Quiz 3

Problem 1 (15 points; 3 points each)**.** Decide if each of the following are true or false and provide a small proof or counterexample in each case. All vector spaces are assumed to be finite-dimensional here.

(a) ______ If V is an 5-dimensional vector space and $U, W \subseteq V$ are subspaces with $\dim(U) = 3$ and $\dim(W) = 2$, then $V = U \oplus W$.

Recall: $V = U + W$ means that each $\boldsymbol{v} \in V$ can be written or decomposed as $\boldsymbol{u} + \boldsymbol{w}$ for some $\boldsymbol{u} \in U$ and $\boldsymbol{w} \in W$. $V = U \oplus W$ means $V = U + W$ and moreover the decomposition is unique, equivalently $U \cap W = \{0\}$.

(b) ______ Suppose V is a vector space and $U \subseteq V$ is a subspace. For any $\boldsymbol{v} \in V$, there is a **unique** $\boldsymbol{u} \in U$ so that $\boldsymbol{v} = \boldsymbol{u} + (\boldsymbol{v} - \boldsymbol{u})$, that is, there is a unique "projection" of V into U.

(c) ______ If A is a 5×7 matrix and $\text{rank}(A) = 5$, then the columns of A span $\mathbb{R}^5$.

(d) ______ If A is a 5×7 matrix and $\text{rank}(A) = 5$, then the rows of A are linearly dependent.

Problem 2 (10 pts)**.** Show that the collection, UT_3, of upper triangular 3×3 matrices is a subspace of $\mathbb{R}^{3\times 3}$ (the space of all 3×3 matrices). Give a basis for UT_3.

Problem 3 (10 pts)**.** Either verify that $(0, \infty) = \mathbb{R}^+$ is a vector space (over $\mathbb{R}$) with vector addition $a + b = e^{(a+b)} = e^a e^b$ and $c \cdot a = (e^a)^c = e^{(ca)}$ for $c \in \mathbb{R}$ or else verify that this is not a vector space.