Homework

① Consider a domain with $\rho_\ell = K_\ell = 1$ $(x<0)$ and $\rho_r = K_r$ $(x>0)$, with initial data

$$q(x,t=0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} f(x), \text{ where } f(x) = 0 \text{ for } x>0.$$

This is a purely right-going solution, like the simulations in class. What will be the nature of the solution in the limit $\left\{ \begin{matrix} \rho_r \to \infty \\ K_r \to \infty \end{matrix} \right\}$ with $c_r = \sqrt{\frac{K_r}{\rho_r}} = 1$?

② Now consider the Cauchy problem with $\rho = K = 1$ everywhere, and initial data

$$q(x,t=0) = \begin{cases} \begin{bmatrix} 1 \\ 1 \end{bmatrix} f(x) & x<0 \\ \begin{bmatrix} -1 \\ 1 \end{bmatrix} f(-x) & x>0 \end{cases}$$

Show that, for $x<0$, the solution of this problem is the same as the solution of part ①.