1) **Common Data for Questions 1 & 2**

A function k is defined as follows.

$k : \mathbb{R}^d \times \mathbb{R}^d \to \mathbb{R}$
$k(x_1, x_2) = x_1^T x_2$

Is k a valid kernel?

Yes

No

2) If k is the valid kernel, we apply it to the three-dimensional dataset to run the kernel PCA. Select the correct options.

We cannot run the PCA as k is not a valid kernel.

It will be the same as PCA with no kernel.

It will be the same as the polynomial transformation of degree 2 and then run the PCA.

It will be the same as the polynomial transformation of degree 3 and then run the PCA.

3) Consider ten data points lying on a curve of degree two in a two-dimensional space. We run a kernel PCA with a polynomial kernel of degree two on the same data points. Choose the correct options.

- [x] The transformed data points will lie on a 5-dimensional subspace of $\mathbb{R}^6$.
- [ ] The transformed data points will lie on a 6-dimensional subspace of $\mathbb{R}^{10}$
- [x] There will be some $w \in \mathbb{R}^6$ that all of the data points are orthogonal to.
- [ ] There will be some $w \in \mathbb{R}^{10}$ that all of the data points are orthogonal to.

4) Which of the following matrices can not be appropriate matrix $K = X^T X$ for some data matrix X?

$\begin{bmatrix} 1 & 8 \\ 8 & -1 \end{bmatrix}$

$\begin{bmatrix} 1 & 8 \\ 8 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 8 \\ -8 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

5) A function k is defined as

$k : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$
$k(x_1, x_2) = (x_1^T x_2)^2$
Is k a valid kernel?

- (●) Yes
- ( ) No

6) Kernel PCA was run on the four data points $[1, 2]^T$, $[2, 3]^T$, $[2, -3]^T$, and $[4, 4]^T$ with the polynomial kernel of degree 2. What will be the shape of the matrix K? Notations are used as per lectures.

- ( ) 2×2
- (●) 4×4
- ( ) 6×6
- ( ) None of the above

7) Find the element at the index $(2, 3)$ of the matrix K defined in Question 6. Take the points in the same order.

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8) A dataset containing 200 examples in four-dimensional space has been transformed into higher dimensional space using the polynomial kernel of degree two. What will be the dimension of transformed feature space?

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9) Let $x_1, x_2, ..., x_n$ be d-dimensional data points $(d > n)$ and X be the matrix of shape $d \times n$ containing the data points. The k^{th} largest eigenvalue and corresponding unit eigenvector of $X^T X$ is λ and α_k, respectively. What will be the projection of x_i on the k^{th} principal component? **1 po**

- $x_i^T \alpha_k$
- $\frac{x_i^T \alpha_k}{\lambda}$
- $\frac{x_i^T X \alpha_k}{\sqrt{\lambda}}$
- $\frac{x_i^T X \alpha_k}{\sqrt{n\lambda}}$

10) Let k_1 and k_2 be two valid kernels. Is $3k_1 + 5k_2$ a valid kernel?

- Yes
- No