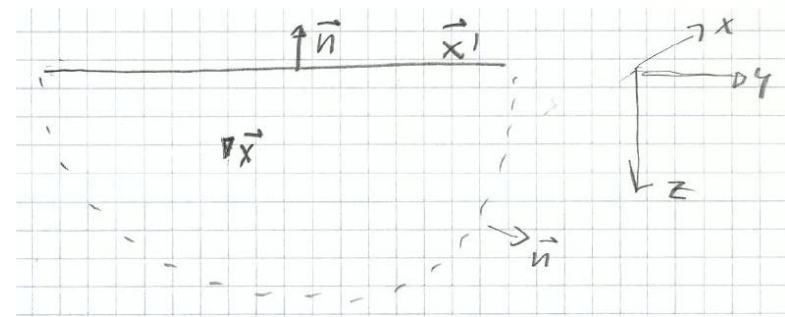


FK migration FD migration

KetilHokstad, January 2016

Kirchhoff integral recap



The Kirchhoff integral:

$$P(\mathbf{x}, \omega) = \int \int \int d^3x' G(\mathbf{x}, \mathbf{x}') S(\mathbf{x}', \omega)$$

Body forces

$$+ \int \int dx' dy' [\partial'_z G(\mathbf{x}, \mathbf{x}') P(\mathbf{x}') - G(\mathbf{x}, \mathbf{x}') \partial'_z P(\mathbf{x}')]]$$

boundary conditions

Broadband seismic: P and v_z recorded

$$i\omega\rho v_i(\mathbf{x}) = -\partial_i P(\mathbf{x}) \text{ (Newton)}$$

$$P(\mathbf{x}, \omega) = \int \int \int d^3x' G(\mathbf{x}, \mathbf{x}') S(\mathbf{x}', \omega)$$

$$- \int \int dx' dy' [\partial'_z G(\mathbf{x}, \mathbf{x}') P(\mathbf{x}') + i\omega\rho(\mathbf{x}') G(\mathbf{x}, \mathbf{x}') v_z(\mathbf{x}')]]$$

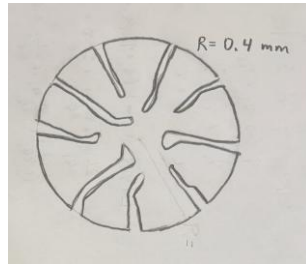
We can derive Kirchhoff integrals for acoustic and elastic wave equations, electric and magnetic fields (Maxwell), and diffusion equation

Diffusive intermezzo

Diffusion equation for a spherical particle

$$\frac{\partial C}{\partial t} - D \nabla^2 C = S$$

$$\nabla^2 C = \frac{1}{r} \frac{\partial^2}{\partial r^2} (rC)$$



$$\frac{\partial f(r,t)}{\partial t} - D \frac{\partial^2 f(r,t)}{\partial r^2} = J(r,t)$$

$$f = rC$$

$$J = rS$$

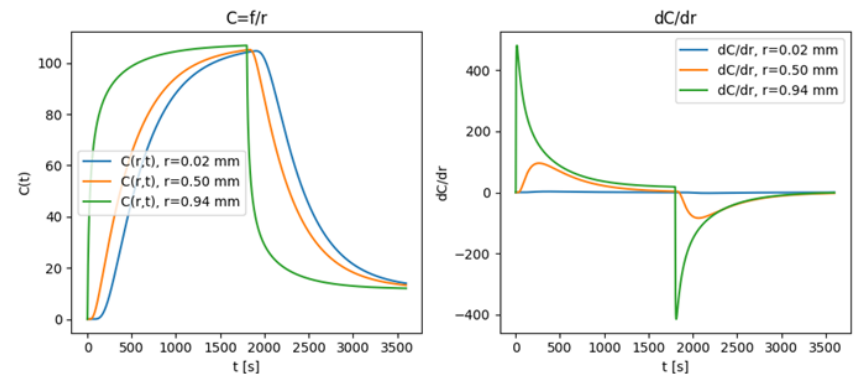
Green's function for diffusion

$$\frac{\partial G(r,t;r',t')}{\partial t} - D \frac{\partial^2 G(r,t;r',t')}{\partial r^2} = \delta(r-r')\delta(t-t')$$

$$G(r,t;r',t') = \sqrt{\frac{4\pi}{D(t-t')}} \left[e^{-\frac{(r-r')^2}{4D(t-t')}} - e^{-\frac{(r+r')^2}{4D(t-t')}} \right]$$

Kirchhoff integral (representation theorem) for diffusion

$$\begin{aligned} f(r,t) &= \int_0^R G(r,r',t) * J(r',t) dr' \\ &+ D \left[G(r,R,t) * \frac{\partial f(R,t)}{\partial R} - \frac{\partial G(r,R,t)}{\partial R} * f(R,t) \right] \\ &+ \int_0^R f(r',0) G(r,r',t) dr' \end{aligned}$$



Reference: Morse and Feshbach, 1953,
Methods of Theoretical Physics, Ch. 7.

Helmholtz equation

$$\partial_i \partial_i F + \kappa_0^2 F = 0$$

Seismic:

$$\begin{aligned} F &= P \\ \kappa_0 &= \frac{\omega}{v} \end{aligned}$$

EM:

$$\begin{aligned} F &= E_i \\ \kappa_0^2 &= i\omega\mu_0\sigma \\ \kappa_0 &= \left(\frac{1+i}{\sqrt{2}} \right) \sqrt{\omega\mu_0\sigma} \end{aligned}$$

Fourier transform

FK-domain: $F(z, k_x, k_y, \omega)$

Forward transform:

$$F(\mathbf{k}, \omega) = \frac{1}{(\sqrt{2\pi})^4} \int \int \int \left[\int F(\mathbf{x}, t) e^{-i(\mathbf{k} \cdot \mathbf{x} - \omega t)} dt \right] dx dy dz$$

Inverse transform:

$$F(\mathbf{x}, t) = \frac{1}{(\sqrt{2\pi})^4} \int \left[\int \int \int F(\mathbf{k}, \omega) e^{i(\mathbf{k} \cdot \mathbf{x} - \omega t)} dk_x dk_y dk_z \right] d\omega$$

Be aware of sign conventions

FK-domain

Derivatives:

$$\partial_t \rightarrow -i\omega \quad \partial_j \rightarrow ik_j = i\omega p_j$$

Helmholtz in the FK domain:

$$\partial_z^2 + k_z^2 F = 0$$

$$k_z = \sqrt{\kappa_0^2 - k_x^2 - k_y^2}$$

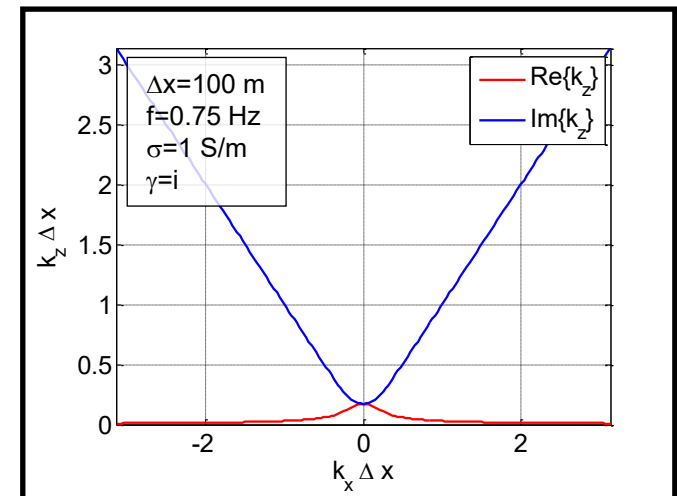
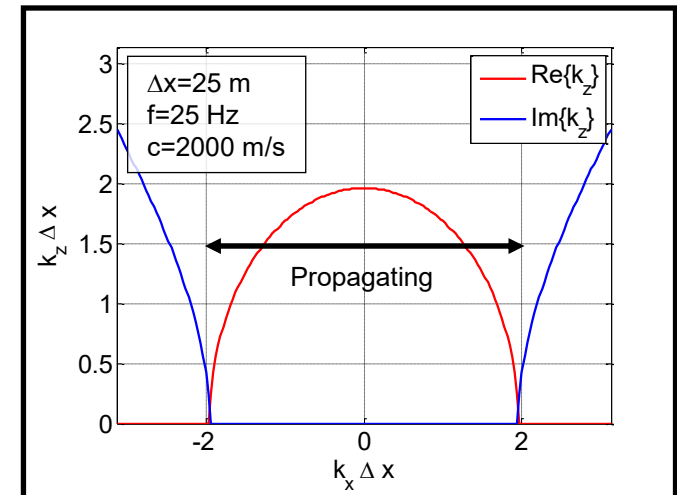
Two solutions:

$$F(z) = e^{\pm ik_z z} F(0)$$

One-way equations satisfied by F:

$$\partial_z D = ik_z D \text{ (downgoing waves)}$$

$$\partial_z U = -ik_z U \text{ (upgoing waves)}$$



Dispersion relations:
Seismic (top) and EM (bottom)

Gazdag migration

Depth stepping of U and D:

$$\begin{aligned}D(z + \Delta z, k_x, k_y) &= e^{ik_z(z)\Delta z} D(z, k_x, k_y) \\U(z + \Delta z, k_x, k_y) &= e^{-ik_z(z)\Delta z} U(z, k_x, k_y)\end{aligned}$$

Vertical wavenumber (positive z downwards)

$$k_z(z) = \sqrt{\kappa_0^2(z) - k_x^2 - k_y^2}$$

Imaging condition:

$$\begin{aligned}I(\mathbf{x}) &= \int d\omega \frac{U(\mathbf{x}, \omega)}{D(\mathbf{x}, \omega)} \\I(\mathbf{x}) &= \int d\omega \frac{U(\mathbf{x}, \omega) D^*(\mathbf{x}, \omega)}{D(\mathbf{x}, \omega) D^*(\mathbf{x}, \omega) + \epsilon}\end{aligned}$$

Inverse FFT from (k_x, k_y) to (x, y) :

$$\begin{aligned}D(\mathbf{x}, \omega) &= FFT^{-1}[D(z, k_x, k_y, \omega)] \\U(\mathbf{x}, \omega) &= FFT^{-1}[U(z, k_x, k_y, \omega)]\end{aligned}$$

$I(\mathbf{x})$ is related to gradient and Hessian, as we will see later

Claerbout's 15-degree equation

Gazdag migration:

$$\partial_z P = \pm i \sqrt{\kappa_0^2 - k_x^2 - k_y^2} F$$

Inverse Fourier transform:

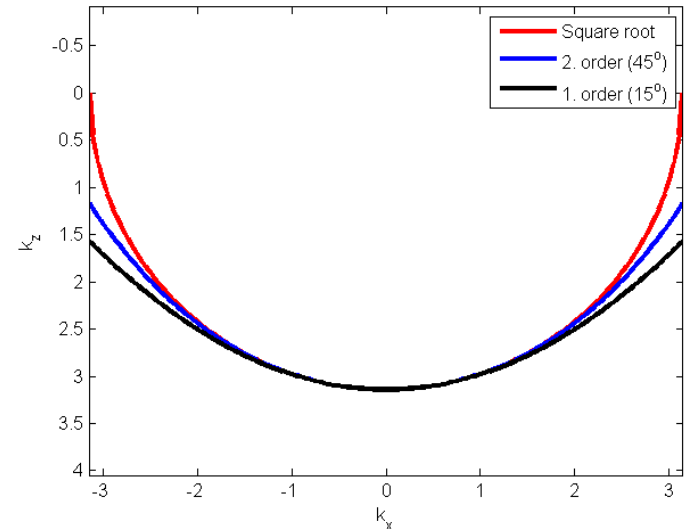
$$\partial_z P = \pm i \sqrt{\kappa_0^2 + \partial_x^2 + \partial_y^2} F$$

Series expansion: 15-degree equation

$$\partial_z P \simeq \pm i \kappa_0 \left(1 + \frac{\partial_x^2 + \partial_y^2}{2\kappa_0^2} \right) F$$

«Schrödinger equation»:

$$-i \partial_z P \simeq \pm \left(\kappa_0 F + \frac{\partial_x^2 F + \partial_y^2 F}{2\kappa_0} \right)$$



45° and 60° equations can be developed along the the same lines

Explicit one-way FD migration

«Wave-equation migration»

Gazdag:

$$F(z + \Delta z, k_x, k_y) = e^{\gamma \Delta z \sqrt{\kappa_0^2 - k_x^2 - k_y^2}} F(z, k_x, k_y)$$

Gazdag FK migration works for (approximately) 1D media
We want to get around this limitation

Rewrite in terms of normalized wave numbers:

$$F(z + \Delta z, k_x, k_y) = e^{\gamma \frac{\Delta z}{\Delta x} \sqrt{\hat{\kappa}_0^2 - \hat{k}_x^2 - \hat{k}_y^2}} F(z, k_x, k_y)$$

Normalized wavenumbers:

$$\hat{\kappa}_0 = \Delta x \kappa_0$$

$$\hat{k}_x = \Delta x k_x$$

$$\hat{k}_y = \Delta y k_y = \Delta x k_y$$

$$k_x^{Nyq} = \frac{\pi}{\Delta x}$$

$$\hat{k}_x^{Nyq} = \pi$$

und zu weiter

Lateral velocity variations

Assume a 3D medium: $\kappa_0 = \kappa_0(x, y, z)$.

Multiplication in FK fomain \Leftrightarrow convolution in FX domain

Replace the phase shift operator by a discrete convolutional filter:

$$F(x, y, z + \Delta z) = \sum_{m, n=-L}^L W(m\Delta x, n\Delta y, \hat{\kappa}_0, g) F(x - m\Delta x, y - n\Delta y, z)$$

The complex filter W depends on

$$\begin{aligned}\hat{\kappa}_0 &= \kappa_0 \Delta x \\ g &= \frac{\Delta z}{\Delta x}\end{aligned}$$

Wavefield extrapolation filter

Compute $W(m\Delta x, n\Delta y, \hat{\kappa}_0, g)$ by minimization of

$$J = ||W(i\Delta\hat{k}_x, j\Delta\hat{k}_y, \hat{\kappa}_0, g) - e^{\gamma \frac{\Delta z}{\Delta x} \hat{k}_z}||^4$$
$$\hat{k}_z = \sqrt{\hat{\kappa}_0^2 - (i\Delta\hat{k}_x)^2 - (j\Delta\hat{k}_y)^2}$$

where

$$W(i\Delta\hat{k}_x, j\Delta\hat{k}_y, \hat{\kappa}_0, g) = DFT[W(m\Delta x, n\Delta y, \hat{\kappa}_0, g)]$$

- Create look-up tables for W
- Can be computed once and for all
- Max propagation angle depends on operator length L

Illustration of 3D FD Depth Migration Scheme

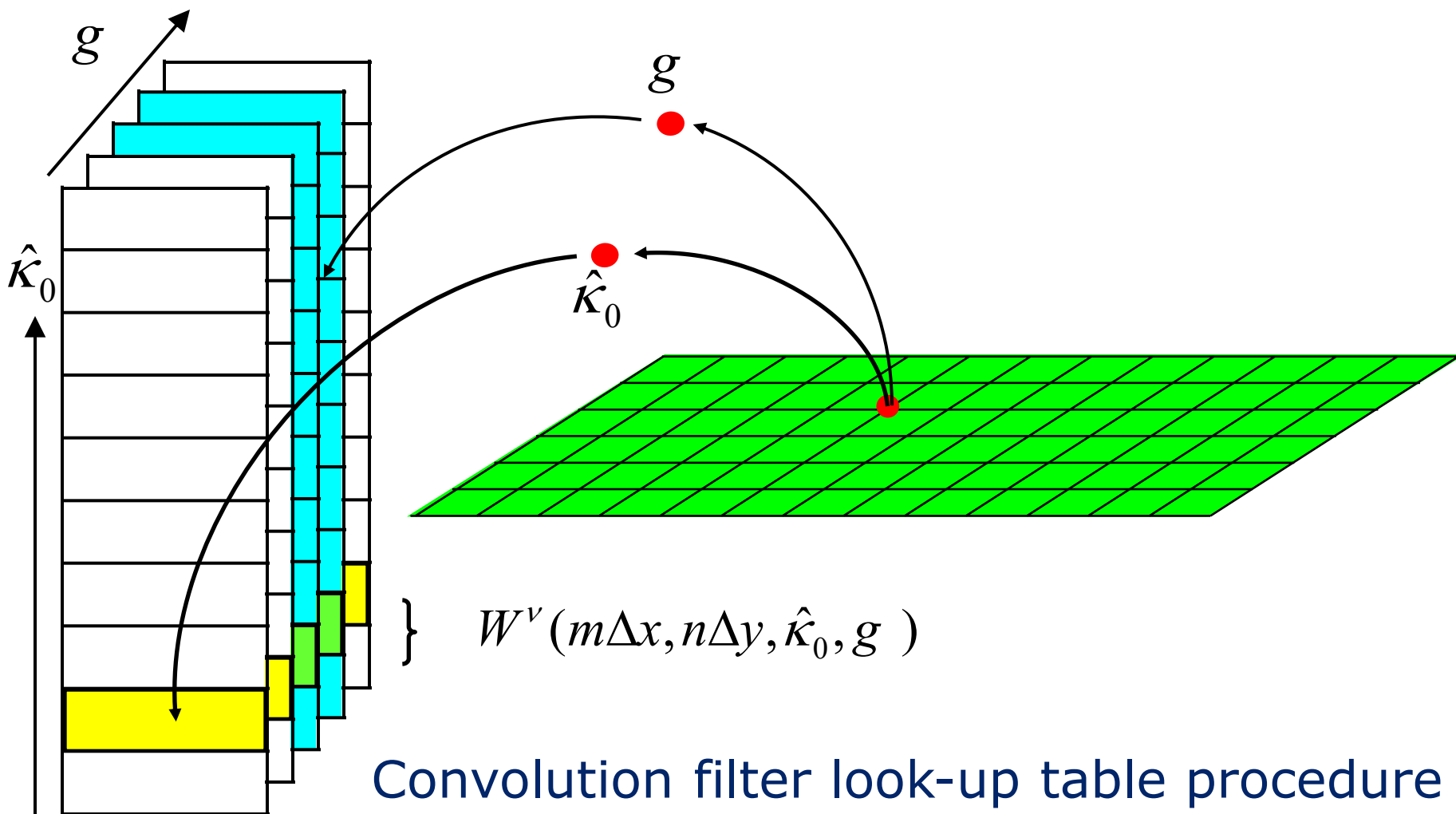
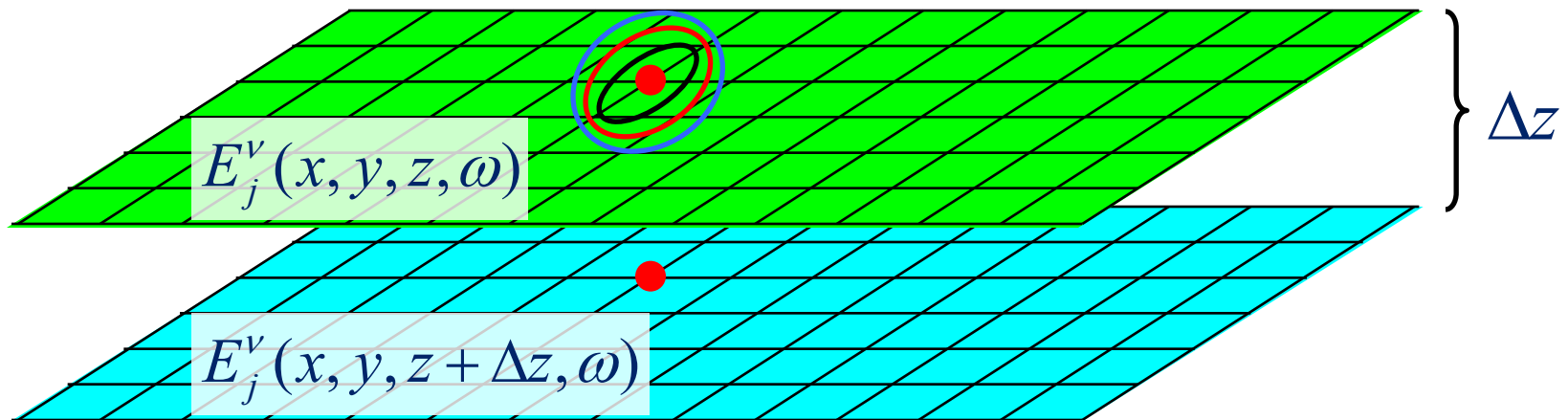


Illustration of 3D FD Depth Migration Scheme

$$E_{(m)}^v(x, y, z + \Delta z, \omega) = \sum_{m,n=-L}^L \left[W^v(m\Delta x, n\Delta y, \hat{\kappa}_0, g) \right] \\ \times E_{(m)}^v(x - m\Delta x, y - n\Delta y, z, \omega)$$



Wave field extrapolation by filter convolution

FK and FD migration summary

- FK migration:
 - Accurate up to 90 degrees dip
 - 1D medium: $v(z)$
- Explicit FD migration:
 - Dip accuracy depends on operator length (60 degree is common)
 - 3D medium: $v(x,y,z)$

