The Reduced Ostrovsky Equation

$$u_t + \mu u u_x = \gamma v$$

$$v_x = u$$

Rescaling:

$$\tilde{x} = \gamma x$$

$$\tilde{t} = t$$

$$\tilde{u} = \mu\gamma u$$

$$\tilde{v} = \mu\gamma^2 v$$

$$u_x = \frac{1}{\mu\gamma}\gamma\,\tilde{u}_{\tilde{x}} = \frac{1}{\mu}\tilde{u}_{\tilde{x}}$$

$$u_t = \frac{1}{\mu\gamma}\,\tilde{u}_{\tilde{t}}$$

$$v = \frac{1}{\mu\gamma^2}\tilde{v}$$

$$v_x = \frac{\gamma}{\mu\gamma^2}\tilde{v}_{\tilde{x}} = \frac{1}{\mu\gamma}\tilde{v}_{\tilde{x}}$$

$$u = \frac{1}{\mu\gamma}\tilde{u}$$

$$\frac{\partial \tilde{u}}{\partial \tilde{x}} = \frac{\partial \tilde{u}}{\partial x}\frac{dx}{d\tilde{x}} = \gamma\frac{\partial \tilde{u}}{\partial x}$$

Substitute:

$$\frac{1}{\mu\gamma}\tilde{u}_{\tilde{t}} + \cancel{\mu}\,\frac{1}{\mu\gamma}\tilde{u}\cdot\cancel{\frac{1}{\mu}}\tilde{u}_{\tilde{x}} = \frac{\cancel{\gamma}}{\mu\gamma^{\cancel{2}}}\tilde{v}$$

$$\frac{1}{\mu\gamma}\left(\tilde{u}_{\tilde{t}} + \tilde{u}\tilde{u}_{\tilde{x}}\right) = \frac{1}{\mu\gamma}\tilde{v}$$

$$\tilde{u}_{\tilde{t}} + \tilde{u}\tilde{u}_{\tilde{x}} = \tilde{v} \qquad \tilde{v}_{\tilde{x}} = \tilde{u}$$

So we can take $\mu = \gamma = 1$ without loss of generality.

$$u_t + uu_x = v$$
$$v_x = u$$

We assume

$$\int_{-\infty}^{\infty} u\,dx = 0$$

$$\int_{-\infty}^{\infty} v\,dx = 0.$$

Characteristics

$$X(t) = x_0 + \int_0^t u(X(\tau),\tau)\,d\tau$$

$$\frac{d}{dt}u(X(t),t) = u_t + u_x X'(t)$$
$$= u_t + uu_x = v$$

Along characteristics, u is NOT constant

$$u(X(t),t) = u(x_0,0) + \int_0^t v(X(\tau),\tau)\,d\tau.$$

$$u_t + uu_x = v \qquad v_x = u$$

$$u_{txx} + (u_x^2 + uu_{xx})_x = v_{xx} = u_x$$

$$u_{txx} + \underline{2u_x u_{xx}} + \underline{u_x u_{xx}} + uu_{xxx} = \underline{u_x}$$

$$\underline{u_{xxt}} + u_x(3u_{xx} - 1) + uu_{xxx} = 0$$

Let $F^3 = 1 - 3u_{xx}$

$$F^2 F_x = -\ u_{xxx}$$

$$F^2 F_t = -\ u_{xxt}$$

$$-F^2 F_t - F^3 u_x - uF^2 F_x = 0$$

$$F_t + Fu_x + uF_x = 0$$

$$F_t + (uF)_x = 0$$

If $F(u(x_0, 0))$ vanishes, then $F = 0 \ \forall t$ along this characteristic.

As $\min(F(u(x,t=0)) \to 0^-$ the time of shock formation grows (probably going to ∞)

If $1-3u_{xx}>0$, no shock forms.

Traveling waves

$$u_t + uu_x = v$$
$$v_x = u$$

$$u \to u(x-ct)$$
$$v \to v(x-ct)$$

$$-cu' + uu' = v$$

$$\boxed{\begin{aligned} v' &= u \\ u' &= \frac{v}{u-c} \end{aligned}}$$

$$u' = \frac{v}{u-c}$$
$$v' = u$$

Equilibrium solutions: $u=0$, $v=0$

$$J = \begin{bmatrix} -\frac{v}{(u-c)^2} & \frac{1}{u-c} \\ 1 & 0 \end{bmatrix}$$

$$J_{u=v=0} = \begin{bmatrix} 0 & \frac{-1}{c} \\ 1 & 0 \end{bmatrix}$$

$$\lambda = \pm i\sqrt{\frac{1}{c}}$$

$$\lambda^2 - \frac{-1}{c} = 0$$

$$\lambda^2 = -\frac{1}{c}$$

11.1

(11.11) $q(x,t) = \mathring{q}(\xi) \xrightarrow[\text{w.r.t } x]{\text{diff}} q_x = \mathring{q}' \xi_x$

(11.12) $x = \xi + f'(\mathring{q}(\xi))t \xrightarrow[\text{w.r.t } x]{\text{diff.}} 1 = \xi_x + f''(\mathring{q}(\xi))\mathring{q}'(\xi)\xi_x t$

$$1 = \left(1 + f''(\mathring{q}(\xi))\mathring{q}'(\xi)t\right)\xi_x$$

$$\Rightarrow \xi_x = \frac{1}{1 + f''(\mathring{q}(\xi))\mathring{q}'(\xi)t}$$

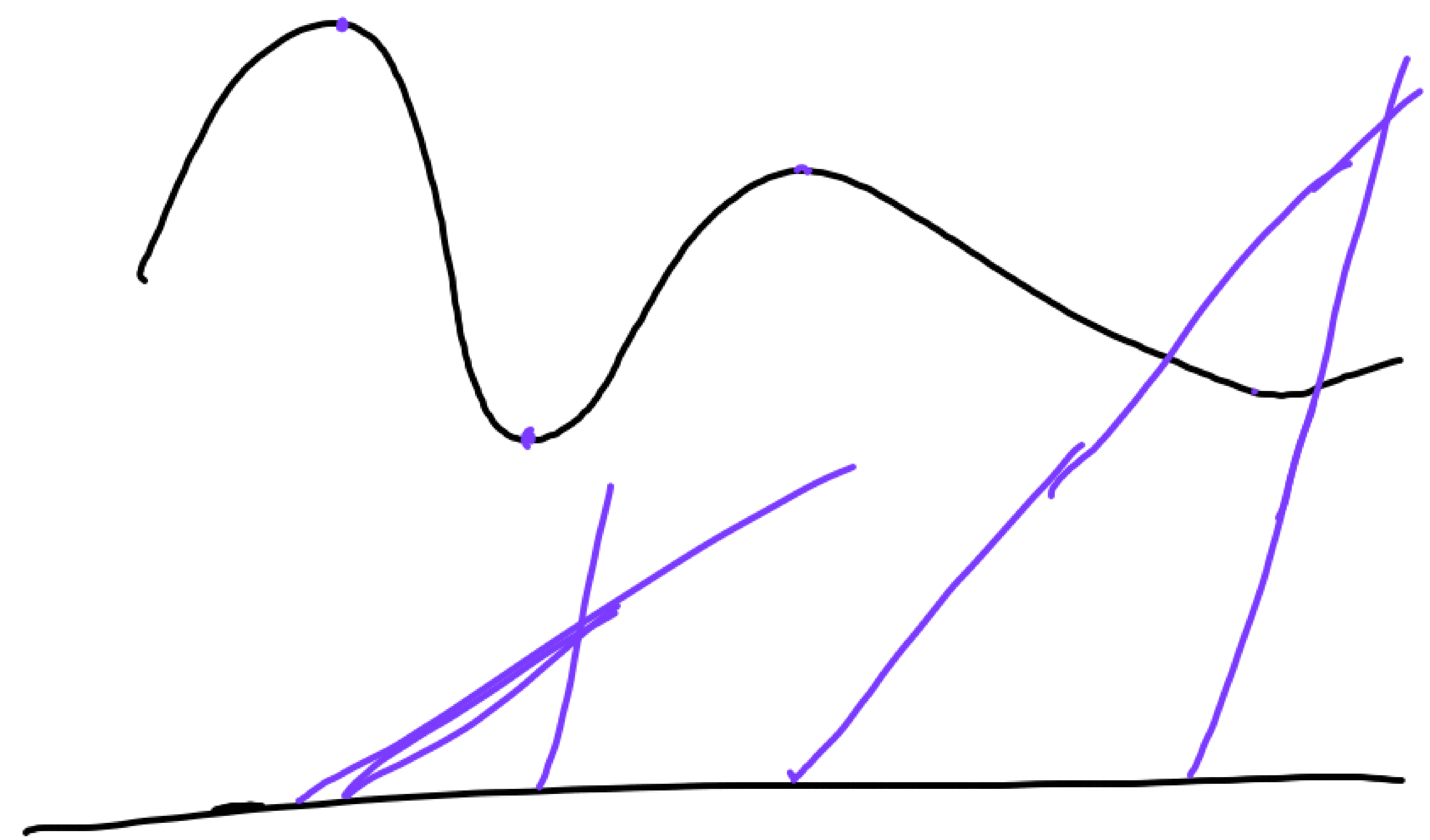
$$q_x = \frac{\mathring{q}_x}{1 + f''(\mathring{q}(\xi))\mathring{q}'(\xi)t}, \quad 1 + f''(\mathring{q}(\xi))\mathring{q}'(\xi)t \neq 0 \qquad t = T_b$$

$$T_b = \frac{-1}{f''(q(\xi))\,q'(\xi)} = Z(\xi)$$

$\xi \in [a,b]$, $\exists\, \xi_m \Rightarrow Z(\xi_m) = \max Z$

Burgers: $f(q) = \frac{1}{2}q^2$

$f''(q) = 1$



$$T_b = \frac{-1}{\min_x f''(q(\xi))\,q'(\xi)}$$

$$= \frac{-1}{\min_x f''(q(x,0))\,q_x(x,0)}.$$