Name: ____ Quiz 1 - MAT345

Problem 1 (15 points; 3 points each)**.** Decide if each of the following is true or false and provide a justification or counterexample in each case.

(a) _____ If A and B are $n \times n$ symmetric matrices, then AB is symmetric.

This is false. $(AB)^T = B^T A^T = BA$, but for symmetry, we would need $(AB)^T = AB$, and there is no reason that $AB = BA$.

In fact, generating two random 2×2 symmetric matrices will most likely give an example. Here is the first pair I generated:

$$A = \begin{bmatrix} 6 & -2 \\ -2 & -4 \end{bmatrix} \quad B = \begin{bmatrix} -4 & 4 \\ 4 & 2 \end{bmatrix} \quad AB = \begin{bmatrix} -32 & 20 \\ -8 & -16 \end{bmatrix}$$

Clearly, A and B are symmetric, while AB is not.

(b) _____ An echelon form of a matrix is unique.

This is false. There is a unique reduced echelon form. Both

$$E_1 = \begin{bmatrix} 2 & 3 & 1 \\ 0 & 3 & 2 \end{bmatrix}$$

and

$$E_2 = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 3 & 2 \end{bmatrix}$$

are both echelon and each can be got from the other by an elementary row operation.

(c) _____ If A is a 4×2 matrix, then $A\boldsymbol{x} = \mathbf{0}$ has at least two free variables.

This is true. This is true, there can be at most two pivots, and hence at least two variables will correspond to non-pivots.

(d) _____ $A^2 - B^2 = (A - B)(A + B)$ for all square $n \times n$ matrices A and B.

This is false. $(A-B)(A+B) = (A-B)A + (A-B)B = A^2 = A^2 - BA + AB - B^2$, but as $AB \neq BA$ can occur, there is no reason that $AB - BA = 0$.

(e) _____ For A and 4×6 matrix, let E be the matrix so that EA is the result of the row op $R_1 - 3R_2 \to R_1$ applied to A. Then the first row of E is $\begin{bmatrix} 1 & -3 & 0 \end{bmatrix}$.

This is true.

$$\begin{bmatrix} 1 & -3 & 0 \\ \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} \boldsymbol{a}_1 \\ \boldsymbol{a}_2 \\ \boldsymbol{a}_3 \end{bmatrix} = \begin{bmatrix} (1) \cdot \boldsymbol{a}_1 + (-3) \cdot \boldsymbol{a}_2 + (0) \cdot \boldsymbol{a}_3 \\ \vdots \end{bmatrix}$$

Problem 2 (25 points)**.** Solve $A\boldsymbol{x} = \mathbf{0}$ for

$$A = \begin{bmatrix} 4 & 8 & 5 & -3 \\ 5 & 10 & 2 & -8 \\ -3 & -6 & -4 & 2 \end{bmatrix}$$

Follow the procedure discussed in class

- Use elementary row ops to reduce to an echelon matrix.
- Write down the resulting triangular system.
- Use back-substitution to solve.
- Write out your solution as a linear combination of vectors.

The augmented matrix is

$$\begin{bmatrix} 4 & 8 & 5 & -3 \\ -3 & -6 & -4 & 2 \\ 5 & 10 & 2 & -8 \end{bmatrix}$$

Reduction to echelon form requires 3 row manipulations:

$$\begin{bmatrix} 4 & 8 & 5 & -3 \\ -3 & -6 & -4 & 2 \\ 5 & 10 & 2 & -8 \end{bmatrix} \underset{\substack{R_2 \leftarrow R_2 - (-3/4)R_1 \\ R_3 \leftarrow R_3 - (5/4)R_1}}{\longrightarrow} \begin{bmatrix} 4 & 1 & -1 & 3 \\ 0 & 0 & -1/4 & -1/4 \\ 0 & 0 & -17/4 & -17/4 \end{bmatrix}$$

$$\underset{R_3 \leftarrow R_3 - (17)R_1}{\longrightarrow} \begin{bmatrix} 4 & 1 & -1 & 3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Here x_2 and x_4 are free variables, so let $x_2 = s$ and $x_4 = t$, then we get (by back-substitution)

$$x_3 = -t$$
$$4x_1 = -s + (-t) - 3t = -s - 4t \to x_1 = -s/4 - t$$

So we have

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -s/4 - t \\ s \\ -t \\ t \end{bmatrix} = s \begin{bmatrix} -1/4 \\ 1 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} -1 \\ 0 \\ -1 \\ 1 \end{bmatrix}$$