Name: ____________________________ Quiz 4 - MAT345

Question 1 (20 points; 4 points each)**.** No justification required for the this quiz, just the T/F responses.

(a) _____ If all eigenvalues of A are 0, and A is diagonalizable, then $A = O$.

(b) _____ The following matrix is diagonalizable

$$\begin{bmatrix} 0 & 1 & 2 & 3 \\ 0 & 0 & 4 & 5 \\ 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(c) _____ If $\boldsymbol{v}$ is an eigenvector for both A and B, then $\mathbf{v}$ is an eigenvector for AB.

(d) _____ If A and B are similar matrices and $\lambda, \boldsymbol{v}$ is an eigenvalue/eigenvector pair for A, then the same is true for B.

(e) _____ If $\lambda_1 = \frac{1}{2}$ and $\lambda_2 = -2$ are the eigenvalues for $L : \mathbb{R}^2 \to \mathbb{R}^2$ with corresponding eigenvectors $\boldsymbol{v}_1 = (1, -1)$ and $\boldsymbol{v}_2 = (1, 1)$. Then for any $\boldsymbol{x} \in \mathbb{R}^2$, $L^n(\boldsymbol{x})$ approaches the line $E_{\lambda_2} = \text{span}\{\boldsymbol{v}_2\}$ as $n \to \infty$, specifically $\text{dist}(L^n(\boldsymbol{x}), E_{\lambda_2}) = \| \text{proj}^{\perp}_{E_{\lambda_2}}(L^n(\boldsymbol{x}))\| \to 0$ as $n \to \infty$.

Question 2 (20 points)**.** Let $A = \begin{bmatrix} 1/2 & 1 \\ 1/2 & 0 \end{bmatrix}$. Diagonalize A, i.e. write $A = S\Lambda S^{-1}$ where Λ is the diagonal matrix of eigenvalues for A.

i) What is $p_A(t)$?

ii) What are the eigenvalues?

iii) Find eigenvectors for each eigenvalue.

iv) Diagonalize A, i.e., write $A = BDB^{-1}$ where D is diagonal.

Bonus (5pts - Can complete this at home.) Give a formula for $A^n\mathbf{v}$ where $\boldsymbol{v} = \left[\begin{smallmatrix} a \\ b \end{smallmatrix}\right]$ and use this to find $\lim_{n\to\infty} A^n\boldsymbol{v}$.