Quiz 3

Problem 1. Find a basis for span$\{\boldsymbol{u}_1, \ldots, \boldsymbol{u}_5\}$ from among the vectors $\boldsymbol{u}_1, \ldots, \boldsymbol{u}_5$, where

$$\boldsymbol{u}_1 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} \qquad \boldsymbol{u}_2 = \begin{bmatrix} 2 \\ 5 \\ 4 \end{bmatrix} \qquad \boldsymbol{u}_3 = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} \qquad \boldsymbol{u}_4 = \begin{bmatrix} 2 \\ 7 \\ 4 \end{bmatrix} \qquad \boldsymbol{u}_5 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

Make sure to show all work and explain your reasoning.

Problem 2. Suppose U and W are subspaces of $\mathbb{R}^6$, $\dim(U) = 4 = \dim(V)$ and $U + V = \mathbb{R}^6$. What is the dimension of $U \cap V$? You must explain your answer completely.

Problem 3. Let c_1, c_2, and c_3 be three distinct real numbers. Let $p_i = \prod_{\substack{j=1 \\ j \neq i}}^{3} (x - c_j)/(c_i - c_j)$. Show that $\mathcal{B} = \{p_1, p_2, p_3\}$ is a basis for P_2.