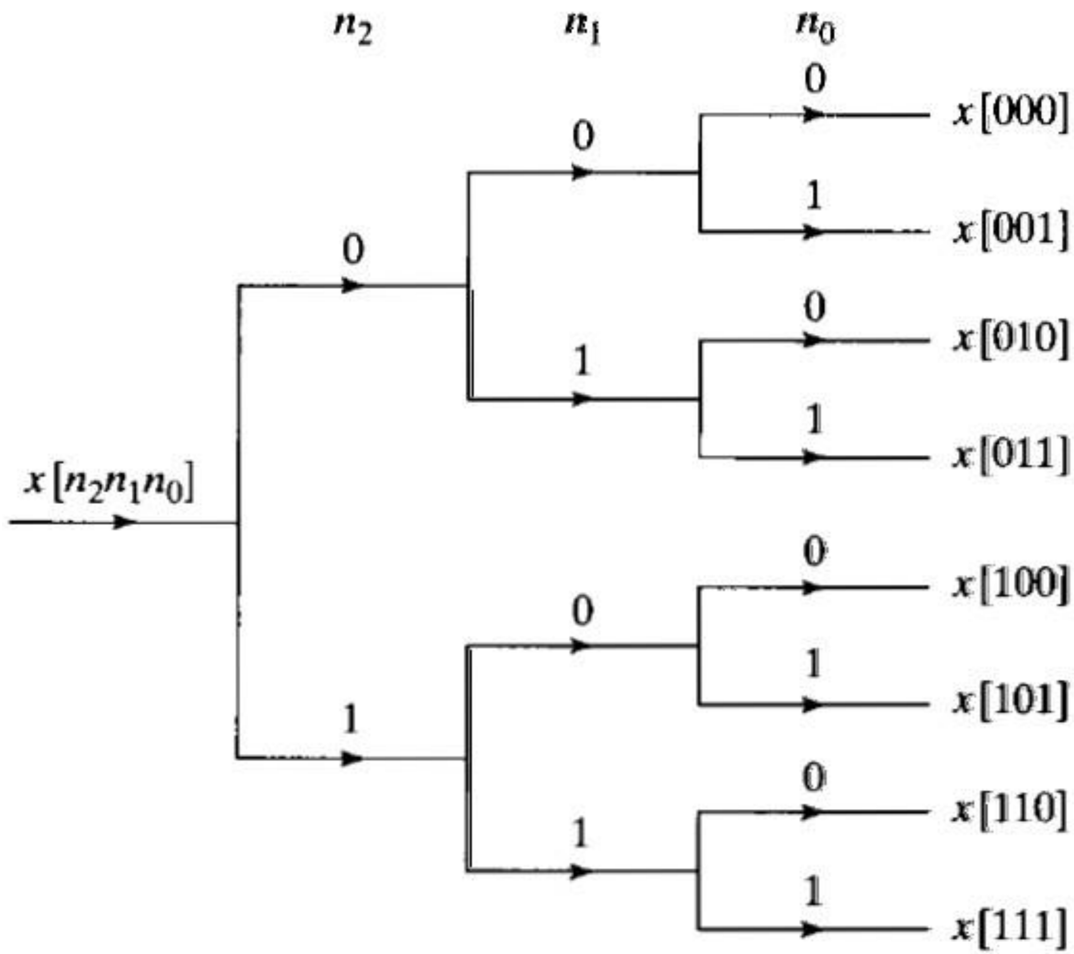


## Example

- Find DFT of $x[n] = \{1, 2, 1, 2\}$ using DITFFT



# Bit-reversed order




$$X_0[000] = x[000]$$
$$X_0[001] = x[100]$$
$$X_0[010] = x[010]$$
$$X_0[011] = x[110]$$
$$X_0[100] = x[001]$$
$$X_0[101] = x[101]$$
$$X_0[110] = x[011]$$
$$X_0[111] = x[111]$$

Tree diagram depicting normal order sorting *[Source: Chapter 9, Digital Signal Processing* by Prokakis]

# 8-point DITFFT

- Let $x[n] = \{x_0,\ x_1\ , x_2\ , x_3, x_4, x_5, x_6, x_7\} \xrightarrow{DFT} X(k) = \{X_0, X_1, X_2, X_3, X_4, X_5, X_6, X_7\}$

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$x_0$

$x_4$

$x_2$

$x_6$

$x_1$

$x_5$

$x_3$

$x_7$

# 8-point DITFFT

- Let $x[n] = \{x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7\} \xrightarrow{DFT} X(k) = \{X_0, X_1, X_2, X_3, X_4, X_5, X_6, X_7\}$

| | |
|---|---|
| $x_0$ | $X_0$ |
| $x_4$ | $X_1$ |
| $x_2$ | $X_2$ |
| $x_6$ | $X_3$ |
| $x_1$ | $X_4$ |
| $x_5$ | $X_5$ |
| $x_3$ | $X_6$ |
| $x_7$ | $X_7$ |

## 8-point DITFFT

- Let $x[n] = \{x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7\} \xrightarrow{DFT} X(k) = \{X_0, X_1, X_2, X_3, X_4, X_5, X_6, X_7\}$

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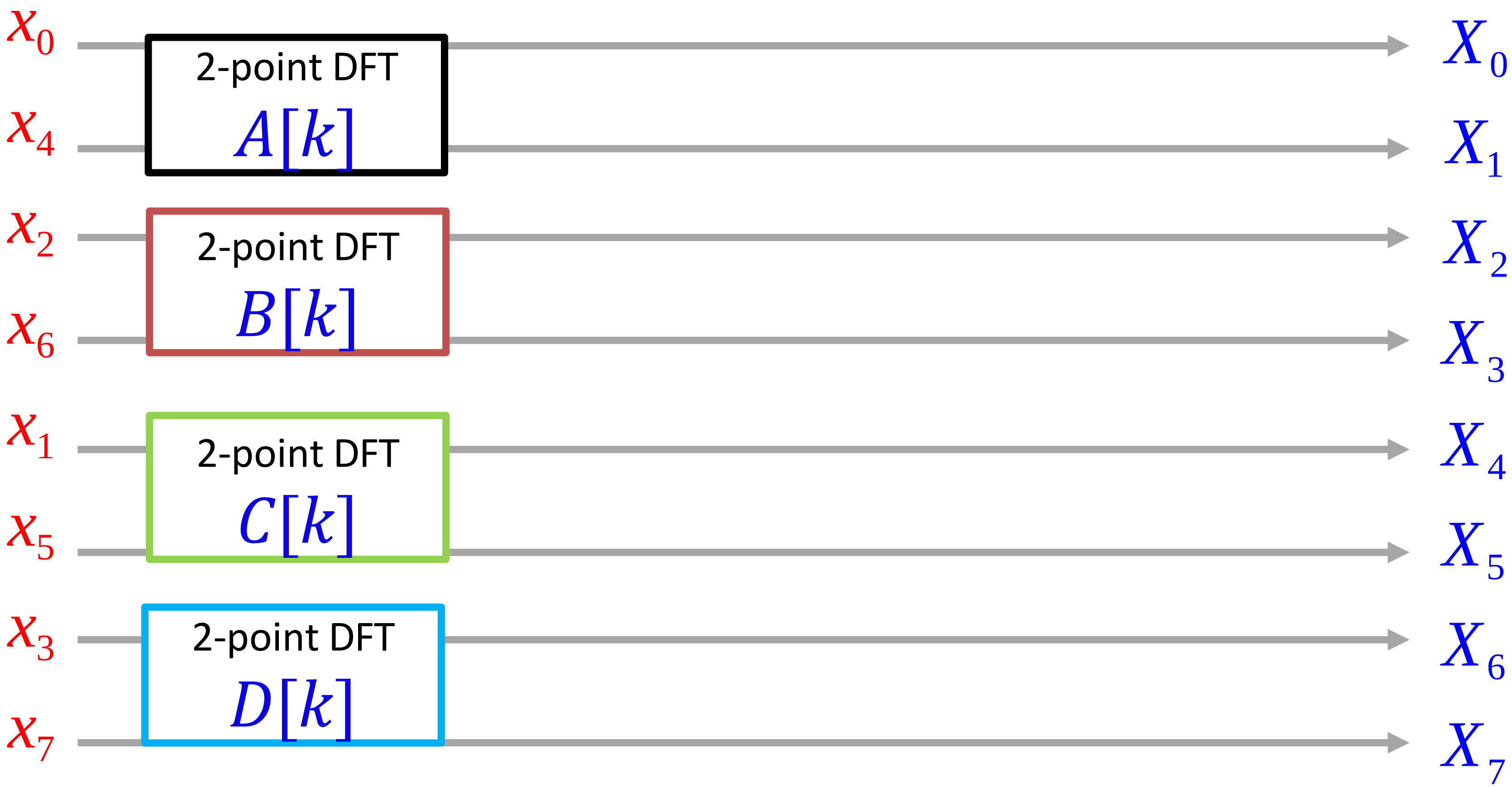

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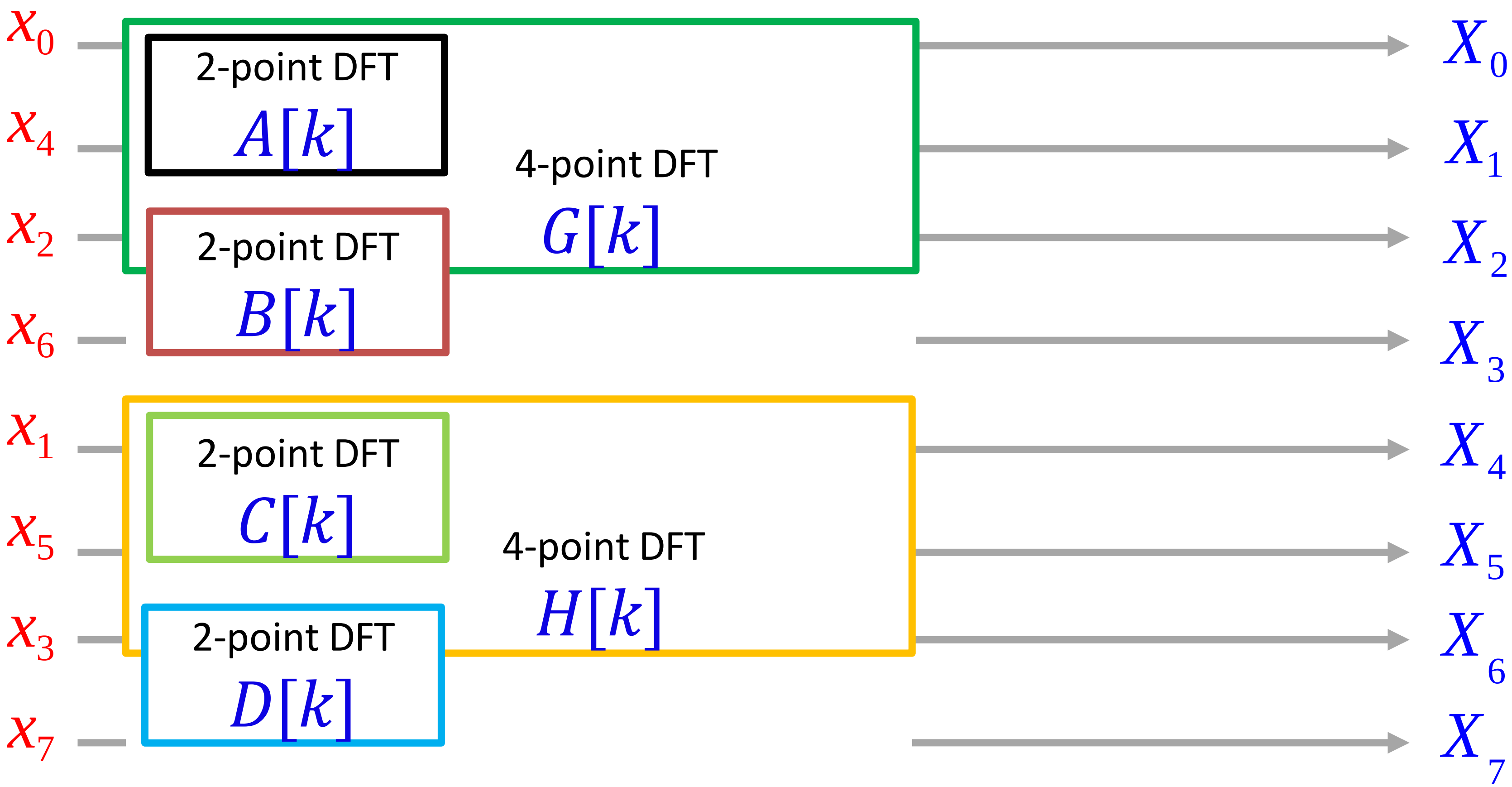

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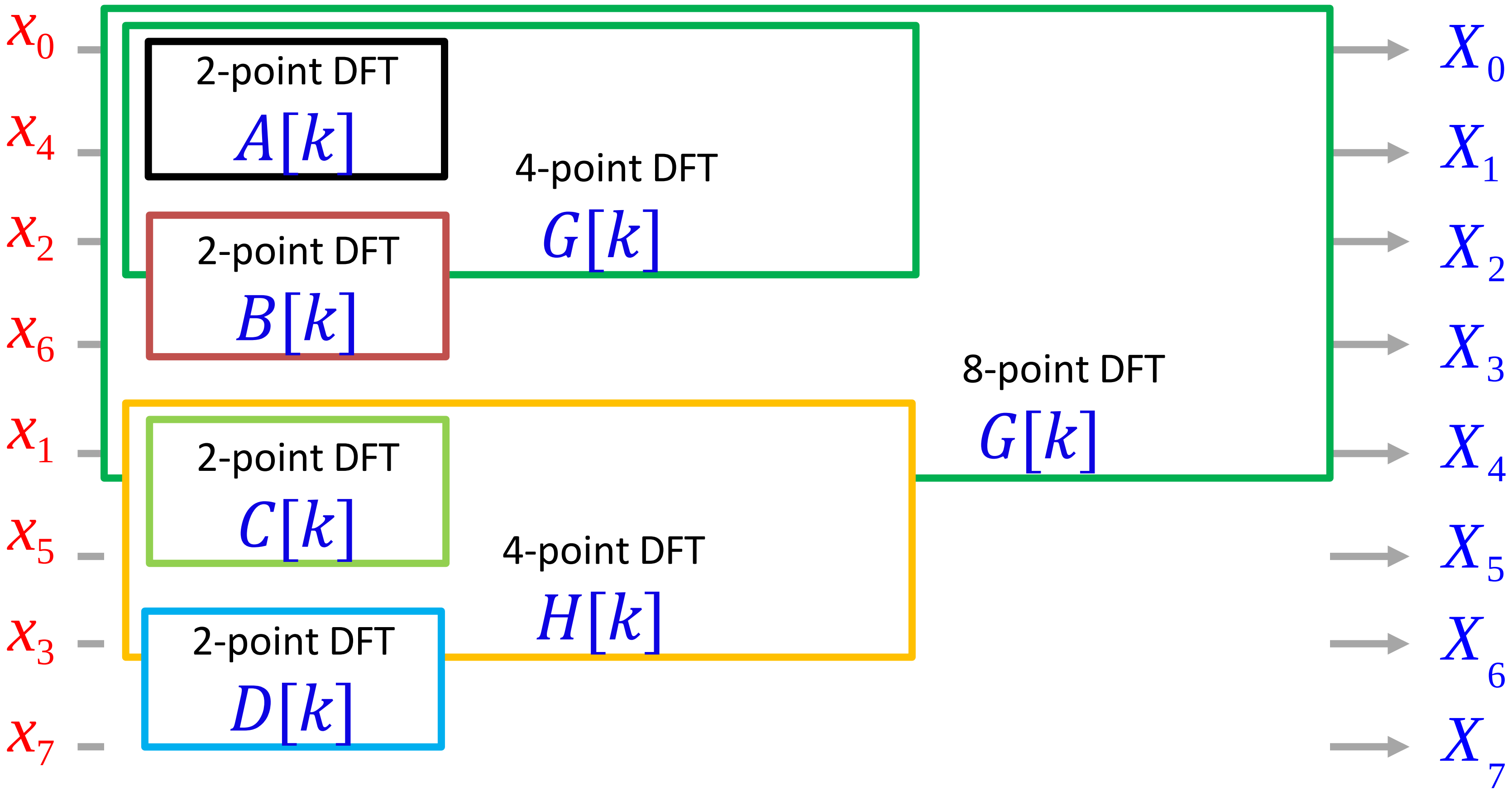

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$x_0$

$x_4$

$x_2$

$x_6$

$x_1$

$x_5$

$x_3$

$x_7$

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| | |
|---|---|
| $x_0$ | $X_0$ |
| $x_4$ | $X_1$ |
| $x_2$ | $X_2$ |
| $x_6$ | $X_3$ |
| $x_1$ | $X_4$ |
| $x_5$ | $X_5$ |
| $x_3$ | $X_6$ |
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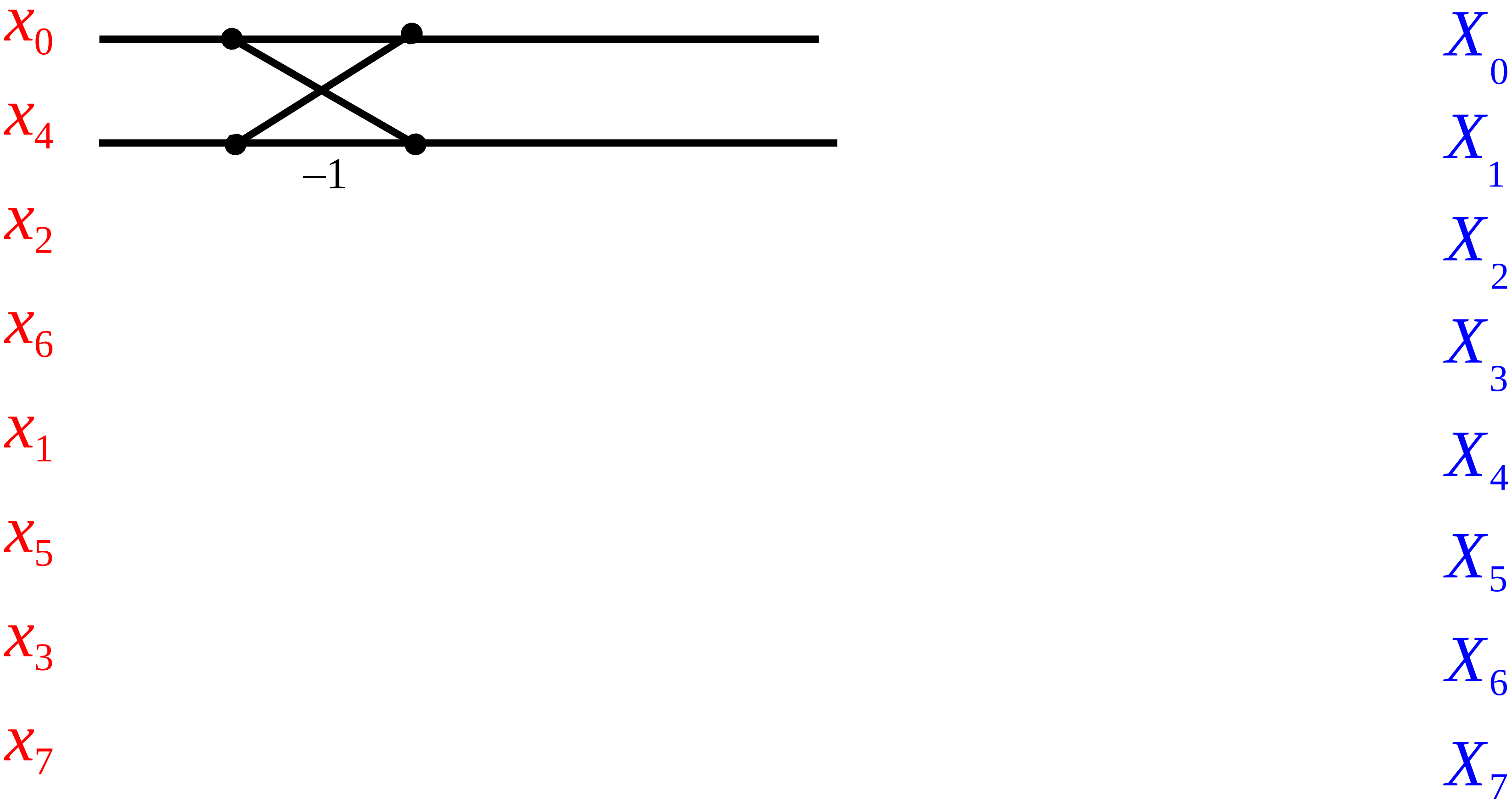

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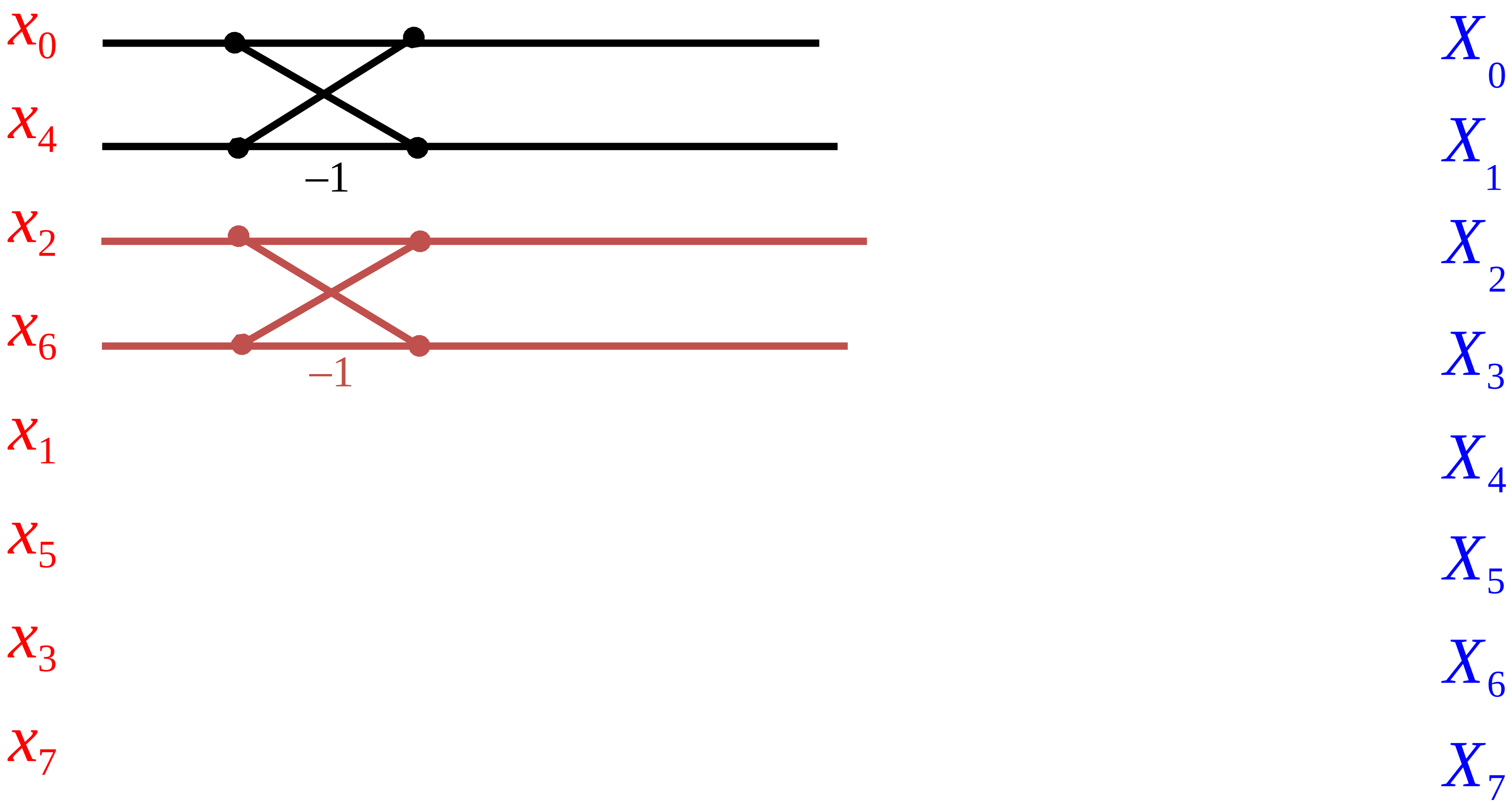

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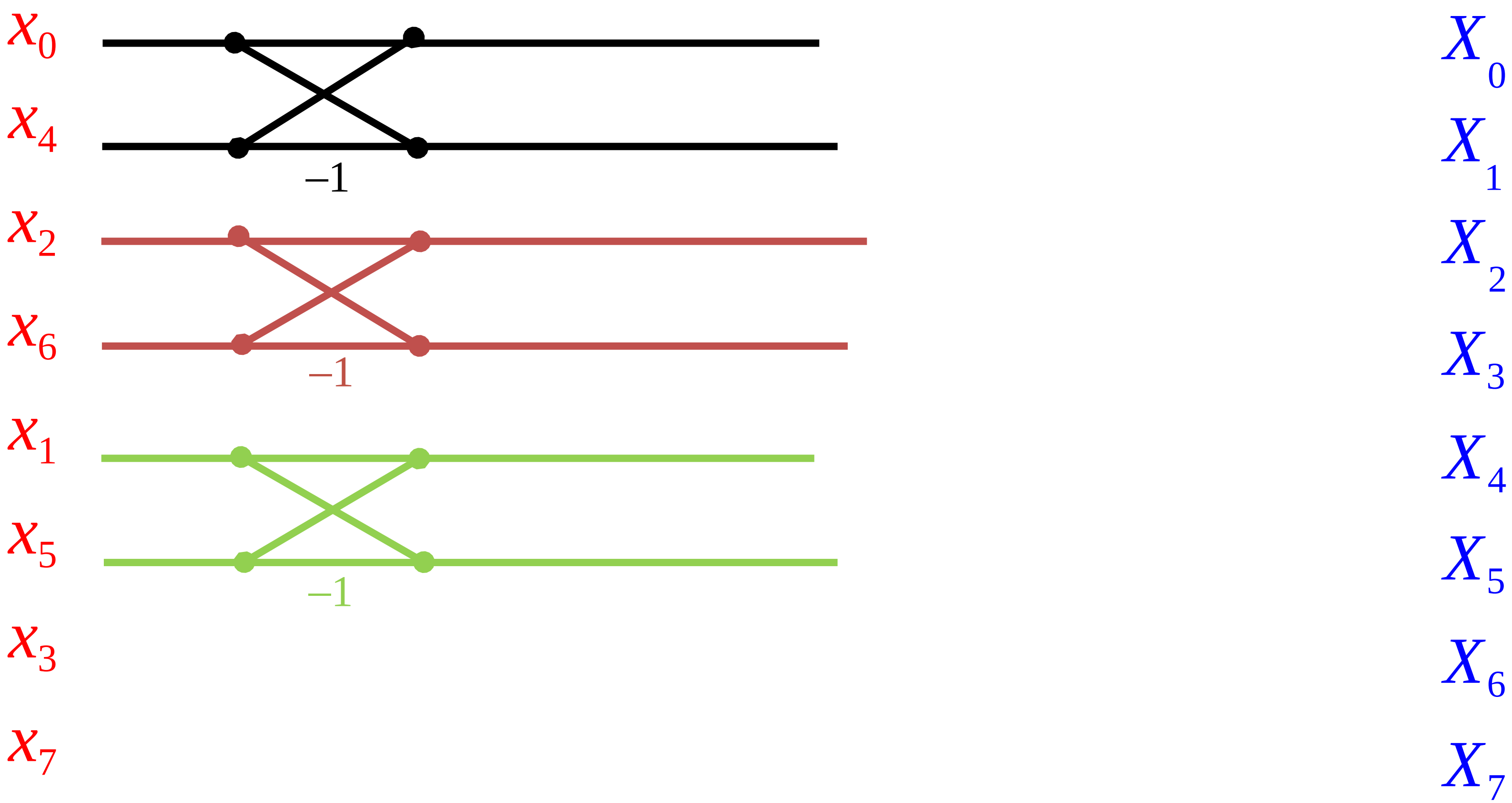

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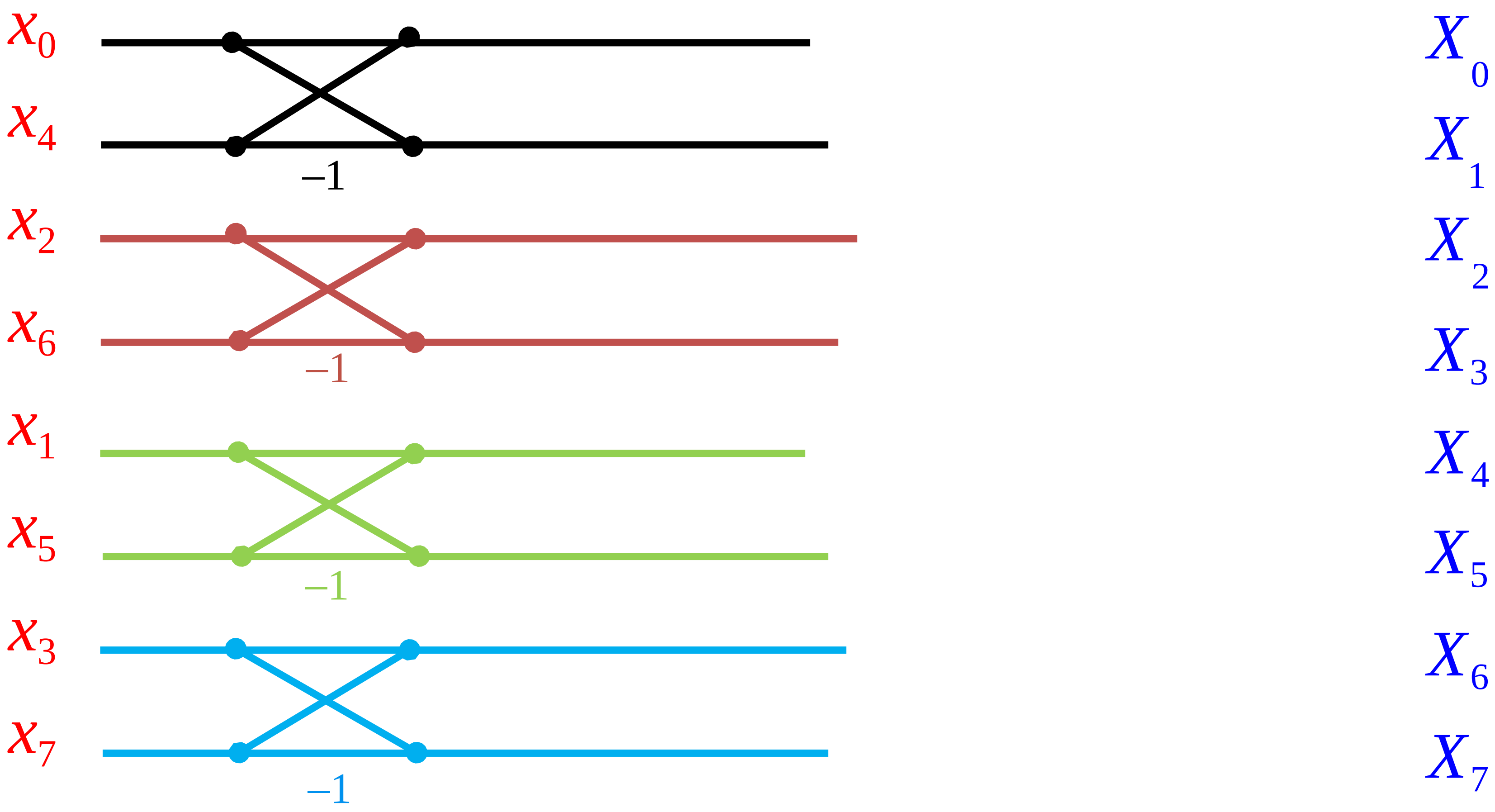

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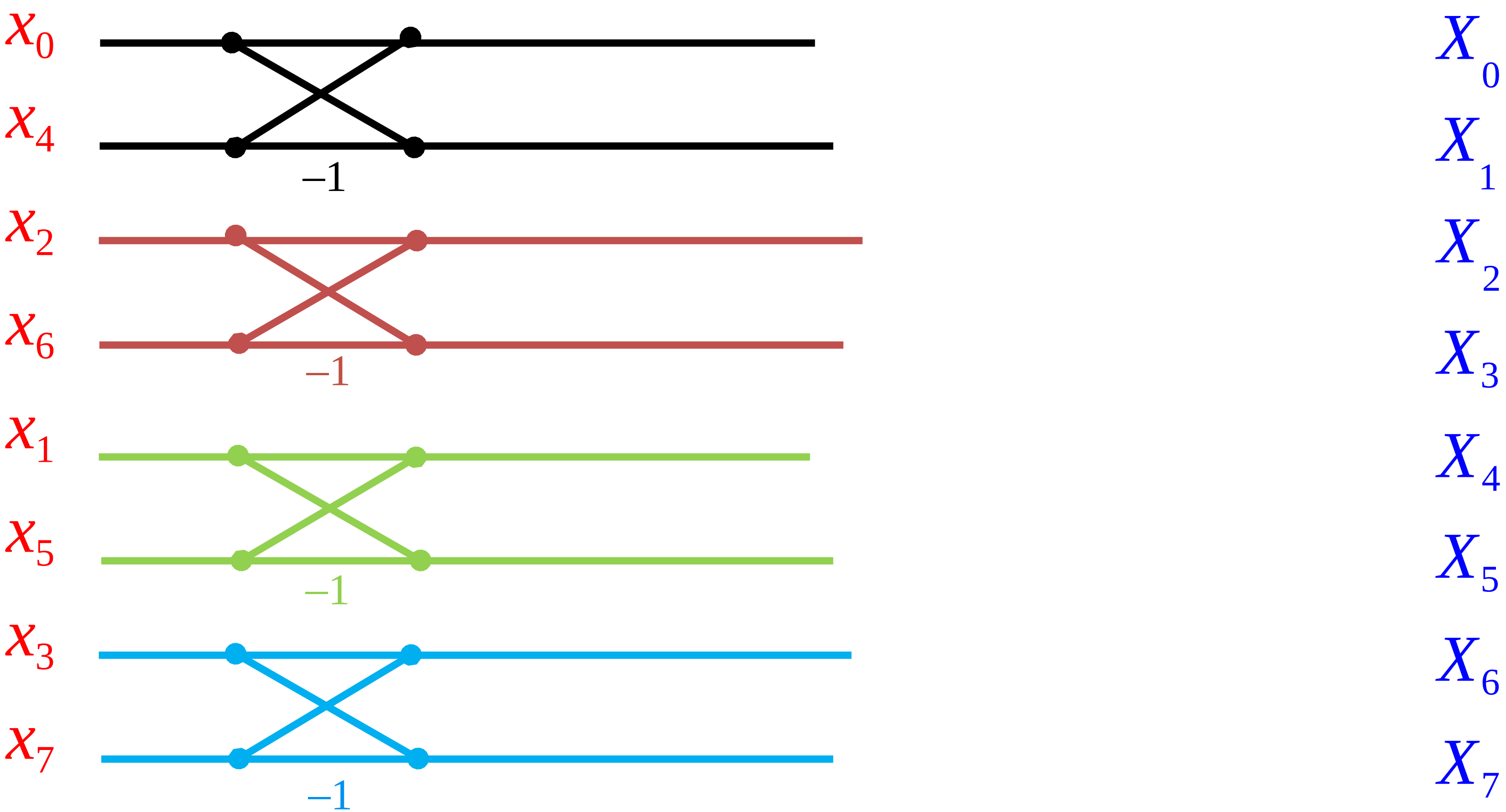

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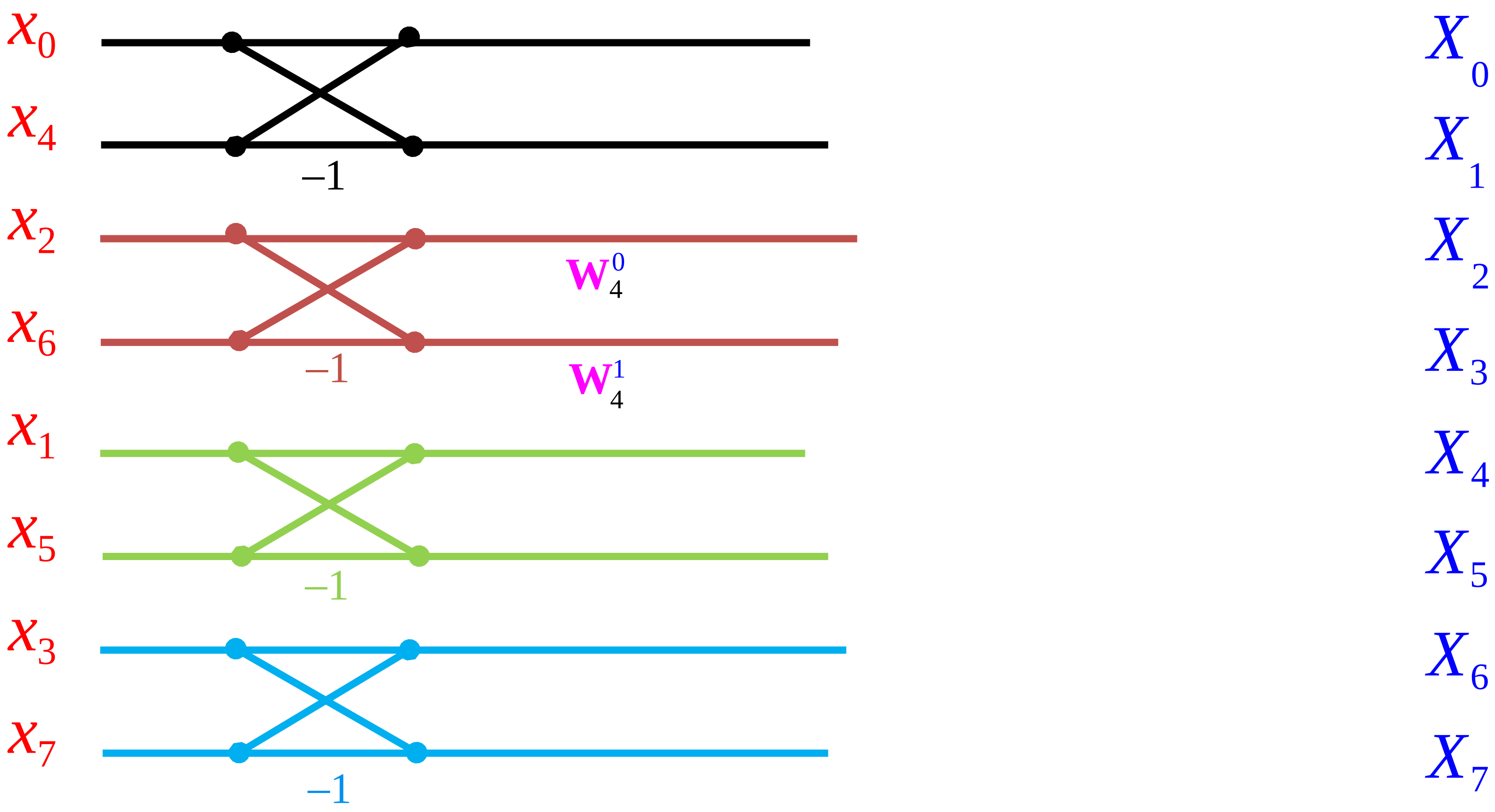

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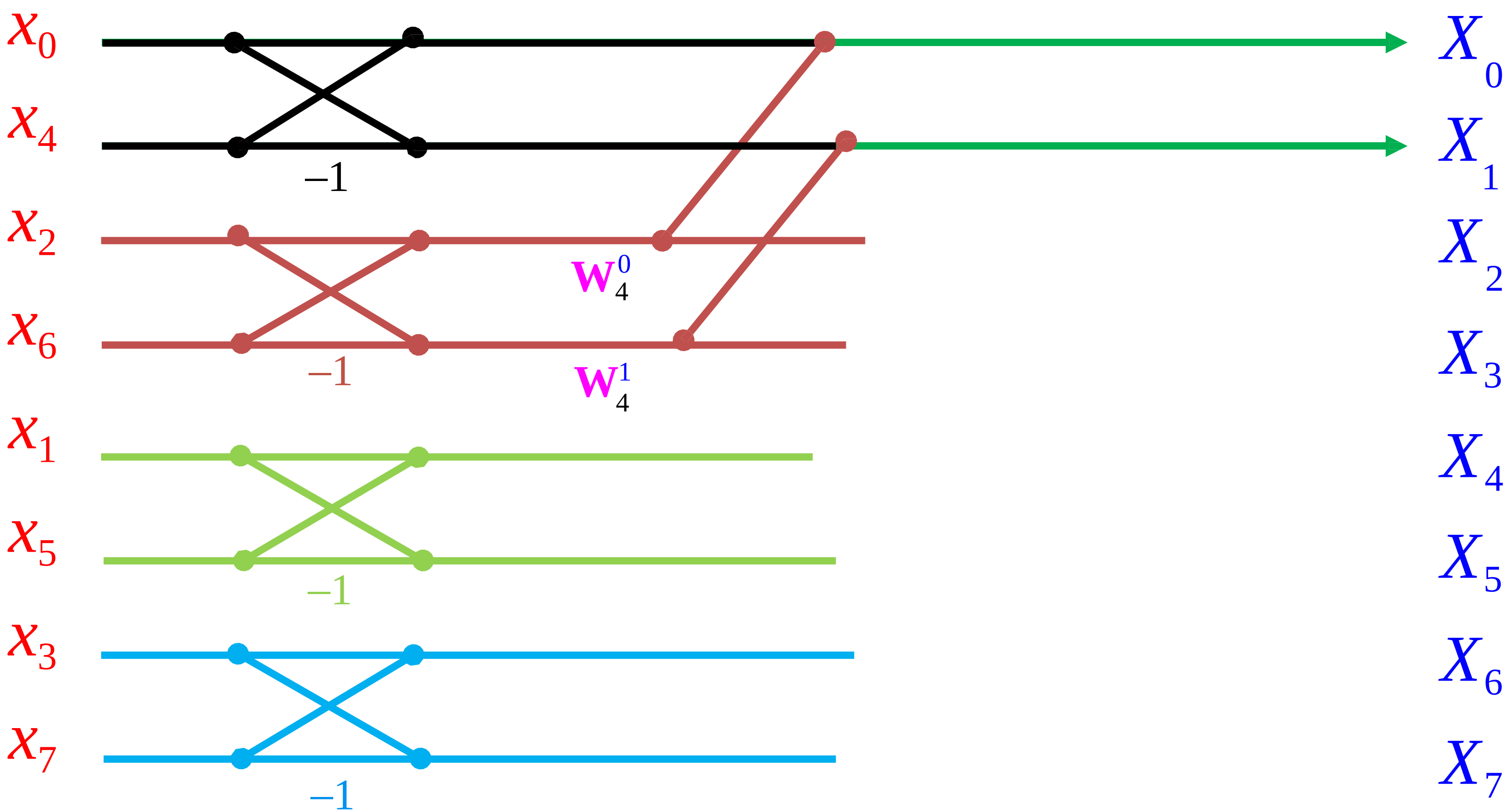

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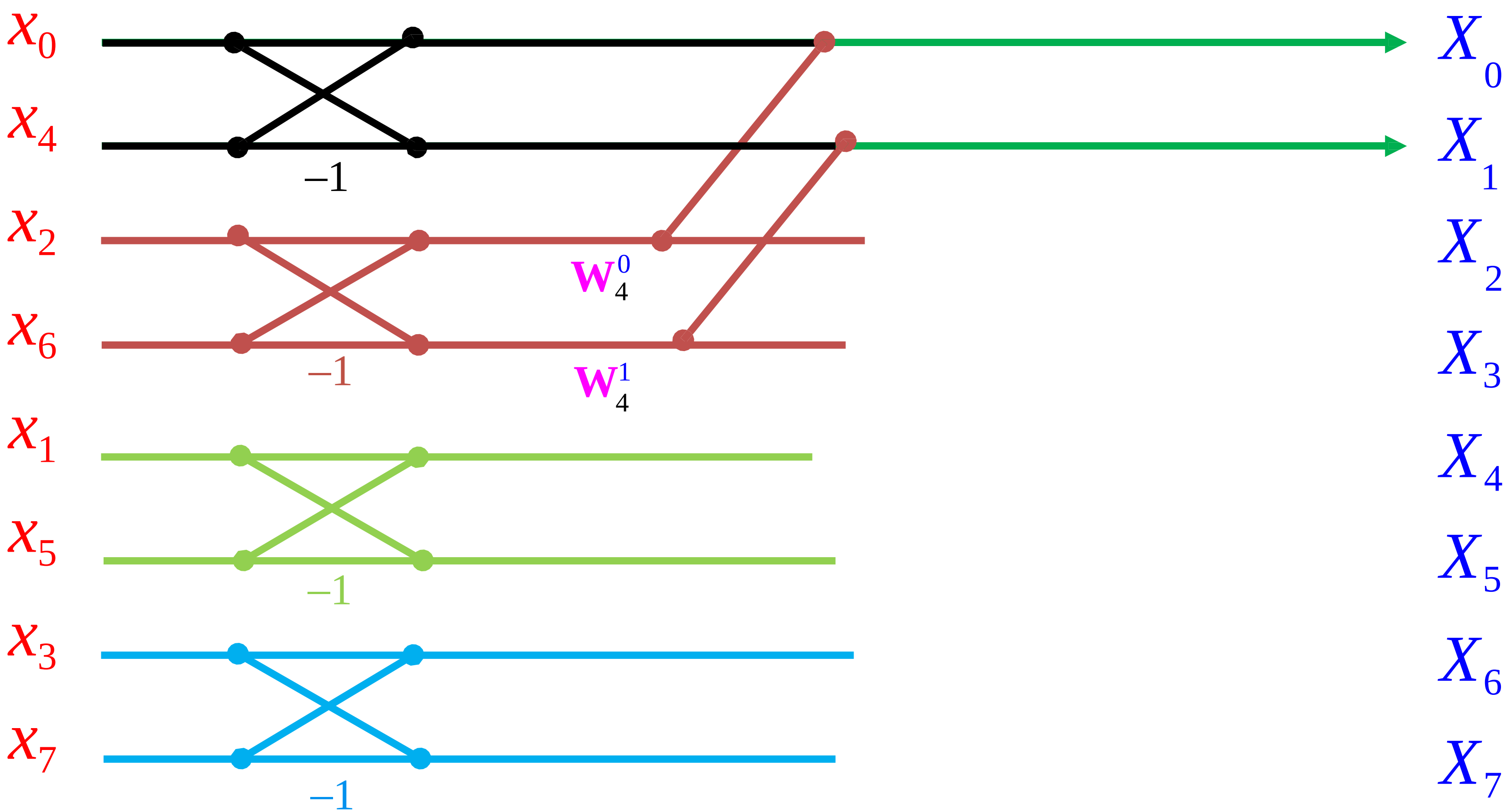

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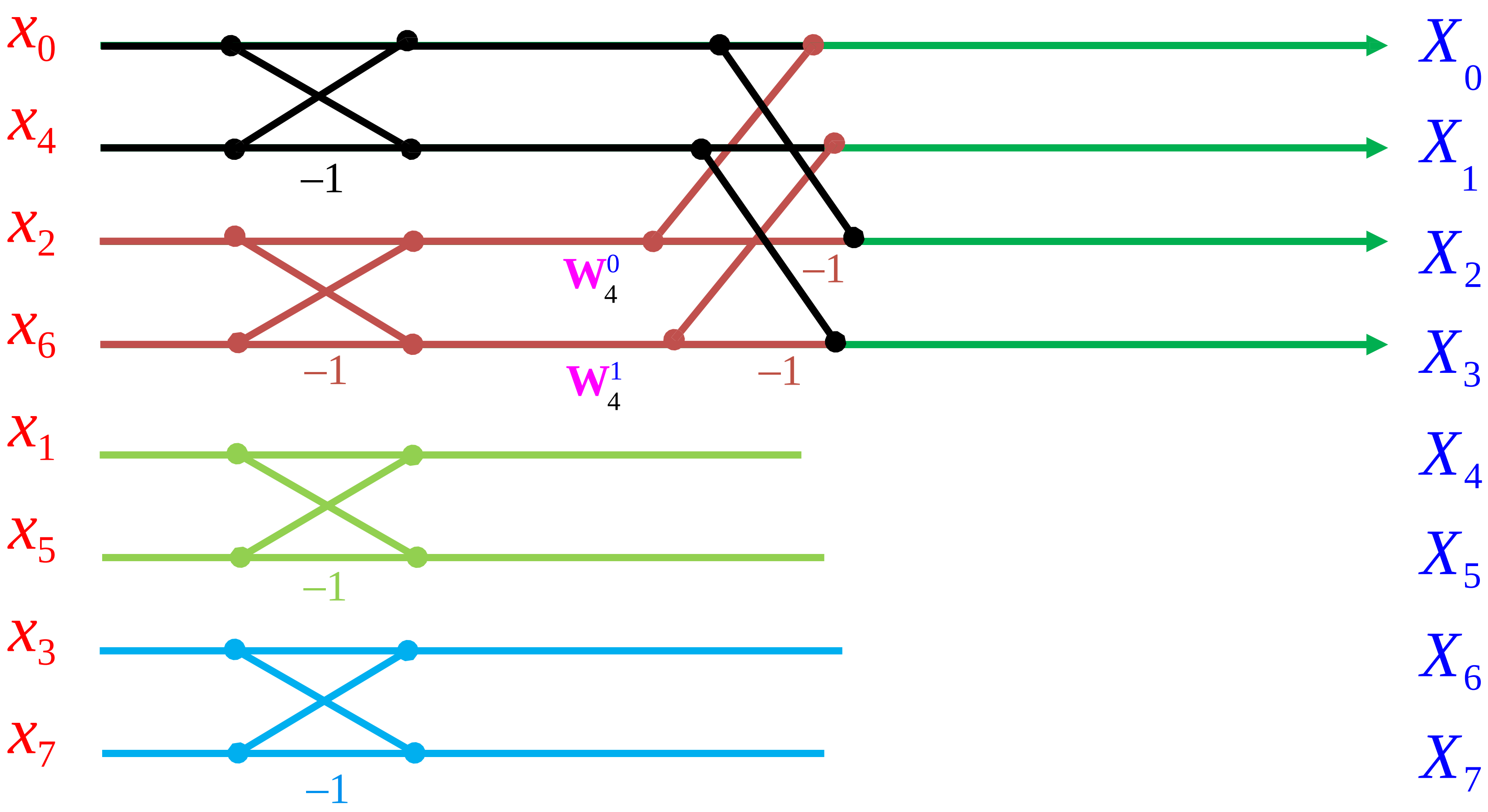

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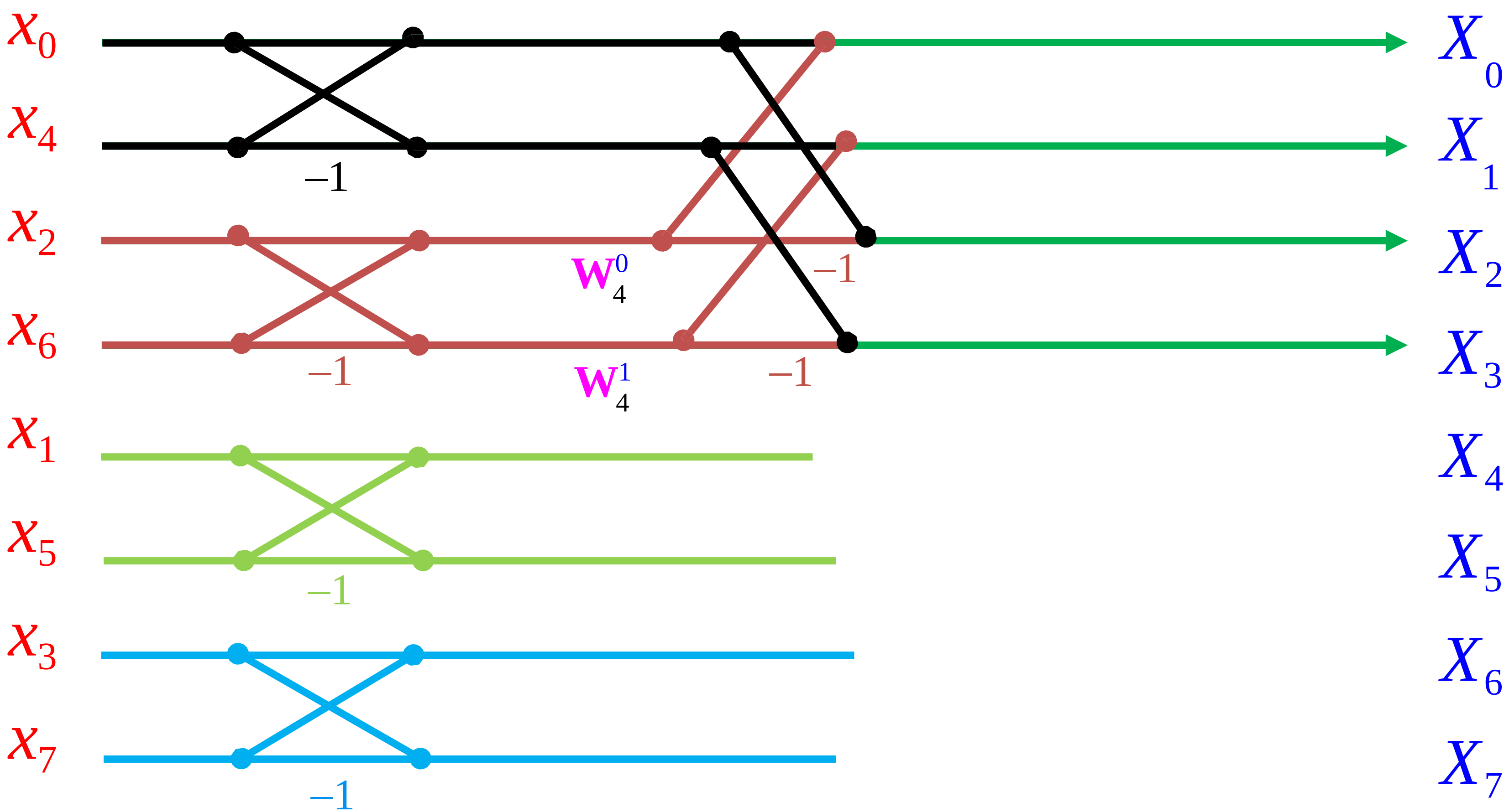

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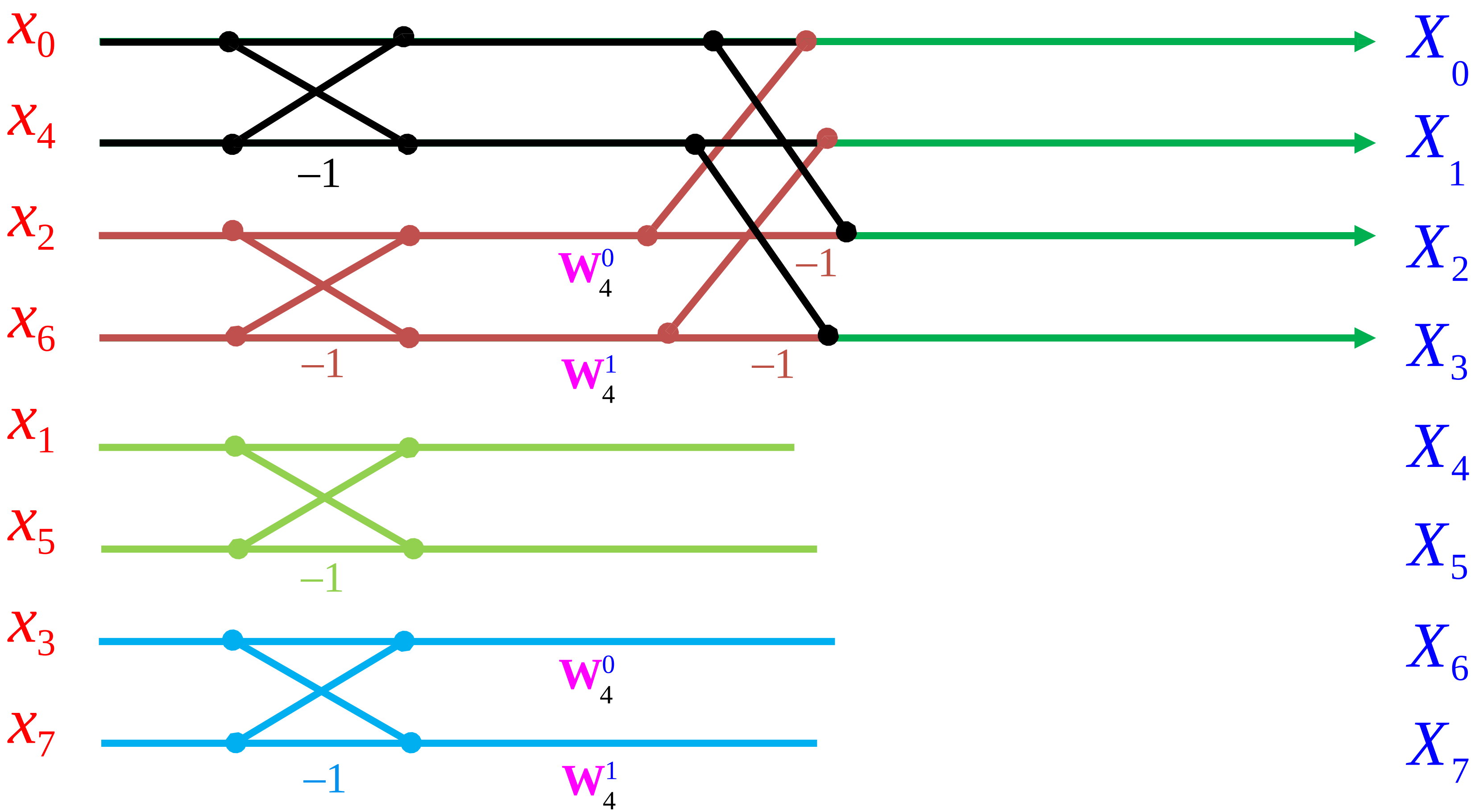

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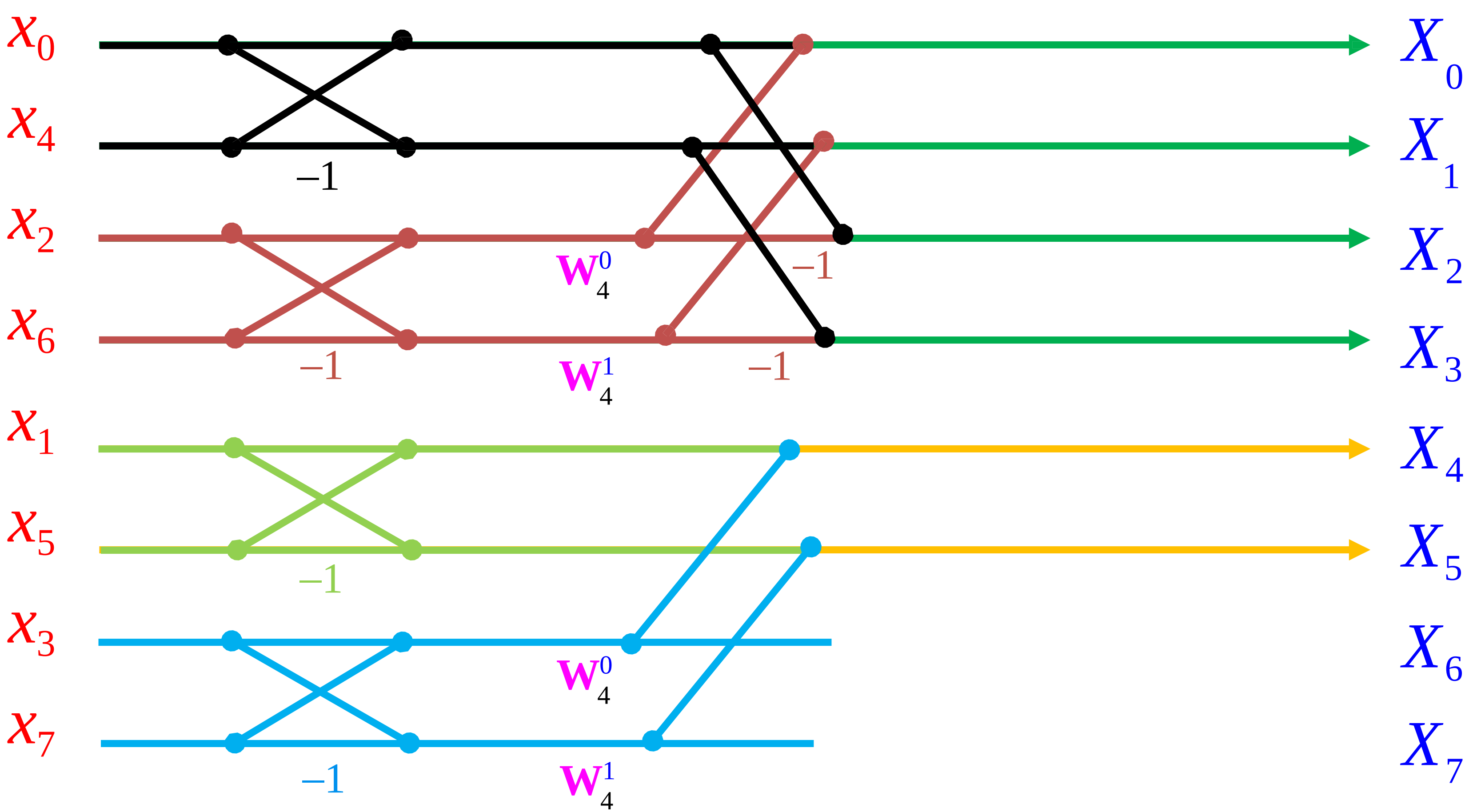

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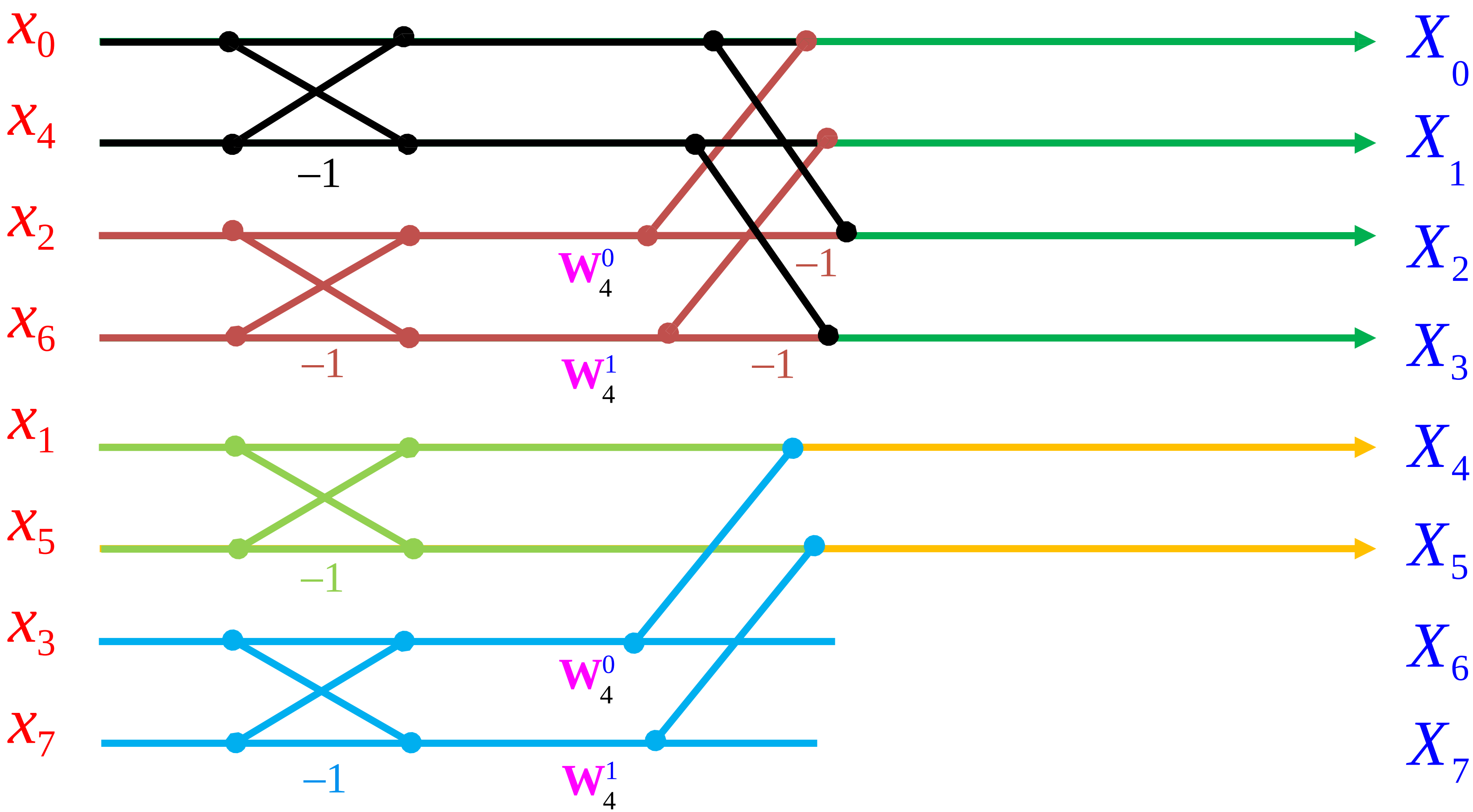

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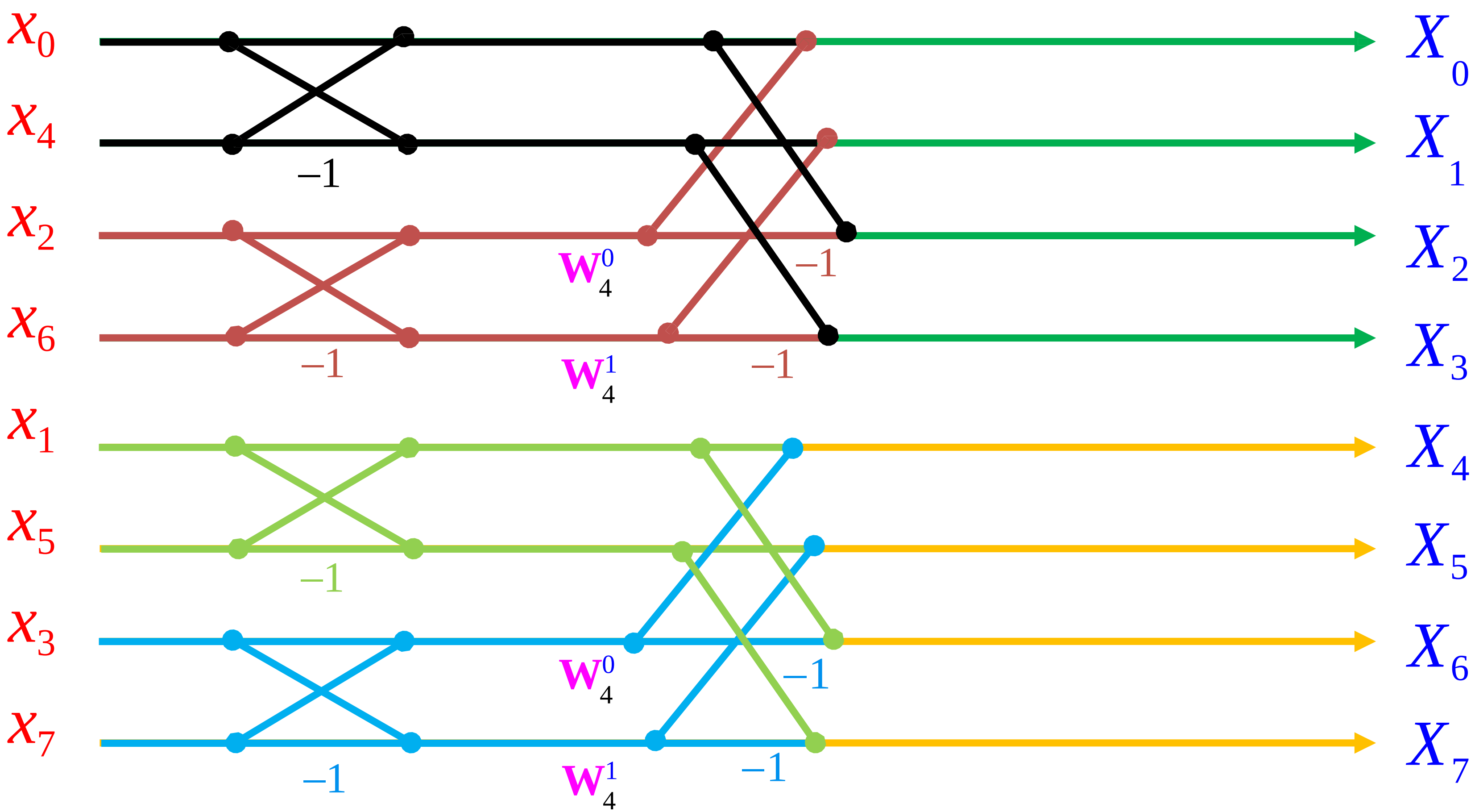
- Let $x[n] = \{x_0,\ x_1, x_2, x_3, x_4, x_5, x_6, x_7\} \xrightarrow{DFT} X(k) = \{X_0, X_1, X_2, X_3, X_4, X_5, X_6, X_7\}$



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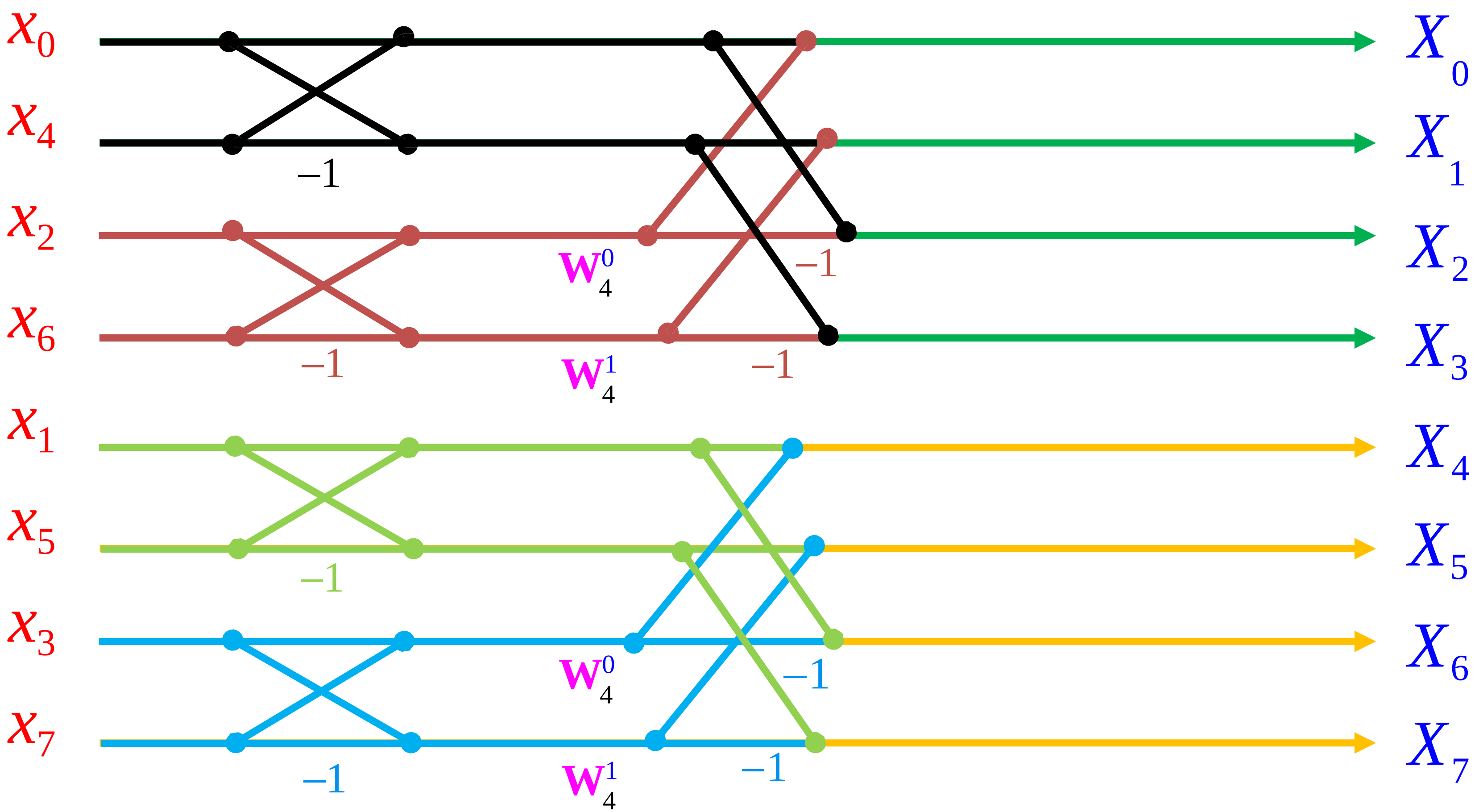

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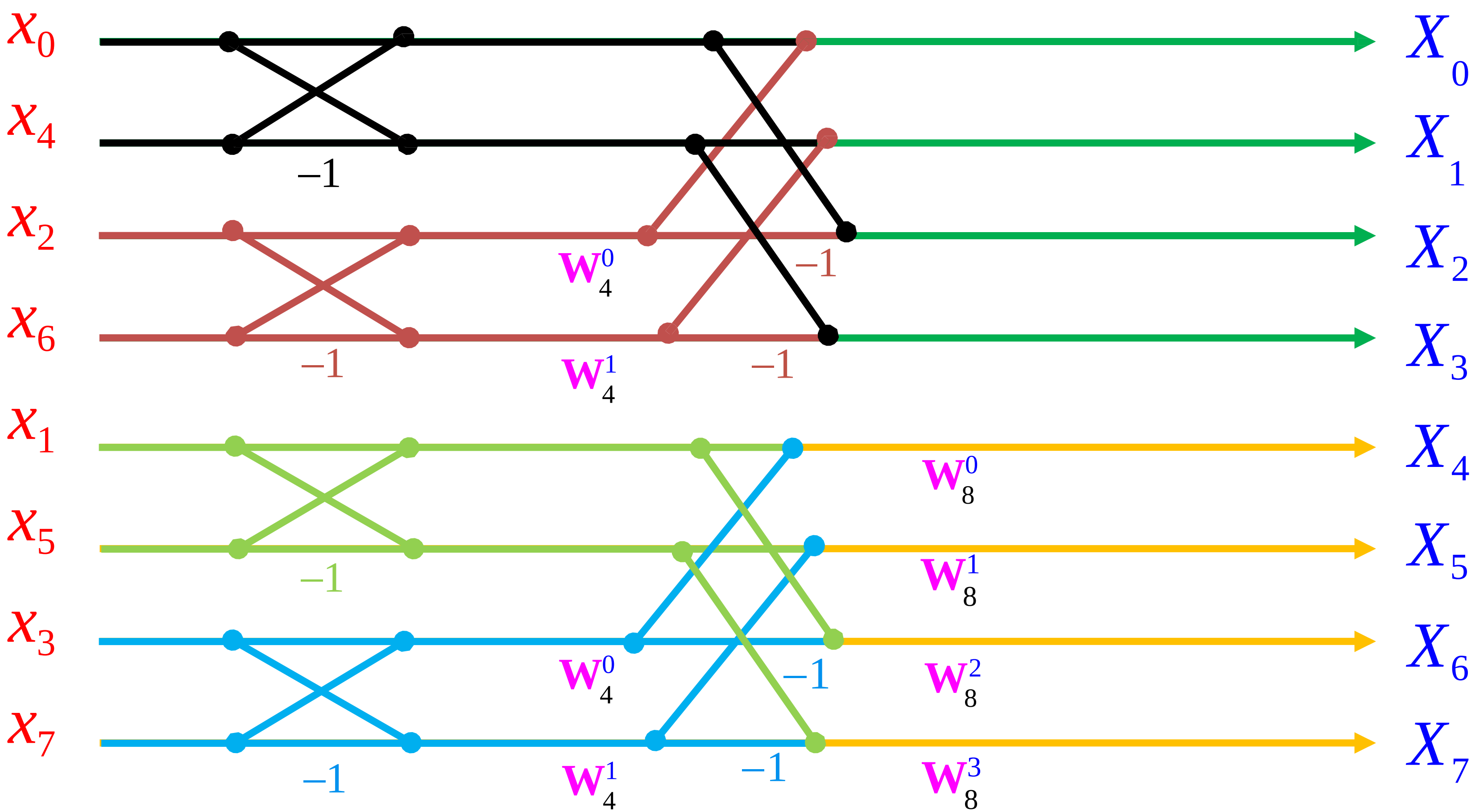

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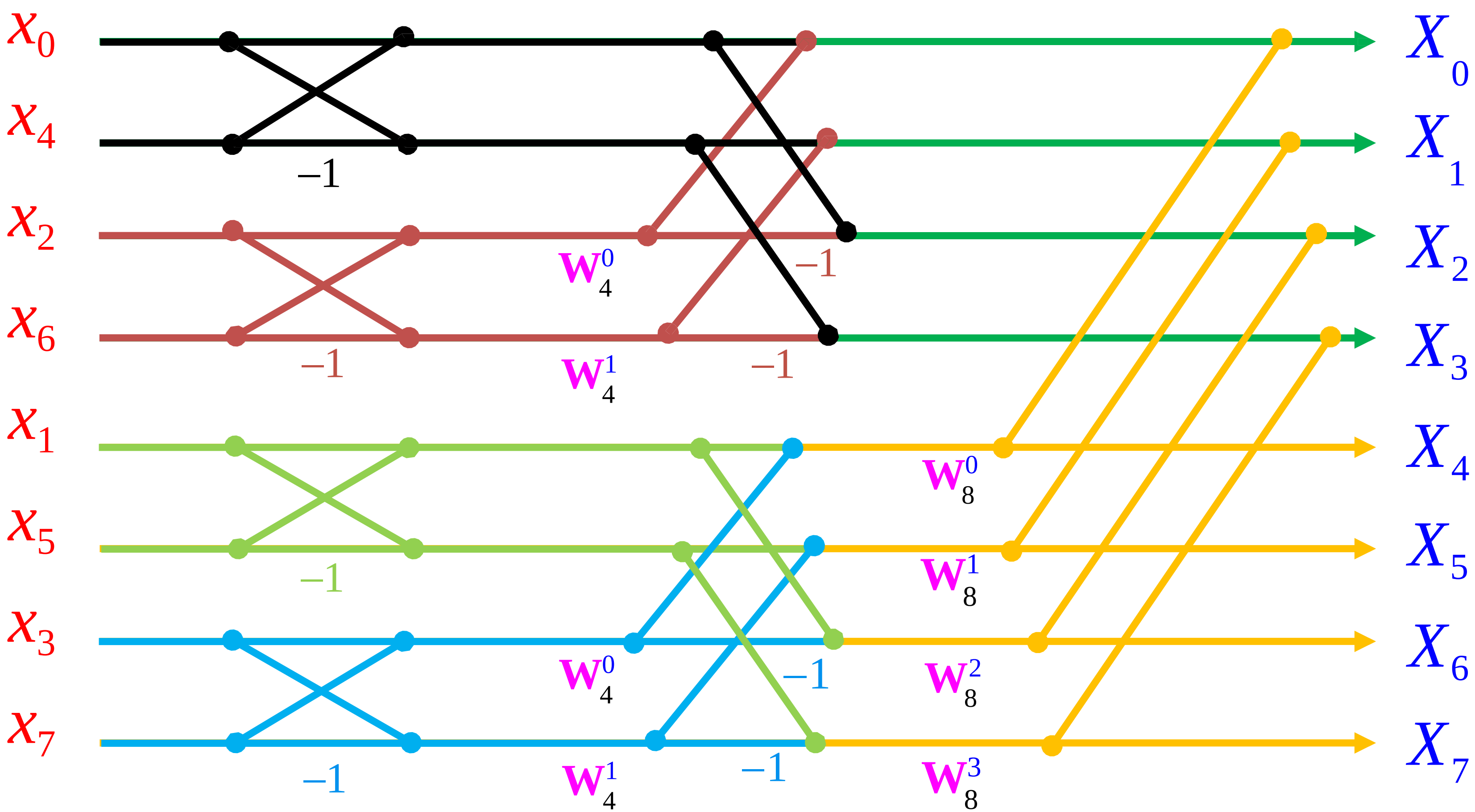

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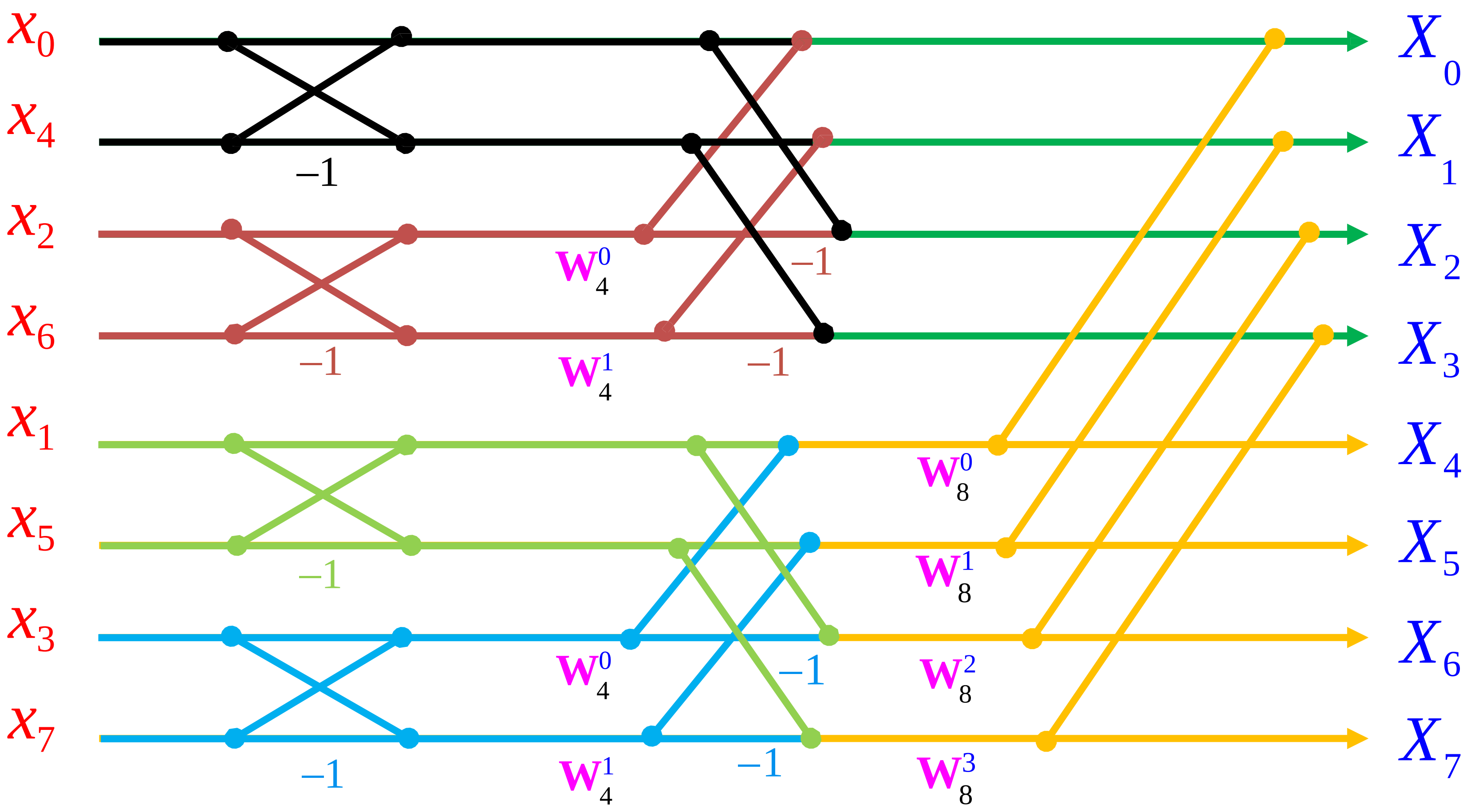

# 8-point DITFFT

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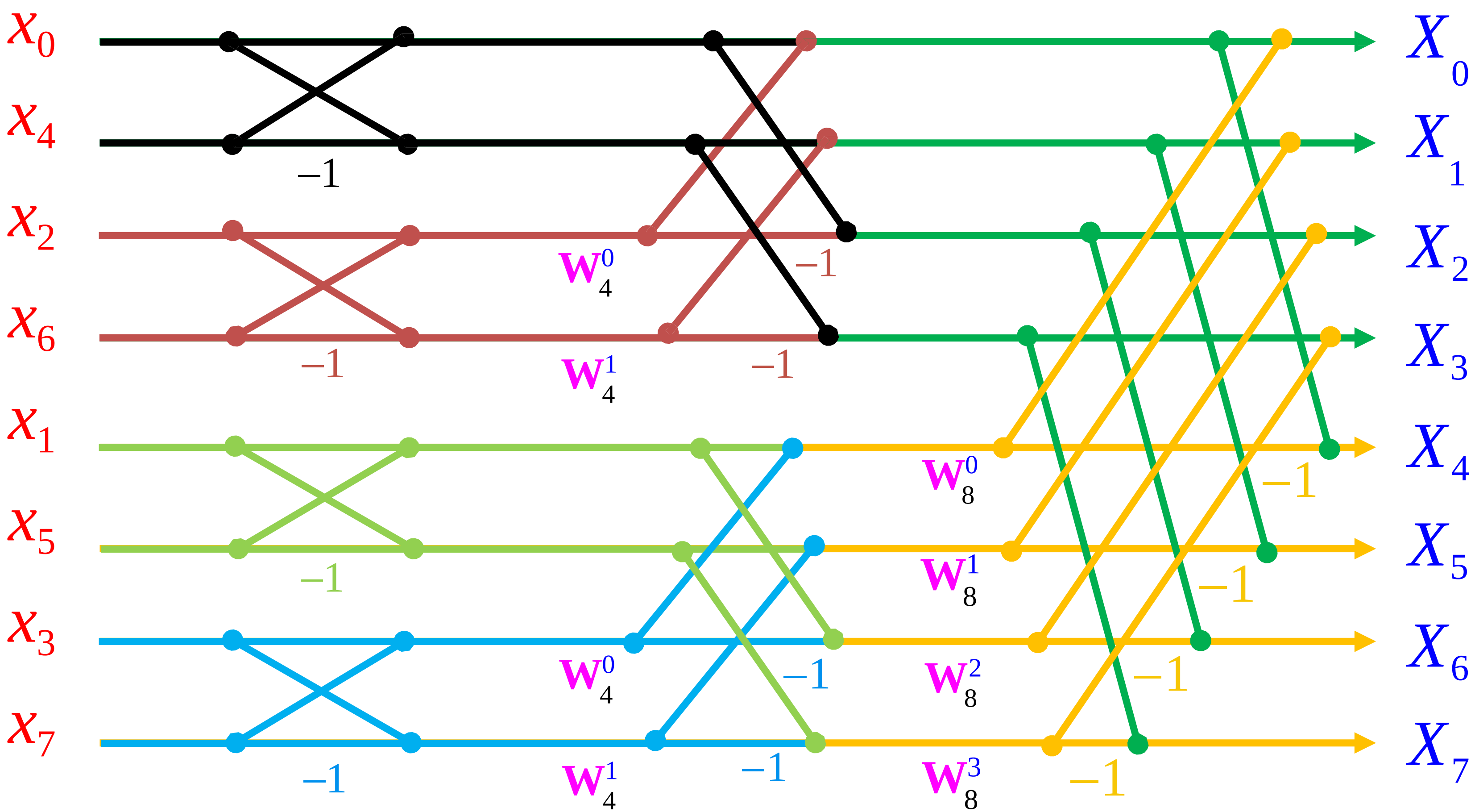

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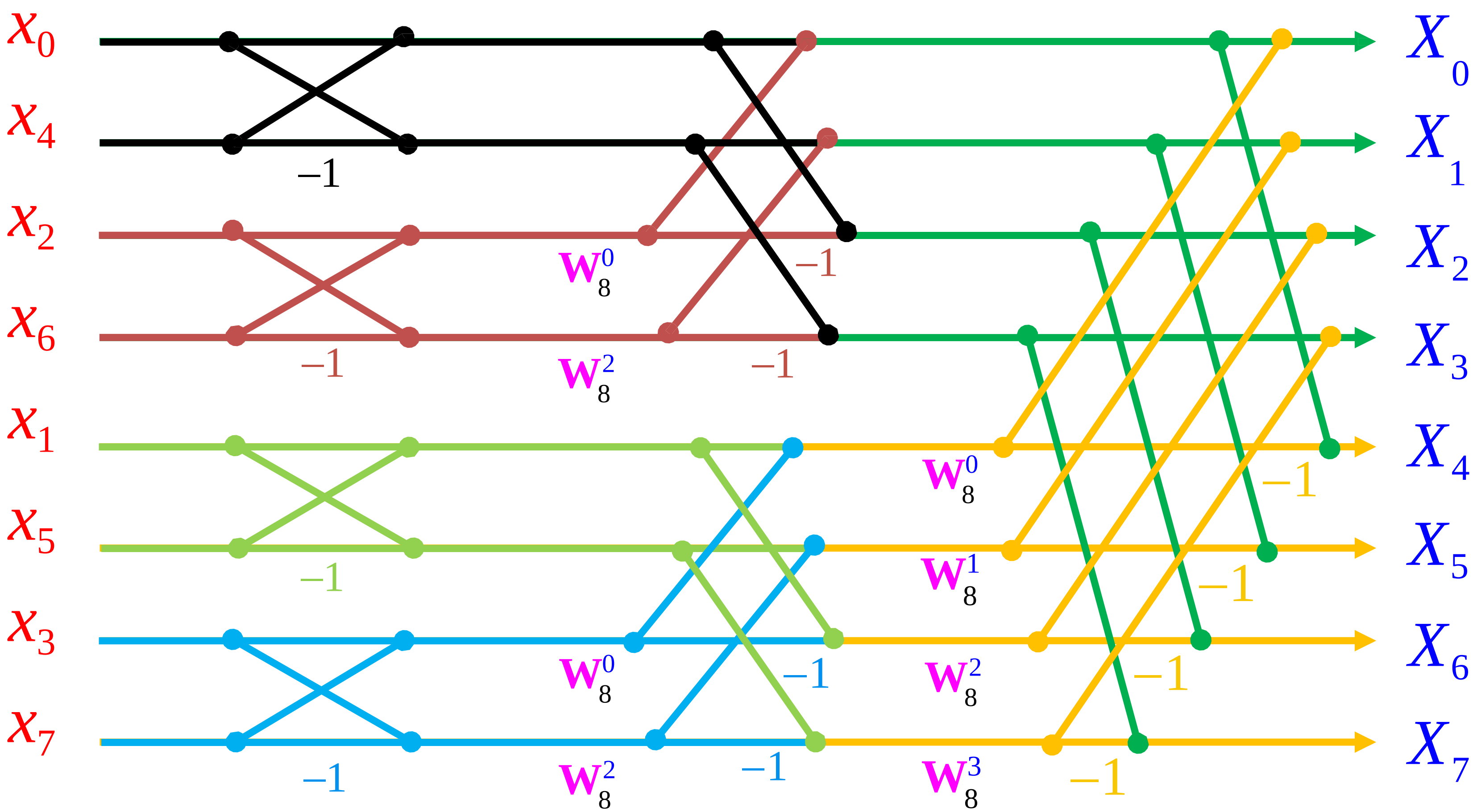

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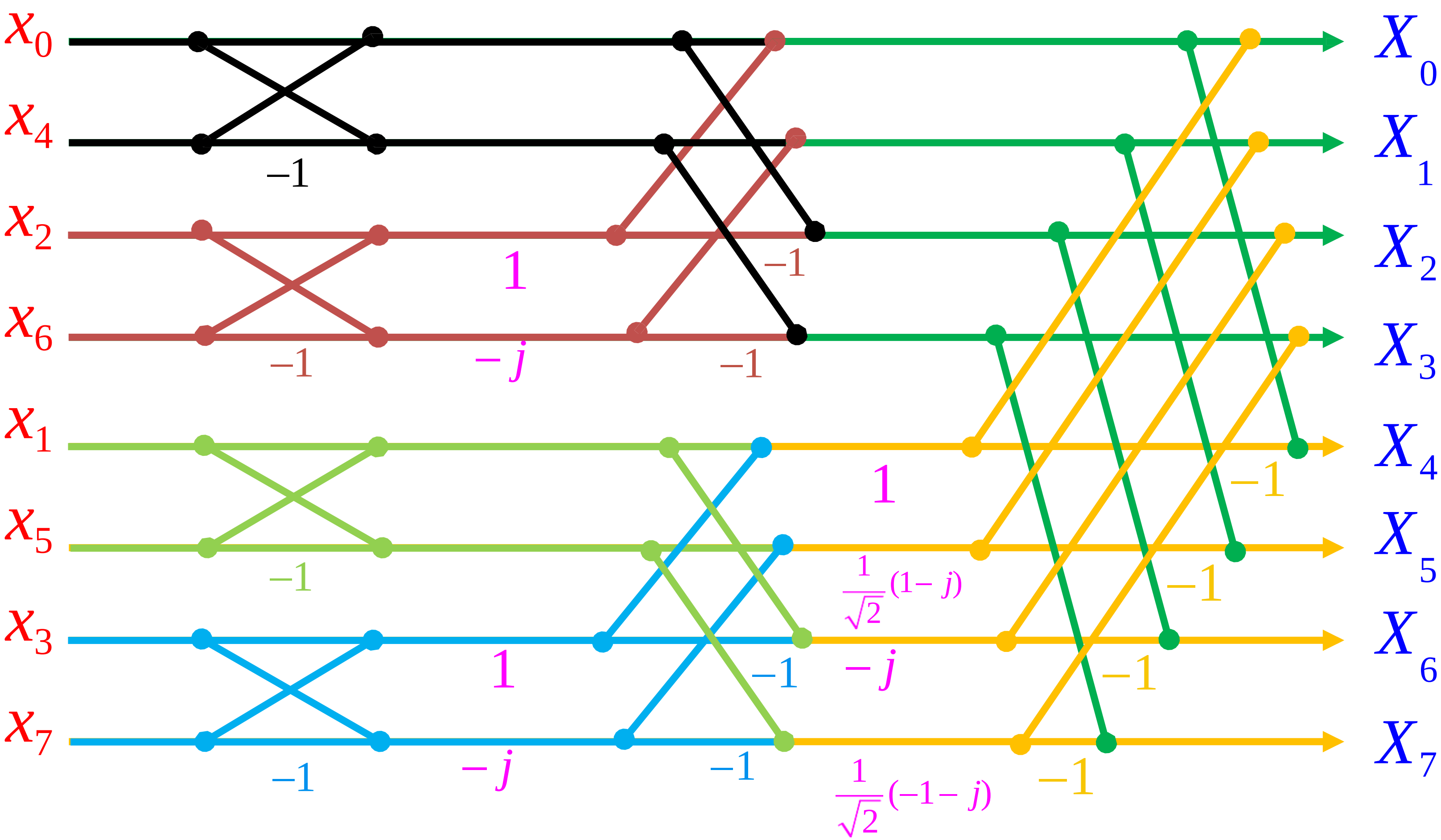
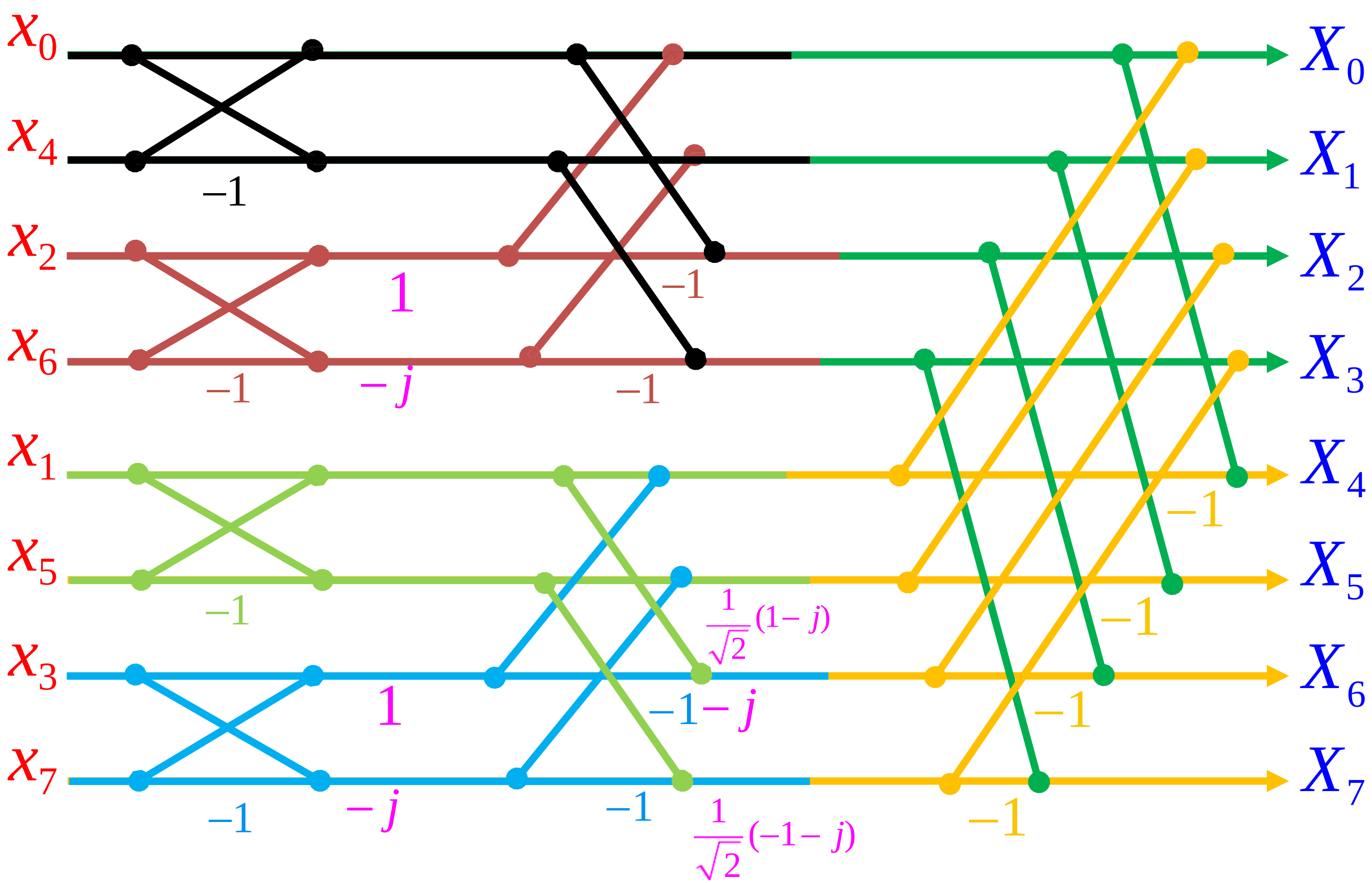


## Example

Compute DFT of $x[n] = \{1,2,3,4,5,6,7,8\}$ using DITFFT algorithm. (May19,10 marks)



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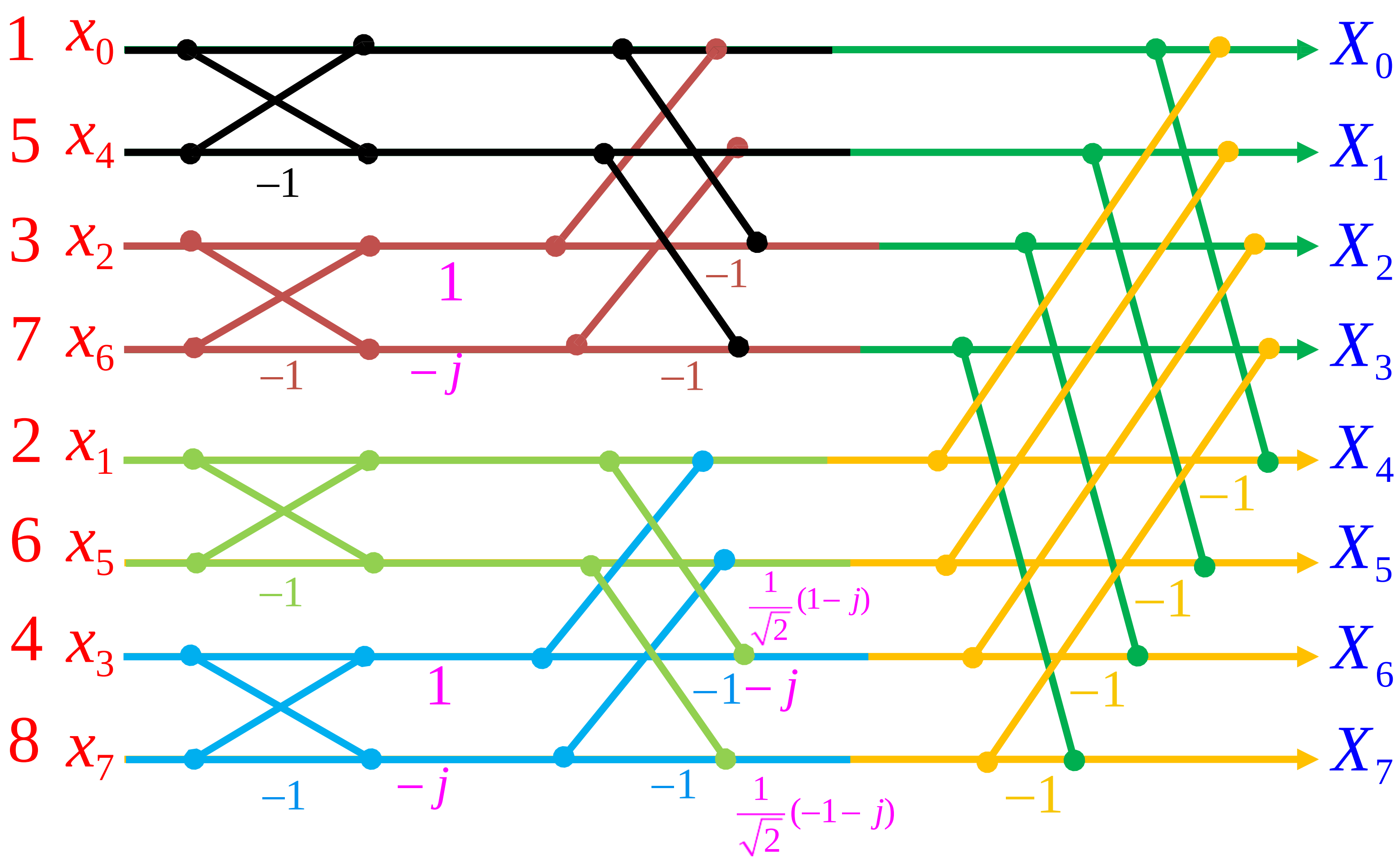

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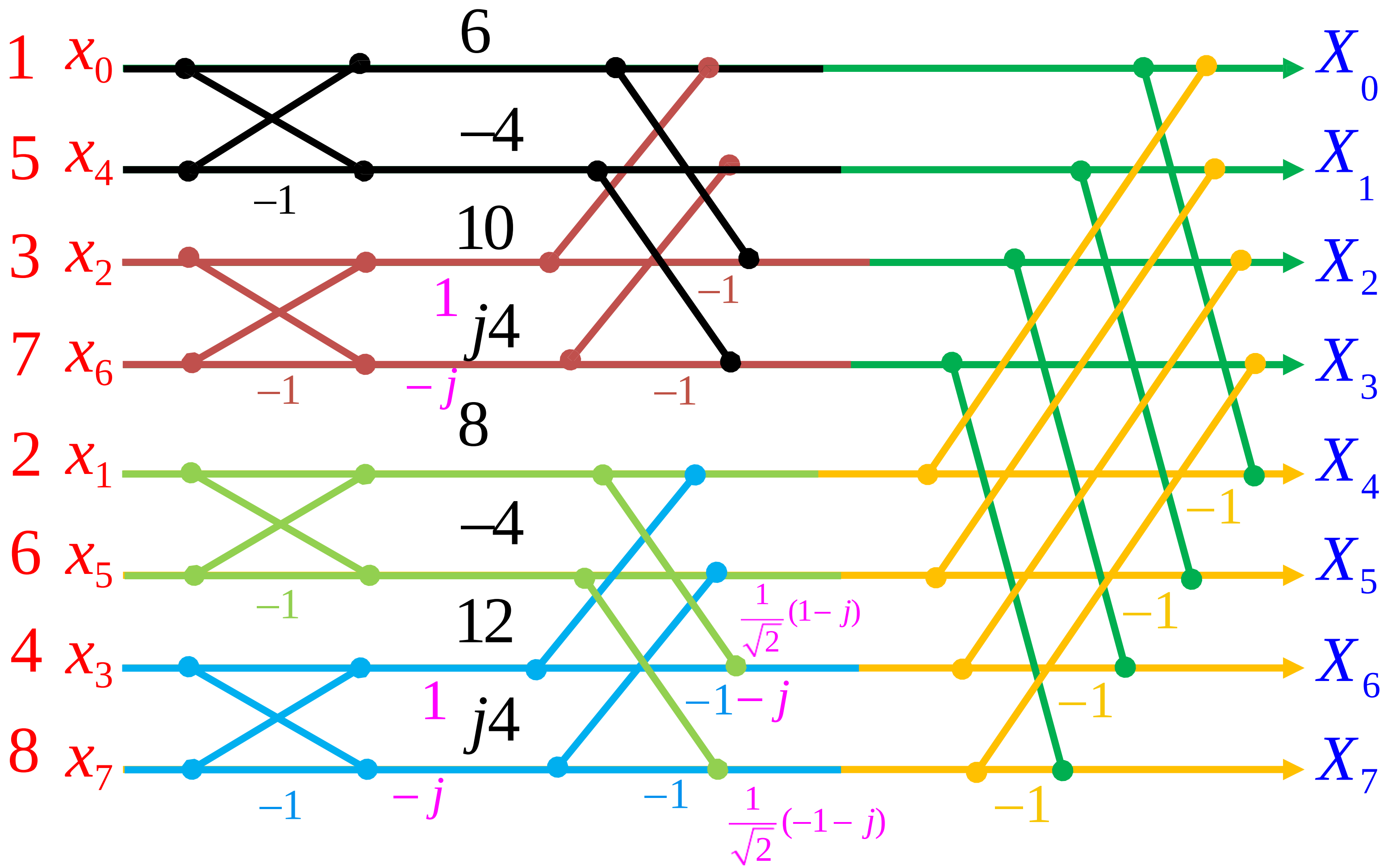

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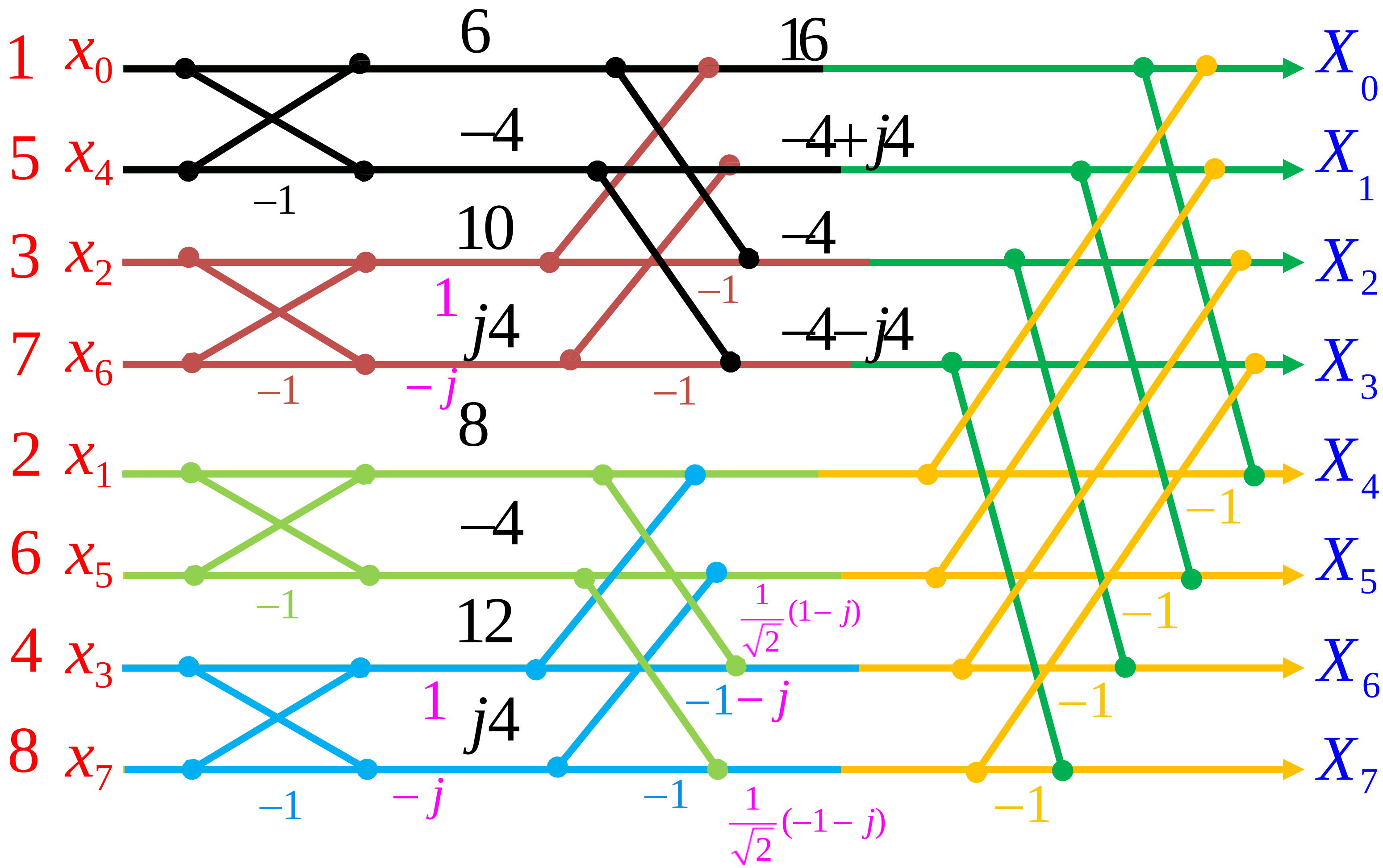

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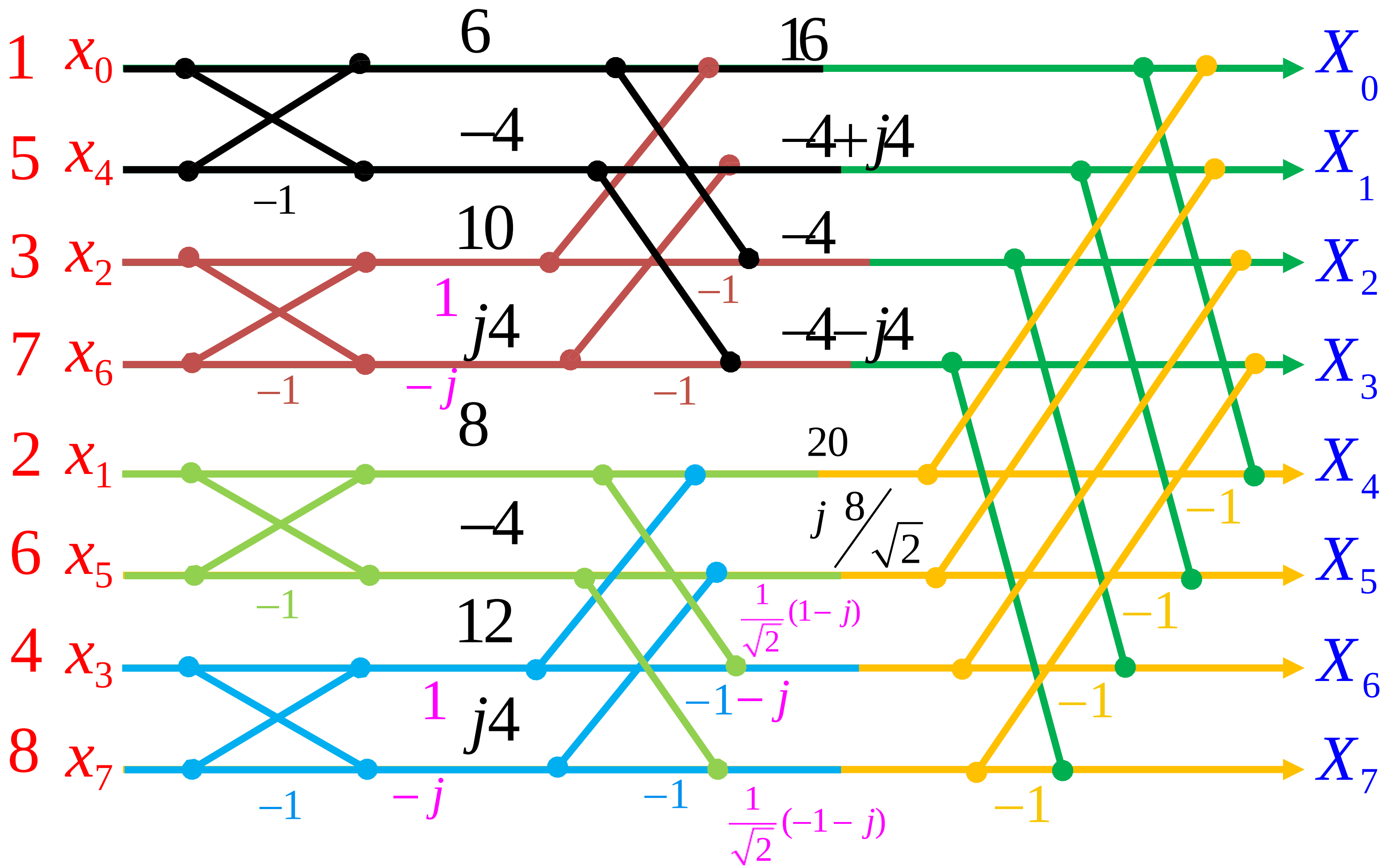

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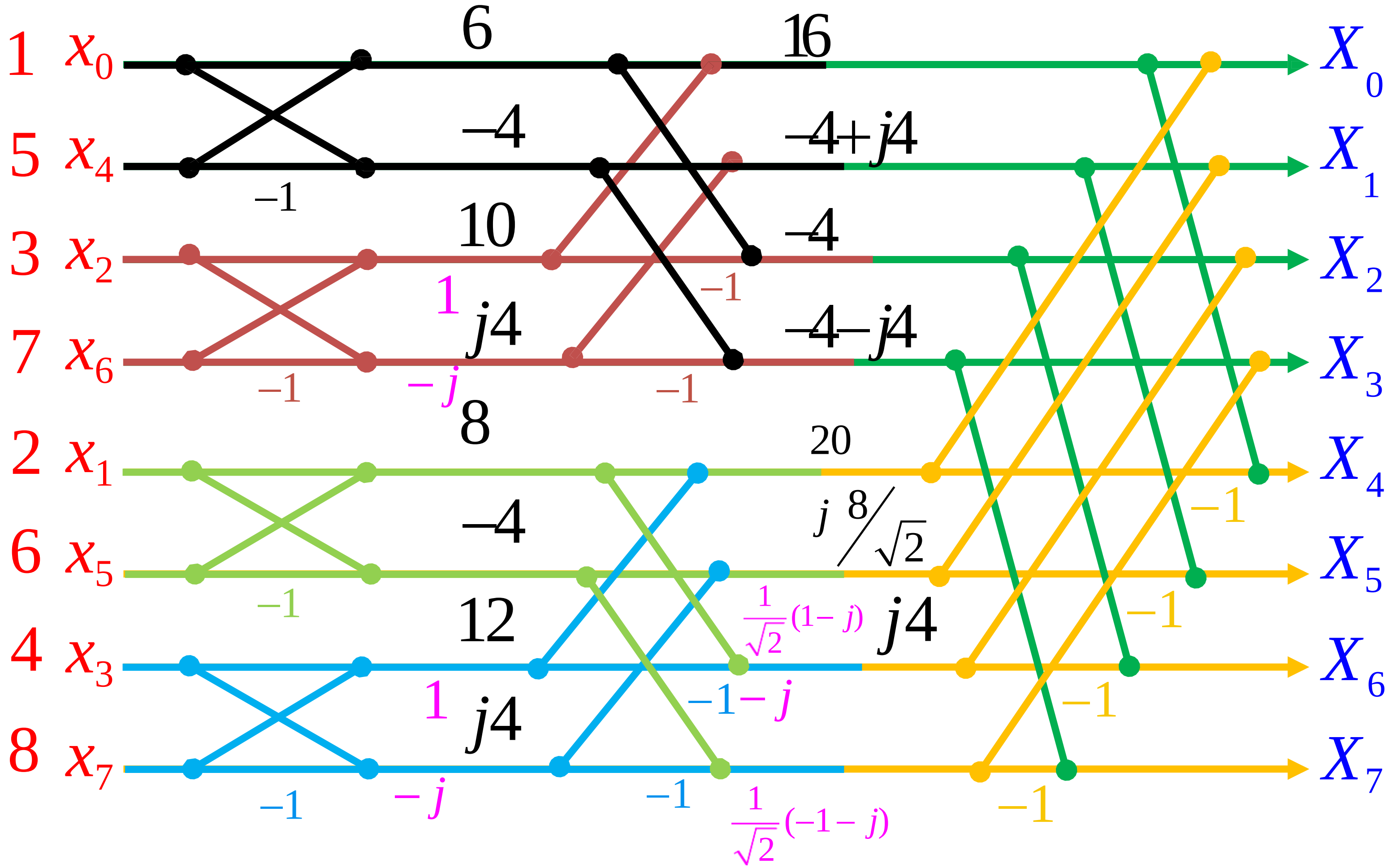

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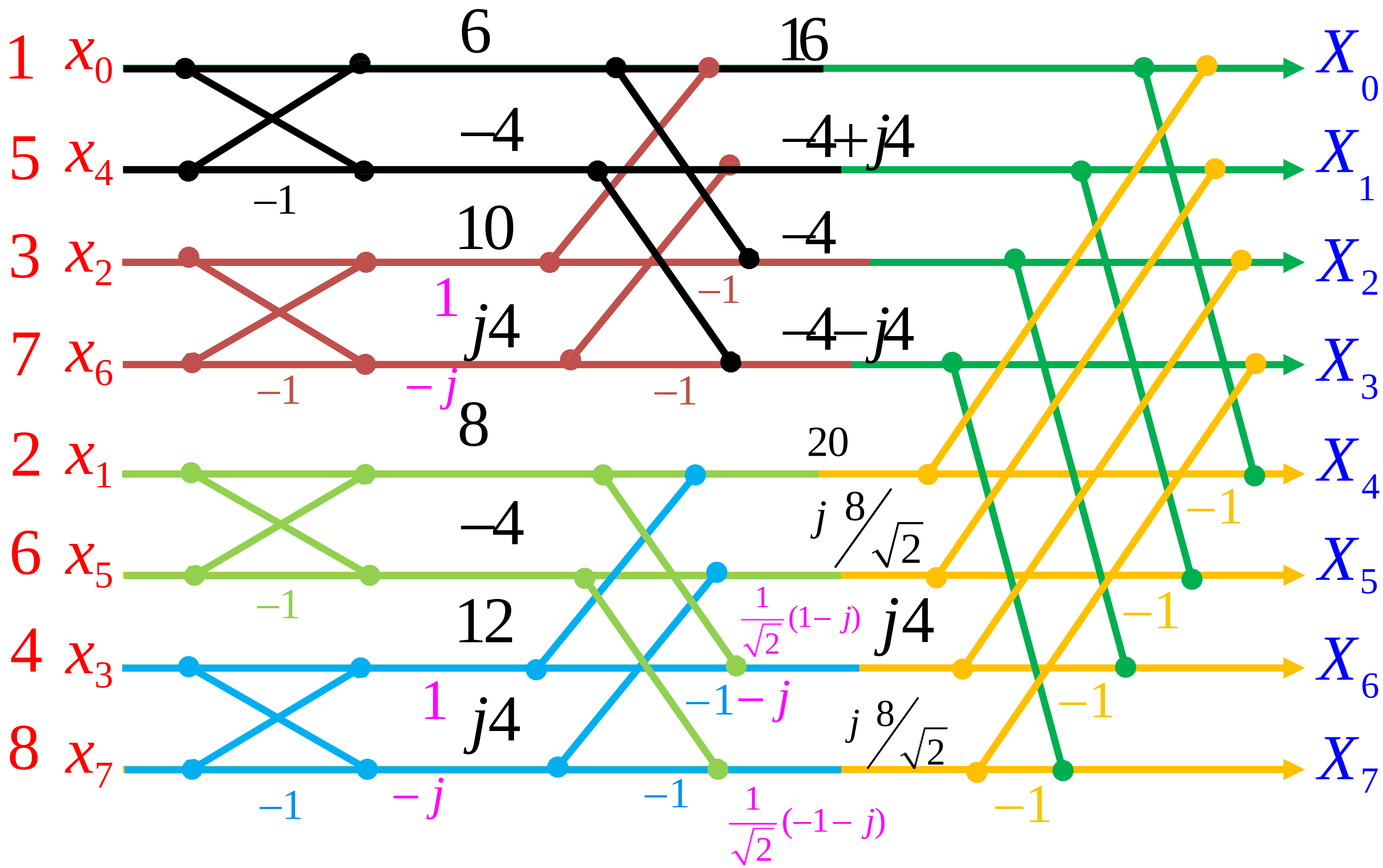

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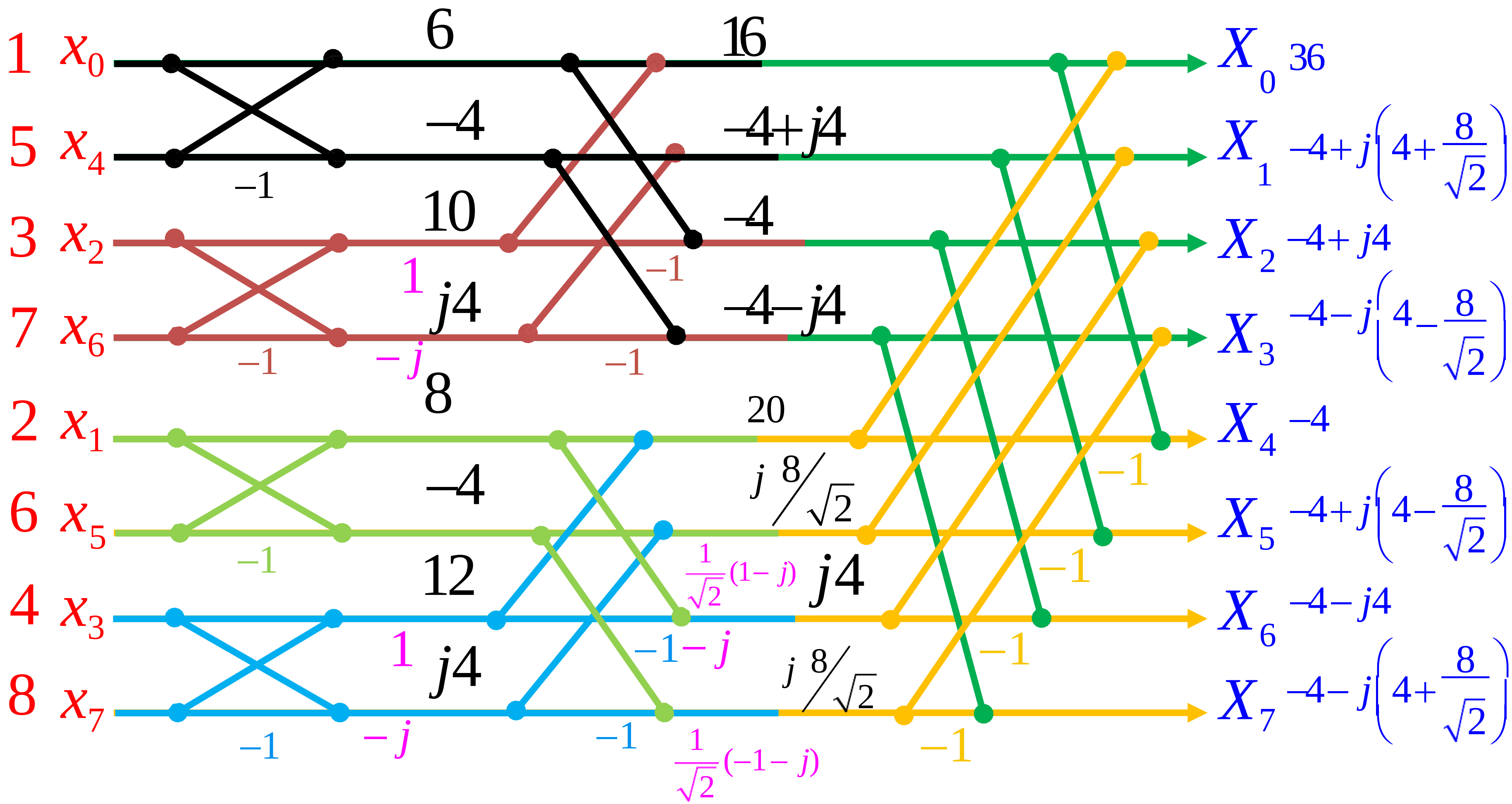

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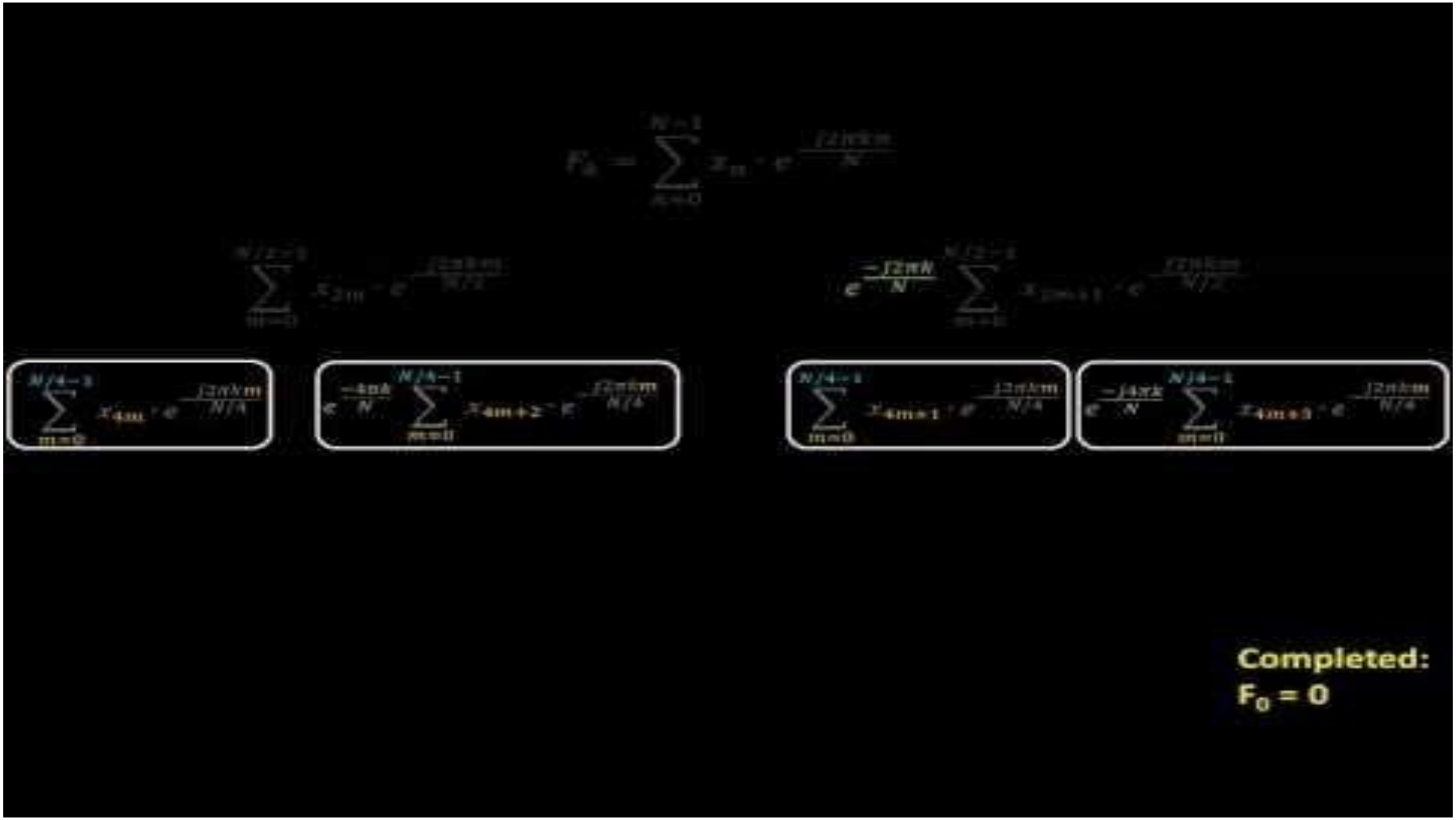


[FFT Algorithm: Step-by-Step]

# Decimation-In-Frequency FFT Algorithm

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- $(N/2)$-point DFT of the $(N/2)$ -point sequence obtained by adding the first half and the last half of the input sequence.
  - Adding the two halves of the input sequence represents **time aliasing**

# Decimation-In-Frequency FFT Algorithm

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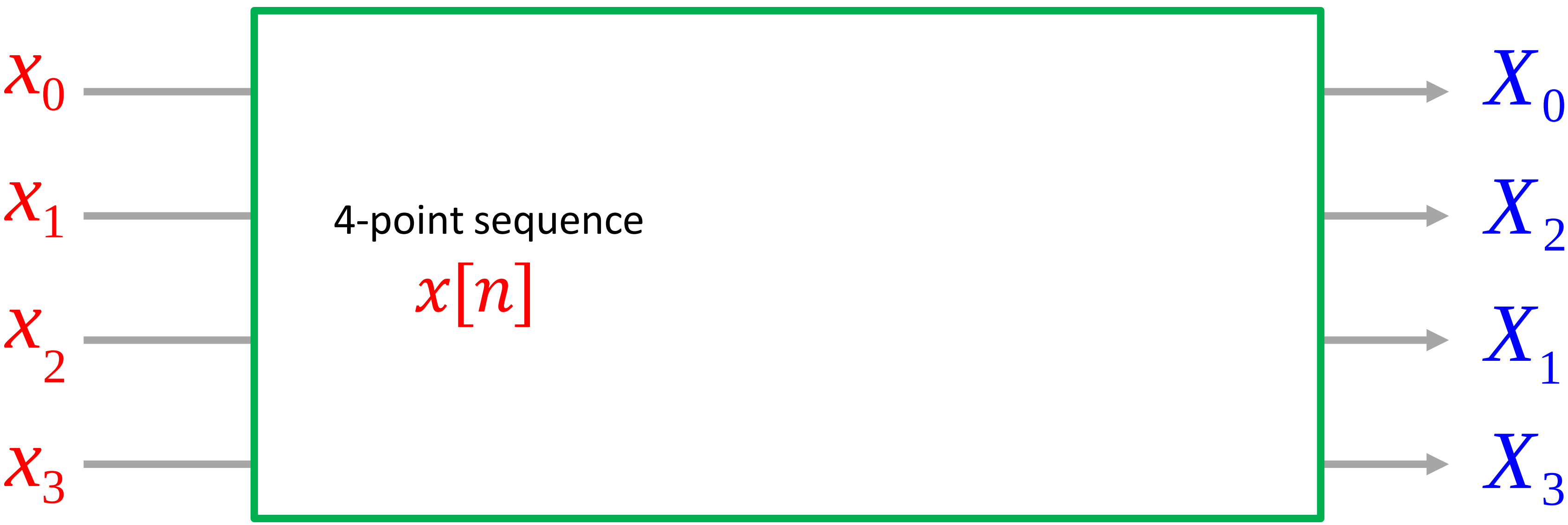

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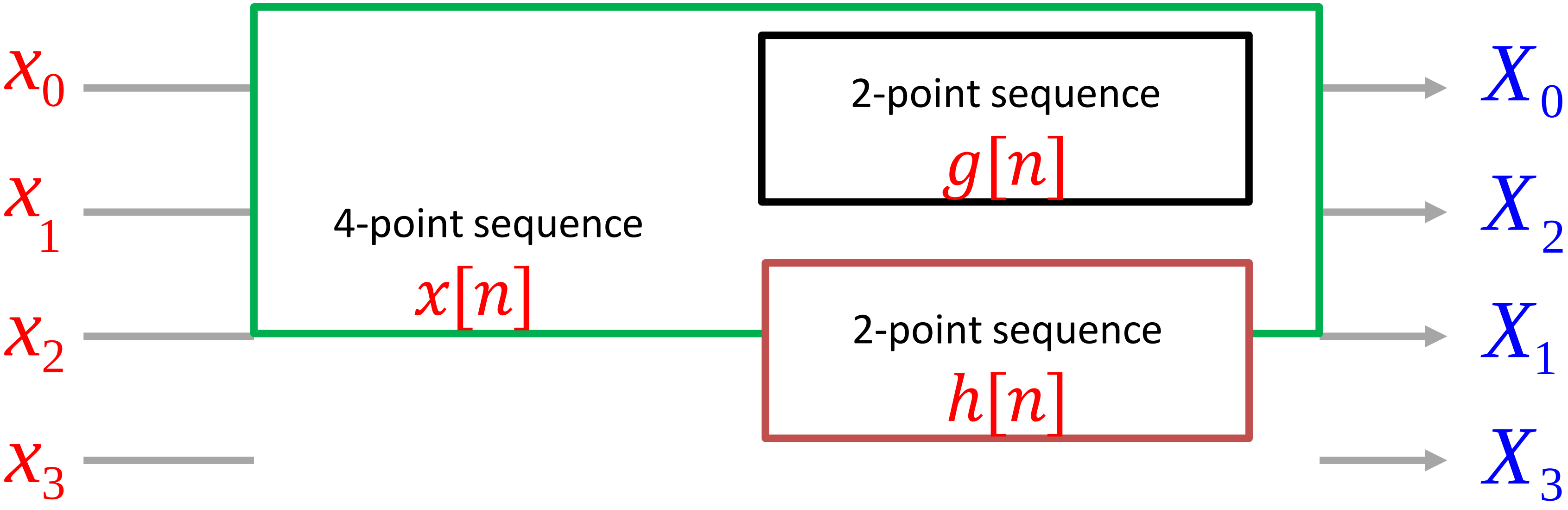

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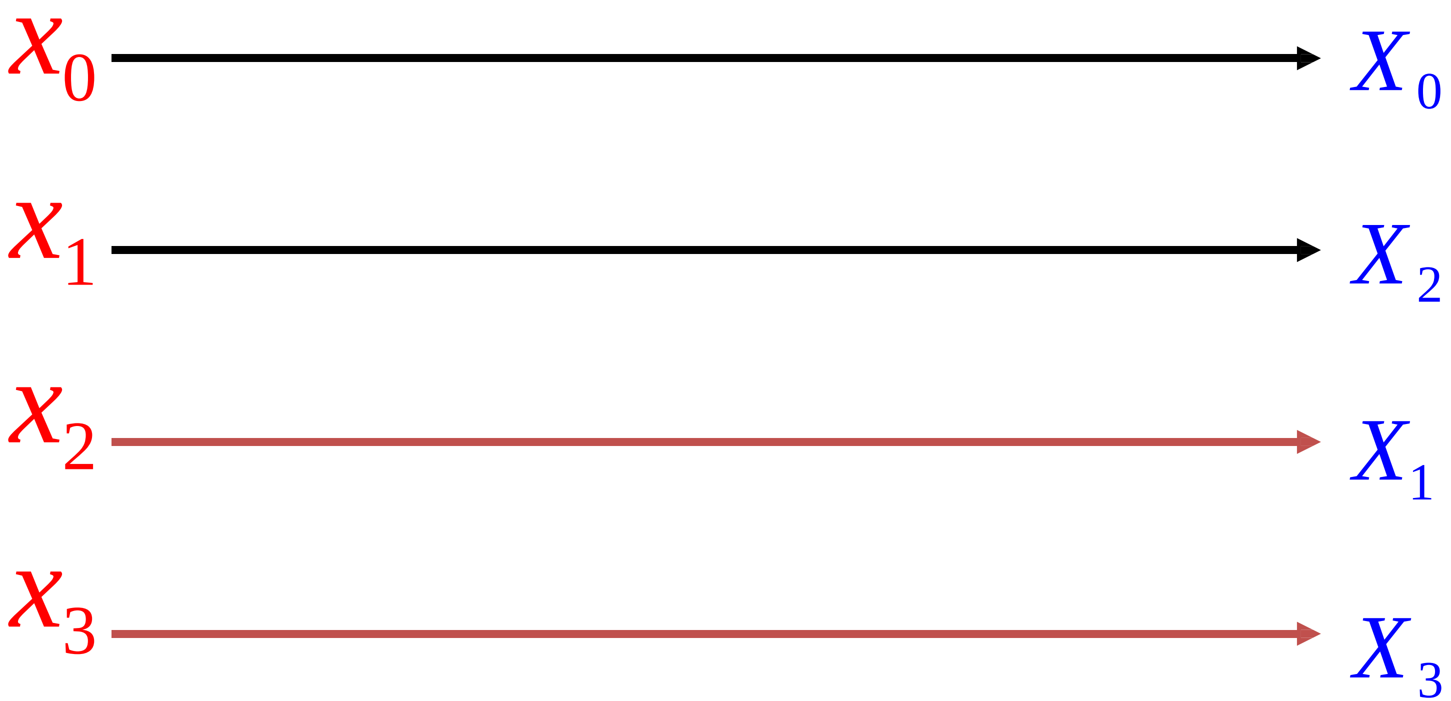

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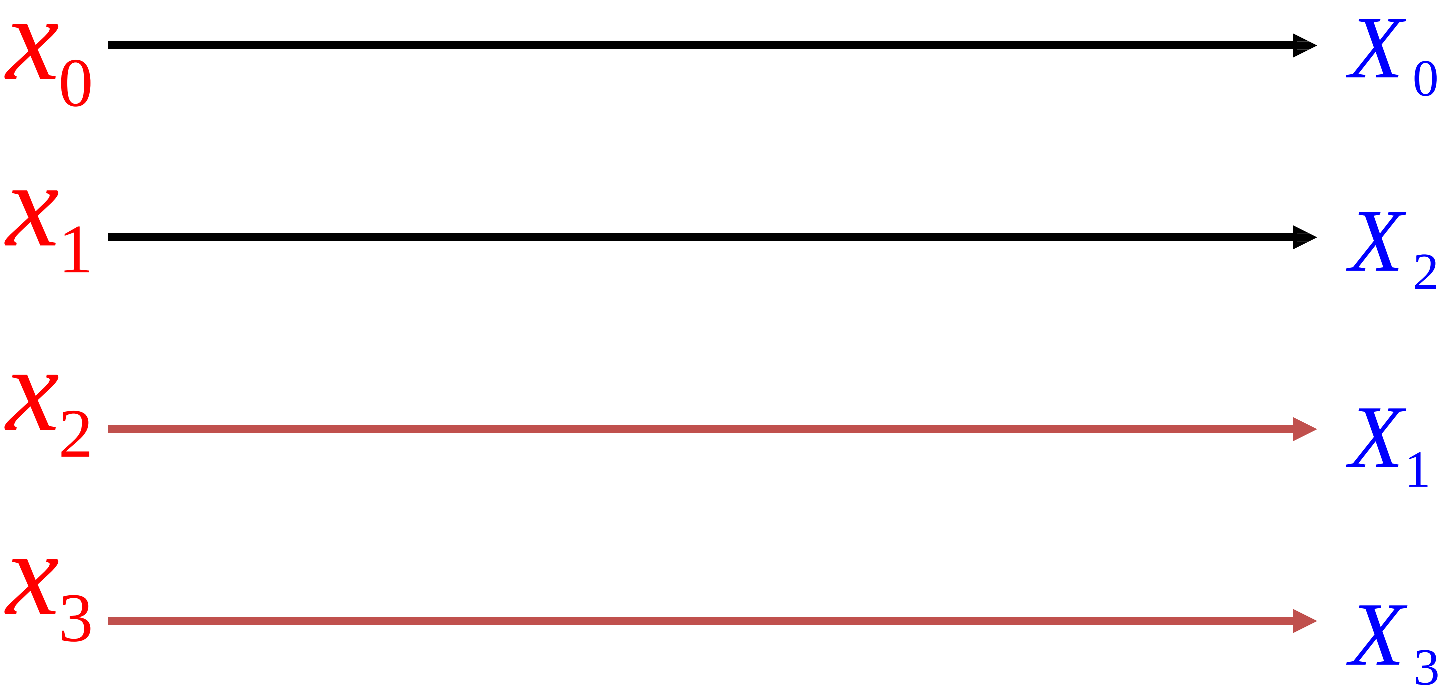

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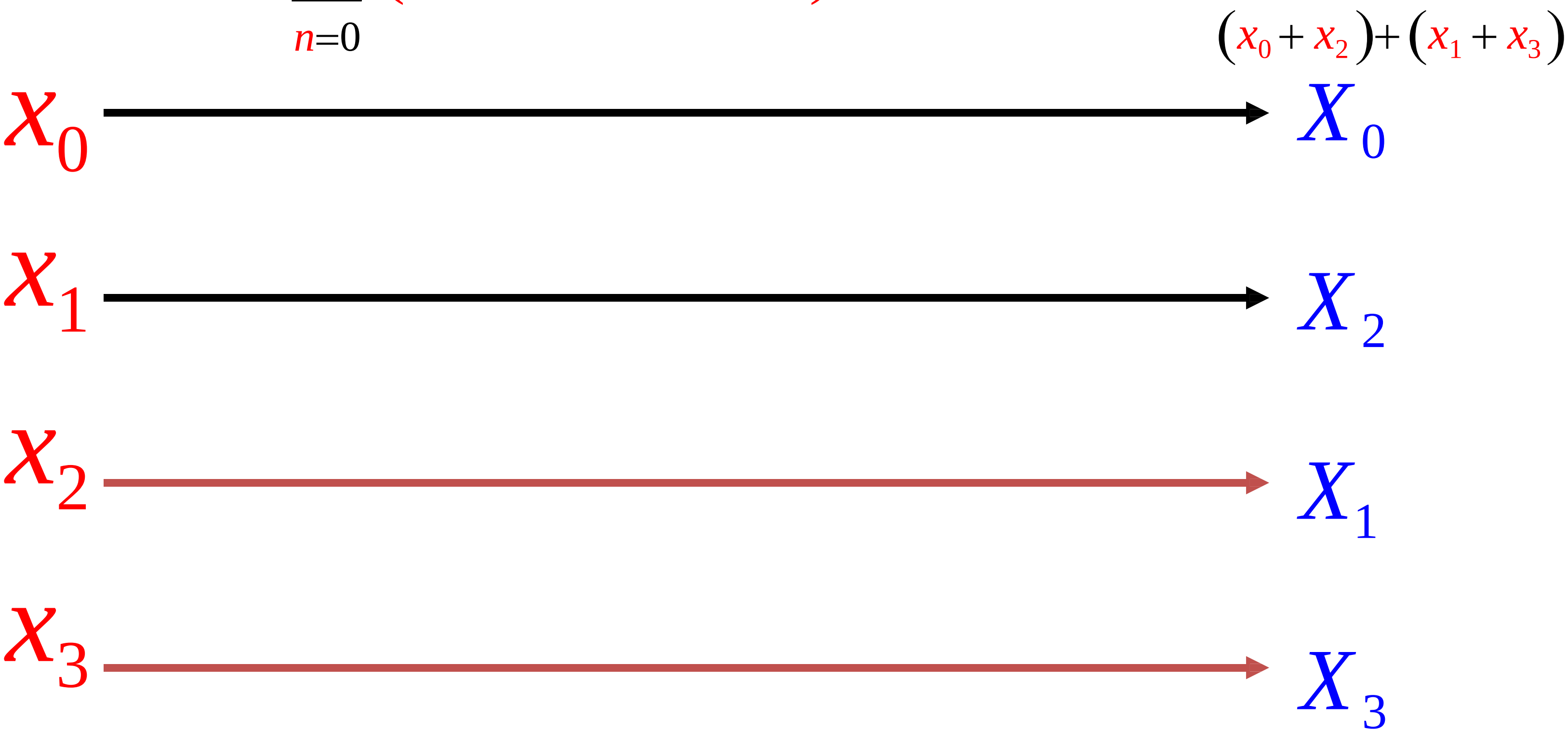

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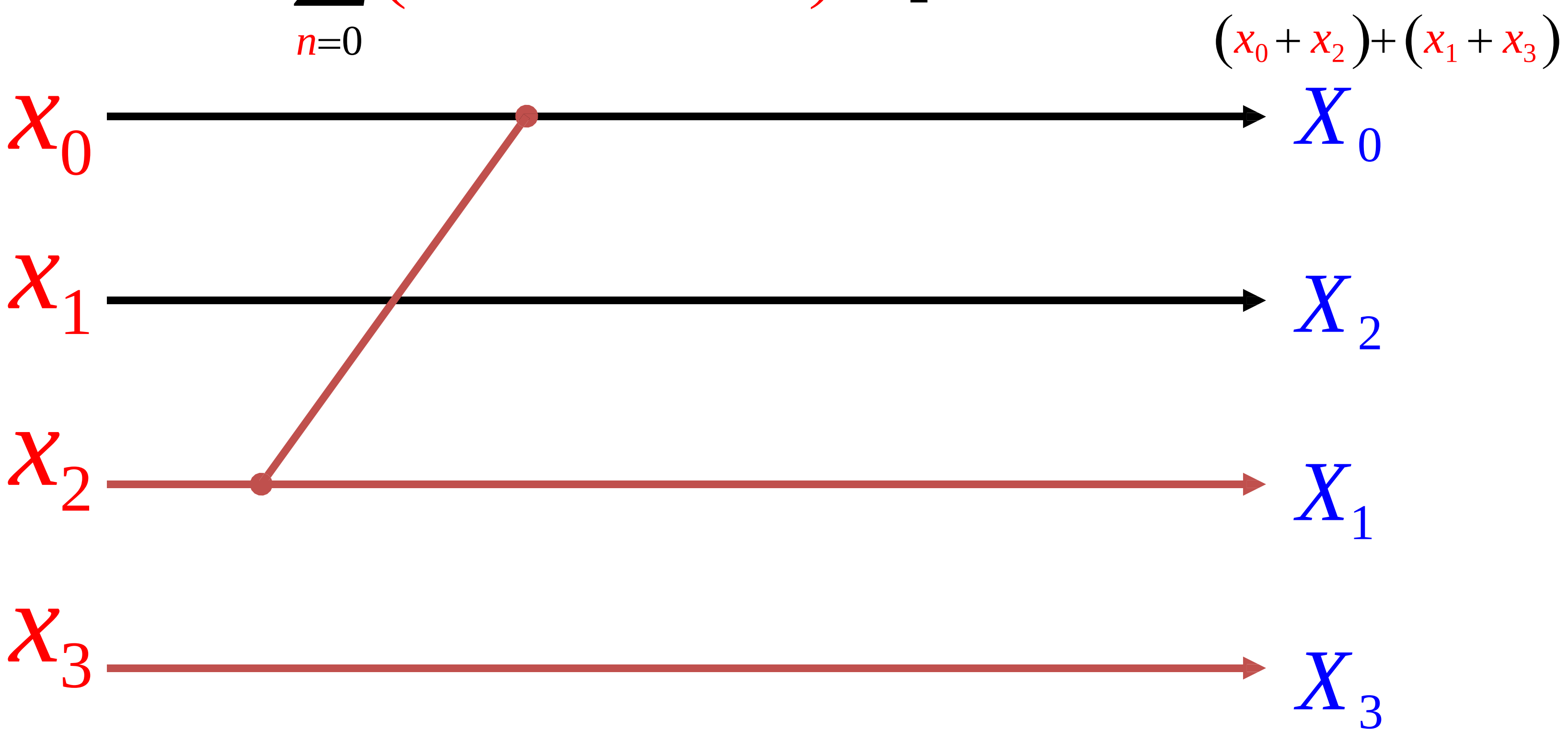

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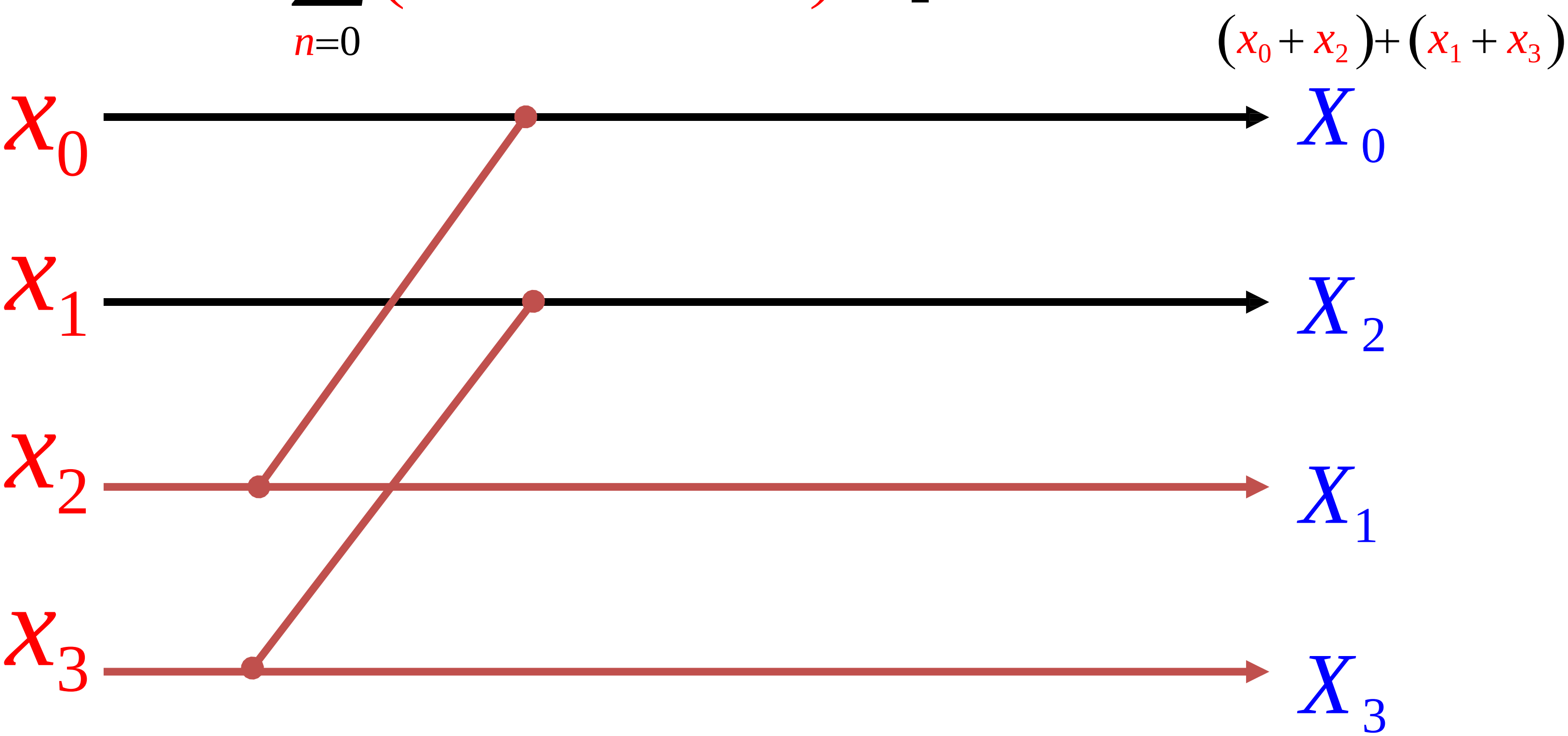

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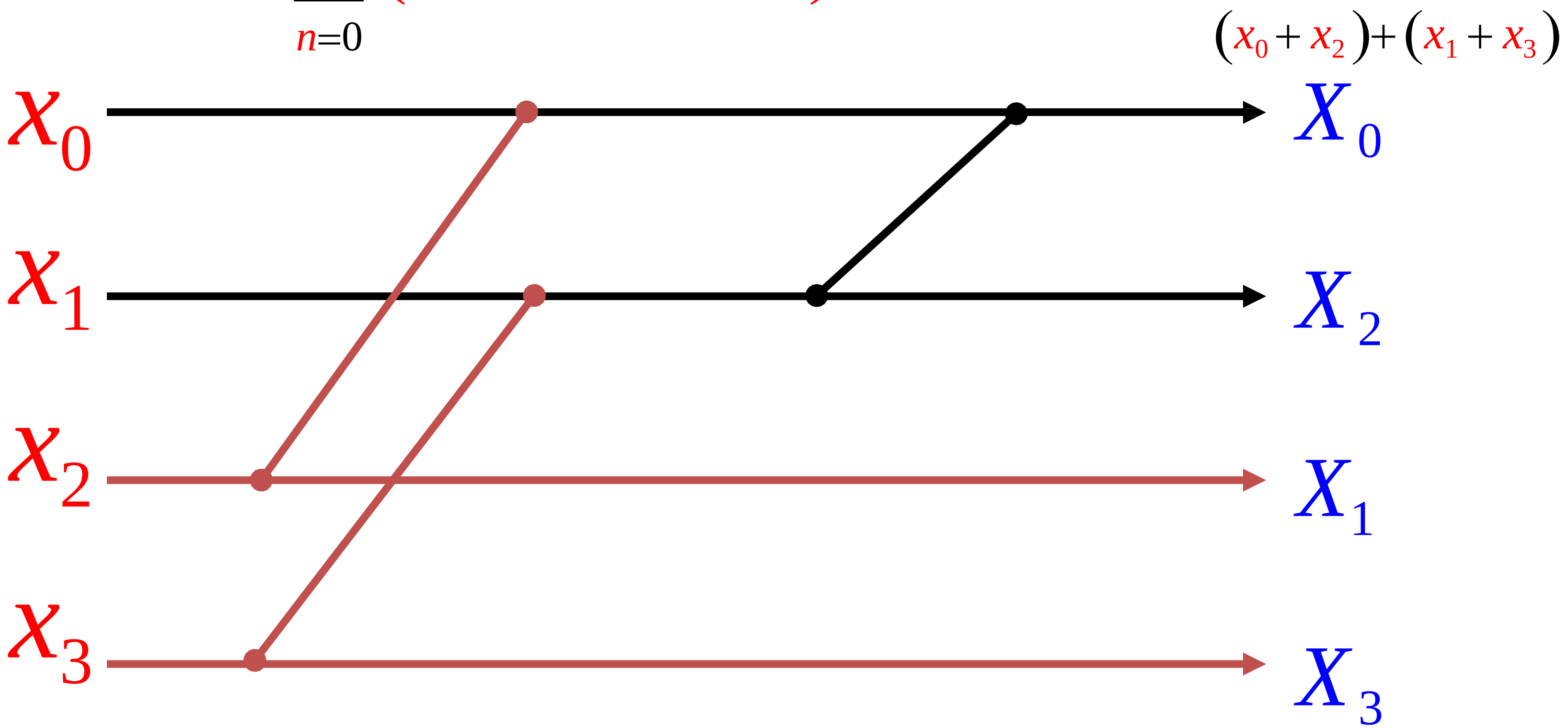
$$X[2r] = \sum_{n=0}^{1}\left(x[n] + x[n+2]\right)W_2^{rn} \qquad r = 0,1$$



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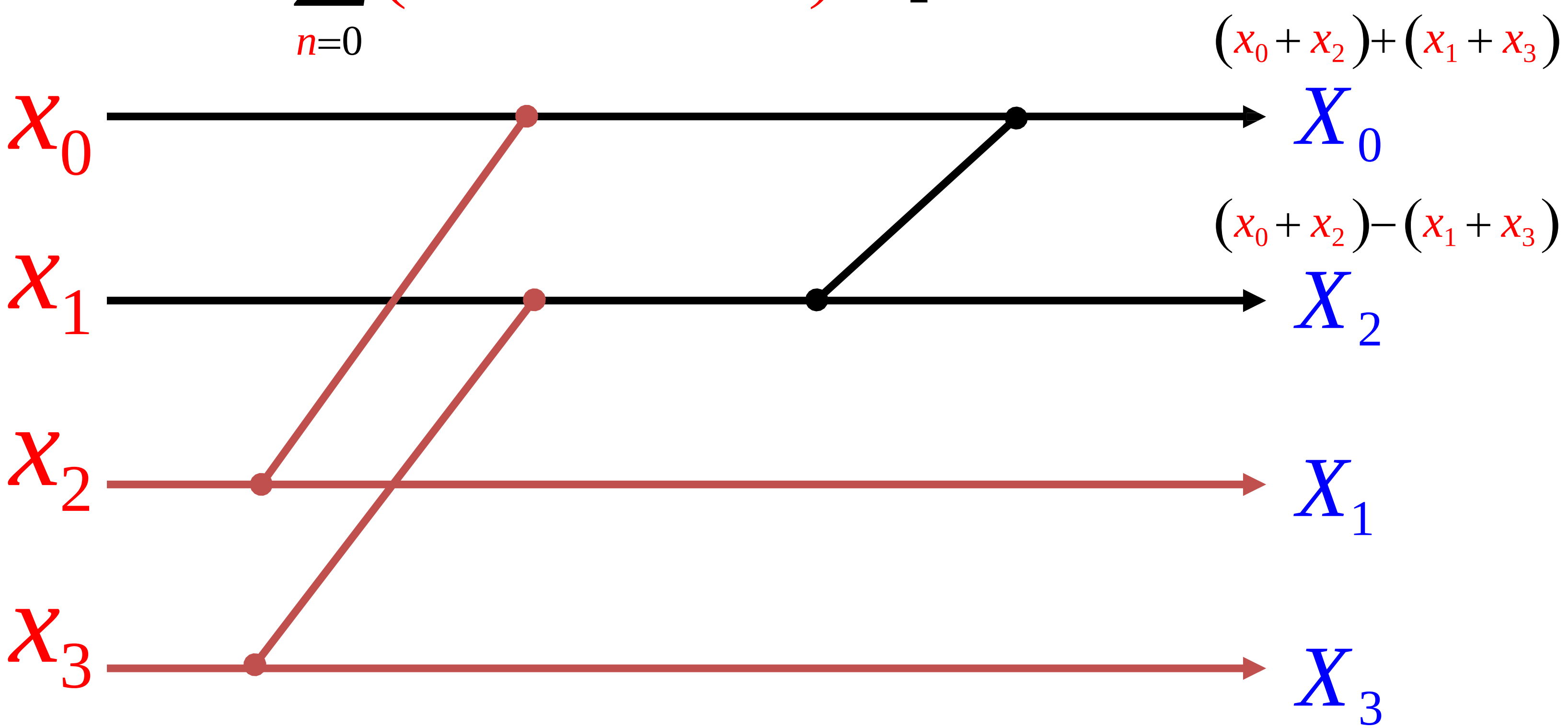

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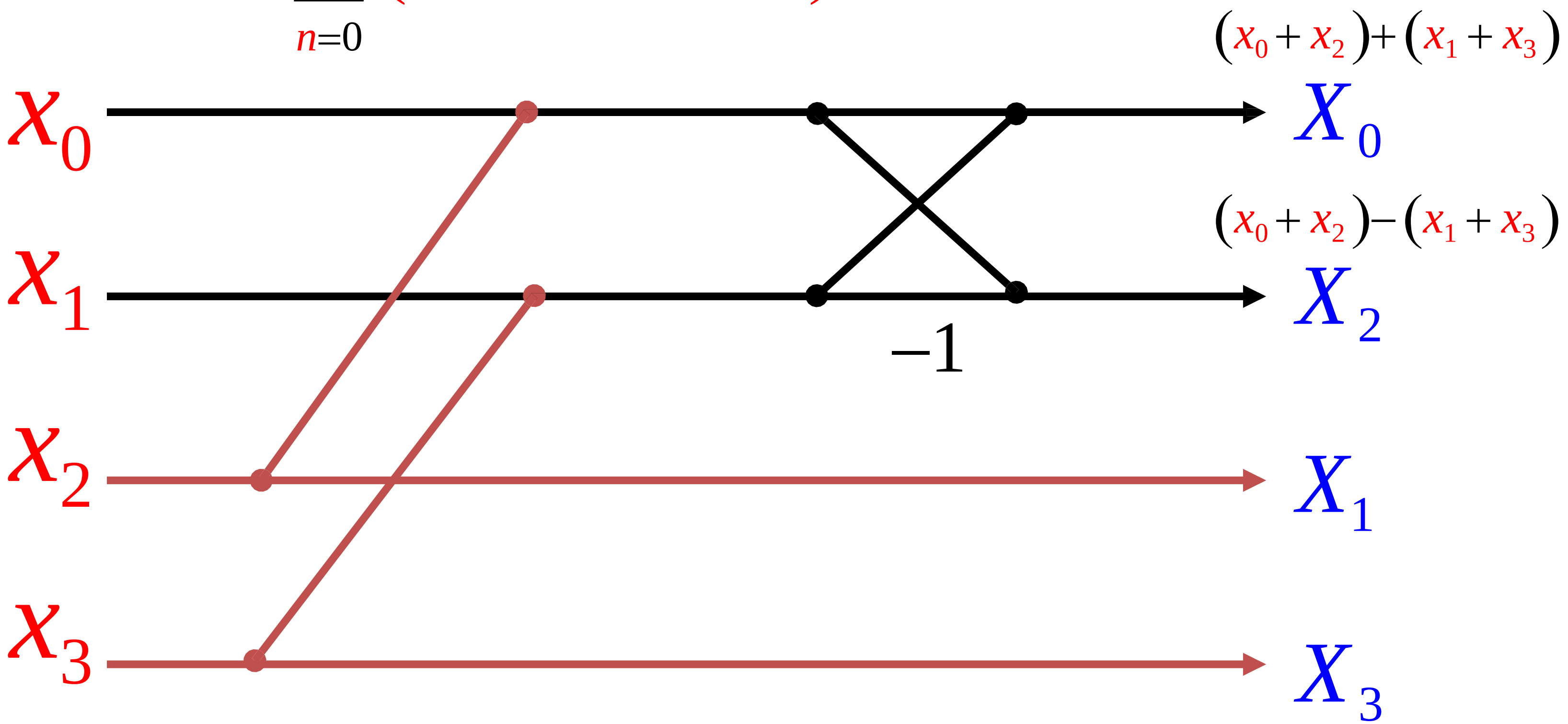

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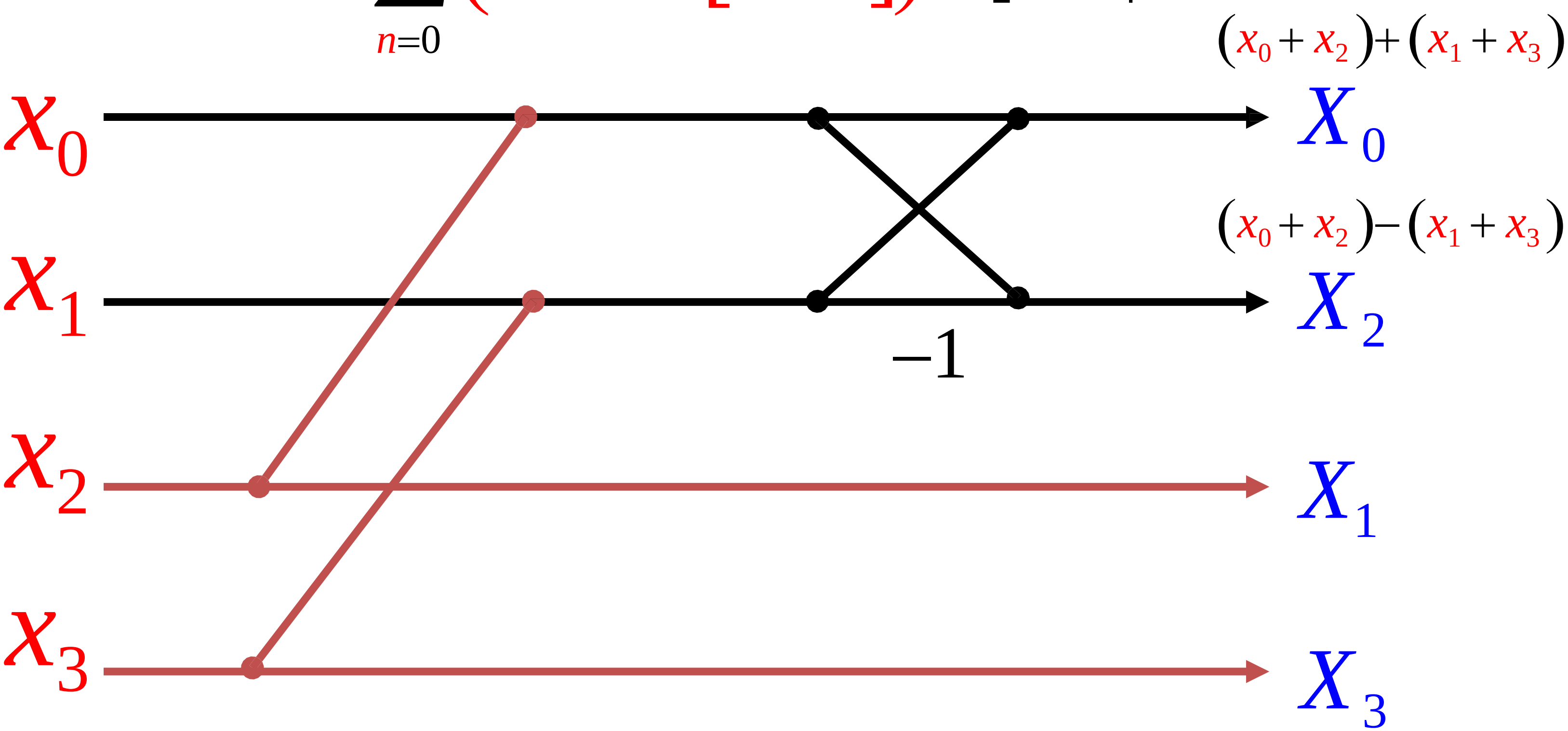

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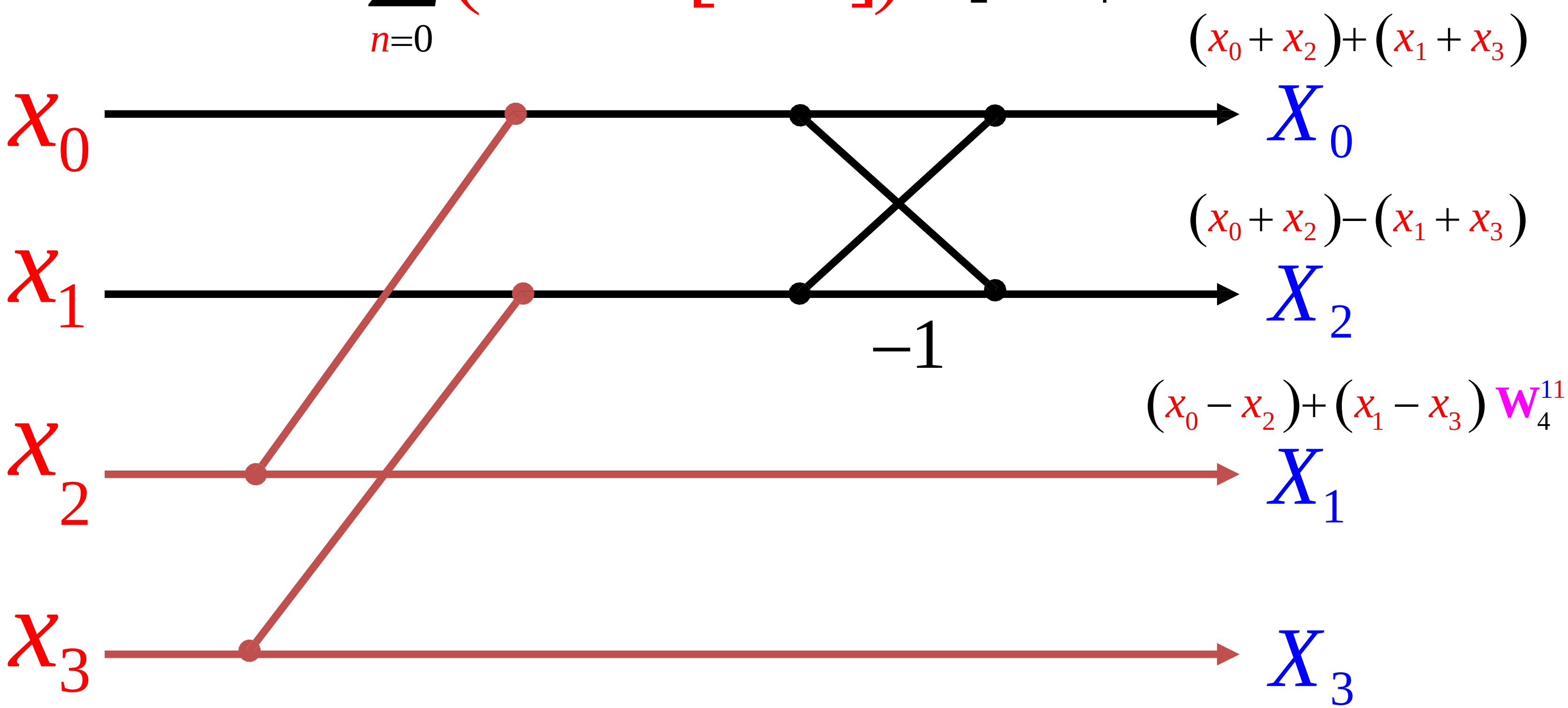

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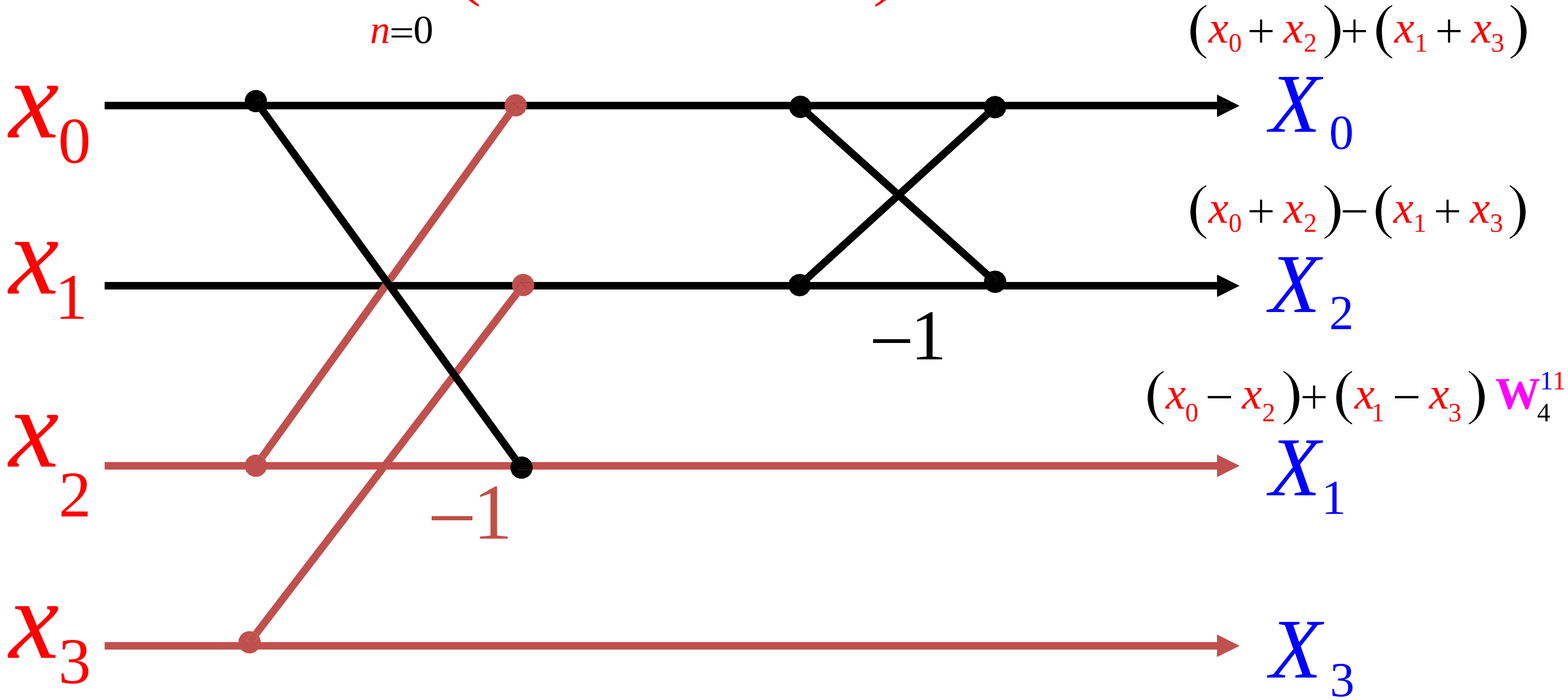

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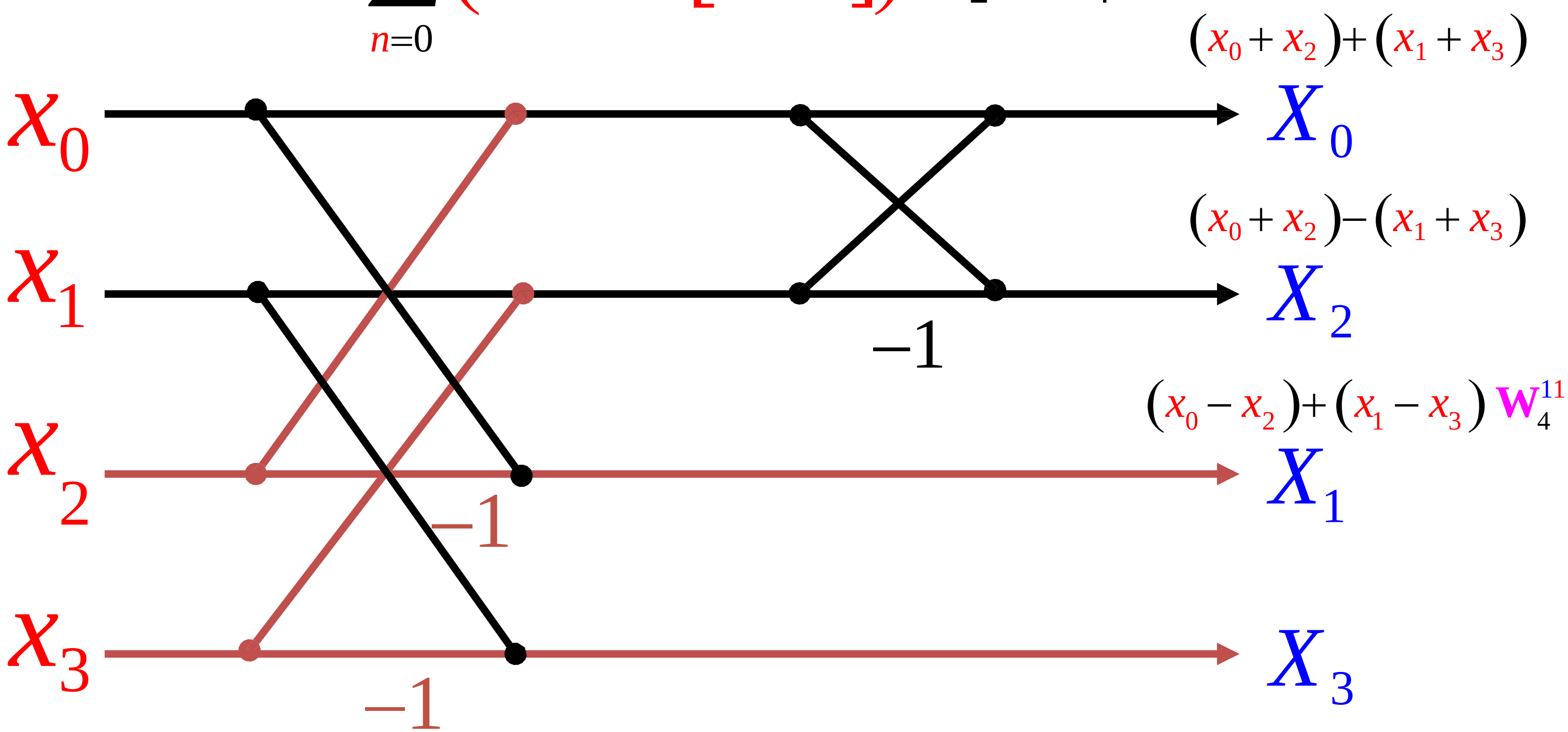

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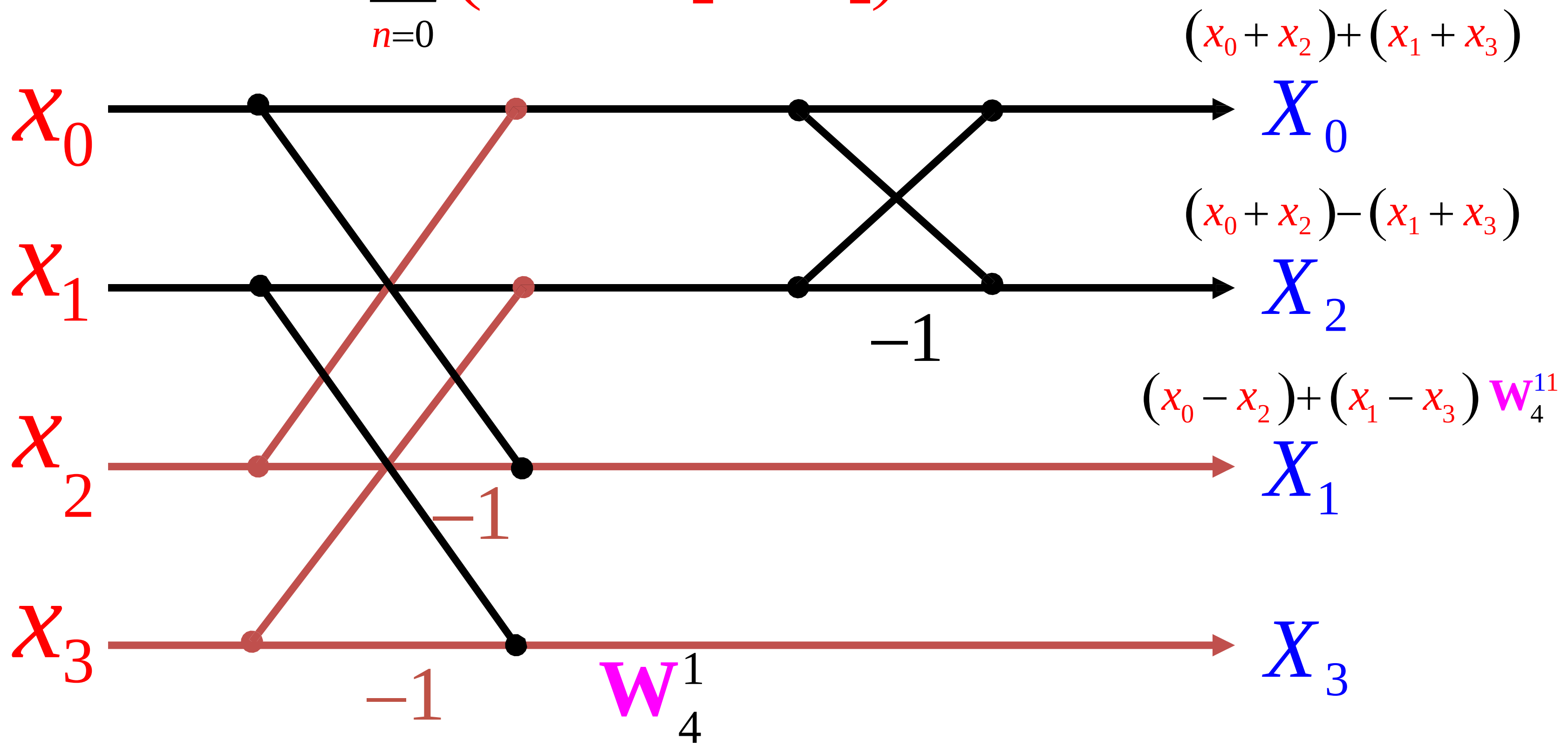

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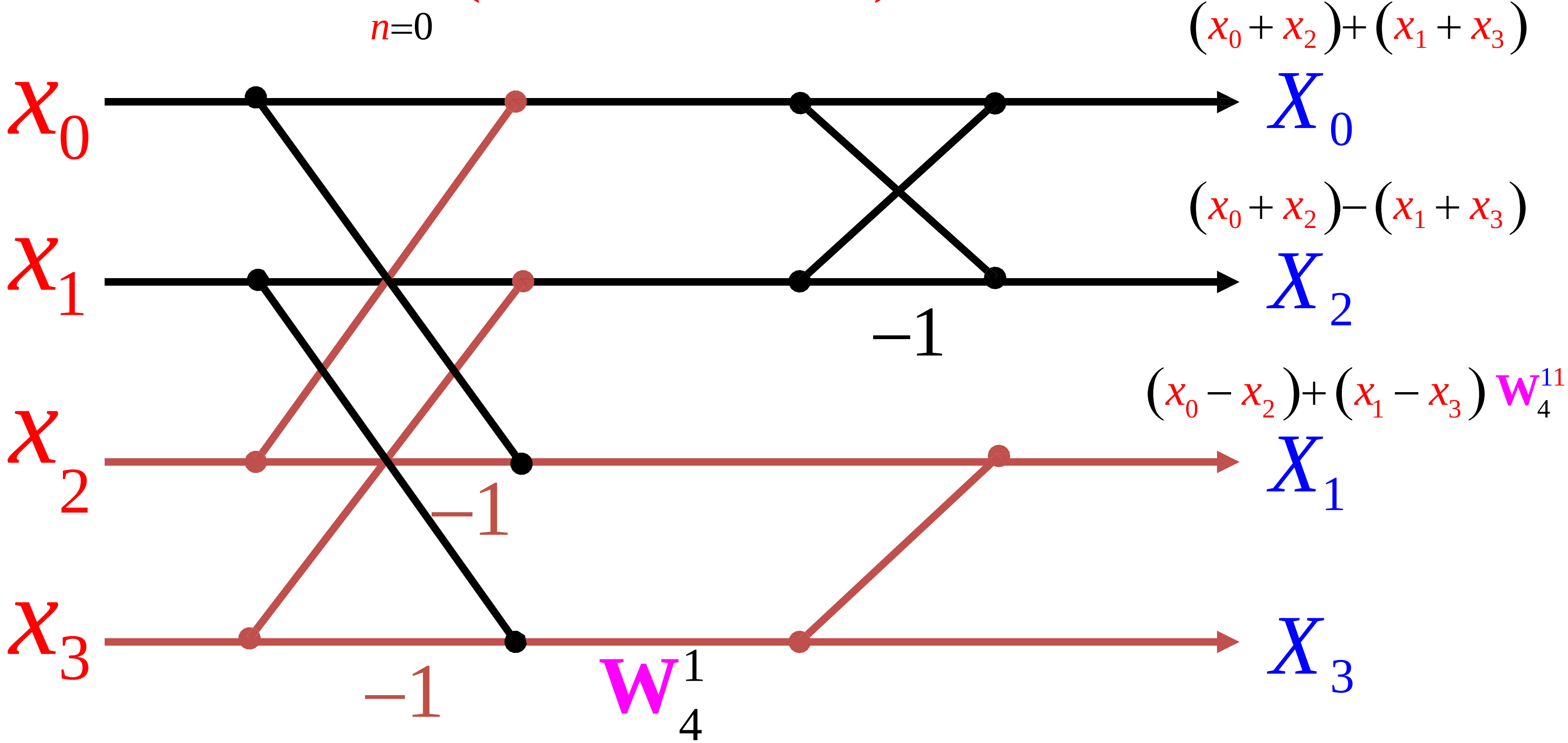

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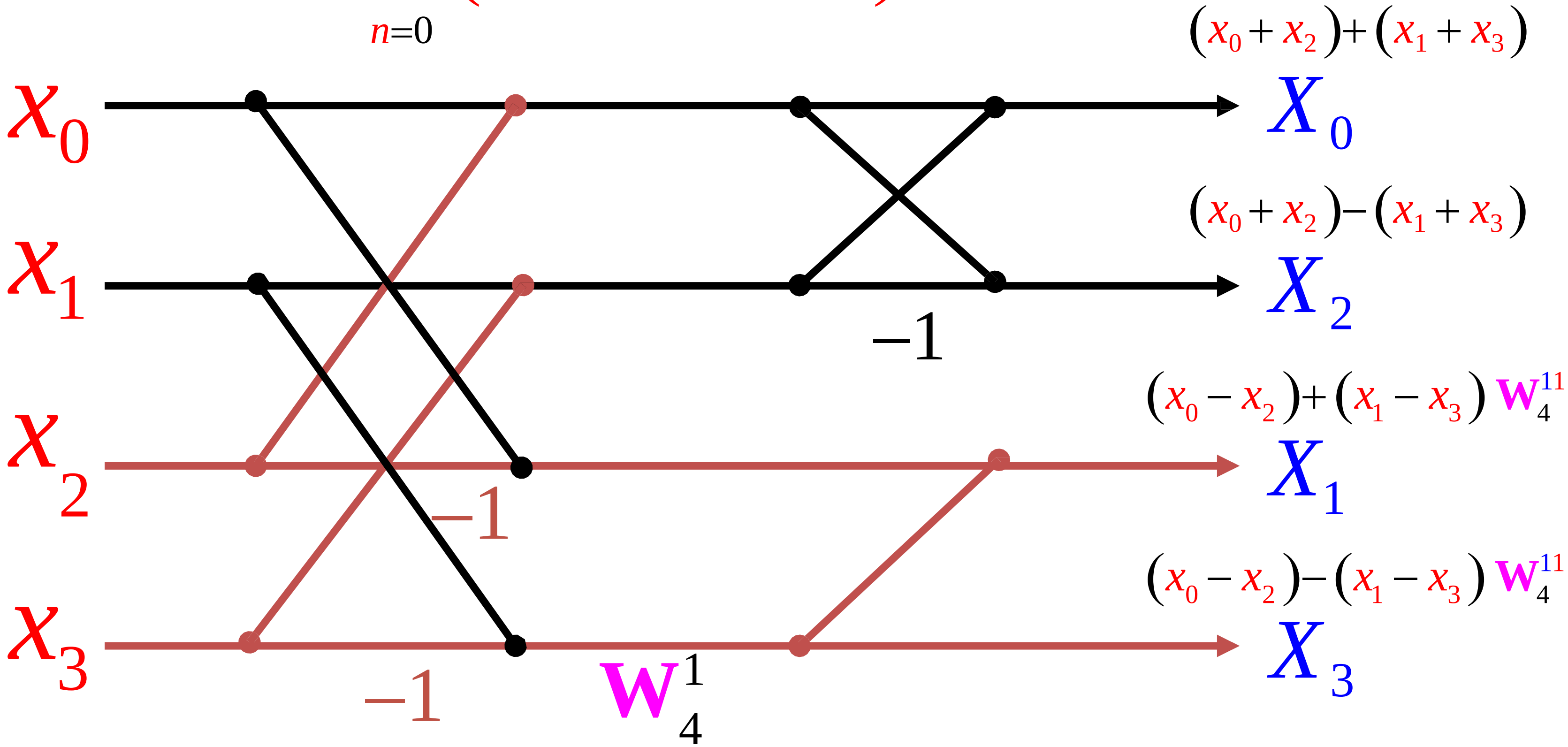

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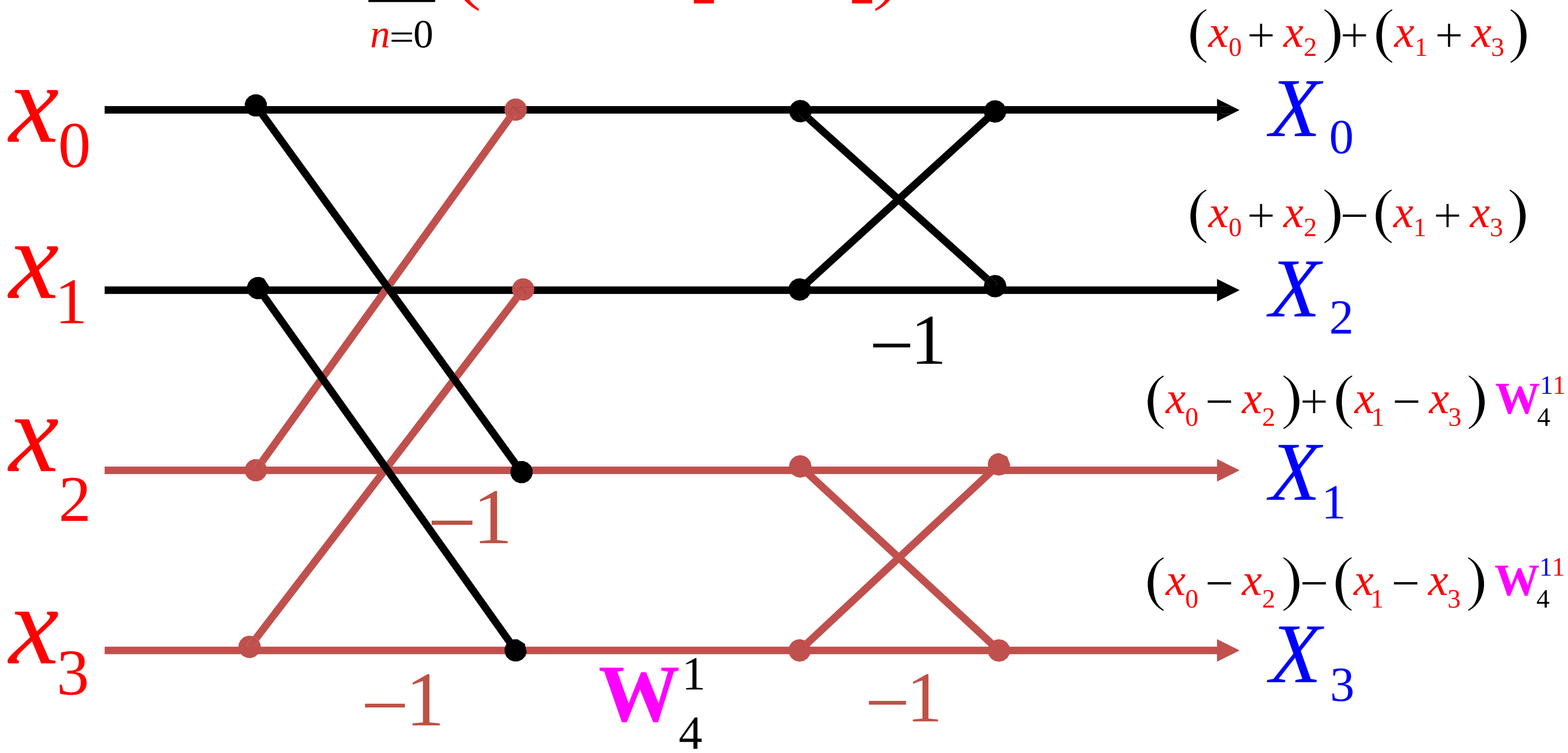

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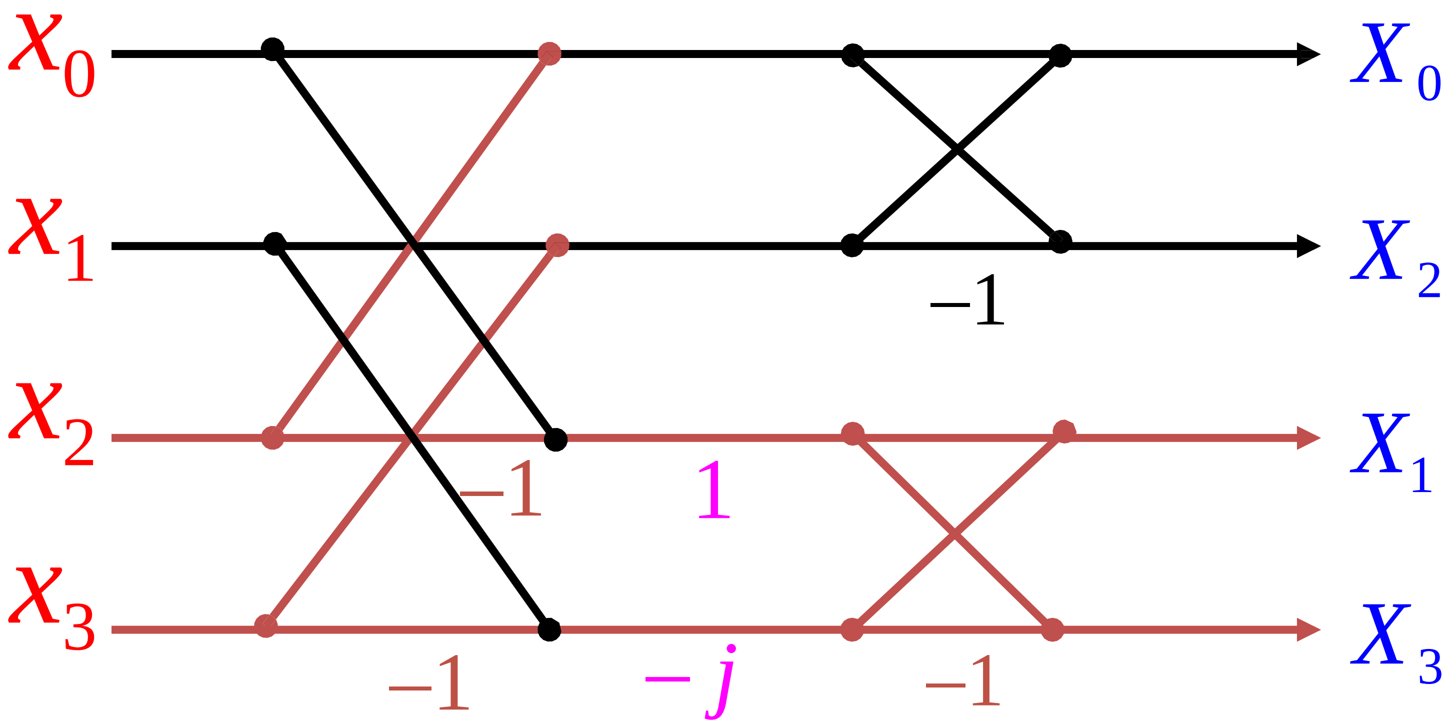
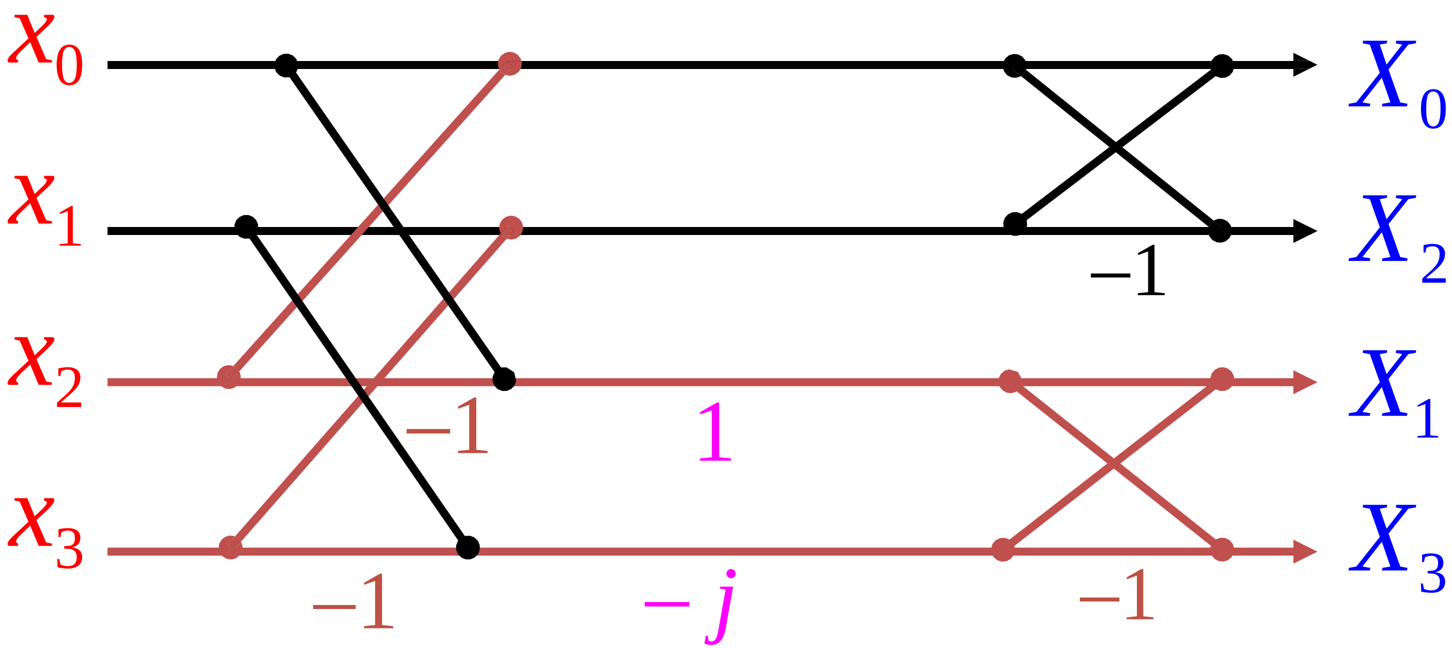


## Example

Compute DFT of $x[n] = \{1,2,1,2\}$ using DIFFFT algorithm.



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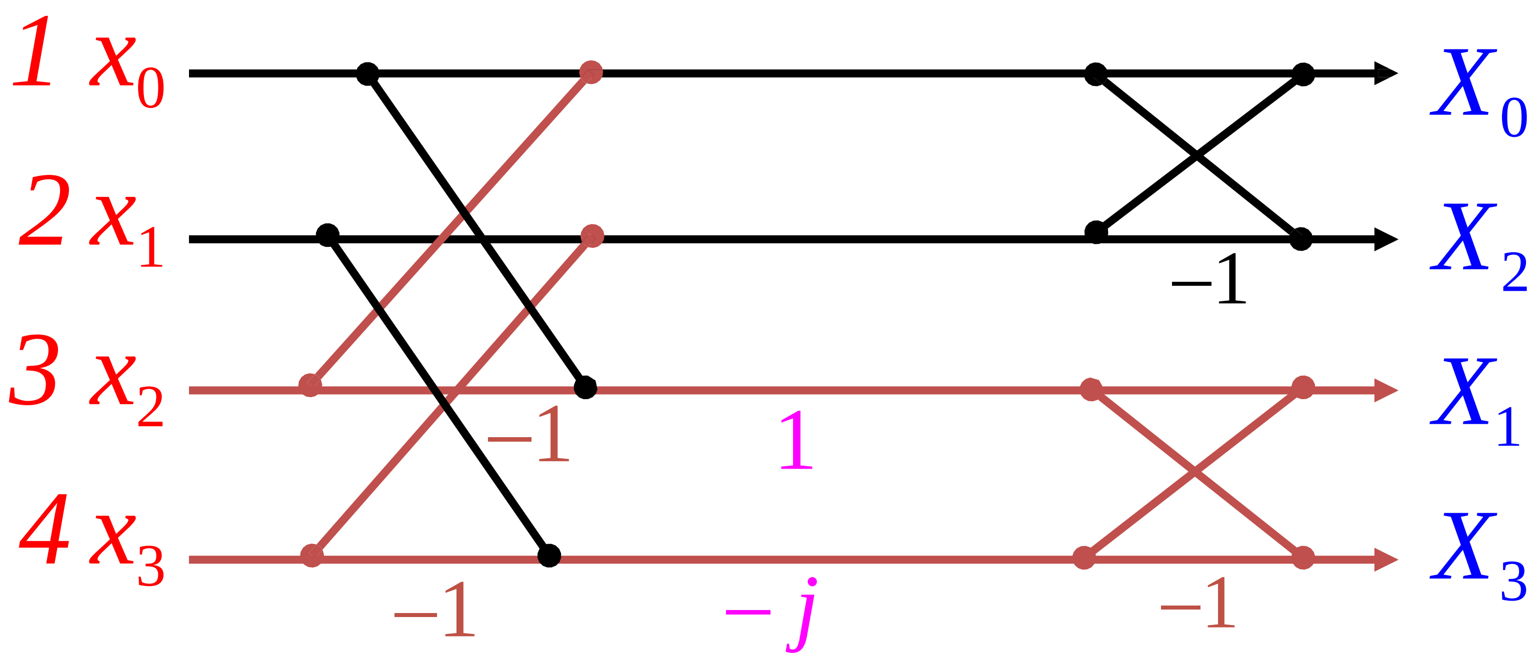

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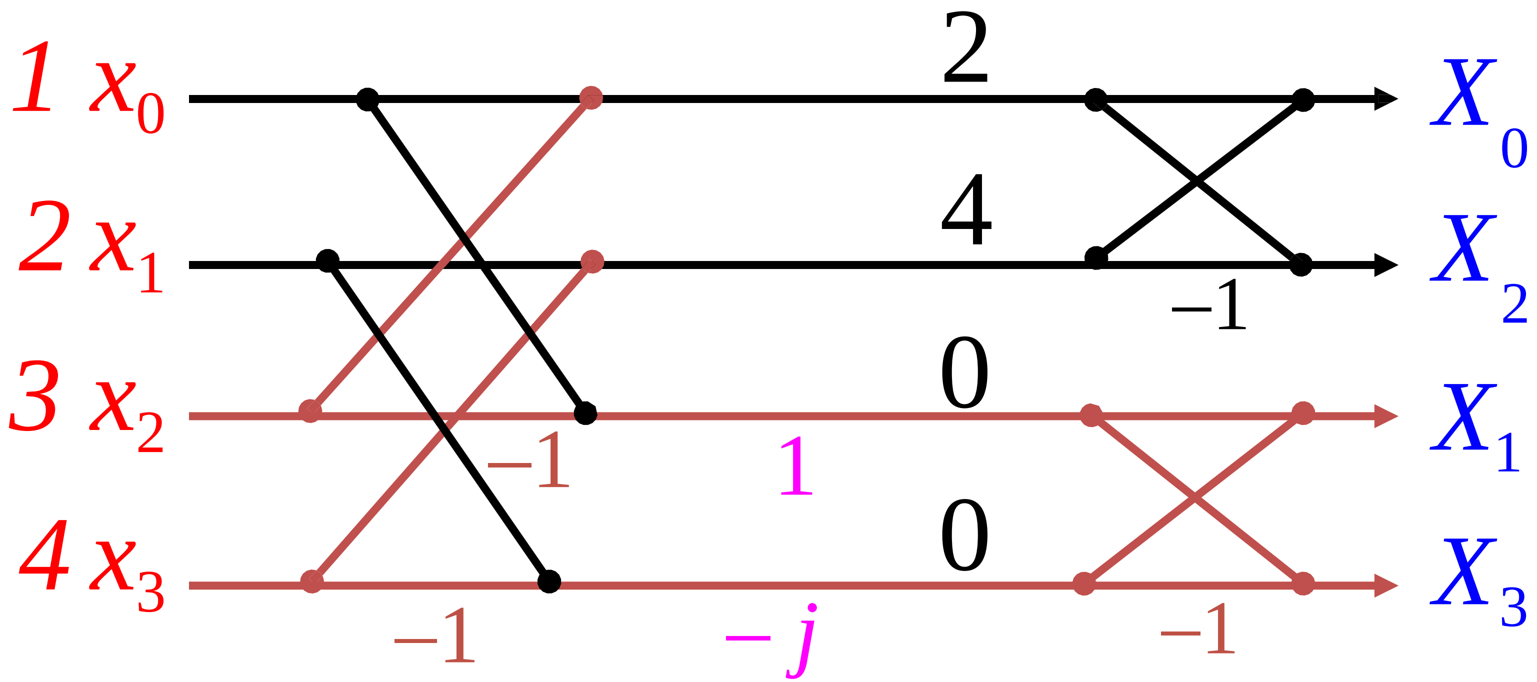

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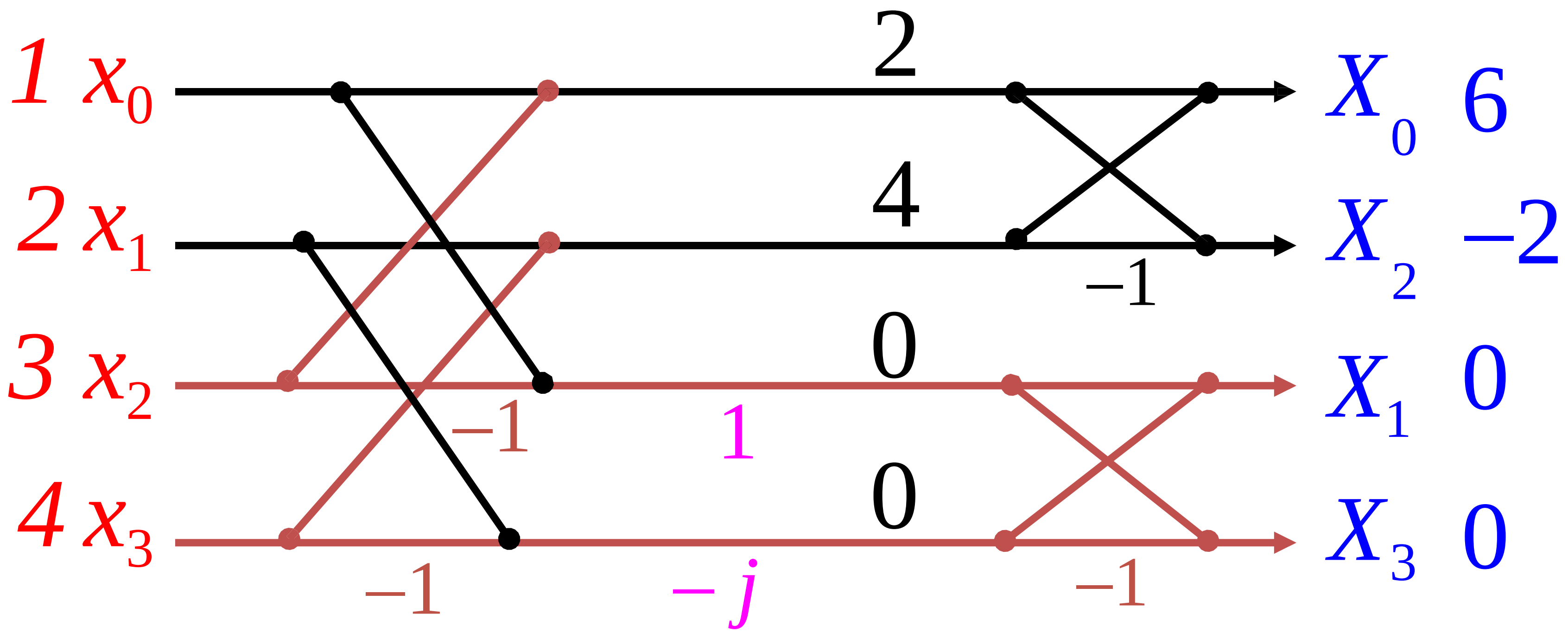

# Thank You