

$$u_i'(t) = \underline{f_i(u)}$$

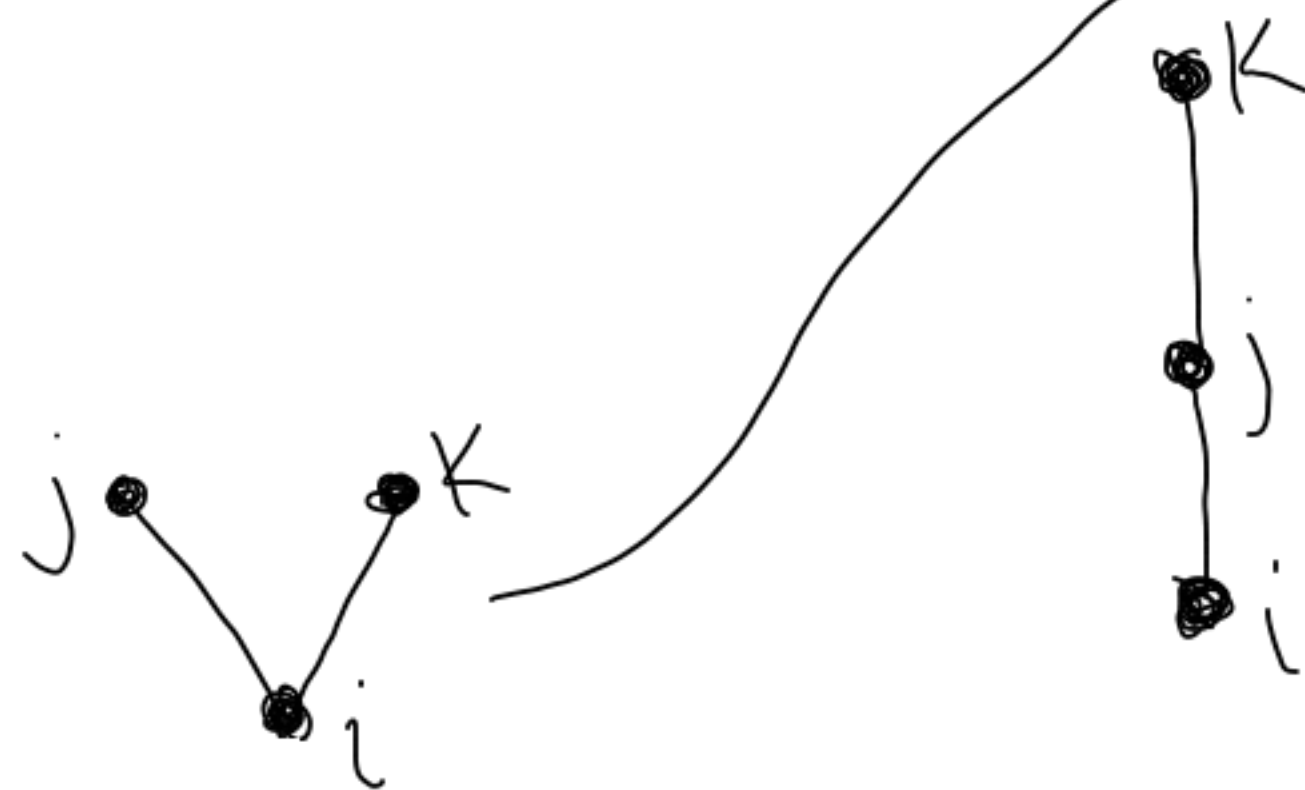
$$u_i''(t) = \sum_j \frac{\partial f_i}{\partial u_j} \frac{du_j}{dt} = \sum_j \frac{\partial f_i}{\partial u_j} \underline{f_j(u)} = \underline{f' f}$$

$$u_i'''(t) = \boxed{\sum_k \sum_j \frac{\partial^2 f_i}{\partial u_k \partial u_j} f_j(u) f_k(u)} + \underbrace{\sum_k \sum_j \frac{\partial f_j}{\partial u_k} \frac{\partial f_i}{\partial u_j} f_k(u)}$$

$$u_i'''(t) = \underline{f''(f, f)} + \underline{f' f' f}$$

$$u_i'''(t) = \underline{f'''}(f, f, f) + \underline{3f''}(f, f' f) + f' f''(f, f) + f' f' f' f$$

These derivatives have
a 1-1 correspondence
with rooted trees.



$$\sum_{j,k} \frac{\partial^2 f_i}{\partial u_j \partial u_k} f_j f_k$$

Elementary differential

$$\sum_{j,k} \frac{\partial f_i}{\partial u_j} \frac{\partial f_j}{\partial u_k} f_k$$

$$\sum_{i,k,j} b_i a_{ik} a_{ij}$$

$$= \sum_i b_i c_i c_i = b^T C^2 = \frac{1}{3}$$

Elementary weight

