$$\left(\frac{\omega}{k}\right)^2 = k^2 + 2$$

$$C_\rho^2 = 2$$

$$\beta = \sqrt{2}$$

$$\rho = 1 + \varepsilon^2 \rho^{(1)} + \varepsilon^4 \rho^{(2)}$$

$$V = \varepsilon^2 V^{(1)} + \varepsilon^4 V^{(2)}$$

$$\begin{cases} \rho_t = -2V_x \rho - 2V\rho_x \\ 2\rho^3 (V_t + \rho_x + 2VV_x) = \rho_x^3 + \rho_{xxx}\rho^2 - 2\rho_x \rho_{xx} \rho \end{cases}$$

$$\partial_x \longrightarrow \varepsilon \partial_\xi$$

$$\partial_t \longrightarrow -\beta \varepsilon \partial_\xi + \varepsilon^3 \partial_\tau$$

$$\Rightarrow \begin{cases} -\beta \rho_\xi + \varepsilon^2 \rho_\tau = -2V_\xi \rho - 2V\rho_\xi \\ 2\rho^3(-\beta V_\xi + \varepsilon^2 V_\tau + \rho_\xi + 2VV_\xi) = \varepsilon^2 \rho_\xi^3 + \varepsilon^2 \rho_{\xi\xi\xi} \rho^2 - 2\varepsilon^2 \rho_\xi \rho_\xi \rho \end{cases}$$

$$e^{A+B} = I + (A+B) + \frac{1}{2}\underbrace{(A+B)^2}_{A^2 + AB + BA + B^2} + \cdots \qquad (A+B)(A+B)$$

$$e^{tf}\, u(0)$$

$$u_t = f(u)$$

The solution of

$$u_t = Au + Bu \quad \text{is} \quad u(t) = e^{(A+B)t}\, u(0)$$

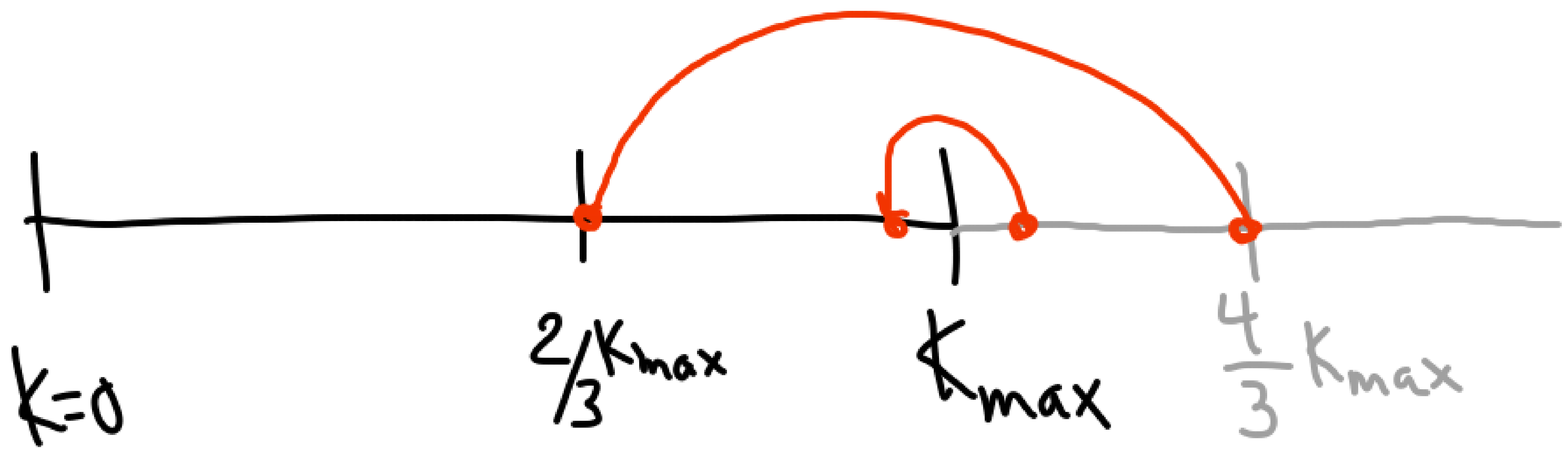
$$u_t = Au$$

$$u_t = Bu$$

We approximate this (Lie-Trotter or Godunov) by

$$u(t) \approx e^{Bt} e^{At} u(0)$$

$$e^{Bt} e^{At} = \left(I + Bt + \frac{B^2}{2}t^2 + \cdots\right)\left(I + At + \frac{A^2 t^2}{2} + \cdots\right) = I + (A+B)t + \frac{t^2}{2}(2BA + A^2 + B^2) + \cdots$$



$$\frac{1}{2}\left(u^2\right)_x$$

$$K_3 = K_1 + K_2$$

$$u_t + u u_x + u_{xxx} = \varepsilon u_{xxxx}$$

$$u_t = f(u) + Lu$$

$$L = F^{-1} D[i\xi^3] F$$

$$e^L = F^{-1} e^{D[i\xi^3]} F$$