Name: _____ Quiz 4 - MAT345

**Question 1** (20 points; 4 points each)**.** No justification required for the this quiz, just the T/F responses.

(a) True If all eigenvalues of $A$ are 0, and $A$ is diagonalizable, then $A = O$.

(b) False The following matrix is diagonalizable

$$\begin{bmatrix} 0 & 1 & 2 & 3 \\ 0 & 0 & 4 & 5 \\ 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(c) True If $\boldsymbol{v}$ is an eigenvector for both $A$ and $B$, then $\mathbf{v}$ is an eigenvector for $AB$.

(d) False If $A$ and $B$ are similar matrices and $\lambda, \boldsymbol{v}$ is an eigenvalue/eigenvector pair for $A$, then the same is true for $B$.

(e) True If $\lambda_1 = \frac{1}{2}$ and $\lambda_2 = -2$ are the eigenvalues for $L : \mathbb{R}^2 \to \mathbb{R}^2$ with corresponding eigenvectors $\boldsymbol{v}_1 = (1, -1)$ and $\boldsymbol{v}_2 = (1, 1)$. Then for any $\boldsymbol{x} \in \mathbb{R}^2$, $L^n(\boldsymbol{x})$ approaches the line $E_{\lambda_2} = \operatorname{span}\{\boldsymbol{v}_2\}$ as $n \to \infty$, specifically $\operatorname{dist}(L^n(\boldsymbol{x}), E_{\lambda_2}) = \| \operatorname{proj}^{\perp}_{E_{\lambda_2}}(L^n(\boldsymbol{x}))\| \to 0$ as $n \to \infty$.

**Question 2** (20 points)**.** Let $A = \begin{bmatrix} 1/2 & 1 \\ 1/2 & 0 \end{bmatrix}$. Diagonalize $A$, i.e. write $A = S\Lambda S^{-1}$ where $\Lambda$ is the diagonal matrix of eigenvalues for $A$.

i) What is $p_A(t)$?

$$p_A(t) = \det \begin{bmatrix} 1/2 - t & 1 \\ 1/2 & -t \end{bmatrix} = (1/2 - t)(-t) - 1/2 = t^2 - 1/2t - 1/2$$

ii) What are the eigenvalues?

$$p_A(t) = 0 \iff 2t^2 - t - 1 = (2t+1)(t-1) = 0 \iff t = -1/2 \text{ or } t = 1$$

So the eigenvalues are $\lambda_1 = -1/2$ and $\lambda_2 = 1$.

iii) Find eigenvectors for each eigenvalue.

$$E_{-1/2} = \text{NS} \begin{bmatrix} 1 & 1 \\ 1/2 & 1/2 \end{bmatrix} = \text{span}\left\{ \begin{bmatrix} -1 \\ 1 \end{bmatrix} \right\}$$

and

$$E_1 = \text{NS} \begin{bmatrix} -1/2 & 1 \\ 1/2 & -1 \end{bmatrix} = \text{span}\left\{ \begin{bmatrix} 1 \\ 1/2 \end{bmatrix} \right\}$$

iv) Diagonalize $A$, i.e., write $A = BDB^{-1}$ where $D$ is diagonal.

$$A = \begin{bmatrix} -1 & 1 \\ 1 & 1/2 \end{bmatrix} \begin{bmatrix} -1/2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 1 & 1/2 \end{bmatrix}^{-1} = \begin{bmatrix} -1 & 1 \\ 1 & 1/2 \end{bmatrix} \begin{bmatrix} -1/2 & 0 \\ 0 & 1 \end{bmatrix} (-\tfrac{2}{3}) \begin{bmatrix} 1/2 & -1 \\ -1 & -1 \end{bmatrix}$$

**Bonus** (5pts - Can complete this at home.) Give a formula for $A^n\mathbf{v}$ where $\boldsymbol{v} = \left[\begin{smallmatrix} a \\ b \end{smallmatrix}\right]$ and use this to find $\lim_{n\to\infty} A^n\boldsymbol{v}$.