Quiz 2 - Make up

Problem 1 (5 points)**.** Use the following three facts about determinants to compute the determinant of a matrix using row operations.

a. If B is diagonal, then $\det(B) = b_{11} \cdot b_{22} \cdots b_{nn}$.

b. If B arises from A by a type I row operation, i.e., interchanging two rows, then $\det(B) = -\det(A)$.

c. If B arises from A by a type III row operation, i.e., $r_i + ar_j \to r_i$, that is, row i is replaced by row i plus a scalar multiple of row j, where $i \neq j$. Then $\det(A) = \det(B)$.

Compute $\det(A)$ by:

1. Reducing A to a diagonal matrix B using only type I and III operations.
2. Keep track of how many row swaps were made.
3. Compute $\det(B)$ by multiplying the diagonal elements of B.

$$A = \begin{bmatrix} 8 & 6 & -3 & 20 \\ 4 & 2 & -5 & -7 \\ 8 & 2 & 7 & 20 \\ 4 & 2 & -11 & -4 \end{bmatrix}$$

Show the work for the above computation here.

On your own, don't include this in the quiz, try computing this determinant by expanding on a row or column.

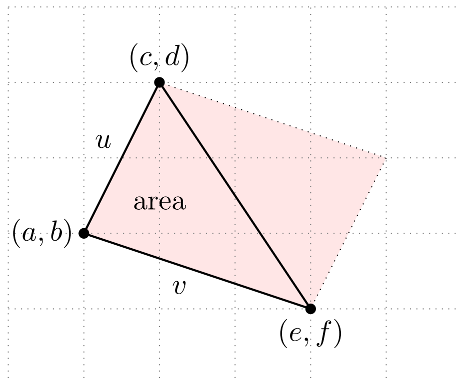
Discuss which method, "expansion along a row or column" or "using elementary row operations" is, in general, a faster method of computing a determinant.

This page left blank

Problem 2 (5 points)**.** Let A be as above, consider $A\boldsymbol{x} = \boldsymbol{b}$ where $\boldsymbol{b} = (31, 2, 21, 11)$. Find x_1 using Cramer's rule. (You may use MATLAB/Octave to compute the determinants, but write out what you are computing.)

]

Problem 3 (5 points)**.** Given vectors $v = \begin{bmatrix} v_0 \\ v_1 \end{bmatrix}$ and $u = \begin{bmatrix} u_0 \\ u_1 \end{bmatrix}$ it is true that $|\det(u, v)| = \left|\det \begin{bmatrix} u_0 & v_0 \\ u_1 & v_1 \end{bmatrix}\right|$ is the area of the parallelogram formed by u and v in the natural way. Thus we know:



$$\text{area} = \frac{1}{2}\det(u, v) = \frac{1}{2}\det \begin{bmatrix} c - a & e - a \\ d - b & f - b \end{bmatrix}$$

Show that the area of a triangle with vertices (a, b), (c, d), and (e, f) is given by

$$\text{area} = \frac{1}{2}\det \begin{bmatrix} 1 & 1 & 1 \\ a & c & e \\ b & d & f \end{bmatrix}$$

Problem 4 (10 points; 2 points each)**.** Decide if each of the following are true or false and provide a small proof or counterexample in each case.

(a) _____ The value of $\det(A)$ is completely determined by

I. Interchanging two rows (or columns) of a matrix changes the sign of the determinant.

II. Multiplying a single row or column of a matrix by a scalar has the effect of multiplying the value of the determinant by that scalar.

III. Adding a multiple of one row (or column) to another does not change the value of the determinant.

and the fact that $\det(I) = 1$.

(b) _____ If A and B are $n \times n$ matrices such that $A = EB$ for some invertible matrix E, then $\det(A) = \det(B)$.

(c) _____ Let A be a 2×2 matrix. Given any "simple" region $R \subset \mathbb{R}^2$ if we transform R by applying A to all points in R to create the transformed region $AR = \{A\boldsymbol{x} \mid \boldsymbol{x} \in R\}$, then $\text{area}(AR) = |\det(A)| \cdot \text{area}(R)$.

Remark: Simple here just means something like the region inside a simple closed smooth (differentiable) curve. You can just imagine a rectangle, circle, oval, or some similar shape.

(d) _____ AB is invertible iff A and B are both invertible.

(e) _____ If $D : (\mathbb{R}^n)^n \to \mathbb{R}^n$ is a multi-linear function such that $D(\boldsymbol{v}_1, \ldots, \boldsymbol{v}_n) = 0$ whenever there are $\boldsymbol{v}_i = \boldsymbol{v}_j$ for some $i \neq j$, then D *alternates*, that is $D(\boldsymbol{v}_1, \ldots, \boldsymbol{v}_n) = -D(\boldsymbol{v}_1', \ldots, \boldsymbol{v}_n')$ where $\boldsymbol{v}_1', \ldots, \boldsymbol{v}_n'$ arises from $\boldsymbol{v}_1, \ldots, \boldsymbol{v}_n$ by swapping two of the vectors.

Problem 5 (10 points)**.** submit the completion certificate for the OnRamp tutorial from MATLAB.