$u(x,t)$

$$u_t = \sum_{j=0}^{n} \alpha_j \frac{\partial^j u}{\partial x^j}$$

$$u(x,t) = \hat{u}_\xi(t)\, e^{i\xi x}$$

$$\hat{u}'_\xi(t) = \underbrace{\left(\sum_{j=0}^{n} \alpha_j (i\xi)^j\right)} \hat{u}_\xi(t) = P(\xi)\, \hat{u}_\xi(t)$$

$$\hat{u}_\xi(t) = e^{P(\xi)t}\, \hat{u}_\xi(0)$$

$$u(x,t) = e^{P(\xi)t}\, e^{i\xi x}\, \hat{u}_\xi(0)$$

Example: $u_t = u_{xxx}$ $\quad P(\xi) = -i\xi^3$

$$u(x,t) = e^{-i\xi^3 t + i\xi x}\, \hat{u}_\xi(0)$$

$$= e^{i\xi(x - \xi^2 t)}\, \hat{u}_\xi(0)$$

Like advection, but high wavenumbers move more quickly. (dispersion)

Example 2: $u_t = -u_{xxxx}$

$p(\xi) = -\xi^4$

$u(x,t) = e^{-\xi^4 t} e^{i\xi x} \hat{u}_\xi(0)$

Like diffusion, but even stronger decay of high wavenumbers.

In general:

- Odd order derivatives lead to dispersion.
- Even order derivatives lead to dissipation/diffusion.

$u_t = \qquad u_{xxx}$