Quiz 5 Make-Up

Problem 1 (15 points; 3 points each)**.** Decide if each of the following are true or false and provide a justification or counterexample in each case. A justification could consist of a theorem from the text. All vector spaces are assumed to be finite-dimensional here.

(a) ______ If A is singular, then $A\boldsymbol{x} = \boldsymbol{b}$, has infinitely many least-square solutions.

(b) ______ If A is singular, there are infinitely many $\hat{\boldsymbol{b}}$ so that $\hat{\boldsymbol{b}} - \boldsymbol{b} \perp \operatorname{rng}(A)$.

(c) ______ Let $\mathcal{U} = \{\boldsymbol{u}_1, \ldots, \boldsymbol{u}_n\}$ is a basis for V with respect to an inner product $\langle \cdot, \cdot \rangle : V \times V \to \mathbb{C}$. Suppose that whenever $\boldsymbol{v} = \sum_{i=1}^{n} \alpha_i \boldsymbol{u}_i$, then $\|\boldsymbol{v}\|_2^2 = \sum_{i=1}^{n} |\alpha_i|^2$. Then $\mathcal{U}$ is orthonormal.

(d) ______ There can be more than one norm on a vector space that is generated by an inner-product.

(e) ______ If $\mathcal{C} = \{\boldsymbol{u}_1, \ldots, \boldsymbol{u}_n\}$ is an orthonormal basis for V with respect to an inner product $\langle \cdot, \cdot \rangle : V \times V \to \mathbb{C}$ and $\boldsymbol{v} \in V$, then for any $(c_1, \ldots, c_n) = [\boldsymbol{v}]_{\mathcal{C}}$, $c_i = \langle v, u_i \rangle$.

Problem 2 (10 points)**.** Using the inner product

$$\langle p, q \rangle = \int_0^1 pq\, dx$$

use Gram-Schmidt to find an orthonormal basis for $U = \operatorname{span}\{1, x^{1/2}, x^2\}$.

Use this to find the orthogonal projection, q, of $p = x$ onto U.

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Problem 3 (10 points). Submit your Linear Algebra Tutorial MATLAB Certificate to the shared MATLAB drive.