Quiz 1

Problem 1 (15 points; 3 points each). Decide if each of the following are true or false and provide a justification or counterexample in each case. A justification could consist of a theorem from the text. All vector spaces are assumed to be finite-dimensional here.

(a) _____ The inverse of a Type III elementary matrix need not be elementary.

(b) _____ An echelon form of a matrix is unique.

(c) _____ A matrix is invertible if and only if it can be written as the product of elementary matrices.

(d) _____ Right cancellation holds for matrix multiplication: If $AC = BC$, then $A = B$.

(e) _____ Suppose for all matrices C, $AC = BC$, then $A = B$.

Problem 2 (10 points)**.** Write down the augmented matrix associated to this system and then use the elementary row operations I and III to find an equivalent echelon, upper-triangular, form of this matrix. (See the note below.) Show every step of the reduction and indicate what operation you use.

Note: The book requires the leading entries in non-zero rows be 1's. Many, including myself, drop this and just describe the pivot as the leading non-zero entry. If you like, call this more general form, "*upper-triangular.*"

Eliminate all the non-pivot leading coefficients in column 1 first, then work on column 2, etc. You can combine all operations for one column in a single step, so this should require 2 or 3 steps depending on how you write things.

$$\begin{aligned} 4x_1 + x_2 - x_3 + 3x_4 &= 5 \\ -4x_1 - 4x_2 + x_3 &= 4 \\ -2x_1 + 4x_2 + 4x_3 - 4x_4 &= -12 \end{aligned}$$

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Problem 3 (10 points)**.** Using your echelon form above re-write the initial system as an equivalent triangular system. Let the variable that correspond to non-pivot element be the independent variable and solve for the remaining three variables in terms of this one. This is the "**back substitution**" step. Finally, write the solution set as $\{t\boldsymbol{v} + \boldsymbol{u} \,|\, t \in \mathbb{R}\}$ for some $\boldsymbol{v}, \boldsymbol{u} \in \mathbb{R}^4$. This way it is clear that the solution set is a line in $\mathbb{R}^4$.