$$V = \begin{pmatrix} p \\ u \end{pmatrix} \Rightarrow A v = -B v_x + C v_{xx} \qquad O(\delta)$$

$$\hat{V}_t = \underbrace{-A^{-1}(i\xi B + \xi^2 C)}\hat{V}$$

$$A^{-1} = \begin{pmatrix} \tilde{K} & 0 \\ 0 & \tilde{\rho} \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & \delta\alpha \hat{c}^2 \\ -\delta_2 \hat{c}^2 & 0 \end{pmatrix}$$

$$\tilde{K} = \overline{K^{-1}}^{-1} \qquad \tilde{\rho} = \overline{\rho}^{-1}$$

$$\hat{V}_t = -A^{-1}(i\xi B + \xi^2 C + i\xi^3 D)\hat{V} \qquad O(\delta^2)$$

$$D = \begin{pmatrix} 0 & \delta^2 C_{12} \\ \delta^2 C_{22} & 0 \end{pmatrix}$$

With $\mathcal{O}(\delta)$ terms

we found

$$\omega = \pm \hat{c}\xi\sqrt{1+\hat{c}^2\xi^2\delta^2\alpha^2}$$

With $\mathcal{O}(\delta^2)$ terms:

$$\omega = \pm \hat{c}\xi\sqrt{1+\hat{c}^2\alpha^2\delta^2\xi^2 - \mu^2\xi^4}$$

For large ξ, ω is complex

So $e^{-i\omega t}$ blows up

(for some ξ values).

$$[\![f]\!] = \int_0^y \{f(\xi)\} - \int_0^1\int_0^y \{f(\xi)\}d\xi\,dy$$

where $\{f\} = f - \int_0^1 f(\tau)\,d\tau$

$$u_t + u u_x - u_{xxt} = 0$$

$$c_p = \frac{u_0}{1+k^2}$$

$$u_t + u u_x + v_x = 0$$

$$\frac{v_t}{c} + u_x = w$$

$$w_t + \frac{w_x}{\hat{c}} = -v$$

$$c \to \infty : \quad w \approx u_x$$

$$\hat{c} \to \infty : \quad v \approx -w_t \approx -u_{xt}$$