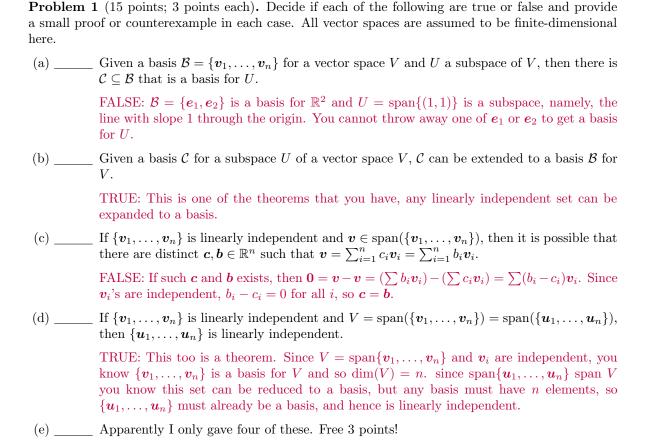
Quiz 3



Problem 2 (10 pts). Find a basis for span $\{u_1, \ldots, u_5\}$ from among the vectors u_1, \ldots, u_5 , where

$$oldsymbol{u}_1 = egin{bmatrix} 1 \ 2 \ 2 \end{bmatrix} \qquad oldsymbol{u}_2 = egin{bmatrix} 2 \ 5 \ 4 \end{bmatrix} \qquad oldsymbol{u}_3 = egin{bmatrix} 1 \ 3 \ 2 \end{bmatrix} \qquad oldsymbol{u}_4 = egin{bmatrix} 2 \ 7 \ 4 \end{bmatrix} \qquad oldsymbol{u}_5 = egin{bmatrix} 1 \ 1 \ 0 \end{bmatrix}$$

Do this by building the matrix consisting of the u_i 's as rows or columns (you must choose correctly) and use Gaussian elimination. This is described carefully in the notes.

Using the notation $A \sim B$ to mean A and B are related by performing elementary row operations, or B = EA where E is invertible. Let $A = \begin{bmatrix} \mathbf{u}_1 & \cdots & \mathbf{u}_5 \end{bmatrix}$, then

$$A = \begin{bmatrix} 1 & 2 & 1 & 2 & 1 \\ 2 & 5 & 3 & 7 & 1 \\ 2 & 4 & 2 & 4 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 1 & 2 & 1 \\ 0 & 1 & 1 & 3 & -1 \\ 0 & 0 & 0 & 0 & -2 \end{bmatrix}$$

B is an echelon form of A with pivots in columns 1, 2, and 5. Thus $\{u_1, u_2, u_5\}$ is a basis for $\text{span}\{u_1, \dots, u_5\} = \text{CS}(A)$.

This was all that was asked here. The rest is just something like you have on the exam.

We also know that $v_1 = (1, 2, 1, 2, 1)$, $v_2 = (0, 1, 1, 3, -1)$, and $v_3 = (0, 0, 0, 0, -1)$ is a basis for RS(A).

By back substitution, letting $x_3 = s$ and $x_4 = t$ we have $x_5 = 0$, $x_2 = -s - 3t$, and $x_1 = -2x_2 - s - 2t = 2(s + 3t) - s - 2t = s + 4t$, so NS(A) consists of vectors of the form (s + 4t, -s - 3t, s, t, 0) = s(1, -1, 1, 0, 0) + t(4, -3, 0, 1, 0), so NS(A) = span $\{v + 4, v_5\}$, where $v_4 = (1, -1, 1, 0, 0)$ and $v_5 = (4, -3, 0, 1, 0)$.

Quick check: Check that v_4 and v_5 are orthogonal to all rows of A.

Problem 3 (10 pts). Let c_1, c_2, \ldots, c_n be n distinct real numbers. Let $p_i = \prod_{\substack{j=1 \ j \neq i}}^n (x - c_j)/(c_i - c_j)$. Show that $\mathcal{B} = \{p_1, p_2, \ldots, p_n\}$ is a basis for P_{n-1} .

Hint: Compute $p_i(c_j)$ and look at what happens when i = j and when $i \neq j$. Use this to argue the independence of \mathcal{B} .

I'll do the n-dimensional case. Suppose c_1, \dots, c_n are distinct reals then define $p_i \in P_{n-1}$

$$p_i = \prod_{\substack{j=1 \ j \neq i}}^{n} (x - c_j) / (c_i - c_j)$$

It is trivial to see that

$$p_i(c_j) = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$$

This shows independence since if $p = \sum_{i=1}^{n} \alpha_i p_i$, then $p_i(c_j) = \alpha_j$ so if p = 0, then $\alpha_j = 0$ for all j.

This all you were asked to do.

Just for fun, for the basis, $\mathcal{B} = \{p_1, p_2, \dots, p_n\}, p = \sum_{i=1}^n p(c_i)p_i$, that is,

$$[p]_{\mathcal{B}} = \begin{bmatrix} p(c_1) \\ p(c_2) \\ \vdots \\ p(c_n) \end{bmatrix}$$

So given n points (x_i, y_i) with the x_i 's distinct. The unique degree (n-1)-polynomial through these points is $p(x) = \sum_{i=1}^{n} y_i \cdot p_i(x)$.