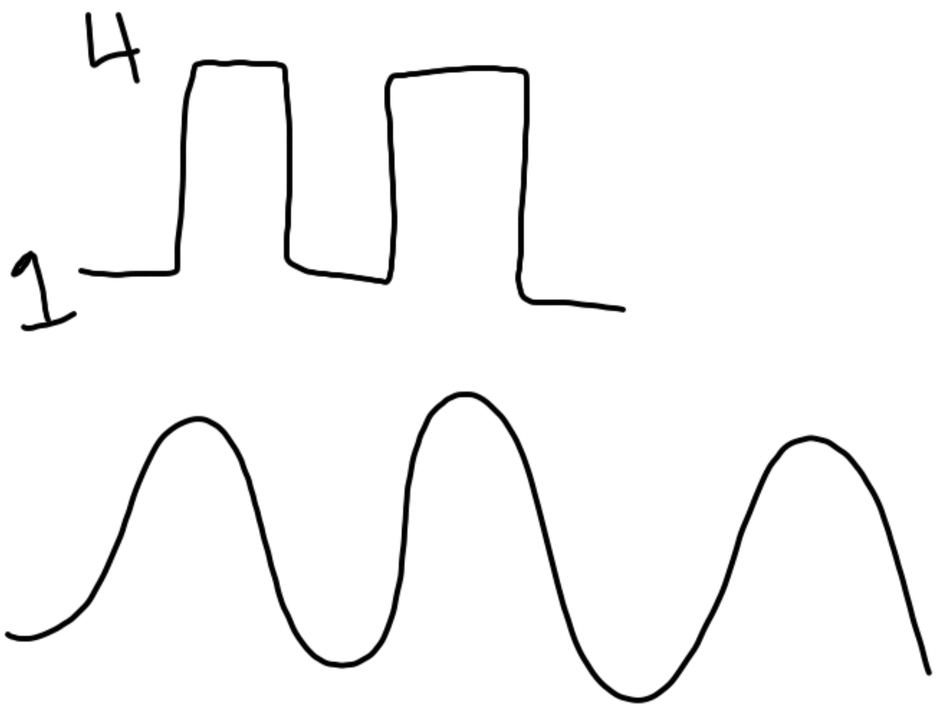
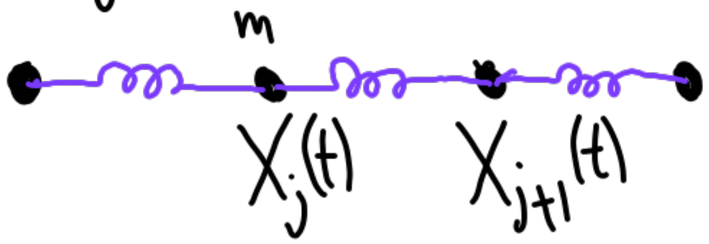


Connection to the Toda lattice

Springs and masses:



Unstretched spring length: Δx

Strain: $\varepsilon_{j+1/2} = \dfrac{X_{j+1} - X_j - \Delta x}{\Delta x}$

$U_j(t) = X_j'(t)$

Differentiate: $\varepsilon'_{j+1/2} = \dfrac{U_{j+1} - U_j}{\Delta x}$

$F = ma$

Let $\rho = m\Delta x$

$m V_j'(t) = \sigma(\varepsilon_{j+1/2}) - \sigma(\varepsilon_{j-1/2})$

$\sigma(\varepsilon)$: stress-strain relation

Toda: $\sigma(\varepsilon) = e^{K\varepsilon} - 1$

$\varepsilon_{j+1/2}' = \dfrac{V_{j+1} - V_j}{\Delta x}$

$\rho V_j' = \dfrac{\sigma(\varepsilon_{j+1/2}) - \sigma(\varepsilon_{j-1/2})}{\Delta x}$

ODE System
Has semi-discrete solitons
Is integrable

Looks like a semi-discretization of

$\varepsilon_t = u_x$

$\rho u_t = \sigma(\varepsilon)_x$