Name: ______________________ Quiz 2 - MAT345

Problem 2.1 (20 points; 4 points each)**.** Decide if each of the following are true or false and provide a small proof or counterexample in each case. All vector spaces are assumed to be finite-dimensional here.

(a) _____ Given a basis $\mathcal{B} = \{\boldsymbol{v}_1, \ldots, \boldsymbol{v}_n\}$ for a vector space V and U a subspace of V, then there is $\mathcal{C} \subseteq \mathcal{B}$ that is a basis for U.

(b) _____ Given a basis $\mathcal{C}$ for a subspace U of a vector space V, $\mathcal{C}$ can be extended to a basis $\mathcal{B}$ for V.

(c) _____ If $\boldsymbol{v} \in \operatorname{span}\{\boldsymbol{v}_1, \ldots, \boldsymbol{v}_n\}$, then it is guaranteed that there are unique scalars $\alpha_1, \ldots, \alpha_n$ so that $\boldsymbol{v} = \sum \alpha_i \boldsymbol{v}_i$.

(d) _____ If $\{\boldsymbol{v}_1, \ldots, \boldsymbol{v}_n\}$ span V and $\{\boldsymbol{u}_1, \ldots, \boldsymbol{u}_n\} \subseteq V$ is linearly independent, then $\{\boldsymbol{u}_1, \ldots, \boldsymbol{u}_n\}$ span V.

(e) _____ Suppose V is a vector space and $U \subseteq V$ is a subspace. For any $\boldsymbol{v} \in V$, there is a **unique** $\boldsymbol{u} \in U$ so that $\boldsymbol{v} = \boldsymbol{u} + (\boldsymbol{v} - \boldsymbol{u})$, that is, there is a unique "projection" of V into U.

Problem 2.2 (10 pts)**.** A square matrix A is called **horizontally-symmetric** if $\text{flip}(A) = A$ where $\text{flip}(A)$ is the matrix you obtain from A by flipping it horizontally, for example,

$$\text{flip}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 1 \\ 6 & 5 & 4 \\ 9 & 8 & 7 \end{bmatrix}$$

a) Show that the flip-symmetric 3×3 matrices form a subspace of all 3×3 matrices.

b) Give a basis, $\mathcal{B}$, for the 3×3 flip-symmetric matrices.

c) Give representation $[\boldsymbol{v}]_{\mathcal{B}}$ for $\boldsymbol{v} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ -2 & -3 & -2 \end{bmatrix}$ with respect to the basis that you gave.

Problem 2.3. Suppose U and W subspaces of a vector space V such that

$$U + W = V, \text{ and } U \cap W = \{\mathbf{0}\}.$$

Then for every $\boldsymbol{v} \in V$, there is a **unique pair** $\boldsymbol{u} \in U, \boldsymbol{w} \in W$ so that $u + w = v$.

Recall: $U + W = \{\boldsymbol{u} + \boldsymbol{w} \mid \boldsymbol{u} \in U \text{ and } \boldsymbol{w} \in W\}$.