Math 571 - Exam 1 (20 points)

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Question 1 (20 points)**.** For the exam, you need only indicate True or False. No justification is required. If you want to earn back some points, you can supply full justifications for **all** of the problems. You may earn back 50% of lost points.

(a) False Let $X = (0,1] \subseteq \mathbb{R}$. In the induced metric, X is closed and bounded, so X is compact.

(b) True A discrete space is compact iff it is finite.

(c) True $\mathrm{Cl}(A \cup B) = \mathrm{Cl}(A) \cup \mathrm{Cl}(B)$.

(d) False $\mathrm{Cl}(A \cap B) = \mathrm{Cl}(A) \cap \mathrm{Cl}(B)$.

(e) False For X a metric space, to show that a set $F \subseteq X$ is closed, it is necessary and sufficient to show that every sequence from F has a subsequence that converges to a point in F.

(f) False For X a metric space, to show that a set $K \subseteq X$ is compact, it is necessary and sufficient to show that every sequence from K has a subsequence that converges.

(g) False If A is connected, then ∂A is connected.

(h) False Let (Y, d_Y) be a metric space and $f : X \to Y$. Define $d_f : X \times X \to [0, \infty)$ by $d_f(x, x') = d_Y(f(x), f(x'))$. d_f will always give a metric on X for all X, Y, and f.

(i) False On $\mathbb{R}^* = \mathbb{R} - \{0\}$, $d^*(x, y) = \left|\frac{1}{x} - \frac{1}{y}\right| = \frac{|x-y|}{|xy|}$ is a metric on $\mathbb{R}^*$. In this metric, $\left(\frac{1}{n} \mid n = 1, 2, \ldots\right)$ has a limit.

(j) True Let $d(x, y) = |x - y|$ be the standard metric on $\mathbb{R}$ and let d^* be as in part (i). A little work gives that for $\delta|x_0| < 1$, letting $\delta' = |x_0|\left(1 - \frac{1}{\delta|x_0|+1}\right)$ and $\delta'' = |x_0|\left(\frac{1}{1-\delta|x_0|} - 1\right)$ we have that

$$|x - x_0| < \delta' \implies \left|\frac{1}{x} - \frac{1}{x_0}\right| < \delta$$

and

$$\left|\frac{1}{x} - \frac{1}{x_0}\right| < \delta \implies |x - x_0| < \delta''.$$

So $(\mathbb{R}^*, d^*)$ and $(\mathbb{R}^*, d)$ have the same open sets, and hence the two metrics induce the same topological space.