PG8108 course topics

Seismic depth migration

- Kirchhoff integral
- Sources and boundary conditions
- Kirchhoff family of migrations
  - Kirchhoff migration
  - Gaussian beam
  - Angle migration (GRT)
  - Demigration/remigration
- One-way migrations
  - Gazdag migration
  - Explicit FD migration
- Least-squares migration
- Reverse-time migration

Ray theory

- Hamilton's equations
- Kinetic ray-tracing system
- Dynamic ray-tracing system

Linear inversion

- Seismic tomography
- Bayesian AVO inversion
- Gravity and magnetic inversion
- In-field referencing

Machine learning

- ML vs inversion

Non-linear inversion

- Gradient and Hessian
- Gauss Newton (magnetic)
- Steepest decent
- Conjugate gradient
- Least squares
- Minimum norm
- Contrast-source inversion
- Sparse inversion
- EM inversion (CSEM and MT)
- Seismic full-waveform inversion
- Elastic FD modeling; staggering
- Joint inversion

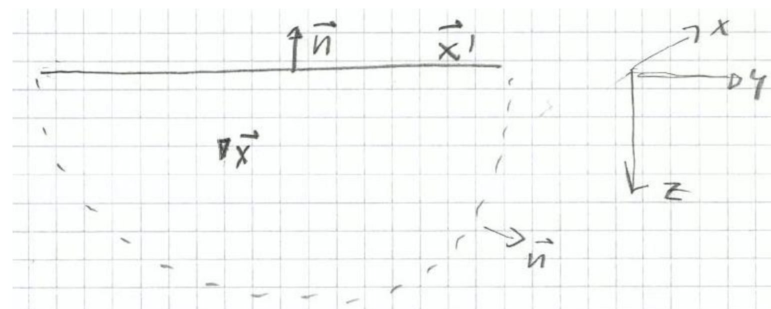
Kirchhoff integral
Kirchhoff migration

KetilHokstad, January 2016

Helmholtz equation and Kirchhoff integral

Wave equation and Helmholtz equation:

$$
\begin{aligned}
\partial_i \partial_i P(x,t) - \frac{1}{v^2}\partial_t^2 P(x,t) &= S(x,t) \\
\partial_i \partial_i P(x,\omega) + \frac{\omega^2}{v^2} P(x,\omega) &= S(x,\omega)
\end{aligned}
$$



Green's function (omitting ω):

$$
\partial_i \partial_i G(x,x') + \frac{\omega^2}{v^2} G(x,x') = \delta(x-x')
$$

The Kirchhoff integral:

$$
P(x,\omega) = \int\int\int d^3x' G(x,x') S(x',\omega)
$$

Volume integral; body forces

$$
+ \int\int dS' [G(x,x')\partial_i' P(x') - P(x')\partial_i' G(x,x')] n_i
$$

Surface integral; boundary conditions

$S(x) = \delta(x - x_s) S(\omega),$ (point source; body force)

$$\nabla^2 p(x) + k_0^2 p(x) = -S(x)$$

$$\nabla^2 G(x,x') + k_0^2 G(x,x') = -\delta(x-x')$$

$$G\nabla^2 p + G k_0^2 p = -GS$$

$$p\nabla^2 G + p k_0^2 G = -p\delta$$

$$G\nabla^2 p - p\nabla^2 G + G k_0^2 p - p k_0^2 G = -(GS - p\delta)$$

$$\int p(x')\,\delta(x-x')\,d^3x' = \int G(x,x')\,S(x')\,d^3x'$$

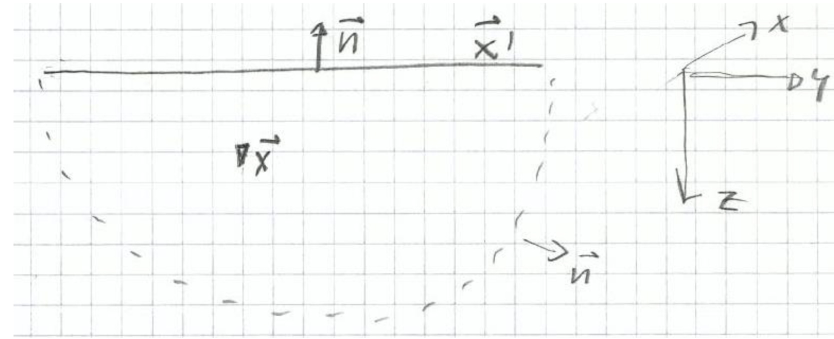
$$+ \int\!\!\int \Big[\, G(x,x')\,\nabla'^2 p(x') - p(x')\,\nabla'^2 G(x,x')\,d^3x'$$

$$P(x) = \int G(x,x')\, S(x')\, d^3x' + \oint \left[G(x,x')\, \nabla' P(x') - P(x')\, \nabla' G(x,x') \right] d\vec{S}'$$

$$\nabla (G \nabla P - P \nabla G) = \cancel{\nabla G \nabla P} + G \nabla^2 P - \cancel{\nabla P \nabla G} - P \nabla^2 G$$

$$= G \nabla^2 P - P \nabla^2 G$$

Acoustic wave equation and Kirchhoff integral



- Conventional surface seismic: Only P is recorded
  => we know only half the boundary condition
- This is OK if we know which side of surface S the wave field is coming from

Then, assuming S is the horizontal surface at $z = 0$:

$$P(x,\omega) = \int\int\int d^3x' G(x,x') S(x',\omega)$$

Volume integral; body forces

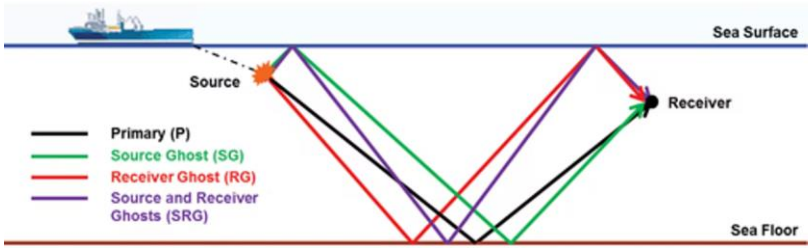
$$-2 \int\int dx'dy' P(x') \partial_i' G(x,x')] n_i$$

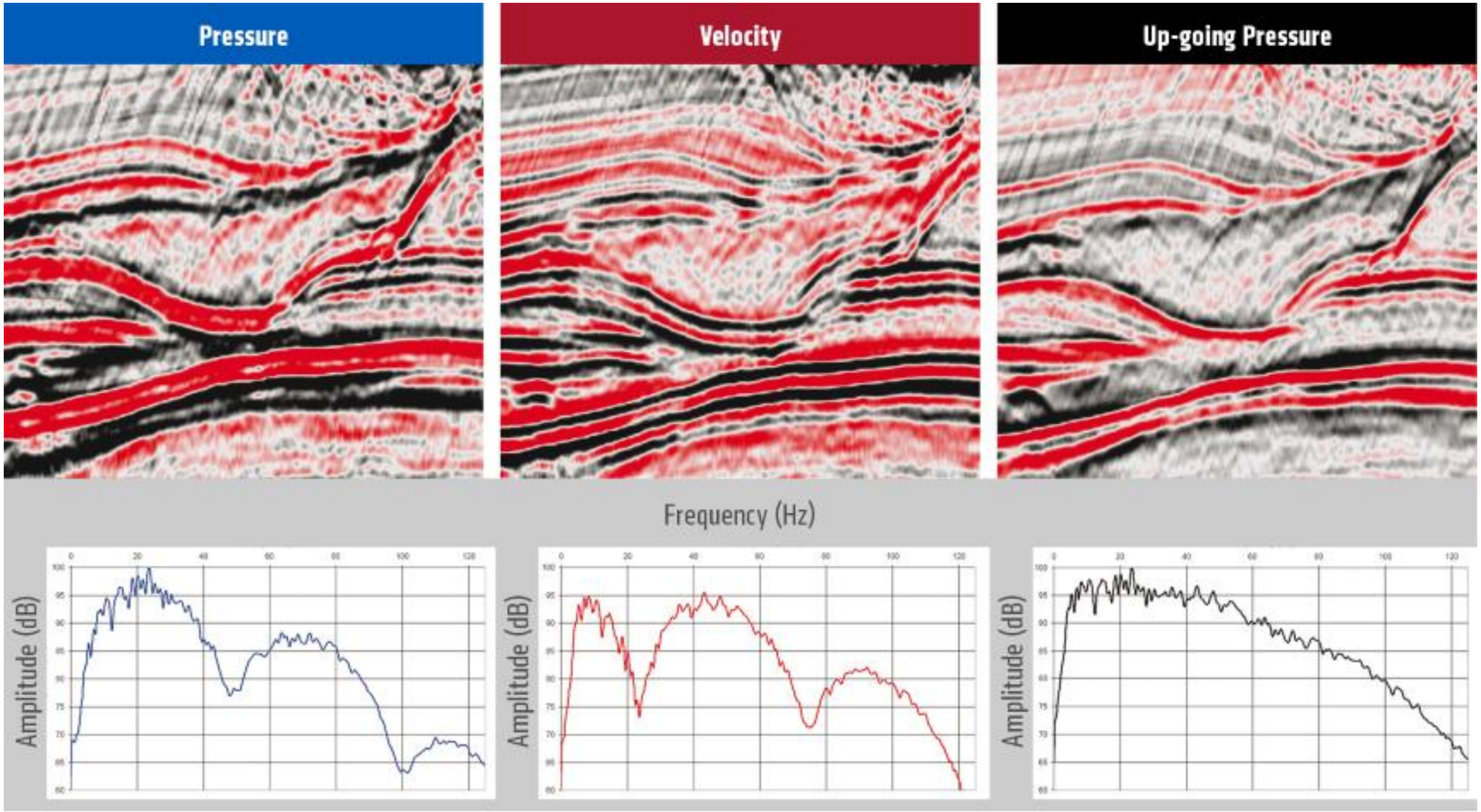
Surface integral; half boundary condition

The integral over the half-sphere vanish by the Sommerfeld radiation condition

- (Broadband seismic: Both P and Vz (the derivative of P) are recorded)

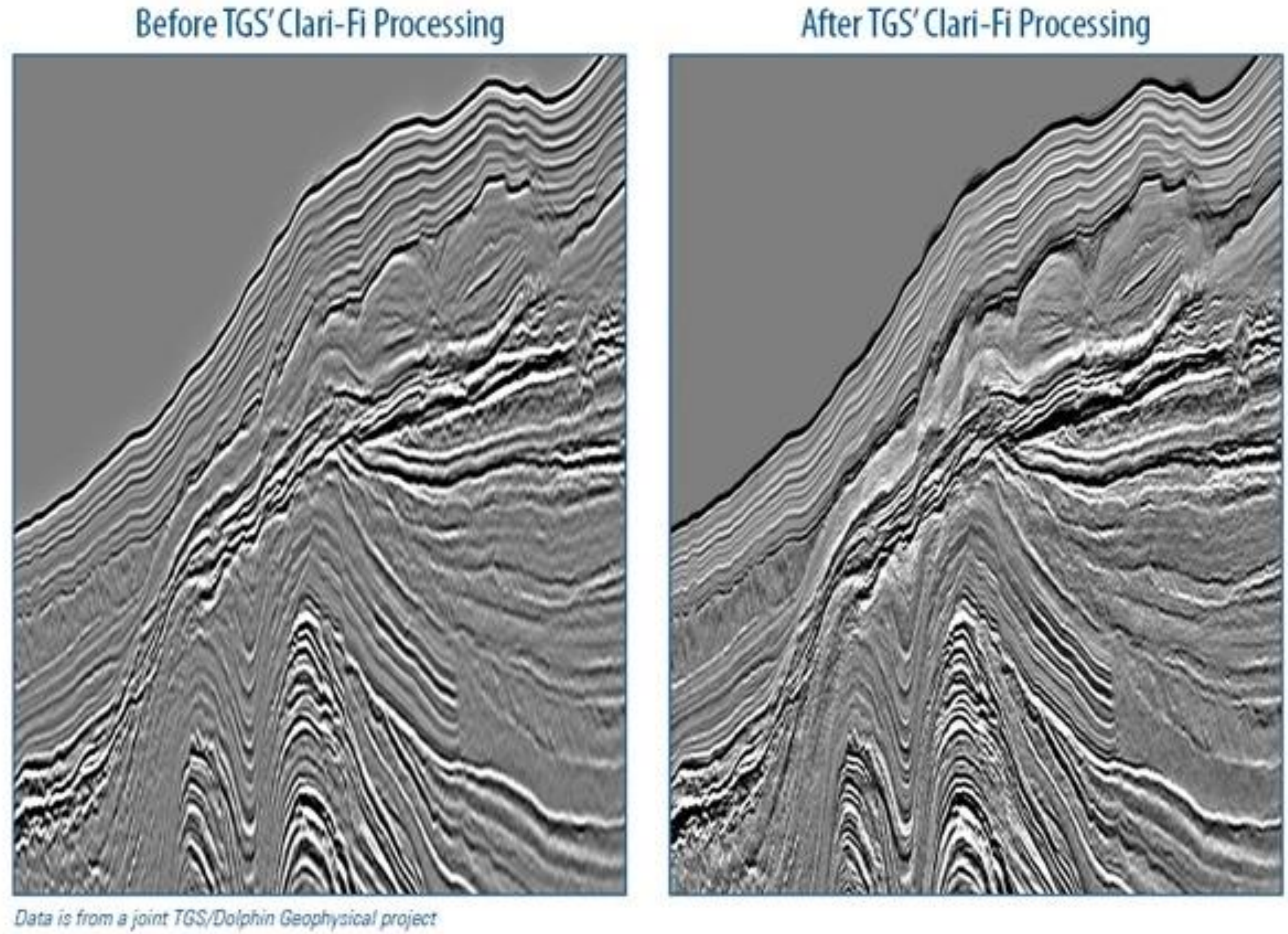
Broadband seismic





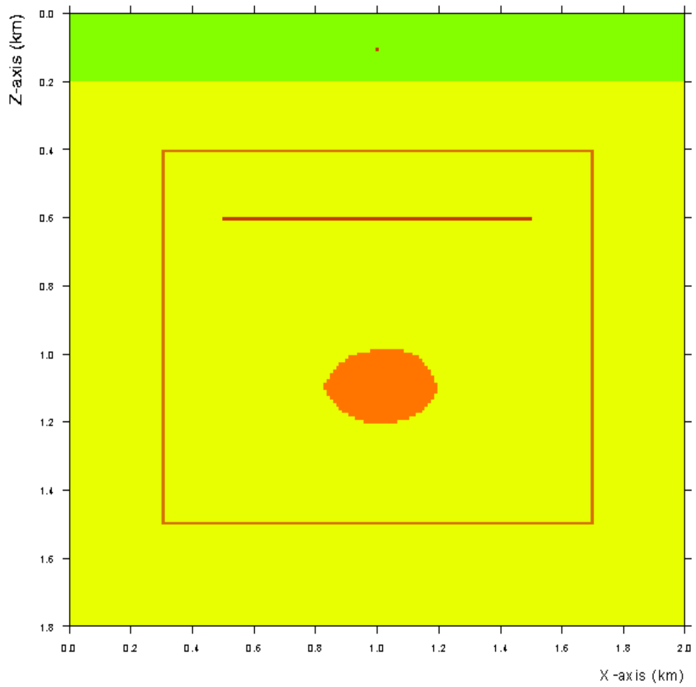
Picture from PGS webs site (pgs.com)

Deghosting

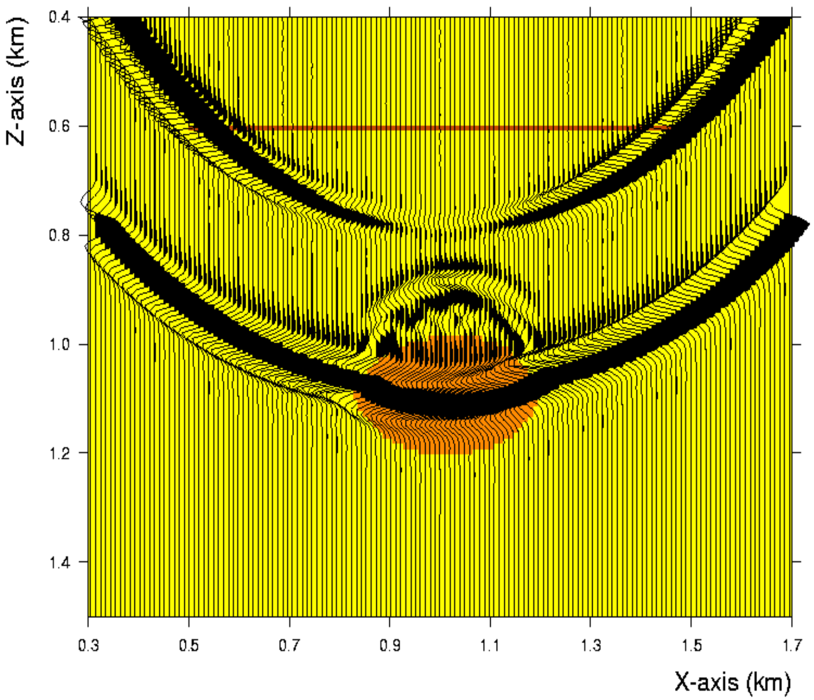


Picture from TGS webs site (tgs.com)

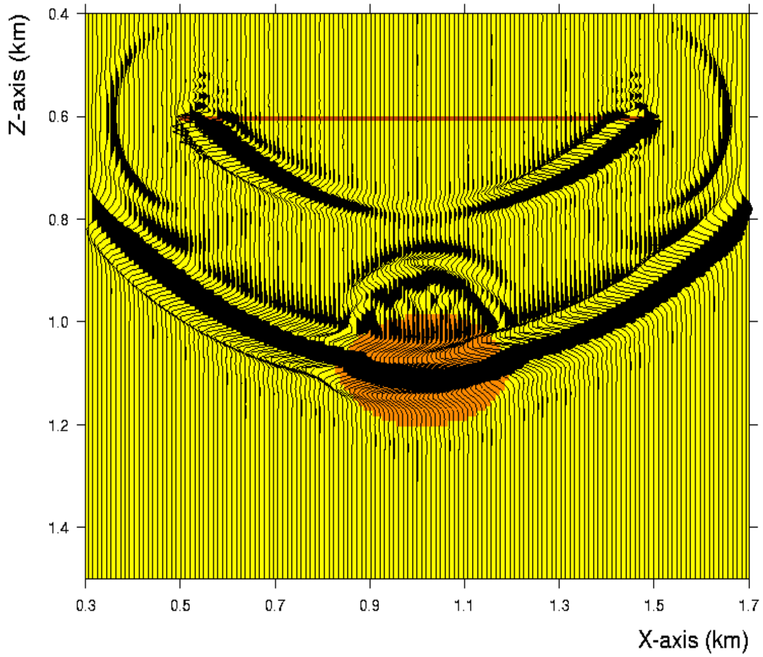
Scatter model:



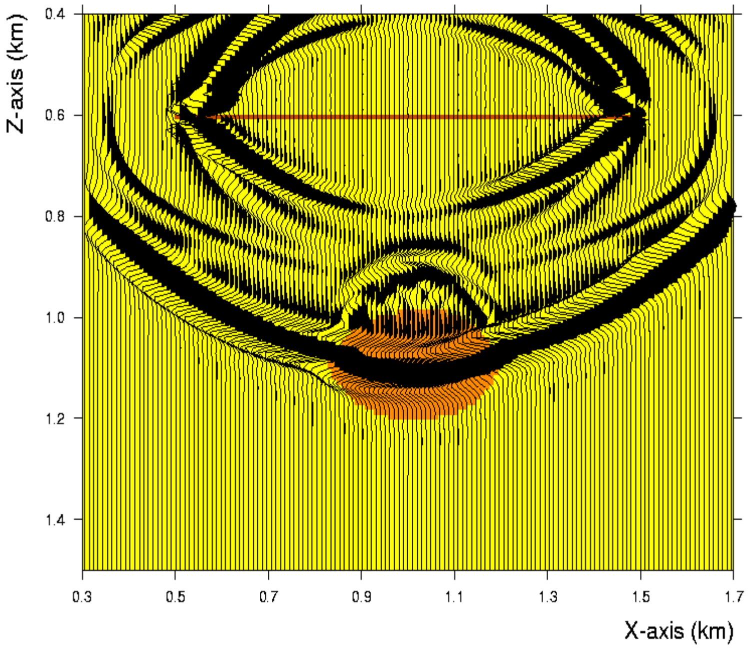
FD Reference (A_z):



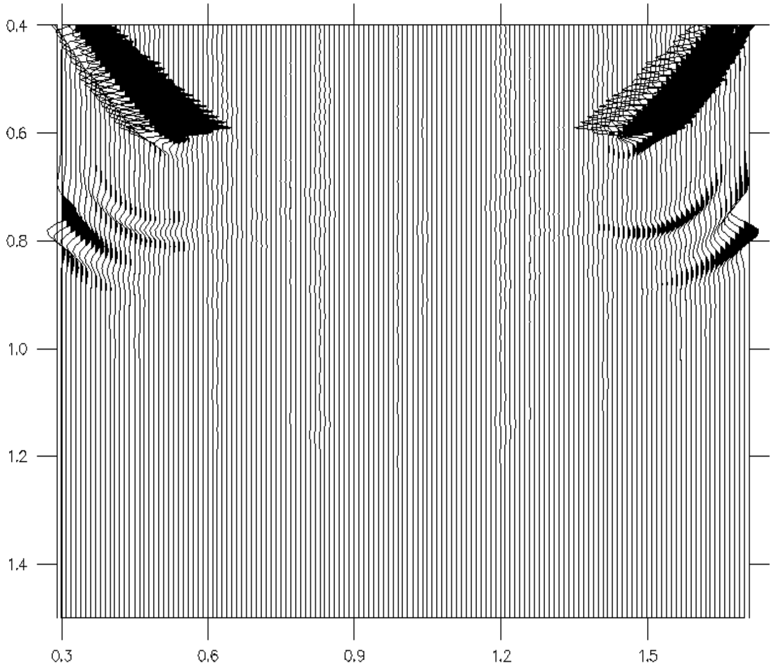
Complete b.c:



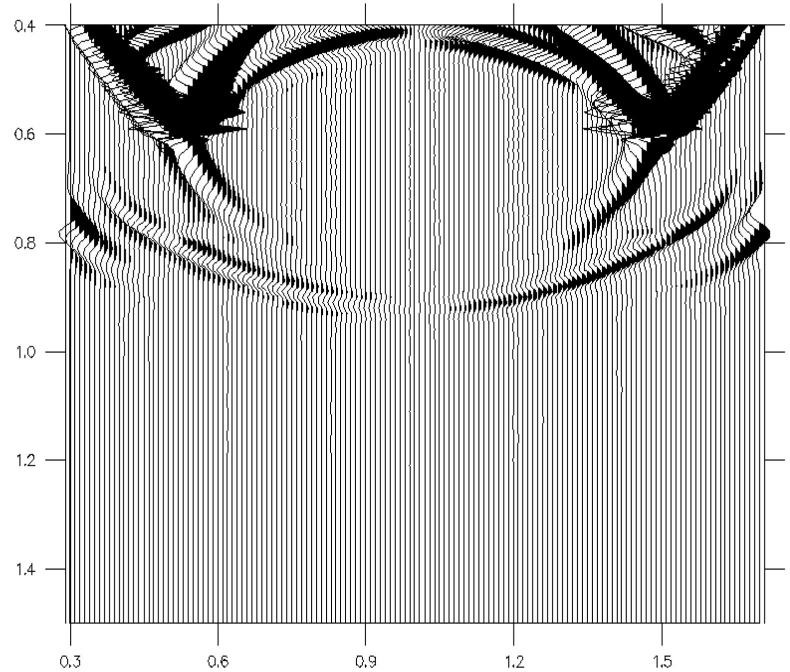
Incomplete b.c.:



Snapshot residuals:



Complete b.c.



Incomplete b.c.

Time reversal (G*) and back propagation

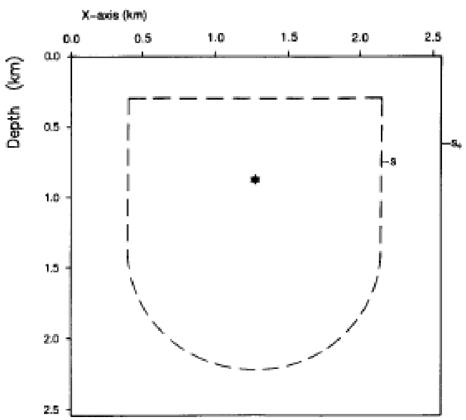
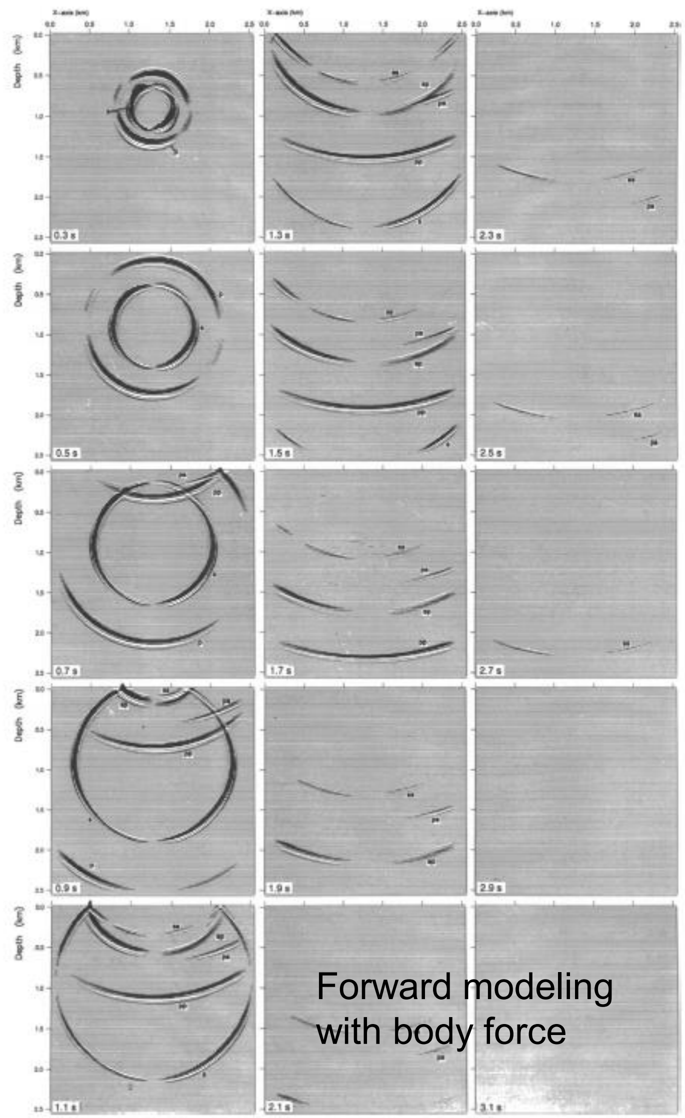
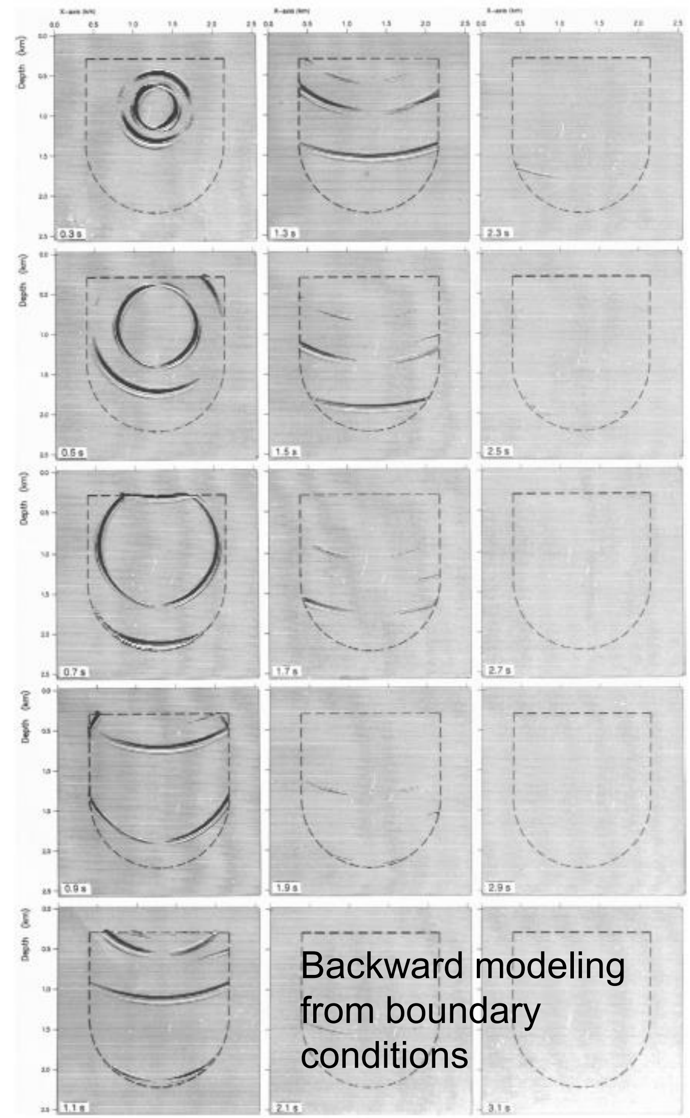


FIG. 1. The model used in the numerical experiment. P-wave velocity is 2000 m/s, S-wave velocity is 1200 m/s, and density is 2.0 g/cm^3. The top surface is a free surface, the rest of the model is surrounded by absorbing boundaries. The source position is marked with a star. The fields $a_x^n(i_r, j_r)$, $a_z^n(i_r, j_r)$, $t_x^n(i_r, j_r)$, and $t_z^n(i_r, j_r)$ are recorded on the surface S. The numerical grid is contained within the surface S_0.

From Mittet (1994)



Forward modeling with body force



Backward modeling from boundary conditions

Migration: U and D

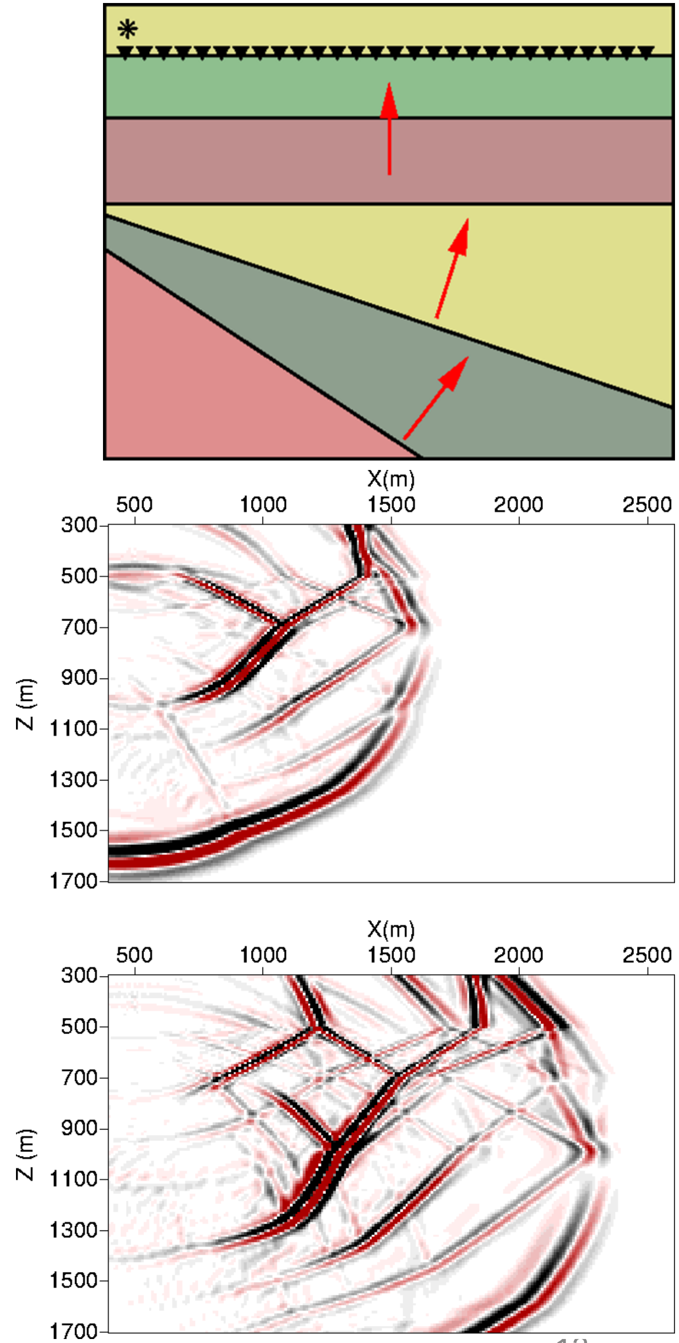
- D = down-going field
- U = up-going field

Ignoring the source spectrum; assume $S(\omega) = 1$:

$$D(x) \quad = \quad G(x, x_s)$$

Time reversal: $G \rightarrow G^*$ in the BC:

$$U(x) = 2 \int \int dx_r dy_r P(x_r) \partial_z^r G^*(x, x_r)$$



Ray Green's function

Pase shift due to caustics; ignore

Ray approximation for G and G^*:

$$
\begin{aligned}
G(x,x_s) &= \frac{\mathcal{R}(x,x_s)e^{-i(\pi/2)sgn(\omega)\sigma_s}}{4\pi\mathcal{L}(x,x_s)}e^{i\omega T(x,x_s)} \\
\partial_z^r G^*(x,x_r) &= -i\omega p_z(x_r)\frac{\mathcal{R}(x,x_r)e^{+i(\pi/2)sgn(\omega)\sigma_r}}{4\pi\mathcal{L}(x,x_r)}e^{-i\omega T(x,x_r)}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{R}(x,x_s) &\simeq 1 \\
\mathcal{R}(x,x_r) &\simeq 1
\end{aligned}
$$

Assume smooth medium; neglect transmission losses

$$p_z(x_r) = \partial_z T(x,x_r) = \frac{\cos\theta_r}{v}$$

Slowness; phase angle and phase velocity in anisotropic media

Ray Green's function

Substitute ray approximation in source and BC terms:

$$
\begin{aligned}
D(x) &= \frac{1}{4\pi\mathcal{L}(x, x_s)} e^{i\omega T(x,x_s)} \\
U(x) &= \frac{i\omega}{2\pi} \int\int dx_r dy_r \frac{p_z(x_r)}{\mathcal{L}(x, x_r)} e^{-i\omega T(x,x_r)} P(x_r)
\end{aligned}
$$

Imaging condition (reintroducing ω):

$$
I(x) = \int d\omega \frac{U(x,\omega)}{D(x,\omega)}
$$

The idea behind the imaging condition:

$$
\begin{aligned}
U(x,\omega) &\sim R(x)D(x,\omega) \\
I(x) &= \hat{R}(x)
\end{aligned}
$$

The kinematics are correct

Kirchhoff migration

Substitute U and D in imaging condition:

$$I(x;x_s) \quad = \quad \int d\omega \int\int dx_r dy_r p_z(x_r) \frac{\mathcal{L}(x,x_s)}{\mathcal{L}(x,x_r)} e^{-i\omega[T(x,x_r)+T(x,x_s)]} i\omega P(x_r,\omega)$$

The integral over ω is the inverse Fourier transform:

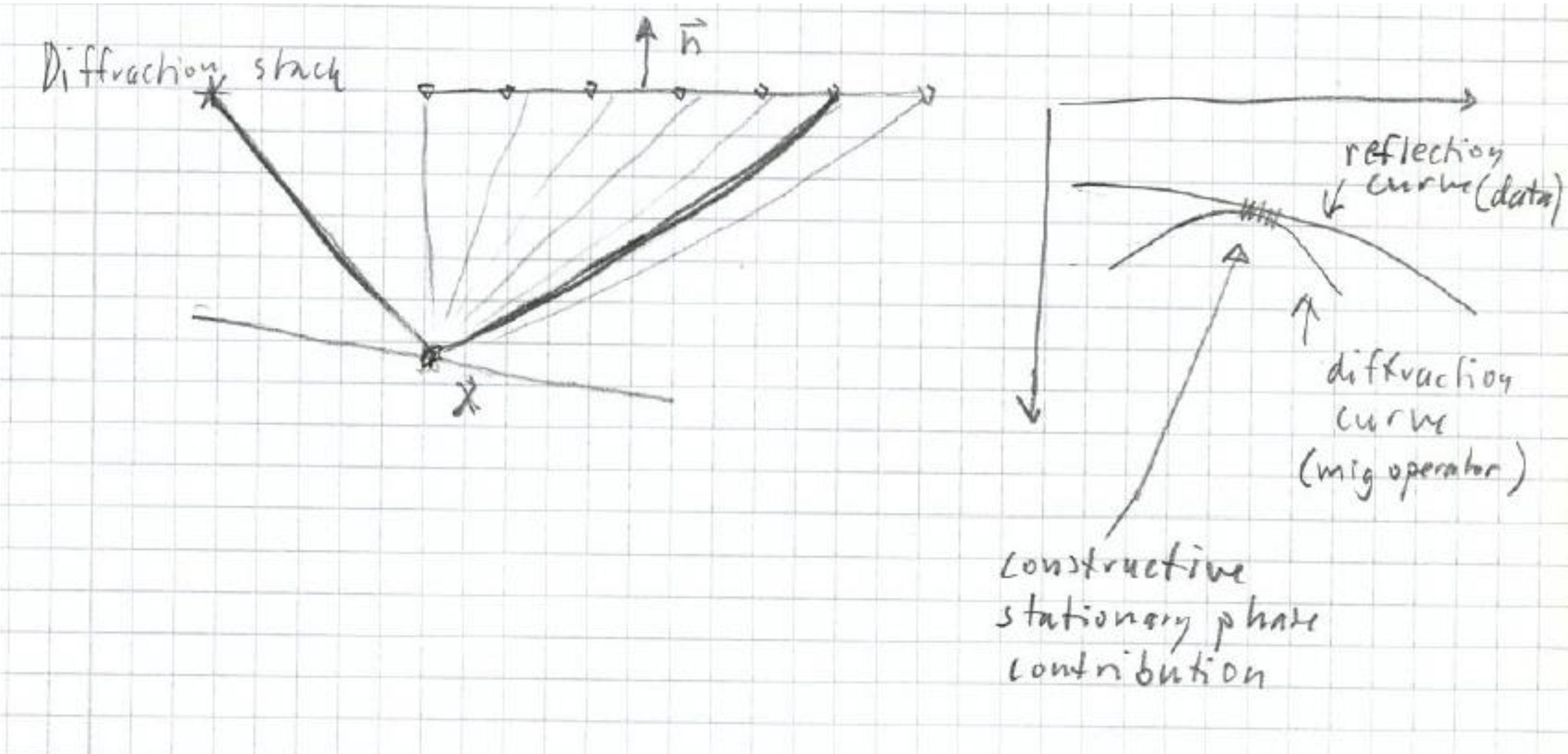
$$I(x;x_s) \quad = \quad -2\int\int dx_r dy_r W(x,x_r,x_s)\partial_t P(x_r,\tau)$$

Common-shot
Kirchhoff migration

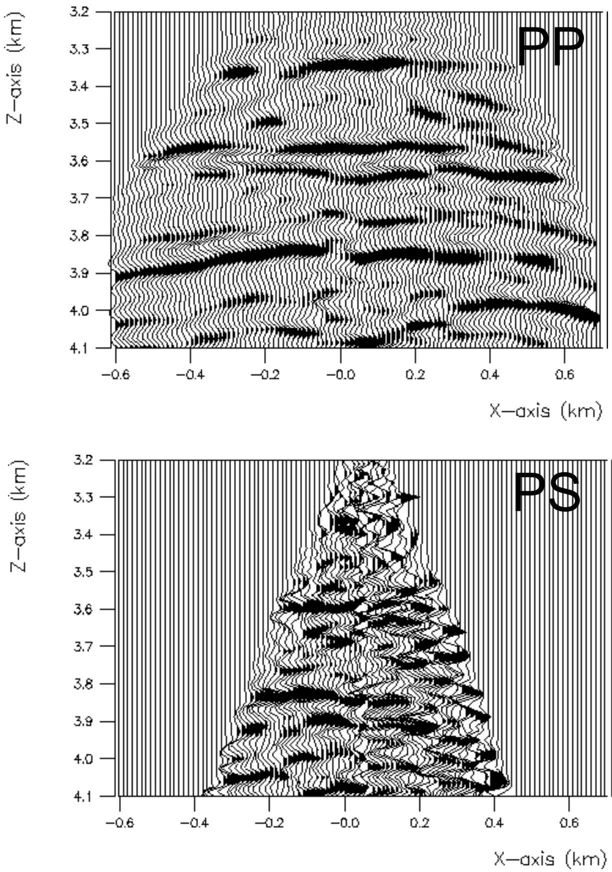
The integral over ω is the inverse Fourier transform:

$$\begin{aligned} \tau \quad &= \quad T(x,x_r)+T(x,x_s) = \text{total traveltime} \\ W \quad &= \quad \frac{\cos\theta}{v}\frac{\mathcal{L}(x,x_s)}{\mathcal{L}(x,x_r)} e^{-i\delta\tau} = \text{weight function} \\ \delta\tau \quad &= \quad \text{phase shift due to caustics} \end{aligned}$$

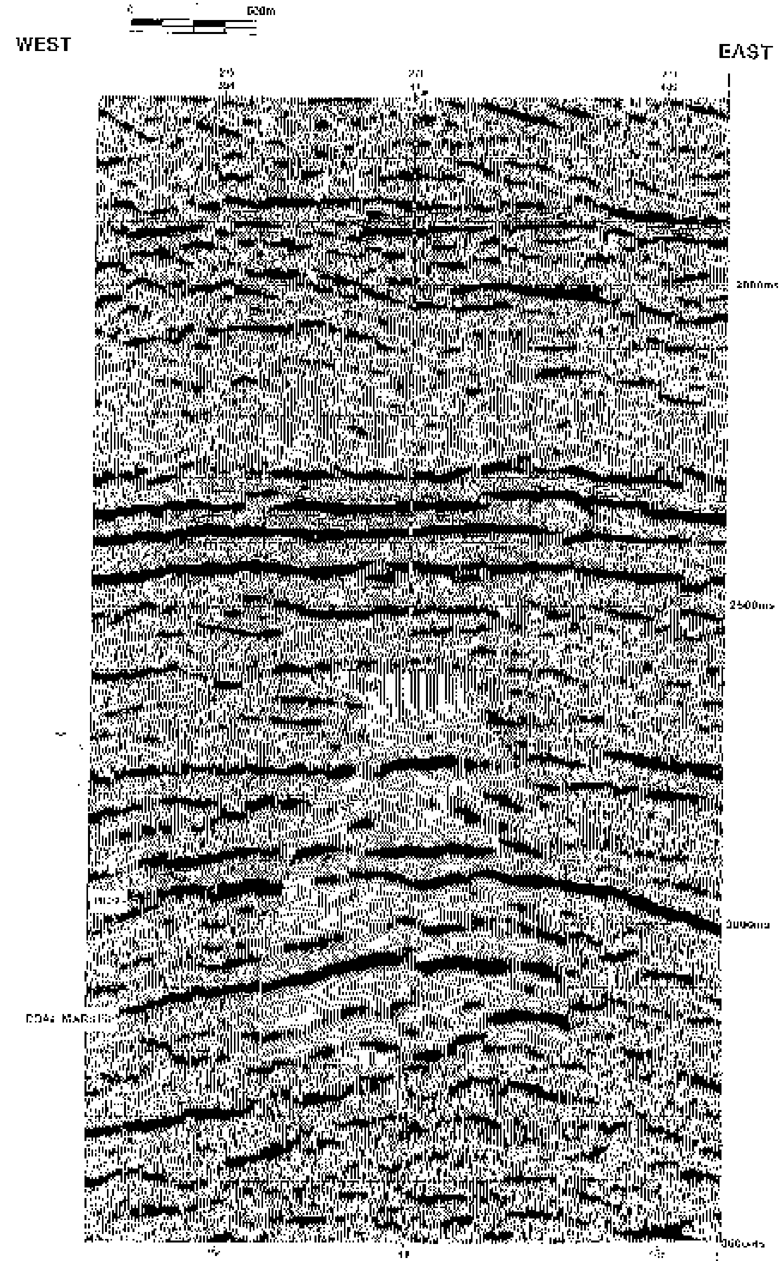
Diffraction stack



Common-receiver elastic Kirchhoff migration Walk-away VSP data

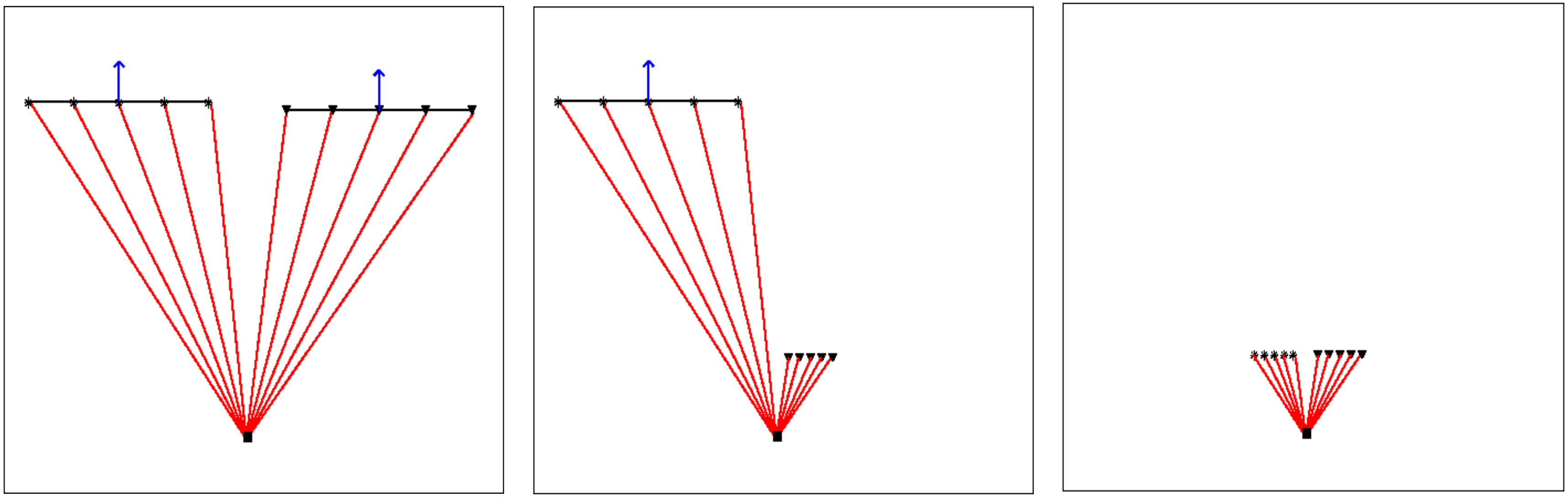


(Example from Hokstad, Geophysics, 2000)



Alternative derivation: Survey sinking + MZO

- Assume multi-shot, multi-receiver experiment
- Downward continuation of sources and receivers
- Migration to zero offset (MZO) imaging condition



Double downward continuation

Downard continue receivers (as before):

$$P(x_a, x_s) \quad = \quad -\frac{1}{2\pi} \int\int dx_r dy_r \frac{i\omega p_z(x_r)}{\mathcal{L}(x_a, x_r)} e^{-i\omega T(x_a, x_r)} P(x_r, x_s) \tag{1}$$

Reciprocity:

$$\begin{aligned} P(x_a, x_s) \quad &= \quad P(x_s, x_a) \\ \gamma_{ij}(x_a, x_s) \quad &= \quad \gamma_{ji}(x_s, x_a) \quad \text{(elastic)} \end{aligned}$$

Downard continue sources:

$$P(x_b, x_a) \quad = \quad -\frac{1}{2\pi} \int\int dx_s dy_s \frac{i\omega p_z(x_s)}{\mathcal{L}(x_b, x_s)} e^{-i\omega T(x_b, x_s)} P(x_s, x_a) \tag{2}$$

Putting things together

Substitute (1) in (2):

$$P(x_b, x_a) = \frac{-1}{4\pi^2} \int\int\int\int dx_s dy_s dx_r dy_r \frac{\omega^2 p_z(x_s) p_z(x_r)}{\mathcal{L}(x_a, x_r)\mathcal{L}(x_b, x_s)} e^{-i\omega[T(x_b,x_s)+T(x_b,x_s)]} P(x_r, x_s)$$

Introducing weight function and total traveltime:

$$\begin{aligned} P(x_b, x_a; \omega) &= \frac{-1}{4\pi^2} \int\int\int\int dx_s dy_s dx_r dy_r \omega^2 W(x_r, x_s, x) e^{-i\omega\tau} P(x_r, x_s; \omega) \\ \tau &= T(x_b, x_s) + T(x_b, x_s) \\ W &= \frac{p_z(x_s) p_z(x_r)}{\mathcal{L}(x_a, x_r)\mathcal{L}(x_b, x_s)} e^{i\delta\tau} \end{aligned}$$

MZO imaging condition

Inverse Fourier transform from frequency to time domain:

$$
\begin{aligned}
P(x_b, x_a, t) &= \int d\omega\, e^{-i\omega t} P(x_b, x_a; \omega) \\
P(x_b, x_a, t) &= \frac{1}{4\pi^2} \int\int\int\int dx_r dy_r dx_s dy_s W(x_r, x_s, x) \partial_t^2 P(x_r, x_s, t+\tau)
\end{aligned}
$$

MZO imaging condition:

$$
I(x) = \lim_{t \to 0, x_a \to x, x_b \to x} P(x_a, x_b, t+\tau)
$$

$$
I(x) = \frac{1}{4\pi^2} \int\int\int\int dx_r dy_r dx_s dy_s W(x_r, x_s, x) \partial_t^2 P(x_r, x_s, \tau)
$$

Common-offset migration

Change of variables:

$$
\begin{aligned}
h &= \frac{1}{2}(x_g - x_s) = \text{half-offset} \\
x_m &= \frac{1}{2}(x_g + x_s) = \text{midpoint}
\end{aligned}
$$

Common-offset (partial) image:

$$
\begin{aligned}
I(x,h) &= \frac{J}{4\pi^2}\int\int dx_m dy_m W(x_m,h,x)\partial_t^2 P(x_m,h,\tau) \\
J &= \text{Jacobian (trivial)}
\end{aligned}
$$

Full stack (sum over offsets):

$$
I(x) = = \int\int dh_x dh_y I(x,h)
$$

Time and depth migration

Time migration: Neglecting Snell's law

$$\begin{aligned} T(x, x_r) &= \frac{\sqrt{(x - x_r)^2 + z^2}}{V} = \frac{r}{V} \\ 1/\mathcal{L}(x, x_r) &\propto \frac{1}{\sqrt{(x - x_r)^2 + z^2}} = \frac{1}{r} \end{aligned}$$

Depth migration: Account for Snell's law

$T(x, x_r)$ from kinematic ray tracing

$\mathcal{L}(x, x_r)$ from dynamic ray tracing

More about ray theory later

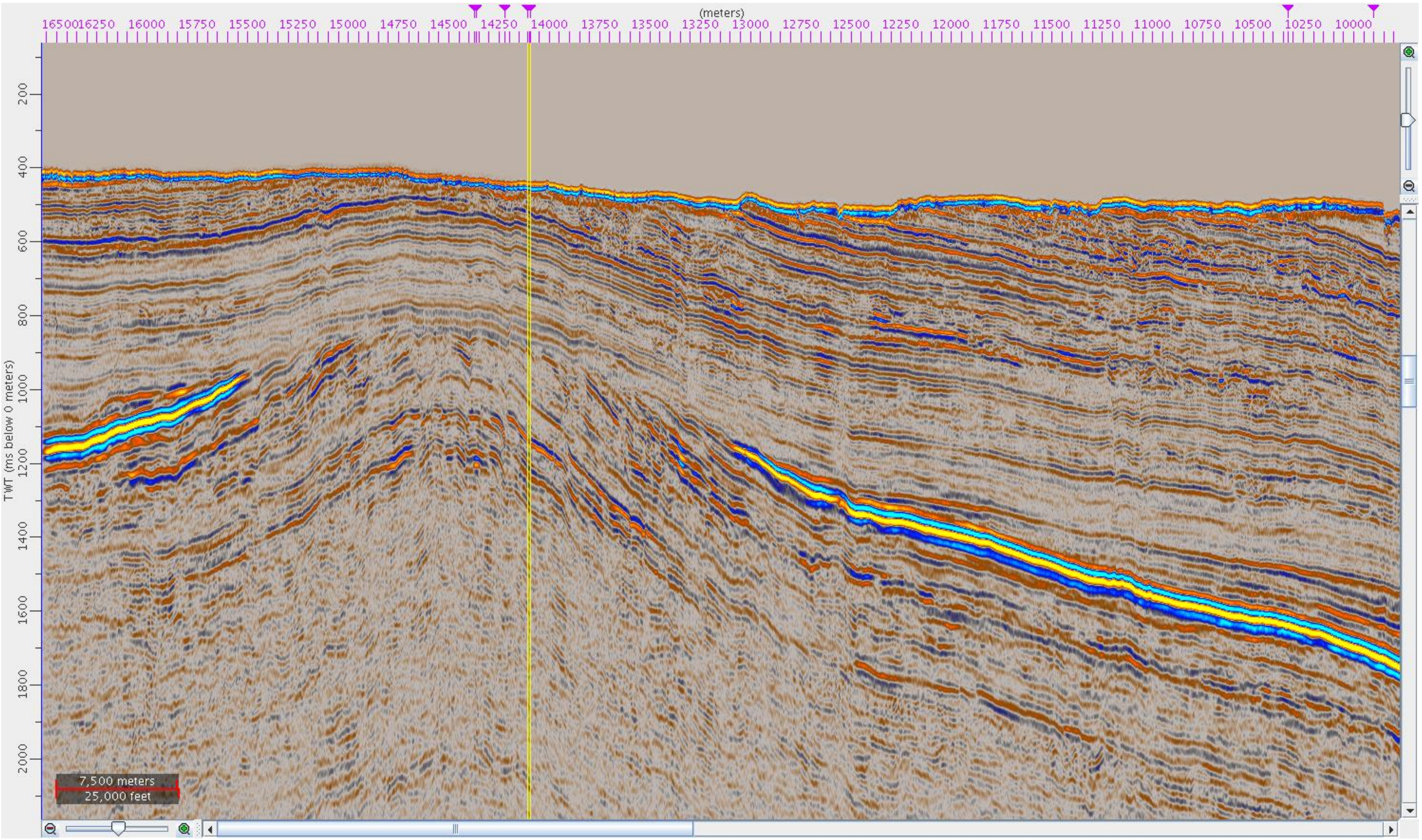
Kirchhoff migration summary

- Advantages:
  - Fast
  - Fexible wrt. Acquisition geometry
  - True amplitude (for AVO analysis)
- Disadvantages
  - Based on ray theory; inaccurate in complex geology

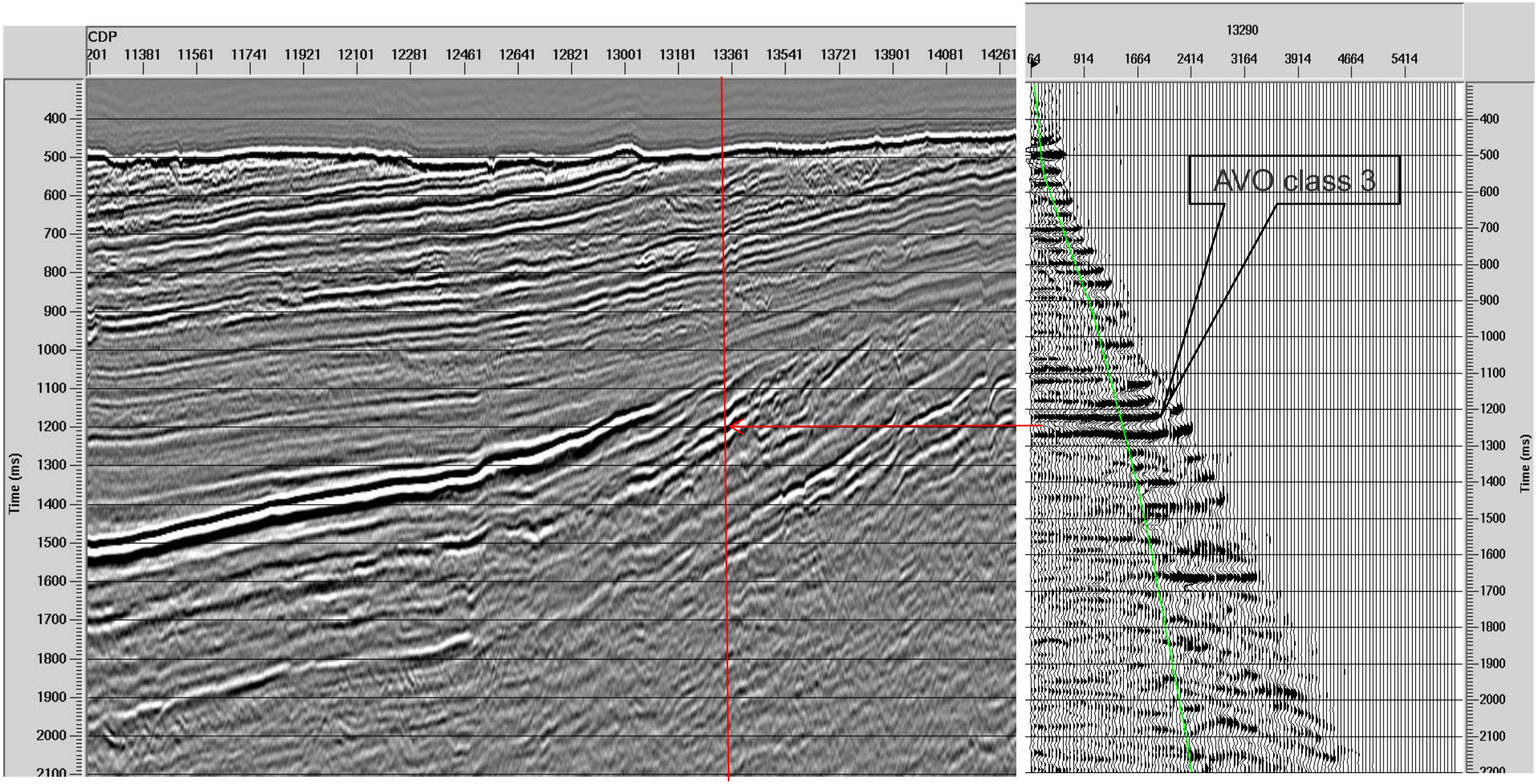
- Common shot, common receiver migration: OBS and VSP
- Common offset migration: for surface seismic data

- Later we will look at two more advanced members of the Kirchhoff family of migrations:
  - Gaussian beam migration
  - Angle migration (GRT)

Kirchhoff migration works fine in simple geology



2D Kirchhoff migration – full stack and CIP gather

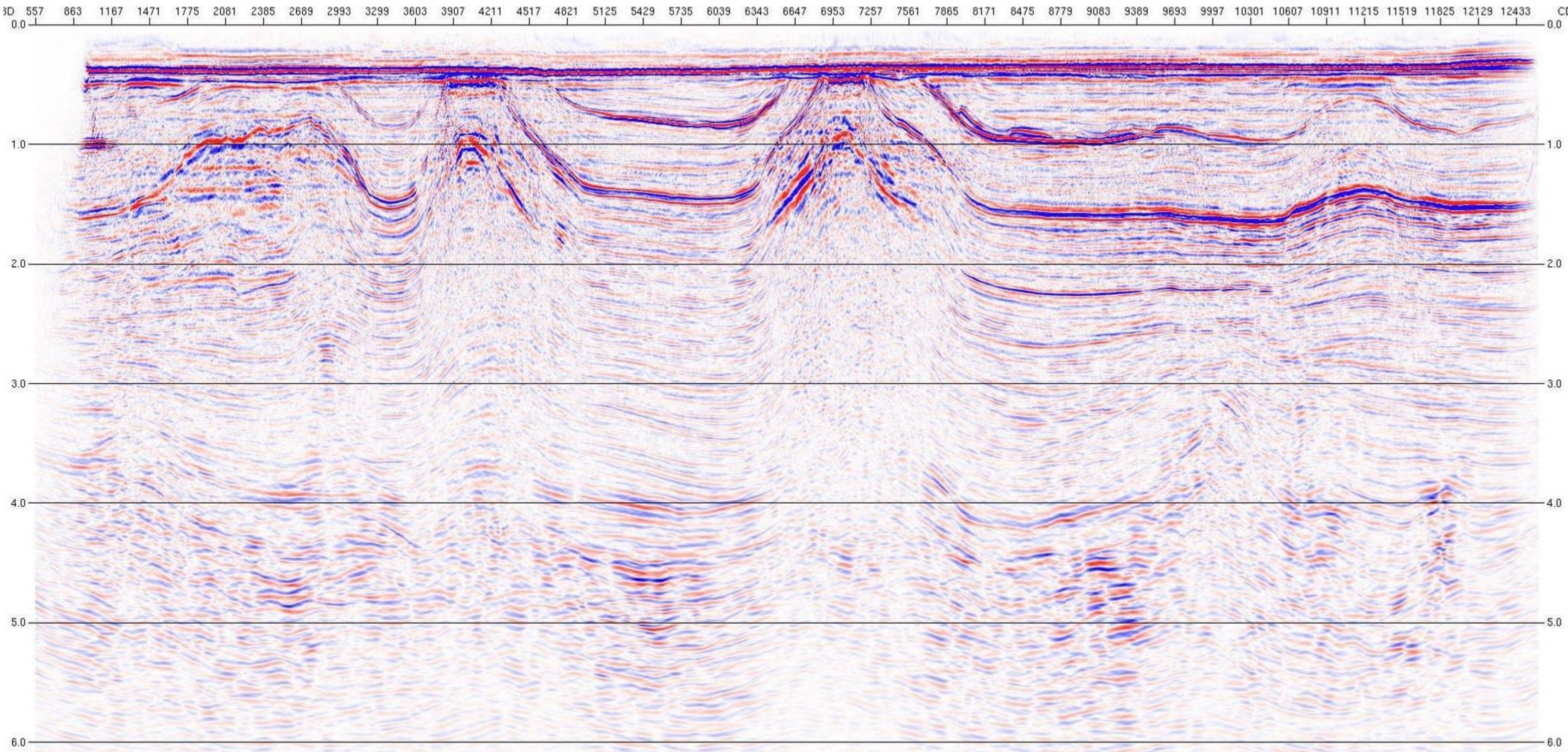


Full stack

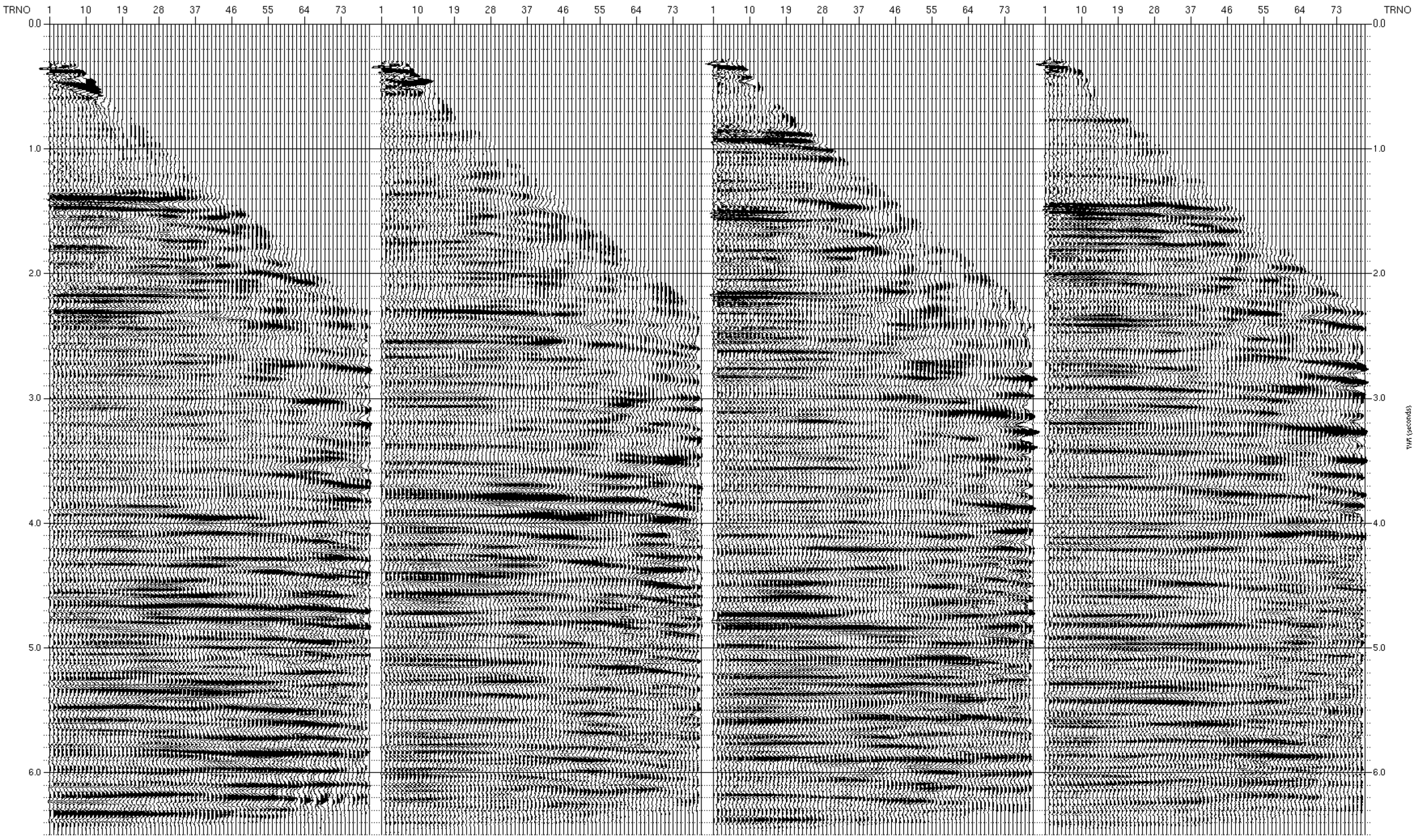
CIP gather

Stratigraphic trap with AVO Class 3 anomaly.

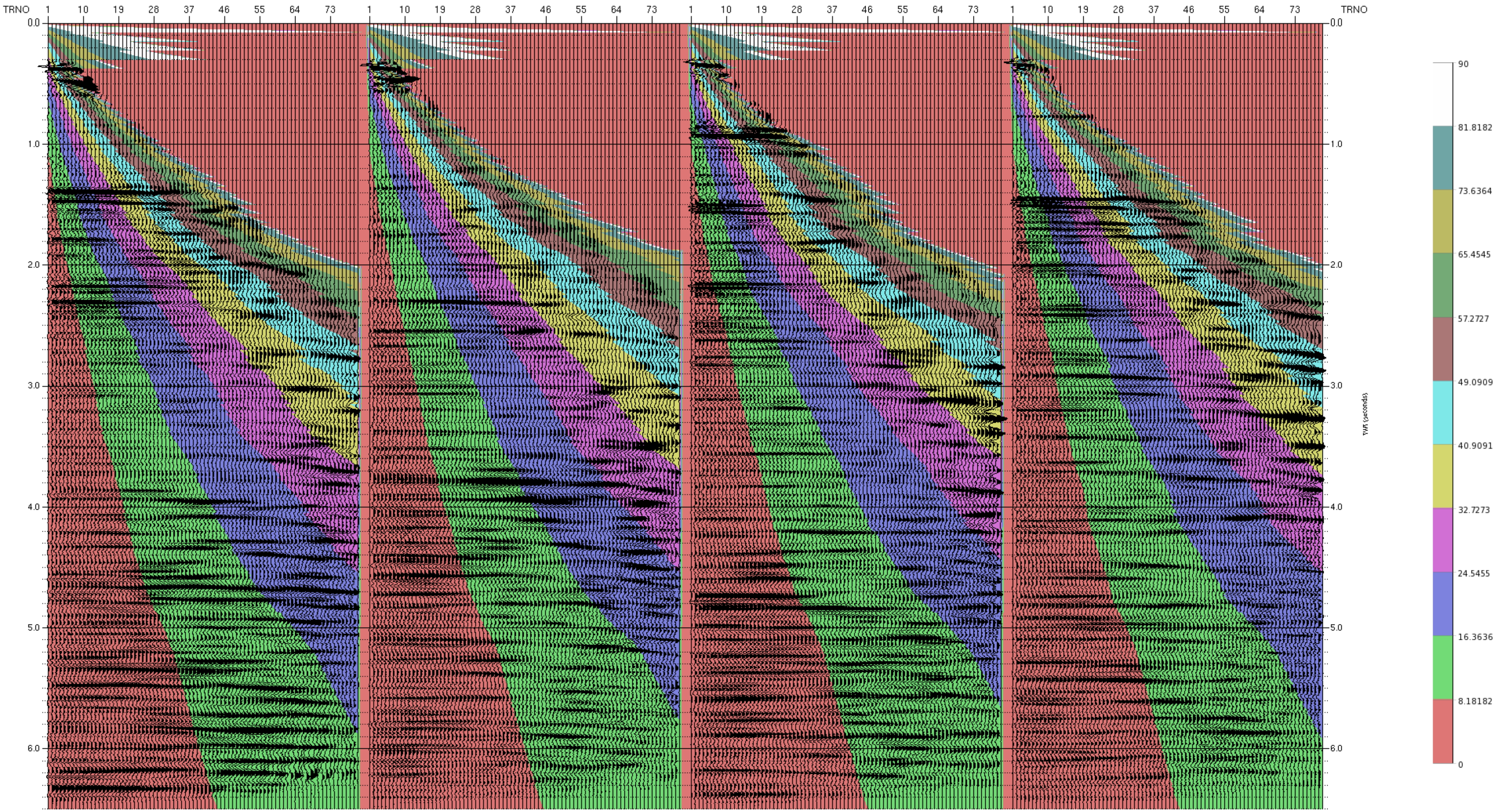
On-board processing: ST10011 Inline 1219
3D Kirchhoff migration



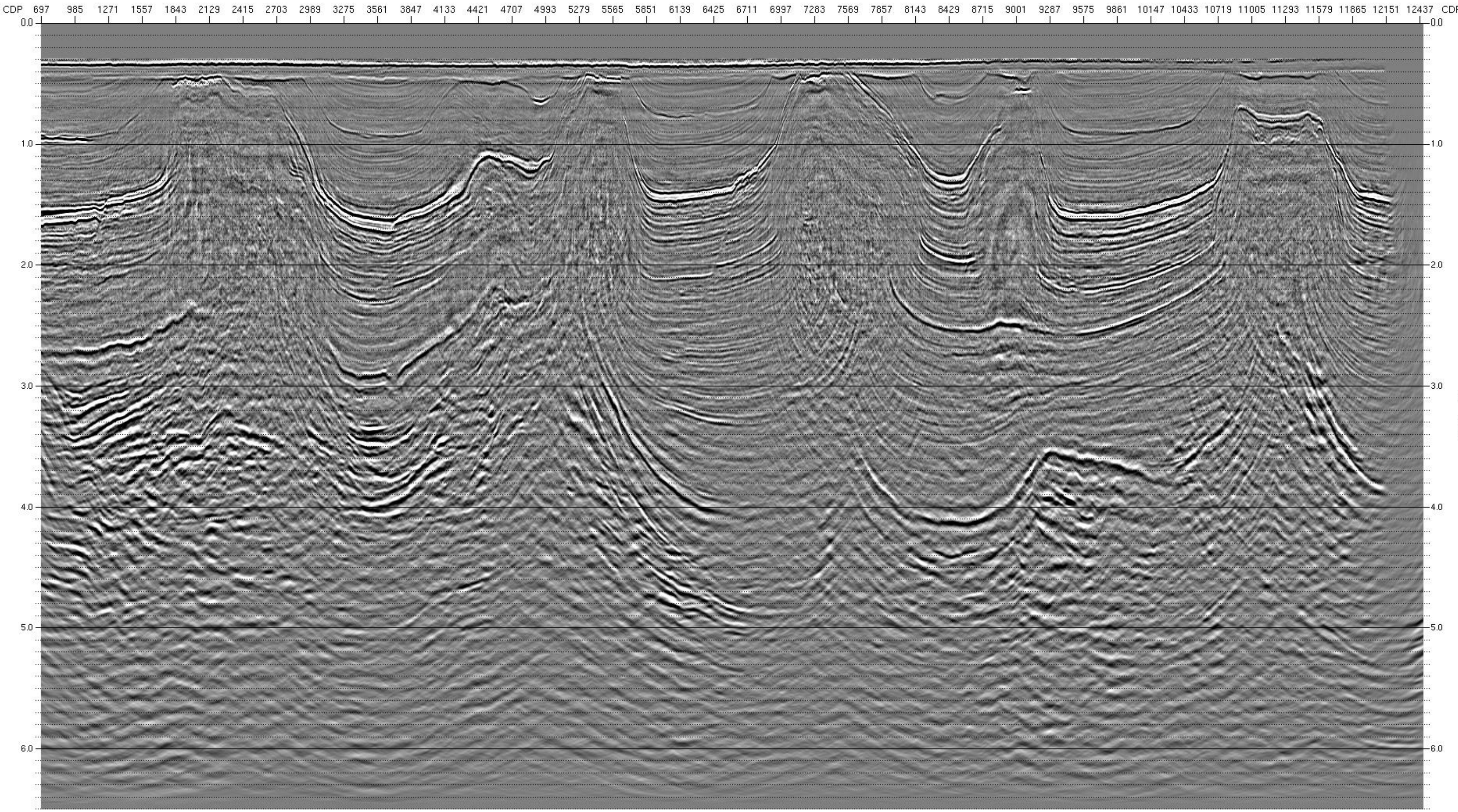
IL 1200 : CDP gathers



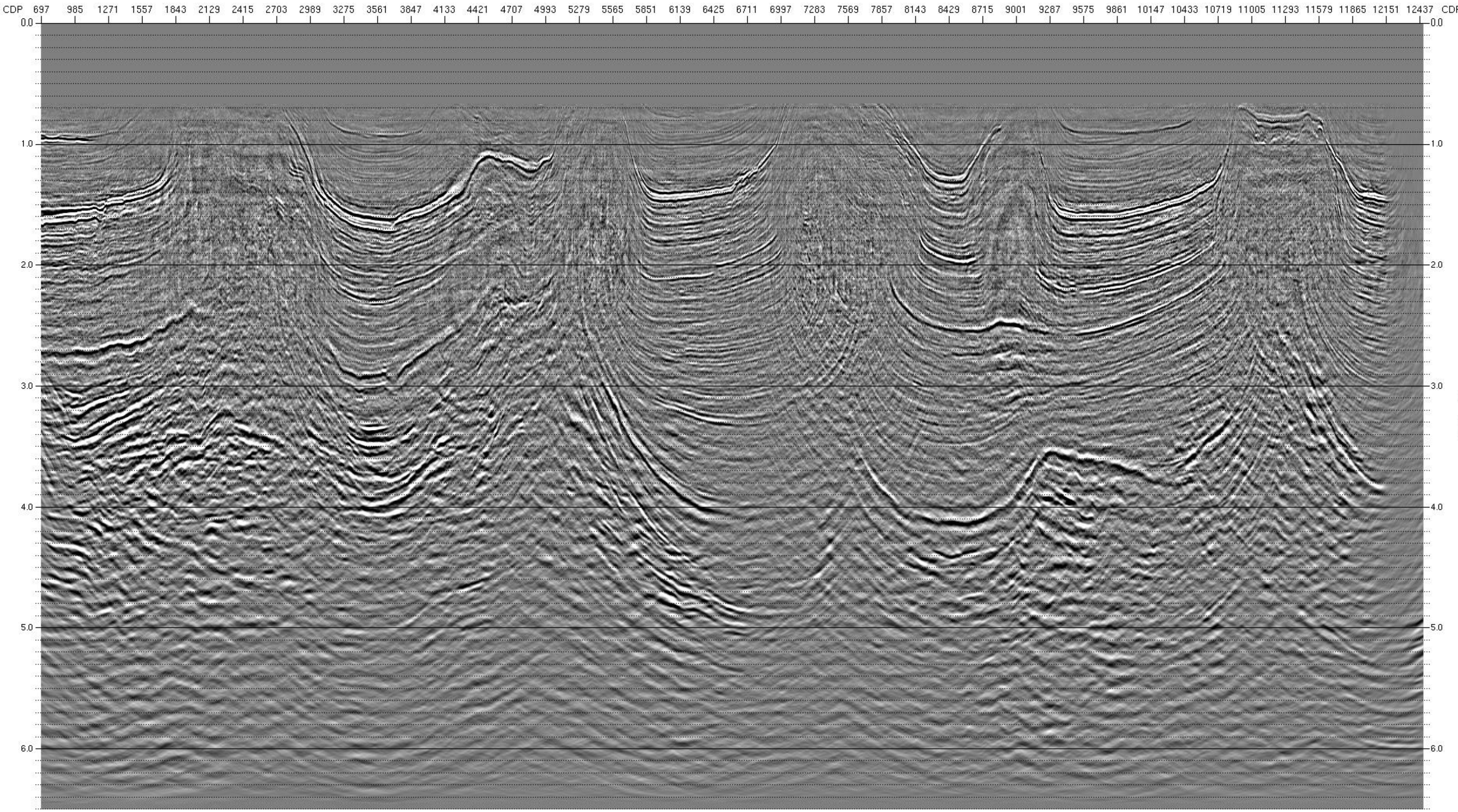
IL 1200 : CDP gathers with incidence angles overlaid



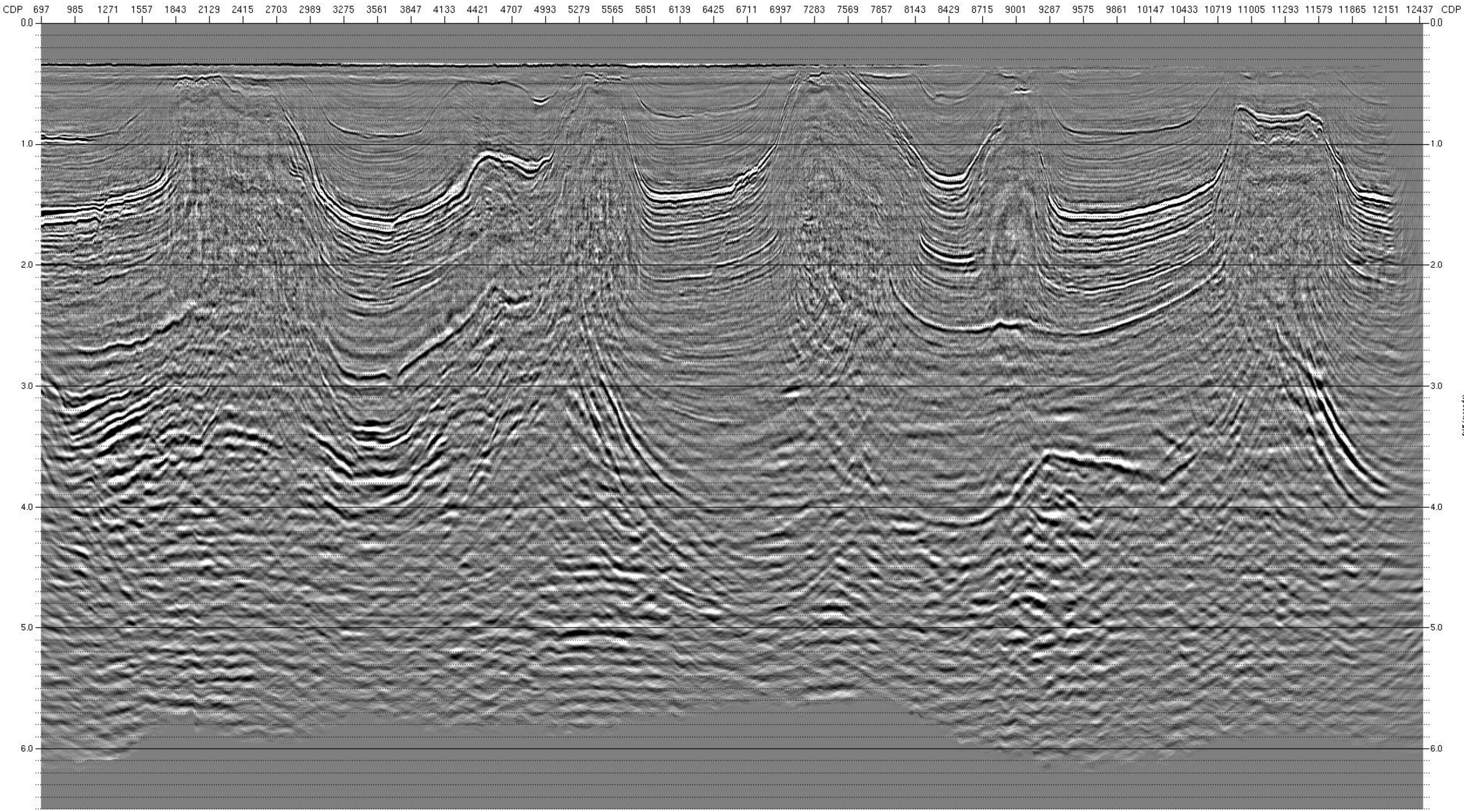
3D Kirchhoff time migration: IL 1400 : full stack after radon production using a time-offset dependant far offset mute



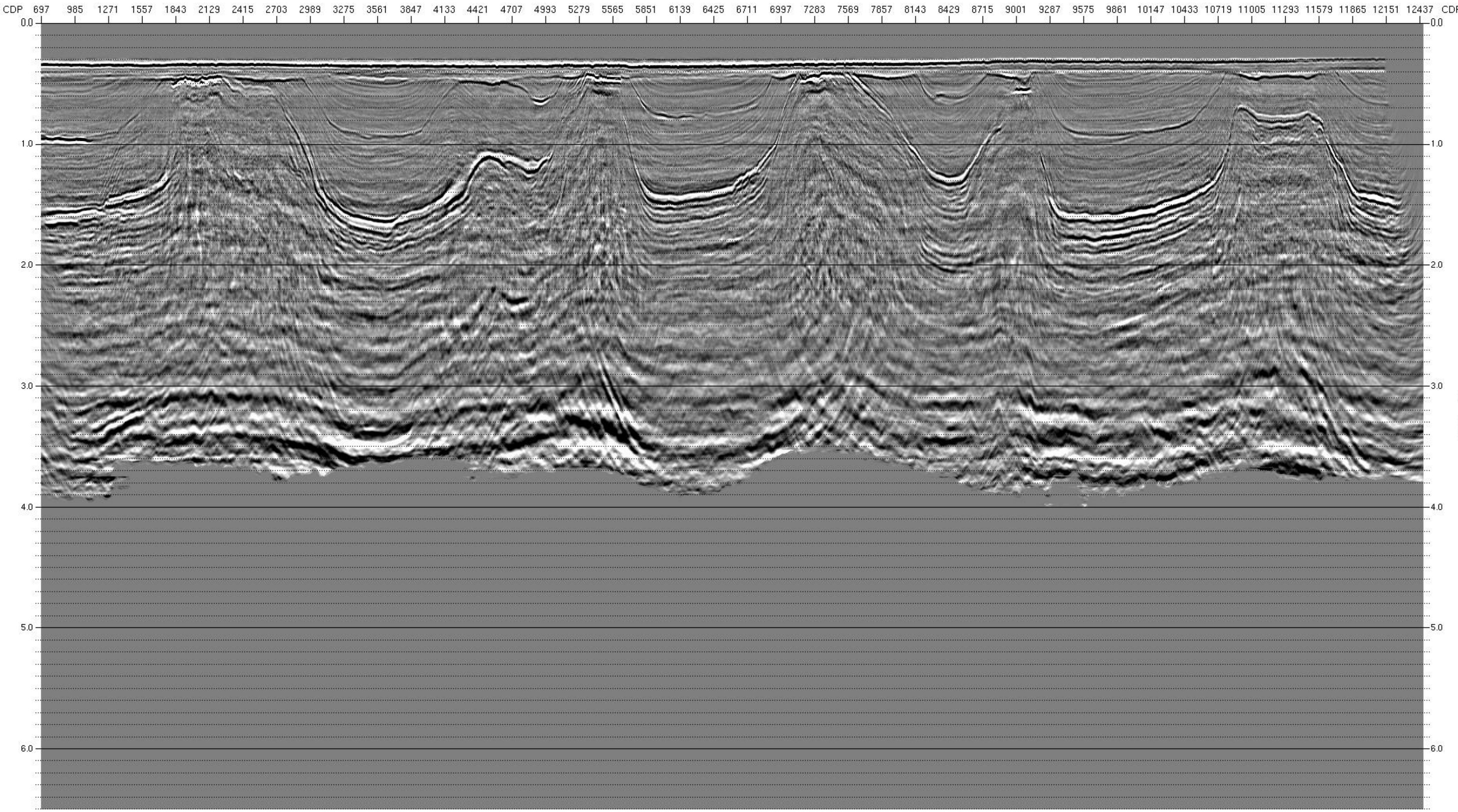
IL 1400 : Near angle stack



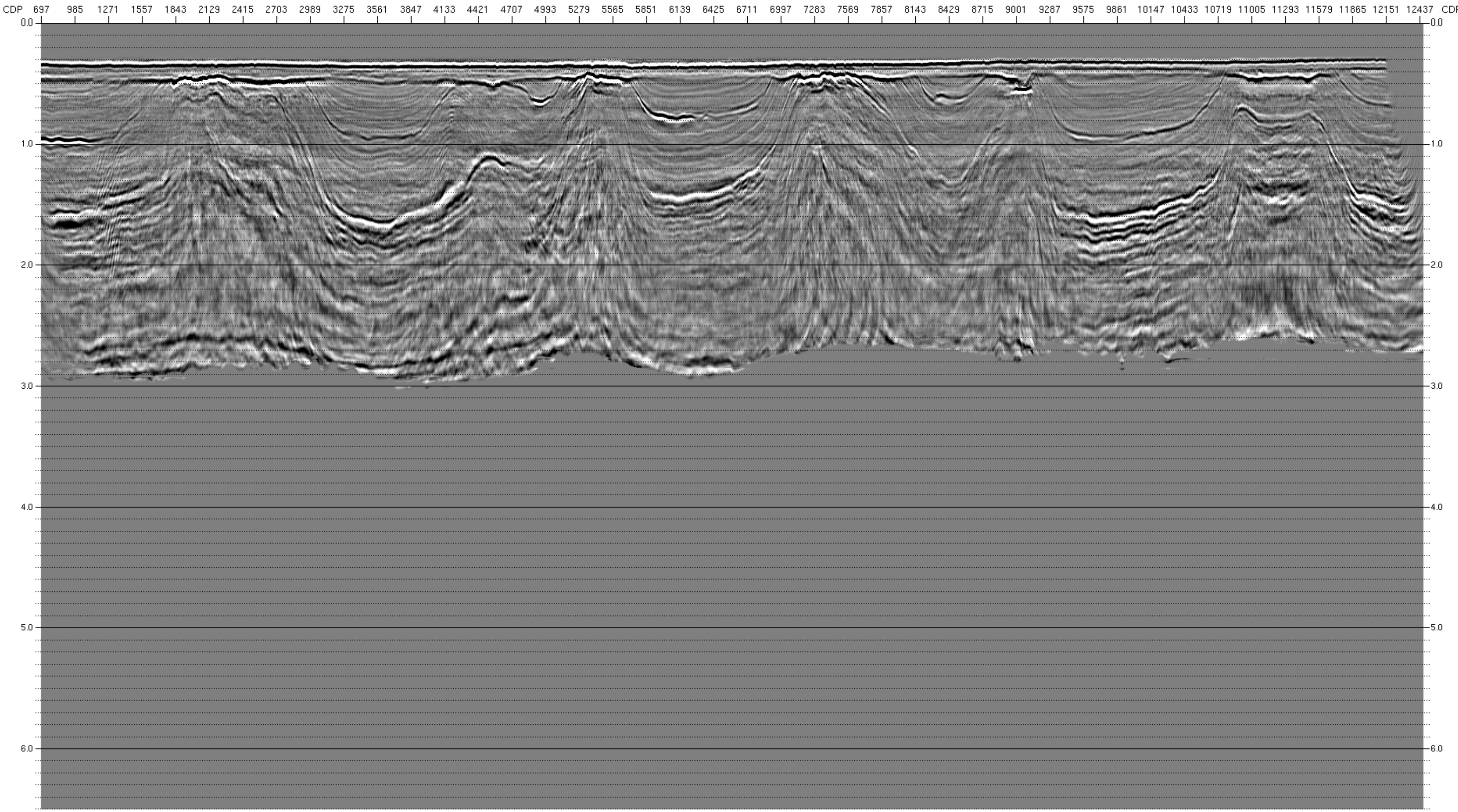
IL 1400 : Near-mid angle stack



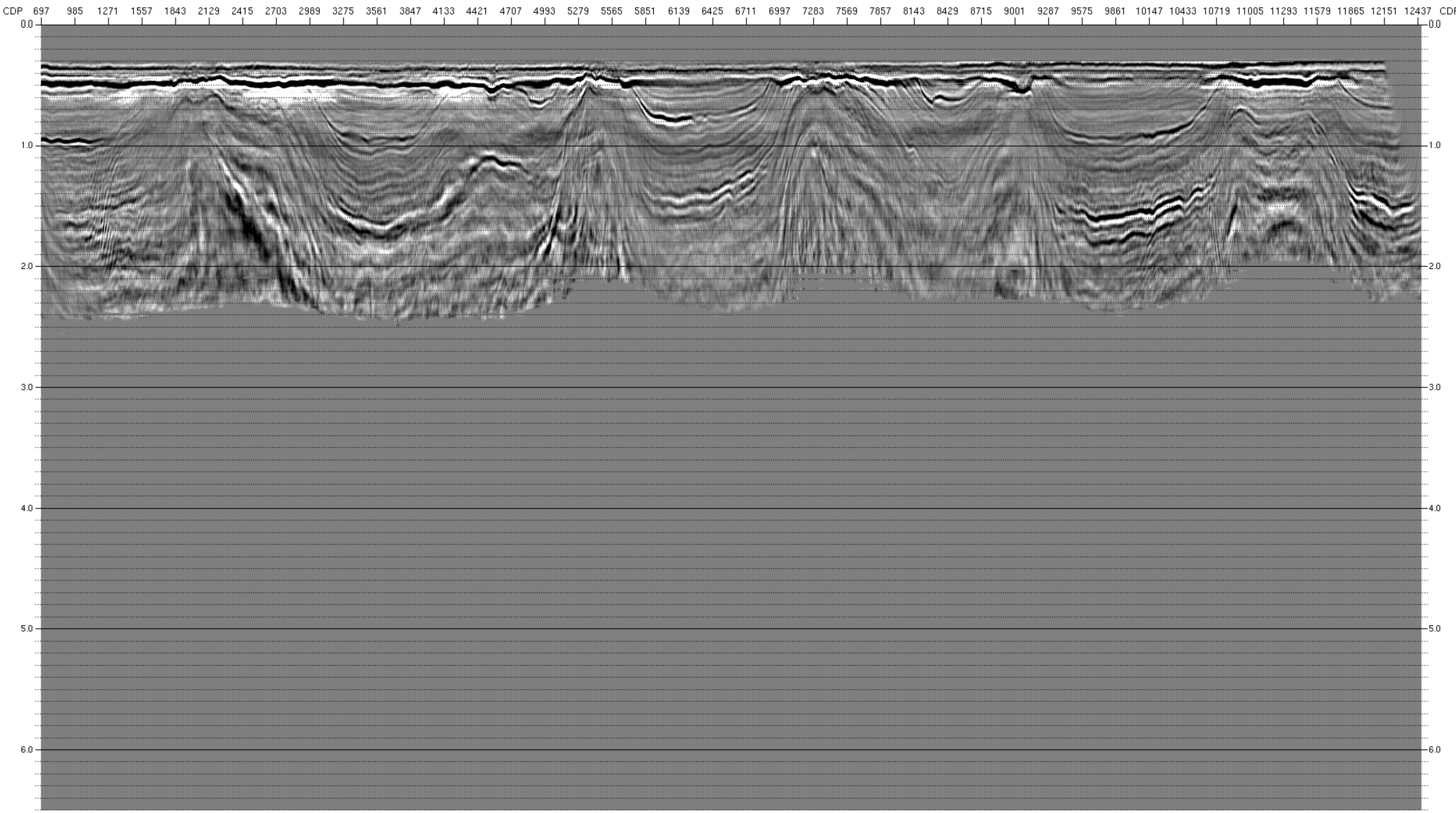
IL 1400 : Mid angle stack



IL 1400 : Mid-far angle stack



IL 1400 : Far angle stack



3D Kirchhoff time migration: IL 1400 : full stack after radon production using a time-offset dependant far offset mute