

Finite difference Methods

$$\frac{df}{dx} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) \approx \frac{f(x+h) - f(x)}{h}$$

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2}f''(x) + O(h^3)$$

$$\frac{\cancel{f(x)} + \cancel{h}f'(x) + \frac{h^2}{2}f''(x) + O(h^3) - \cancel{f(x)}}{\cancel{h}}$$

$$= f'(x) + \underbrace{\frac{h}{2}f''(x)}_{\text{Leading truncation}} + O(h^2)$$

$g(h) = O(f(h))$ means

there exists C, h_0
such that

$$\frac{g(h)}{f(h)} < C \text{ for } h < h_0$$

Read Ch. 1
and appendix A.