Quiz 2

Problem 1 (7 points)**.** Use the following three facts about determinants to compute the determinant of a matrix using row operations.

a If B is diagonal, then $\det(B) = b_{11} \cdot b_{22} \cdots b_{nn}$.

b If B arises from A by a type I row operation, i.e., interchanging two rows, then $\det(B) = -\det(A)$.

c If B arises from A by a type III row operation, i.e., $r_i + ar_j \to r_i$, that is, row i is replaced by row i plus a scalar multiple of row j, where $i \neq j$. Then $\det(A) = \det(B)$.

Compute $\det(A)$ by:

1 Reducing A to a diagonal matrix B using only type I and III operations.

2 Keep track of how many row swaps were made.

3 Compute $\det(B)$ by multiplying the diagonal elements of B.

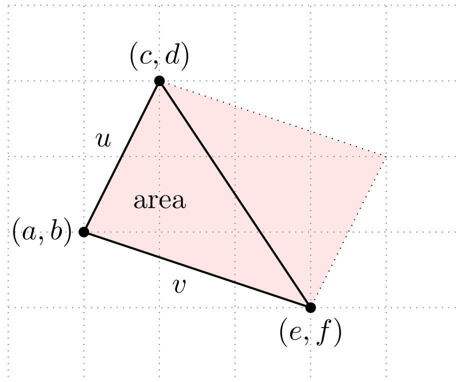
$$A = \begin{bmatrix} 8 & 6 & -3 & 20 \\ 4 & 2 & -5 & -7 \\ 8 & 2 & 7 & 20 \\ 4 & 2 & -11 & -4 \end{bmatrix}$$

Show the work for the above computation. On your own, don't include this in the quiz, try computing this determinant by expanding on a row or column.

Discuss which method, in general, seems to be a faster method of computing a determinant, expanding the determinant along a row or column, or using row operations.

Problem 2 (7 points)**.** Let A be as above, consider $A\boldsymbol{x} = \boldsymbol{b}$ where $\boldsymbol{b} = (31, 2, 21, 11)$. Find x_1 using Cramer's rule. (You may use MATLAB/Octave to compute the determinants, but write out what you are computing.)

Problem 3 (7 points)**.** Given vectors $v = \begin{bmatrix} v_0 \\ v_1 \end{bmatrix}$ and $u = \begin{bmatrix} u_0 \\ u_1 \end{bmatrix}$ it is true that $\det(u, v) = \det \begin{bmatrix} u_0 & v_0 \\ u_1 & v_1 \end{bmatrix}$ is the area of the parallelogram formed by u and v in the natural way. Thus we know:



$$\text{area} = \frac{1}{2}\det(u, v) = \frac{1}{2}\det\begin{bmatrix} c-a & e-a \\ d-b & f-b \end{bmatrix}$$

Show that the area of a triangle with vertices (a, b), (c, d), and (e, f) is given by

$$\text{area} = \frac{1}{2}\det\begin{bmatrix} 1 & 1 & 1 \\ a & c & e \\ b & d & f \end{bmatrix}$$

Problem 4 (14 points)**.** submit the completion certificate for the OnRamp tutorial from MATLAB.