Stiffness

$$U'(t) = \lambda(U - \cos(t)) - \sin(t)$$

$$U(0) = 1$$

$$\Rightarrow U(t) = \cos(t)$$

$$K \leq \frac{2}{|\lambda|}$$

$$\lambda = -10$$

$$K = 0.21$$

With initial Condition

$$U(t_0) = U_0$$

the exact solution

$$U(t) = e^{\lambda(t-t_0)}(U - \cos(t_0)) + \cos(t)$$

The step size K may be limited by:

(local)
① accuracy K_{acc}

② stability K_{stab}

$$K \leq \min(K_{acc}, K_{stab})$$

If $K_{acc} \hat{\sim} K_{stab}$, we can use an explicit method.

If $K_{stab} \lll K_{acc}$ for an explicit method, we should use a method with a larger K_{stab}.

Absolute stability:

$$K_{stab} = \max_K \{K \lambda \in S \quad \forall \lambda \}$$

(Unconditional Stability)

We want $K_{stab} = \infty$. Then we can take $K = K_{acc}$.

A(α)-Stability