Name: ______________________________ Quiz 1 - MAT345

**Problem 1.1** (15 points; 3 points each)**.** Decide if each of the following is true or false.

(a) True For $A$ and $B$ invertible matrices,

$$((AB)^{-1})^T = (A^{-1})^T(B^{-1})^T$$

(b) True (i) implies (ii) where:

i. $B = EA$ for some invertible matrix $E$.

ii. $A\mathbf{x} = \mathbf{0} \iff B\mathbf{x} = \mathbf{0}$, that is, $\mathrm{NS}(A) = \mathrm{NS}(B)$.

(c) False If $A$ is a $3 \times 5$ matrix, then $\mathrm{rref}(A)$ has at lest 3 pivot columns.

(d) False If $A^2 = O$, then $A = O$.

(e) False For $A$ a $3 \times 6$ matrix, let $E$ be the matrix so that $EA$ is the result of the row operation $2R_2 - 3R_1 \to R_3$ applied to $A$. Then the second row of $E$ is $\begin{bmatrix} -3 & 2 & 0 \end{bmatrix}$.

**Problem 1.2** (25 points)**.** Solve $A\boldsymbol{x} = \mathbf{0}$ for

$$A = \begin{bmatrix} 1 & 1 & 1 & 0 & 3 \\ 5 & 4 & 6 & -3 & 14 \\ 5 & 9 & 1 & 12 & 19 \\ 4 & 3 & 5 & -3 & 11 \end{bmatrix}$$

Follow the procedure discussed in class

- (10 points) Use elementary row ops to reduce to an echelon matrix.
- (10 points) Write down the resulting triangular system and use back-substitution to solve.
- (5 points) Write out your solution as a linear combination of vectors.

Reduction to echelon form requires 3 row manipulations:

This part is 10/25 points.

$$A = \begin{bmatrix} 1 & 1 & 1 & 0 & 3 \\ 5 & 4 & 6 & -3 & 14 \\ 5 & 9 & 1 & 12 & 19 \\ 4 & 3 & 5 & -3 & 11 \end{bmatrix} \underset{\substack{R_2-5R_1\to R_2 \\ R_3-5R_1\to R_3 \\ R_4-4R_1\to R_4}}{\longrightarrow} \begin{bmatrix} 1 & 1 & 1 & 0 & 3 \\ 0 & -1 & 1 & -3 & -1 \\ 0 & 4 & -4 & 12 & 4 \\ 0 & -1 & 1 & -3 & -1 \end{bmatrix}$$

$$\underset{\substack{R_3+4R_2\to R_2 \\ R_4-R_2\to R_4}}{\longrightarrow} \begin{bmatrix} 1 & 1 & 1 & 0 & 3 \\ 0 & -1 & 1 & -3 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

So $x_3$, $x_4$, and $x_5$ are all free. Back substitution gives

$$\begin{aligned} x_5 &= r \\ x_4 &= s \\ x_3 &= t \\ -x_2 + t - 3s - r = 0 &\to x_2 = t - 3s - r \\ x_1 + (t - 3s - r) + t + 3r = 0 &\to x_1 = -2t + 3s - 2r \end{aligned}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -2t + 3s - 2r \\ t - 3s - r \\ t \\ s \\ r \end{bmatrix} = t \begin{bmatrix} -2 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} 3 \\ -3 \\ 0 \\ 1 \\ 0 \end{bmatrix} + r \begin{bmatrix} -2 \\ -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

This final answer is the remaining 5/25 points.