SPRING 2020, MAT225: Final Exam

This exam is closed book and closed notes. Stand-alone Calculators are allowed (no iPhone/iPad/Laptops as calculator). Make sure to show all of your work. By taking this exam, you are agreeing to the EMCC code of conduct. You can use technology to perform some calculations unless the problems asks to perform calculations by hand. State anytime you use technology to find RREF. By submitting this exam you also confirm that you have neither given nor received any unauthorized assistance on this exam. Furthermore, you agree not to discuss this exam with anyone until the exam testing period is over (May 15th).

Problem 1 [20 Pts]

*Find AB and BA (***by hand, show your work***):*

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 5 & 0 \end{bmatrix}, \quad and \;\; B = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$$

Problem 2 [20 Pts]

Test if the following vectors are Linearly Independent and explain your reasoning. Find a minimal set of vectors that spans S and write it as a linear combination.

$$S = \left\{ \begin{bmatrix} 1 \\ 3 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ 6 \\ 6 \\ 9 \end{bmatrix} \right\}$$

Problem 3 [20 Pts]

Are these matrices linearly independent in M_{32}, the vector space of all 3×2 matrices?

$$W = \left\{ \begin{bmatrix} 3 & -1 \\ 1 & 4 \\ 6 & -6 \end{bmatrix}, \begin{bmatrix} -2 & 3 \\ 1 & -3 \\ -2 & -6 \end{bmatrix}, \begin{bmatrix} 6 & -6 \\ -1 & 0 \\ 7 & -9 \end{bmatrix}, \begin{bmatrix} 7 & 9 \\ -4 & -5 \\ 2 & 5 \end{bmatrix} \right\}$$

Problem 4 [20 Pts]

Find the solution of the system. Explain your work.

$$\begin{aligned}
-x_1 + 5x_2 &= -8 \\
-2x_1 + 5x_2 + 5x_3 + 2x_4 &= 9 \\
-3x_1 - x_2 + 3x_3 + x_4 &= 3 \\
7x_1 + 6x_2 + 5x_3 + x_4 &= 30
\end{aligned}$$

Problem 5 [20 Pts]

For the matrix A:

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 3 \\ -1 & 1 & 1 \end{bmatrix}$$

Find

(a) *Find a basis for $\mathcal{N}(A)$.*
(b) *Find the row space of A.*
(c) *Find column space of A.*
(d) *Find the rank of A.*
(e) *Find the nullity of A.*

Problem 6 **[20 Pts]**

For the matrix A below

$$A = \begin{bmatrix} 29 & 14 & 2 & 6 & -9 \\ -47 & -22 & -1 & -11 & 13 \\ 19 & 10 & 5 & 4 & -8 \\ -19 & -10 & -3 & -2 & 8 \\ 7 & 4 & 3 & 1 & -3 \end{bmatrix}$$

(a) *Find the eigenvalues;*

(b) *Find the algebraic and geometric multiplicity of the eigenvalues*

(c) *Can the matrix be diagonalized? Why?*

Problem 7 **[20 Pts]**

Diagonalize the matrix below:

$$A = \begin{bmatrix} -1 & 0 & 1 \\ 5 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

Problem 8 [20 Pts]

Find the eigenvalues and eigenvectors of the following matrix by hand

$$A = \begin{bmatrix} 3 & -2 \\ 4 & -1 \end{bmatrix}$$

Problem 9 **[20 Pts]**

A linear transformation has the following matrix representation:

$$A = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 2 & -1 & 1 & 0 & 1 \\ 1 & 2 & -1 & -2 & 1 \\ 1 & 3 & 2 & 1 & 2 \end{bmatrix}$$

(a) *Find Kernel to determine if T is injective.*

(b) *Discuss why this result is not suprising (HINT: use a dimensional argument).*

Problem 10 [20 Pts]

Consider the vectors space $S = \{(x_1, x_2) | x_1, x_2 \in \mathbb{R}\}$*, with the two operation:*

Vector Addition *:* $(x_1, x_2) + (y_1, y_2) = (x_1 + y_1 - 1, x_2 + y_2 + 1)$.

Scalar Multiplication *:* $\alpha(x_1, x_2) = (\alpha x_1 - \alpha + 1, \alpha x_2 + \alpha - 1)$.

Find:

$$3(2, -5) - 2(3, 1)$$

SOLUTION

Problem 11 [20 Pts]

Given the set S of vectors, use (by-hand) Gram-Schmidt procedure to find an orthogonal basis:

$$S = \left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 8 \\ 1 \\ -6 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\},$$

Problem 12 [20 Pts]

Find the determinant by hand for the matrix below:

$$A = \begin{bmatrix} -1 & 0 & 1 \\ 5 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

Problem 13 [60 Pts]

Given these vectors:

$$\mathbf{v}_1 = \begin{bmatrix} 0 \\ -4 \\ -1 \\ -1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 3 \\ -5 \\ -4 \\ 4 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} -3 \\ 2 \\ -5 \\ -1 \end{bmatrix}, \quad \mathbf{v}_4 = \begin{bmatrix} -3 \\ 6 \\ 29 \\ -15 \end{bmatrix},$$

Find (if possible) a nontrivial linear combination of these vectors that adds up to 0. If not possible, clearly state why.

Problem 14 [60 Pts]

Find eigenspace of matrix with repeatingeigenvalues show work to find e-vec and e-space

Problem 15 **[20 Pts]**

Consider the set of matrices of the form

$$\begin{bmatrix} \star & 0 & 0 \\ 0 & \star & 0 \\ 0 & 0 & \star \end{bmatrix}$$

where $\star$ *could be any real number. Determine whether or not this set forms a vector subspace of* M_{33} *with the usual operation of addition and scalar multiplication.*

Problem 16 [20 Pts]

Consider the vectors space $S = \{(x_1, x_2) | x_1, x_2 \in \mathbb{R}\}$, *with the two operation:*

Vector Addition *:* $(x_1, x_2) + (y_1, y_2) = (x_1 + y_1 - 1, x_2 + y_2 + 1)$.

Scalar Multiplication *:* $\alpha(x_1, x_2) = (\alpha x_1 - \alpha + 1, \alpha x_2 + \alpha - 1)$.

Find:

$$3(2, -5) - 2(3, 1)$$

Problem 17 [20 Pts]

Is it a vector space????????? the vectors space $S = \{(x_1, x_2)|x_1, x_2 \in \mathbb{R}\}$*, with the two operation:*

Vector Addition *:* $(x_1, x_2) + (y_1, y_2) = (x_1 + y_1 - 1, x_2 + y_2 + 1)$.

Scalar Multiplication *:* $\alpha(x_1, x_2) = (\alpha x_1 - \alpha + 1, \alpha x_2 + \alpha - 1)$.

Find:

$$3(2, -5) - 2(3, 1)$$

Problem 18 **[20 Pts]**

Given the matrix:

$$A = \begin{bmatrix} -1 & 0 & 0 & 1 \\ 5 & 1 & 0 & -5 \\ 7 & 1 & 5 & -7 \\ 2 & 0 & 1 & -2 \end{bmatrix}$$

(a) *Is it singular?*

(b) *What is its determinant?.*

(c) *Based on the results of (a) and (b), what can you say about the eigenvalues and the eigenvectors?*

Problem 19 [20 Pts]

Given the two functions below, :

$$T_1\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} -x_1 + 5x_2 + 2x_3 \\ x_2 - x_3 + 5 \end{bmatrix} \qquad \textit{and} \qquad T_2\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 7x_2 - 2x_3 \\ x_1 + x_3 \\ 0 \end{bmatrix}$$

(a) *What are domain and codomain of each?*

(b) *Are they both Linear Transformations? Why?*