Quiz 1 - Make-Up

Problem 1 (15 points; 3 points each)**.** Decide if each of the following are true or false and provide a justification or counterexample in each case. A justification could consist of a theorem from the text. All vector spaces are assumed to be finite-dimensional here.

(a) _____ Given a matrix with rational entries, it is possible to use elementary row operations with only rational constants in II and III to reduce the matrix to *reduced row echelon form (RREF)*?

(b) _____ Write $A \underset{\text{row-op}}{\sim} B$ iff there is a sequence of elementary row operations which when applied to A result in B. Is it true that $A \underset{\text{row-op}}{\sim} B$ iff $\text{rref}(A) = \text{rref}(B)$?

(c) _____ Every matrix has an LU decomposition, that is $A = LU$ where L is a lower triangular matrix with 1's on the diagonal and U is echelon. If A is square, then U is upper triangular.

(d) _____ Define $A \heartsuit B = AB - BA$ for $n \times n$ matrices A and B. Then $A \heartsuit B + B \heartsuit A = 0$.

(e) _____ Again, define $A \heartsuit B = AB - BA$ for $n \times n$ matrices A and B. $\heartsuit$ is associative, that is, $(A \heartsuit B) \heartsuit C = A \heartsuit (B \heartsuit C)$?

Problem 2 (10 points)**.** Given

$$A = \begin{bmatrix} 1 & -2 & -4 & 3 \\ 4 & 0 & 1 & 2 \\ 0 & -2 & 2 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 3 & -2 & 9 & 3 \\ -1 & 4 & 2 & -5 \\ -8 & -2 & 0 & -2 \end{bmatrix}$$

Prove or disprove that $A \underset{\text{row-op}}{\sim} B$ (You might make use of something from the T/F section above to help here.)

Show all steps, that is indicate exactly what row operations are used.

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Problem 3 (10 points)**.** For A as in the previous problem, find the solution set to $A\boldsymbol{x} = \mathbf{0}$. Write the solution set as $\{t\boldsymbol{v} | \, t \in \mathbb{R}\}$ for some $\boldsymbol{v} \in \mathbb{R}^4$. This way it is clear that the solution set is a line in $\mathbb{R}^4$.