Derivation of BBMH

$$\omega = \frac{u_0}{1+k^2}$$

BBM: $u_t + u u_x - u_{xxt} = 0$

We want:
- First-order part should be hyperbolic
- $\omega(k) \in \mathbb{R}$ (all roots) so $e^{-i\omega t}$ is bounded.

$u_x - w \approx 0$ $\quad w \approx u_x$

$v \approx -u_{xt} \approx -w_t$

$w_t + v \approx 0$

$$\left.\begin{aligned} u_t + u u_x - v_x &= 0 \\ v_t + c(u_x - w) &= 0 \\ (w_t + v) + \frac{w_x}{\hat{c}} &= 0 \end{aligned}\right\}$$

$\omega(k) \in \mathbb{R}$ iff $\hat{c}, c \geq 0$ (for $u_0 = 0$)

Some similar ideas:
- "Divergence cleaning" to enforce $\nabla \cdot u = 0$
- Relaxation Riemann solvers

KdVH

$\int u\,dx = \text{const.}$

$\int u^2 dx \stackrel{?}{=} \text{cost.}$

$$u_t + uu_x + u_{xxx} = 0$$

$v \approx u_x \Rightarrow u_{xxx} \approx w_x$

$w \approx v_x$

$$u_t + uu_x + w_x = 0$$

$$v_t + c_1(v_x - w) = 0$$

$$w_t + c_2(u_x - v) = 0$$

or

$$v_t + c_1(u_x - v) = 0$$

$$w_t + c_2(v_x - w) = 0$$

$$u_t + uu_x + w_x = 0$$

$$v_t + c_1 v_x = c_1 w$$

$$w_t + c_2 u_x = c_2 v$$

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix}_t + \begin{bmatrix} u & 0 & 1 \\ 0 & c_1 & 0 \\ c_2 & 0 & 0 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix}_x = \begin{bmatrix} 0 \\ c_1 w \\ c_2 v \end{bmatrix}$$

$\lambda = c_1, u \pm \sqrt{u^2 + 4c_2}$

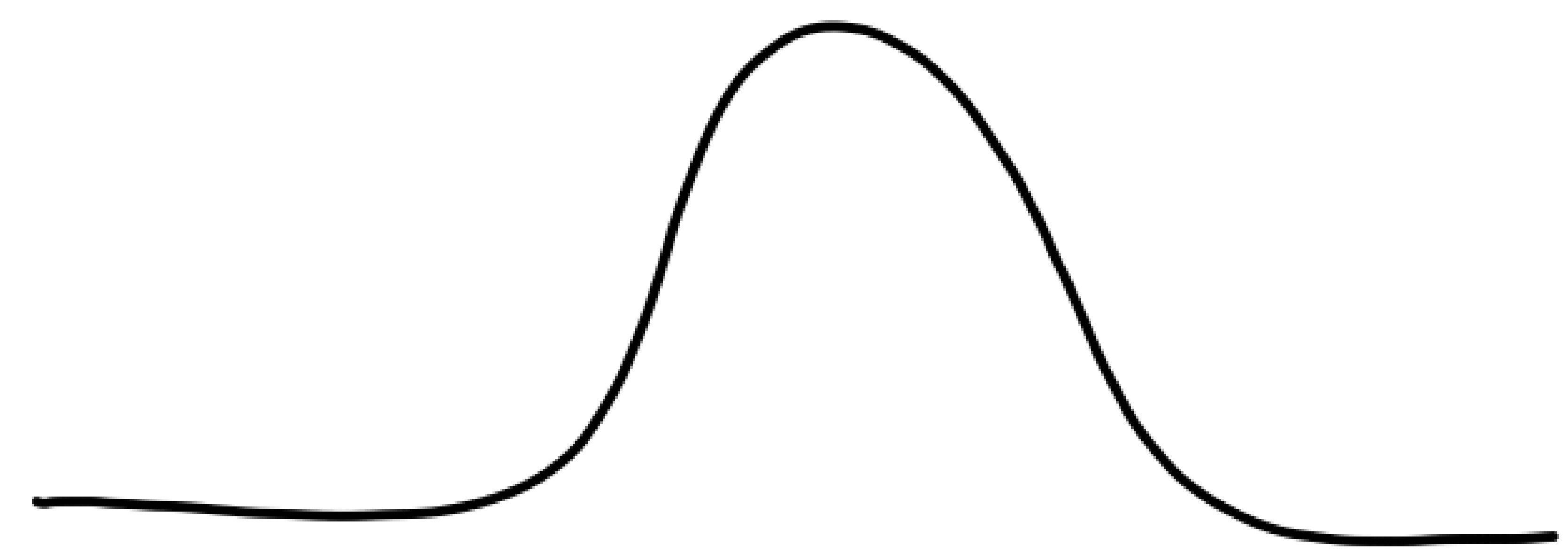
$\Rightarrow c_2 > 0$

$$\begin{bmatrix} u_0 ik - i\omega & 0 & ik \\ 0 & ikq - i\omega & -c_1 \\ ikc_2 & -c_2 & -i\omega \end{bmatrix} \begin{bmatrix} \hat{u} \\ \hat{v} \\ \hat{w} \end{bmatrix} = 0$$

NLSH

$$u_t - \frac{i}{2} v_x = K|u|^2 u$$

$$v_t + icu_x = icv$$



$u=0$

$$u_t + uu_x = v$$

$$v_t + cv_x = cu$$