Formulas and Notation

$$sgm(x) = \frac{1}{1 - e^{-x}}$$

$$Loss = \frac{1}{2} * \sum_{i=1}^{2} E_i, \ \ where\ E_i is\ the\ MSE\ for\ i^{th} term$$

$$E_i = \frac{1}{2} * (y_i - y_i')^2$$

In general we can write that

$$\frac{dE_i}{d\emptyset} = \sum_{k=1}^{i} \frac{dE_i}{dy_i'} * \frac{dy_i'}{dh_i} * \frac{dh_i}{dh_k} * \frac{dh_k}{d\emptyset}$$

Thus, for 1st error term this would yield:

$$\frac{dE_1}{d\emptyset} = \frac{dE_1}{dy'} * \frac{dy'}{dh_1} * \frac{dh_1}{d\emptyset}$$

For 2nd error term this would yield:

$$\frac{dE_2}{d\emptyset} = \sum_{k=1}^{2} \frac{dE_2}{dy_2'} * \frac{dy_2'}{dh_2} * \frac{dh_2}{dh_k} * \frac{dh_k}{d\emptyset}$$

1. ***For w***

$$\frac{dE_1}{dw} = \frac{1}{2} * 2 * (y_1 - y_1') * sgm(wh_1 + b_2) * \big(1 - sgm(wh_1 + b_2)\big) * h1$$

$$\frac{dE_2}{dw} = \frac{1}{2} * 2 * (y_2 - y_2') * sgm(wh_2 + b_2) * \big(1 - sgm(wh_2 + b_2)\big) * h2$$

$$\frac{dLoss}{dw} = \frac{1}{2} * \sum_{i=1}^{2} \frac{dE_i}{dw}, \ \ so\ it\ is\ the\ sum\ of\ the\ above\ terms\ times\ \frac{1}{2}$$

2. ***For w_x***

$$\frac{dE_1}{dw_x} = \frac{1}{2} * 2 * (y_1 - y_1') * sgm(wh_1 + b_2) * \big(1 - sgm(wh_1 + b_2)\big) * w$$
$$* sgm(w_x * x_1 + w_h * h_0 + b_1) * \big(1 - sgm(w_x * x_1 + w_h * h_0 + b_1)\big) * x_1$$

$$\frac{dE_2}{dw_x} = \frac{1}{2} * 2 * (y_2 - y_2') * sgm(wh_2 + b_2) * \big(1 - sgm(wh_2 + b_2)\big) * w$$
$$* sgm(w_x * x_2 + w_h * h_1 + b_1) * \big(1 - sgm(w_x * x_2 + w_h * h_1 + b_1)\big)$$
$$* \big[w_h * sgm(w_x * x_1 + w_h * h_0 + b_1) * \big(1 - sgm(w_x * x_1 + w_h * h_0 + b_1)\big) * x_1$$
$$+ x_2\big]$$

$$\frac{dLoss}{dw_x} = \frac{1}{2} * \sum_{i=1}^{2} \frac{dE_i}{dw}, \text{ so it is the sum of the above terms times } \frac{1}{2}$$

3. For w_h

$$\frac{dE_1}{dw_h} = \frac{1}{2} * 2 * (y_1 - y_1') * sgm(wh_1 + b_2) * \big(1 - sgm(wh_1 + b_2)\big) * w$$
$$* sgm(w_x * x_1 + w_h * h_0 + b_1) * \big(1 - sgm(w_x * x_1 + w_h * h_0 + b_1)\big) * h_0$$

$$\frac{dE_2}{dw_h} = \frac{1}{2} * 2 * (y_2 - y_2') * sgm(wh_2 + b_2) * \big(1 - sgm(wh_2 + b_2)\big) * w$$
$$* sgm(w_x * x_2 + w_h * h_1 + b_1) * \big(1 - sgm(w_x * x_2 + w_h * h_1 + b_1)\big)$$
$$* \big[w_h * sgm(w_x * x_1 + w_h * h_0 + b_1) * \big(1 - sgm(w_x * x_1 + w_h * h_0 + b_1)\big) * h_0 + h_1\big]$$

$$\frac{dLoss}{dw_h} = \frac{1}{2} * \sum_{i=1}^{2} \frac{dE_i}{dw}, \text{ so it is the sum of the above terms times } \frac{1}{2}$$

4. For b_2

$$\frac{dE_1}{db_2} = \frac{1}{2} * 2 * (y_1 - y_1') * sgm(wh_1 + b_2) * \big(1 - sgm(wh_1 + b_2)\big) * 1$$

$$\frac{dE_2}{db_2} = \frac{1}{2} * 2 * (y_2 - y_2') * sgm(wh_2 + b_2) * \big(1 - sgm(wh_2 + b_2)\big) * 1$$

5. For b_1

$$\frac{dE_1}{db_1} = \frac{1}{2} * 2 * (y_1 - y_1') * sgm(wh_1 + b_2) * \big(1 - sgm(wh_1 + b_2)\big) * w$$
$$* sgm(w_x * x_1 + w_h * h_0 + b_1) * \big(1 - sgm(w_x * x_1 + w_h * h_0 + b_1)\big) * 1$$

$$\frac{dE_2}{dw_h} = \frac{1}{2} * 2 * (y_2 - y_2') * sgm(wh_2 + b_2) * \big(1 - sgm(wh_2 + b_2)\big) * w$$
$$* sgm(w_x * x_2 + w_h * h_1 + b_1) * \big(1 - sgm(w_x * x_2 + w_h * h_1 + b_1)\big)$$
$$* \big[w_h * sgm(w_x * x_1 + w_h * h_0 + b_1) * \big(1 - sgm(w_x * x_1 + w_h * h_0 + b_1)\big) * 1 + 1\big]$$

6. Building Blocks

$$h_1 = sgm(w_x * x_1 + w_h * h_0 + b_1) = sgm(1 * 1 + 1 * 0 + 1) = 0.88$$

$$\big(1 - sgm(w_x * x_1 + w_h * h_0 + b_1)\big) = 0.12$$

$$y_1 = sgm(wh_1 + b_2) = 0.867$$

$$y_2 = sgm(wh_2 + b_2) = 0.866$$

7. *Calculations*

$$\frac{dLoss}{dw} = 0.015$$

$$\frac{dLoss}{dw_x} = 0.001122$$

$$\frac{dLoss}{dw_h} = 0.0020$$

$$\frac{dLoss}{db_2} = 0.0168$$

$$\frac{dLoss}{db_1} = 0.00215$$