Quiz 2

Problem 1 (10 points)**.** Use the following three facts about determinants to compute the determinant of a matrix using row operations.

a. If B is diagonal, then $\det(B) = b_{11} \cdot b_{22} \cdots b_{nn}$.

b. If B arises from A by a type I row operation, i.e., interchanging two rows, then $\det(B) = -\det(A)$.

c. If B arises from A by a type III row operation, i.e., $r_i + ar_j \to r_i$, that is, row i is replaced by row i plus a scalar multiple of row j, where $i \neq j$. Then $\det(A) = \det(B)$.

Compute $\det(A)$ by:

1. Reducing A to a diagonal matrix B using only type I and III operations.
2. Keep track of how many row swaps were made.
3. Compute $\det(B)$ by multiplying the diagonal elements of B.

$$A = \begin{bmatrix} 2 & 6 & 3 & 2 \\ 4 & 2 & 3 & 2 \\ 2 & 2 & 2 & 1 \\ 4 & 2 & 1 & 5 \end{bmatrix}$$

Show the work for the above computation here.

On your own, don't include this in the quiz, try computing this determinant by expanding on a row or column.

Discuss which method, "expansion along a row or column" or "using elementary row operations" is, in general, a faster method of computing a determinant.

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Problem 2 (5 points)**.** Let A be as above, consider $A\boldsymbol{x} = \boldsymbol{b}$ where $\boldsymbol{b} = (-3, -3, -2, 1)$. Find x_1 using Cramer's rule. (You may use MATLAB/Octave to compute the determinants, but write out what you are computing.)

Problem 3 (10 points; 2 points each)**.** Decide if each of the following are true or false and provide a small proof or counterexample in each case.

(a) _____ If A is an $n \times n$ matrix all of whose entries are integers and $\det(A) = \pm 1$, then A^{-1} also has only integer entries.

(b) _____ If A and B are similar, then $\det(A) = \det(B)$.

Here two $n \times n$ matrices A and B are called *similar* iff $A = SBS^{-1}$ for some invertible S.

(c) _____ Three vectors in $\mathbb{R}^3$, $\boldsymbol{v}_1$, $\boldsymbol{v}_2$, and $\boldsymbol{v}_3$ are co-planar iff $\det(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3) = 0$.

(d) _____ $\det(A^2 + B^2) \geq 0$ for all $n \times n$ matrices A and B with real entries that commute.

(e) _____ The determinant can be viewed as a multilinear function $\det : (\mathbb{R}^n)^n \to \mathbb{R}$ with the properties that $\det(\boldsymbol{e}_1, \ldots, \boldsymbol{e}_n) = 1$ and $\det(\boldsymbol{v}_1, \ldots, \boldsymbol{v}_n) = \det(\boldsymbol{v}'_1, \ldots, \boldsymbol{v}'_n)$, where $(\boldsymbol{v}'_1, \ldots, \boldsymbol{v}_n)'$ is the result of swapping two of the vectors in $(\boldsymbol{v}_1, \ldots, \boldsymbol{v}_n)$.

Problem 4 (10 points)**.** Submit the completion certificate for the OnRamp tutorial from MATLAB in the MATLAB shared drive.