FK migration
FD migration

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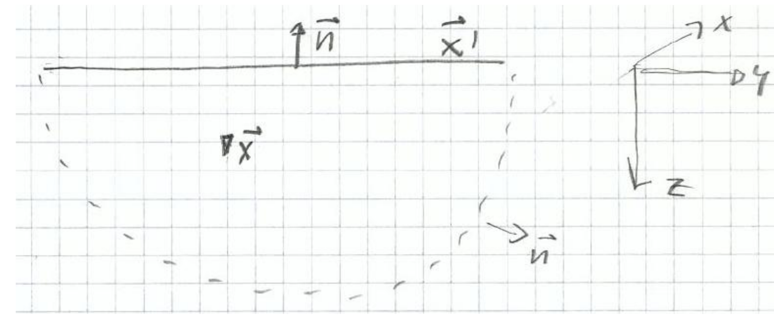
Kirchhoff integral recap



The Kirchhoff integral:

$$\begin{aligned} P(\mathbf{x},\omega) &= \int\int\int d^3x' G(\mathbf{x},\mathbf{x}')S(\mathbf{x}',\omega) \\ &+ \int\int dx'dy'[\partial_z' G(\mathbf{x},\mathbf{x}')P(\mathbf{x}') - G(\mathbf{x},\mathbf{x}')\partial_z' P(\mathbf{x}')] \end{aligned}$$

Body forces

boundary conditions

Broadband seismic: P and v_z recorded

$$i\omega\rho v_i(\mathbf{x}) = -\partial_i P(\mathbf{x}) \quad \text{(Newton)}$$

$$\begin{aligned} P(\mathbf{x},\omega) &= \int\int\int d^3x' G(\mathbf{x},\mathbf{x}')S(\mathbf{x}',\omega) \\ &- \int\int dx'dy'[\partial_z' G(\mathbf{x},\mathbf{x}')P(\mathbf{x}') + i\omega\rho(\mathbf{x}')G(\mathbf{x},\mathbf{x}')v_z(\mathbf{x}')] \end{aligned}$$

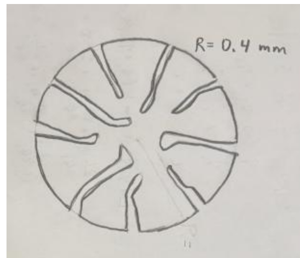
We can derive Kirchhoff integrals for acoustic and elastic wave equations, electric and magnetic fields (Maxwell), and diffusion equation

Diffusive intermezzo

Diffusion equation for a spherical particle

$$\frac{\partial C}{\partial t} - D\nabla^2 C = S$$

$$\nabla^2 C = \frac{1}{r}\frac{\partial^2}{\partial r^2}(rC)$$



$$\frac{\partial f(r,t)}{\partial t} - D\frac{\partial^2 f(r,t)}{\partial r^2} = J(r,t)$$

$$\begin{aligned} f &= rC \\ J &= rS \end{aligned}$$

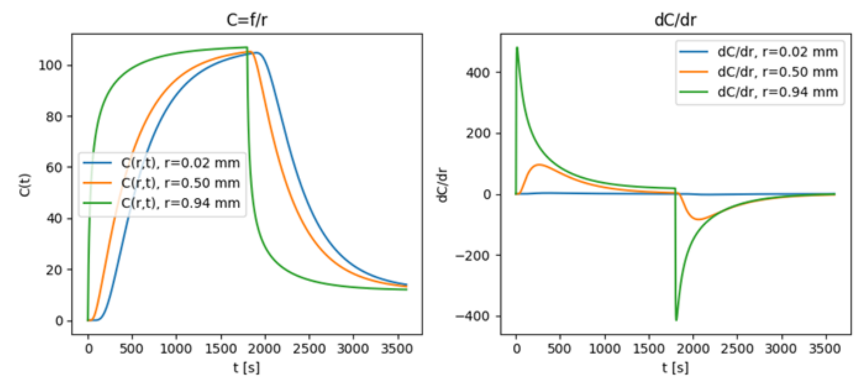
Green's function for diffusion

$$\frac{\partial G(r,t;r',t')}{\partial t} - D\frac{\partial^2 G(r,t;r',t')}{\partial r^2} = \delta(r-r')\delta(t-t')$$

$$G(r,t;r',t') = \sqrt{\frac{4\pi}{D(t-t')}}\left[e^{-\frac{(r-r')^2}{4D(t-t')}} - e^{-\frac{(r+r')^2}{4D(t-t')}}\right]$$

Kirchhoff integral (representation theorem) for diffusion

$$\begin{aligned} f(r,t) &= \int_0^R G(r,r',t) * J(r',t)\, dr' \\ &+ D\left[G(r,R,t) * \frac{\partial f(R,t)}{\partial R} - \frac{\partial G(r,R,t)}{\partial R} * f(R,t)\right] \\ &+ \cancel{\int_0^R f(r',0) G(r,r',t) dr'} \end{aligned}$$



Reference: Morse and Feshbach, 1953, Methods of Theoretical Physics, Ch. 7.

Basic equations: Seismic and EM

Newton:

$$\rho\partial_t^2 u_i = \rho a_i = -\partial_i P$$

Hooke:

$$\begin{aligned} P &= -M\partial_j u_j \\ \partial_t^2 P &= -M\partial_j a_j \end{aligned}$$

Wave equation:

$$\partial_i\partial_i P - \frac{1}{v^2}\partial_t^2 P = 0$$

$$\partial_i\partial_i P + \frac{\omega^2}{v^2} P = 0$$

$$v = \sqrt{M/\rho}$$

Maxwell:

$$\begin{aligned} \nabla \times \mathbf{E} &= -\mu_0\partial_t\mathbf{H} \\ \nabla \times \mathbf{H} &= \mathbf{j}^{\mathbf{E}} \end{aligned}$$

Ohm's law:

$$\mathbf{j}^E = \sigma\mathbf{E}$$

Vector manipulation:

$$\begin{aligned} \nabla \times (\nabla \times \mathbf{E}) &= -\mu_0\sigma\partial_t\mathbf{E} \\ \nabla \times (\nabla \times \mathbf{E}) &= \nabla(\nabla \cdot \mathbf{E}) - \nabla^2\mathbf{E} \\ \nabla \cdot \mathbf{E} &= 0 \end{aligned}$$

(Anisotropic: $\nabla \cdot \mathbf{j} = \partial_i(\sigma_{ij}E_j) = 0$)

Diffusive "wave equation" (isotropic):

$$\partial_j\partial_j E_i - \mu_0\sigma\partial_t E_i = 0$$

$$\partial_j\partial_j E_i + i\omega\mu_0\sigma E_i = 0$$

Helmholtz equation

$$\partial_i \partial_i F + \kappa_0^2 F = 0$$

Seismic:

$$\begin{aligned} F &= P \\ \kappa_0 &= \frac{\omega}{v} \end{aligned}$$

EM:

$$\begin{aligned} F &= E_i \\ \kappa_0^2 &= i\omega\mu_0\sigma \\ \kappa_0 &= \left(\frac{1+i}{\sqrt{2}}\right)\sqrt{\omega\mu_0\sigma} \end{aligned}$$

Fourier transform

FK-domain: $F(z, k_x, k_y, \omega)$

Forward transform:

$$F(\mathbf{k}, \omega) \quad = \quad \frac{1}{(\sqrt{2\pi})^4} \int\int\int \left[\int F(\mathbf{x}, t) e^{-i(\mathbf{k}\cdot\mathbf{x} - \omega t)} dt \right] dx\, dy\, dz$$

Inverse transform:

$$F(\mathbf{x}, t) \quad = \quad \frac{1}{(\sqrt{2\pi})^4} \int \left[\int\int\int F(\mathbf{k}, \omega) e^{i(\mathbf{k}\cdot\mathbf{x} - \omega t)} dk_x\, dk_y\, dk_z \right] d\omega$$

Be aware of sign conventions

FK-domain

Derivatives:

$$\partial_t \to -i\omega \qquad \partial_j \to ik_j = i\omega p_j$$

Helmholtz in the FK domain:

$$\partial_z^2 + k_z^2 F = 0$$

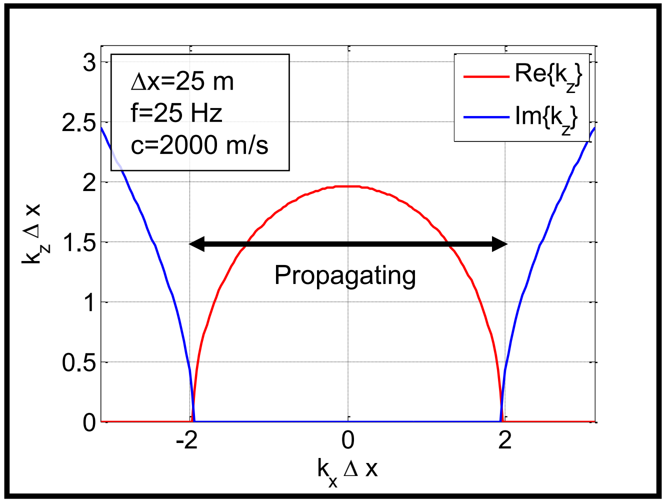
$$k_z = \sqrt{\kappa_0^2 - k_x^2 - k_y^2}$$

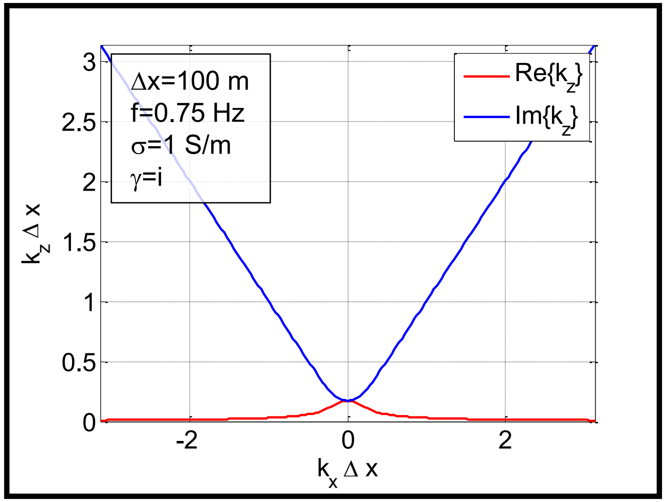
Two solutions:

$$F(z) = e^{\pm i k_z z} F(0)$$

One-way equations satisfied by F:

$$\begin{aligned} \partial_z D &= ik_z D \text{ (downgoing waves)} \\ \partial_z U &= -ik_z U \text{ (upgoing waves)} \end{aligned}$$





Dispersion relations:
Seismic (top) and EM (bottom)

Gazdag migration

Depth stepping of U and D:

$$\begin{aligned} D(z+\Delta z, k_x, k_y) &= e^{ik_z(z)\Delta z} D(z, k_x, k_y) \\ U(z+\Delta z, k_x, k_y) &= e^{-ik_z(z)\Delta z} U(z, k_x, k_y) \end{aligned}$$

Vertical wavenumber (positiv z downwards)

$$k_z(z) = \sqrt{\kappa_0^2(z) - k_x^2 - k_y^2}$$

Imaging condition:

$$\begin{aligned} I(\mathbf{x}) &= \int d\omega \frac{U(\mathbf{x},\omega)}{D(\mathbf{x},\omega)} \\ I(\mathbf{x}) &= \int d\omega \; \frac{U(\mathbf{x},\omega) D^*(\mathbf{x},\omega)}{D(\mathbf{x},\omega) D^*(\mathbf{x},\omega) + \epsilon} \end{aligned}$$

Inverse FFT from (k_x, k_y) to (x, y):

$$\begin{aligned} D(\mathbf{x},\omega) &= FFT^{-1}[D(z,k_x,k_y,\omega)] \\ U(\mathbf{x},\omega) &= FFT^{-1}[U(z,k_x,k_y,\omega)] \end{aligned}$$

$I(\mathbf{x})$ is related to gradient and Hessian, as we will see later

Claerbout's 15-degree equation

Gazdag migration:

$$\partial_z F = \pm i\sqrt{\kappa_0^2 - k_x^2 - k_y^2 F}$$

Inverse Fourier transform:

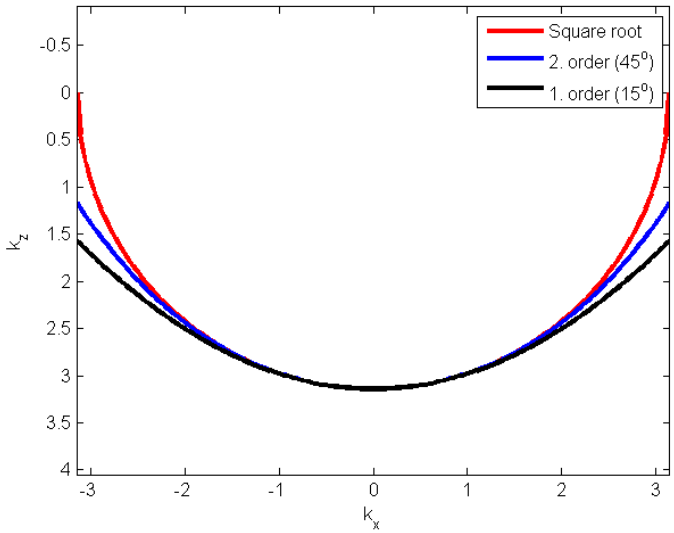
$$\partial_z F = \pm i\sqrt{\kappa_0^2 + \partial_x^2 + \partial_y^2 F}$$

Series expansion: 15-degree equation

$$\partial_z F \simeq \pm i\kappa_0 \left(1 + \frac{\partial_x^2 + \partial_y^2}{2\kappa_0^2}\right) F$$

«Schrödinger equation»:

$$-i\partial_z F \simeq \pm\left(\kappa_0 F + \frac{\partial_x^2 F + \partial_y^2 F}{2\kappa_0}\right)$$



45° and 60° equations can be developed along the the same lines

Explicit one-way FD migration «Wave-equation migration»

Gazdag:

$$F(z+\Delta z, k_x, k_y) \quad = \quad e^{\gamma \Delta z \sqrt{\kappa_0^2 - k_x^2 - k_y^2}} F(z, k_x, k_y)$$

Gazdag FK migration works for (approximately) 1D media
We want to get around this limitation

Rewrite in terms of normalized wave numbers:

$$F(z+\Delta z, k_x, k_y) \quad = \quad e^{\gamma \frac{\Delta z}{\Delta x} \sqrt{\hat{\kappa_0}^2 - \hat{k}_x^2 - \hat{k}_y^2}} F(z, k_x, k_y)$$

Normalized wavenumbers:

$$\begin{aligned} \hat{\kappa}_0 &= \Delta x \kappa_0 \\ \hat{k}_x &= \Delta x k_x \\ \hat{k}_y &= \Delta y k_y = \Delta x k_y \end{aligned}$$

$$\begin{aligned} k_x^{Nyq} &= \frac{\pi}{\Delta x} \\ \hat{k}_x^{Nyq} &= \pi \end{aligned}$$

und zu weiter

Lateral velocity variations

Assume a 3D medium: $\kappa_0 = \kappa_0(x, y, z)$.
Multiplication in FK fomain $\Leftrightarrow$ convolution in FX domain
Replace the phase shift operator by a discrete convolutional filter:

$$F(x, y, z + \Delta z) = \sum_{m,n=-L}^{L} W(m\Delta x, n\Delta y, \hat{\kappa}_0, g) F(x - m\Delta x, y - n\Delta y, z)$$

The complex filter W depends on

$$\begin{aligned} \hat{\kappa}_0 &= \kappa_0 \Delta x \\ g &= \frac{\Delta z}{\Delta x} \end{aligned}$$

Wavefield extrapolation filter

Compute $W(m\Delta x, n\Delta y, \hat{\kappa}_0, g)$ by minimization of

$$\begin{aligned} J &= ||W(i\Delta\hat{k}_x, j\Delta\hat{k}_y, \hat{\kappa}_0, g) - e^{\gamma\frac{\Delta z}{\Delta x}\hat{k}_z}||^4 \\ \hat{k}_z &= \sqrt{\hat{\kappa}_0^2 - (i\Delta\hat{k}_x)^2 - (j\Delta\hat{k}_y)^2} \end{aligned}$$

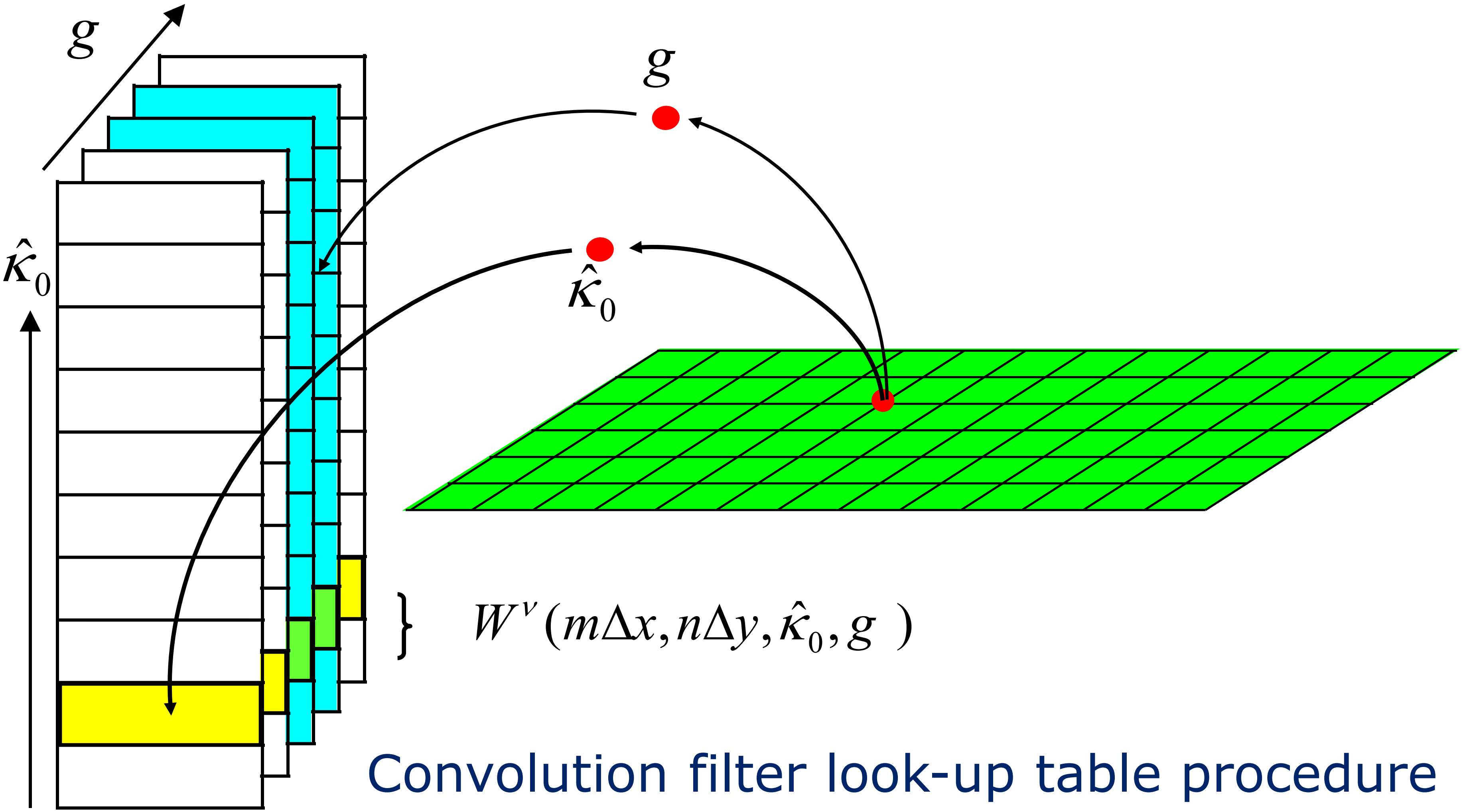
where

$$W(i\Delta\hat{k}_x, j\Delta\hat{k}_y, \hat{\kappa}_0, g) = DFT[W(m\Delta x, n\Delta y, \hat{\kappa}_0, g]$$

- Create look-up tables for *W*
- Can be computed once and for all

- Max propagation angle depends on operator length *L*

Illustration of 3D FD Depth Migration Scheme

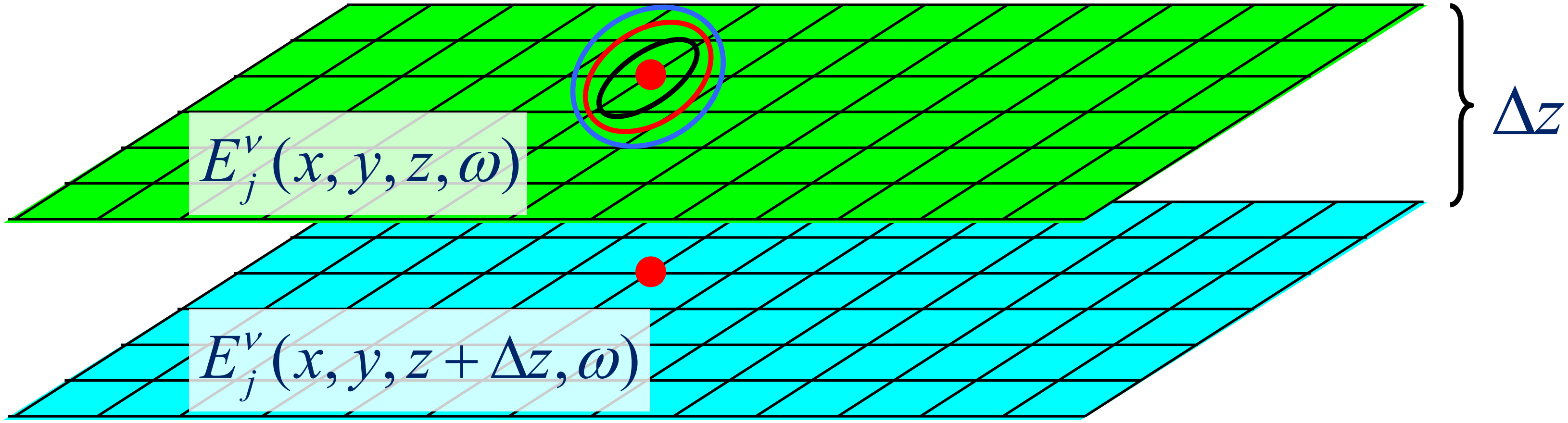


Convolution filter look-up table procedure

Illustration of 3D FD Depth Migration Scheme

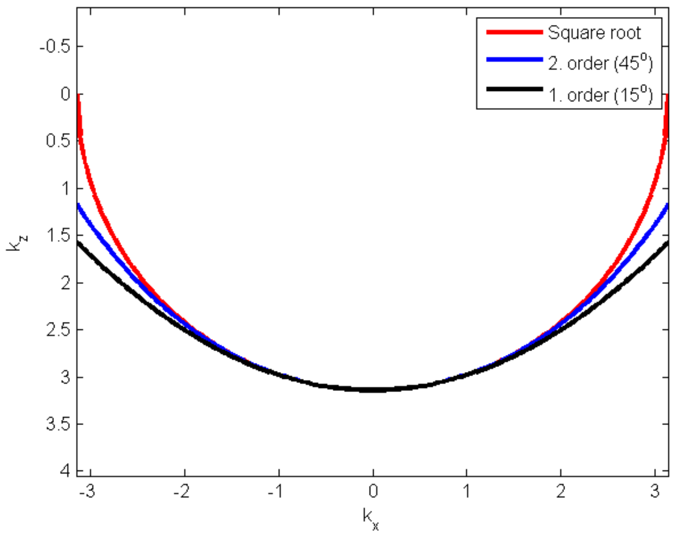
$$E^{\nu}_{(m)}(x,y,z+\Delta z,\omega)=\sum_{m,n=-L}^{L}\left[W^{\nu}(m\Delta x,n\Delta y,\hat{\kappa}_0,g)\right]$$

$$\times E^{\nu}_{(m)}(x-m\Delta x,y-n\Delta y,z,\omega)$$



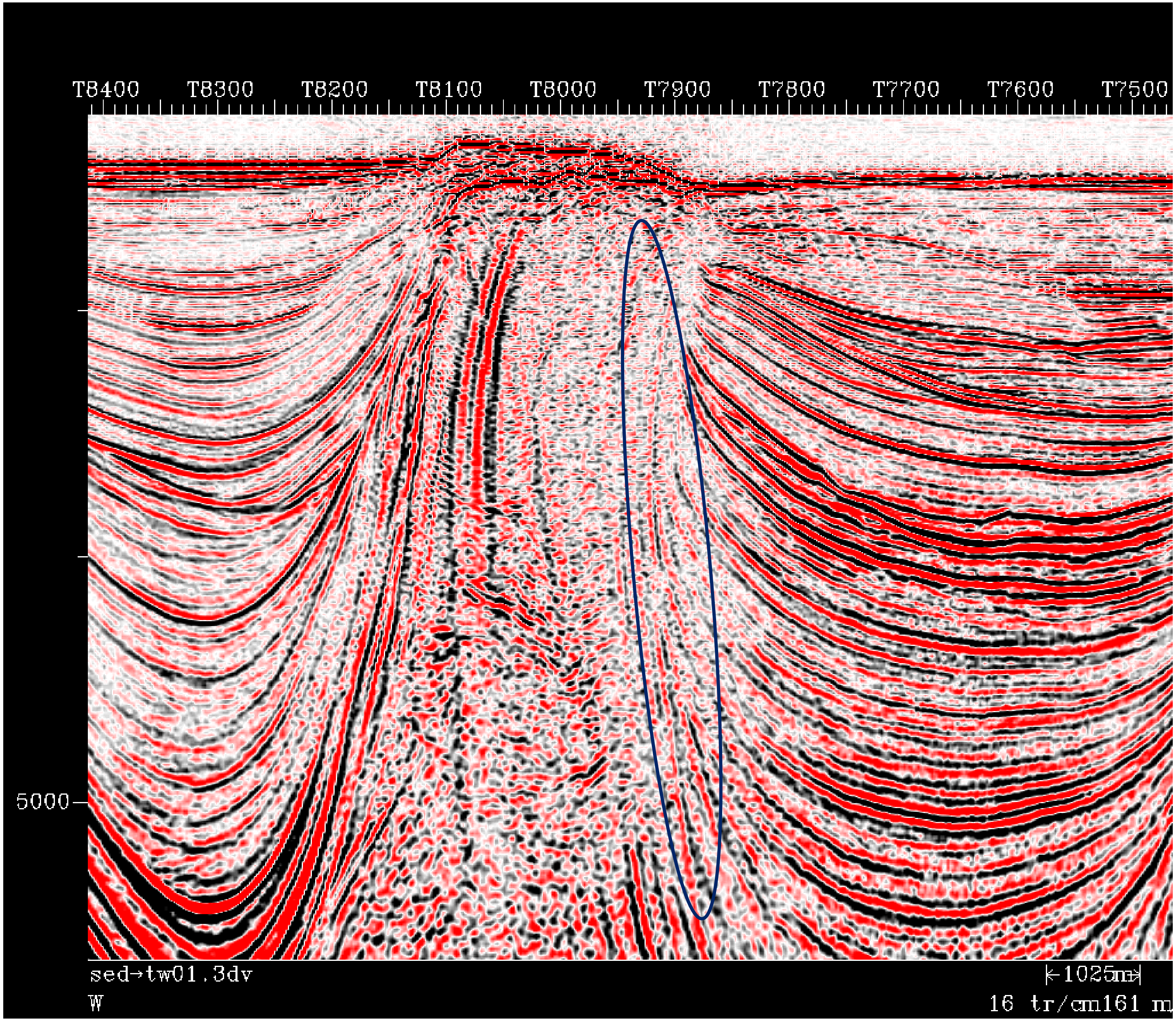
Wave field extrapolation by filter convolution

FK and FD migration summary



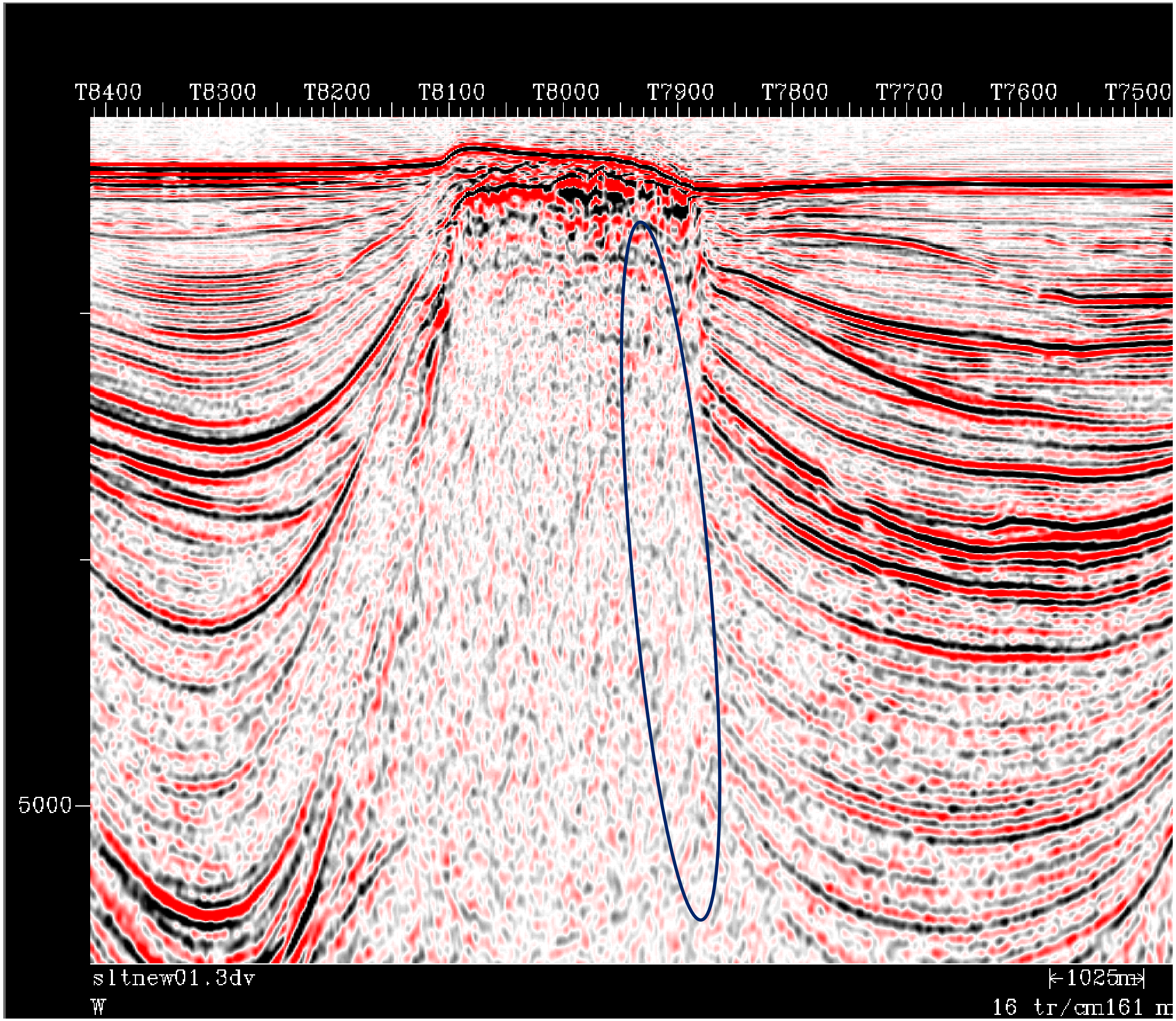
- FK migration:
  - Accurate up to 90 degrees dip
  - 1D medium: v(z)

- Explicit FD migration:
  - Dip accuracy depends on operator length (60 degree is common)
  - 3D medium: v(x,y,z)



TW Kirchhoff – sediment only velocity model – Line 10825

STATOIL



Wave Equation – salt velocity model – Line 10825

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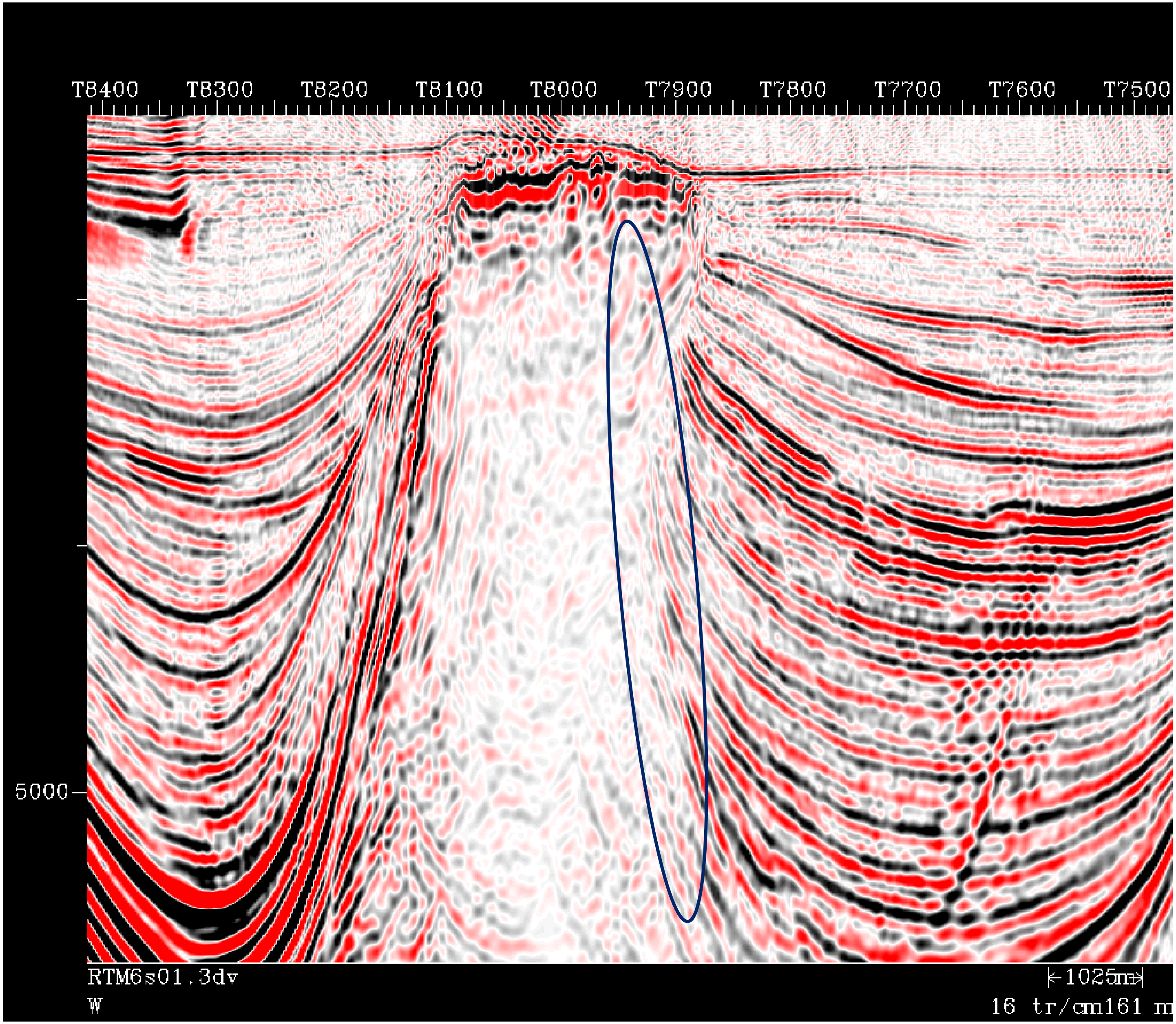

RTM – salt velocity model – Line 10825

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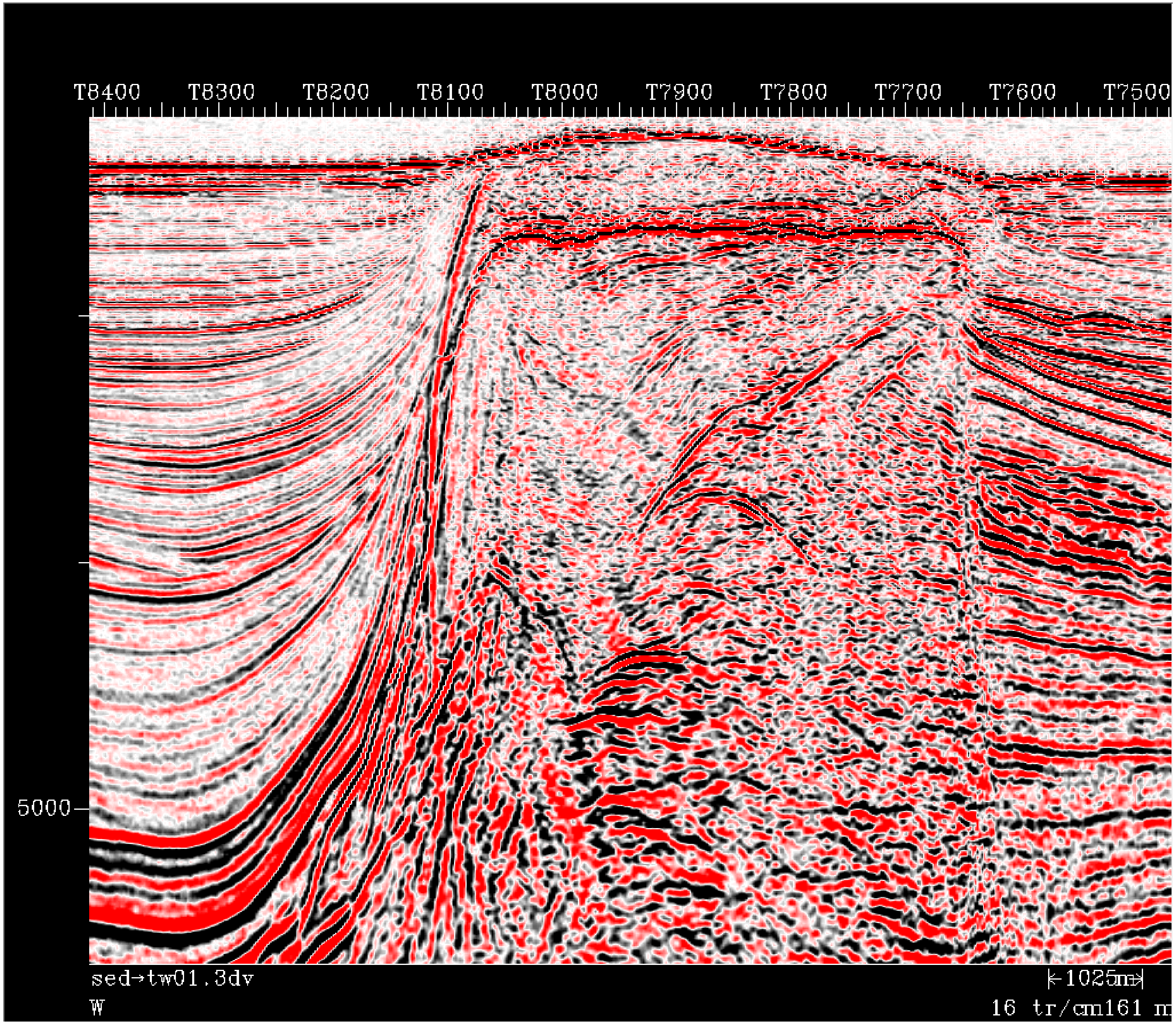

TW Kirchhoff – sediment only velocity model – Line 11050

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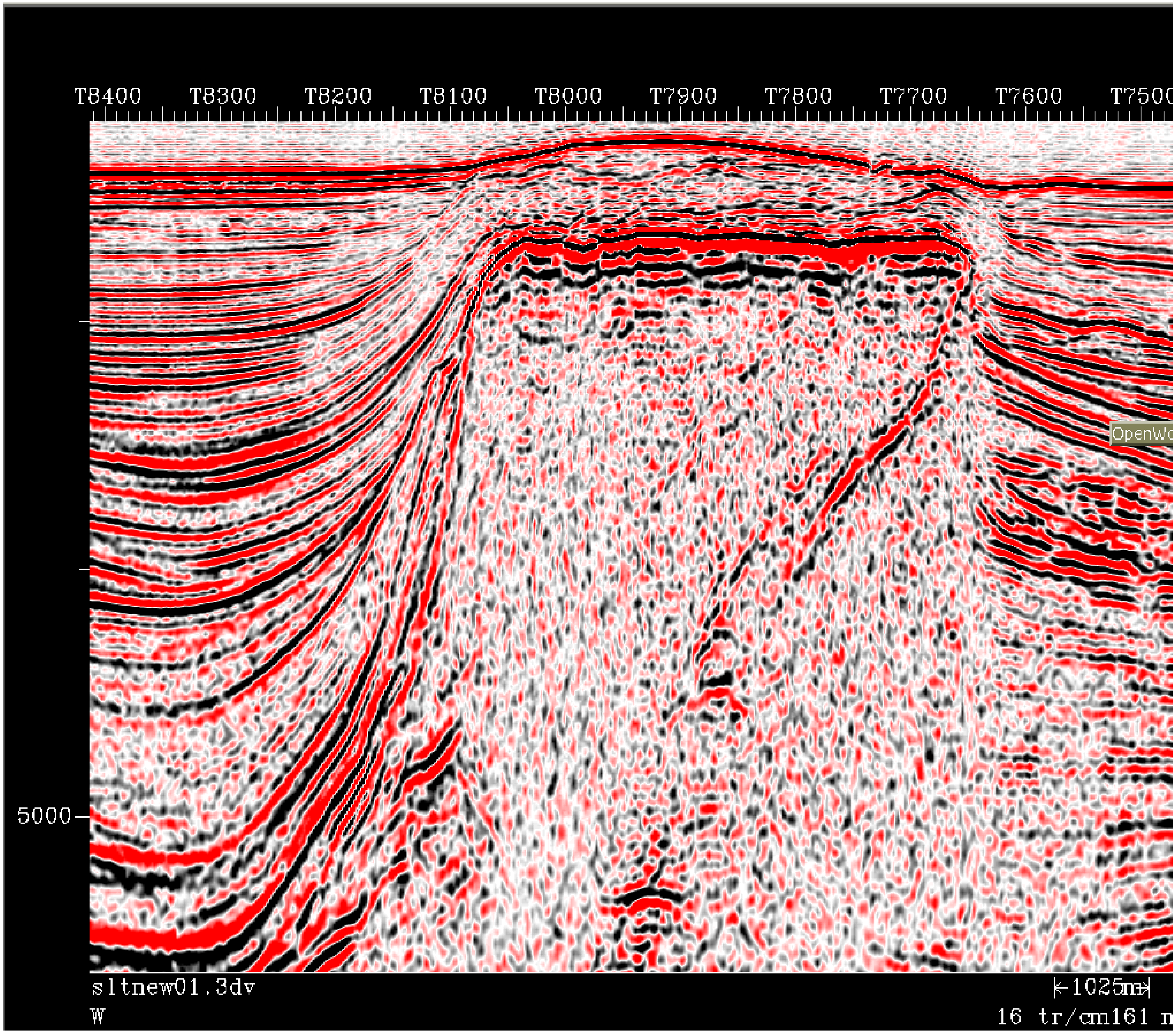
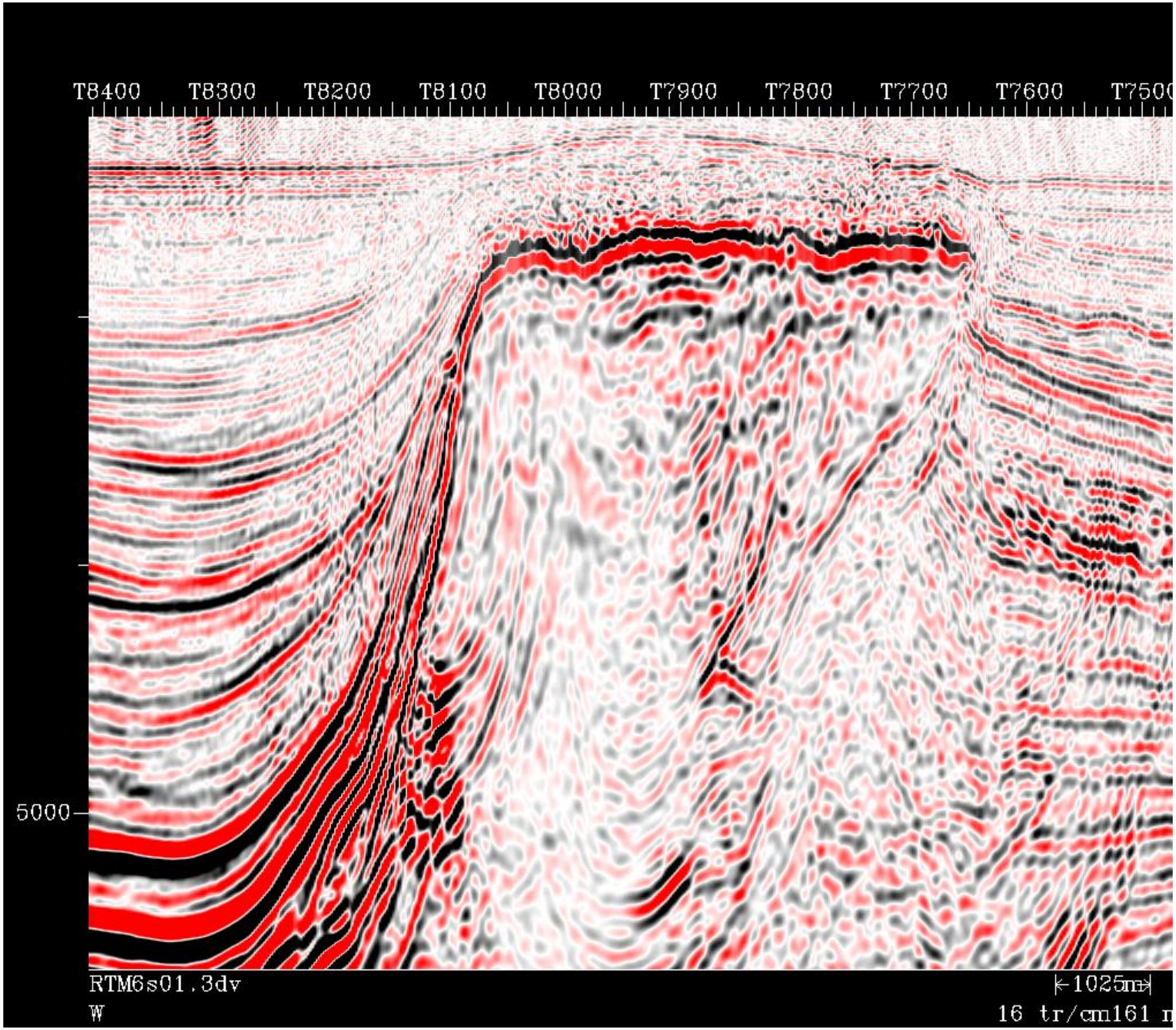


Wave Equation – salt velocity model – Line 11050

STATOIL



RTM – salt velocity model – Line 11050