

Quiz 6

Question 1 (12 points). Let $A = \begin{bmatrix} 5/4 & -3/4 \\ -3/4 & 5/4 \end{bmatrix}$, write $A = U\Lambda U^{-1}$ where U is unitary, columns are orthonormal basis for \mathbb{R}^2 and $\Lambda = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$ with $\lambda_1 > \lambda_2$.

Recall: $U^{-1} = U^T$ for unitary U .

Question 2 (12 points). Suppose the matrix A from Question 1 is used to transform points in the plane iteratively. That is, given a point \mathbf{v} , consider the sequence $\mathbf{v}_n = A^n \mathbf{v}$. Letting $U = [\mathbf{u}_1 \ \mathbf{u}_2]$ so that \mathbf{u}_i is an eigenvector associated to λ_i and letting $\mathbf{v} = c_1 \mathbf{u}_1 + c_2 \mathbf{u}_2$ what is a simple expressions for a_n and b_n so that $\mathbf{v}_n = A^n \mathbf{v} = a_n \mathbf{u}_1 + b_n \mathbf{u}_2$.

Question 3 (12 points). If A and B are similar square matrices and say S witnesses this, that is $A = SBS^{-1}$, show that if (λ, \mathbf{v}) is an eigenvalue/eigenvector pair for A , then $(\lambda, S^{-1}\mathbf{v})$ is an eigenvalue/eigenvector pair for B .