Name: ____________________________ Quiz 5 - MAT345

Problem 1 (15 points; 3 points each)**.** Decide if each of the following are true or false and provide a justification or counterexample in each case. A justification could consist of a theorem from the text. All vector spaces are assumed to be finite-dimensional here.

(a) _____ There is a unique least squares solution to $A\boldsymbol{x} = \boldsymbol{b}$.

(b) _____ There is a unique $\boldsymbol{y}$ so that $\|\boldsymbol{y} - \boldsymbol{b}\|$ is minimal and $A\boldsymbol{x} = \boldsymbol{y}$.

(c) ______ If $\{\boldsymbol{u}_1, \ldots, \boldsymbol{u}_n\}$ is an orthonormal basis for V with respect to an inner product $\langle \cdot, \cdot \rangle : V \times V \to \mathbb{C}$ and $\boldsymbol{v} = \sum_{i=1}^{n} \alpha_i \boldsymbol{u}_i$, then $\|\boldsymbol{v}\|_2^2 = \sum_{i=1}^{n} |\alpha_i|^2$.

(d) ______ All norms $\|\cdot\| : \mathbb{R}^n \to [0, \infty)$ on $\mathbb{R}^n$ come from an inner product.

(e) ______ If $\mathcal{C} = \{\boldsymbol{u}_1, \ldots, \boldsymbol{u}_n\}$ is an orthonormal basis for V with respect to an inner product $\langle \cdot, \cdot \rangle : V \times V \to \mathbb{C}$ and $\boldsymbol{v} \in V$, then for any $(c_1, \ldots, c_n) = [\boldsymbol{v}]_{\mathcal{C}}$, $c_i = \langle v, u_i \rangle$.

Problem 2 (25 points)**.** You are given some data points $\{(x_i, y_i) \mid i = 1, \ldots, N\}$ and want to model the data by a function of the form $f(x) = a + bx + c\cos(x) + d\sin(x)$. This involves setting up a matrix A and finding a least-squares solution to $A\mathbf{x} = \mathbf{b}$.

a) (5 points) What is $\mathbf{b}$? (In terms of the data.)

b) (8 points) What is A? (Again, in terms of the data.)

c) (7 points) Suppose you have the least-squares solution $\hat{\mathbf{x}}$. What is $f(x)$? (In terms of $\hat{\mathbf{x}}$)

d) (5 points) What is the relationship between $\hat{\mathbf{b}} = A\hat{\mathbf{x}}$ and $\mathbf{b}$?

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