Quiz 1

Problem 1. Solve the following systems simultaneously:

$$\begin{aligned} x_1 + 2x_2 &= 1 \\ 3x_1 + 4x_2 &= 0 \end{aligned} \qquad\qquad \begin{aligned} x_1 + 2x_2 &= 0 \\ 3x_1 + 4x_2 &= 1 \end{aligned}$$

Do this by forming

$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 3 & 4 & 0 & 1 \end{array}\right] = [\, A \mid I \,]$$

where A is the coefficient matrix. Use row operations to reduce A to reduced row echelon form. You end up with

$$\left[\begin{array}{cc|cc} 1 & 0 & b_{11} & b_{12} \\ 0 & 1 & b_{21} & b_{22} \end{array}\right] = [\, I \mid B \,]$$

The vectors $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_{11} \\ b_{21} \end{bmatrix}$ and $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_{12} \\ b_{22} \end{bmatrix}$ are solutions to the initial system. Verify this by showing $AB = I$. Also show that $BA = I$.

Problem 2. There are three distinct ways to interpret matrix multiplication, each important in different contexts. Let's borrow notation from MATLAB for this problem. Given a matrix A let $A_{i,:}$ be the i^{th} row of A and $A_{:,j}$ be the j^{th} column. The definitions of AB are as follows for A an $m \times k$ and B a $k \times n$ matrix.

$$AB = \begin{bmatrix} A_{1,:}B_{:,1} & A_{1,:}B_{:,2} & \cdots & A_{1,:}B_{:,n} \\ A_{2,:}B_{:,1} & A_{2,:}B_{:,2} & \cdots & A_{2,:}B_{:,n} \\ \vdots & \vdots & \ddots & \vdots \\ A_{m,:}B_{:,1} & A_{m,:}B_{:,2} & \cdots & A_{m,:}B_{:,n} \end{bmatrix}$$

$(AB)_{ij} = \sum_{l=1}^{k} A_{il}B_{lj}$ is the inner-product of the i$^{\text{th}}$ row of A and the j$^{\text{th}}$ column of B

$$AB = \begin{bmatrix} A_{1,:}B \\ A_{2,:}B \\ \vdots \\ A_{m,:}B \end{bmatrix}$$

The i$^{\text{th}}$ row of AB is the result of row operations determined by $A_{i,:}$ applied to B, that is $B_{i,:} = \sum_{l=1}^{k} A_{il}B_{j,:}$

$$= \begin{bmatrix} AB_{:,1} & AB_{:,2} & \cdots & AB_{:,n} \end{bmatrix}$$

The j$^{\text{th}}$ column of AB is the result of column operations determined by $B_{:,j}$ applied to A, that is $(AB)_{:,j} = \sum_{l=1}^{k} A_{:,l} B_{l,j}$

Example:

$$\begin{aligned}
\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 6 & 7 \end{bmatrix} &= \begin{bmatrix} \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} 4 \\ 6 \end{bmatrix} & \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} 5 \\ 7 \end{bmatrix} \\ \begin{bmatrix} 3 & 4 \end{bmatrix} \begin{bmatrix} 4 \\ 6 \end{bmatrix} & \begin{bmatrix} 3 & 4 \end{bmatrix} \begin{bmatrix} 5 \\ 7 \end{bmatrix} \end{bmatrix} && \text{(inner product)} \\
&= \begin{bmatrix} \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 6 & 7 \end{bmatrix} \\ \begin{bmatrix} 3 & 4 \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 6 & 7 \end{bmatrix} \end{bmatrix} && \text{(row ops)} \\
&= \begin{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 4 \\ 6 \end{bmatrix} & \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 \\ 7 \end{bmatrix} \end{bmatrix} && \text{(column ops)} \\
&= \begin{bmatrix} 16 & 36 \\ 19 & 43 \end{bmatrix}
\end{aligned}$$

Compute AB in each of the three ways described above where

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \qquad B = \begin{bmatrix} -1 & 1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}$$