1. Write a spectral solver for the homogenized equations we derived in class:

$$\overline{K^{-1}}\overline{p}_t + \overline{u}_x = \delta\alpha\hat{c}^2\overline{u}_{xx} \tag{1}$$

$$\overline{\rho}\ \overline{u}_t + \overline{p}_x = -\delta\alpha\hat{c}^2\overline{u}_{xx} \tag{2}$$

where

$$\alpha = \overline{K^{-1}[\![\rho]\!]}$$

with

$$[\![f]\!](y) = \int_0^y \{f(\xi)\}\, d\xi - \int_0^1 \int_0^\tau \{f(\xi)\}\, d\xi d\tau.$$

where $\{f(y)\} = f(y) - \int_0^1 f$, and δ is the period of the coefficient functions $K(x), \rho(x)$.

Since these equations are linear, you don't need the pseudospectral approach and you don't need to discretize in time.

2. Compare the solution from your spectral solver with the direct solution of the variable-coefficient wave equation. For the latter, you may use the code in the notebook presented in class. How does the agreement between these two change as you:

- Vary the final time?
- Vary the length of the spatial period?
- Vary the "wavelength" of the initial data?
- Vary the amount of variation in the impedance: $Z(x) = \sqrt{K(x)\rho(x)}$?

If you're interested, here are some other questions you could consider:

3. Work out the next order terms using perturbation theory. Add these to your spectral solver. How much does the agreement between the homogenized system and the variable-coefficient system improve?

Can you conjecture the general form of the corrections that will appear at subsequent orders?

4. In class, we wrote the homogenized system (without exchanging t-derivatives for x-derivatives) as a set of 4 first-order equations. What is the dispersion relation for this system, and how does it differ from the dispersion relation we derived in class?

Write a solver for this system using either operator splitting or an exponential method, and compare the results it gives with those obtained in part (2) above.