Quiz 5

Problem 1 (15 points; 3 points each)**.** Decide if each of the following are true or false and provide a justification or counterexample in each case. A justification could consist of a theorem from the text. All vector spaces are assumed to be finite-dimensional here.

(a) _____ There is a unique least squares solution $\hat{x} = (A^T A)^{-1} A^T \boldsymbol{b}$ to $A\boldsymbol{x} = \boldsymbol{b}$.

(b) _____ For A and $m \times n$ matrix,

$$\hat{\boldsymbol{x}} \text{ is a least squares solution to } A\boldsymbol{x} = \boldsymbol{b} \text{ iff } A\hat{\boldsymbol{x}} = \hat{b}$$

where $\hat{\boldsymbol{b}} = A(A^T A)^{-1} A^T \boldsymbol{b}$ is the unique vector satisfying $\|\boldsymbol{b} - \hat{\boldsymbol{b}}\|^2 = \min\{\|A\boldsymbol{x} - \boldsymbol{b}\|^2 \mid \boldsymbol{x} \in \mathbb{R}^n\}$.

(c) ______ If $\{\boldsymbol{u}_1, \ldots, \boldsymbol{u}_n\}$ is an orthonormal basis for V with respect to an inner product $\langle \cdot, \cdot \rangle : V \times V \to \mathbb{C}$ and $\boldsymbol{v} = \sum_{i=1}^n \alpha_i \boldsymbol{u}_i$, then $\|\boldsymbol{v}\|_2^2 = \sum_{i=1}^n |\alpha_i|^2$.

(d) ______ All norms $\|\cdot\| : \mathbb{R}^n \to [0, \infty)$ on $\mathbb{R}^n$ come from an inner product by $\|\boldsymbol{x}\|^2 = \langle \boldsymbol{x}, \boldsymbol{x} \rangle$.

(e) ______ If $\mathcal{C} = \{\boldsymbol{u}_1, \ldots, \boldsymbol{u}_n\}$ is an orthonormal basis for V with respect to an inner product $\langle \cdot, \cdot \rangle : V \times V \to \mathbb{C}$ and $\boldsymbol{v} \in V$, then for any $(c_1, \ldots, c_n) = [\boldsymbol{v}]_{\mathcal{C}}$, $c_i = \langle v, u_i \rangle$.

Problem 2 (10 points)**.** Using the inner product

$$\langle p, q \rangle = \int_0^1 pq \, dx$$

use Gram-Schmidt to find an orthonormal basis for $\mathbb{P}_2[x]$, the space of all polynomials of degree 2 or less.

Use this to find the projection, q, of $p = x^{2/3}$ onto $\mathbb{P}_2[x]$.

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Problem 3 (10 points)**.** Submit your Linear Algebra Tutorial MATLAB Certificate to the shared MATLAB drive.