Quiz 5

Problem 1. Find the equation of the circle that best fits (in the sense of least squares) the points (−1,−2), (0, 2.4), (1.1,−4), and (2.4,−1.6).

Hint: The equation of an arbitrary circle is of the form $x^2 + y^2 + c_1x + c_2y + c_3 = 0$ which can be rearranged as $c_1x + c_2y + c_3 = -(x^2 + y^2)$. Now for each point (x_i, y_i) you have one equation of the form $c_1x_i + c_2y_i + c_3 = -(x_i^2 + y_i^2)$. You want to find c_1, c_2 and c_3 that "best" do the job.

Problem 2. Using the inner product

$$\langle p, q\rangle = \int_0^1 pq\,dx$$

use Gram-Schmidt to find an orthonormal basis for $\mathbb{P}_2[x]$, the space of all polynomials of degree 2 or less.

Use this to find the projection, q, of $p = x^3 - x$ onto $\mathbb{P}_2[x]$.

Problem 3. Find a QR decomposition of the matrix

$$A = \begin{bmatrix} -1 & 4 & -3 \\ -1 & 0 & -1 \\ -1 & 4 & 3 \\ -1 & 0 & 5 \end{bmatrix}$$

Do this all by hand, show all work. You should check your answer, easy since $A = QR$ must be true.