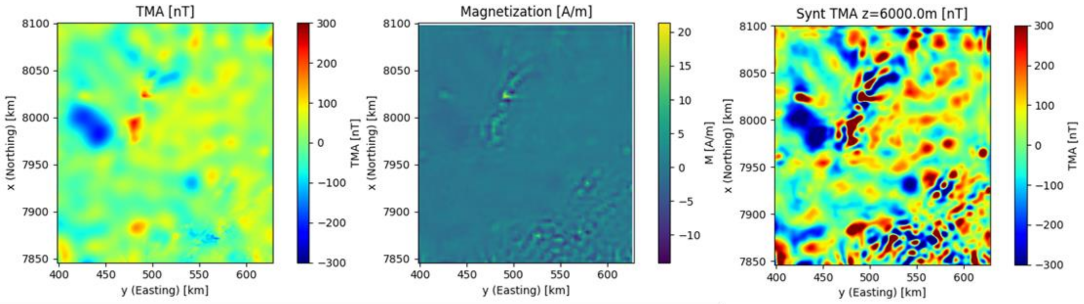


Equivalent source method for magnetic extrapolation

KetilH, 23. October 2025

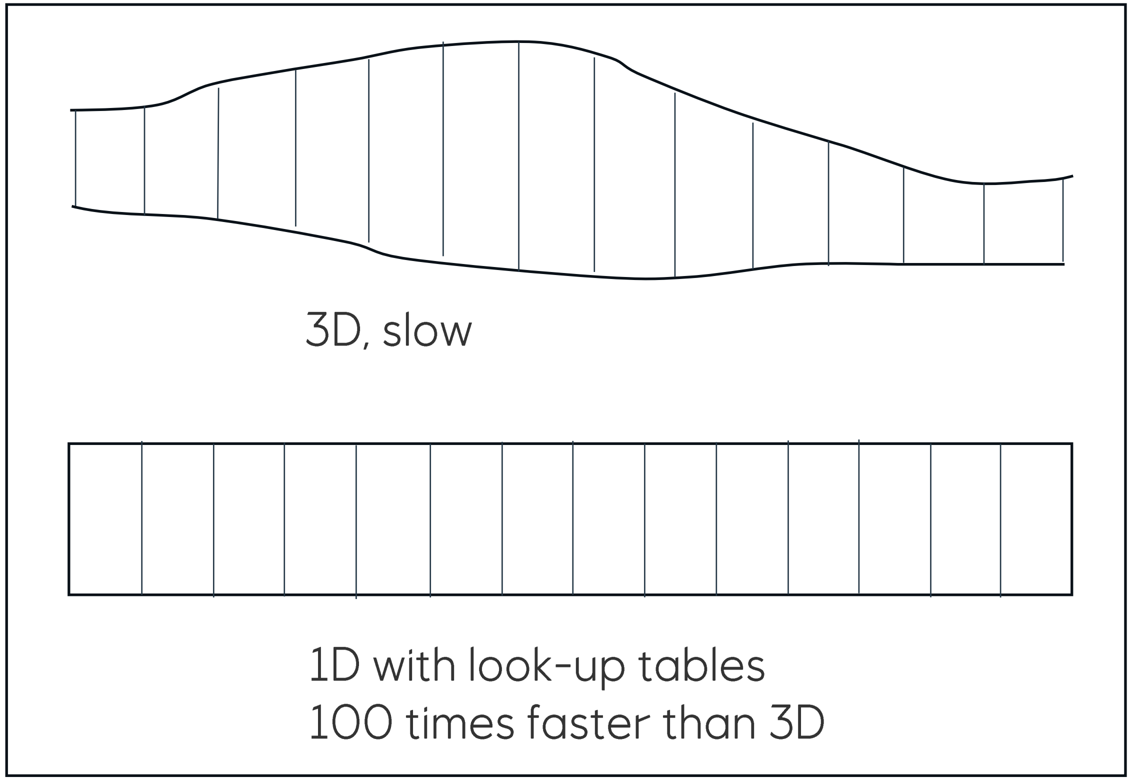
Magnetic map inversion

What the code was made for:

- Deep-sea minerals (mid-ocean ridges)
- Geothermal exploration (Iceland)
- Basalt (MORB); $Q_K \sim 20$

- However, the code is also suitabe for equivalent-source data extrapolation

- Implemented in Python
- The main numerical cost is computation of square-roots
- Can be improved by a compiled C/Fortran function with Python API (?)



References

Hokstad, K., Alasonati-Tašárová, Z., Sæther, B.M., and Tänavsuu-Milkeviciene, K., 2020, Inversion of Magnetic Data for Subsurface Temperature: Proceedings of the World Geothermal Congress 2020+1.

Hokstad, K. and Kruber, C., 2023, Multigeophysical Inversion for Geothermal Exploration. GRC Transactions **47**, 2939.

Magnetic inversion scheme

Magnetic anomaly

$$\boldsymbol{B}_A(\boldsymbol{x}) = -\frac{\mu_0}{4\pi}\boldsymbol{\nabla}\boldsymbol{\nabla}\cdot\iiint\frac{\boldsymbol{M}(\boldsymbol{x}')}{|\boldsymbol{x}'-\boldsymbol{x}|}dV'$$

$$\boldsymbol{M} = \boldsymbol{M}_R + \boldsymbol{M}_I = \boldsymbol{M}_R + \chi\boldsymbol{H}$$

Assume $\boldsymbol{M}$ is independent of z or average over z

$$\boldsymbol{M}(\mathrm{x},\mathrm{y}) = \frac{1}{z_2-z_1}\int_{z_1}^{z_2}\boldsymbol{M}(x,y,z)\,dz$$

$$\boldsymbol{M} = M\boldsymbol{e} \simeq M\boldsymbol{t}$$

$$\boldsymbol{t} = \frac{\boldsymbol{B}_0}{|\boldsymbol{B}_0|}$$

Analytic integration over z in the equation for $\boldsymbol{B}_A$

$$\int_{z_1}^{z_2}\frac{1}{r}dz' = \ln[(z'-z)+r]\,|_{z_1}^{z_2} = F(z_2) - F(z_1)$$

$$F(z') = \ln[(z'-z)+r]$$

$$r = |\boldsymbol{x}'-\boldsymbol{x}|$$

Forward model

$$B_A(\boldsymbol{x}) = \frac{\mu_0}{4\pi}\sum_{i=1}^{3}\sum_{j=1}^{3}\iint t_i A_{ij}(\boldsymbol{x},\boldsymbol{x}')M_j(x',y')\,dx'dy'$$

$$B_A(\boldsymbol{x}_i) = |\boldsymbol{B}_0 + \boldsymbol{B}_A| - |\boldsymbol{B}_0| \simeq \boldsymbol{t}\cdot\boldsymbol{B}_A(\boldsymbol{x})$$

$$A_{ij} = \frac{\partial^2 F(z_2)}{\partial x_i \partial x_j} - \frac{\partial^2 F(z_1)}{\partial x_i \partial x_j}$$

Vector-matrix form

$$\mathbf{d} = \mathbf{Lm} + \mathbf{n}$$

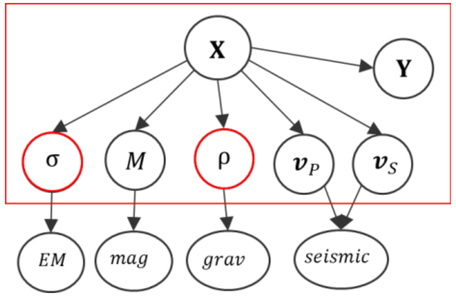
$$L_{ij} = \frac{\mu_0}{4\pi}\sum_{k=1}^{3}\sum_{l=1}^{3}t_k A_{kl}(x_i,y_i,z_i,x_j',y_j')t_l$$

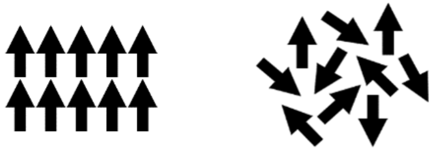
Marquardt-Levenberg

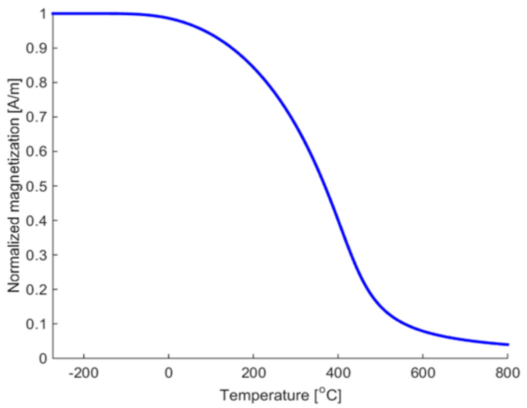
$$\mathbf{m} = \left[\mathbf{L}^{\mathrm{T}}\boldsymbol{\Sigma}_{\mathrm{d}}^{-1}\mathbf{L} + \boldsymbol{\Sigma}_{\mathrm{m}}^{-1}\right]^{-1}\mathbf{L}^{\mathrm{T}}\boldsymbol{\Sigma}_{\mathrm{d}}^{-1}\mathbf{d}$$

$$\boldsymbol{\Sigma}_{\mathrm{m}}^{-1} = \lambda\,\mathrm{diag}\left(\mathbf{L}^{\mathrm{T}}\mathbf{L}\right)$$

Part of multigeophysical inversion (MGI)



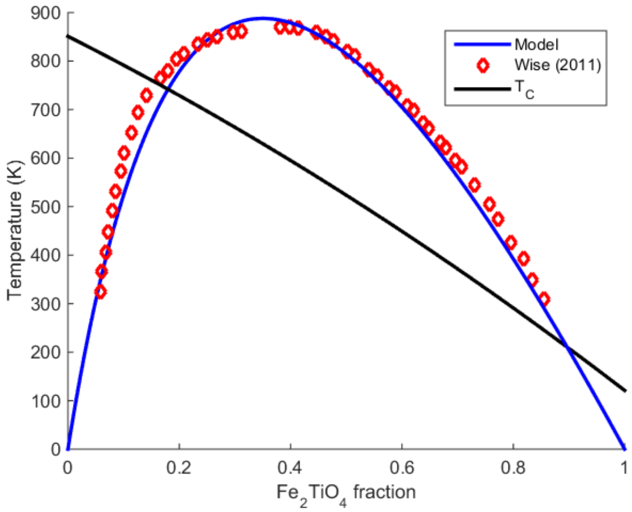




Ising (1925) model

Macroscopic magnetization

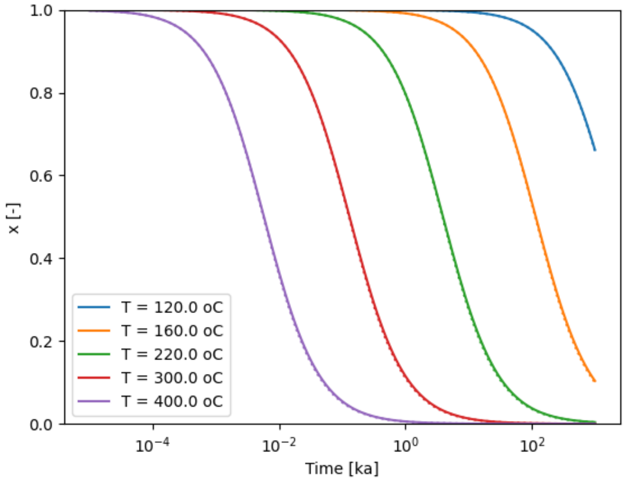
$$\frac{M}{M_0} = \tanh\left[\frac{T_C M}{T M_0} + \frac{CH}{T M_0}\right]$$



Spinodal exsolution

$$\frac{\partial^2 G}{\partial u^2} = -2W + k_B T\left[\frac{(1+h)}{u} + \frac{(1-h)}{v}\right] = 0$$

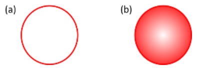
- Curie temperature from Lattard et al. (2006)
- Calibration using data from Wise et al. (2011)



Oxidation: Mixed 1st/2nd order Arrhenius

$$\frac{dx}{dt} = -\left(\frac{k_1 - k_2}{x_0}\right)x^2 - k_2 x$$

$$k_i = A e^{-\frac{E_i}{RT}}$$

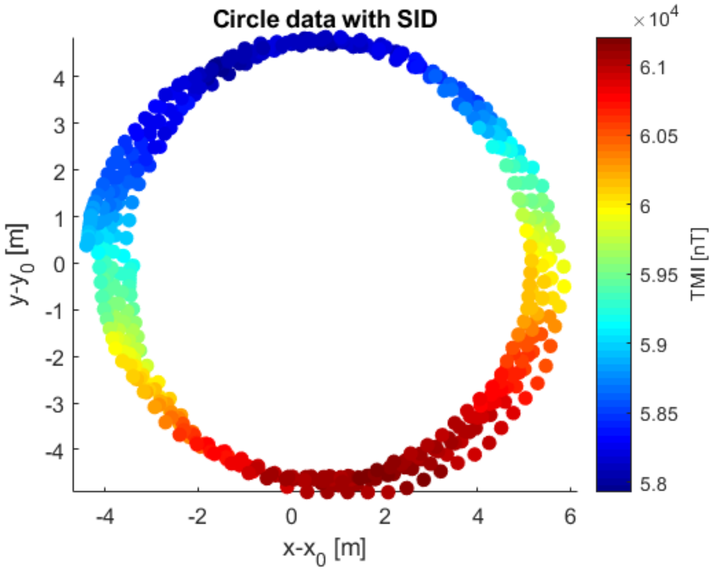


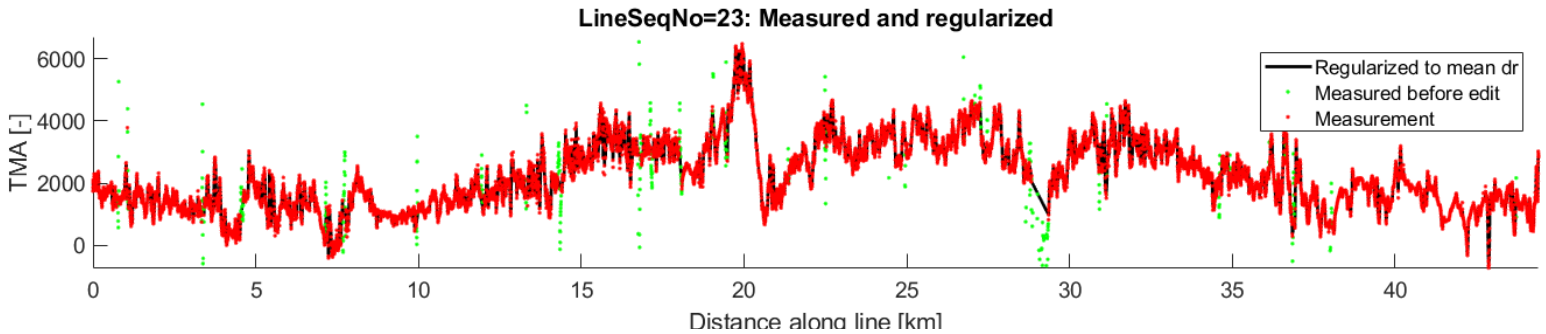
Geothermal Iceland

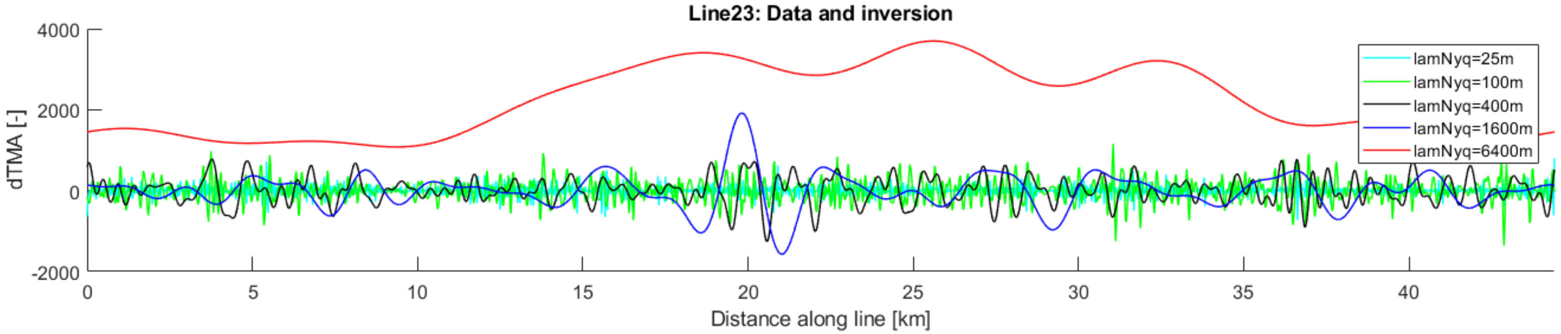
Reykjavik

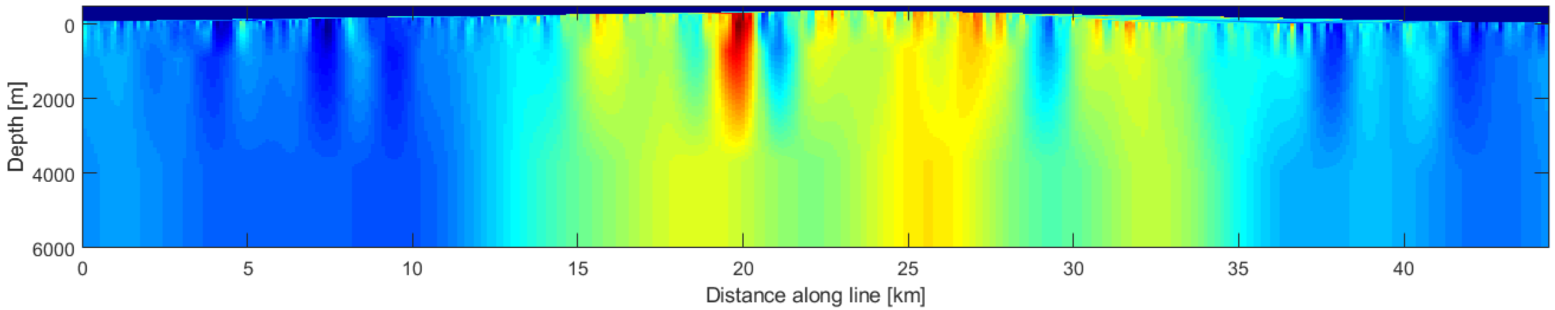
Hellisheiði

Selfoss



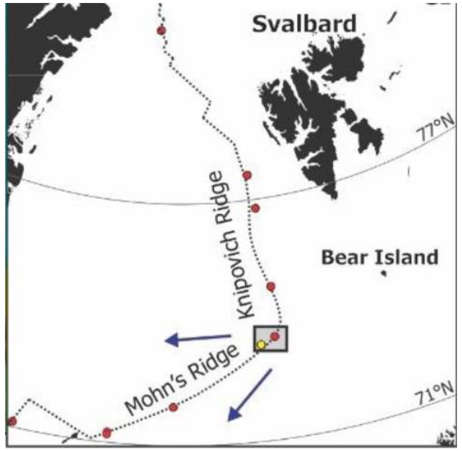


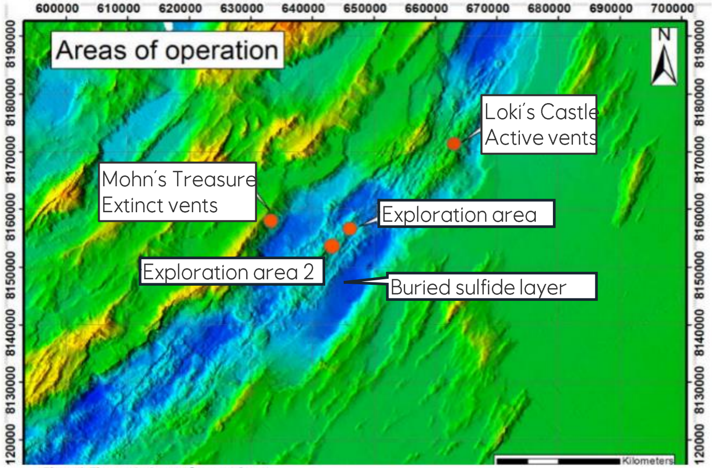




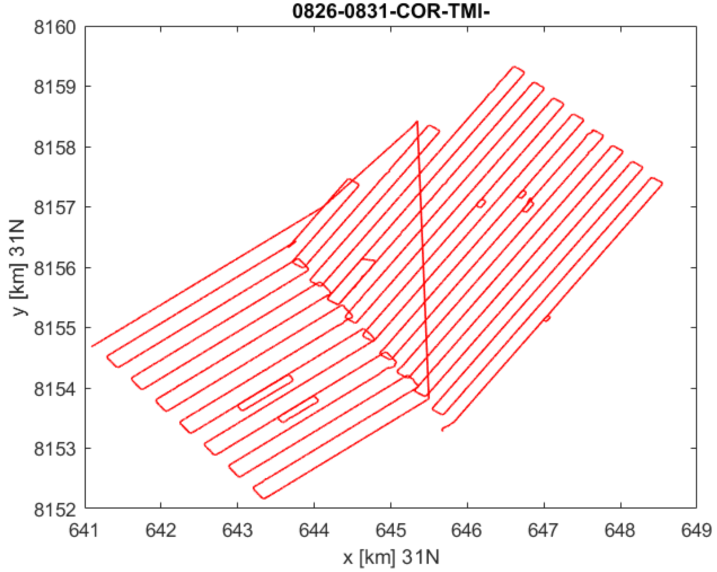
Deep-sea minerals Mohn's Ridge

Data from NTNU MarMine cruise to Mohn's Ridge 2016 (Lim et al., 2019; Lim PhD thesis, 2020)

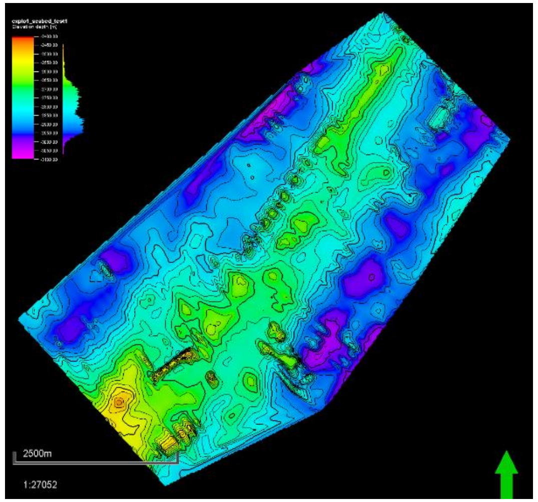




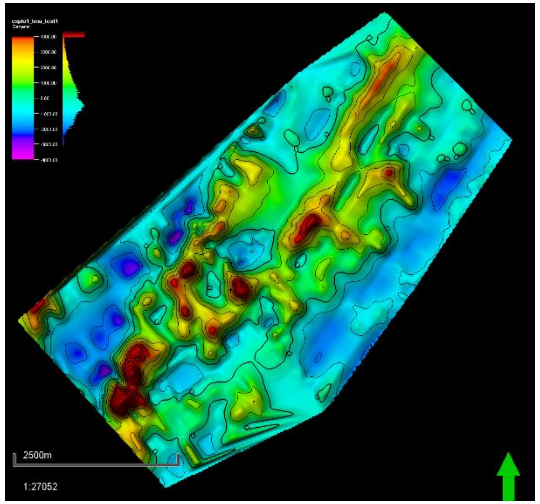
Figur 1 Figure 1. Area of operations.



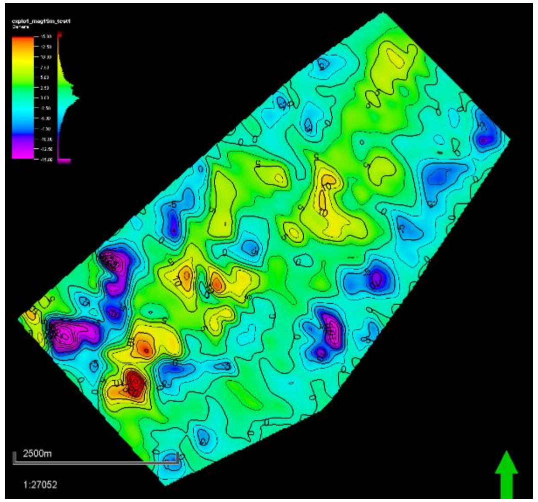
AUV track explo-area 1



Bathymetry

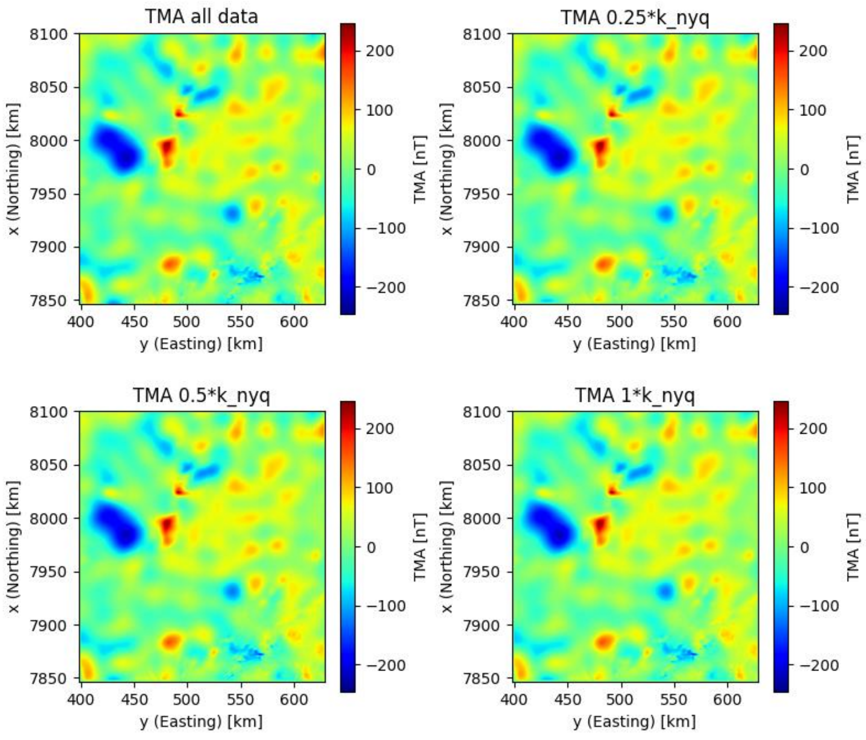


Total magnetic anomaly (TMA)



Magnetization from inversion

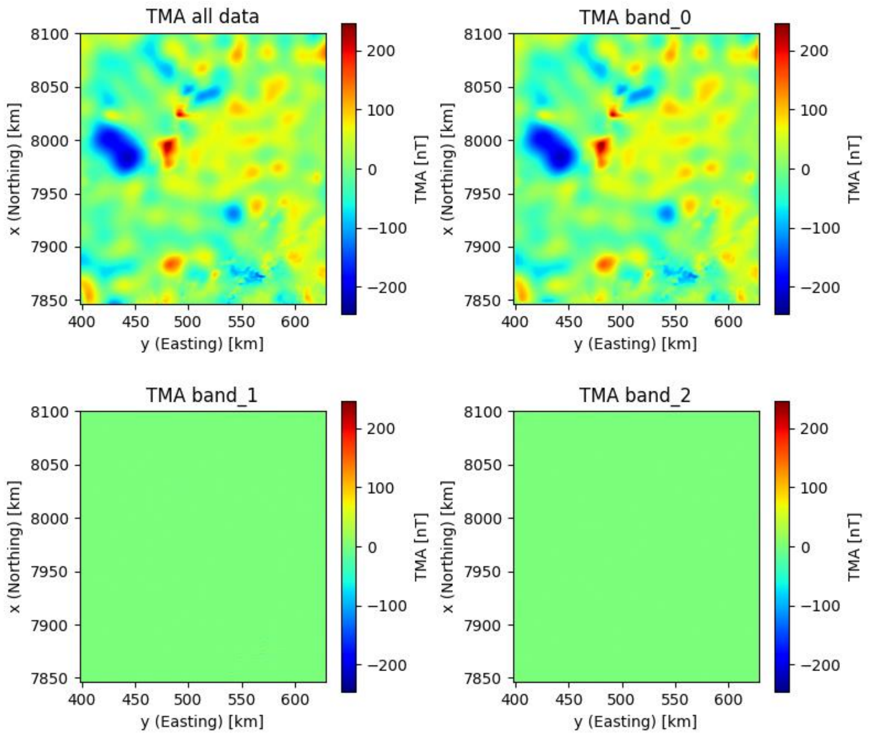
Equivalent-source test data analysis



Radial wavenumber filtering

$$k_r = \sqrt{k_x^2 + k_y^2}$$

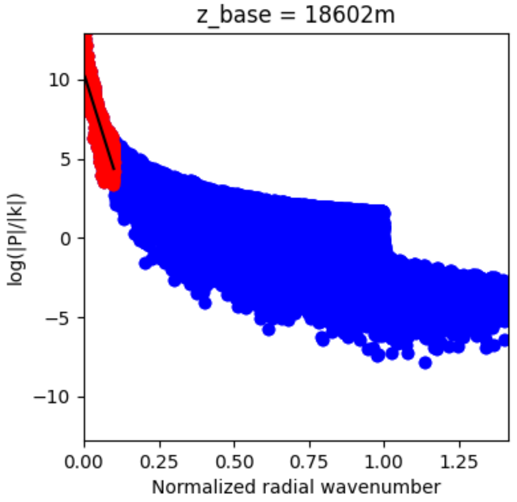
$$k_{nyq} = \frac{\pi}{\Delta x}$$



Wavenumber band decomposition in fractions of k_{nyq}

band_0 : 0 – ¼
band_1 : ¼ - ½
band_2 : ½ - 1

Data input to analysis
DX=DY=500m



Approximate base of magnetic layer from power-band analysis (Blakely, 1996)

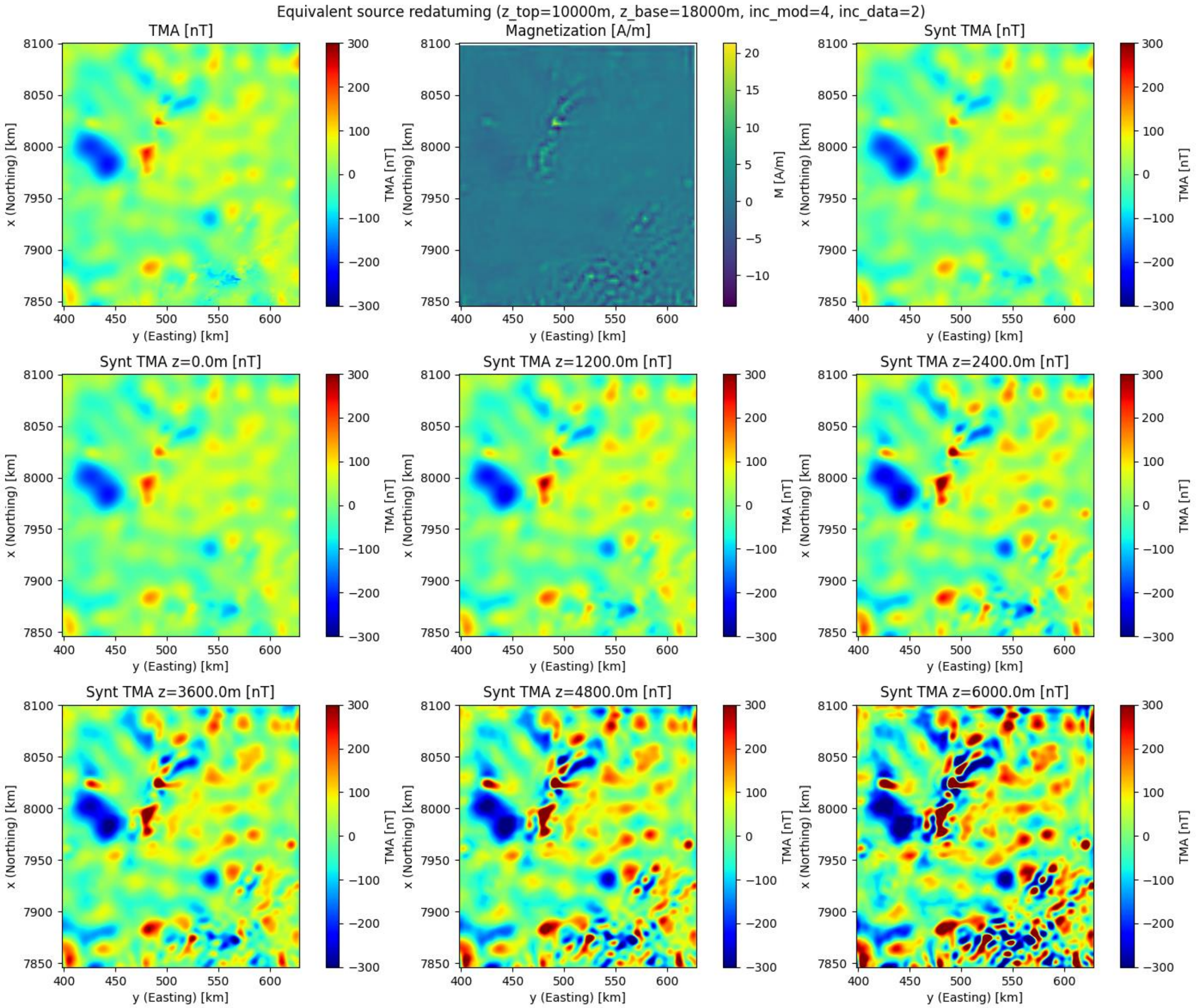
Equivalent-source extrapolation

Data
- DX=DY=1000m
- Receiver altitude: 240m

Model:
- DX=DY=4000m
- Top magnetic layer: 10km
- Base magnetic layer: 18km

Extrapolation depths:
- z0 = 0m
- z1 = 1200m
- z2 = 2400m
- z3 = 3600m
- z4 = 4800m
- z5 = 6000m

Compute time
- Inversion: 28 sec
- Modeling: 108 sec
- 1D model is 100 times faster than 3D



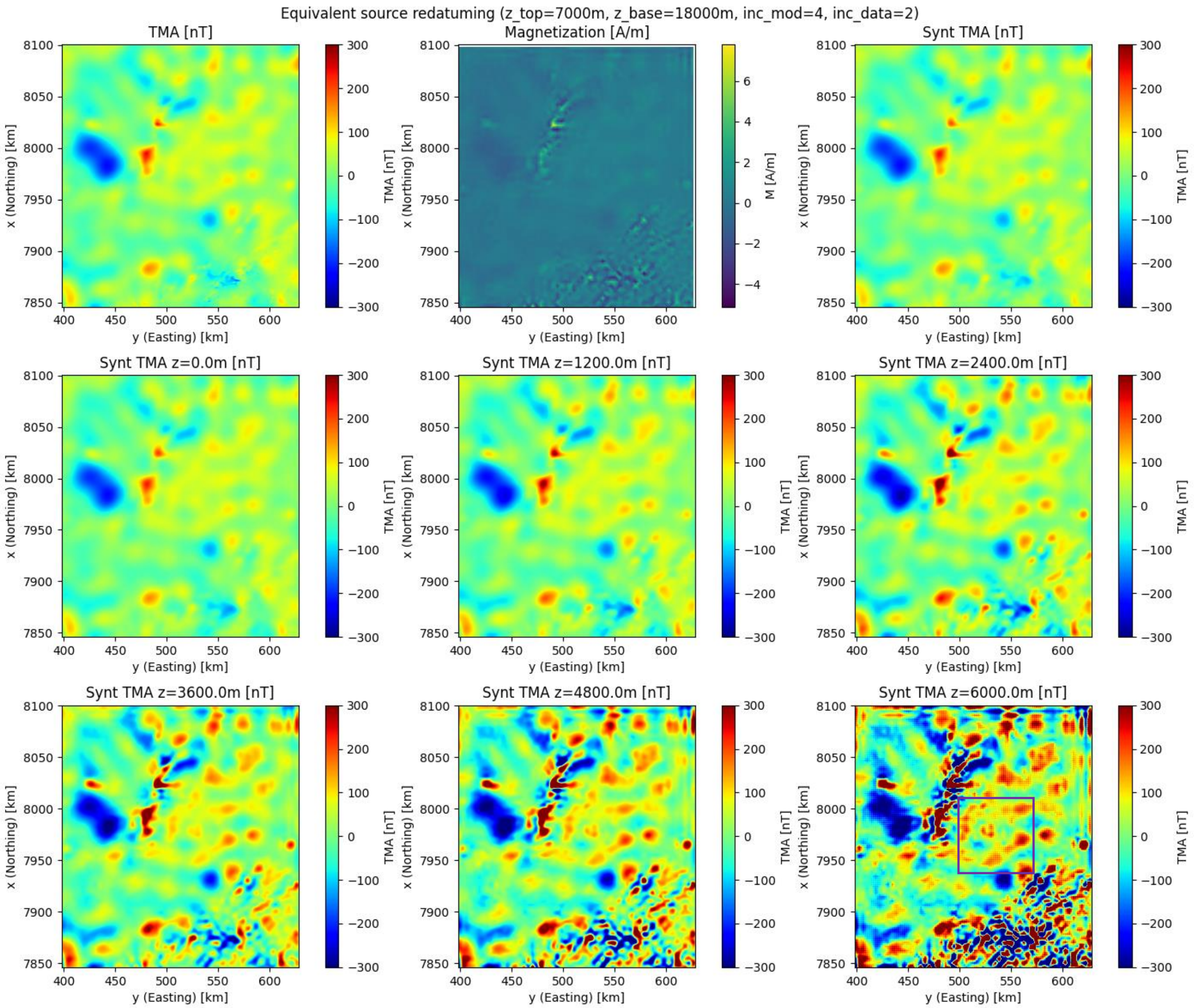
Distance to top magnetic layer

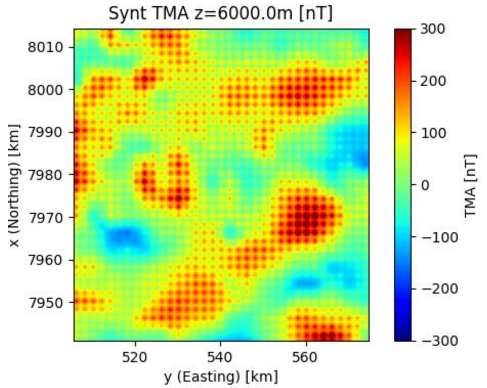
Model:

Top magnetic layer: 7km
Max extrapolation depth: 6km

Top magnetic layer too close to max extrapolation depth

- More edge effects
- Grid-effects are visible





Internal

Removing edge effects - mirroring

Mirroring data on the edges:

- Extend data
- Extend model (important)

Comes at the cost of increased compute time

No mirroring

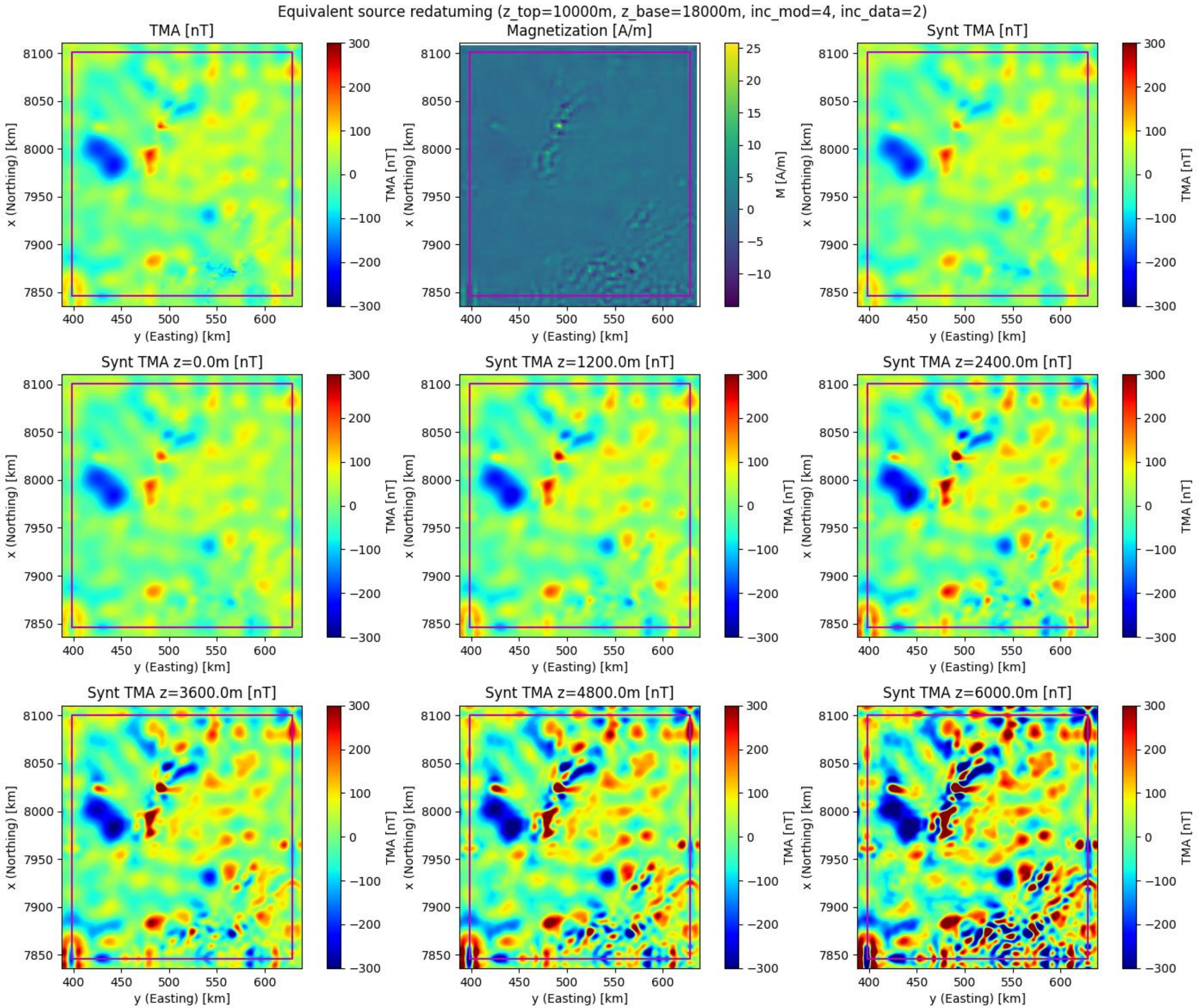
- Inversion: 28 sec
- Modeling: 108 sec

20 nodes mirroring

- Inversion: 44 sec
- Modeling: 186 sec

60 nodes mirroring

- Inversion: 103 sec
- Modeling: 1160 sec



Conclusions and recommendations

Conclusions

- Map inversion works well for equivalent source method
- Some challenges with edge effects
- Python code

- Fast option:
  - 1D model, with look up tables
  - Compute time: ~ 3 min
  - 100 times faster than 3D model

- Slow option
  - 3D model
  - Compute time: ~5 hours

Recommendations

- Select $\Delta x, \Delta y$ for data input to inversion by wavenumber spectral analysis
- Grid spacing for equivalent source model:

$$\Delta x_m = \Delta y_m = 4\Delta x = 4\Delta y$$

- Magnetic equivalent source layer:

$$z_{top} \geq z_n + dx_m$$

z_{base} from power spectral method (Blakely, 1996)

- Dealing with edge effects
  - Expand data area (if possible)
  - Data mirroring at the edges of the grid

- Monitor these numbers in the inversion:
  - Rank of the Marquardt-Levenberg matrix
  - Condition number (OK if ~10**4 or less)

Equivalent source method for magnetic extrapolation

Ketil Hokstad
kehok@equinor.com
+47 90971837