

PG8108 course topics

Seismic depth migration

- Kirchhoff integral
- Sources and boundary conditions
- Kirchhoff family of migrations
 - Kirchhoff migration
 - Gaussian beam
 - Angle migration (GRT)
 - Demigration/remigration
- One-way migrations
 - Gazdag migration
 - Explicit FD migration
- Least-squares migration
- Reverse-time migration

Ray theory

- Hamilton's equations
- Kinetic ray-tracing system
- Dynamic ray-tracing system

Linear inversion

- Seismic tomography
- Bayesian AVO inversion
- Gravity and magnetic inversion

Machine learning

- Basic concepts
- ML vs inversion
- Neural networks

Non-linear inversion

- Gradient and Hessian
- Gauss Newton (magnetic)
- Steepest decent
- Conjugate gradient
- Least squares
- Minimum norm
- Contrast-source inversion
- Sparse inversion
- EM inversion (CSEM and MT)
- Seismic full-waveform inversion
- Elastic FD modeling; staggering
- Joint inversion

Kirchhoff integral Kirchhoff migration

KetilHokstad, January 2016

Helmholtz equation and Kirchhoff integral

Wave equation and Helmholtz equation:

$$\partial_i \partial_i P(x, t) - \frac{1}{v^2} \partial_t^2 P(x, t) = S(x, t)$$

$$\partial_i \partial_i P(x, \omega) + \frac{\omega^2}{v^2} P(x, \omega) = S(x, \omega)$$

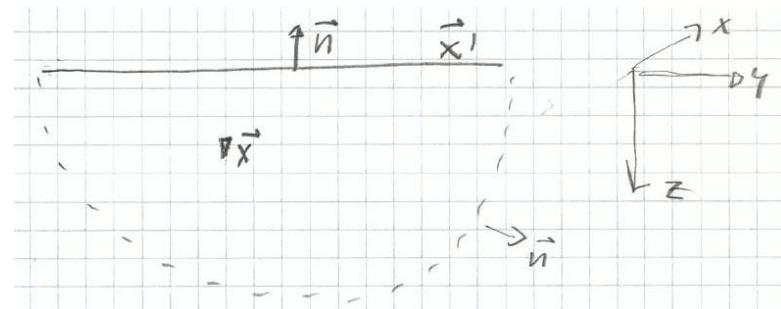
Green's function (omitting ω):

$$\partial_i \partial_i G(x, x') + \frac{\omega^2}{v^2} G(x, x') = \delta(x - x')$$

The Kirchhoff integral:

$$P(x, \omega) = \int \int \int d^3 x' G(x, x') S(x', \omega) + \int \int dS' [G(x, x') \partial'_i P(x') - P(x') \partial'_i G(x, x')] n_i$$

$$S(x) = \delta(x - x_s) S(\omega), \quad (\text{point source; body force})$$



Volume integral; body forces

Surface integral;
boundary conditions

$$\nabla^2 P(x) + k_0^2 P(x) = -S(x)$$

$$\nabla^2 \zeta(x, x') + k_0^2 \zeta(x, x') = -\delta(x - x')$$

$$\zeta \nabla^2 P + \zeta k_0^2 P = -\zeta S$$

$$P \nabla^2 \zeta + P k_0^2 \zeta = -P \delta$$

$$\zeta \nabla^2 P - P \nabla^2 \zeta + \zeta k_0^2 P - P k_0^2 \zeta = -(\zeta S - P \delta)$$

$$\int P(x') \delta(x - x') d^3x' = \int \zeta(x, x') S(x') d^3x'$$

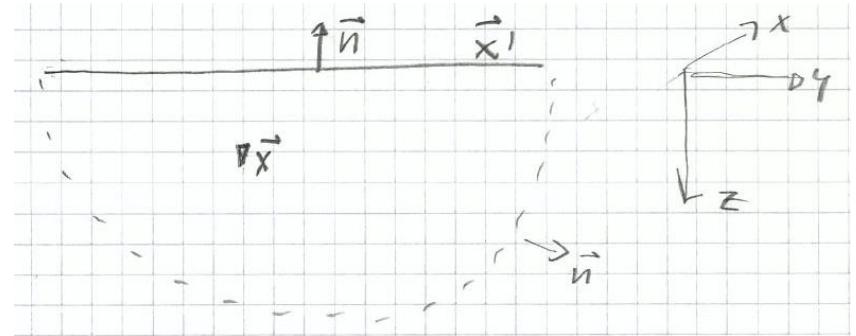
$$+ \int \left[\zeta(x, x') \nabla'^2 P(x') - P(x') \nabla'^2 \zeta(x, x') \right] d^3x'$$

$$P(x) = \int G(x, x') S(x') d^3x'$$

$$+ \left[\left[G(x, x') \nabla' P(x') - P(x') \nabla' G(x, x') \right] \right] d^3x'$$

$$\begin{aligned} \nabla(G \nabla P - P \nabla G) &= \cancel{\nabla G \nabla P} + G \nabla^2 P \\ &\quad - \cancel{\nabla P \nabla G} - P \nabla^2 G \\ &= G \nabla^2 P - P \nabla^2 G \end{aligned}$$

Acoustic wave equation and Kirchhoff integral



- Conventional surface seismic: Only P is recorded
=> we know only half the boundary condition
- This is OK if we know which side of surface S the wave field is coming from

Then, assuming S is the horizontal surface at $z = 0$:

$$P(x, \omega) = \int \int \int d^3 x' G(x, x') S(x', \omega) - 2 \int \int dx' dy' P(x') \partial_i' G(x, x') n_i$$

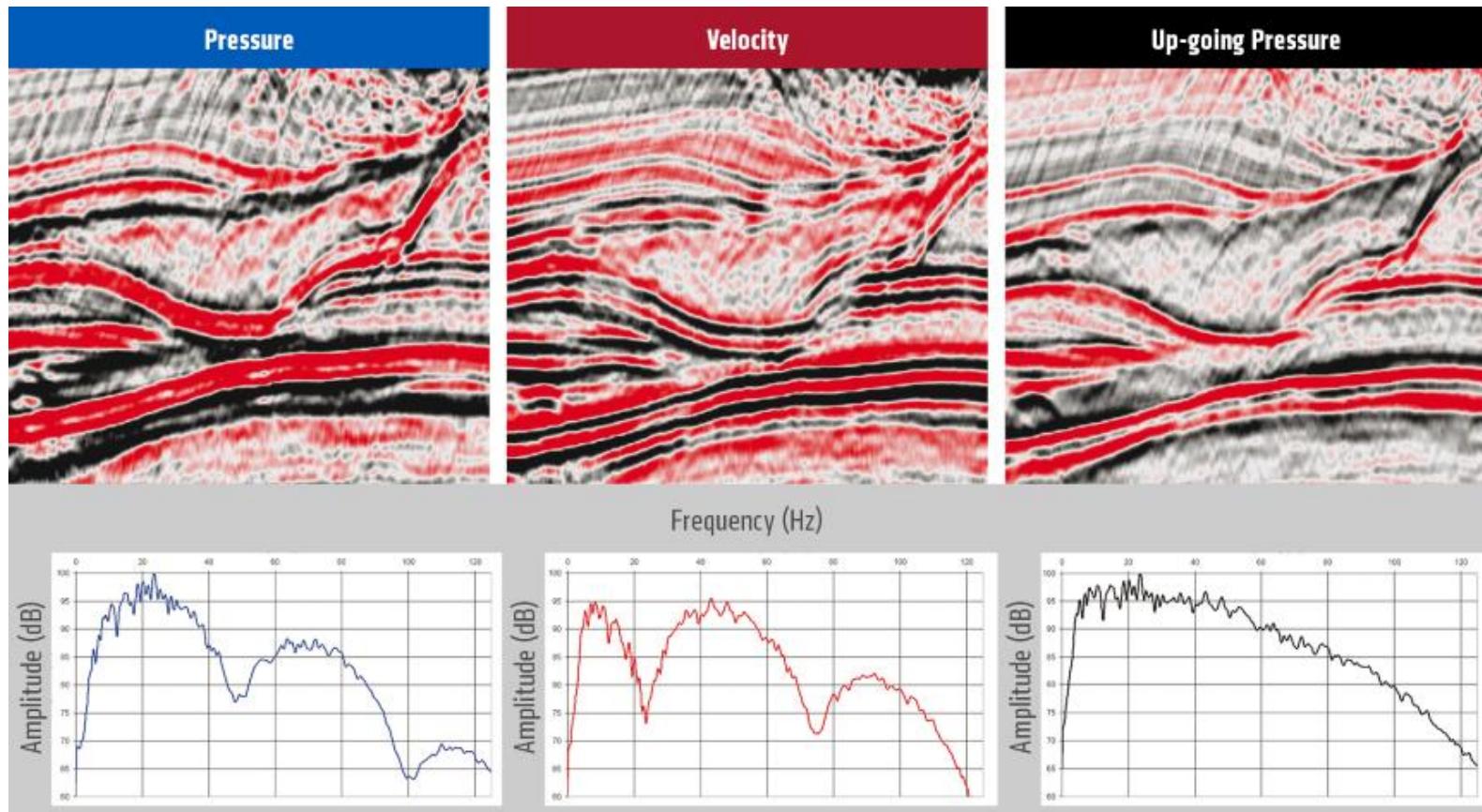
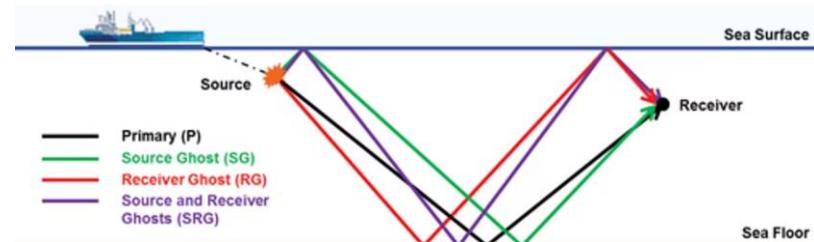
Volume integral; body forces

Surface integral; half boundary condition

The integral over the half-sphere vanish by the Sommerfeld radiation condition

- (Broadband seismic: Both P and Vz (the derivative of P) are recorded)

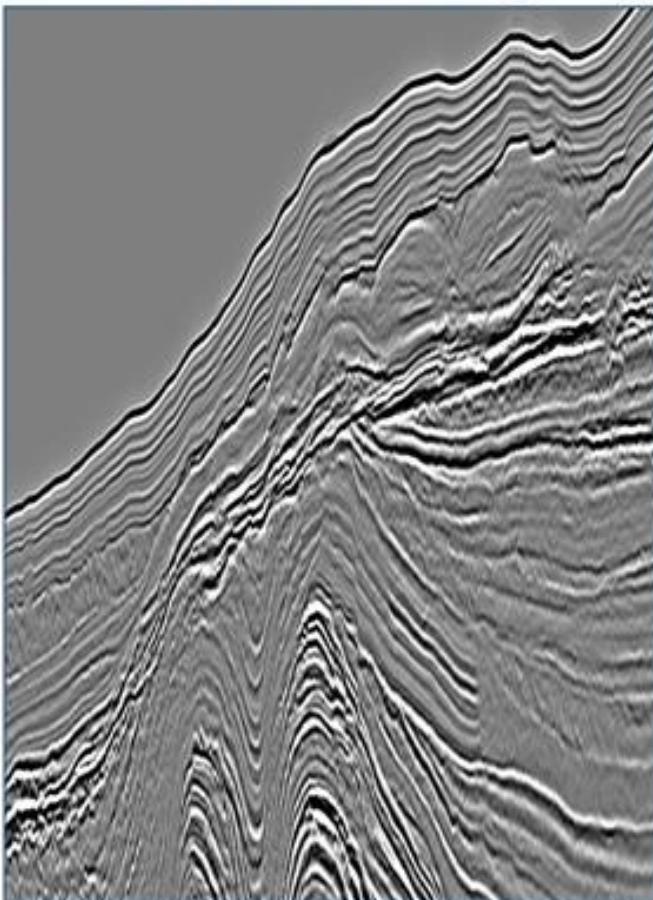
Broadband seismic



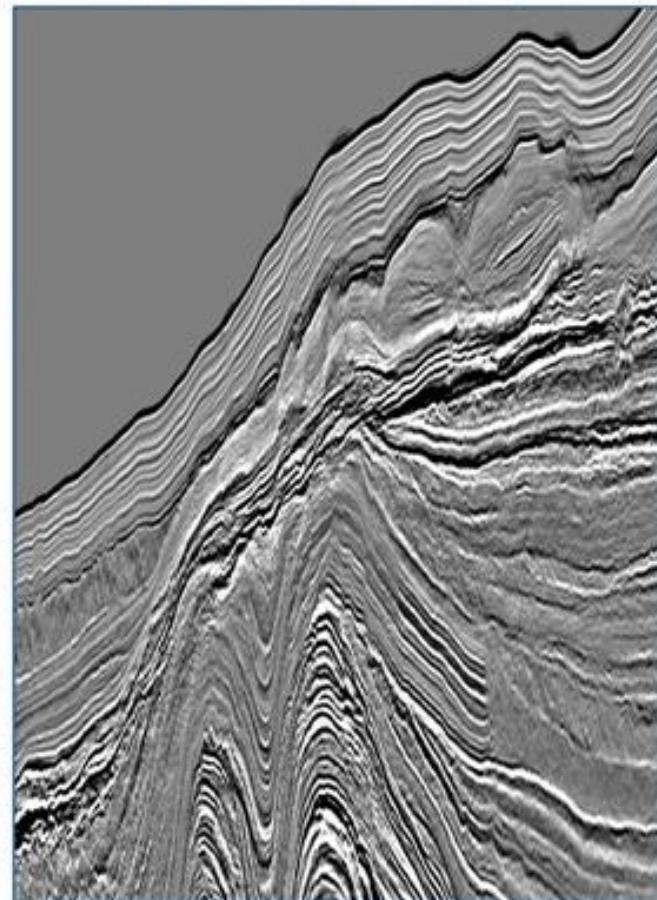
Picture from PGS webs site (pgs.com)

Deghosting

Before TGS' Clari-Fi Processing



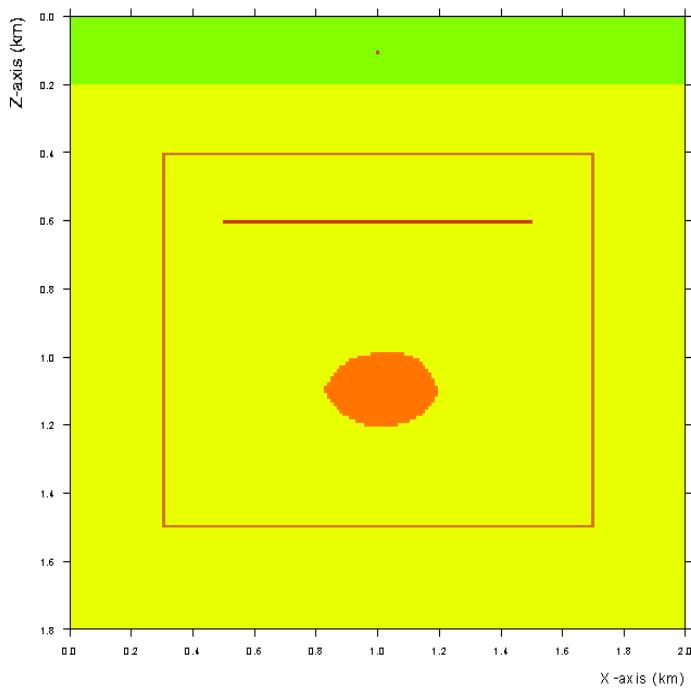
After TGS' Clari-Fi Processing



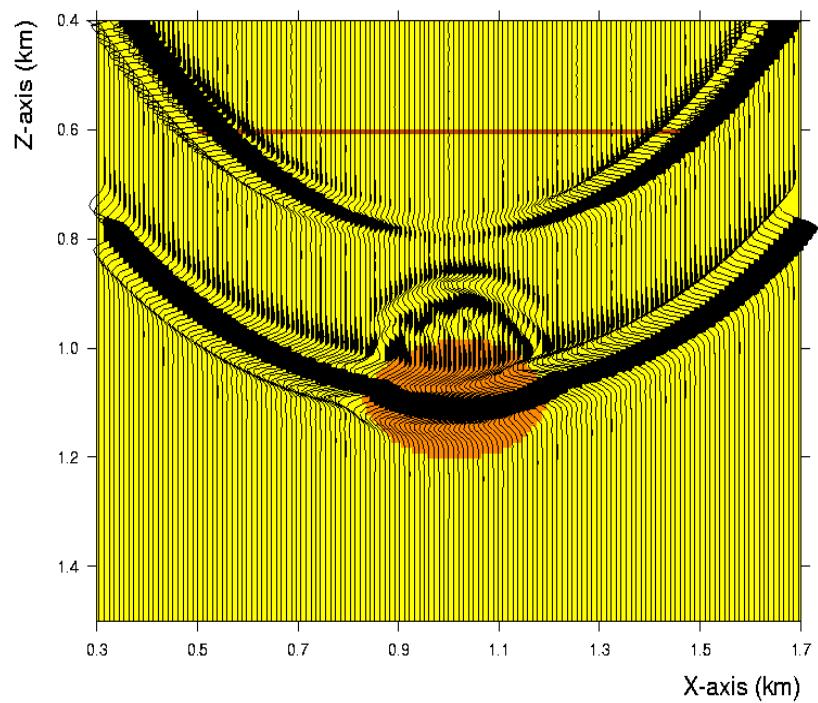
Data is from a joint TGS/Dolphin Geophysical project

Picture from TGS webs site (tgs.com)

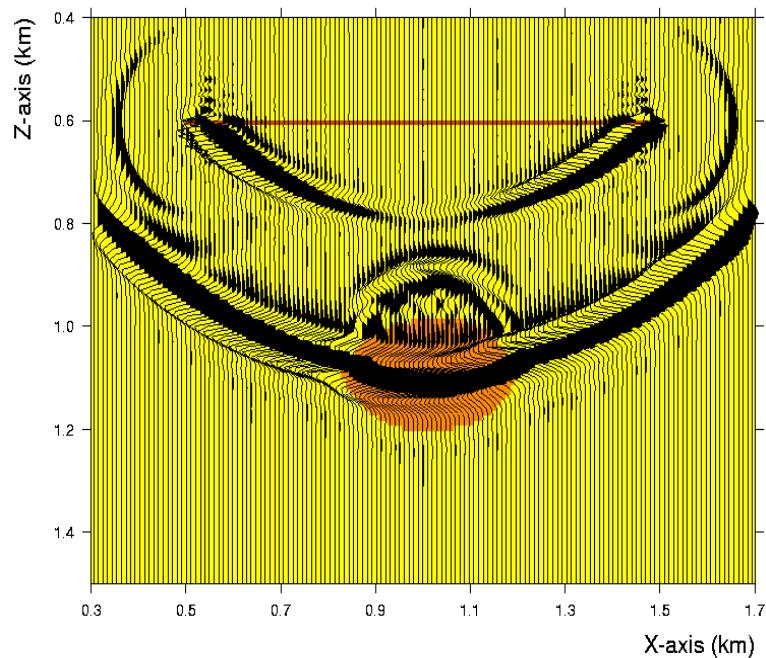
Scatter model:



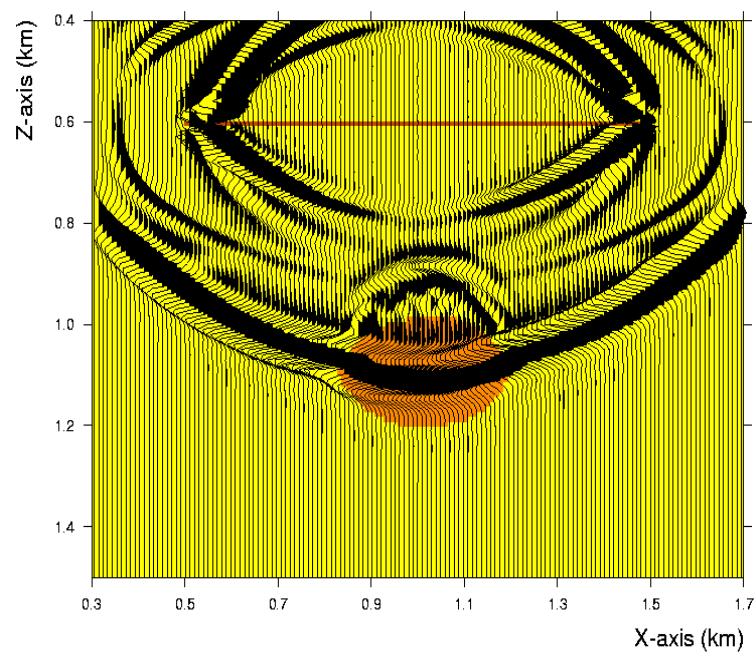
FD Reference (A_z):



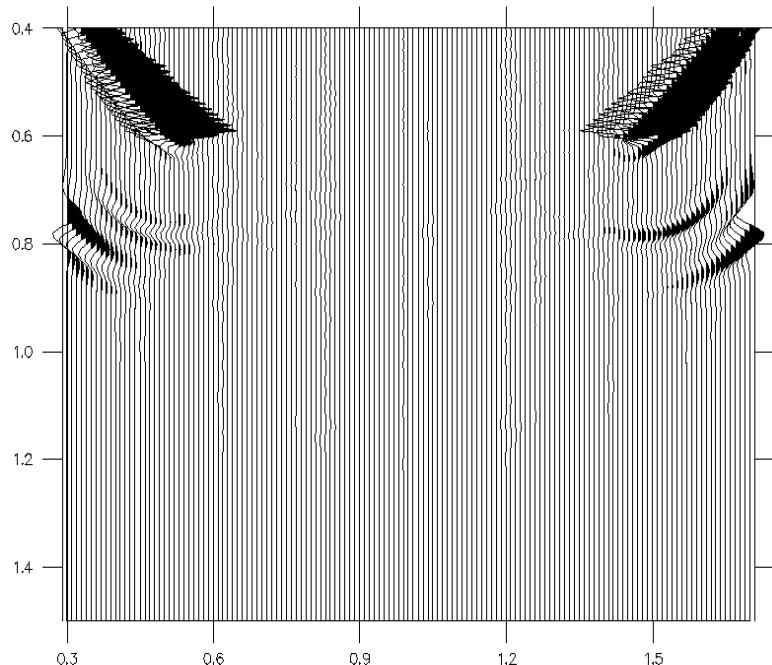
Complete b.c.:



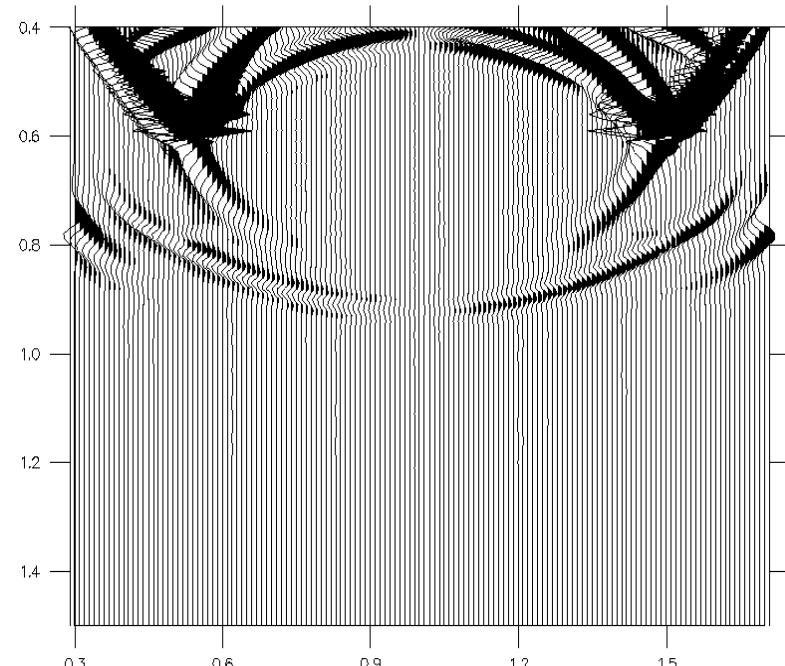
Incomplete b.c.:



Snapshot residuals:



Complete b.c.



Incomplete b.c.

Time reversal (G^*) and back propagation

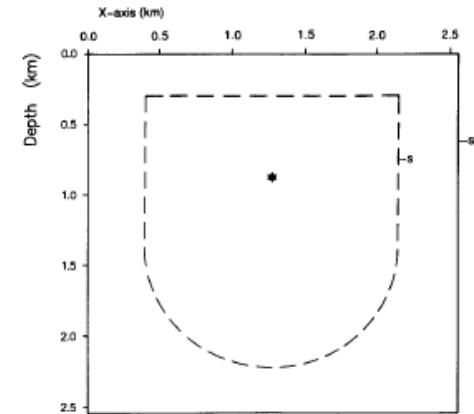
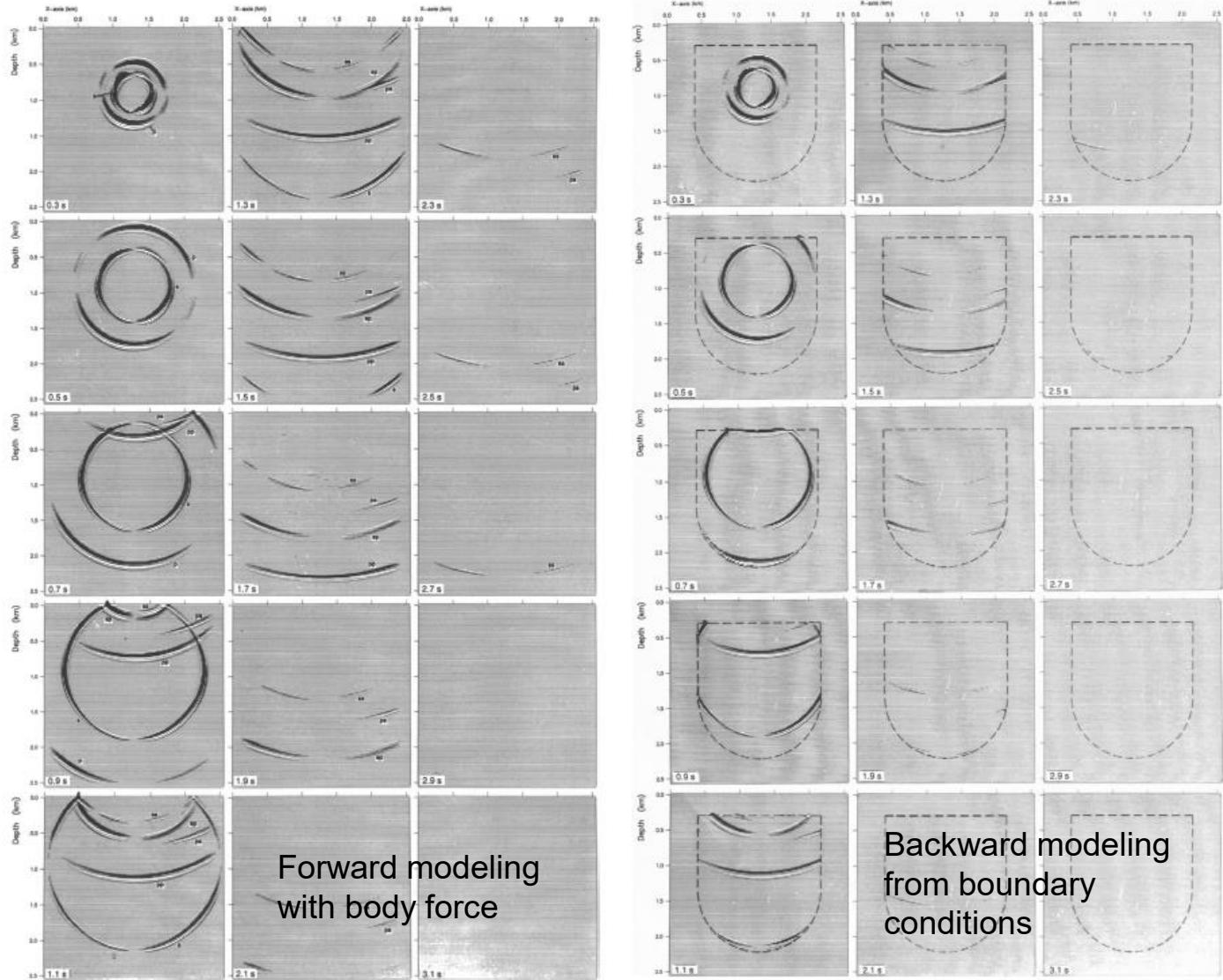


FIG. 1. The model used in the numerical experiment. P -wave velocity is 2000 m/s, S -wave velocity is 1200 m/s, and density is 2.0 g/cm³. The top surface is a free surface, the rest of the model is surrounded by absorbing boundaries. The source position is marked with a star. The fields $a_x^n(i_r, j_r)$, $a_z^n(i_r, j_r)$, $t_x^n(i_r, j_r)$, and $t_z^n(i_r, j_r)$ are recorded on the surface S_0 . The numerical grid is contained within the surface S_0 .



From Mittet (1994)

Migration: U and D

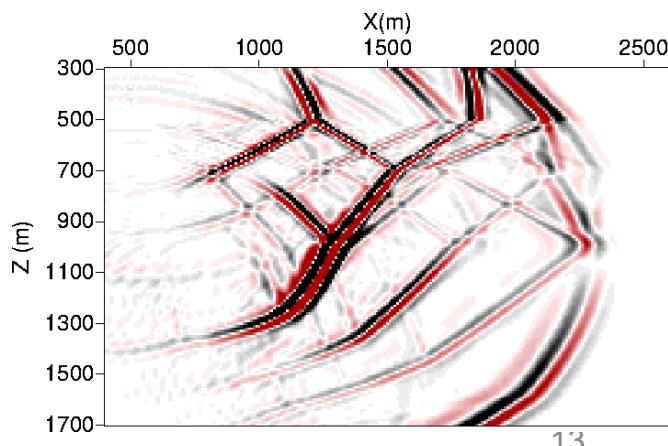
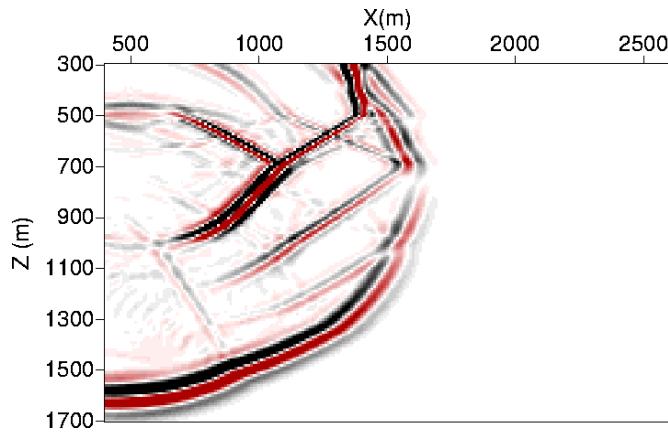
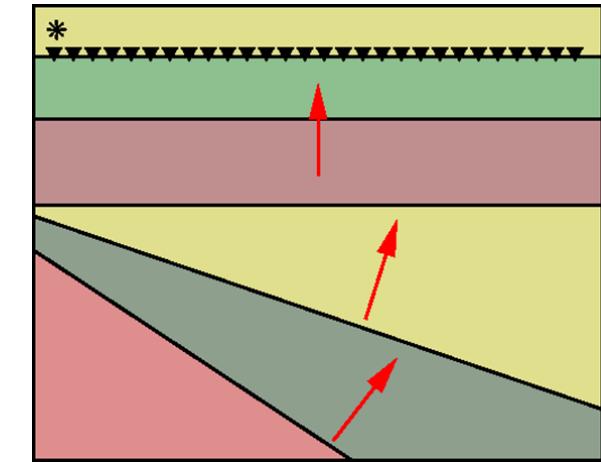
- D = down-going field
- U = up-going field

Ignoring the source spectrum; assume $S(\omega) = 1$:

$$D(x) = G(x, x_s)$$

Time reversal: $G \rightarrow G^*$ in the BC:

$$U(x) = 2 \int \int dx_r dy_r P(x_r) \partial_z^r G^*(x, x_r)$$



Ray Green's function

Ray approximation for G and G^* :

Phase shift due to
caustics; ignore

$$G(x, x_s) = \frac{\mathcal{R}(x, x_s) e^{-i(\pi/2) \operatorname{sgn}(\omega) \sigma_s}}{4\pi \mathcal{L}(x, x_s)} e^{i\omega T(x, x_s)}$$

$$\partial_z^r G^*(x, x_r) = -i\omega p_z(x_r) \frac{\mathcal{R}(x, x_r) e^{+i(\pi/2) \operatorname{sgn}(\omega) \sigma_r}}{4\pi \mathcal{L}(x, x_r)} e^{-i\omega T(x, x_r)}$$

$$\mathcal{R}(x, x_s) \simeq 1$$

Assume smooth medium; neglect
transmission losses

$$\mathcal{R}(x, x_r) \simeq 1$$

$$p_z(x_r) = \partial_z T(x, x_r) = \frac{\cos \theta_r}{v}$$

Slowness; phase angle and phase
velocity in anisotropic media

Ray Green's function

Substitute ray approximation in source and BC terms:

$$\begin{aligned} D(x) &= \frac{1}{4\pi \mathcal{L}(x, x_s)} e^{i\omega T(x, x_s)} \\ U(x) &= \frac{i\omega}{2\pi} \int \int dx_r dy_r \frac{p_z(x_r)}{\mathcal{L}(x, x_r)} e^{-i\omega T(x, x_r)} P(x_r) \end{aligned}$$

Imaging condition (reintroducing ω):

$$I(x) = \int d\omega \frac{U(x, \omega)}{D(x, \omega)}$$

The idea behind the imaging condition:

$$\begin{aligned} U(x, \omega) &\sim R(x)D(x, \omega) \\ I(x) &= \hat{R}(x) \end{aligned}$$

The kinematics are correct

Kirchhoff migration

Substitute U and D in imaging condition:

$$I(x; x_s) = \int d\omega \int \int dx_r dy_r p_z(x_r) \frac{\mathcal{L}(x, x_s)}{\mathcal{L}(x, x_r)} e^{-i\omega[T(x, x_r) + T(x, x_s)]} i\omega P(x_r, \omega)$$

The integral over ω is the inverse Fourier transform:

$$I(x; x_s) = -2 \int \int dx_r dy_r W(x, x_r, x_s) \partial_t P(x_r, \tau)$$

Common-shot
Kirchhoff migration

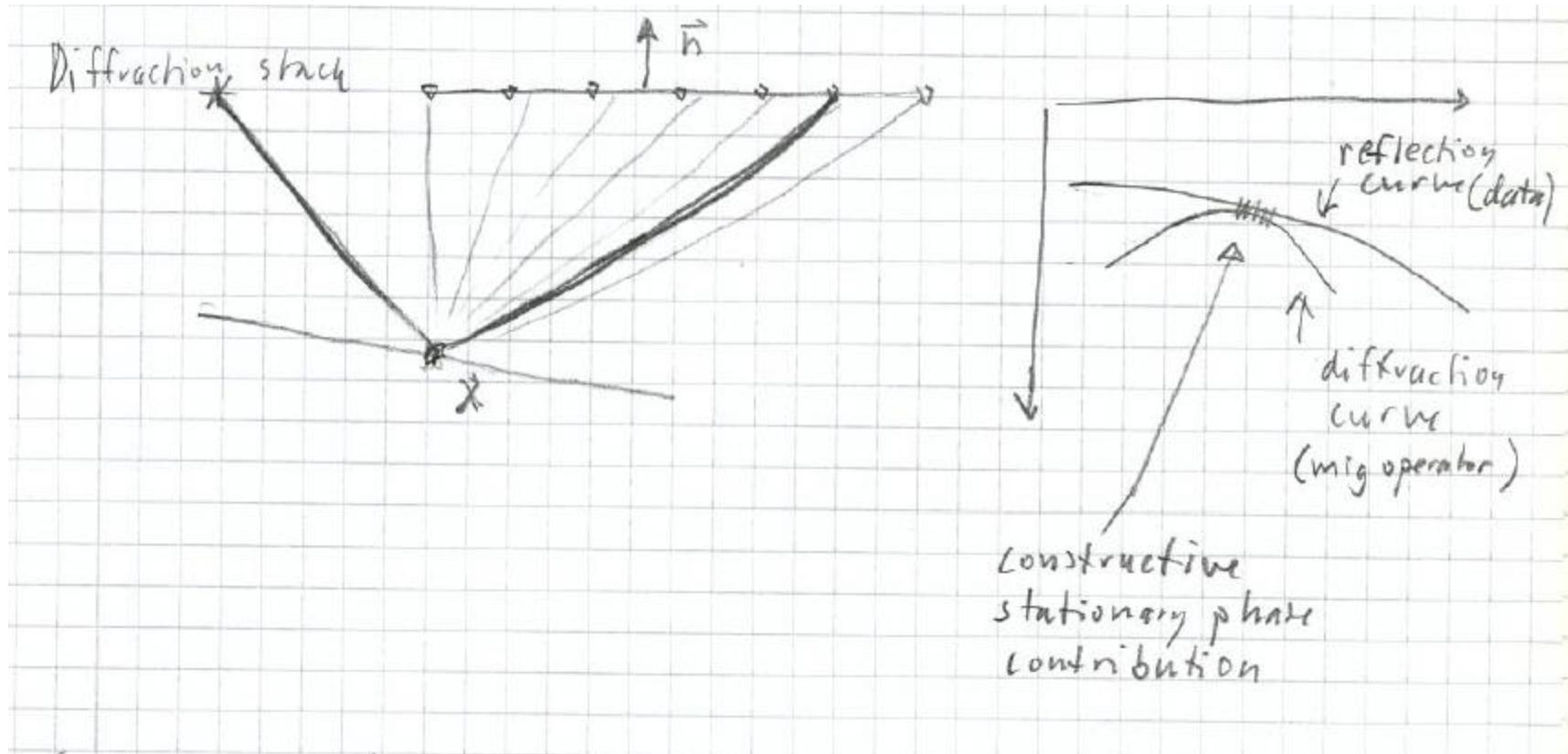
The integral over ω is the inverse Fourier transform:

$$\tau = T(x, x_r) + T(x, x_s) = \text{total traveltime}$$

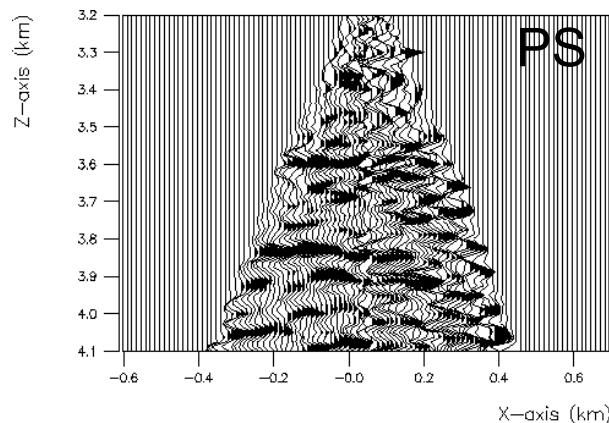
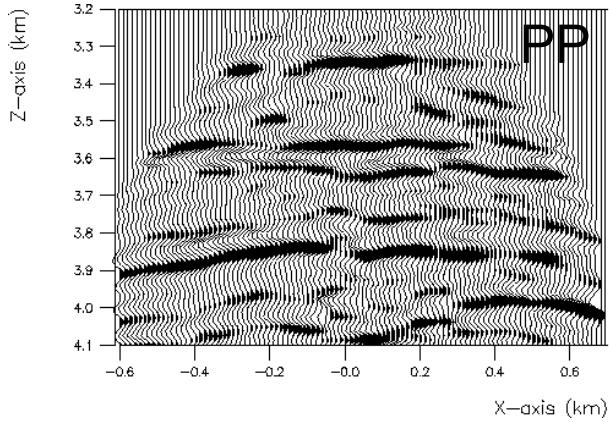
$$W = \frac{\cos \theta}{v} \frac{\mathcal{L}(x, x_s)}{\mathcal{L}(x, x_r)} e^{-i\delta\tau} = \text{weight function}$$

$$\delta\tau = \text{phase shift due to caustics}$$

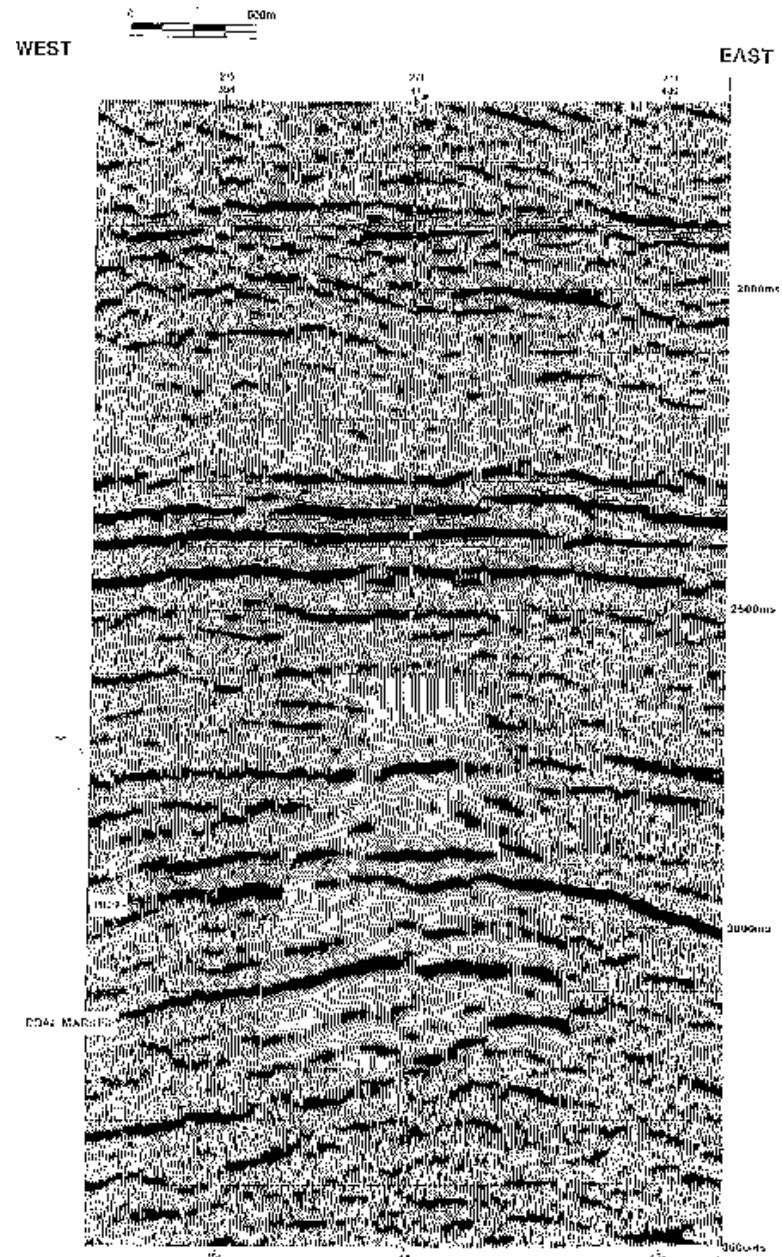
Diffraction stack



Common-receiver elastic Kirchhoff migration Walk-away VSP data

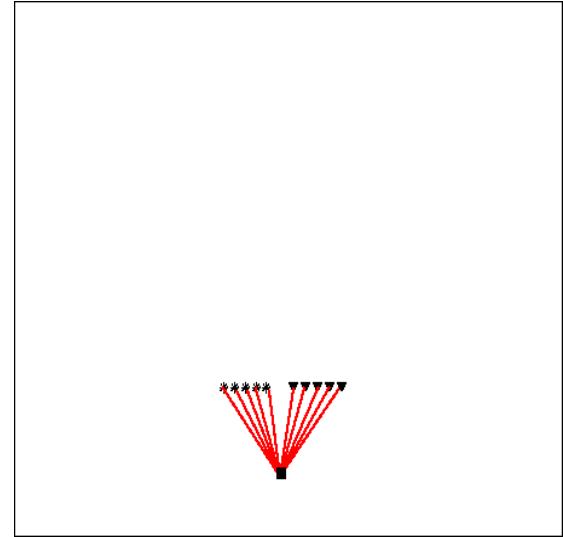
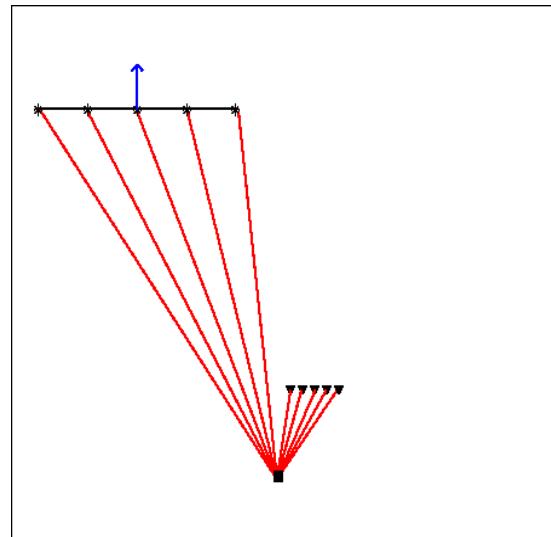
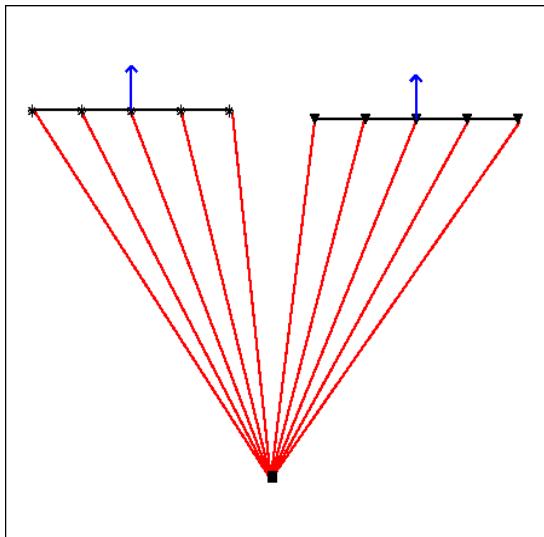


(Example from Hokstad, Geophysics, 2000)



Alternative derivation: Survey sinking + MZO

- Assume multi-shot, multi-receiver experiment
- Downward continuation of sources and receivers
- Migration to zero offset (MZO) imaging condition



Double downward continuation

Downward continue receivers (as before):

$$P(x_a, x_s) = -\frac{1}{2\pi} \int \int dx_r dy_r \frac{i\omega p_z(x_r)}{\mathcal{L}(x_a, x_r)} e^{-i\omega T(x_a, x_r)} P(x_r, x_s) \quad (1)$$

Reciprocity:

$$\begin{aligned} P(x_a, x_s) &= P(x_s, x_a) \\ \gamma_{ij}(x_a, x_s) &= \gamma_{ji}(x_s, x_a) \quad (\text{elastic}) \end{aligned}$$

Downward continue sources:

$$P(x_b, x_a) = -\frac{1}{2\pi} \int \int dx_s dy_s \frac{i\omega p_z(x_s)}{\mathcal{L}(x_b, x_s)} e^{-i\omega T(x_b, x_s)} P(x_s, x_a) \quad (2)$$

Putting things together

Substitute (1) in (2):

$$P(x_b, x_a) = \frac{-1}{4\pi^2} \int \int \int \int dx_s dy_s dx_r dy_r \frac{\omega^2 p_z(x_s) p_z(x_r)}{\mathcal{L}(x_a, x_r) \mathcal{L}(x_b, x_s)} e^{-i\omega[T(x_b, x_s) + T(x_b, x_s)]} P(x_r, x_s)$$

Introducing weight function and total traveltime:

$$\begin{aligned} P(x_b, x_a; \omega) &= \frac{-1}{4\pi^2} \int \int \int \int dx_s dy_s dx_r dy_r \omega^2 W(x_r, x_s, x) e^{-i\omega\tau} P(x_r, x_s; \omega) \\ \tau &= T(x_b, x_s) + T(x_b, x_s) \\ W &= \frac{p_z(x_s) p_z(x_r)}{\mathcal{L}(x_a, x_r) \mathcal{L}(x_b, x_s)} e^{i\delta\tau} \end{aligned}$$

MZO imaging condition

Inverse Fourier transform from frequency to time domain:

$$P(x_b, x_a, t) = \int d\omega e^{-i\omega t} P(x_b, x_a; \omega)$$

$$P(x_b, x_a, t) = \frac{1}{4\pi^2} \int \int \int \int dx_r dy_r dx_s dy_s W(x_r, x_s, x) \partial_t^2 P(x_r, x_s, t + \tau)$$

MZO imaging condition:

$$I(x) = \lim_{t \rightarrow 0, x_a \rightarrow x, x_b \rightarrow x} P(x_a, x_b, t + \tau)$$

$$I(x) = \frac{1}{4\pi^2} \int \int \int \int dx_r dy_r dx_s dy_s W(x_r, x_s, x) \partial_t^2 P(x_r, x_s, \tau)$$

Common-offset migration

Change of variables:

$$\begin{aligned} h &= \frac{1}{2}(x_g - x_s) = \text{half-offset} \\ x_m &= \frac{1}{2}(x_g + x_s) = \text{midpoint} \end{aligned}$$

Common-offset (partial) image:

$$\begin{aligned} I(x, h) &= \frac{J}{4\pi^2} \int \int dx_m dy_m W(x_m, h, x) \partial_t^2 P(x_m, h, \tau) \\ J &= \text{Jacobian (trivial)} \end{aligned}$$

Full stack (sum over offsets):

$$I(x) = \int \int dh_x dh_y I(x, h)$$

Time and depth migration

Time migration: Neglecting Snell's law

$$T(x, x_r) = \frac{\sqrt{(x - x_r)^2 + z^2}}{V} = \frac{r}{V}$$
$$1/\mathcal{L}(x, x_r) \propto \frac{1}{\sqrt{(x - x_r)^2 + z^2}} = \frac{1}{r}$$

Depth migration: Account for Snell's law

$T(x, x_r)$ from kinematic ray tracing

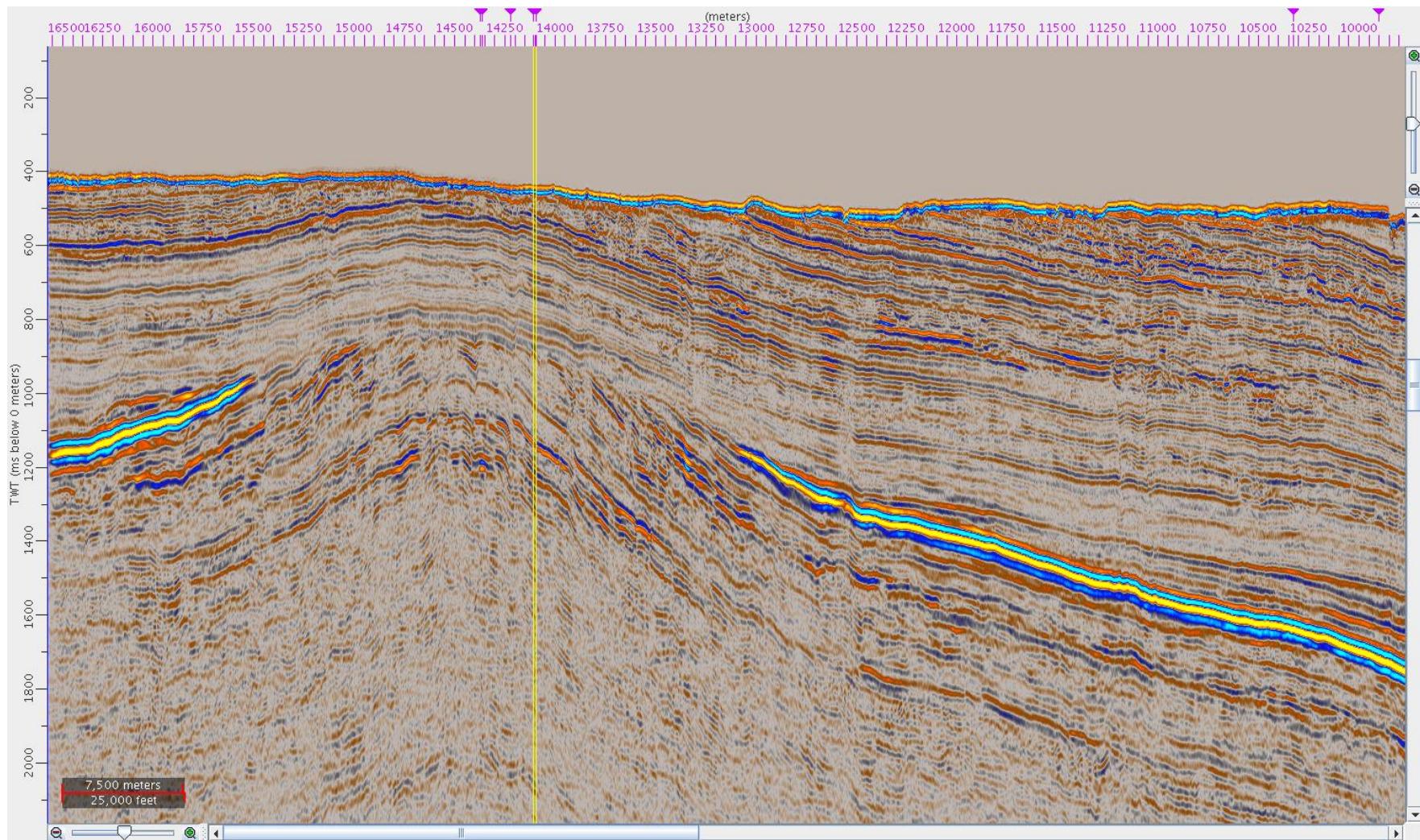
$\mathcal{L}(x, x_r)$ from dynamic ray tracing

More about ray theory later

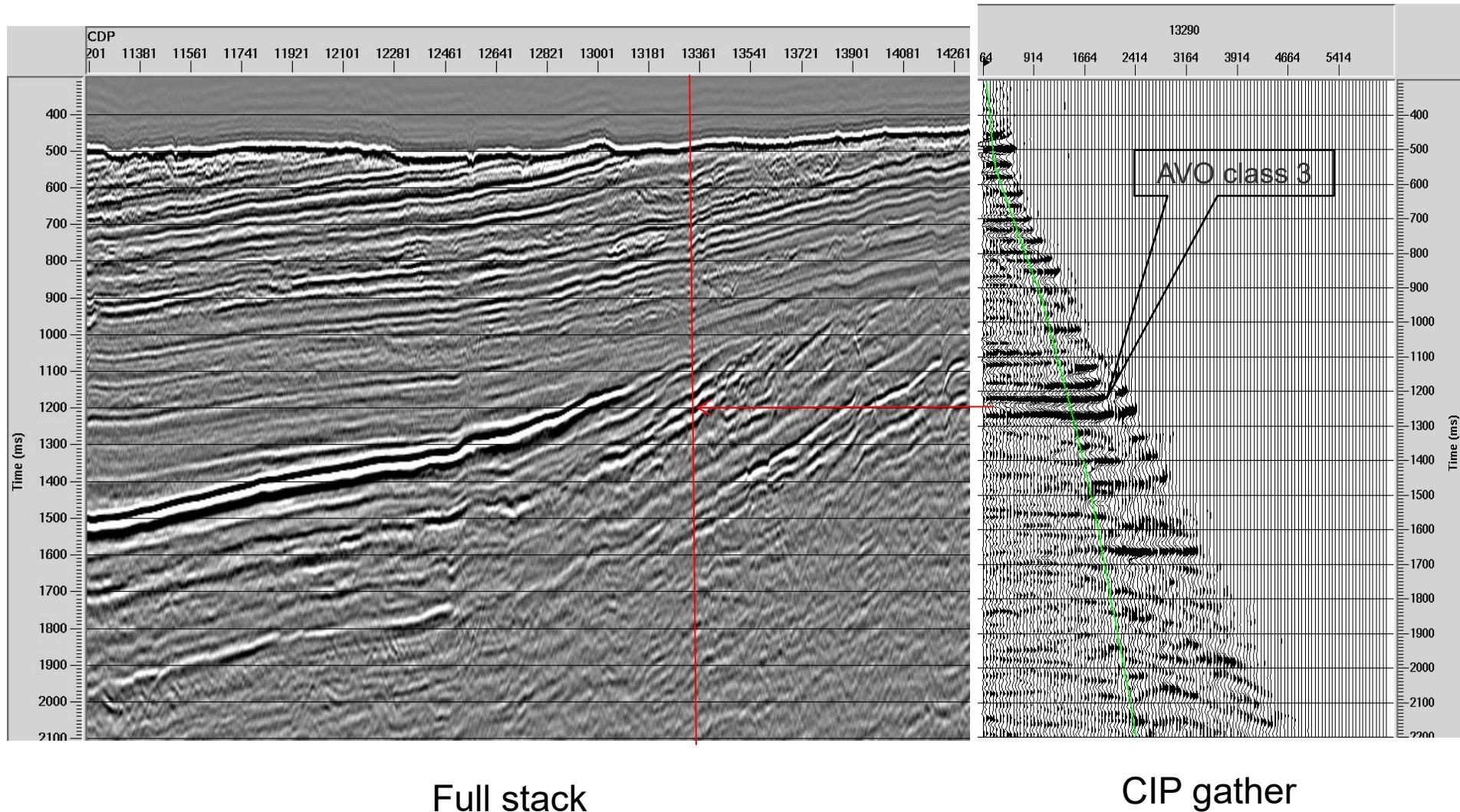
Kirchhoff migration summary

- Advantages:
 - Fast
 - Flexible wrt. Acquisition geometry
 - True amplitude (for AVO analysis)
- Disadvantages
 - Based on ray theory; inaccurate in complex geology
- Common shot, common receiver migration: OBS and VSP
- Common offset migration: for surface seismic data
- Later we will look at two more advanced members of the Kirchhoff family of migrations:
 - Gaussian beam migration
 - Angle migration (GRT)

Kirchhoff migration works fine in simple geology



2D Kirchhoff migration – full stack and CIP gather

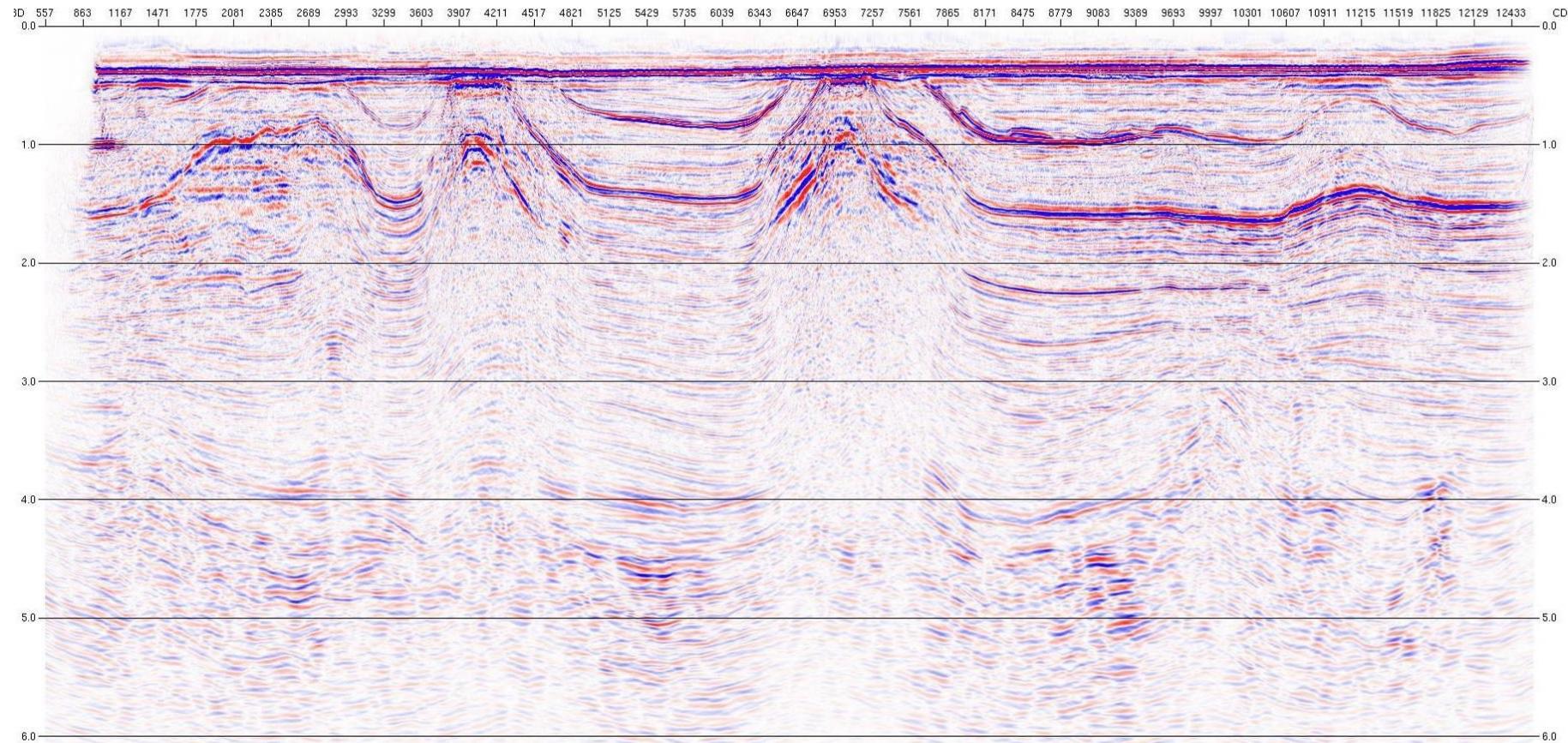


Full stack

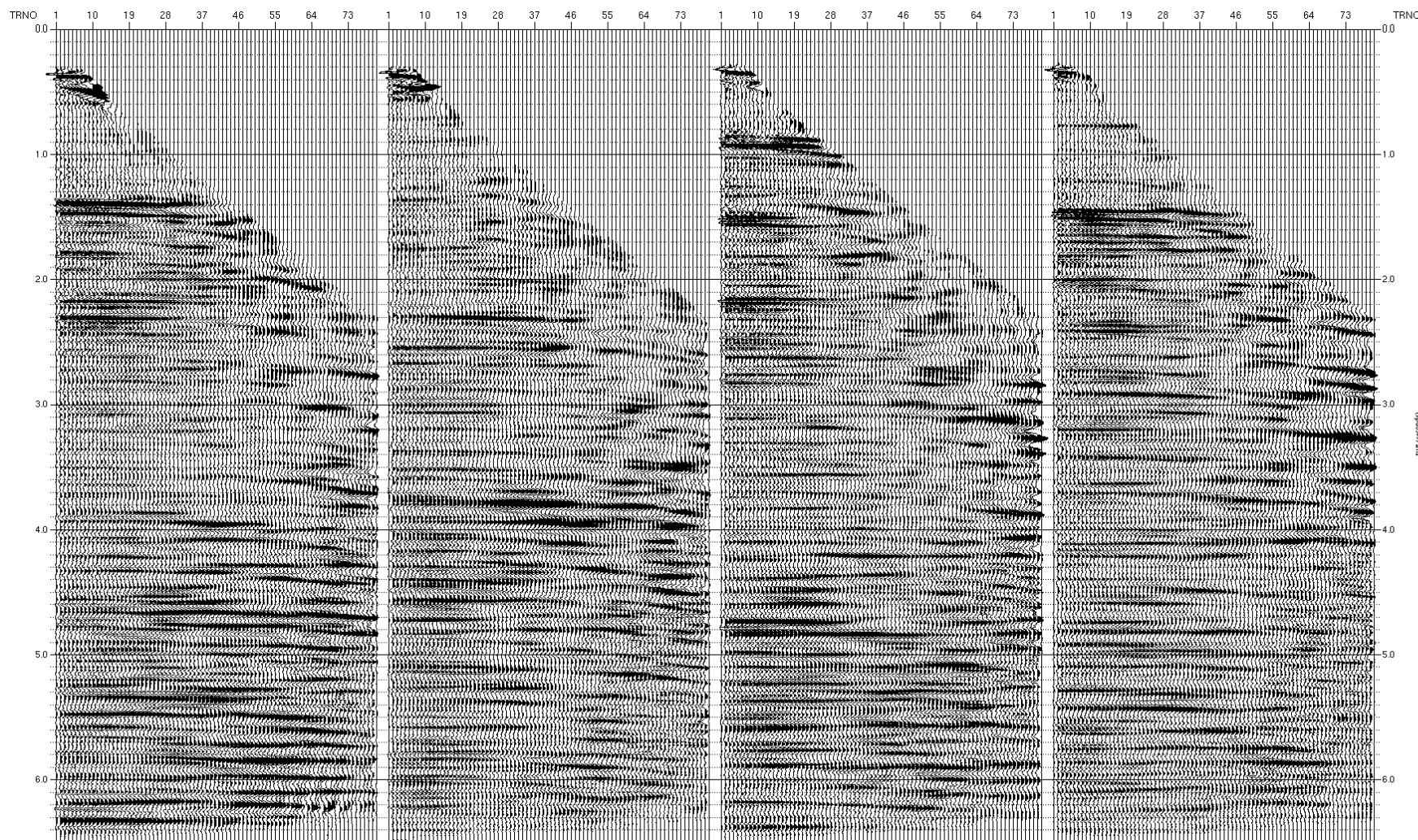
CIP gather

Stratigraphic trap with AVO Class 3 anomaly.

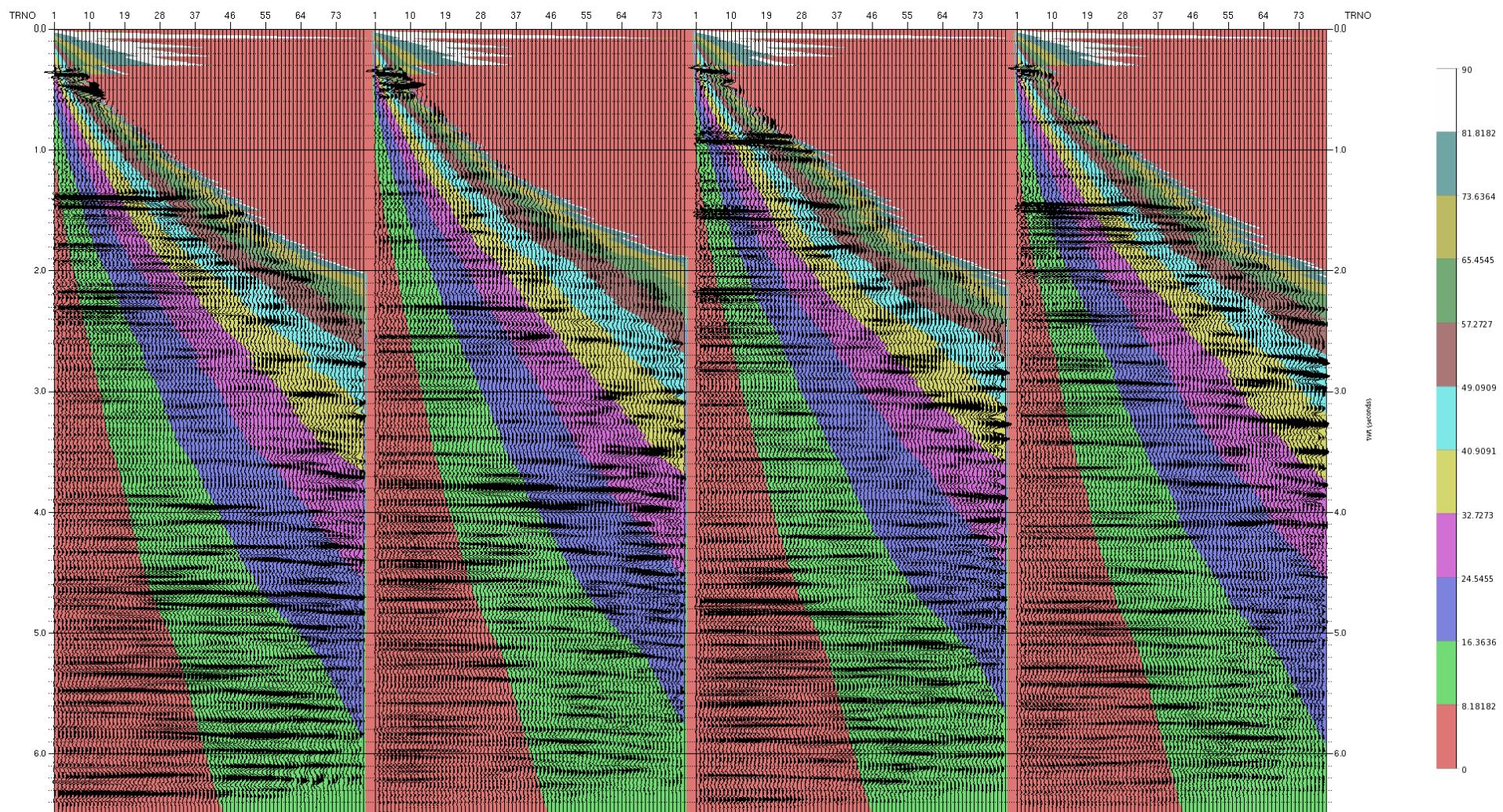
On-board processing: ST10011 Inline 1219 3D Kirchhoff migration



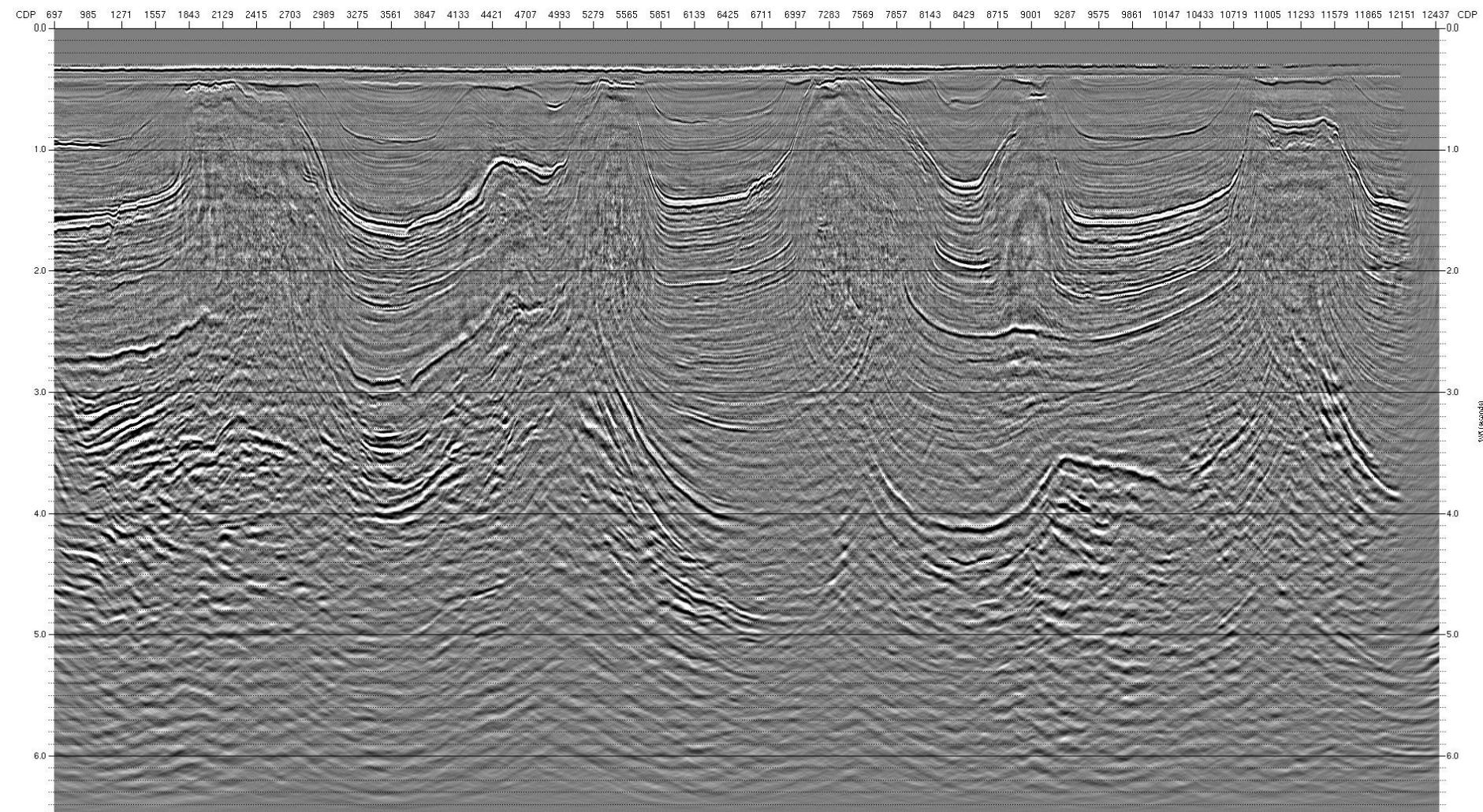
IL 1200 : CDP gathers



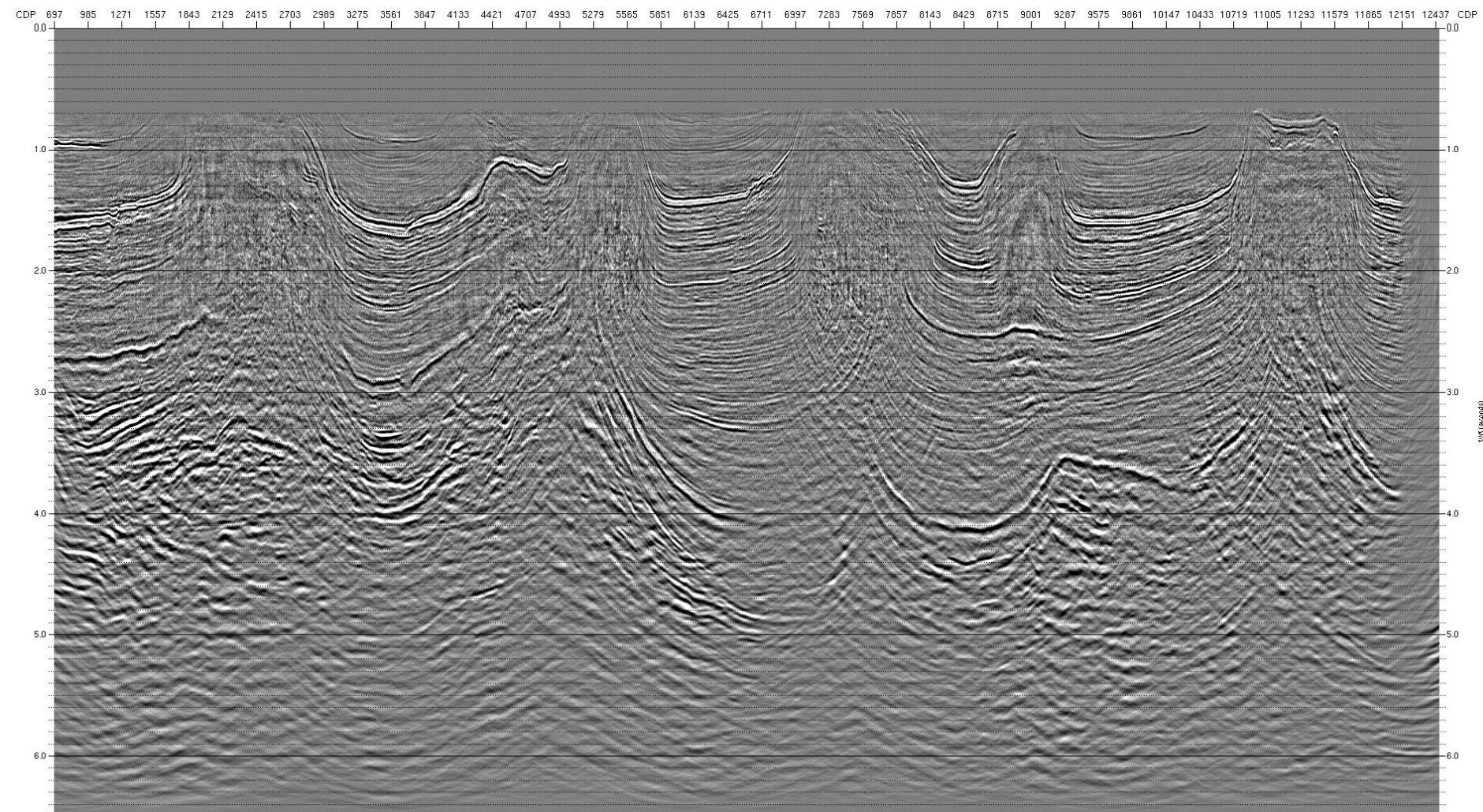
IL 1200 : CDP gathers with incidence angles overlaid



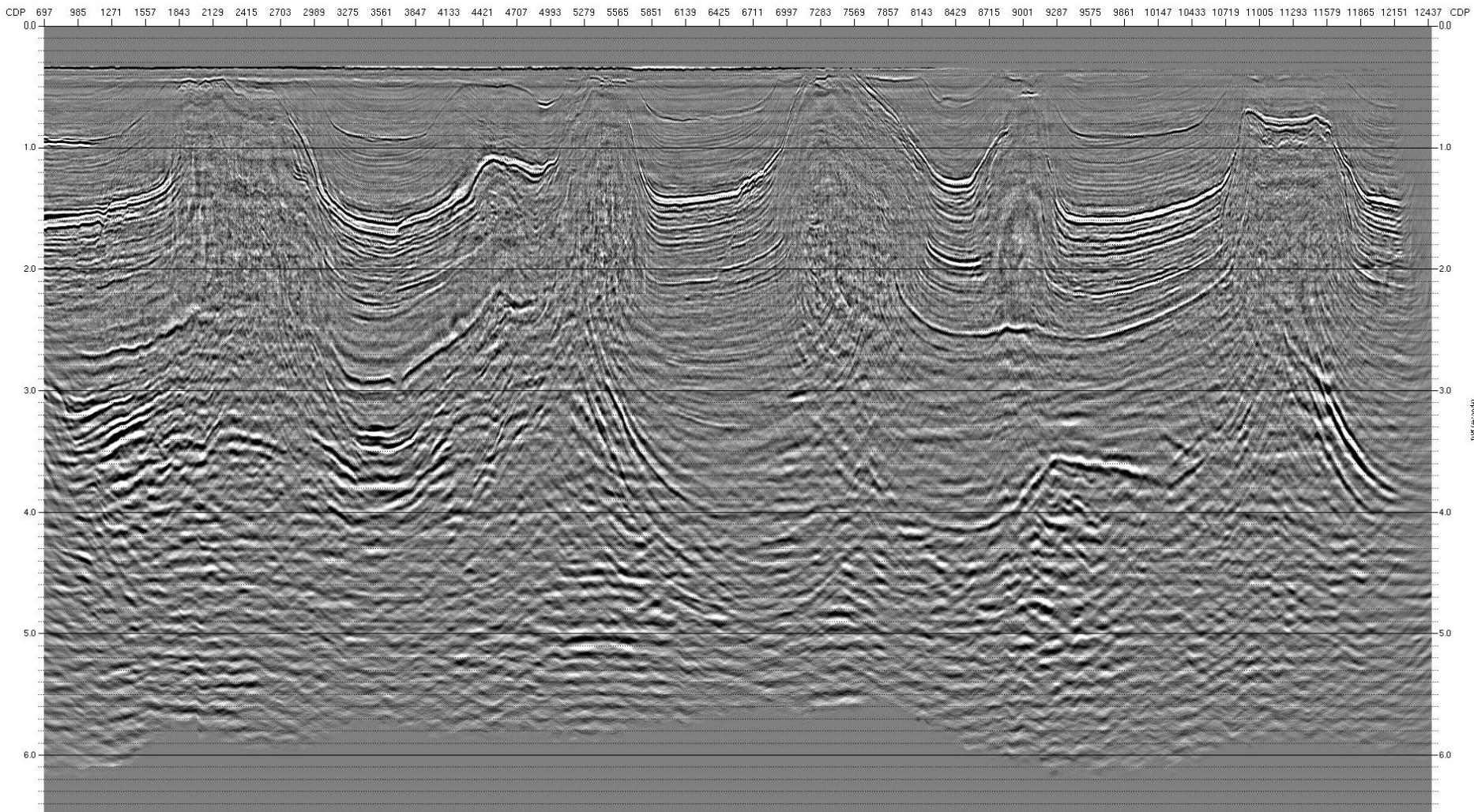
3D Kirchhoff time migration: IL 1400 : full stack after radon production using a time-offset dependant far offset mute



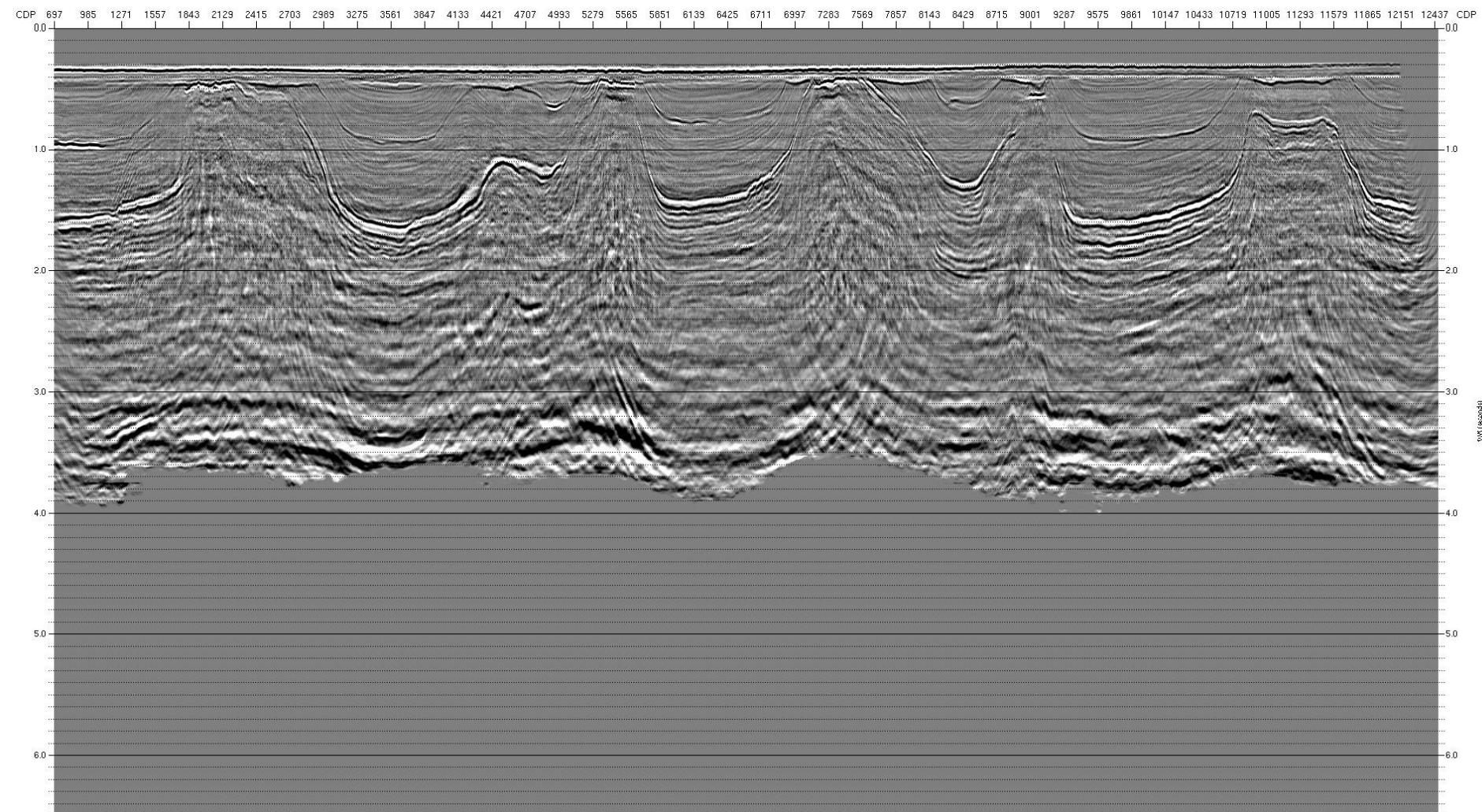
IL 1400 : Near angle stack



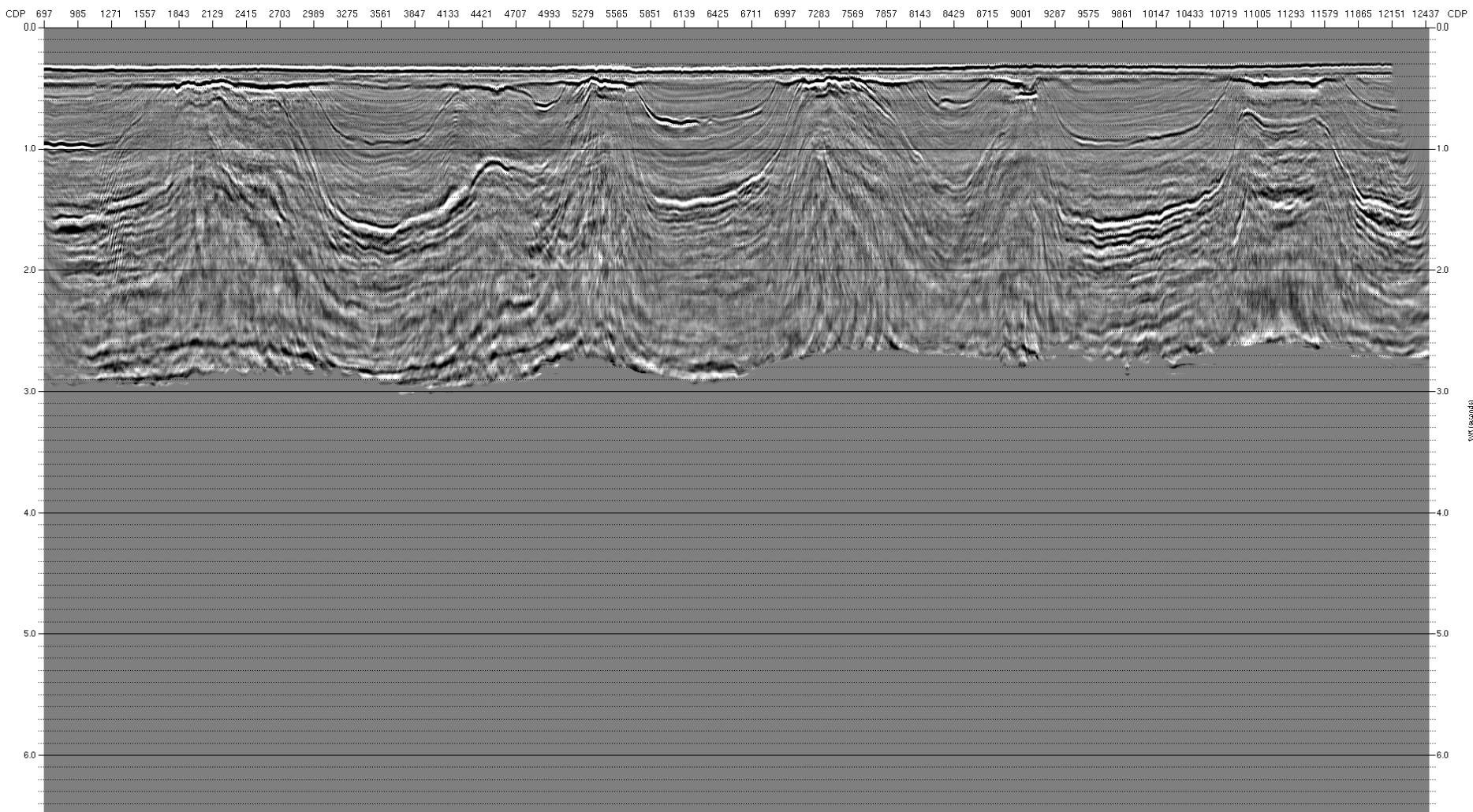
IL 1400 : Near-mid angle stack



IL 1400 : Mid angle stack



IL 1400 : Mid-far angle stack



IL 1400 : Far angle stack

