GEOPHYSICS, VOL. 57, NO. 5 (MAY, 1992); P. 680–692, 14 FIGS.

Reflection tomography in the postmigrated domain

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ABSTRACT

Reflection tomography is an inversion method that adjusts a velocity and reflector depth model to be consistent with the prestack time data. This tomography approach minimizes the misfit of the data and model in the premigrated domain. Generally, the data are represented by the traveltimes of reflection events, which has made the technique problematic and unpopular.

Techniques generally known as "migration velocity analysis" have a similar objective but use the postmigrated domain. For a variety of practical reasons, this postmigrated domain has advantages over the premigrated domain. With slight modifications, the reflection tomography approach can be implemented in the postmigrated domain. In this domain, a model is determined by optimizing the consistency of imaged reflection events on what has been called a common reflection point (CRP) gather.

Extending reflection tomography to the postmigrated domain allows much of the knowledge developed for migration velocity analysis to be coupled with that of reflection tomography. As a result, many of the practical techniques developed for migration velocity analysis can be used to improve the robustness and efficiency of reflection tomography. Similarly, much of the reflection tomography work done on analysis and solution of the linear system can be applied to postmigrated domain optimization.

INTRODUCTION

A variety of techniques has been proposed for using prestack data from a surface reflection survey for performing velocity analysis in areas of lateral velocity variations. These techniques share the approach of finding a model that is consistent with the prestack reflection data, but differ in how they measure consistency and what procedure is used to find the appropriate model. The techniques proposed in the literature can be grouped into two categories, migration velocity analysis and tomography, based on the domain in which consistency of the model with the data is measured. The tomographic techniques measure consistency in the premigrated domain using, for instance, common-midpoint (CMP) gathers, while migration velocity analysis techniques measure consistency in the postmigrated domain using what has been called common reflection point (CRP) gathers. They have also been called coherency panels, but I prefer the term CRP gathers.

The simplest implementation of tomography is a traveltime inversion that is referred to as reflection tomography (Bishop et al., 1985; Williamson, 1986, 1990; Bording et al., 1987; Scales, 1987; Kennett et al., 1988; Farra and Madariaga, 1988; Dyer and Worthington, 1988; and Stork and Clayton, 1991). Figure 1 demonstrates an implementation of this process. As developed in the literature, reflection tomography works by first taking a reference velocity and reflector depth model over a seismic line produced by conventional methods. Rays are traced through the reference model to compute the predicted traveltimes for arrivals from one or several reflectors in the model. The raypaths are also stored. The predicted time of an arrival on a trace is then compared with the actual arrival time in the prestack data. This step generally involves picking the time of the arrival in the data and subtracting it from the predicted time. By treating each trace separately, any moveout function on CMP gathers can be represented, including those that are nonhyperbolic and hyperbolic ones corresponding to nonphysical velocities. The velocity and reflector depths of the reference model are then adjusted to minimize the difference between the predicted traveltimes and the actual traveltimes of the prestack time data. The raypaths determine what part of the velocity and reflector model to adjust by how much to minimize the differences.

The process of adjustment is a large inverse problem that is solved by one of a variety of iterative techniques that can

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efficiently handle the size of the problem. Much of the reflection tomography literature is focused on analysis of this inverse problem to determine its complications and on methods for addressing them. It is probably because this literature focuses on the analysis of the inverse problem that it develops its methods using the most direct form for representing the data, the premigrated time domain, such as shot gathers or CMP gathers.

Other techniques that also determine a model based on how well it predicts the premigrated time data can also be called tomography: Tarantola (1984) and Mora (1987) propose an iterated Born inversion approach of the waveform data; Santosa and Symes (1989) investigate Born inversion using the WKBJ approximation and a quasi-Newton nonlinear iteration for the purpose of improving the inversion behavior in resolving the broad velocity components as well as the smaller-scale components. Thorson (1984) proposes a layer-stripping approach using nonhyperbolic stack optimization; Sneider et al. (1988) propose using a Monte-Carlo or a Simplex search algorithm to optimize waveform correlation between data and synthetic seismograms from a model. Toldi (1985, 1990) and Sherwood (1986) present semblance methods for using stacking velocities to determine the interval velocity field. In this paper, I will concentrate on the simplest form of tomography, reflection tomography.

The other class of prestack velocity analysis techniques, "migration velocity analysis," is based on optimizing prestack migration. (Gardner et al., 1974; Fowler, 1988; Deregowski, 1990; van Trier, 1990; Etgen, 1990; verWest, 1990; Jeannot et al., 1986; Julien et al., 1988; Yilmaz and Chambers, 1984; MacBain, 1989). Figure 2 demonstrates one implementation of migration velocity analysis. In a reflection survey, each point on a reflector is generally illuminated by the data recorded from several shots, giving redundant images of the subsurface. (With flat reflectors, fold is a measure of this redundancy.) When the prestack data is migrated to its point of reflection, there will be multiple images of that reflector, which are summed to produce the final migrated section. If the correct velocity model is used in a prestack depth migration, these multiple images are iden-

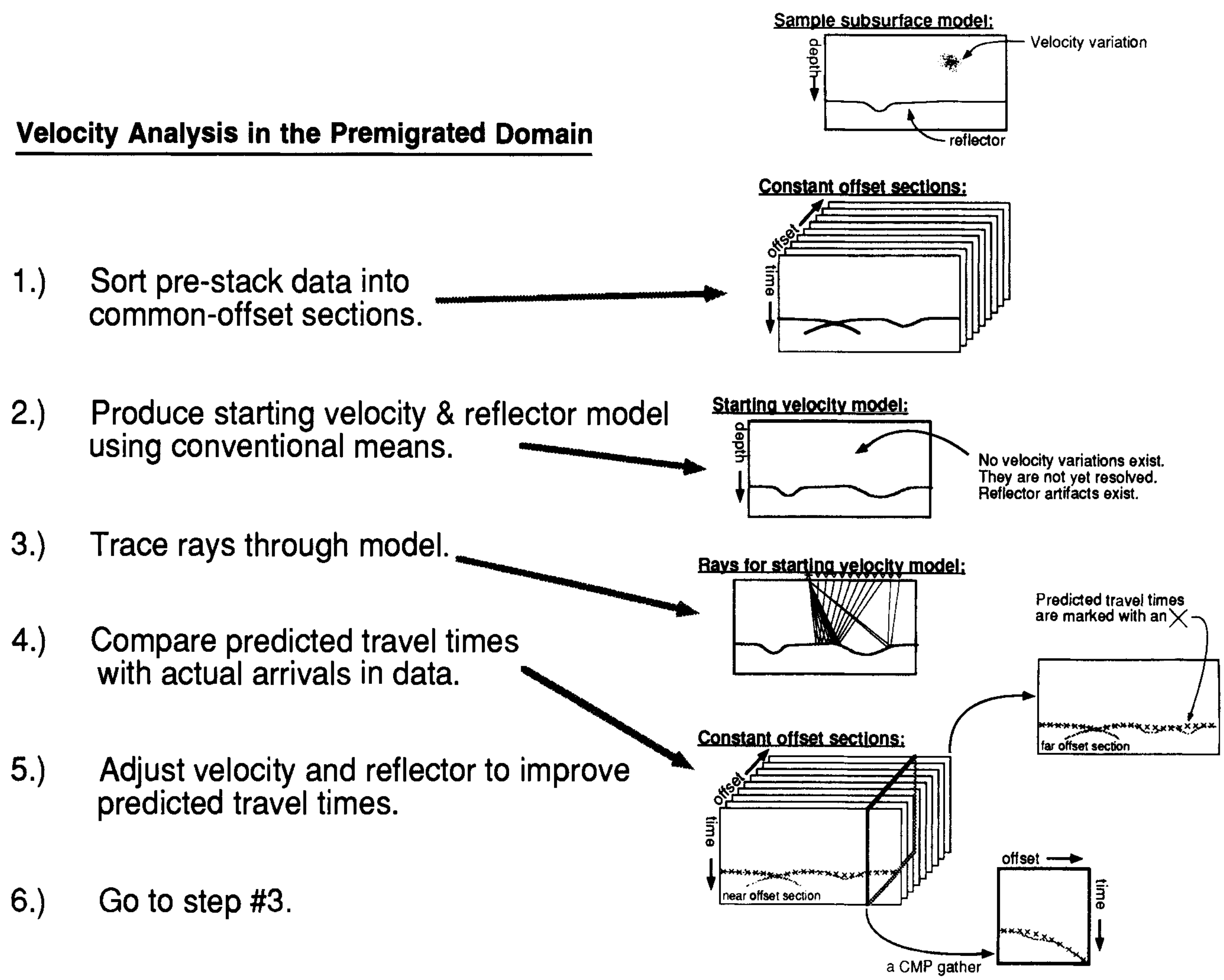

FIG. 1. A description of velocity analysis in the premigrated time domain. This is the domain used by reflection tomography. In this domain, predicted traveltimes are computed through a reference velocity and reflector model. The differences between the predicted traveltimes of the events and the actual traveltimes represent errors in the reference model. Reflection tomography adjusts the model to optimize the predicted traveltimes.

tical and improve S/N on summing (Gardner et al., 1974). However, when the velocity model has errors, these multiple images are not identical and the summation is not optimal. The process of migration velocity analysis alters the velocity field so that these multiple images are more identical and produce a better summation.

A plot of the CRP gathers is an effective method of analyzing how identical the multiple images are (van Trier, 1990; and Etgen, 1990) and hence how accurate the velocity model is. A CRP gather is the collection of traces selectively taken from the multiple images of the reflector that are to be summed into one trace of the final migrated section. These traces correspond to the same horizontal position in space, hence the name "common reflection point," or CRP gather. The CRP gather can be considered to be the postmigrated analogy of an NMO-corrected CMP gather. If an accurate velocity model is used in the migration, all the traces of a CRP gather have a reflector located at the same depth, producing a flat event that sums constructively. When the events are not flat, the multiple images are not identical and the velocity model can be improved. The differences between the traces of a CRP gather occur because the energy traveling to the common point of reflection transmits through different regions of the velocity field.

Prestack seismic data generally includes significant correlated noise that may corrupt the signal. Much of the migration velocity analysis literature addresses the stability of velocity analysis with noisy data and the ability of a person to quality control the process. For instance, Yilmaz and Chambers (1984), Jeannot et al. (1986), Deregowski (1990), Fowler (1988), and Etgen (1990) employ semblance methods to measure the degree of similarity between the redundant images of the subsurface. Gardner et al. (1974), MacBain (1989), Julien et al. (1988), verWest (1990), and van Trier (1990) show how a person can quality control the similarity between prestack migration panels. In addition, Fowler (1988),

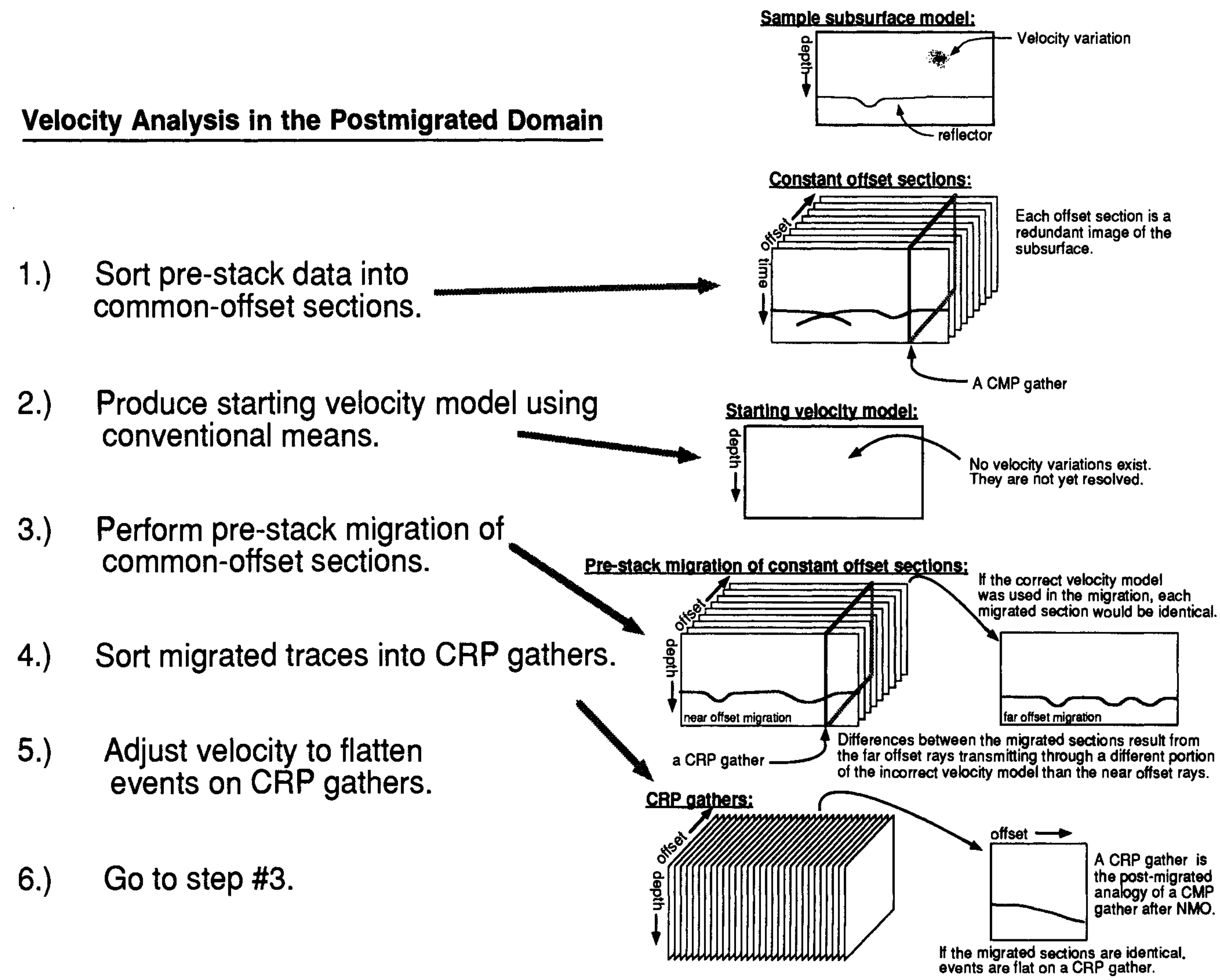

FIG. 2. A description of velocity analysis in the postmigrated depth domain. This is the domain used by migration velocity analysis. In this domain, the prestack data is migrated through the velocity field but not stacked. The multiple images of the reflector, which in this case are migrations of constant offset sections, would be identical if the correct velocity field was used in the migration. As a result, the event would be flat on a CRP gather. When an incorrect velocity field is used, events are not flat on a CRP gather. Migration velocity analysis adjusts the model to flatten the events on a CRP gather.

Etgen (1990), and van Trier (1990) employ automatic, inverse methods for migration velocity analysis. The automatic approach of Sword (1987) can be considered to be a ray-based method analogous to migration velocity analysis that uses the migrated traveltimes for the objective function. It is probably because this group of literature focuses on the practical aspects of velocity analysis, such as stability and human quality control, that it prefers the postmigrated domain for optimization.

These two domains for measuring the consistency of a model with data, premigrated time, and postmigrated depth, share significant similarities. They rely on the same information in the data: energy transmitted through the velocity variations and reflected back to the surface from underlying reflectors. In an ideal case, if a velocity field matches the data in one domain, it will also match it in the other.

The advantages of reflection tomography result from the understanding gained to date on the inverse problem. Once the data have been formulated for use by tomography, such as by the picking of traveltimes, the determination of the velocity field is fast and can easily be controlled and constrained. Much is known of the resolution of the reflection tomography inverse problem. The disadvantages of tomography result from its use of the premigrated time domain. Data in the premigrated time domain are frequently distorted by wave propagation effects, making the picking of traveltimes or other data formulation techniques problematic and tedious. These distortions are most likely to exist in areas of structural complexity, which is where we generally are most interested in applying the technique. Moreover, in the presence of correlated noise, it can be difficult to quality control the data in the premigrated domain to ensure that an aspect of a result is real and not an artifact.

The advantages of migration velocity analysis correspond to the disadvantages of reflection tomography. The postmigrated domain used by migration velocity analysis methods generally has less distorted data, making the analysis of the data more efficient and less susceptible to noise. It is also generally easier for an explorationist to interpret the data in the postmigrated domain, making quality control easier. These considerations are important practical advantages for the postmigrated domain.

With minor modification to the present formulation, reflection tomography can be applied to the postmigrated domain. As a result, the data formulation for tomography in the postmigrated domain is simpler and it is easier for a user to quality control the process. Using tomography in the postmigrated domain, however, does not alleviate the need for data formulation, of which picking is an example.

Fowler (1988), Etgen (1990), and van Trier (1990) employ automatic inverse methods for updating the velocity field in the postmigrated domain. The application of reflection tomography to the postmigrated domain shares some significant similarities with their formulation. By relating these approaches, the experience base gained with reflection tomography can be complemented with their work.

I demonstrate below the practical advantages of the postmigrated domain with a geologic model and show the modification necessary to adapt reflection tomography to this domain.

ADVANTAGES OF THE POSTMIGRATED DOMAIN

The advantages of the postmigrated domain result mainly from the continuity and uniform reflection strength of reflectors in the subsurface. When these reflectors have structure, the time data (CMP gathers or constant offset sections) are generally not continuous and do not have uniform reflection strength. Thus, the analysis and formulation of the data is simpler in the postmigrated domain.

Synthetic data produced from the model of Figure 3 demonstrate the difference between data in the premigrated domain and the postmigrated domain. The sample subsurface model of Figure 3 contains identical reflectors with a syncline and stratigraphic sequence on the left side, an anticline with a pinch out occurring near the center, and mild, small scale folding on the right. Within a background velocity consisting of a vertical gradient, velocity variations of about 15 percent exist above the reflectors. The resolution of these velocity variations might be the objective of a velocity analysis technique.

Figures 4a and b show two common-offset sections from these data. The data were produced using finite-difference modeling of a reflection survey. Both sections contain a

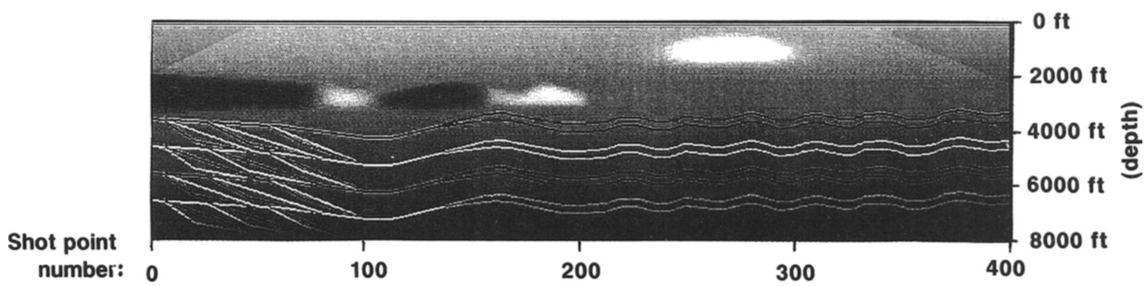

FIG. 3. Model used to demonstrate the advantages of the postmigrated domain for the analysis and formulation of data. The model contains identical reflectors with a syncline and stratigraphic sequence on the left side, an anticline with a pinch out occurring near the center, and mild, small scale folding on the right. Within a smooth background velocity, velocity variations of about 15 percent exist above the reflectors.

significant amount of crossing arrivals, diffractions, and signal strength variations. The right side of the section is especially corrupted by the small scale folding of the reflector. Figure 4c shows CMP gathers after NMO. Identifying arrivals are difficult. In some cases, it is not possible to follow an arrival across an entire CMP gather. The multiple arrivals from one reflector on a trace complicate ray tracing, and it may not be clear as to which arrival corresponds to which ray of a reference model. Figure 4d shows a series of velocity semblance plots for some of the CMP gathers. The semblance peaks are poorly defined and scattered making their use difficult.

Using the velocity field of Figure 5, which consists of the correct vertical gradient without the velocity variations, the synthetic data are prestack migrated. The result of the migration, Figure 6, shows an image washed out under the velocity variations and positioned at the wrong depth. Figures 7a and b show the near-offset and far-offset sections after migration through the velocity field. Figure 7c is the associated CRP gathers. The effect of unresolved velocity variations is readily apparent from the nonflat events on the CRP gathers.

By migrating the data, many of the problems from the premigrated time sections have been removed. Most of the crossing arrivals, diffractions, and varying signal strength in the common-offset sections have been collapsed, and the reflector has fairly uniform strength. Identifying arrivals should be easier in the postmigrated domain rather than in the premigrated domain. However, minor distortion of the postmigrated data will invariably be caused by unresolved velocity variations.

THE APPLICATION OF REFLECTION TOMOGRAPHY TO THE POSTMIGRATED DOMAIN

In the standard reflection tomography formulation, the traveltime deviation of an arrival between the data and that predicted from a best guess reference model is the linear integral

$$\Delta t = \int_{\text{raypath}} \Delta s \cdot d\ell, \qquad (1a)$$

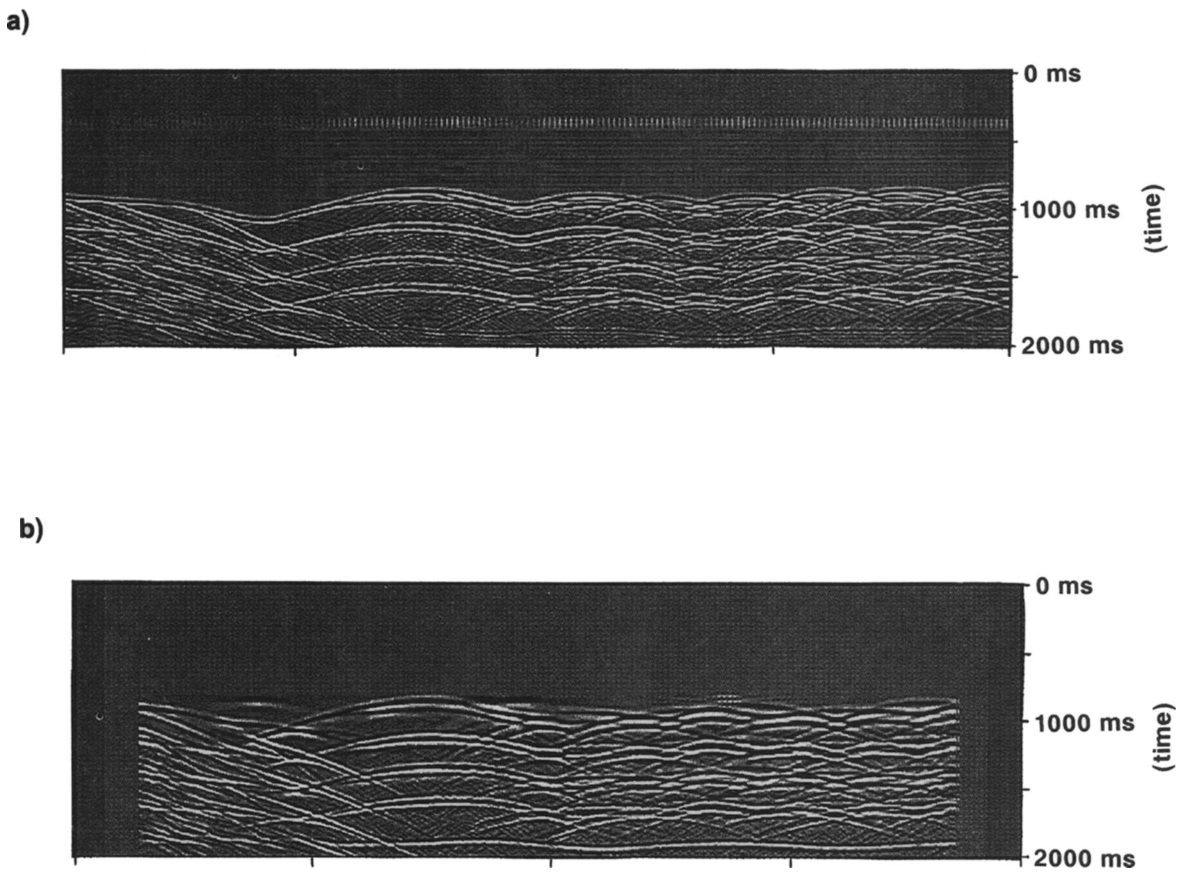

FIG. 4. Premigrated time data from the model in Figure 3. (a) Is a near-offset section. (b) is a far-offset section. The reflection events are distorted by diffractions, overlapping arrivals, and varying amplitudes. Moreover, it is difficult to interpret whether the differences between these two sections are caused by the different offsets or by an incorrect velocity field. Resorting the data into CMP gathers in (c) does not significantly improve the data. Arrivals are still difficult to identify. The velocity semblance plots in (d) show poorly defined peaks positioned at the wrong velocities.

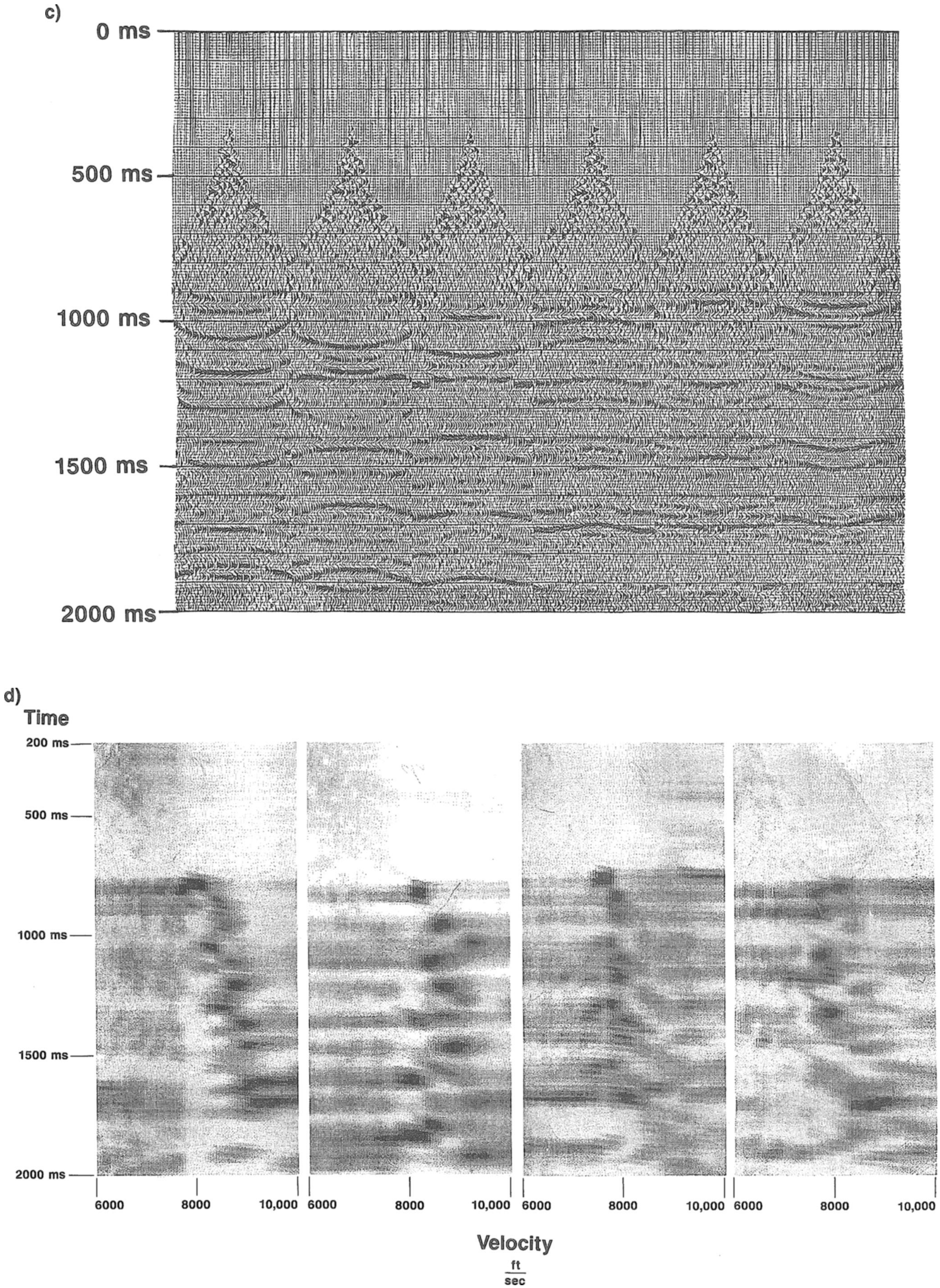

FIG. 4. continued

where

Δt is the travel deviation,
$d\ell$ is the ray segment length along the raypath, and
Δs is the slowness deviation between the reference model and true model along the raypath. (Slowness is the inverse of velocity.)

By discretizing the slowness field $\Delta \mathbf{s}$ (such as into cells), the collection of all the raypaths can be written as the matrix equation

$$\Delta \mathbf{t} = \underset{\sim}{\mathbf{L}} \Delta \mathrm{s}, \tag{1b}$$

where

$\underset{\sim}{\mathbf{L}}$ is the matrix of the segment lengths of a given ray in a given cell.

The matrix $\underset{\sim}{\mathbf{L}}$ is generally approximated by ray tracing from shot locations to receiver locations through the reference model. These are the same rays that are used to compute the predicted traveltimes for the reference model.

This equation is the standard formulation of the forward problem used in inverse theory. Much work in reflection tomography addresses how to invert this forward problem to produce the slowness field $\Delta \mathbf{s}$ from the data $\Delta \mathbf{t}$. Reflector depth can be included in the formulation by the approach in Bishop et al. (1985) or Stork and Clayton (1991).

This forward problem uses the traveltime deviations from the premigrated domain as the data. In the postmigrated domain, the data that we are trying to optimize are the depth positions of the multiple images of the reflector locations. The multiple images at one location of a given reflector will have depth deviations from a reference depth at that location. We define the depth deviations from the reference as Δz. The actual choice of the reference depth is not critical since it is a parameter that is adjusted in the inverse problem (Stork and Clayton, 1991).

To adapt these depth deviations to reflection tomography, they must be converted to traveltime deviations. For constant offset sections, the conversion is:

$$\Delta t = 2 \cdot s \cdot \Delta z \cdot \cos(\phi) \cdot \cos(\gamma) \tag{1c}$$

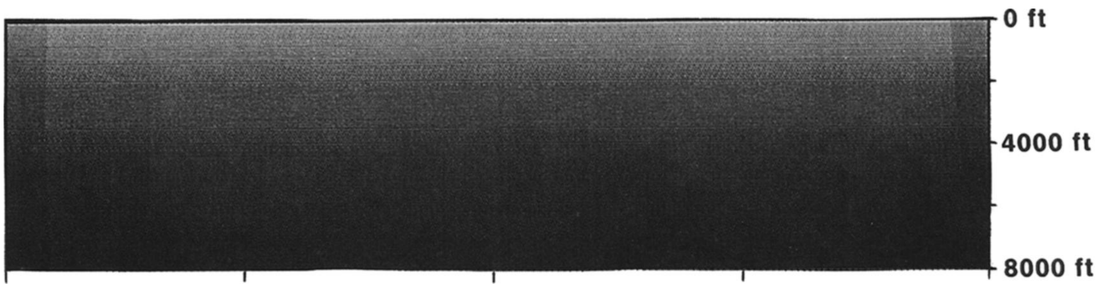

FIG. 5. Initial velocity model used for migration. It consists of the correct vertical gradient without any lateral velocity variations.

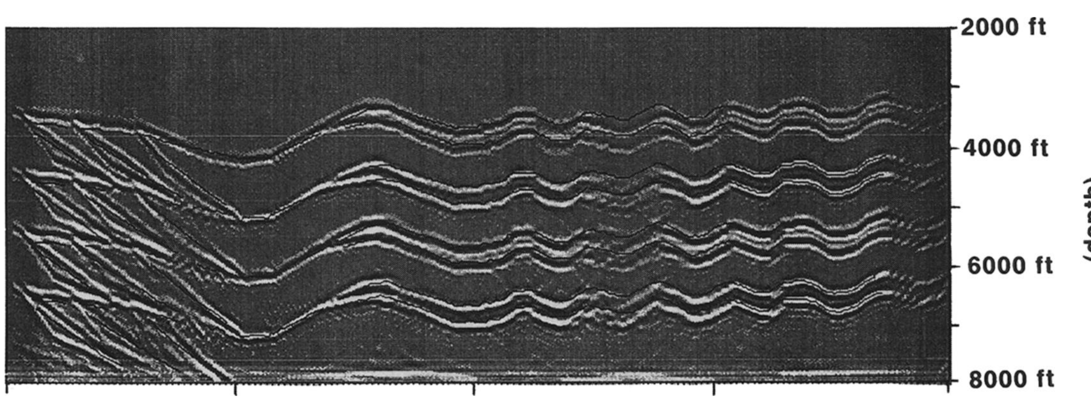

FIG. 6. Prestack depth migration through the initial velocity model of Figure 5 shows that the data is washed out below the velocity variations. The thin black lines mark the correct reflector depth. The mismatch between the lines and the migrated image indicates that the reflector is also located at the wrong depth.

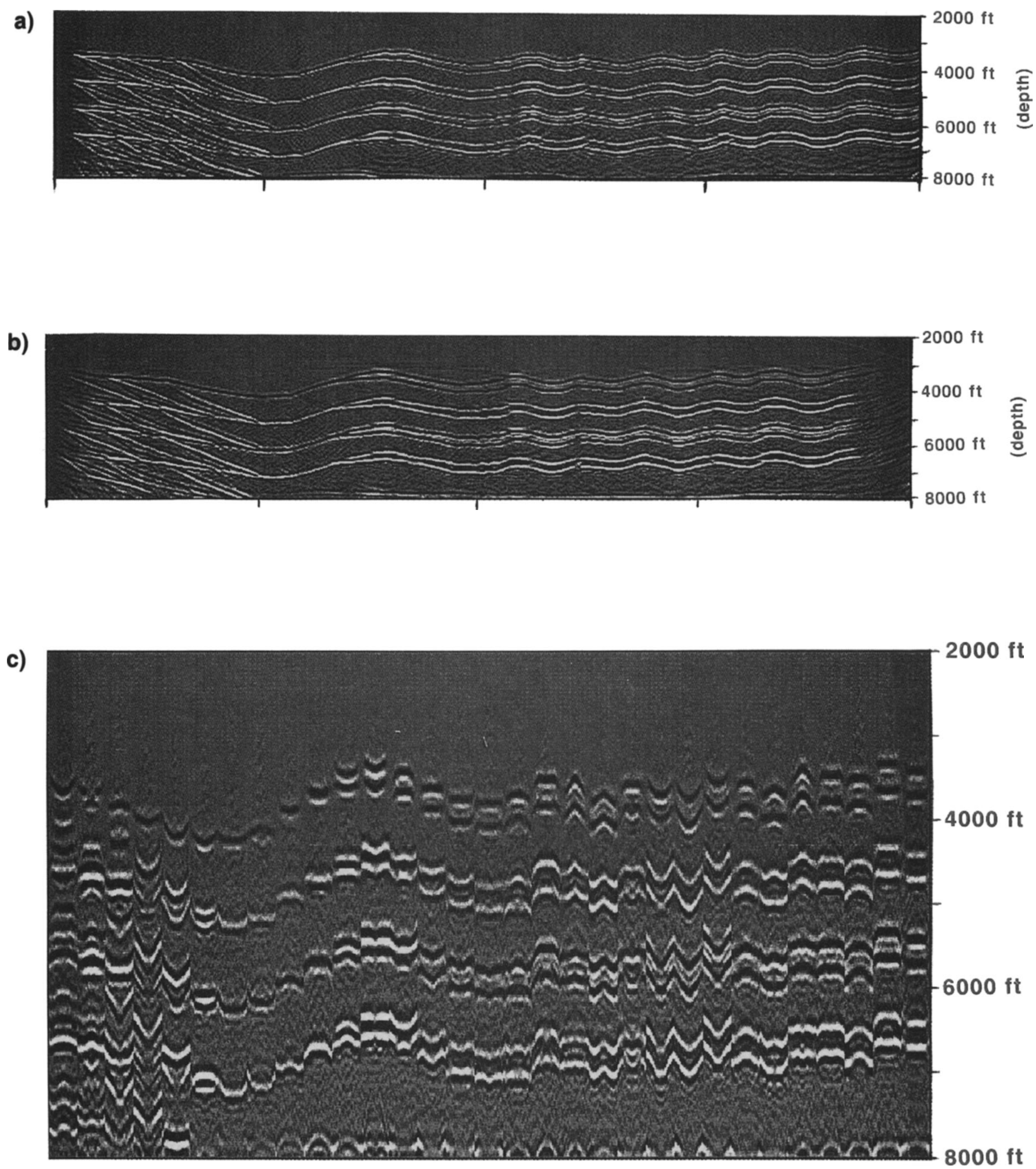

FIG. 7. Migration of the common-offset sections of Figure 4a and b using the initial velocity model of Figure 5. (a) Is the migration of the near-offset section. (b) is the migration of the far-offset section. The reflection events have little distortion compared with the common-offset time sections in Figure 4a and b. If the correct velocity model was used for migration, these two sections would be identical. However, the sections are different because an incorrect velocity model was used for migration. The differences are more apparent when the data are displayed as CRP gathers in (c). The residual moveout of the events on the CRP gathers indicate that the migrated common-offset sections are not identical. We seek a velocity model that will remove the residual moveout of the events on the CRP gathers. The events on the CRP gathers are clearer than on the CMP gathers of Figure 4c.

where

s is the local slowness above the reflector at the point of reflection; and as defined in Figure 8,

ϕ is the angle of the reflector slope from horizontal, and

γ is the angle of incidence of the ray on the reflector measured from the reflector's perpendicular.

This conversion from Δz to Δt computes the traveltime deviation caused by the extra path length that a ray must travel due to the depth deviation. The $\cos(\gamma)$ term is the dominant one that is also known as residual NMO (Etgen, 1990). The dependence of Δt on the reflector dip ϕ is mild for small dips but becomes significant at larger ones. This dependence is related to residual DMO (Etgen, 1990).

To compute the Δt values, rays need only to be traced to determine the angle γ. They are not needed to determine predicted traveltimes for a reference model. As a result, the Δt values computed from CRP gathers are insensitive to raypath errors, allowing use of a simpler and faster ray-tracing algorithm.

Rays are used mainly to compute the $\underset{\sim}{\mathbf{L}}$ matrix which determines what part of the slowness field needs to be adjusted to reduce the Δt's and hence the Δz's. These rays will correspond to the path taken by the arrivals on the traces of a CRP gather. There are several ways to trace these rays depending on how the CRP gather is produced. Approaches for how to trace the rays corresponding to the traces of a CRP gather are presented in van Trier (1990) and Etgen (1990).

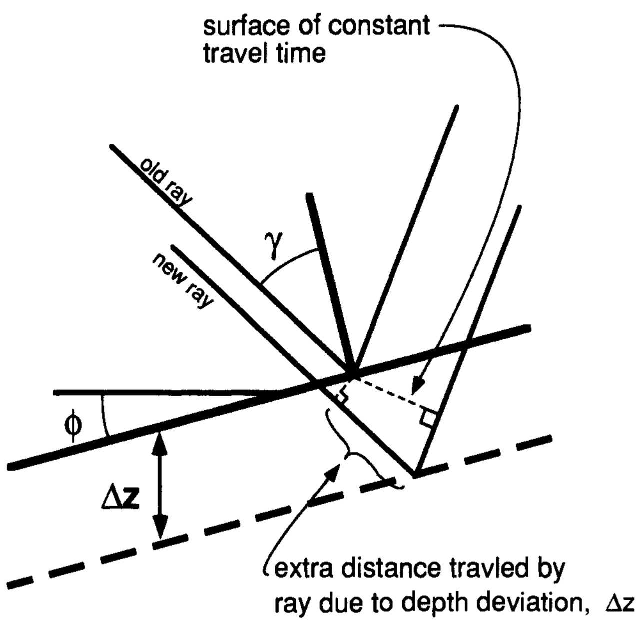

FIG. 8. The Δz perturbation values from CRP gathers are converted to Δt perturbation values used by reflection tomography using Fermat's principle, which states that surfaces of constant traveltime are perpendicular to the raypath in isotropic media. Comparing with Stork and Clayton (1991), it is evident that $\Delta t = (2/\text{velocity}) \cdot \Delta z \cdot \cos(\phi) \cdot \cos(\gamma)$.

DISCUSSION

Implementing reflection tomography in the postmigrated domain on the synthetic data produces the resulting velocity inversion in Figure 9. The prestack migration Figure 10 using this velocity field positions the reflector at the proper depth and images clear, continuous reflectors. The CRP gathers of the migration, Figure 11, show mostly flat events, indicating that the tomographic inversion was successful.

Much of the published work on reflection tomography is focused on determining the complications of the inverse problem and on methods for addressing them (Bishop et al., 1985; Williamson, 1986, 1990; Ivansson, 1986; Bording et al., 1987; Scales, 1987; Kennett et al., 1988; Farra and Madariaga, 1988; and Stork and Clayton, 1991). These complications include: (1) the efficient inversion of a very large but sparse matrix, (2) inversion to small eigenvalue, (3) the use of weights to address heterogeneous ray coverage, (4) the relative weighting of velocity and reflector depth in the inverse problem, (5) the resolution between reflector depth and certain velocity variations, (6) methods for handling nonlinear aspects, (7) methods for automatic picking, (8) procedures for giving the user control of the inversion, and (9) constraints that help adapt the inversion to the unique characteristics of a data set. Many of these complications of reflection tomography may effect velocity analysis in the postmigrated domain.

For example, there are indications that the tomographic inverse problem for velocity analysis, whether applied in the premigrated or postmigrated domain, is ill-conditioned and poorly behaved (Stork, 1988). It is ill-conditioned because

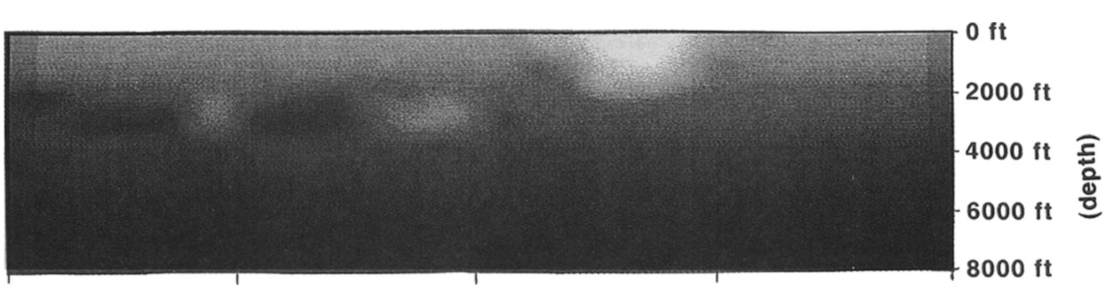

FIG. 9. Velocity inversion 1 produced using reflection tomography applied in the postmigrated domain to alter the initial velocity model of Figure 5. The velocity variations of the correct model (Figure 3) are identifiable in the inversion, but are smeared. This inversion was produced using mild constraints to position the velocity variations in their approximate locations.

several aspects of the model cannot be resolved with the given data. It is poorly behaved because the inversion may not even be able to find the solution for aspects of the model that can be resolved with the data. It can be said that there are an infinite number of solutions to the inverse problem, all equally bad.

For example, an alternate velocity inversion of Figure 12 is produced using reflection tomography in the postmigrated domain. The prestack migration using this velocity field, shown in Figure 13, positions the reflector at significantly wrong depths. However, the CRP gathers of this migration, shown in Figure 14, are mostly flat. As a result, two different velocity fields, that of Figures 14 and 11, are able to flatten events on the CRP gathers and produce significantly different reflector depths. The tomographic inverse problem for velocity analysis is nonunique.

As is generally the case with seismic data, inversion will not replace the need for the user to be involved with the data and make subjective decisions on the result. Methods that provide the user with an appreciation of and control over the process are desirable. Kennett et al. (1988), Farra and Madariaga (1988), and Stork and Clayton (1991) address this problem from a reflection tomography perspective, and van Trier (1990) addresses it from a migration velocity analysis

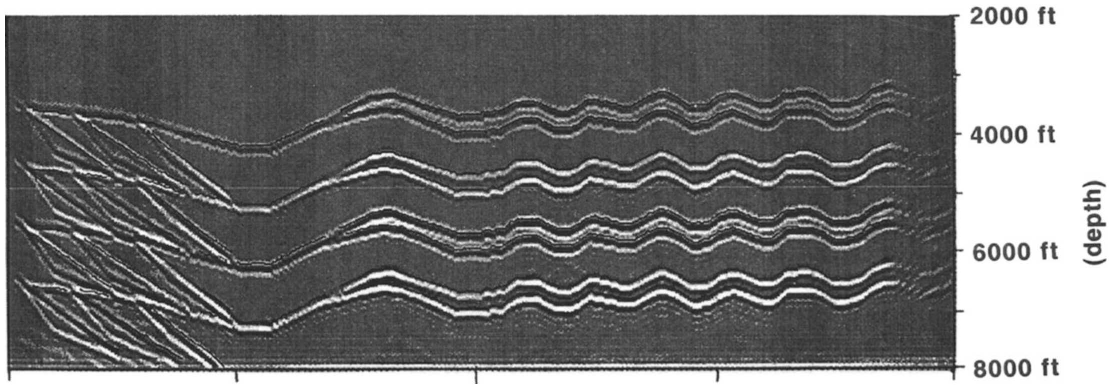

FIG. 10. The prestack depth migration using velocity inversion 1 produces continuous and well imaged reflectors positioned at the correct depth (as marked by the thin black line.) This migration should be compared to that of Figure 6.

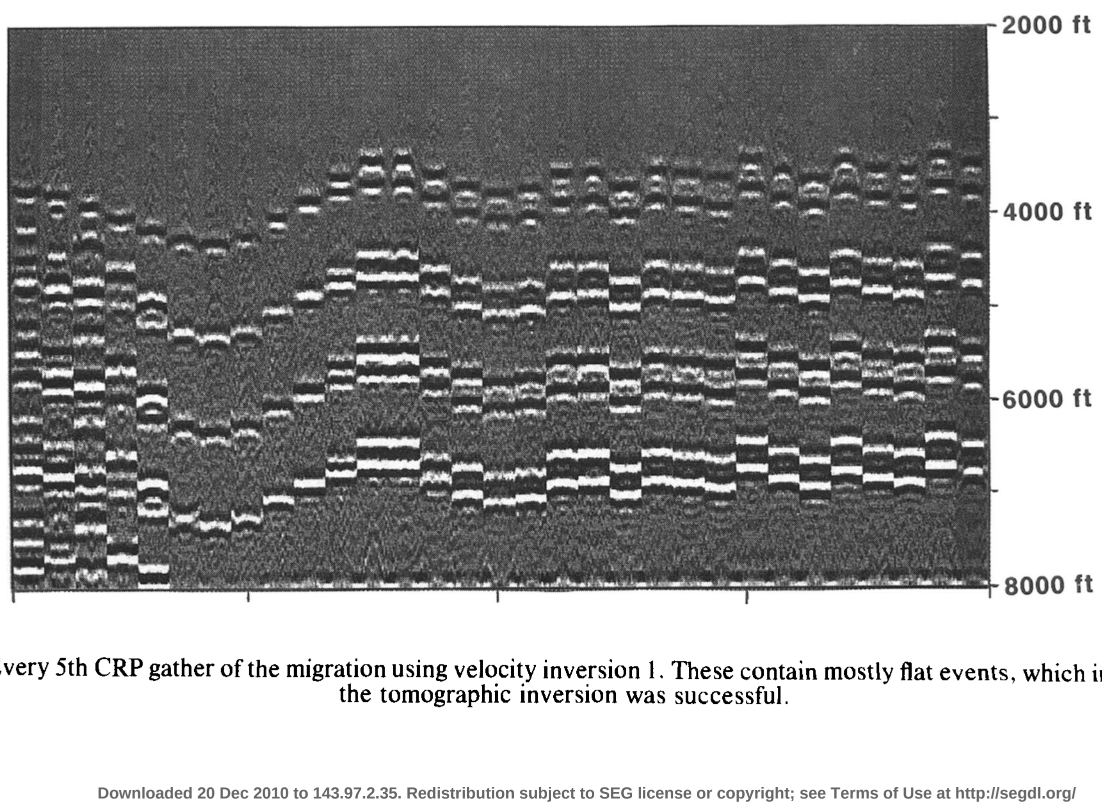

FIG. 11. Every 5th CRP gather of the migration using velocity inversion 1. These contain mostly flat events, which indicates that the tomographic inversion was successful.

perspective. Methods that allow the user control over the process let him explore the range of possible solutions, produce a result according to his geologic knowledge, and understand the reliability of the result.

The formulation presented here for reflection tomography in the postmigrated domain requires that values of Δz be determined from the migrated traces. To determine these values, the location of an event on a trace must be identified. A straightforward method for locating events is by picking, which is expected to be considerably simpler in the postmigrated domain than in the premigrated domain. Another method for locating events is a semblance approach (Toldi, 1989; and Etgen, 1990). Since human interpretation is crucial for separating signal from artifact in this critical step, it is unlikely that the process of identifying events can ever be fully automated for many data sets. Nevertheless, developing an efficient and robust procedure for locating events and hence determining the Δz values is undoubtedly an important step for making the application of velocity analysis on data from complex areas a routine procedure.

This conversion of the reflection tomography inverse problem to the postmigrated domain is accomplished by developing a back-projection operator for the reflector perturbations Δz We wonder whether the several waveform back-projection operators that have been developed for the premigrated time domain, such as those in Tarantola (1984), Mora (1987), Woodward (1988), and Luo and Schuster (1991), can be converted to the postmigrated domain.

CONCLUSION

The linear inverse problem used in reflection tomography can be easily adapted to the postmigrated domain. There are significant advantages for adapting it to the postmigrated domain as used in approaches called migration velocity analysis.

By relating the fields of reflection tomography and migration velocity analysis, the experience and methods learned

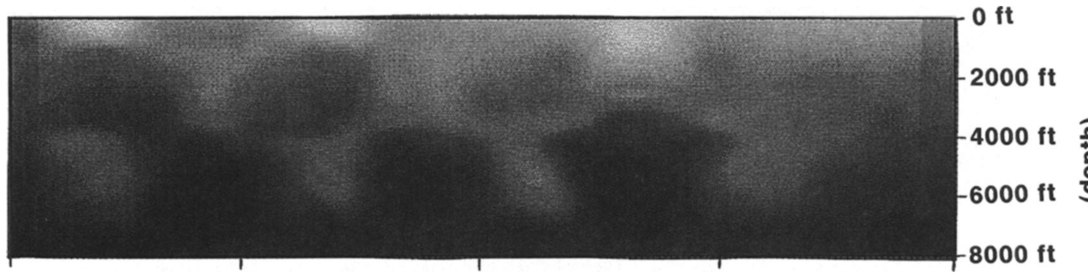

FIG. 12. Velocity inversion 2 produced using reflection tomography applied in the postmigrated domain. This inversion was produced using constraints to push the result away from the correct solution and cause spurious velocity variations. This velocity field differs significantly from the correct velocity field of Figure 3.

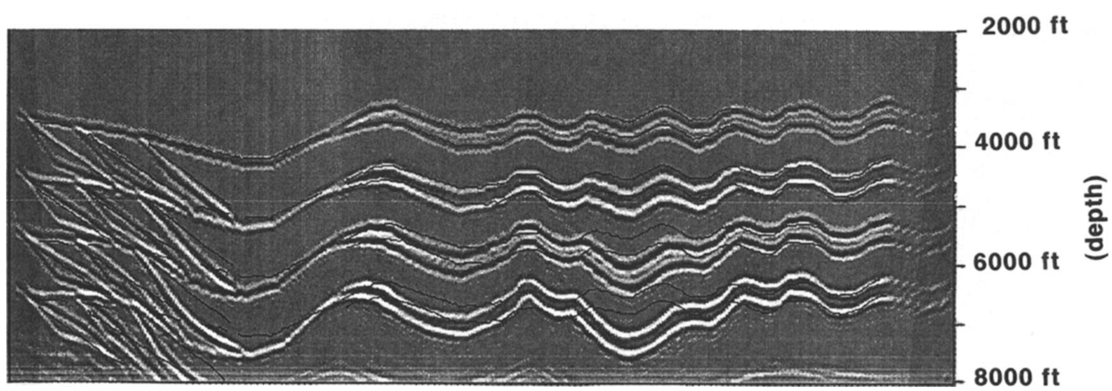

FIG. 13. The prestack depth migration using velocity inversion 2 produces continuous and well imaged reflectors with significant depth errors.

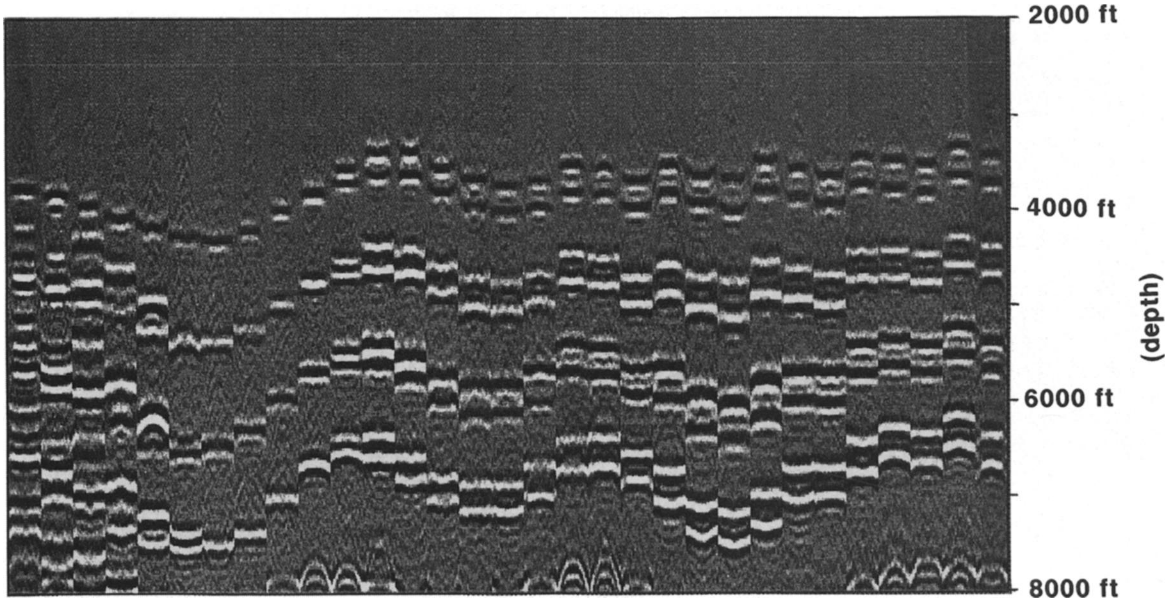

FIG. 14. Every 5th CRP gather of the migration using velocity inversion 2. These contain mostly flat events even though the inverted velocity model contains significant errors.

from each can be combined to help solve the problems of velocity analysis in laterally varying media.

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