Quiz 1

Problem 1 (15 points; 3 points each)**.** Decide if each of the following are true or false and provide a justification or counterexample in each case. A justification could consist of a theorem from the text. All vector spaces are assumed to be finite-dimensional here.

(a) _____ Given a matrix with integer entries, it is possible to use elementary operations I – III with only integer constants in II and III. So that the final echelon matrix and all matrices along the way have only integer entries.

(b) _____ There is only one echelon form of a matrix.

(c) _____ The inverse of an elementary row operation is another elementary row operation of exactly the same type.

(d) _____ If $AC = BC$, then $A = B$.

(e) _____ If A and B are invertible $n \times n$ matrices, then

$$((AB)^T)^{-1} = ((AB)^{-1})^T = (A^T)^{-1}(B^T)^{-1} = (A^{-1})^T(B^{-1})^T$$

Problem 2 (10 points)**.** Write down the augmented matrix associated to this system and then use the elementary row operations I and III to find an equivalent echelon form of this matrix. Show every step of the reduction and indicate what operation you use. Use only integer constants in II – III.

Eliminate all the non-pivot leading coefficients in column 1 first, then work on column 2, etc. You can combine all operations for one column in a single step, so this should require 2 or 3 steps depending on how you write things.

$$\begin{aligned} 4x_1 + x_2 - x_3 + 3x_4 &= 5 \\ -4x_1 - 4x_2 + x_3 &= 4 \\ -2x_1 + 4x_2 + 4x_3 - 4x_4 &= -12 \end{aligned}$$

Problem 3 (10 points)**.** Using your echelon form above re-write the initial system as an equivalent triangular system. Let the variable that correspond to non-pivot element be the independent variable and solve for the remaining three variables in terms of this one. This is the "**back substitution**" step. Finally, write the solution set as $\{t\boldsymbol{v} + \boldsymbol{u} \,|\, t \in \mathbb{R}\}$ for some $\boldsymbol{v}, \boldsymbol{u} \in \mathbb{R}^4$. This way it is clear that the solution set is a line in $\mathbb{R}^4$.