Quiz 2

Problem 1 (10 points; 2 points each)**.** Decide if each of the following are true or false and provide a justification or counterexample in each case. A justification could consist of a theorem from the text. All vector spaces are assumed to be finite-dimensional here.

(a) _____ For A and B $n \times n$ matrices, $\det(AB) = \det(A)\det(B)$

(b) _____ For A and B $n \times n$ matrices, $\det(AB - BA) = \det(AB) - \det(BA)$

(c) ______ Performing type III elementary row operations on a square matrix does not change the value of the determinant.

(d) ______ If A is a 4×4 matrix with rows $\boldsymbol{a}_1$, $\boldsymbol{a}_2$, $\boldsymbol{a}_3$, and $\boldsymbol{a}_4$, and $\boldsymbol{a}_1 = 2\boldsymbol{a}_2 - 3\boldsymbol{a}_3 + 4\boldsymbol{a}_4$, then $\det(A) = 2 - 3 + 4 = 3$.

(e) ______ $\det(A)$ has a geometric interpretation.

Problem 2 (10 points)**.** Use the following three facts about determinants to compute the determinant of a matrix using row operations.

a. If B is diagonal, then $\det(B) = b_{11} \cdot b_{22} \cdots b_{nn}$.

b. If B arises from A by a type I row operation, i.e., interchanging two rows, then $\det(B) = -\det(A)$.

c. If B arises from A by a type III row operation, i.e., $r_i + ar_j \to r_i$, that is, row i is replaced by row i plus a scalar multiple of row j, where $i \neq j$. Then $\det(A) = \det(B)$.

Compute $\det(A)$ by:

1. Reducing A to a triangular matrix B using only type I and III operations. (I would say echelon form, except for the issue with pivots being 1).
2. Keep track of how many row swaps were made.
3. Compute $\det(B)$ by multiplying the diagonal elements of B.

$$A = \begin{bmatrix} 2 & 6 & 3 & 2 \\ 4 & 2 & 3 & 2 \\ 2 & 2 & 2 & 1 \\ 4 & 2 & 1 & 5 \end{bmatrix}$$

Show the work for the above computation here.

On your own, don't include this in the quiz, try computing this determinant by expanding on a row or column.

Discuss which method, "expansion along a row or column" or "using elementary row operations" is, in general, a faster method of computing a determinant.

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Problem 3 (5 points)**.** Let A be as above, consider $A\boldsymbol{x} = \boldsymbol{b}$ where $\boldsymbol{b} = (-3, -3, -2, 1)$. Find x_1 using Cramer's rule. (You may use MATLAB/Octave to compute the determinants, but write out what you are computing.)

Problem 4 (5 points)**.** Submit the completion certificate for the OnRamp tutorial from MATLAB in the MATLAB shared drive.