Name: ______________________________ Quiz 1 - MAT345

Problem 1.1 (15 points; 3 points each)**.** Decide if each of the following is true or false.

(a) <u>False</u> If A and B are $n \times n$ symmetric matrices, then $(A+B)(A-B)$ is symmetric.

It is clear that $A-B$ and $A+B$ are symmetric, since $(A \pm B)^T = A^T \pm B^T = A \pm B$, so

$$((A+B)(A-B))^T = (A-B)^T(A+B)^T = (A-B)(A+B)$$

But there is no reason that $(A-B)(A+B) = (A+B)(A-B)$.

In particular, let $A = \left[\begin{smallmatrix} 1 & 3 \\ 3 & -1 \end{smallmatrix}\right]$ and $B = \left[\begin{smallmatrix} -1 & 2 \\ 2 & 1 \end{smallmatrix}\right]$ so $A - B = \left[\begin{smallmatrix} 2 & 1 \\ 1 & -2 \end{smallmatrix}\right]$ and $A + B = \left[\begin{smallmatrix} 0 & 5 \\ 5 & 0 \end{smallmatrix}\right]$. Then $(A-B)(A+B) = \left[\begin{smallmatrix} 5 & 10 \\ -10 & 5 \end{smallmatrix}\right]$ which is not symmetric.

(b) <u>True</u> Given matrices A and B that are equivalent, $A\mathbf{x} = \mathbf{0}$ iff $B\mathbf{x} = \mathbf{0}$.

Recall: A and B are equivalent iff there is a series of row operations that transform A into B.

This is the whole point of row operations; they do not change the solution set to $A\mathbf{x} = \mathbf{0}$.

(c) <u>True</u> If A is a 2×4 matrix, then $A\boldsymbol{x} = \mathbf{0}$ has at least two free variables.

A 2×4 matrix corresponds to an *underdetermined system* with at least two free variables.

(d) <u>False</u> If $(A-I)(A+I) = O$, then $A = I$ or $A = -I$.

Consider $A = \left[\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}\right]$, then $(A-I)(A+I) = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}\left[\begin{smallmatrix} 1 & 1 \\ 1 & 1 \end{smallmatrix}\right] = \left[\begin{smallmatrix} 0 & 0 \\ 0 & 0 \end{smallmatrix}\right]$.

(e) <u>True</u> For A a 3×6 matrix, let E be the matrix so that EA is the result of the row operation $R_2 - 3R_1 \to R_2$ applied to A. Then the second row of E is $\begin{bmatrix} -3 & 1 & 0 \end{bmatrix}$.

$$\begin{bmatrix} ? & ? & ? \\ -3 & 1 & 0 \\ ? & ? & ? \end{bmatrix} \begin{bmatrix} \boldsymbol{a}_1 \\ \boldsymbol{a}_2 \\ \boldsymbol{a}_3 \end{bmatrix} = \begin{bmatrix} \cdots \\ (-3) \cdot \boldsymbol{a}_1 + (1) \cdot \boldsymbol{a}_2 + (0) \cdot \boldsymbol{a}_3 \\ \cdots \end{bmatrix}$$

Problem 1.2 (25 points)**.** Solve $A\boldsymbol{x} = \mathbf{0}$ for

$$A = \begin{bmatrix} 5 & 5 & 15 & -3 \\ -3 & 0 & -3 & 0 \\ 5 & 4 & 13 & 5 \end{bmatrix}$$

Follow the procedure discussed in class

- (10 points) Use elementary row ops to reduce to an echelon matrix.
- (10 points) Write down the resulting triangular system and use back-substitution to solve.

- (5 points) Write out your solution as a linear combination of vectors.

Reduction to echelon form requires 3 row manipulations:

This part is 10/25 points.

$$\begin{bmatrix} 5 & 5 & 15 & -3 \\ -3 & 0 & -3 & 0 \\ 5 & 4 & 13 & 5 \end{bmatrix} \underset{\substack{R_2 \leftarrow R_2 - (-3/5)R_1 \\ R_3 \leftarrow R_3 - (5/5)R_1}}{\longrightarrow} \begin{bmatrix} 5 & 5 & 15 & -3 \\ 0 & 3 & 6 & -9/5 \\ 0 & -1 & -2 & 8 \end{bmatrix}$$

$$\underset{R_3 \leftarrow R_3 - (-1/3)R_2}{\longrightarrow} \begin{bmatrix} 5 & 5 & 15 & -3 \\ 0 & 3 & 6 & -9/5 \\ 0 & 0 & 0 & 37/5 \end{bmatrix}$$

So x_3 is the only free variable. Let $x_3 = t$. Back substitution gives

$$\begin{aligned} x_4 &= 0 \\ x_3 &= t \\ 3x_2 + 6t &= 0 \rightarrow x_2 = -2t \\ 5x_1 + 5(-2t) + 15t &= 0 \rightarrow x_1 = -t \end{aligned}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -t \\ -2t \\ t \\ 0 \end{bmatrix} = t \begin{bmatrix} -1 \\ -2 \\ 1 \\ 0 \end{bmatrix}$$

This final answer is the remaining 5/25 points.