Quiz 6

Question 1 (15 points; 3 points each)**.** Decide if each of the following are true or false and provide a justification or counterexample in each case. A justification could consist of a theorem from the text. All vector spaces are assumed to be finite-dimensional here. All vector spaces are now over $\mathbb{C}$ unless otherwise stated.

(a) ______ If the characteristic polynomial of a 4×4 matrix is $p(t) = (t-1)^2 t^2$, then there must be an invertible matrix S so that $A = SDS^{-1}$ where

$$D = \operatorname{diag}(1, 1, 0, 0) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(b) ______ If A and B are $n \times n$ matrices and λ_A and λ_B are eigenvalues for A and B respectively with respect to the same eigenvector $\boldsymbol{v}$, then $AB\boldsymbol{v} = BA\boldsymbol{v}$.

(c) ______ If A and B are $n \times n$ matrices and λ_A is an eigenvalue of A and λ_B is an eigenvalue of B, then $\lambda_A \lambda_B$ is an eigenvalue of AB.

(d) ______ If A and B are diagonalizable $n \times n$ matrices, then AB is diagonalizable.

(e) ______ Suppose A is diagonalizable, then e^A is diagonalizable and e^λ is an eigenvalue of e^A iff λ is and eigenvalue of A.

Question 2 (10 points)**.** Let $A = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 3 & 0 \\ 2 & 0 & 0 \end{bmatrix}$, write $A = U\Lambda U^{-1}$ where U is unitary, columns are orthonormal basis for $\mathbb{R}^3$ and $\Lambda = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$ with $\lambda_1 > \lambda_2 > \lambda_3$.

Recall: $U^{-1} = U^T$ for unitary U.

Question 3 (10 points)**.** Suppose the matrix $A = \frac{1}{12}\left[\begin{smallmatrix} 5 & -1 \\ -1 & 5 \end{smallmatrix}\right]$ is used to transform points in the plane iteratively. That is, given a point $\boldsymbol{v}$, consider the sequence $\boldsymbol{v_n} = A^n\boldsymbol{v}$. Letting $U = \begin{bmatrix}\boldsymbol{u_1} & \boldsymbol{u_2}\end{bmatrix}$ so that $\boldsymbol{u_i}$ is an eigenvector associated to λ_i and letting $\boldsymbol{v} = c_1\boldsymbol{u_1} + c_2\boldsymbol{u_2}$ what is a simple expressions for a_n and b_n so that $\boldsymbol{v_n} = A^n\boldsymbol{v} = a_n\boldsymbol{u_1} + b_n\boldsymbol{u_2}$.