Name: __________________________ Quiz 4 - MAT345

Question 1 (20 points; 4 points each)**.** You do need to add your justifications for each for full credit. The justifications will count 50%.

(a) _____ If all eigenvalues of A are 0, then $A = O$ (the 0 matrix).

(b) _____ Let $L : \mathbb{R}^4 \to \mathbb{R}^4$ scale all vectors in a plane P_1 by $\frac{1}{2}$ and scale all points in a plane P_2 by 3. Then $[L]_{\mathcal{C}}$ is diagonalizable for any basis $\mathcal{C}$ for $\mathbb{R}^4$.

$$\begin{bmatrix} 0 & 1 & 2 & 3 \\ 0 & 4 & 5 & 6 \\ 0 & 0 & 7 & 8 \\ 0 & 0 & 0 & 9 \end{bmatrix}$$

(c) _____ If λ_1 is an eigenvalue for A and λ_2 is an eigenvalue for B, then $\lambda_2\lambda_1$ is an eigenvalue for AB.

(d) _____ There is a 3×3 matrix with eigenvalues $\lambda_1 = 1/2$ and $\lambda_2 = -1/3$ with

$$E_{\lambda_1} = \text{span}\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\} \text{ and } E_{\lambda_2} = \text{span}\left\{ \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} \right\}$$

(e) _____ If $\lambda_1 = \frac{1}{2}$ and $\lambda_2 = 1$ are the eigenvalues for $L : \mathbb{R}^3 \to \mathbb{R}^3$ with corresponding eigenspaces $E_{\frac{1}{2}} = \text{span}\{(1, -1, 0), (0, 1, 1)\}$ and $E_1 = \text{span}\{(1, 1, 1)\}$. Then $L^n(\boldsymbol{x}) \to S\boldsymbol{x}$ as $n \to \infty$ where $S : \mathbb{R}^3 \to \mathbb{R}^3$ is the projection of points in $\mathbb{R}^3$ onto the line $L = E_1$ along the plane $P = E_{\frac{1}{2}}$.

Question 2 (20 points)**.** The *Turkey Table Co-op* is planning to distribute turkeys to three local shelters for Thanksgiving. Each shelter starts with a certain number of turkeys, and they redistribute a portion of their turkeys to other shelters each day according to a fixed sharing matrix.

Let the number of turkeys at the three shelters be represented by the vector

$$\mathbf{x}_n = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix},$$

where x_1, x_2, and x_3 are the number of turkeys at shelters 1, 2, and 3, respectively, on day n.

The redistribution process is governed by the matrix A, where:

$$A = \begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.3 & 0.5 & 0.2 \\ 0.1 & 0.3 & 0.6 \end{bmatrix}.$$

Each column of A represents how turkeys are redistributed to the shelters as a fraction of the turkeys at that shelter.

1. **Find the steady-state distribution:** Determine the eigenvector corresponding to the eigenvalue $\lambda = 1$, and normalize it to find the steady-state turkey distribution as $n \to \infty$.
2. **Initial condition:** Suppose the initial turkey distribution is $\mathbf{x}_0 = \begin{bmatrix} 100 \\ 50 \\ 150 \end{bmatrix}$. Compute the turkey distribution after one day ($\mathbf{x}_1 = A\mathbf{x}_0$), two days ($\mathbf{x}_2 = A^2\mathbf{x}_0$), and n-days ($\mathbf{x}_3 = A^3\mathbf{x}_0$). Is this what you would expect from (1.)
3. **Interpret the results:** Explain how the shelters can expect turkey distributions to stabilize over time. Do any shelters receive a disproportionately large or small share of turkeys?