

Name: _____

Quiz 1 - MAT345

Problem 1.1 (15 points; 3 points each). Decide if each of the following is true or false.

(a) False If A and B are $n \times n$ symmetric matrices, then $(A + B)(A - B)$ is symmetric.

(b) True Given matrices A and B that are equivalent, $A\mathbf{x} = \mathbf{0}$ iff $B\mathbf{x} = \mathbf{0}$.

Recall: A and B are equivalent iff there is a series of row operations that transform A into B .

(c) True If A is a 2×4 matrix, then $A\mathbf{x} = \mathbf{0}$ has at least two free variables.

(d) False If $(A - I)(A + I) = O$, then $A = I$ or $A = -I$.

(e) True For A a 3×6 matrix, let E be the matrix so that EA is the result of the row operation $R_2 - 3R_1 \rightarrow R_2$ applied to A . Then the second row of E is $[-3 \ 1 \ 0]$.

Problem 1.2 (25 points). Solve $A\mathbf{x} = \mathbf{0}$ for

$$A = \begin{bmatrix} 5 & 5 & 15 & -3 \\ -3 & 0 & -3 & 0 \\ 5 & 4 & 13 & 5 \end{bmatrix}$$

Follow the procedure discussed in class

- (10 points) Use elementary row ops to reduce to an echelon matrix.
- (10 points) Write down the resulting triangular system and use back-substitution to solve.
- (5 points) Write out your solution as a linear combination of vectors.

Reduction to echelon form requires 3 row manipulations:

This part is 10/25 points.

$$\begin{aligned} \begin{bmatrix} 5 & 5 & 15 & -3 \\ -3 & 0 & -3 & 0 \\ 5 & 4 & 13 & 5 \end{bmatrix} &\xrightarrow{\substack{R_2 \leftarrow R_2 - (-3/5)R_1 \\ R_3 \leftarrow R_3 - (5/5)R_1}} \begin{bmatrix} 5 & 5 & 15 & -3 \\ 0 & 3 & 6 & -9/5 \\ 0 & -1 & -2 & 8 \end{bmatrix} \\ &\xrightarrow{R_3 \leftarrow R_3 - (-1/3)R_2} \begin{bmatrix} 5 & 5 & 15 & -3 \\ 0 & 3 & 6 & -9/5 \\ 0 & 0 & 0 & 37/5 \end{bmatrix} \end{aligned}$$

So x_3 is the only free variable. Let $x_3 = t$. Back substitution gives

$$\begin{aligned} x_4 &= 0 \\ x_3 &= t \\ 3x_2 + 6t &= 0 \rightarrow x_2 = -2t \\ 5x_1 + 5(-2t) + 15t &= 0 \rightarrow x_1 = -t \end{aligned}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -t \\ -2t \\ t \\ 0 \end{bmatrix} = t \begin{bmatrix} -1 \\ -2 \\ 1 \\ 0 \end{bmatrix}$$

This final answer is the remaining 5/25 points.