Name: ______________________________ Quiz 1 - MAT345

Problem 1 (15 points; 3 points each)**.** Decide if each of the following is true or false.

(a) _____ If A and B are $n \times n$ symmetric matrices, then $AB + BA$ is symmetric.

This is true. $(AB+BA)^T = (AB)^T + (BA)^T = B^T A^T + A^T B^T = BA + AB = AB + BA$.

(b) _____ Given a matrix A, there might be more than one reduced row echelon form of A.

This is false. There is a unique reduced echelon form.

(c) _____ If A is a 2×4 matrix, then $A\boldsymbol{x} = \mathbf{0}$ has at least two free variables.

This is true. A 2×4 matrix corresponds to an *underdetermined system* with at least two free variables.

(d) _____ $A^2 + 2AB + B^2 = (A + B)^2$ for all square $n \times n$ matrices A and B.

This is false. $(A+B)(A+B) = (A+B)A + (A+B)B = A^2 + BA + AB + B^2$, but as $AB \neq BA$ can occur, there is no reason that $AB + BA = 2AB$.

(e) _____ For A a 3×6 matrix, let E be the matrix so that EA is the result of the row op $R_2 - 3R_1 \to R_2$ applied to A. Then the second row of E is $\begin{bmatrix} -3 & 1 & 0 \end{bmatrix}$.

This is true.

I did not intend this to be a trick problem, so I will accept either answer here.

$$\begin{bmatrix} ? & ? & ? \\ -3 & 1 & 0 \\ ? & ? & ? \end{bmatrix} \begin{bmatrix} \boldsymbol{a}_1 \\ \boldsymbol{a}_2 \\ \boldsymbol{a}_3 \end{bmatrix} = \begin{bmatrix} \cdots \\ (-3) \cdot \boldsymbol{a}_1 + (1) \cdot \boldsymbol{a}_2 + (0) \cdot \boldsymbol{a}_3 \\ \cdots \end{bmatrix}$$

Problem 2 (25 points)**.** Solve $A\boldsymbol{x} = \mathbf{0}$ for

$$A = \begin{bmatrix} 5 & 5 & 15 & -3 \\ -3 & 0 & -3 & 0 \\ 5 & 4 & 13 & 5 \end{bmatrix}$$

Follow the procedure discussed in class

- (10 points) Use elementary row ops to reduce to an echelon matrix.
- (10 points) Write down the resulting triangular system and use back-substitution to solve.
- (5 points) Write out your solution as a linear combination of vectors.

Reduction to echelon form requires 3 row manipulations:

This part is 10/25 points.

$$\begin{bmatrix} 5 & 5 & 15 & -3 \\ -3 & 0 & -3 & 0 \\ 5 & 4 & 13 & 5 \end{bmatrix} \underset{\substack{R_2 \leftarrow R_2 - (-3/5)R_1 \\ R_3 \leftarrow R_3 - (5/5)R_1}}{\longrightarrow} \begin{bmatrix} 5 & 5 & 15 & -3 \\ 0 & 3 & 6 & -9/5 \\ 0 & -1 & -2 & 8 \end{bmatrix}$$

$$\underset{R_3 \leftarrow R_3 - (-1/3)R_2}{\longrightarrow} \begin{bmatrix} 5 & 5 & 15 & -3 \\ 0 & 3 & 6 & -9/5 \\ 0 & 0 & 0 & 37/5 \end{bmatrix}$$

So x_3 is the only free variable. Let $x_3 = t$. Back substitution gives

$$\begin{aligned} x_4 &= 0 \\ x_3 &= t \\ 3x_2 + 6t &= 0 \to x_2 = -2t \\ 5x_1 + 5(-2t) + 15t &= 0 \to x_1 = -t \end{aligned}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -t \\ -2t \\ t \\ 0 \end{bmatrix} = t \begin{bmatrix} -1 \\ -2 \\ 1 \\ 0 \end{bmatrix}$$

This final answer is the remaining 5/25 points.