Quiz 5

Question 1 (12 points)**.** Let $A = \begin{bmatrix} 5/4 & -3/4 \\ -3/4 & 5/4 \end{bmatrix}$, write $A = U\Lambda U^{-1}$ where U is unitary, columns are orthonormal basis for $\mathbb{R}^2$ and $\Lambda = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$ with $\lambda_1 > \lambda_2$.

Recall: $U^{-1} = U^T$ for unitary U.

Question 2 (12 points)**.** Suppose the matrix A from Question 1 is used to transform points in the plane iteratively. That is, given a point $\boldsymbol{v}$, consider the sequence $\boldsymbol{v_n} = A^n\boldsymbol{v}$. Letting $U = \begin{bmatrix} \boldsymbol{u_1} & \boldsymbol{u_2} \end{bmatrix}$ so that $\boldsymbol{u_i}$ is an eigenvector associated to λ_i and letting $\boldsymbol{v} = c_1\boldsymbol{u_1} + c_2\boldsymbol{u_2}$ what is a simple expressions for a_n and b_n so that $\boldsymbol{v_n} = A^n\boldsymbol{v} = a_n\boldsymbol{u_1} + b_n\boldsymbol{u_2}$.

Question 3 (11 points)**.** If A and B are similar square matrices and say S witnesses this, that is $A = SBS^{-1}$, show that $(\lambda, \boldsymbol{v})$ is an eigenvalue/eigenvector pair for A if, and only if, $(\lambda, S^{-1}\boldsymbol{v})$ is an eigenvalue/eigenvector pair for B.