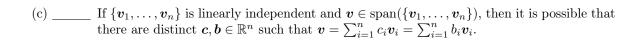
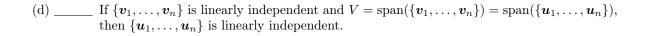
Quiz 3

Problem 1 (15 points; 3 points each). Decide if each of the following are true or false and provide a small proof or counterexample in each case. All vector spaces are assumed to be finite-dimensional here.

(a) _____ Given a basis $\mathcal{B} = \{v_1, \dots, v_n\}$ for a vector space V and U a subspace of V, then there is $C \subseteq \mathcal{B}$ that is a basis for U.

(b) _____ Given a basis $\mathcal C$ for a subspace U of a vector space V, $\mathcal C$ can be extended to a basis $\mathcal B$ for V.





(e) ______ Suppose V is a vector space and $U \subseteq V$ is a subspace. For any $v \in V$, there is a **unique** $u \in U$ so that v = u + (v - u), that is, there is a unique "projection" of V into U.

Problem 2 (10 pts). Show that the collection, U, of upper triangular 3×3 matrices is a subspace of $\mathbb{R}^{3\times 3}$ (the space of all 3×3 matrices). Give a basis \mathcal{B} for U and for $\mathbf{v} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$, give $[\mathbf{v}]_{\mathcal{B}}$.

Problem 3 (10 pts). Let c_1, c_2, \ldots, c_n be n distinct real numbers. Let $p_i = \prod_{\substack{j=1 \ j \neq i}}^n (x - c_j)/(c_i - c_j)$. Show that $\mathcal{B} = \{p_1, p_2, \ldots, p_n\}$ is a basis for P_{n-1} .

Hint: Compute $p_i(c_j)$ and look at what happens when i=j and when $i\neq j$. Use this to argue the independence of \mathcal{B} .