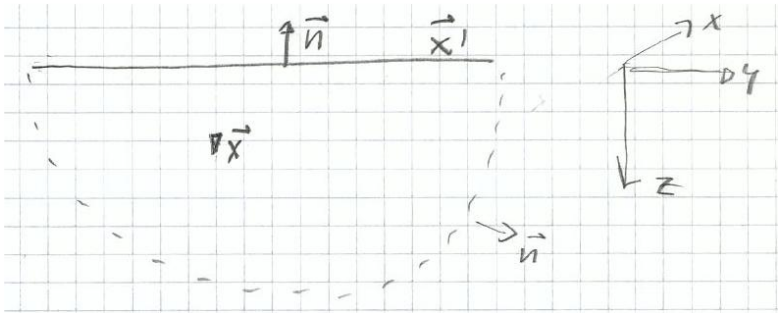
FK migration
FD migration

KetilHokstad, January 2016

Classification: Open

Kirchhoff integral recap



The Kirchhoff integral:

$$P(\mathbf{x},\omega) = \int\int\int d^3x' G(\mathbf{x},\mathbf{x}')S(\mathbf{x}',\omega)$$

Body forces

$$+ \int\int dx'dy'[\partial'_z G(\mathbf{x},\mathbf{x}')P(\mathbf{x}') - G(\mathbf{x},\mathbf{x}')\partial'_z P(\mathbf{x}')]$$

boundary conditions

Broadband seismic: P and v_z recorded

$$i\omega\rho v_i(\mathbf{x}) = -\partial_i P(\mathbf{x}) \text{ (Newton)}$$

$$P(\mathbf{x},\omega) = \int\int\int d^3x' G(\mathbf{x},\mathbf{x}')S(\mathbf{x}',\omega)$$

$$- \int\int dx'dy'[\partial'_z G(\mathbf{x},\mathbf{x}')P(\mathbf{x}') + i\omega\rho(\mathbf{x}')G(\mathbf{x},\mathbf{x}')v_z(\mathbf{x}')]$$

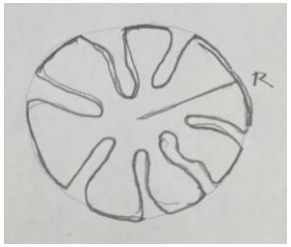
We can derive Kirchhoff integrals for acoustic and elastic wave equations, electric and magnetic fields (Maxwell), and diffusion equation

Diffusive intermezzo

Diffusion equation for a spherical particle

$$\frac{\partial C}{\partial t} - D\nabla^2 C = S$$

$$\nabla^2 C = \frac{1}{r}\frac{\partial^2}{\partial r^2}(rC)$$



$$\frac{\partial f(r,t)}{\partial t} - D\frac{\partial^2 f(r,t)}{\partial r^2} = J(r,t)$$

$$\begin{aligned} f &= rC \\ J &= rS \end{aligned}$$

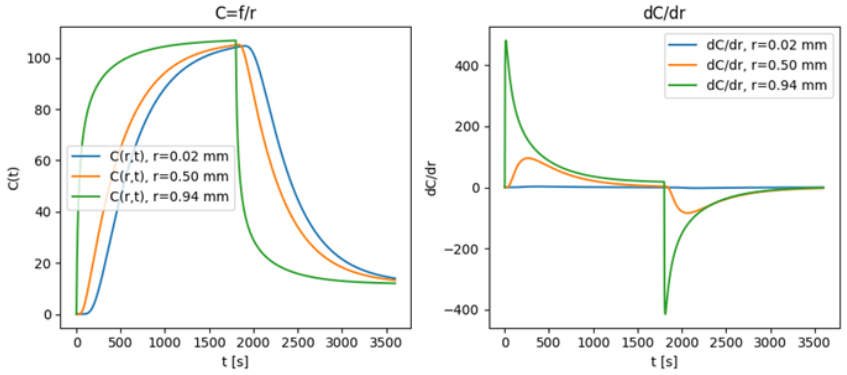
Green's function for diffusion

$$\frac{\partial G(r,t;r',t')}{\partial t} - D\frac{\partial^2 G(r,t;r',t')}{\partial r^2} = \delta(r-r')\delta(t-t')$$

$$G(r,t;r',t') = \sqrt{\frac{4\pi}{D(t-t')}}\left[e^{-\frac{(r-r')^2}{4D(t-t')}} - e^{-\frac{(r+r')^2}{4D(t-t')}}\right]$$

Kirchhoff integral (representation theorem) for diffusion

$$\begin{aligned} f(r,t) &= \int_0^R G(r,r',t) * J(r',t)\, dr' \\ &+ D\left[G(r,R,t) * \frac{\partial f(R,t)}{\partial R} - \frac{\partial G(r,R,t)}{\partial R} * f(R,t)\right] \\ &+ \int_0^R f(r',0)G(r,r',t)dr' \end{aligned}$$



Reference: Morse and Feshbach, 1953, Methods of Theoretical Physics, Ch. 7.

Helmholtz equation

$$\partial_i \partial_i F + \kappa_0^2 F = 0$$

Seismic:

$$\begin{aligned} F &= P \\ \kappa_0 &= \frac{\omega}{v} \end{aligned}$$

EM:

$$\begin{aligned} F &= E_i \\ \kappa_0^2 &= i\omega\mu_0\sigma \\ \kappa_0 &= \left(\frac{1+i}{\sqrt{2}}\right)\sqrt{\omega\mu_0\sigma} \end{aligned}$$

Fourier transform

FK-domain: $F(z, k_x, k_y, \omega)$

Forward transform:

$$F(\mathbf{k}, \omega) = \frac{1}{(\sqrt{2\pi})^4} \int\int\int \left[\int F(\mathbf{x}, t) e^{-i(\mathbf{k}\cdot\mathbf{x} - \omega t)} dt \right] dx\, dy\, dz$$

Inverse transform:

$$F(\mathbf{x}, t) = \frac{1}{(\sqrt{2\pi})^4} \int \left[\int\int\int F(\mathbf{k}, \omega) e^{i(\mathbf{k}\cdot\mathbf{x} - \omega t)} dk_x\, dk_y\, dk_z \right] d\omega$$

Be aware of sign conventions

FK-domain

Derivatives:

$$\partial_t \rightarrow -i\omega \qquad \partial_j \rightarrow ik_j = i\omega p_j$$

Helmholtz in the FK domain:

$$\partial_z^2 + k_z^2 F = 0$$

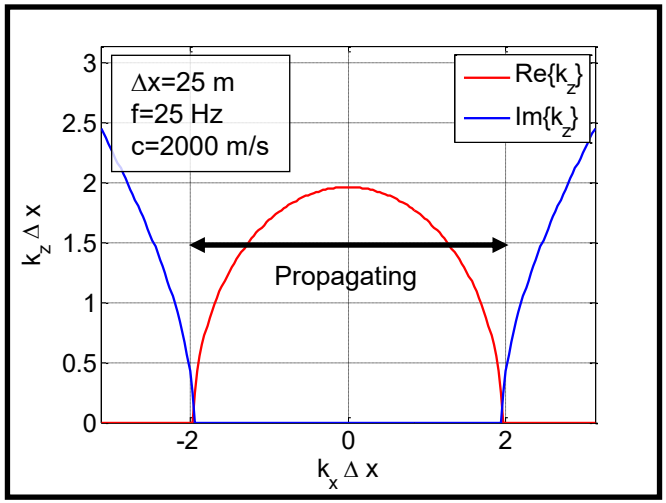
$$k_z = \sqrt{\kappa_0^2 - k_x^2 - k_y^2}$$

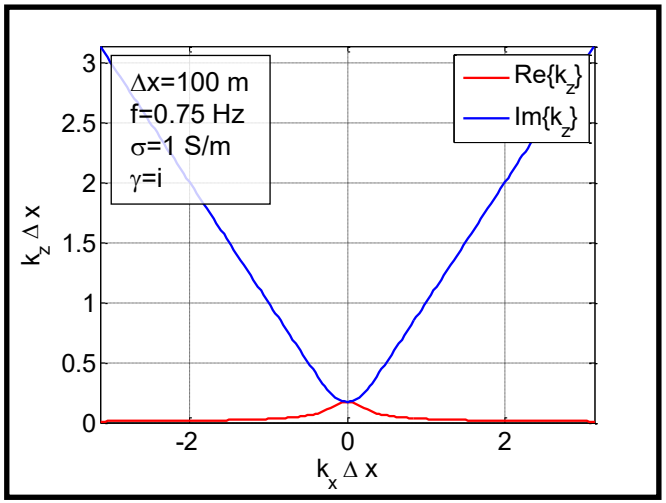
Two solutions:

$$F(z) = e^{\pm ik_z z} F(0)$$

One-way equations satisfied by F:

$$\partial_z D = ik_z D \text{ (downgoing waves)}$$
$$\partial_z U = -ik_z U \text{ (upgoing waves)}$$





Dispersion relations:
Seismic (top) and EM (bottom)

Gazdag migration

Depth stepping of U and D:

$$
\begin{aligned}
D(z+\Delta z, k_x, k_y) &= e^{ik_z(z)\Delta z} D(z, k_x, k_y) \\
U(z+\Delta z, k_x, k_y) &= e^{-ik_z(z)\Delta z} U(z, k_x, k_y)
\end{aligned}
$$

Vertical wavenumber (positiv z downwards)

$$
k_z(z) = \sqrt{\kappa_0^2(z) - k_x^2 - k_y^2}
$$

Imaging condition:

$$
\begin{aligned}
I(\mathbf{x}) &= \int d\omega \frac{U(\mathbf{x},\omega)}{D(\mathbf{x},\omega)} \\
I(\mathbf{x}) &= \int d\omega \frac{U(\mathbf{x},\omega) D^*(\mathbf{x},\omega)}{D(\mathbf{x},\omega) D^*(\mathbf{x},\omega) + \epsilon}
\end{aligned}
$$

Inverse FFT from (k_x, k_y) to (x, y):

$$
\begin{aligned}
D(\mathbf{x},\omega) &= FFT^{-1}[D(z, k_x, k_y, \omega)] \\
U(\mathbf{x},\omega) &= FFT^{-1}[U(z, k_x, k_y, \omega)]
\end{aligned}
$$

$I(\mathbf{x})$ is related to gradient and Hessian, as we will see later

Claerbout's 15-degree equation

Gazdag migration:

$$\partial_z P = \pm i \sqrt{\kappa_0^2 - k_x^2 - k_y^2} F$$

Inverse Fourier transform:

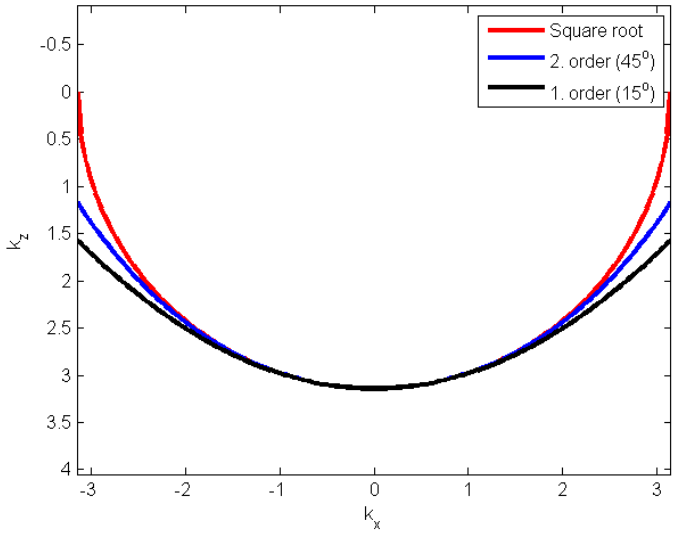
$$\partial_z P = \pm i \sqrt{\kappa_0^2 + \partial_x^2 + \partial_y^2} F$$

Series expansion: 15-degree equation

$$\partial_z P \simeq \pm i \kappa_0 \left(1 + \frac{\partial_x^2 + \partial_y^2}{2\kappa_0^2} \right) F$$

«Schrödinger equation»:

$$-i\partial_z P \simeq \pm \left(\kappa_0 F + \frac{\partial_x^2 F + \partial_y^2 F}{2\kappa_0} \right)$$



45° and 60° equations can be developed along the the same lines

Explicit one-way FD migration «Wave-equation migration»

Gazdag:

$$F(z+\Delta z, k_x, k_y) \quad = \quad e^{\gamma \Delta z \sqrt{\kappa_0^2 - k_x^2 - k_y^2}} F(z, k_x, k_y)$$

Gazdag FK migration works for (approximately) 1D media
We want to get around this limitation

Rewrite in terms of normalized wave numbers:

$$F(z+\Delta z, k_x, k_y) \quad = \quad e^{\gamma \frac{\Delta z}{\Delta x} \sqrt{\hat{\kappa_0}^2 - \hat{k}_x^2 - \hat{k}_y^2}} F(z, k_x, k_y)$$

Normalized wavenumbers:

$$\begin{aligned} \hat{\kappa}_0 &= \Delta x \kappa_0 \\ \hat{k}_x &= \Delta x k_x \\ \hat{k}_y &= \Delta y k_y = \Delta x k_y \end{aligned}$$

$$\begin{aligned} k_x^{Nyq} &= \frac{\pi}{\Delta x} \\ \hat{k}_x^{Nyq} &= \pi \end{aligned}$$

und zu weiter

Lateral velocity variations

Assume a 3D medium: $\kappa_0 = \kappa_0(x, y, z)$.
Multiplication in FK fomain $\Leftrightarrow$ convolution in FX domain
Replace the phase shift operator by a discrete convolutional filter:

$$F(x, y, z + \Delta z) = \sum_{m,n=-L}^{L} W(m\Delta x, n\Delta y, \hat{\kappa}_0, g) F(x - m\Delta x, y - n\Delta y, z)$$

The complex filter W depends on

$$\begin{aligned} \hat{\kappa}_0 &= \kappa_0 \Delta x \\ g &= \frac{\Delta z}{\Delta x} \end{aligned}$$

Wavefield extrapolation filter

Compute $W(m\Delta x, n\Delta y, \hat{\kappa}_0, g)$ by minimization of

$$
\begin{aligned}
J &= ||W(i\Delta\hat{k}_x, j\Delta\hat{k}_y, \hat{\kappa}_0, g) - e^{\gamma \frac{\Delta z}{\Delta x}\hat{k}_z}||^4 \\
\hat{k}_z &= \sqrt{\hat{\kappa}_0^2 - (i\Delta\hat{k}_x)^2 - (j\Delta\hat{k}_y)^2}
\end{aligned}
$$

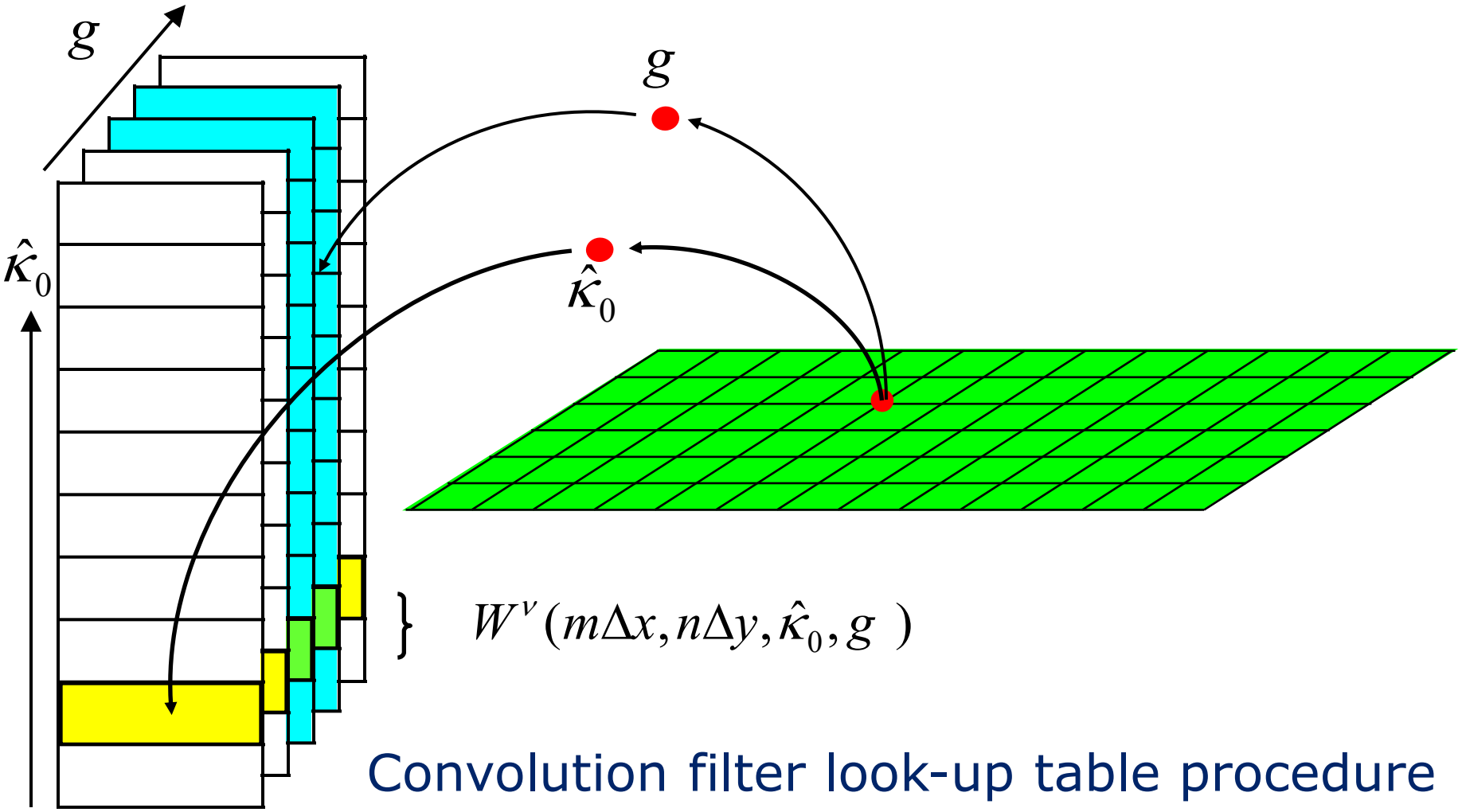
where

$$
W(i\Delta\hat{k}_x, j\Delta\hat{k}_y, \hat{\kappa}_0, g) = DFT[W(m\Delta x, n\Delta y, \hat{\kappa}_0, g]
$$

- Create look-up tables for *W*
- Can be computed once and for all

- Max propagation angle depends on operator length *L*

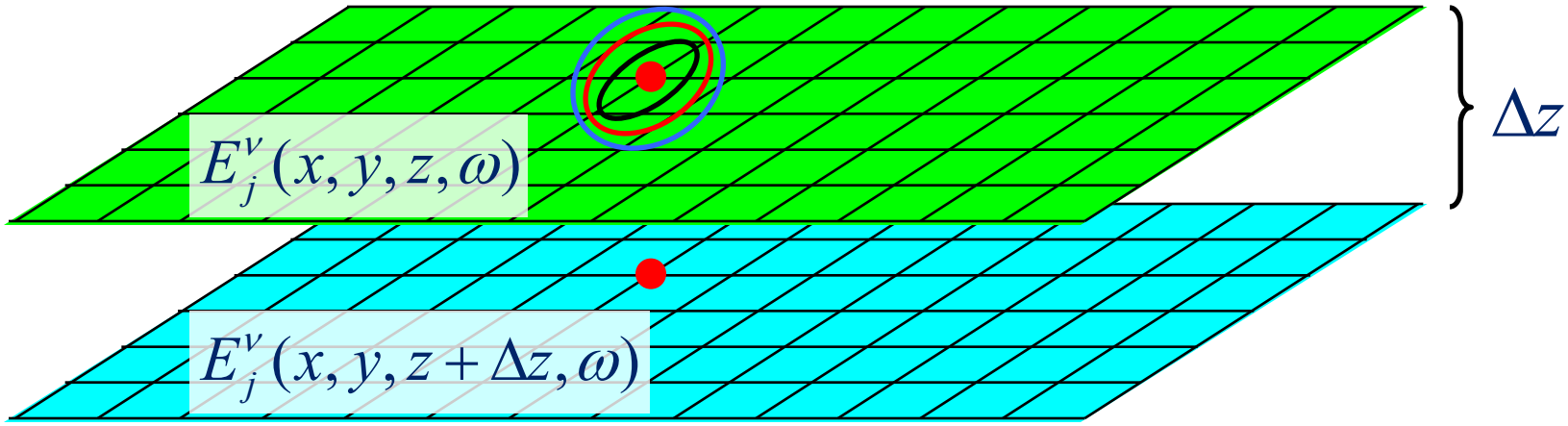
Illustration of 3D FD Depth Migration Scheme



Convolution filter look-up table procedure

Illustration of 3D FD Depth Migration Scheme

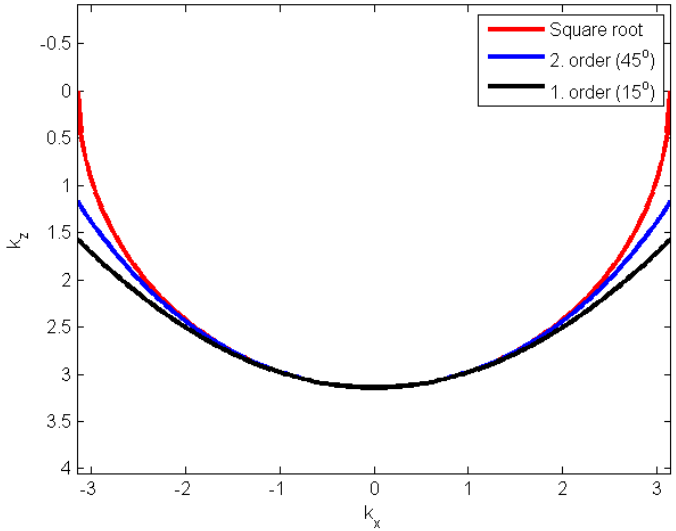
$$E^{\nu}_{(m)}(x,y,z+\Delta z,\omega) = \sum_{m,n=-L}^{L}\left[W^{\nu}(m\Delta x, n\Delta y, \hat{\kappa}_0, g)\right] \times E^{\nu}_{(m)}(x-m\Delta x, y-n\Delta y, z, \omega)$$



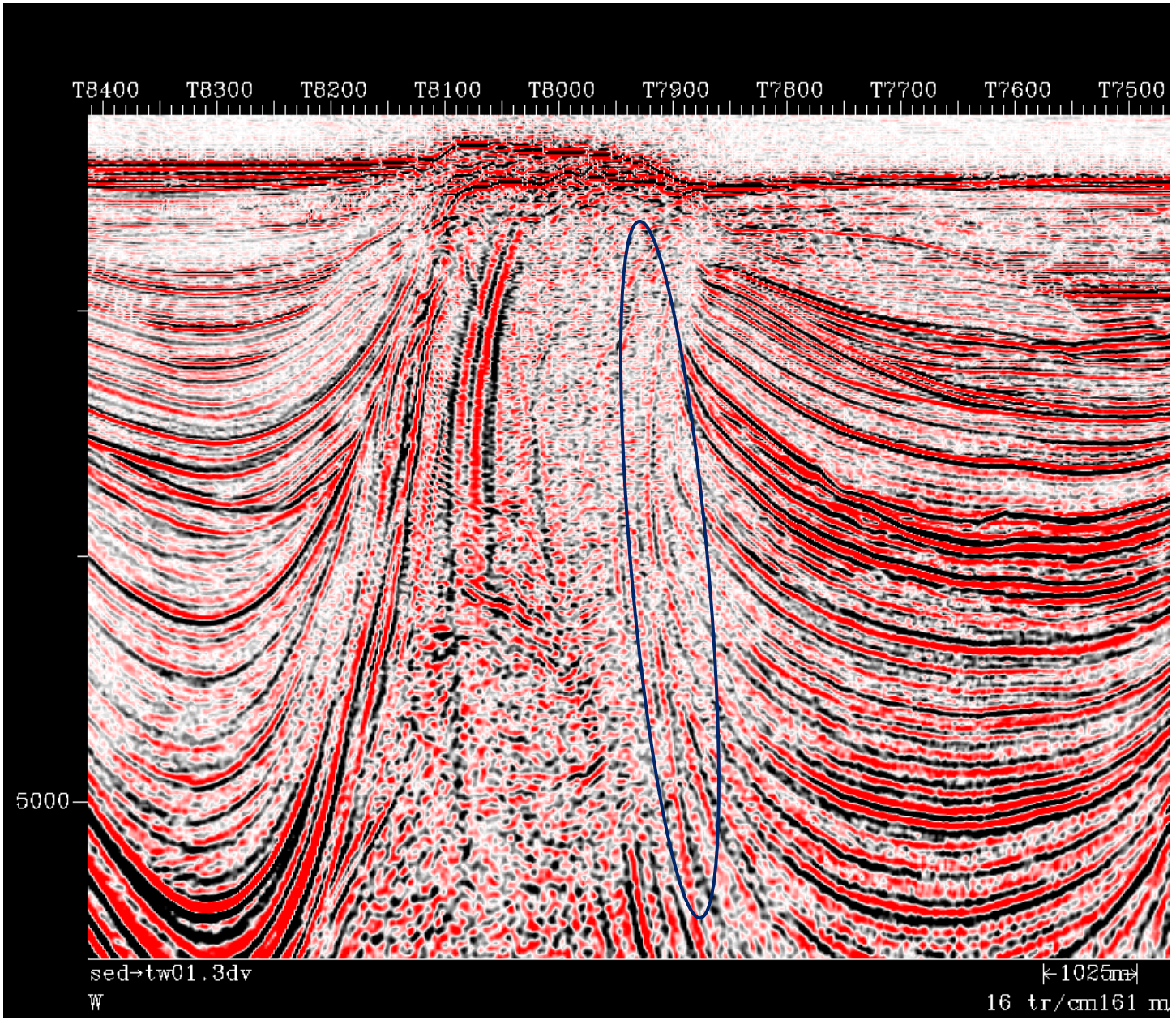
Wave field extrapolation by filter convolution

FK and FD migration summary

- FK migration:
  - Accurate up to 90 degrees dip
  - 1D medium: v(z)

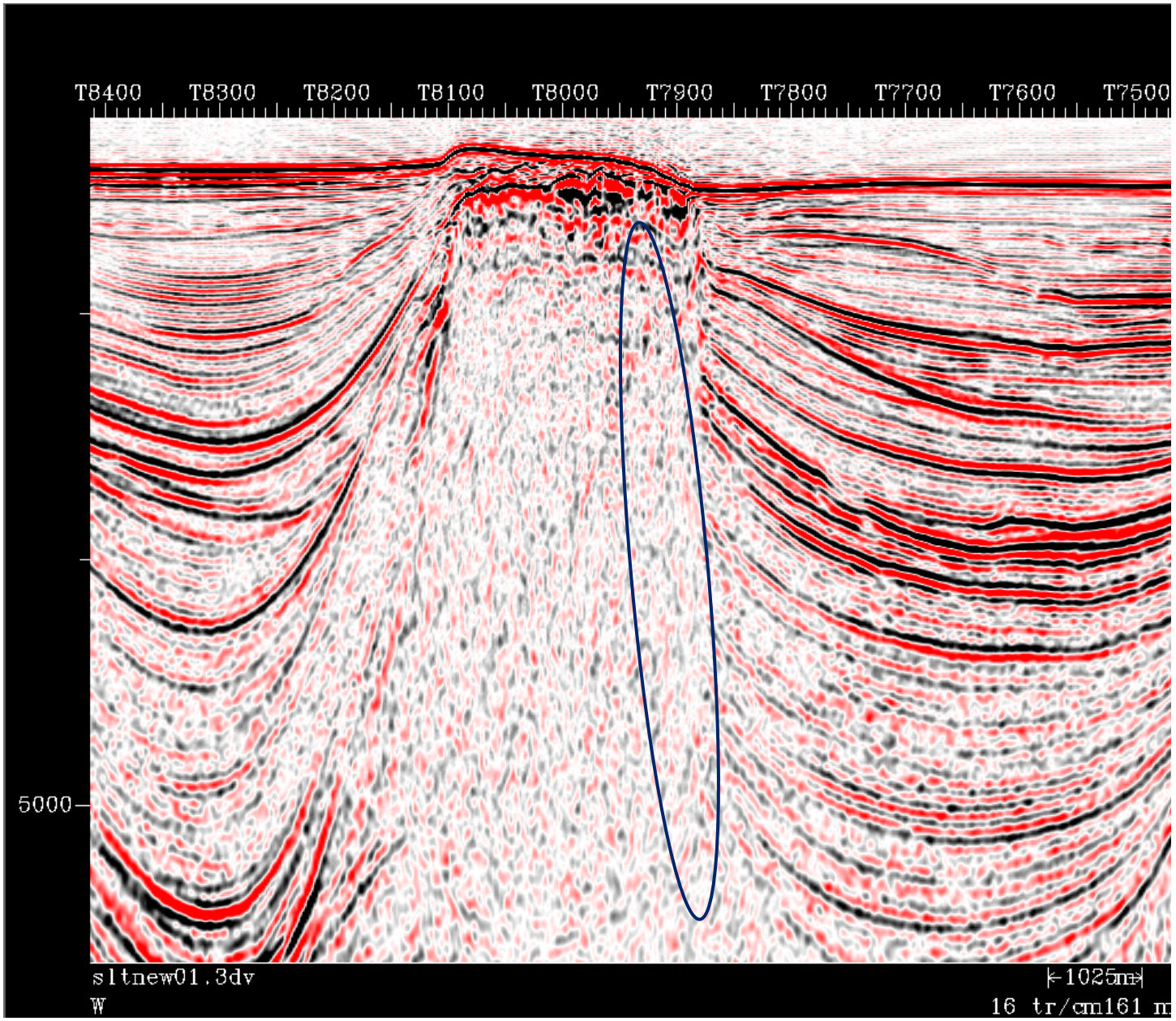


- Explicit FD migration:
  - Dip accuracy depends on operator length (60 degree is common)
  - 3D medium: v(x,y,z)



TW Kirchhoff – sediment only velocity model – Line 10825

STATOIL



Wave Equation – salt velocity model – Line 10825

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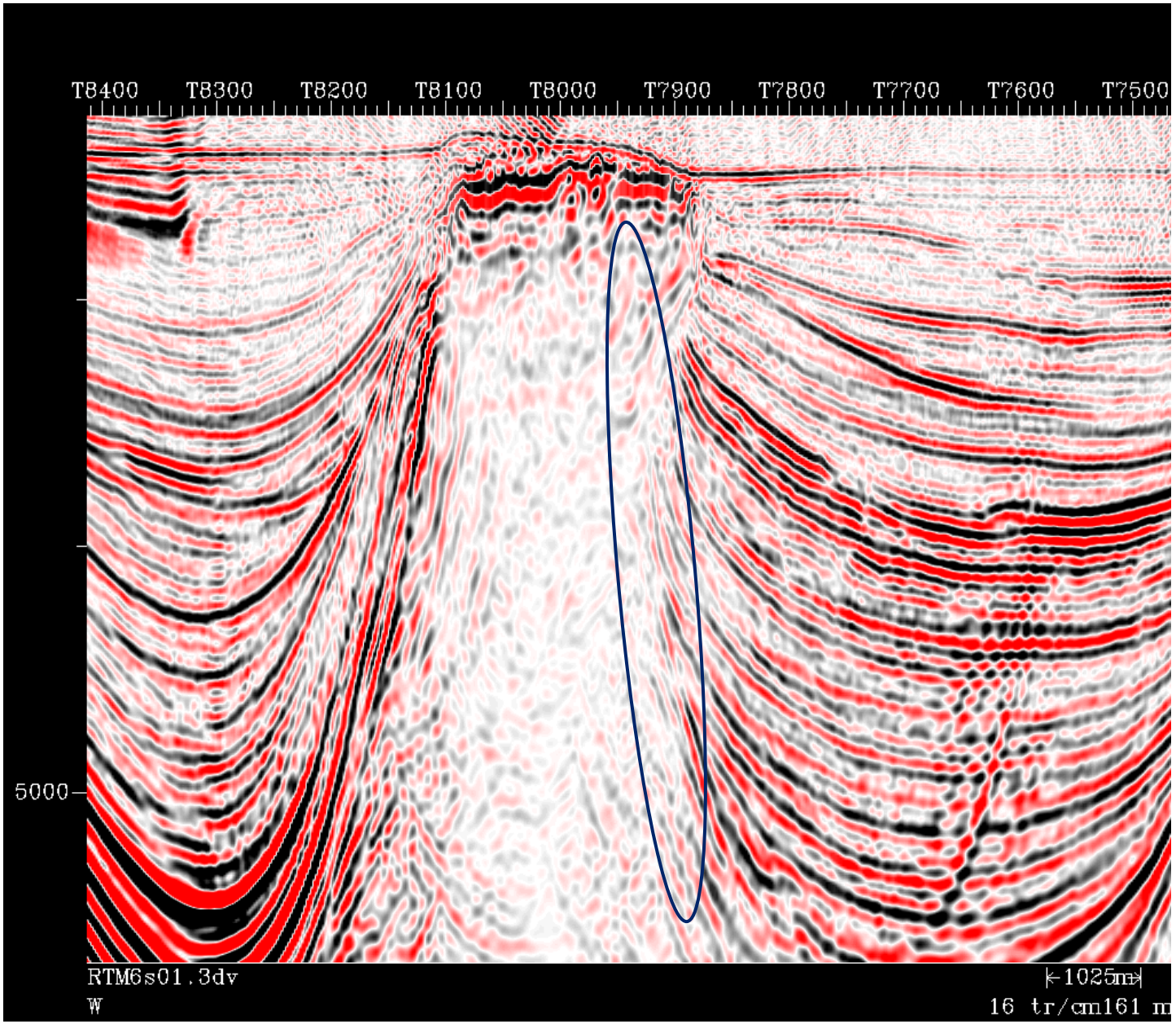

RTM – salt velocity model – Line 10825

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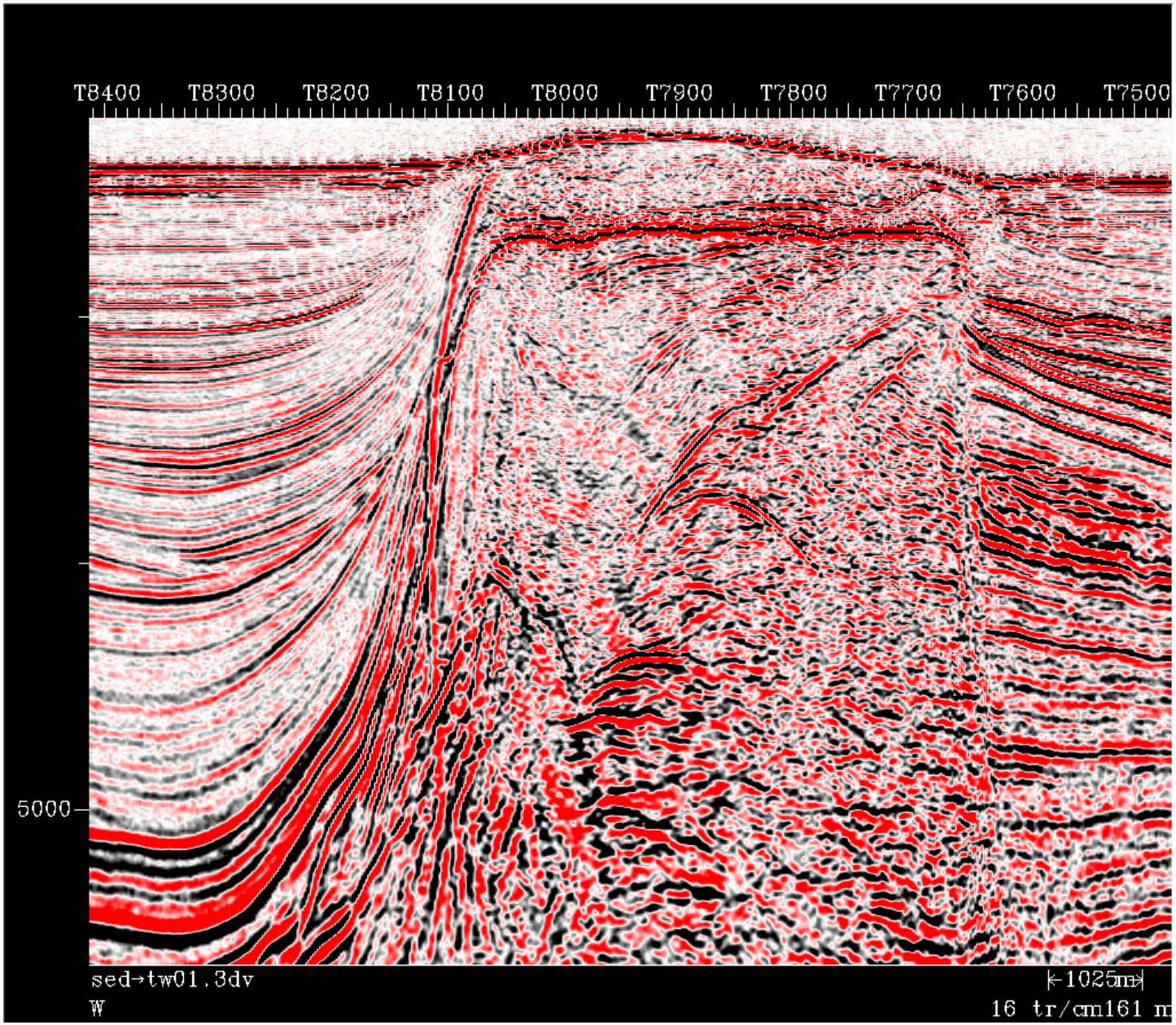

TW Kirchhoff – sediment only velocity model – Line 11050

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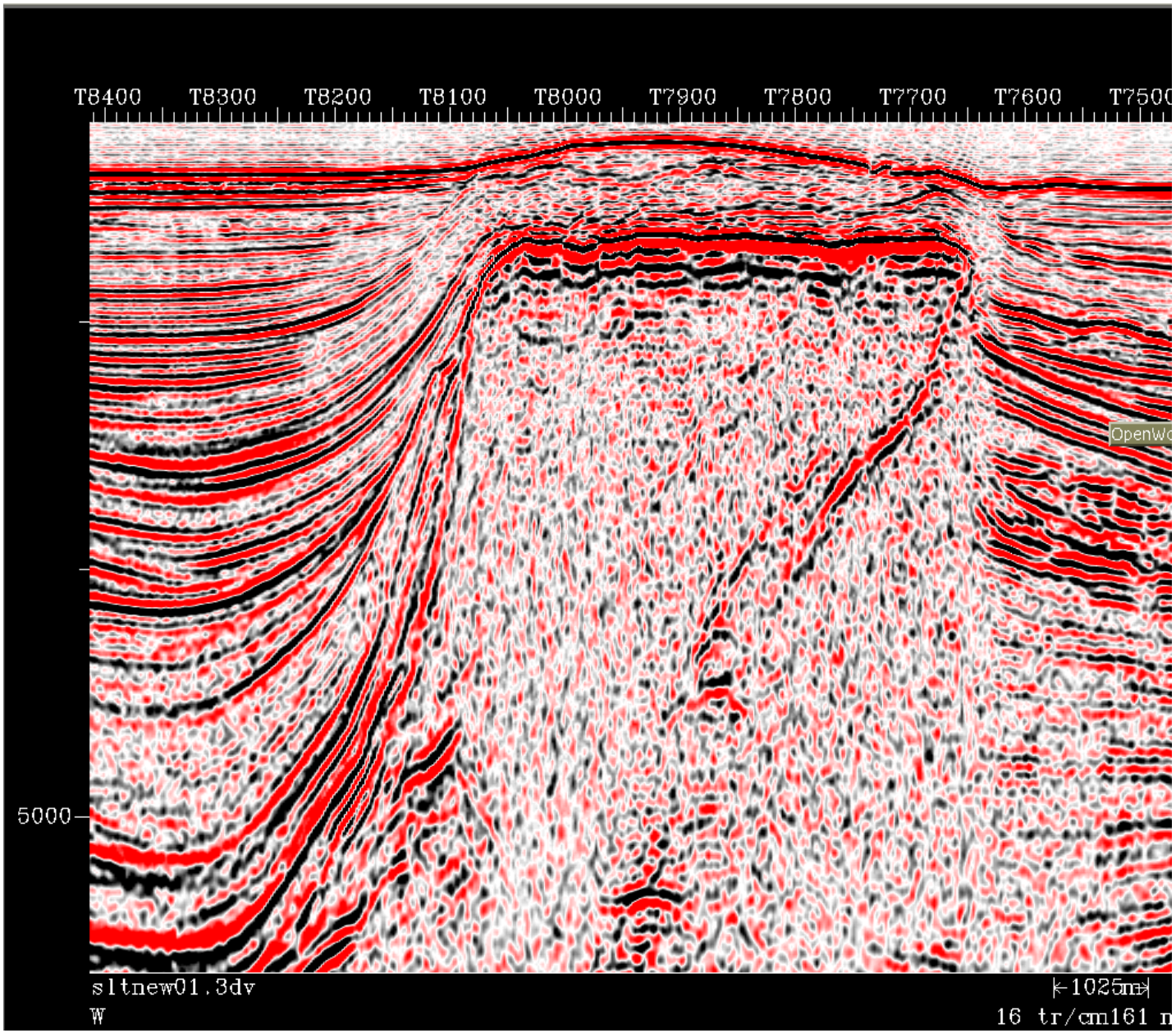

Wave Equation – salt velocity model – Line 11050

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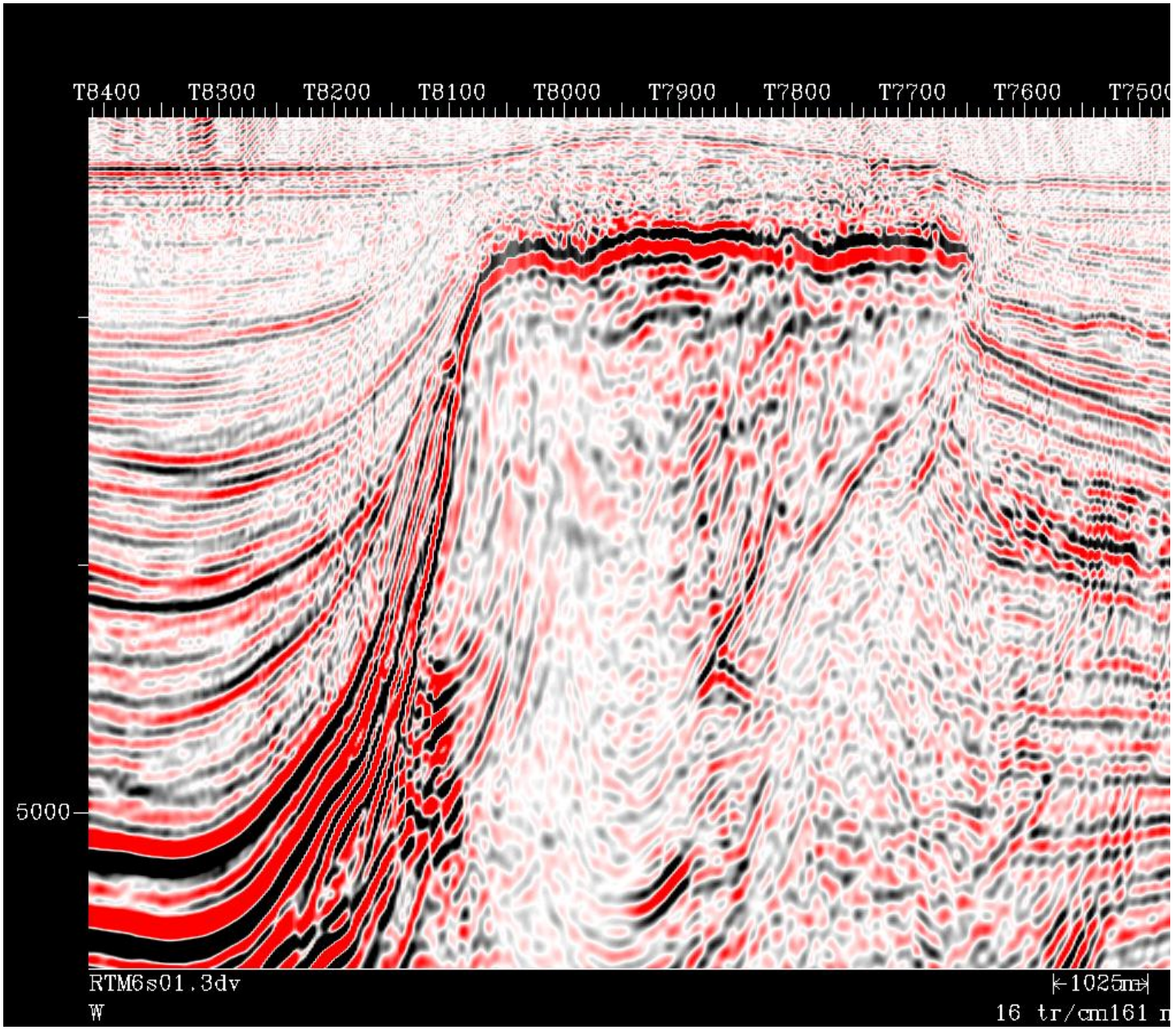

RTM – salt velocity model – Line 11050

STATOIL