

Exercise 1

Characteristics for nonlinear scalar conservation laws

In class we saw that, for the advection equation

$$q_t + a q_x = 0 \tag{1}$$

the solution is constant along straight lines (*characteristics*) in the $x-t$ plane with slope a .

Now consider the nonlinear equation

$$q_t + f(q)_x = 0. \tag{2}$$

Show that the solution of (2) is also constant along straight lines (i.e., characteristics) and determine their slope. Explain how to find the solution of (2) at a given point (x^*, t^*) using these characteristics (i.e., write down the equation that must be solved to find $q(x^*, t^*)$).

Hint: consider a straight line $X(t)$ and the total derivative dq/dt along this line.