AMCS 333

Spring 2019

Time: 2:30–4:00 pm Monday and Wednesday

Location: Building 9 room 3128

Instructor: David Ketcheson
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Instructor’s office: Al-Khawarizmi building, Room 4202
Office hour: By appointment

Texts:

- R. J. LeVeque: Numerical Methods for Conservation Laws (NMCL)
- R. J. LeVeque: Finite volume methods for hyperbolic problems (FVMHP)
- D. Serre, Systems of Conservation laws (2 vols.)
- G. B. Whitham: Linear and nonlinear waves
- Ketcheson, LeVeque, and del Razo Sarmina: Riemann problems and Jupyter solutions (RPJS) https://github.com/clawpack/riemann_book

Additional resources:

- Clawpack software: http://www.clawpack.org
- HyperPython short course: https://github.com/ketch/HyperPython

Overview and preparation

This course will introduce you to advanced concepts in the theory and numerical analysis of first-order hyperbolic PDEs. Beginning with background on linear wave equations and conservation laws, we will study the breakdown of characteristics and the need for weak solutions; non-uniqueness and entropy solutions; the Riemann problem and its solution, including rarefaction waves and shock waves. On the numerical side we will focus on high-resolution Godunov-type methods that make use of limiters and approximate Riemann solvers. The ideas will be investigated and illustrated through applications in fluid dynamics, water waves, and traffic modeling.

You should already have had courses in PDEs and numerical analysis (AMCS 231 +

AMCS 252 or their equivalent). You should also be comfortable programming simple numerical algorithms. The programming language for the course is Python. Many examples will be given in Python and you will be required to code solutions to some homework problems in Python.

Evaluation

As this is an advanced topics course, I expect that most students will be enrolled because it is relevant to their research. Thus evaluation is a secondary concern (the primary one being learning things that are useful to your research!) However, evaluation is often a useful motivator to make sure you stay engaged with the course at a level that will benefit you.

Exercises (50%)

Exercises will be assigned regularly (usually one or two at the end of each lecture), and students will be asked to present solutions in class. Typically solutions will be presented at the next class meeting or one week after being assigned. Your grade for this portion depends on both (a) presenting solutions; and (b) actively participating in discussions of other students' solutions.

You are encouraged to work with other students on the exercises. Exercises will involve both programming and analysis.

Course project (50%)

The remainder of your grade for the course will be based on completion of a research project. This project should involve reading one or more relevant articles or book chapters (beyond the regular reading for the course), and typically will include development of a basic software implementation. One course session will be devoted to a discussion of a paper from your project; you will lead this discussion. To finalize the project, you have the option of writing a report or giving a presentation to the class.

Important project dates:

- Project proposal due: February 20
- Progress report due: April 15
- Project presentations/reports due: May 15

You are encouraged to come up with your own project topic. Guidelines for the proposal will be distributed during the first week of the course. Your project topic should not be very closely related to your thesis research.

Scheduling

There will be no class sessions during the following times:

- February 24–28 (SIAM CSE conference)
- March 24–28 (spring break)
- May 8

Some potential project topics

Applications:

- Tsunami or flood modeling - The shallow water equations are frequently used in both
- Waves in periodic or random materials
- Detonation waves - the mathematics of explosions
- Magnetohydrodynamics - Magnetized ideal gases, important in many astrophysical applications as well as in fusion experiments
- Compressible fluid dynamics applications (e.g. astrophysics)

Theory:

- Glimm's random choice method - see Serre vol. 1, Ch. 5
- Non-conservative hyperbolic systems - see FVMHP Section 16.5
- Non-convex hyperbolic systems - see FVMHP Section 16.1
- Nonlinear geometric optics - see Serre vol. 2, Ch. 11
- Relaxation systems - see FVMHP Sections 17.17–17.18
- Compensated compactness - a method for proving global (in time) existence of solutions; see Serre vol. 2, Ch. 9
- Oleinik's entropy condition - see FVMHP Section 11.13

Numerics:

- Discontinuous Galerkin methods - finite element methods based on piecewise continuous function spaces
- Problems with spatially varying fluxes - see FVMHP Section 16.4 and RPJS
- Well-balanced numerical methods - for solving problems with source terms that are near a steady state
- Time integration for hyperbolic PDEs - efficiency, stability, accuracy, storage issues

- Comparison of multidimensional algorithms in Clawpack - do transverse Riemann solves really pay off? See FVMHP Chaps. 20–21
- Adaptive Mesh Refinement - to resolve fine structures without using a fine grid everywhere
- Instabilities near stationary shocks (the “carbuncle” phenomenon)