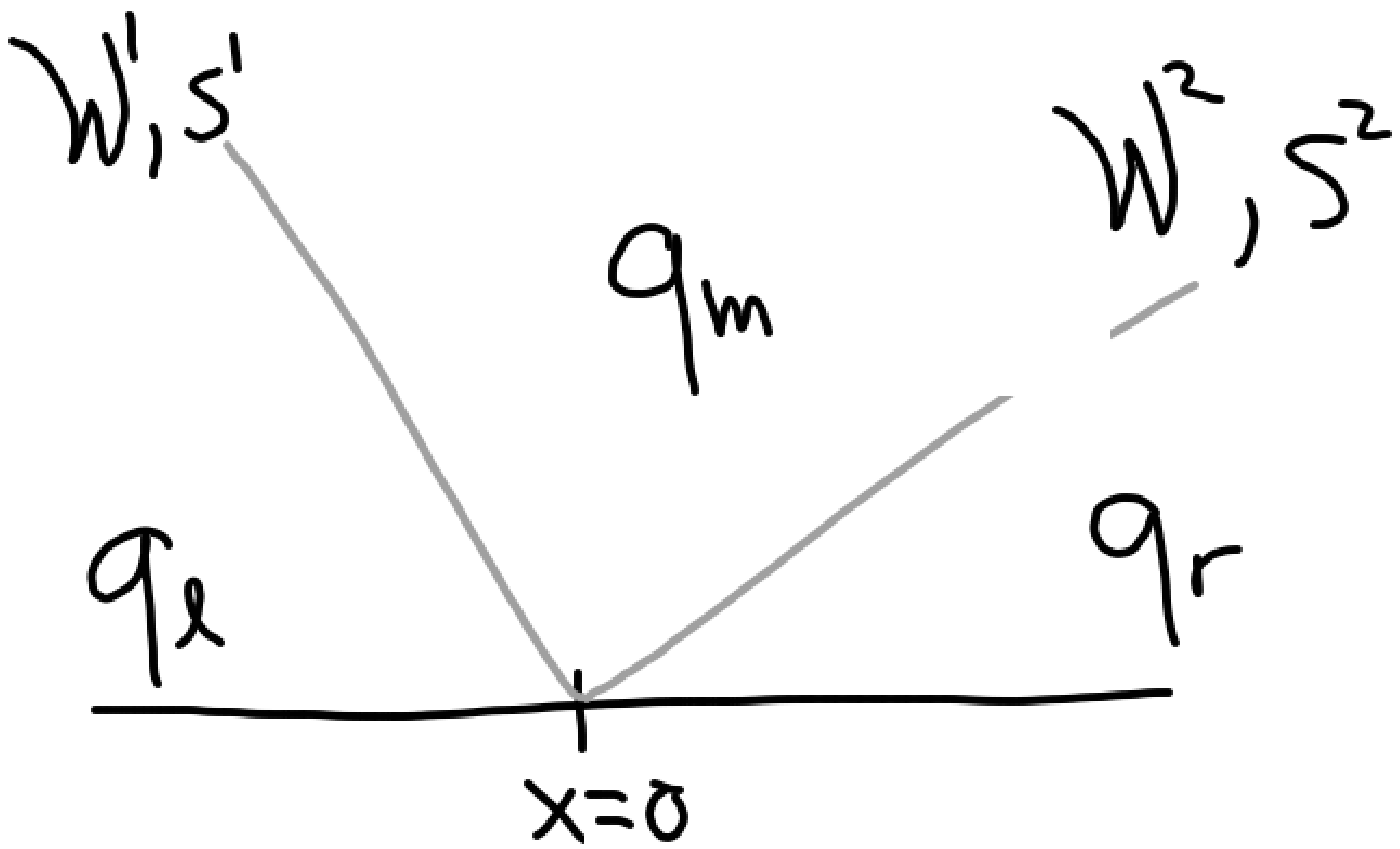
Two-wave solvers



$$W^1 = q_m - q_\ell$$

$$W^2 = q_r - q_\ell$$

$$s^1 W^1 + s^2 W^2 = f(q_r) - f(q_\ell)$$

$$s^1 (q_m - q_\ell) + s^2 (q_r - q_m) = f(q_r) - f(q_\ell)$$

$$q_m = \frac{f(q_r) - f(q_\ell) + s^1 q_\ell - s^2 q_r}{s^1 - s^2}$$

Need to choose s^1, s^2

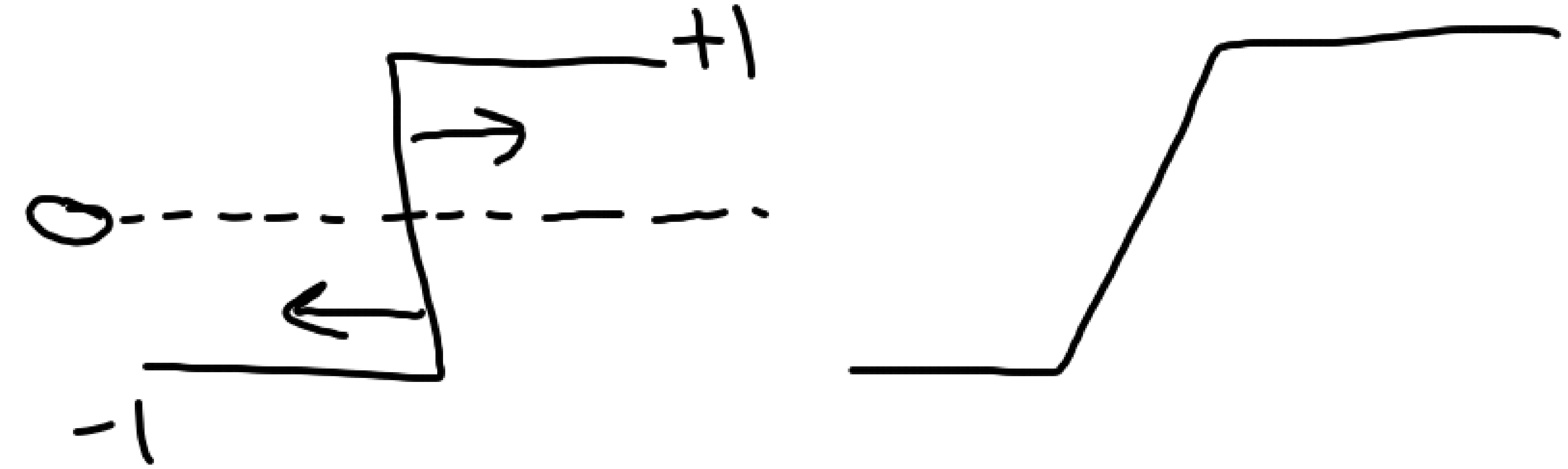
For stability, these should bound the exact wave speeds.

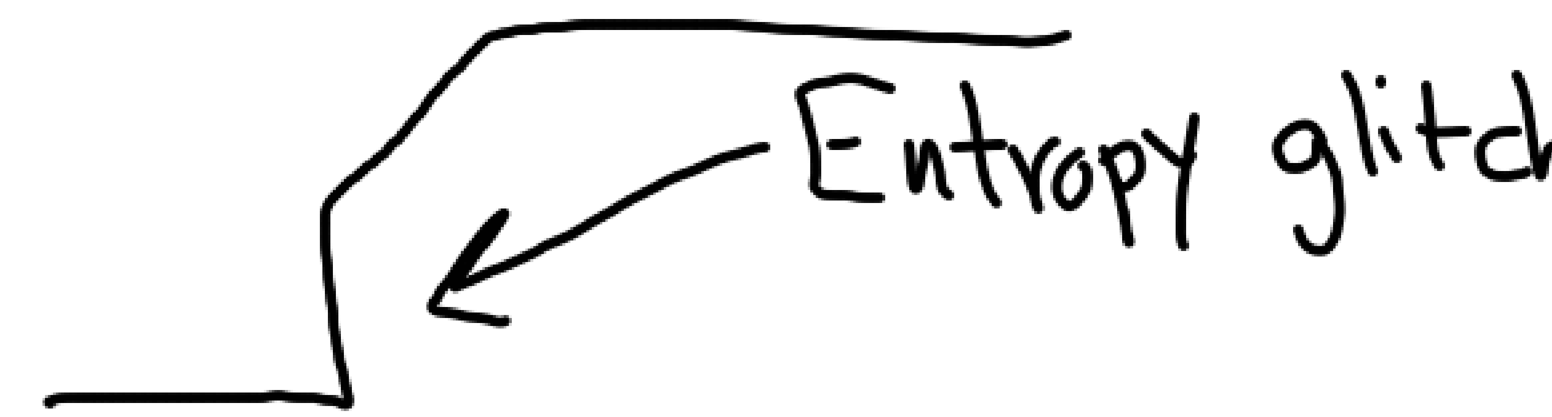
Rusanov/(local) Lax-Friedrichs

Choose

$$-s^1 = s^2 = \max_x |f'(q(x))|$$

$$= \max_i |f'(Q_i)| \quad \text{LF}$$





LF and LLF don't need an entropy fix (because they are dissipative)

Local Lax-Friedrichs:

$$-s^1_{i-1/2} = s^2_{i-1/2} = \max\left(|f'(q_r)|, |f'(q_\ell)|\right)$$

LLF is less dissipative.

If f is convex, this gives speeds that bound those of the the true solution.

Harten-Lax-van Leer (HLL)

$$s^1 = \min(f'(q_\ell), f'(q_r))$$
$$s^2 = \max(f'(q_\ell), f'(q_r))$$

This is less dissipative.

2-wave solvers for shallow water

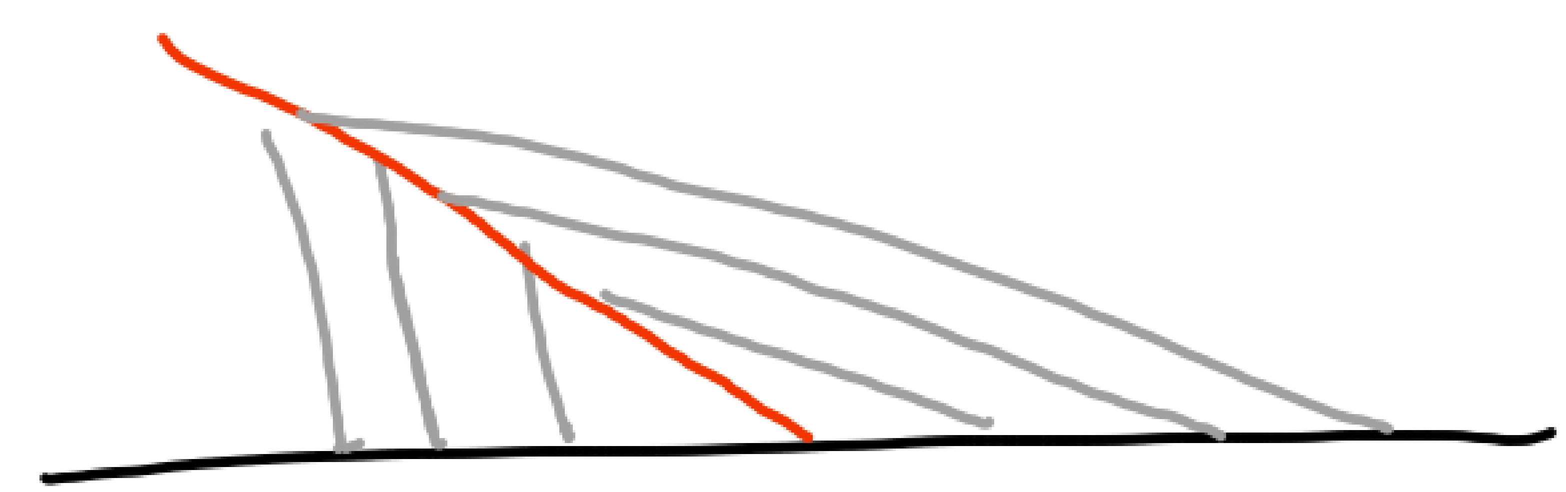
$$u - \sqrt{gh} = \lambda^1(q) \leq \lambda^2(q) = u + \sqrt{gh}$$

So we could take

$$s^1 = \min(\lambda^1(q_\ell), \lambda^1(q_m))$$
$$s^2 = \max(\lambda^2(q_m), \lambda^2(q_r))$$

But we don't know q_m.

If $\min(\lambda^1(q_\ell), \lambda^1(q_m)) = \lambda^1(q_m)$ then the 1-wave is a shock:



HLLE (Einfeldt)

$$S^1 = \min\left(\lambda^1(q_\ell), \underbrace{\hat{u} - \sqrt{g\hat{h}}}_{\text{Roe averages}}\right)$$

$$S^2 = \max\left(\lambda^2(q_r), \hat{u} + \sqrt{g\hat{h}}\right)$$

This is exact for a single shock.

No need for entropy fix.

Homework:

Derive a Roe solver for the p-system (isothermal gas)

$$\rho_t + (\rho u)_x = 0$$

$$(\rho u)_t + (\rho u^2 + a^2 \rho)_x = 0$$