Quiz 1

Problem 1. Solve the following systems simultaneously:

$$\begin{aligned} x_1 + 2x_2 &= 1 \\ 3x_1 + 4x_2 &= 0 \end{aligned} \qquad\qquad \begin{aligned} x_1 + 2x_2 &= 0 \\ 3x_1 + 4x_2 &= 1 \end{aligned}$$

Do this by forming

$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 3 & 4 & 0 & 1 \end{array}\right] = [\, A \mid I \,]$$

where A is the coefficient matrix. Use row operations to reduce A to reduced row echelon form Show all the steps. You end up with

$$\left[\begin{array}{cc|cc} 1 & 0 & b_{11} & b_{12} \\ 0 & 1 & b_{21} & b_{22} \end{array}\right] = [\, I \mid B \,]$$

The vectors $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_{11} \\ b_{21} \end{bmatrix}$ and $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_{12} \\ b_{22} \end{bmatrix}$ are solutions to the initial systems. Verify this by showing $AB = I$. Also show that $BA = I$.

Problem 2. Consider a new operation on matrices $A \otimes B \stackrel{\text{df}}{=} AB - BA$.

Decide if each of the following are true or false and provide a small proof or counterexample in each case.

(a) ______ $\otimes$ is commutative. For all $A, B \in \mathbb{R}^{n \times n}$, $A \otimes B = B \otimes A$.

(b) ______ $\otimes$ is associative. For all $A, B, C \in R^{n \times n}$, $(A \otimes B) \otimes C = A \otimes (B \otimes C)$.

(c) ______ For all $A \in \mathbb{R}^{n \times n}$, there is an $\otimes$-identity, E so that $E \otimes A = A \otimes E = A$.

(d) ______ For all $A, B \in \mathbb{R}^{n \times n}$, $(A \otimes B)^T = B^T \otimes A^T$.

Notation: "$\stackrel{\text{df}}{=}$" means "*is defined to be*" and $\mathbb{R}^{n \times n}$ is the set/space of all $n \times n$ matrices with real entries.