Linear dispersion relation

$$\rho_{tt} = -2V_{xt}$$

$$V_{tx} = \frac{\rho_{xxxx}}{2} - \rho_{xx} \Rightarrow -\rho_{tt} = \rho_{xxxx} - 2\rho_{xx}$$

$$\omega^2 \rho = k^4 \rho + 2k^2 \rho$$

$$\omega^2 = k^4 + 2k^2$$

$$\rho = e^{i(xk - \omega t)}$$

$$c_p^2 = \frac{\omega^2}{k^2} = \cancel{k^2}^{\,0} + 2$$

$$c_p^2 \approx 2$$

$$\beta = c_p$$

$$c\left(-\beta\rho^{(2)} + 2V^{(2)}\right) = 2\rho^{(2)} - 2\beta V^{(2)}$$

$$-c\beta = 2 \Rightarrow c = -\frac{2}{\beta} \underset{\beta=\sqrt{2}}{=} -\sqrt{2}$$

$$2c = -2\beta \Rightarrow c = -\beta \underset{\beta=\sqrt{2}}{=} -\sqrt{2}$$

$$\rho_t = -2(V\rho)_x = -2V_x\rho - 2V\rho_x$$

$$V_t = \left(-V^2 - \frac{\rho_x^2}{4\rho^2} + \frac{\rho_{xx}}{2\rho} - \rho\right)_x = -2VV_x + \frac{\rho_x^3 + \rho_{xxx}\rho^2 - 2\rho_x\rho_{xx}\rho}{2\rho^3} - \rho_x$$

$$\boxed{\begin{aligned} &\rho_t = -2V_x\rho - 2V\rho_x \\ &2\rho^3(V_t + \rho_x + 2VV_x) = \rho_x^3 + \rho_{xxx}\rho^2 - 2\rho_x\rho_{xx}\rho \end{aligned}}$$

$$-\beta\epsilon\rho_\xi + \epsilon^3\rho_\tau = -2\epsilon V_\xi\rho - 2\epsilon V\rho_\xi$$

$$2\rho^3(-\beta\epsilon V_\xi + \epsilon^3 V_\tau + \epsilon\rho_\xi + 2\epsilon VV_\xi)$$
$$= \epsilon^3\rho_\xi^3 + \epsilon^3\rho_{\xi\xi\xi}\rho^2 - 2\epsilon^3\rho_\xi\rho_{\xi\xi}\rho$$

$$\partial_x \to \epsilon\partial_\xi$$

$$\partial_t \to -\beta\epsilon\partial_\xi + \epsilon^3\partial_\tau$$

$$-\beta\rho_\xi + \epsilon^2\rho_\tau = -2V_\xi\rho - 2V\rho_\xi$$

$$2\rho^3(-\beta V_\xi + \epsilon^2 V_\tau + \rho_\xi + 2VV_\xi) = \epsilon^2\rho_\xi^3 + \epsilon^2\rho_{\xi\xi\xi}\rho^2 - 2\epsilon^2\rho_\xi\rho_{\xi\xi}\rho.$$

$$\rho = 1 + \varepsilon^2 \rho^{(1)} + \varepsilon^4 \rho^{(2)}$$

$$V = \varepsilon^2 V^{(1)} + \varepsilon^4 V^{(2)}$$

$$\rho_\xi = \epsilon^2 \rho_f^{(1)} + \epsilon^4 \rho_f^{(2)}$$

$$\rho^3 = 1 + 3\epsilon^2 \rho^{(1)} + 3\epsilon^4 \rho^{(2)} + 3\epsilon^4 \rho^{2(1)} + 6\epsilon^6 \rho^{(1)} \rho^{(2)}$$

$$+ \epsilon^6 \rho^{3(1)} + 3\epsilon^8 \rho^{2(1)} \rho^{(2)} + 3\epsilon^8 \rho^{2(2)} + 3\epsilon^{10} \rho^{(1)} \rho^{2(2)}$$

$$+ \epsilon^{12} \rho^{3(2)}$$

$$\rho_f^3 = \epsilon^6 \rho_f^{3(1)} + 3\epsilon^8 \rho_f^{2(1)} \rho_f^{(2)} + 3\epsilon^{10} \rho_f^{(1)} \rho_f^{(2)2} + \epsilon^{12} \rho_f^{3(2)}$$

$$\rho^2 = 1 + 2\epsilon^2 \rho^{(1)} + 2\epsilon^4 \rho^{(2)} + \epsilon^4 \rho^{2(1)} + 2\epsilon^6 \rho^{(1)} \rho^{(2)}$$

$$+ \epsilon^8 \rho^{2(2)}$$

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• Collecting:

0 th order:

1st order:

* 2nd order:

$$-\beta \rho_{\xi}^{(1)} = -2 V_{\xi}^{(1)} \Rightarrow \boxed{\rho_{\xi}^{(1)} = \frac{2}{\beta} V_{\xi}^{(1)}}$$

$$-2\beta V_{\xi}^{(1)} + 2\rho_{\xi}^{(1)} = 0 \xRightarrow{\int d\xi} \rho_{1}^{(1)} = \frac{2}{\beta} V^{(1)} + C \;\star$$

$$\Rightarrow -2\beta V_{\xi}^{(1)} + \frac{4}{\beta} V_{\xi}^{(1)} = 0$$

$$-2\beta + \frac{4}{\beta} = 0 \Rightarrow -2\beta^2 + 4 = 0$$

$$\Rightarrow \beta^2 = 2 \Rightarrow \beta = \sqrt{2}$$

4th order:

$$-\beta \rho_{\xi}^{(2)} + \rho_{\tau}^{(1)} = -2 V_{\xi}^{(2)} - 2 V_{\xi}^{(1)} \rho^{(1)} - 2 V^{(1)} \rho_{\xi}^{(1)}$$

$$-2\beta V_{\xi}^{(2)} + 2 V_{\tau}^{(1)} + 2\rho_{\xi}^{(2)} + 4 V^{(1)} V_{\xi}^{(1)} - 6\beta \rho^{(1)} V_{\xi}^{(1)} + 6\rho^{(1)} \rho_{\xi}^{(1)} = \rho_{\xi\xi\xi}^{(1)}$$

$$-\beta \rho^{(2)}_{\xi} + 2V^{(2)}_{\xi} = -\rho^{(1)}_{\tau} - 12V^{(1)}_{\xi}\rho^{(1)} - 2V^{(1)}\rho^{(1)}_{\xi} \qquad (1)$$

$$2\rho^{(2)}_{\xi} - 2\beta V^{(2)}_{\xi} = \rho_{\xi\xi\xi} + 6\beta\rho^{(1)}V^{(1)}_{\xi} - 6\rho^{(1)}\rho^{(1)}_{\xi} - 2V^{(1)}_{\tau} - 4V^{(1)}V^{(1)}_{\xi} \qquad (2)$$

$(1)\ (-\sqrt{2})$ $\quad 2\rho^{(2)}_{\xi} - 2\sqrt{2}V^{(2)}_{\xi} = \sqrt{2}\rho^{(1)}_{\tau} + 2\sqrt{2}V^{(1)}_{\xi}\rho^{(1)} + 2\sqrt{2}V^{(1)}\rho^{(1)}_{\xi}$

$(2)\ (-1)$ $\quad -2\rho^{(2)}_{\xi} + 2\sqrt{2}V^{(2)}_{\xi} = -\rho^{(1)}_{\xi\xi\xi} - 6\sqrt{2}\rho^{(1)}V^{(1)}_{\xi} + 6\rho^{(1)}\rho^{(1)}_{\xi} + 2V^{(1)}_{\tau} + 4V^{(1)}V^{(1)}_{\xi}$

$(1)+(2)$ $\quad 0 = \underline{2V^{(1)}_{\tau}} + 8V^{(1)}_{\xi}V^{(1)} - \sqrt{2}V^{(1)}_{\xi\xi\xi} - \cancel{12V^{(1)}V^{(1)}_{\xi}} + \cancel{12V^{(1)}V^{(1)}_{\xi}} + \underline{2V^{(1)}_{\tau}} + 4V^{(1)}V^{(1)}_{\xi}$

$$0 = 4V^{(1)}_{\tau} + 12V^{(1)}_{\xi}V^{(1)} - \sqrt{2}V^{(1)}_{\xi\xi\xi}$$

$$V^{(1)}_{\tau} = -3V^{(1)}_{\xi}V^{(1)} + \frac{\sqrt{2}}{4}V^{(1)}_{\xi\xi\xi}$$